Information Aggregation and Design in Asset Markets with Adverse Selection

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Abstract

How effectively does a decentralized marketplace aggregate information that is dispersed throughout the economy? We study this question in a dynamic setting where asset sellers have private information that is correlated with an unobservable aggregate state. A common feature of all equilibria is that each seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. We ask whether the state is revealed as the number of informed traders grows large. Perhaps surprisingly, the answer is no; we provide generic conditions under which information aggregation necessarily fails. In another region of the parameter space, aggregating and non-aggregating equilibria can coexist. We solve for the optimal information policy of a social planner who observes trading behavior and chooses what information to reveal. The information generated in a laissez-faire economy is always inefficient when aggregation fails; the optimal policy conceals some information from the agents in order to accelerate trade.

JEL: G14, G18, D47, D53, D82, D83.

Keywords: Information Aggregation; Information Design; Decentralized Markets; Adverse Selection; Optimal Information Policy; Transparency.

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1 Introduction

Since the seminal work of Hayek (1945), the question of whether markets effectively aggregate dispersed information has been a central one in economics. Formal investigations of this question are plentiful (see Section 1.1 for a discussion). Yet, they are typically conducted in a setting with a single (perhaps divisible) asset about which traders have dispersed information. Whether information is aggregated then usually boils down to whether the equilibrium price reveals the value of the asset conditional on the union of traders' information. This broad class of models is natural for many applications from static common-value auctions to dynamic trading in financial markets. For other applications (e.g., real estate, OTC markets), information dispersion arises due to dispersion in ownership, and one is interested in the extent to which aggregate trading behavior across heterogeneous assets reveals information about the underlying state of the economy. In this paper, we explore such a setting.

Building on the framework of Asriyan, Fuchs, and Green (2017), we investigate the question of information aggregation in a dynamic setting with many assets, whose values are independently and identically drawn from a distribution that depends on an underlying aggregate state. The value of each asset is privately observed by its seller, who receives offers each period from competitive buyers. We ask whether the history of all transactions reveals the aggregate state as the number of informed sellers in the economy (denoted by N) grows large.

To answer this question, we begin by characterizing the set of equilibria for arbitrary N. Due to a complementarity between the amount of information collectively revealed by others and the optimal strategy of an individual seller, multiple equilibria can exist. A feature common to all equilibria is that each individual seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. Therefore, one might intuitively expect that, by the law of large numbers, the state would be revealed as the number of sellers tends to infinity.

Our first main result shows that the intuition is incorrect. We provide necessary and sufficient conditions under which there does not exist any sequence of equilibria that reveal the state as $N \to \infty$. The reason why aggregation fails is that the information content of each individual seller's behavior tends to zero at a rate of $1/N$, just fast enough to offset the additional number of observations. As a result, some information is revealed by the limiting trading behavior, but not enough to precisely determine the underlying state. Roughly speaking, the conditions for non-aggregation require that the correlation of asset values is sufficiently high and that agents are sufficiently patient. Intuitively, these conditions guarantee that if the aggregate state were to be revealed with certainty tomorrow, then the option value of delaying trade today is relatively high. An immediate corollary is that information aggregation always obtains in a static model.

That is, dynamic considerations are a necessary ingredient for non-aggregation.

When the non-aggregation conditions are not satisfied, there exists a sequence of equilibria such that information about the state is aggregated as $N \to \infty$. However, even in this case, information aggregation is not guaranteed. Our second main result shows that there exists a region of the parameter space in which there is coexistence of equilibria that reveal the state with equilibria that do not. The key difference across the two types of equilibria is the rate at which trade declines as the number of informed sellers grows. In the non-aggregating equilibria, trade declines at rate $1/N$ whereas in aggregating equilibria, the rate of trade declines slower than $1/N$. We are not aware of analogous coexistence results in the literature.

Whether information aggregates has important implications for welfare, prices, and trading behavior. To understand them, it is useful to draw comparison to a fictitious economy in which the state is exogenously revealed after the first trading period. When information aggregates, both trading volume and welfare converge to their levels in the fictitious economy and the volatity of prices conditional on the true state goes to zero. In contrast, along a sequence of non-aggregating equilibria, trading volume and welfare are strictly lower than in the fictitious economy and the conditional price volatility remains strictly positive even as $N \to \infty$.

Two immediate implications follow. First, from a social welfare perspective, aggregating equilibria are always preferable to non-aggregating equilibria when they co-exist. Thus, among laissez-faire outcomes, aggregation is optimal. Second, if all equilibria are non-aggregating, then a planner could improve overall welfare by learning and revealing the true state. Of course, it is not obvious how the planner would learn the true state. It is more natural to think that the planner is uninformed, but can learn about the true state by observing the trading behavior of market participants. The problem facing the planner is then how best to reveal this information to other agents in the economy.

We address this question in Section 4. Doing so involves formulating and solving an information design problem that is related to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). One key difference is that the planner's problem in our model must take into account the fact that her policy influences the information content of trading behavior and therefore the information content of whatever is revealed. In other words, the statistical properties of the information that is revealed by the planner endogenously depends on the planner's revelation policy. Our solution technique involves two steps. First, we solve the information design problem for an "informed" planner who (exogenously) learns the aggregate state at $t = 1$. The solution to this problem helps us derive an upper bound on the surplus that can be attained by an uninformed planner. We then construct a policy under which the payoffs converge to the upper bound as $N \to \infty$.

To describe the optimal information policy, it is useful to draw comparison with the *laissez-*

faire outcomes, i.e., the set of equilibrium outcomes in which all agents observe the full history of trading behavior. We characterize precisely when the optimal information policy coincides with one of the *laissez-faire* outcomes. Our characterization amounts to a threshold discount rate that depends on model parameters, below which the optimal information policy coincides with one of the laisses-faire outcomes. Above the threshold, laissez-faire equilibria generate information inefficiently; the optimal information policy involves concealing some information in order to promote a greater volume of trade in the first period. Moreover, the optimal information policy Pareto dominates laissez-faire, i.e. all market participants would agree to delegate information dissemination about past trades to the social planner. These findings have implications for policies aimed at promoting market transparency.

Recently, there has been a strong regulatory push towards making financial markets more transparent (i.e., disclosing more information about trading activity to market participants). For example, one of the stated goals of the Dodd-Frank Act of 2010 is to increase transparency and information dissemination in the financial system. The European Commission is considering revisions to the Markets in Financial Instruments Directive (MiFID), in part to improve the transparency of European financial markets. Our results highlight a potential trade-off for such policies and provide a justification for limiting the amount of information available to market participants.

The introduction of benchmarks that reveal some aggregate trading information has also received recent attention by policy makers and academics. Duffie et al. (2017) analyze the role of benchmarks (e.g., LIBOR) in revealing information about fundamentals and suggest that the introduction of benchmarks is welfare enhancing. Our analysis highlights an important consideration that is absent in their setting. Namely, that the informational content of the benchmark may change once it is published due to endogenous responses by market participants.

1.1 Related Literature

In addition to the welfare implications studied in this paper, there are also variety of other reasons for why information aggregation may be a desirable property. For instance, such information may be useful for informing firms' investment decisions (Fishman and Hagerty, 1992; Leland, 1992; Dow and Gorton, 1997; Camargo et al., 2015), government interventions (Bond et al., 2009; Bond and Goldstein, 2015; Boleslavsky et al., 2017), and monetary policy (Bernanke and Woodford, 1997). Markets that convey more information can also be more useful for providing better incentives to managers (Baumol, 1965; Fishman and Hagerty, 1989) and mitigating the winner's curse in common-value auctions (Milgrom and Weber, 1982). As documented by a number of papers in this literature, the feedback loop between real decisions and price informativeness may undermine the ability of markets to aggregate information and lead to aggregation failures.[1] To highlight how our mechanism differs from this literature, we abstract from any such considerations here and leave the value of aggregation unspecified.

Within static environments, there is a large literature that studies questions regarding information aggregation. Seminal works on this topic include Grossman (1976), Wilson (1977), Milgrom (1979), Hellwig (1980), and Kyle (1989). More recent progress on this question has been made by Pesendorfer and Swinkels (1997), Kremer (2002), Rostek and Weretka (2012), Lauermann and Wolinsky (2013), Bodoh-Creed (2013), Albagli et al. (2015), Axelson and Makarov (2017), and Siga and Mihm (2018), among others.[2] By and large, this literature is largely defined by a centralized trading environment in which there is a single asset about which agents have dispersed information. The question of information aggregation is whether the price is a sufficient statistic for the union of this dispersed information. In contrast, we explore a decentralized trading environment with heterogeneous assets and ask whether the history of trading behavior is sufficient to infer the underlying state. Moreover, our results pertaining to non-aggregation crucially rely on dynamic considerations—with a single opportunity to trade, information is *always* aggregated.

Kyle (1985) studies a dynamic insider trading model and shows that the insider fully reveals his information as time approaches the end of the trading interval. Foster and Viswanathan (1996) and Back et al. (2000) extend this finding to a model with multiple strategic insiders with different information. Ostrovsky (2012) further generalizes these findings to a broader class of securities and information structures. He considers a dynamic trading model with finitely many partially informed traders and provides necessary and sufficient conditions on security payoffs for information aggregation to obtain. Our paper differs from these works in that we study a setting with heterogeneous but correlated assets owned by privately informed sellers. We ask whether information aggregates as the number of sellers becomes arbitrarily large. Despite the fact that we look at the limit as $N \to \infty$, the strategic considerations do not vanish in our model since there is an idiosyncratic component to the value of each asset.

Golosov et al. (2014) consider an environment in which a fraction of agents has private information about an asset while the other fraction are uninformed. Agents trade in a decentralized anonymous market through bilateral matches, i.e., signaling with trading histories is not possible. They find that information aggregation obtains in the long run. In contrast, in our setting observing trading histories plays a crucial role: signaling through delay diminishes the amount of trade, thus reducing the information content of the market, leading to the possibility that

[1]See Bond et al. (2012) for a survey of both the theoretical and empirical literature on the real effects of information conveyed through markets.

[2]Palfrey (1985) and Vives (1988) explore this question within a Cournot setting.

information aggregation fails.

Lauermann and Wolinsky (2016) study information aggregation in a search market, in which an informed buyer sequentially solicits offers from sellers who have noisy information about the buyer's value. They provide conditions under which information aggregation fails, and they trace this failure to a strong form of winner's curse that arises in a search environment. Although our model is quite different, we share the common feature that the fear of adverse selection hinders trade and thus reduces information generation in markets.

Also, within a search framework, Lester et al. (2018) look at how equilibrium trade, margins and information changes as the probability of meeting a dealer is increased. They show that as the meeting frequency increases the information flow to the market might decrease. Roughly speaking, this corresponds to our finding that information aggregation is more likely to fail as we increase the discount factor.[3]

Babus and Kondor (2016) explore how the network structure affects information diffusion in a static OTC model with a single divisible asset. They show that strategic considerations do not influence the degree of information diffusion. However, the network structure combined with a private value component leads to an informational externality that constrains the informativeness of prices and hence the informational efficiency of the economy.

Finally, our paper is related to a growing literature that studies dynamic markets with adverse selection (e.g., Janssen and Roy (2002), Hörner and Vieille (2009), Fuchs and Skrzypacz (2012), Fuchs et al. (2016), Daley and Green (2012, 2016)). Our innovation is the introduction of asset correlation, which allows us to study the information aggregation properties of these markets. This paper builds upon our previous work, Asriyan et al. (2017) (henceforth AFG), which demonstrates that multiple equilibria can exist in a model with two informed sellers. In this paper, we focus on the information aggregation properties of equilibria. In order to do so, we extend the two-seller model of AFG to a model with an arbitrary number of sellers. Characterizing the set of equilibria in the two-period model with an arbitrary number of sellers (Sections 2.4-2.5) follows closely from AFG. The main contribution of this paper is twofold. First, we explore the information aggregation properties of equilibria as the market grows large, and their implications for trade and welfare (Section 3). Second, we study the normative implications by considering the information design problem of a social planner who observes trading behavior and chooses what information to communicate to the traders (Section 4). Here, we also contribute to the recently growing literature on information design (Bergemann and Morris, 2013, 2016). In addition to the information policy affecting the ex-ante incentives as in Boleslavsky and Kim (2018), we show how the endogeneity of the planner's information set puts constraints on optimal policy.

[3]A higher discount factor translates into a lower expected time until the next opportunity to trade.

2 The Model

There are $N+1$ sellers indexed by $i \in \{1, ..., N+1\}$, with $N \geq 1$. Each seller is endowed with an indivisible asset and is privately informed of her asset's type, denoted by $\theta_i \in \{L, H\}$. Seller i has a value c_{θ_i} for her asset, where $c_L < c_H$. The value of a type-θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$.

We start by considering a model in which there are two trading periods: $t \in \{1, 2\}$. We generalize our results to an infinite-horizon model in the Appendix.[4] In each period, each seller is matched with two competing buyers who make private offers to the seller. Each buyer can make one offer; a buyer whose offer is rejected gets a payoff of zero and exits the game. The payoff to a buyer who purchases an asset of type θ at price p is $v_\theta - p$. Sellers discount future payoffs by a factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ, who agrees to trade at a price p in period t is

$$\left(1 - \delta^{t-1}\right) c_\theta + \delta^{t-1} p. \tag{1}$$

If the seller does not trade at either date, his payoff is c_θ. One can interpret c_θ and v_θ as the present value of the flow payoffs from owning the asset to the seller and buyer respectively.[5] All players are risk neutral.

Asset values are correlated with an unobservable underlying state, S, that takes values in $\{l, h\}$. The unconditional distribution of θ_i is $\mathbb{P}(\theta_i = H) = \pi \in (0, 1)$. Assets are mutually independent conditional on the state, but their conditional distributions are given by $\mathbb{P}(\theta_i = L|S = l) = \lambda \in (1-\pi, 1)$. To allow for arbitrarily high level of correlation, we set $\mathbb{P}(S = h) = \pi$. Our correlation structure introduces the possibility that trades of some assets convey relevant information about the aggregate state and therefore the value of other assets. To capture this possibility, we assume that all transactions are observable. Therefore, prior making offers in the second period, each buyer observes the set of assets that traded in the first period.

Notice that by virtue of knowing her asset quality, each seller has a private and conditionally independent signal about the aggregate state of nature. Thus, if each seller were to report her information truthfully to a central planner, the planner would learn the aggregate state with probability one as $N \to \infty$. Our interest is to explore under what conditions the same information can be gleaned from the transaction data of a decentralized market. To ensure that strategic interactions remain relevant, we focus on primitives which satisfy the following assumptions.

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[4]The two-period model facilitates a more precise characterization of the set of equilibria and thus a sharper intuition for our main results.

[5]Alternatively, we could specify the seller's payoff in (1) as $\delta^{t-1}(v_\theta - c_\theta)$ and interpret c_θ as the seller's production cost.

Assumption 1 $\pi v_H + (1-\pi)v_L < c_H$.

Assumption 2 $v_L < (1-\delta)c_L + \delta c_H$.

The first assumption, which we refer to as the "lemons" condition, asserts that the adverse selection problem is severe enough to rule out the efficient equilibrium in which all sellers trade immediately. In this equilibrium, trade is uninformative about the underlying state (regardless of N). The second assumption implies a lower bound on the discount factor and ensures that dynamic considerations remain relevant. Our main results do not rely on this assumption but it simplifies exposition and rules out fully separating equilibria, which are also independent of N.

2.1 Remarks on Modeling Assumptions

To illustrate the key mechanism for our findings as clearly as possible, we have made several rather stark assumptions regarding the buyers' side of the market. In particular, buyers are short-lived in that they can only make one (private) offer, they have identical values and information, and they compete in Bertrand fashion. This ensures that (1) a buyer makes zero (expected) profits on any accepted offer (2) there do not exist mutually agreeable unrealized trades (except possibly when both the buyer and the seller are indifferent). The primary purpose of these assumptions is to isolate the reason by which trade is delayed due to the seller's strategic considerations and to ensure that prices (i.e., buyers' offers) respond to new information.

While these features seem natural and are not strictly necessary for most our results, it is worth discussing them in a bit more detail. First, that buyers make offers in only one period is a fairly standard assumption in this literature (e.g., Swinkels, 1999; Kremer and Skrzypacz, 2007; Hörner and Vieille, 2009). The set of equilibrium outcomes we identify remain equilibrium outcomes in a model where buyers make offers over multiple periods provided those offers are publicly observable (though it is possible that other equilibria also exist). Complications arise when buyers are long-lived and offers are private as then a buyer may have incentive to experiment in the first period by making an offer that loses money if it is accepted in order to make profits conditional on a rejection.

If buyers' values or information is not identical, then a seller may have incentive to delay trade in the first period in order to meet a more favorable buyer in the second period. The assumption that each buyer is matched to a single seller in a given period is purely for convenience and can easily be relaxed. In what follows, we will also assume that buyers in the second period only observe first-period transactions but not transaction prices. This too is simply for convenience.

Because buyers are uninformed and make the offers, no additional information (beyond whether a transaction occured) is revealed by the price.

2.2 Strategies and Equilibrium Concept

A strategy of a buyer is a mapping from his information set to a probability distribution over offers. In the first period (i.e., at $t = 1$), a buyer's information set is empty. In the second period, buyers' information set is a vector in $\{0,1\}^{N+1}$ which indicates whether each asset traded in the first period. If asset i trades in the first period, then it is efficiently allocated and it is without loss to assume that buyers do not make offers for it in the second period (Milgrom and Stokey, 1982). The strategy of each seller is a mapping from her information set to a probability of acceptance. Seller i's information includes her type, the set of previous and current offers as well as the information set of buyers.

We use Perfect Bayesian Equilibria (PBE) as our solution concept. This has three implications. First, each seller's acceptance rule must maximize her expected payoff at every information set taking buyers' strategies and the other sellers' acceptance rules as given (*Seller Optimality*). Second, any offer in the support of the buyer's strategy must maximize his expected payoff given his beliefs, other buyers' strategy and the seller's strategy (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated according to Bayes' rule whenever possible (*Belief Consistency*).

2.3 Updating

Since buyers in the second period do not observe the level of offers in the first period, they update their beliefs based on whether each asset traded. Let σ_i^θ denote the probability that buyers assign to seller i trading in the first period if her asset is type θ. There are two ways by which the prior about seller i is updated between the first and second periods. First, conditional on rejecting the offer in the first period, the buyers' interim belief is given by

$$\pi_i^{Int} \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1) = \frac{\pi(1-\sigma_i^H)}{\pi(1-\sigma_i^H) + (1-\pi)\,(1-\sigma_i^L)}. \tag{2}$$

Second, before making offers in the second period, buyers learn about any other trades that took place in the first period. How this information is incorporated into the posterior depends on buyer beliefs about the trading strategy of the other sellers (i.e., σ_j^θ, $j \neq i$). Let $z^j \in \{0,1\}$ denote the indicator for whether seller j trades in the first period, and let $\mathbf{z} = (z^j)_{j=1}^{N+1}$ and $\mathbf{z}_{-i} = (z^j)_{j \neq i}$. Denote the probability of $\mathbf{z}_{-i}$ conditional on seller i being of type θ by $\rho_\theta^i(\mathbf{z}_{-i})$, which can be written as

$$\rho^i_\theta(\mathbf{z}_{-i}) \equiv \sum_{s\in\{l,h\}} \mathbb{P}\left(S = s|\theta_i = \theta\right) \cdot \prod_{j\neq i} \mathbb{P}\left(z^j|S = s\right), \tag{3}$$

where $\mathbb{P}\left(z^j = 1|S = s\right) = \sum_{\theta\in\{L,H\}} \sigma^\theta_j \cdot \mathbb{P}\left(\theta_j = \theta|S = s\right)$ is the probability that buyers assign to seller j trading in state s. Provided there is positive probability that i rejects the bid at $t = 1$ and $\mathbf{z}_{-i}$ is realized, we can use equations (2) and (3) to express the posterior probability of seller i being high type conditional on these two events:

$$\pi_i(\mathbf{z}_{-i}) \equiv \mathbb{P}(\theta_i = H|z^i = 0, \mathbf{z}_{-i}) = \frac{\pi^{Int}_i \cdot \rho^i_H\left(\mathbf{z}_{-i}\right)}{\pi^{Int}_i \cdot \rho^i_H\left(\mathbf{z}_{-i}\right) + (1-\pi^{Int}_i) \cdot \rho^i_L(\mathbf{z}_{-i})}. \tag{4}$$

2.4 Equilibrium Properties

AFG establish several properties that must hold in any equilibrium of the two-seller model. It is rather straightforward to show that these properties extend to the model studied here with an arbitrary number of sellers. However, developing an intuition for them will be useful for understanding our main results in Sections 3 and 4, so we provide some explanation of them here. The interested reader may wish to reference AFG for further intuition.

In order to introduce them, we will use the following definitions and notation. We refer to the *bid* for asset i at time t as the maximal offer made across all buyers for asset i at time t. Let $V(\tilde{\pi}) \equiv \tilde{\pi}v_H + (1-\tilde{\pi})v_L$ denote buyers' expected value for an asset given an arbitrary belief $\tilde{\pi}$. Let $\bar{\pi} \in (\pi, 1)$ be such that $V(\bar{\pi}) = c_H$, and recall that π_i denotes the probability that buyers assign to $\theta_i = H$ prior to making offers in the second period.

Property 1 (Second period) *If seller i does not trade in the first period, then in the second period:*

(i) If $\pi_i > \bar{\pi}$ then the bid is $V(\pi_i)$, which the seller accepts w.p.1.

(ii) If $\pi_i < \bar{\pi}$ then the bid is v_L, which the high type rejects and the low type accepts w.p.1.

(iii) If $\pi_i = \bar{\pi}$, then the bid is $c_H = V(\pi_i)$ with some probability $\phi_i \in [0,1]$ and v_L otherwise.

Note that a high type will only accept a bid higher than c_H. When the expected value of the asset is above c_H (as in *(i)*), competition forces the equilibrium offer to be the expected value. When the expected value of the asset is below c_H (as in *(ii)*), buyers cannot attract both types without making a loss. Thus, only the low type will trade and competition pushes the bid to v_L. Finally, when the expected value of the asset is exactly c_H (as in *(iii)*), buyers are indifferent between offering c_H and trading with both types or offering v_L and only trading with the low type.

Notice that Property 1 implies a second period payoff to a type-θ seller i as a function of (π_i, ϕ_i), which we denote by $F_\theta(\pi_i, \phi_i)$, where

$$F_H\left(\pi_i, \phi_i\right) \equiv \max\left\{c_H, V(\pi_i)\right\}, \tag{5}$$

and

$$F_L(\pi_i, \phi_i) \equiv \begin{cases} v_L & \text{if } \pi_i < \bar{\pi} \\ \phi_i c_H + (1-\phi_i) v_L & \text{if } \pi_i = \bar{\pi} \\ V(\pi_i) & \text{if } \pi_i > \bar{\pi}. \end{cases} \tag{6}$$

From seller i's perspective, the strategy of seller $j \neq i$ in the first period is relevant because it influences the distribution of news $\mathbf{z}_{-i}$ and therefore the distribution of π_i. In particular, the (expected) continuation value of a seller from rejecting an offer in the first period can be written as

$$Q_\theta^i \equiv (1-\delta)c_\theta + \delta \sum_{\mathbf{z}_{-i}} \rho_\theta^i(\mathbf{z}_{-i}) F_\theta(\pi_i(\mathbf{z}_{-i}), \phi_i). \tag{7}$$

It depends on seller i's own trading strategy σ_i^θ through the interim belief. But, importantly, it also depends on (i) the correlation of types with the state and (ii) the strategies of sellers $j \neq i$, since both influence the distribution of "news" ρ_θ^i that the buyers receive about seller i in the second period.

Property 2 (Skimming) *In any equilibrium, the expected continuation value of the high type is strictly greater than that for the low type: $Q_H^i > Q_L^i$.*

This result, often referred to in the literature as a "skimming" property, is due to the fact that both the flow payoff c_θ and the continuation payoff F_θ are higher for the high type, and because the high type rationally expects a (weakly) better distribution of buyer posteriors (thus prices) in the second period.

Property 3 (First period) *In the first period, the bid for each asset is v_L. The high-type seller rejects this bid with probability 1. The low-type seller accepts it with probability $\sigma_i < 1$.*

By Property 2, any offer that is acceptable to a high type in the first period is accepted by the low type w.p.1. But Assumption 1 implies that any such offer yields negative profits for the buyers. Hence, in equilibrium only low types trade in the first period and competition pushes the bid to v_L. Finally, if $\sigma_i = 1$, then the bid in the second period must be v_H (Property 1). But then the low-type seller i would strictly prefer to delay trade to the second period (Assumption 2), a contradiction.

Property 4 (Symmetry) *In any equilibrium, $\sigma_i = \sigma > 0$ for all i. If buyer mixing is part of the equilibrium then $\phi_i = \phi$ for all i.*

The key step to prove symmetry is to show that if $\sigma_i > \sigma_j \geq 0$, then $Q_L^i > Q_L^j$. This follows from the fact that, due to imperfect correlation, π_i (and therefore Q_L^i) is more sensitive to i's own trading probability than it is to that of the other players. Note that if $Q_L^i > Q_L^j$, then the low-type seller i strictly prefers to wait, which contradicts $\sigma_i > 0$ being consistent with an equilibrium. That there must be strictly positive probability of trade then follows: if $\sigma_i = 0$ for all i, then no news arrives and buyers in the second period would have the same beliefs as buyers in the first period. This would imply that the second period bid is v_L but in that case the low-type sellers would be strictly better off by accepting v_L in the first period, which contradicts $\sigma_i = 0$.

2.5 Equilibria

Given Properties 1–4, we can drop the subscripts and denote a candidate equilibrium by the pair (σ, ϕ). Because all equilibria are symmetric, any information about seller i that is contained in news $\mathbf{z}_{-i}$ does not depend on the identity of those who sold but only on the number (or fraction) of other sellers that traded. For example, suppose that $\mathbf{z}_{-i} = \mathbf{z}(K)$ where $\mathbf{z}(K)$ is such that $\sum_{j \neq i} z^j = K \leq N$. Then

$$\rho_\theta^i(\mathbf{z}(K)) = \sum_{s \in \{l,h\}} p_s^K \cdot (1 - p_s)^{N-K} \cdot \mathbb{P}(S = s | \theta_i = \theta),$$

where $p_s \equiv \sigma \cdot \mathbb{P}(\theta_i = L | S = s)$ is the probability that any given seller trades in state s. Naturally, the probability of observing K trades among sellers $j \neq i$ is $\binom{N}{K} \cdot \rho_\theta^i(\mathbf{z}(K))$.

Furthermore, since any equilibrium involves $\sigma \in (0, 1)$, a low-type seller must be indifferent between accepting v_L in the first period and waiting until the second period. The set of equilibria can thus be characterized by the solutions to

$$Q_L^i(\sigma, \phi) = v_L, \tag{8}$$

where we now make explicit the dependence of the continuation value on the strategy (σ, ϕ).

As we show in the next proposition, there can be multiple solutions to (8) and hence multiple equilibria.

Proposition 1 (Existence and Multiplicity) *An equilibrium always exists. If λ and δ are sufficiently large, there exist multiple equilibria.*

Intuitively, a higher σ has two opposing effects on the seller's continuation value. On the one hand, the posterior beliefs and thus prices in the second period are increasing in σ, which increases the expected continuation value Q^i_L. On the other hand, as other low types trade more aggressively, the distribution over buyers' posteriors shifts towards lower posteriors, thus decreasing Q^i_L. The latter force generates complementarities in sellers' trading strategies, which results in multiple equilibria when the correlation between assets is high and traders care sufficiently about the future.

We now turn to our main question, specifically, whether information about the underlying state is aggregated as the number of informed participants. To understand the essence of this question, first notice that the trading behavior of each seller provides an informative signal about the aggregate state. If the seller trades in the first period, than she reveals her asset's type is L, which is more likely when the aggregate state is l than when it is h. Conversely, if the seller does not trade, then buyers update their beliefs about the asset toward H and their belief about the aggregate state toward h. Moreover, the amount of information revealed by each seller is increasing in the low-type's trading probability, which we now denote by σ_N (in order to explicitly indicate its dependence on the number of other informed participants).

If the information content of each individual trade were to converge to some positive level (i.e., $\lim_{N\to\infty} \sigma_N = \bar{\sigma} > 0$), then information about the state would aggregate. The reason is that by the law of large numbers the fraction of assets traded would concentrate around its population mean $\bar{\sigma} \cdot \mathbb{P}(\theta_i = L|S = s)$, which is strictly greater when the aggregate state is l than when it is h. If, on the other hand, σ_N decreases to zero at a rate weakly faster than $1/N$ (i.e., $\lim_{N\to\infty} N \cdot \sigma_N < \infty$), then information would not aggregate. In this case, despite having arbitrarily many signals about the state, the informativeness of each signal goes to zero fast enough that the overall amount of information does not reveal the true state.

Of course, the equilibrium trading behavior of each individual seller is determined endogenously. Therefore, in order to establish information aggregation properties of equilibria, we need to understand how the set of equilibrium values of σ_N changes with N. Moreover, since different equilibria have different σ_N, the limiting information aggregation properties could be different for different sequences of equilibria. As we will see in the next section, neither of the two cases mentioned in the previous paragraph is pathological.

3 Information Aggregation

Consider a sequence of economies indexed by N (standing for $N+1$ assets), and let σ_N denote an equilibrium trading probability in the first period and π_N^{State} be the buyers' posterior belief that the aggregate state is h, conditional on having observed the outcome of trade in the first period. That is, given a trading history $\mathbf{z} = (z^j)_{j=1}^{N+1}$, $\pi_N^{State}(\mathbf{z}) \equiv \mathbb{P}(S = h|\mathbf{z})$. We say that:

Definition 1 *There is* ***information aggregation*** *along a given sequence of equilibria if* $\pi_N^{State} \to^p 1_{\{S=h\}}$ *as* $N \to \infty$*, where* $\to^p$ *denotes convergence in probability.*

Our notion of information aggregation requires that, upon observing the trading history, buyers learn all the information available in the market that is relevant to infer the aggregate state. Asymptotically, this is equivalent to asking whether agents' beliefs about the aggregate state become degenerate at the truth.[6]

3.1 A 'Fictitious' Economy

Before presenting our main results, it will be useful to consider a 'fictitious' economy in which buyers observe the true state S via an exogenous signal before making second period offers. This benchmark economy is useful because it approximates the information revealed in the true economy if there is information aggregation. We proceed by deriving a necessary and sufficient condition under which the fictitious economy supports an equilibrium with trade in the first period (Lemma 1). We then show that the same condition is necessary, though not sufficient, for information aggregation (Theorem 1). Intuitively, information aggregation requires trade. But if the fictitious economy does not support an equilibrium with trade, then (by continuity) there cannot exist a sequence of equilibria along which information aggregates.

First, note that Properties 1, 2, and 3 trivially extend to the fictitious economy. Second, observe that conditional on knowing the true state, the information revealed by other sellers is irrelevant for buyers when forming beliefs about seller i. That is, buyers' posterior belief about seller i following a rejection in the first period and observing the true state is s is given by

$$\pi_i^{fict}(s) = \frac{\pi_i^{Int} \cdot \mathbb{P}(S = s|\theta_i = H)}{\pi_i^{Int} \cdot \mathbb{P}(S = s|\theta_i = H) + (1 - \pi_i^{Int}) \cdot \mathbb{P}(S = s|\theta_i = L)}, \tag{9}$$

where π_i^{Int} is the interim belief given in (2). This implies that seller i's continuation value in the fictitious economy, which we denote by $Q_\theta^{i,fict}(\sigma_i, \phi_i)$ is independent of the trading strategies of

[6]That our definition involves convergence in probability is standard in the literature (see e.g., Kremer (2002)).

the other sellers. Analogous to (7), the continuation value is given by

$$Q_\theta^{i,fict}(\sigma_i, \phi_i) = (1-\delta)c_\theta + \delta \sum_s \mathbb{P}(S = s|\theta_i = \theta) F_\theta \left(\pi_i^{fict}(s), \phi_i\right) \tag{10}$$

Since there are no complementarities between sellers' trading strategies, the fictitious economy has a unique equilibrium, which must be symmetric. As in Daley and Green (2012), due to the exogenous arrival of information, it is possible that the equilibrium of the fictitious economy will involve zero probability of trade in the first period.

Lemma 1 *The unique equilibrium of the fictitious economy involves zero probability of trade in the first period (i.e.,* $\sigma^{fict} = 0$*) if and only if*

$$Q_L^{i,fict}(0,0) \geq v_L. \tag{$\star$}$$

Furthermore, $(\star)$ *holds if and only if* λ *and* δ *satisfy the following:*

$$\lambda \geq \bar{\lambda} \equiv 1 - \frac{\pi(1-\bar{\pi})}{1-\pi}$$

and

$$\delta \geq \bar{\delta}_\lambda \equiv \frac{v_L - c_L}{v_L - c_L + (1-\lambda) \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) \cdot (v_H - v_L)}.$$

This result is intuitive. The equilibrium of the fictitious economy features no trade whenever the low type's option value from delaying trade to the second period is high. This occurs when both the information revealed in the second period is sufficiently informative about the seller's type (i.e., $\lambda \geq \bar{\lambda}$) and for a given correlation the future is sufficiently important (i.e., $\delta \geq \bar{\delta}_\lambda$).

3.2 When Does Information Aggregate?

We now establish our first main result, which shows that $(\star)$ is also the crucial determinate of the information aggregation properties of equilibria.

Theorem 1 (Aggregation Properties)

(i) If $(\star)$ *holds with strict inequality, then information aggregation fails along any sequence of equilibria.*

(ii) If $(\star)$ *does not hold, then there exists a sequence of equilibria along which information aggregates.*

The proof of the first statement uses the observation that if information were to aggregate, then for N large enough the continuation payoffs of the sellers are close to the continuation payoffs in the fictitious economy. Thus, when ($\star$) holds strictly, delay is also uniquely optimal when there are a large but finite number of assets. But this contradicts Property 4, which states that $\sigma_N \in \{0, 1\}$ cannot be part of an equilibrium for any finite N. In fact, when ($\star$) holds strictly, the trading probability σ_N is positive but must go to zero at a rate proportional to $1/N$, which is fast enough to prevent information from aggregating. The rate is also slow enough to ensure that the market does not become completely uninformative in the limit. In that case, the bid for any asset in the second period would be v_L with probability arbitrarily close to one; hence, the low types would strictly prefer to trade in the first period (implying $\sigma_N = 1$), which would contradict Property 4.

On the other hand, when the fictitious economy has an equilibrium with positive trade in the first period (i.e., if ($\star$) does not hold), we can explicitly construct a sequence of equilibria in which the trading probability σ_N is bounded away from zero. Clearly, information is aggregated along such a sequence. Nevertheless, even when aggregating equilibria exist, it is not the case that information will necessarily aggregate along *every* sequence of equilibria.

Theorem 2 (Coexistence) *There exists a $\widehat{\delta} < 1$ such that whenever $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ and λ is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails. If either $\lambda < \bar{\lambda}$ or δ is sufficiently small, then information aggregates along any sequence of equilibria.*

To prove the first statement, we first note that for a given $\delta < 1$, if λ is sufficiently large, then we must have $\delta < \bar{\delta}_\lambda$ and thus by Theorem 1 aggregating equilibria must exist. We then show that if we fix δ above a certain threshold, then for a sufficiently large λ, also non-aggregating equilibria must exist. In particular, we explicitly construct a sequence of equilibria in which the second period bid is v_L for all histories except the one in which no seller has traded in the first period. In these equilibria, the probability of the event that *no seller has traded in the first period* remains bounded away from zero, in *both* states of nature! Thus, even as $N \to \infty$, the uncertainty about the state of nature does not vanish.

The second part of Theorem 2 provides sufficient conditions under which information necessarily aggregates. While this result is not particularly surprising, it is instructive to observe that the possibility of aggregation failure requires the two key ingredients of the model: (1) sufficient correlation across assets (i.e. $\lambda > \bar{\lambda}$) and (2) that strategic delay is relevant (i.e. δ is large enough).

Figure 1 summarizes our main results by illustrating the regions of the parameter space for which aggregation holds and fails as well as the region of coexistence. In the top-right

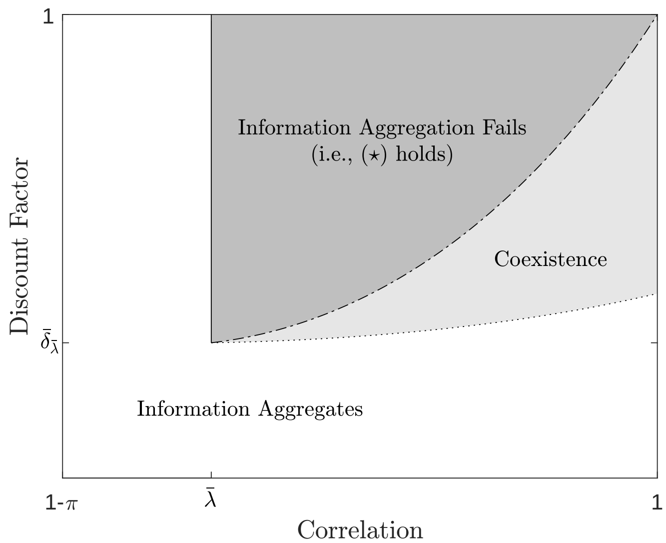


Figure 1: **When does Information Aggregate?** This figure illustrates the regions of the parameter space over which information aggregation obtains or fails.

(darkly shaded) region, ($\star$) holds and hence there do not exist sequences of equilibria that aggregate information. Otherwise, aggregating equilbria exist (Theorem 1). In the bottom-left (unshaded) region, all sequences of equilibria aggregate information and in the middle-right (lightly shaded) region, sequences in which information aggregates coexist with sequences in which information aggregation fails (Theorem 2).

Perhaps surprsingly, our main results can be extended to a model with more than two trading periods. Intuitively, one might expect that with more trading periods there are more opportunities to learn from trading behavior and hence more information will be revealed. However, there is a countervailing force; there are more opportunities for (strategic) sellers to signal through delay. It turns out that two factors essentially cancel each other out. In the Appendix, we extend the model the model to allow for an infinite number of trading periods and demonstrate that the analogs of Theorems 1 and 2 continue to hold.

3.3 Trading Behavior and Welfare in a Large Market

The ex-ante equilibrium surplus of seller i is:

$$W_N^i = (1-\pi)(Q_{L,N}^i - c_L) + \pi(Q_{H,N}^i - c_H),$$

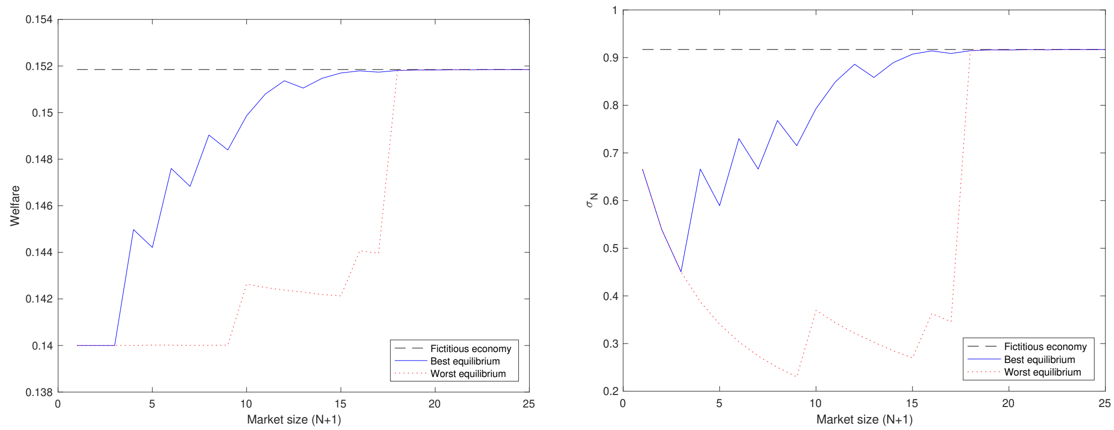


Figure 2: The left panel illustrates how the welfare per trader depends on the number of traders. The right panel shows the corresponding strategy of a low-type seller in the first period. The parameters are such that only aggregating equilibria exist.

where $Q^i_{\theta,N}$ is given by (7) when the market size is $N+1$. Because buyers are competitive and thus break even, W_N is effectively the per trader surplus in our economy. On the other hand, the per trader surplus in the unique equilibrium of the fictitious economy is:

$$W^{i,fict} = (1-\pi)(Q^{i,fict}_L - c_H) + \pi(Q^{i,fict}_H - c_H),$$

where $Q^{i,fict}_\theta$ is given by (10). The following proposition shows that aggregating equilibria behave very much like the fictitious economy.

Proposition 2 (Aggregating Equilibria) *Consider a sequence of equilibria along which information aggregates. Then, along this sequence:*

(i) $\lim_{N\to\infty} \sigma_N = \sigma^{fict}$, *and*

$$\lim_{N\to\infty} \mathbb{P}\left((N+1)^{-1}\sum_{j=1}^{N+1} z^j \le x | S = s\right) = \mathbb{I}\{x \ge \sigma^{fict}\mathbb{P}(\theta_i = L|S=s)\}.$$

(ii) The volatility of prices conditional on the true state goes to zero.

(iii) $\lim_{N\to\infty} W^i_N = W^{i,fict}$.

Figure 2 illustrates this result graphically by plotting the equilibrium trading surplus W_N and the trading probability σ_N against the market size N. For small N, multiple equilibria exist due to strategic complementarities among different sellers, and W^i_N and σ_N can either increase or decrease with N. As N grows large, however, the aggregate state gets learned,

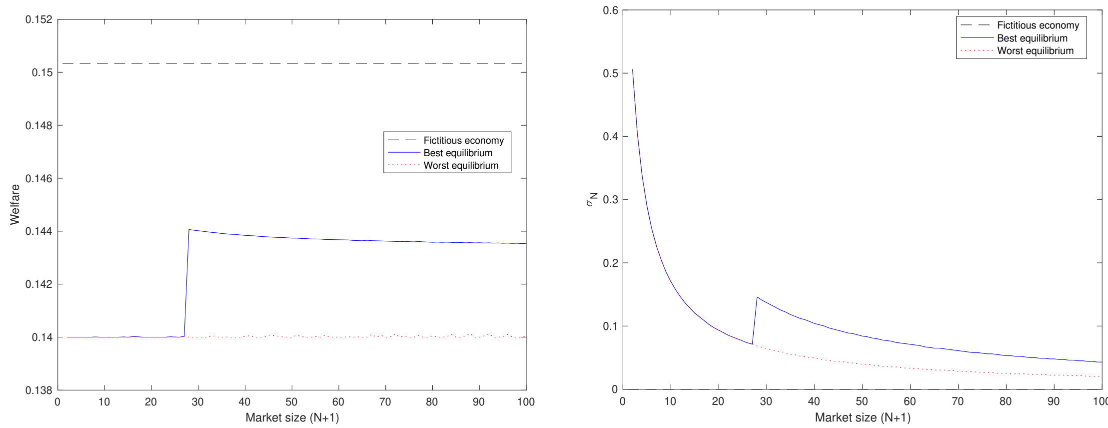


Figure 3: The left panel illustrates how the welfare per trader depends on the number of traders. The right panel shows the corresponding strategy of the seller in the first period. The parameters are such that ($\star$) holds and, hence, absent intervention aggregating equilibria do not exist.

the complementarities vanish, and both welfare and trading behaviour converge to those of the fictitious economy. The implication is that, in this economy, conditional on the aggregate state, the volatility in asset prices and trading volume (in both periods) goes to zero. As we show next, however, the picture changes dramatically in when information fails to aggregate.

Proposition 3 (Non-Aggregating Equilibria) *Consider a sequence of equilibria such that information aggregation fails along any of its subsequences. Then, along this sequence:*

(i) $N\sigma_N \in (\underline{\kappa}, \overline{\kappa})$ *for some constants* $\underline{\kappa}, \overline{\kappa} > 0$*, and for any convergent subsequence* $\{\sigma_{N_k}\}$ *with* $\kappa \equiv \lim_{k\to\infty} \sigma_{N_k} N_k$*,*

$$\lim_{k\to\infty} \mathbb{P}\left(\sum_{j=0}^{N_k+1} z^j \leq x | S = s\right) = \sum_{n=0}^{\lfloor x \rfloor} \frac{1}{n!} \cdot \left(\kappa \mathbb{P}(\theta_i = L|S = s)\right)^n \cdot e^{-\kappa \mathbb{P}(\theta_i = L|S=s)}.$$

(ii) The volatility of prices conditional on the true state remains strictly positive.

(iii) $\limsup_{N\to\infty} W_N^i < W^{i,fict}$.

In non-aggregating equilibria, strategic considerations do not vanish as the market grows large, which leads to (excess) volatility in prices conditional on the state and welfare that is below the fictitious benchmark. Figure 3 illustrates this result graphically.

The contrast between Propositions 2 and 3 demonstrates that aggregating equilibria have several nice properties that are not shared by their non-aggregating counterparts. Two immediate implications follow. First, from a social welfare perspective, aggregating equilibria are always preferable to non-aggregating equilibria when they co-exist. Thus, among laissez-faire outcomes, aggregation is optimal. Second, if the only laissez-faire outcomes are non-aggregating and a social planner could manage to learn the true state at $t = 1$, then she could improve welfare by revealing her information to market participants.

Of course, it is not obvious how a planner would be able to acquire such information. It is more natural to think that the planner is uninformed, but can learn about the true state by observing the trading behavior of market participants. The problem facing the planner is then how best to reveal this information to other agents in the economy. In the next section, we tackle precisely this problem.

4 Optimal Information Policy

How should an (uninformed) planner disclose trading behavior to maximize social welfare? Before analyzing the planner's problem, it is useful to compare the problem we consider to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010) and "information design" problems more generally (Bergemann and Morris, 2013, 2016).[7] On one hand, the problems are quite similar. Both involve designing an information revelation policy to induce other players to take certain actions. On the other hand, the planner's problem in our setting must take into account a novel feedback effect. Namely, the planner's policy influences the information content of trading behavior, and therefore the information content of whatever is revealed. In short, the statistical properties of the information the planner can reveal, which is typically exogenous in a Bayesian persuasion setting, depends on the policy itself.

Our solution method for answering this question will proceed in two steps. First, we consider the information design problem of a planner who (exogenously) learns the aggregate state at the end of period $t = 1$. We refer to this as the *Informed Planner's Problem.* We characterize the solution to this problem, which for δ large enough involves partially concealing the aggregate state in order to increase the volume of trade compared to the laissez-faire outcome. We then return to the problem of interest and provide the necessary and sufficient conditions under which the (uninformed) planner can achieve the same outcome as the informed one. When these conditions do not hold, the planner distorts trade in the first period in order to learn the state, which results in lower welfare than when the planner is informed. Finally, we relate our normative findings to the information aggregation properties of equilibria and discuss their

[7] Bergemann and Morris (2017) provide a more general treatment of information design problems drawing a distinction between whether the designer has an informational advantage (as in Bayesian persuasion) or not (as in communication games). In our model, the planner has no informational advantage ex-ante but has a technology for acquiring one in the interim. Another important distinction of our setting is that the planner has only limited means by which she can elicit information.

policy implications.

4.1 Informed Planner's Problem

In this section, we set up the informed planner's problem and characterize its solution. In doing so, we will assume that the planner (exogenously) learns the aggregate state at $t = 1$ and can design and commit to an information policy ex-ante, the results of which are publicly revealed after trading at $t = 1$. Therefore, buyers of asset i at date $t = 2$ can observe (i) whether asset i traded in the first period, and (ii) any additional information revealed by the planner.

The planner's objective is to maximize the expected discounted gains from trade. Because we focus on a public information policy and all assets are ex-ante identical, it is sufficient to consider the problem of maximizing the expected discounted gains from trade for a single asset. The planner's objective can be written as

$$W = (1 - \pi)(\sigma + \delta(1 - \sigma))(v_L - c_L) + \pi(\mathbb{P}(\pi_i = \bar{\pi}|H)\phi + \mathbb{P}(\pi_i > \bar{\pi}|H))\delta(v_H - c_H), \qquad (11)$$

where π_i is the (random) buyers' posterior belief at $t = 2$ that the seller is H-type.

From Kamenica and Gentzkow (2011), the problem of choosing state-dependent distributions over signals is equivalent to choosing a distribution of posteriors about the state that is Bayes plausible. Let $\tilde{p}$ denote the random variable representing the buyers' posterior about the state conditional on observing the information revealed by the planner and let G denote the distribution of $\tilde{p}$. Bayes plausbility requires that the expected posterior be equal to the prior

$$E_G\{\tilde{p}\} = \pi \qquad (12)$$

Of course, the planner's choice of G will influence both the behavior of the seller and the buyers as captured by (σ, ϕ).

Definition 2 (Informed Planner's Problem) *The informed planner's problem is to choose a triple* (G, σ, ϕ) *to maximize* (11) *subject to two constraints:*

(1) Bayes Plausibility (i.e., (12)*), and*

(2) Given G*,* (σ, ϕ) *must be an equilibrium of the game.*

We say that (G, σ, ϕ) is *feasible* if it satisfies (1) and (2). We let $Q_\theta^G(\sigma, \phi)$ denote the conintuation value to a type-θ seller. For any G, Properties 1-3 must hold in any equilibrim. Moreover, as in Section 2.5, if $\sigma \in (0, 1)$ then constraint (2) requires that the low type's continuation value must equal v_L. However, with an informed planner, it is no longer true that low type sellers must trade with strictly positive probability in the first period (as in Property 4). More specifically, it is possible to design G such that there exist equilibria in which $\sigma = 0$ and $Q_L^G \geq v_L$. Instead of (8), equilibria are characterized by (G, σ, ϕ) such that

$$Q_L^G(\sigma, \phi) \geq v_L \tag{13}$$

$$\sigma \left(Q_L^G(\sigma, \phi) - v_L \right) = 0 \tag{14}$$

The following lemma puts a bound on the set of feasible σ that can be implemented.

Lemma 2 *Define $\bar{\sigma}$ implicitly by $\pi(l, \bar{\sigma}) = \bar{\pi}$ and define $\underline{\sigma} \equiv \inf\{\sigma \in [0,1], \pi(h, \sigma) \geq \bar{\pi}\}$, then σ is feasible only if $\sigma \in [\underline{\sigma}, \bar{\sigma}]$.*

The proof is simple. If $\sigma > \bar{\sigma}$ $(< \underline{\sigma})$, then $Q_L^G > v_L (< v_L)$ regardless of what information is revealed by the planner. The next lemma simplifies the informed planner's problem by showing that, for any candidate σ, it is enough to consider information policies with at most three beliefs in the support.

Lemma 3 *For any feasible σ, the solution to the informed planner's problem can be achieved with a information policy that has a support $S(\sigma) \subseteq \{0, \bar{p}(\sigma), 1\}$, where $\bar{p}(\sigma)$ is such that $\pi_i(\bar{p}(\sigma), \sigma)) = \bar{\pi}$.*

The intuition behind Lemma 3 is as follows. Take any (G, σ, ϕ) such that $\hat{p} \in (0, \bar{p}(\sigma))$ is in the support of G. The low type's payoff in the second period following the realization of $\hat{p}$ is v_L and the high type gets c_H. The same payoffs can be achieved by a policy that reveals either $p = 0$ or $p = \bar{p}(\sigma)$ and where ϕ is adjusted down to keep Q_L unchanged. Thus, it is without loss to restrict attention to policies that do not involve posteriors $p \in (0, \bar{p}(\sigma))$.

Next, consider any policy (G, σ, ϕ) such that $\tilde{p} \in (\bar{p}(\sigma), 1)$ is in the support of G. Let G' be a new information policy that reassigns the weight on $\tilde{p}$ to $\bar{p}(\sigma)$ and 1 (respecting Bayesian plausibility), it can be shown that $Q_H^{G'}(\sigma, \phi) \geq Q_H^G(\sigma, \phi)$ and $Q_L^{G'}(\sigma, \phi) \leq Q_L^G(\sigma, \phi)$. Therefore, it is possible to find $\sigma' \geq \sigma$ such that (G', σ', ϕ) is a feasible policy under which both seller types are weakly better off.

Thus, for any given σ, the information policy (i.e., G) of the informed planner has been reduced to choosing a pair $(\mu_0, \mu_1) \in [0,1]^2$, where $\mu_k = \Pr_G(p = k)$, and the Bayes plausibility constraint reduces to

$$\mu_1 + (1 - \mu_0 - \mu_1)\bar{p}(\sigma) = \pi.$$

Definition 3 *We say that the planner's policy is* fully revealing *if $\mu_0 = 1 - \pi$ and $\mu_1 = \pi$.*

If the policy attaches a strictly positive weight to $\bar{p}(\sigma)$ (i.e., if $\mu_0 + \mu_1 < 1$) then some information is concealed. To further characterize the solution, it is useful to first consider a modified planning problem, in which (13) is required to hold with equality and which we will show arises naturally when the planner is uninformed (see Lemma 5).

When constraint (13) holds with equality, the planner's objective reduces to maximizing the payoff of the high type seller,

$$Q_H^G = c_H + \delta \cdot \frac{\mathbb{P}(S = h|\theta = H)}{\pi} \cdot \mu_1 \cdot \left(V(\pi(h;\sigma)) - c_H\right), \tag{15}$$

where $\pi(h;\sigma)$ is the posterior belief when the state is revealed to be h. Note that Q_H^G is increasing in μ_1 since the planner reveals that the state is h more frequently (and the price in that event is highest), and it is increasing in σ since the pooling price in that state is higher. Crucially, whether the planner faces a tradeoff between μ_1 and σ depends on whether revealing the state more frequently increases the continuation value of the low type.

Lemma 4 *Consider a variant of the informed planner's problem in which the inequality in* (13) *is required to hold with equality. Then the solution to this problem is as follows:*

(i) If $Q_L^{i,fict}(\bar{\sigma}, 0) \leq v_L$, then the optimal information policy is fully revealing, $\sigma^* = \bar{\sigma}$, and ϕ^* is such that $Q_L^{i,fict}(\bar{\sigma}, \phi^*) = v_L$.

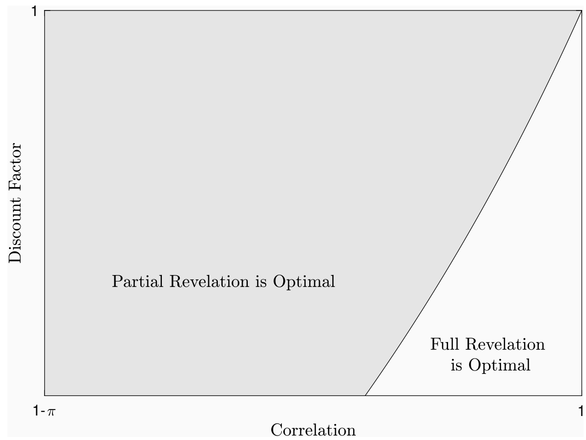
(ii) If $Q_L^{i,fict}(\bar{\sigma}, 0) > v_L$ then the optimal information policy conceals some information: $\mu_0^* = 0, \mu_1^* = \frac{\pi - \bar{p}(\sigma^*)}{1 - \bar{p}(\sigma^*)}$, and σ^* is such that $Q_L^{G^*}(\sigma^*, 0) = v_L$.

Intuitively, the reason why the planner conceals information is closely related to the option value effect of information that we identified in the fictitious economy of Section 3.1: the prospect of the high state being revealed more frequently can generate an increase in the low type's expected future prices and increase inefficient trade delays.

Lemma 4 will be useful in the study of the uninformed planner's problem. Before moving to that problem, however, we complete the characterization of the informed planner's true problem. If (13) is slack, then it must be that the low-type seller trades with probability zero in the first period (by (14)). In this case, the planner's problem reduces to maximizing the probability of trade with the high type in the second period. As the next result demonstrates, this is accomplished by never fully revealing the high state.

Proposition 4 *There exists a $\widetilde{\delta} < 1$ such that the solution to the informed planner's problem is as follows.*

(i) The constraint (13) *holds with equality if and only if $\delta < \widetilde{\delta}$, in which case the optimal policy is characterized by Lemma 4.*



(ii) If $\delta \geq \widetilde{\delta}$ then the optimal policy involves $\sigma^* = \mu_1^* = 0$.

As we will see in the next section, a policy like the one in part (ii) of Proposition 4 is not feasible for the uninformed planner.

4.2 Uninformed Planner's Problem

We are now ready to tackle our problem of interest where, rather than being endowed exogenously with knowledge of the state, the planner must learn it from the trading history she observes. As a result, we also need to keep track of the market size, indexed by N, since it will affect the information the planner observes in the first period. This latter feature also makes it cumbersome to employ the typical Bayesian persuasion approach since the Bayesian plausibility constraint is no longer sufficient. Instead, we work directly on the planner's information policy, which is defined as a mapping $\mathcal{M}_N$ from the trading histories she observes, which elements of $\{0,1\}^N$, to distributions over signals that are publicly observed by agents in the economy.

The planner's objective is to maximize the expected discounted gains from trade,

$$W_N = (1-\pi)(\sigma_N + \delta(1-\sigma_N))(v_L - c_L) + \pi\left(\mathbb{P}^{\mathcal{M}_N}(\pi_i = \bar{\pi}|H)\phi_N + \mathbb{P}^{\mathcal{M}_N}(\pi_i > \bar{\pi}|H)\right)\delta(v_H - c_H), \tag{16}$$

where $\mathbb{P}^{\mathcal{M}_N}(\cdot|\theta)$ denotes the conditional probability distribution over the buyers' posteriors induced by $\mathcal{M}_N$. We let $Q_\theta^{\mathcal{M}_N}(\sigma_N, \phi_N)$ denote the continuation value to a type-θ seller.

Definition 4 (Uninformed Planner's Problem) *The uninformed planner's problem is to choose a triple $(\mathcal{M}_N, \sigma_N, \phi_N)$ to maximize (16) subject to (σ_N, ϕ_N) being an equilibrium of the game.*

The key difference from the informed planner's problem is that the information content of any signal revealed by the planner is endogenous to the equilibrium trading probability σ_N. This has the following important implication.

Lemma 5 *The solution to the uninformed planner's problem must involve* $\sigma_N > 0$ *and*

$$Q_L^{\mathcal{M}_N}(\sigma_N, \phi_N) = v_L. \tag{17}$$

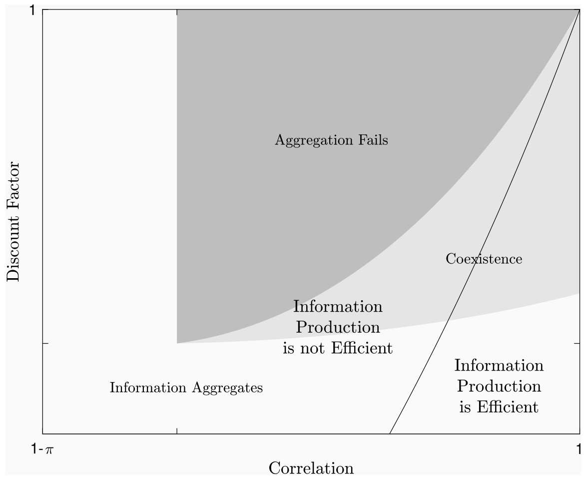
Intuitively, if σ_N were equal to zero, the planner has no relevant information and any signals she reveals are completely uninformative. But then "no trade" cannot be consistent with equilibrium when traders do not expectation arrival of information (see discussion following Property 4).

Combining Lemma 5 with Proposition 4 implies that, when $\delta > \widetilde{\delta}$, the uninformed planner cannot achieve the same level of surplus as the informed planner can attain, even as $N \to \infty$. Futher, Lemma 5 implies that the solution to the modified informed planner's problem characterized in Lemma 4 provides an upper bound on the level of surplus that the uninformed planner can achieve. Clearly, a policy for the uninformed planner that achieves this upper bound must also be optimal for any δ.

Proposition 5 (Pareto Optimum) *There exists a sequence* $\{\mathcal{M}_N, \sigma_N, \phi_N\}$ *along which* $W_N \to W^*$ *and* $\sigma_N \to \sigma^*$*, where* σ^* *and* W^* *are respectively the trading probability and welfare under the information policy of the modified informed planner's problem in Lemma 4.*

To prove this result, we construct an information policy consisting of a binary signal, $\omega_N \in \{b, g\}$, where the planner sends signal $\omega_N = g$ with probability μ_1^* when she observes that the fraction of sellers who traded at $t = 1$ is below some threshold $\tau \in (0, 1)$, and she sends signal $\omega_N = b$ otherwise. We show that, when σ_N is close to σ^* and τ is chosen appropriately, the planner asymptotically learns the state and the information content of her policy converges to that of the information policy of the informed planner as characterized in Lemma 4, which may or may not be fully revealing. Finally, we use continuity arguments to find a sequence of trading probabilities σ_N which both converges to σ^* and is consistent with equilibrium under this information policy, for all N.

It is worth noting that the optimal information policy achieves a Pareto improvement over the laissez-faire outcomes. The reason is that the buyers break even, the low type's payoff is v_L (see Lemma 5) and, thus, all the additional surplus generated by the planner's policy is captured by the high types. The implication of this observation is that all agents would be happy to delegate the information dissemination about past trades to the social planner.



5 Concluding Remarks

We study the information aggregation properties of decentralized dynamic markets in which traders have private information about the value of their asset, which is correlated with some underlying ‘aggregate’ state of nature. We provide necessary and sufficient conditions under which information aggregation necessarily fails. Further, we show that when these conditions are violated, there can be a coexistence of non-trivial equilibria in which information about the state aggregates with equilibria in which aggregation fails. Our findings suggest there are important differences in the aggregation properties of multi-asset decentralized markets (as studied here) and single-asset centralized markets as typically explored in the literature.

We then consider the normative implications of our theory. We solve for the optimal information policy of a social planner who observes the trading behavior and chooses what information to communicate to the traders. We show that the information generated in a laissez-faire economy is always inefficient when aggregation fails. The optimal policy conceals favorable “news” from the traders in order to accelerate trade and increase trading surplus. We discuss the implications of our theory for policies of information dissemination in markets.

References

Albagli, E., A. Tsyvinski, and C. Hellwig (2015): "A Theory of Asset Prices Based on Heterogeneous Information," *Working Paper.*

Asriyan, V., W. Fuchs, and B. Green (2017): "Information Spillovers in Asset Markets with Correlated Values," *American Economic Review*, 107(7), 2007–40.

Axelson, U. and I. Makarov (2017): "Informational Black Holes in Financial Markets," *Working Paper.*

Babus, A. and P. Kondor (2016): "Trading and Information Diffusion in Over-the-Counter Markets," *Working Paper.*

Back, K., C. H. Cao, and G. A. Willard (2000): "Imperfect Competition Among Informed Traders," *Journal of Finance*, 55(5), 2117–2155.

Baumol, W. J. (1965): *The Stock Market and Economic Efficiency*, New York: Fordham University Press.

Bergemann, D. and S. Morris (2013): "Robust Predictions in Games with Incomplete Information," *Econometrica*, 81, 1251–1308.

——— (2016): "Bayes Correlated Equilibrium and the Comparison of Information Structures in Games," *Theoretical Economics*, 11, 487–522.

——— (2017): "Information Design: A Unified Perspective," *Working Paper.*

Bernanke, B. S. and M. Woodford (1997): "Inflation forecasts and monetary policy," Tech. rep., National Bureau of Economic Research.

Bodoh-Creed, A. (2013): "Efficiency and Information Aggregation in Large Uniform-Price Auctions," *Journal of Economic Theory*, 146, 2436–2466.

Boleslavsky, R., D. L. Kelly, and C. R. Taylor (2017): "Selloffs, bailouts, and feedback: Can asset markets inform policy?" *Journal of Economic Theory*, 169, 294–343.

Boleslavsky, R. and K. Kim (2018): "Bayesian persuasion and moral hazard," .

Bond, P., A. Edmans, and I. Goldstein (2012): "The Real Effects of Financial Markets," *Annual Review of Financial Economics*, 4, 339–360.

Bond, P. and I. Goldstein (2015): "Government intervention and information aggregation by prices," *The Journal of Finance*, 70, 2777–2812.

Bond, P., I. Goldstein, and E. S. Prescott (2009): "Market-based corrective actions," *The Review of Financial Studies*, 23, 781–820.

Camargo, B., K. Kim, and B. Lester (2015): "Information spillovers, gains from trade, and interventions in frozen markets," *The Review of Financial Studies*, 29, 1291–1329.

Daley, B. and B. Green (2012): "Waiting for News in the Market for Lemons," *Econometrica*, 80(4), 1433–1504.

——— (2016): "An Information-Based Theory of Time-Varying Liquidity," *Journal of Finance*, 71(2), 809–870.

Dow, J. and G. Gorton (1997): "Stock market efficiency and economic efficiency: Is there a connection?" *The Journal of Finance*, 52, 1087–1129.

Duffie, D., P. Dworczak, and H. Zhu (2017): "Benchmarks in Search Markets," *Journal of Finance (forthcoming).*

Fishman, M. J. and K. M. Hagerty (1989): "Disclosure decisions by firms and the competition for price efficiency," *The Journal of Finance*, 44, 633–646.

——— (1992): "Insider trading and the efficiency of stock prices," *The RAND Journal of Economics*, 106–122.

Foster, F. D. and S. Viswanathan (1996): "Strategic Trading when Agents Forecast the Forecasts of Others," *Journal of Finance*, 51(4), 1437–1478.

Fuchs, W., A. Öery, and A. Skrzypacz (2016): "Transparency and Distressed Sales under Asymmetric Information," *Theoretical Economics*, 11, 1103–1144.

Fuchs, W. and A. A. Skrzypacz (2012): "Costs and benefits of dynamic trading in a lemons market," *Working Paper*, 1–33.

Golosov, M., G. Lorenzoni, and A. Tsyvinski (2014): "Decentralized trading with private information," *Econometrica*, 82, 1055–1091.

Grossman, S. (1976): "On the efficiency of competitive stock markets where trades have diverse information," *The Journal of Finance*, 31, 573–585.

Hayek, F. A. (1945): "The Use of Knowledge in Society," *American Economic Review*, 35, 519–530.

Hellwig, M. F. (1980): "On the aggregation of information in competitive markets," *Journal of economic theory*, 22, 477–498.

Hörner, J. and N. Vieille (2009): "Public vs. Private Offers in the Market for Lemons," *Econometrica*, 77(1), 29–69.

Janssen, M. C. W. and S. Roy (2002): "Dynamic Trading in a Durable Good Market with Asymmetric Information," *International Economic Review*, 43:1, 257–282.

Kamenica, E. and M. Gentzkow (2011): "Bayesian persuasion," *The American Economic*

Review, 101, 2590–2615.

Kremer, I. (2002): "Information Aggregation in Common Value Auctions," *Econometrica*, 70, 1675–1682.

Kremer, I. and A. Skrzypacz (2007): "Dynamic Signaling and Market Breakdown," *Journal of Economic Theory*, 133, 58–82.

Kyle, A. S. (1985): "Continuous auctions and insider trading," *Econometrica*, 53(6), 1315–1335.

——— (1989): "Informed speculation with imperfect competition," *The Review of Economic Studies*, 56, 317–355.

Lauermann, S. and A. Wolinsky (2013): "A Common Value Auction with Bidder Solicitation," *Working Paper*.

——— (2016): "Search with Adverse Selection," *Econometrica*, 84(1), 243–315.

Leland, H. E. (1992): "Insider trading: Should it be prohibited?" *Journal of Political Economy*, 100, 859–887.

Lester, B., A. Shourideh, V. Venkateswaran, and A. Zetlin-Jones (2018): "Market-making with Search and Information Frictions," Tech. rep., National Bureau of Economic Research.

Milgrom, P. and N. Stokey (1982): "Information Trade and Common Knowledge," *Journal of Economic Theory*, 26:1, 17–27.

Milgrom, P. R. (1979): "A Convergence Theorem for Competetive Bidding with Differential Information," *Econometrica*, 47, 679–688.

Milgrom, P. R. and R. J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

Ostrovsky, M. (2012): "Information Aggregation in Dynamic Markets with Strategic Traders," *Econometrica*, 80, 2595–2649.

Palfrey, T. R. (1985): "Uncertainty resolution, private information aggregation and the Cournot competitive limit," *The review of economic studies*, 52, 69–83.

Pesendorfer, W. and J. M. Swinkels (1997): "The Loser's Curse and Information Aggregation in Common Value Auctions," *Econometrica*, 65(6), 1247–1281.

Rayo, L. and I. Segal (2010): "Optimal Information Disclosure," *The Journal of Political Economy*, 118, 949–987.

Rostek, M. and M. Weretka (2012): "Price inference in small markets," *Econometrica*, 80, 687–711.

Siga, L. and M. Mihm (2018): "Information Aggregation in Large Markets," .

Swinkels, J. M. (1999): "Education Signaling with Preemptive Offers," *Review of Economic Studies*, 66, 949–970.

Vives, X. (1988): "Aggregation of information in large Cournot markets," *Econometrica: Journal of the Econometric Society*, 851–876.

Wilson, R. (1977): "A Bidding Model of Perfect Competition," *The Review of Economic Studies*, 44, 511–518.

A Proofs for Sections 2 and 3

Proof of Property 1. For (i) and (ii), see the proof of Lemma 1 in Daley and Green (2016). Conditional on reaching the second trading period and the buyers' belief, π_i, the strategic setting for trading a given asset i is identical to theirs. Moreover, by their Lemma A.3, the bid price must earn zero expected profit. To demonstrate (iii), we will show that the bid price must be either v_L or c_H when $\pi_i = \bar{\pi}$ by ruling out all other bids.

Clearly, at $t = 2$, the reservation price of the low-type seller is c_L and the reservation price of the high-type seller is c_H. Hence, if the bid is strictly above c_H, both types will accept w.p.1 and the winning buyer earns negative expected profit. Next, suppose there is positive probability that the bid is strictly less than v_L. Then, for $\epsilon > 0$ small enough, a buyer could earn strictly positive expected profit by deviating and offering $v_L - \epsilon$. Finally, if the bid is strictly between v_L and c_H, the high type will reject, the low type will accept and the winning buyer makes negative profit. Thus, we have shown that the equilibrium bid price at $t = 2$ when $\pi_i = \bar{\pi}$ must be either v_L or c_H. ■

Proof of Property 2. Since $c_H > c_L$ and $F_H \geq F_L$, the continuation value of the low type seller from rejecting the bid v_L in the first period satisfies:

$$\begin{aligned} Q_L^i &= (1-\delta) \cdot c_L + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &< (1-\delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &\leq (1-\delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}. \end{aligned}$$

Therefore, in order to prove that $Q_H^i > Q_L^i$, it is sufficient to show that $\mathbb{E}_H\{F_H(\pi_i, \phi_i)\} \geq \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}$. Recall that F_H is increasing in π_i and independent of ϕ_i. Hence, the desired inequality is implied by proving that conditional on $\theta_i = H$, the random variable π_i (weakly) first-order stochastically dominates π_i conditional on $\theta_i = L$.

Note that the distribution of π_i in the second period is a function of the trading probabilities of the seller i and of the realization of news from sellers $j \neq i$, $z_i^j \in \{0, 1\}$. Fix the interim belief π_{σ_i}, and consider news $\mathbf{z}'_{-i}$ and $\mathbf{z}''_{-i}$ (which occur with positive probability) such that the posterior π_i satisfies $\pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i})$, i.e., $\mathbf{z}'_{-i}$ is "better news" for seller i than $\mathbf{z}''_{-i}$. But note that:

$$\frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}'_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}'_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}'_{-i})} = \pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i}) = \frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}''_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}''_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}''_{-i})},$$

which implies that $\frac{\rho_H(\mathbf{z}'_{-i})}{\rho_L(\mathbf{z}'_{-i})} \geq \frac{\rho_H(\mathbf{z}''_{-i})}{\rho_L(\mathbf{z}''_{-i})}$, i.e. the ratio of distributions $\frac{\rho_H(\cdot)}{\rho_L(\cdot)}$ satisfies the monotone likelihood ratio property. This in turn implies that $\rho_H(\cdot)$ first-order stochastically dominates

$\rho_H(\cdot)$, which establishes the result. ■

Proof of Property 3. We first show that the bid in the first period is v_L w.p.1. From Property 2, the strict ranking of seller continuation values implies that, in any equilibrium, if the high type is willing to accept an offer with positive probability then the low type must accept w.p.1. Thus, given Assumption 1, any bid at or above c_H would lead to negative expected profit. Any bid in (v_L, c_H) also leads to losses since it is only accepted by the low type. If the bid was strictly less than v_L, a buyer can make strictly positive profits by offering $v_L - \epsilon$, for $\epsilon > 0$ small enough. Thus, any deterministic offer strictly below v_L can be ruled out. The only deterministic bid possible is v_L, at this point there is no profitable deviation for the other buyer than offering v_L as well. The same arguments rule out any mixed strategy equilibrium that has a mass point anywhere other than v_L. Finally, mixing continuously over some interval of offers cannot be an equilibrium. We show this by contradiction. If one of the buyers mixes over some interval $[\underline{b}, \overline{b}]$ with $\overline{b} = v_L$ then the other buyer must be offering v_L with probability 1 because otherwise he would never want to offer v_L, which leads to zero profits w.p.1. If instead $\overline{b} < v_L$, the other buyer's best response can never have $\underline{b}$ (or anything below) as part of its support. This bid will lose with probability 1 and thus earn zero profits, while bidding $\frac{\overline{b}+v_L}{2}$ would lead to strictly positive profits.

Next, it is clear that the high type would reject bid v_L, since $v_L < c_H$. To see that the low type must accept with probability less than one, note that if in equilibrium the low type accepted w.p.1, then the posterior belief would assign probability 1 to the type being high in the next period. The offer in the second period (as argued in Property 1) would then be v_H but, given Assumption 2, the low type seller would then want to deviate and trade in period 2 at v_H rather than at v_L in period 1. ■

Proof of Property 4. The proof that all equilibria involve strictly positive probability of trade in the first period is in the text. We show here that all equilibria must be symmetric. In search of a contradiction, assume there exists an equilibrium in which $\sigma_A > \sigma_B \geq 0$ for some $A, B \in \{1, ..., N\}$. We establish the result by first showing that the beliefs for seller A are more favorable than for seller B, following all news realizations; then we show that good news about seller A are more likely to arrive than good news about seller B.

Let $\pi_{\sigma_i}^{Int}$ denote the interim belief when (low type) seller i trades w.p. σ_i (by Property 3 the high type does not trade in first period). Consider the posterior belief about seller $i \in \{A, B\}$ following some news $\mathbf{z}_{-i} = \left(z_i^j\right)_{j \neq i}$:

$$\pi_i\left(\mathbf{z}_{-i}\right) = \frac{\pi_{\sigma_i}^{Int} \cdot \rho_H^i\left(\mathbf{z}_{-i}\right)}{\pi_{\sigma_i}^{Int} \cdot \rho_H^i\left(\mathbf{z}_{-i}\right) + \left(1 - \pi_{\sigma_i}^{Int}\right) \cdot \rho_L^i\left(\mathbf{z}_{-i}\right)}$$

where we can express $\rho^i_\theta(\mathbf{z}_{-i})$ as:

$$\rho^i_\theta(\mathbf{z}_{-i}) = \sum_{s\in\{l,h\}} \mathbb{P}(S=s|\theta_i=\theta)\cdot\mathbb{P}\left((z^j_i)_{j\neq i,i'}|S=s\right)\cdot\mathbb{P}(z^{i'}_i|S=s)$$

for $i,i'\in\{A,B\}$ and $i'\neq i$. Note that $\rho^i_\theta(\mathbf{z}_{-i})$ depends on $\sigma_{i'}$ only through the term $\mathbb{P}\left(z^{i'}_i|S\right)$. We now show that $\sigma_A>\sigma_B$ implies that:

$$\frac{1-\pi^{Int}_{\sigma_A}}{\pi^{Int}_{\sigma_A}}\cdot\frac{\rho^A_L(\mathbf{z}_{-i})}{\rho^A_H(\mathbf{z}_{-i})} < \frac{1-\pi^{Int}_{\sigma_B}}{\pi^{Int}_{\sigma_B}}\cdot\frac{\rho^B_L(\mathbf{z}_{-i})}{\rho^B_H(\mathbf{z}_{-i})}, \tag{18}$$

which will establish that $\pi_A(\mathbf{z}_{-i})>\pi_B(\mathbf{z}_{-i})$ for all news $\mathbf{z}_{-i}$. There are two cases to consider, depending on whether $z^{i'}_i=0$ or $z^{i'}_i=1$.

If $z^{i'}_i=1$, then $\mathbb{P}\left(z^{i'}_i=1|S=s\right)=\sigma_{i'}\cdot\mathbb{P}(\theta_{i'}=L|S=s)$ and the likelihood ratio $\frac{1-\pi^{Int}_{\sigma_i}}{\pi^{Int}_{\sigma_i}}\cdot\frac{\rho^i_L(\mathbf{z}_{-i})}{\rho^i_H(\mathbf{z}_{-i})}$ decreases in σ_i but is independent of $\sigma_{i'}$. Intuitively, if seller i' traded, her type is revealed to be low, and the intensity with which she trades is irrelevant for updating. But then inequality (18) follows because $\pi^{Int}_{\sigma_i}$ is increasing in σ_i.

If $z^{i'}_i=0$, then $\mathbb{P}\left(z^{i'}_i=0|S=s\right)=1-\sigma_{i'}\cdot\mathbb{P}(\theta_{i'}=L|S=s)$, and now the likelihood ratio $\frac{1-\pi^{Int}_{\sigma_i}}{\pi^{Int}_{\sigma_i}}\cdot\frac{\rho^i_L(\mathbf{z}_{-i})}{\rho^i_H(\mathbf{z}_{-i})}$ decreases in both σ_i and $\sigma_{i'}$. However, given that both i and i' did not trade (both are good news for i), inequality (18) follows because the assets i and i' are imperfectly correlated and $\frac{1-\pi^{Int}_{\sigma_i}}{\pi^{Int}_{\sigma_i}}\cdot\frac{\rho^i_L(\mathbf{z}_{-i})}{\rho^i_H(\mathbf{z}_{-i})}$ is more sensitive to trading probability σ_i than to $\sigma_{i'}$.

Now, note that $\sigma_A>\sigma_B$ also implies that the probability that seller B trades and releases bad news about seller A is lower than the probability that seller A trades and releases bad news about seller B. Since the posteriors following good news are higher than following bad news, this establishes the result.

Finally, the symmetry in ϕ_i follows from monotonicity of Q^i_L in ϕ_i whenever buyer mixing is part of an equilibrium. ∎

Proof of Proposition 1. To prove existence of an equilibrium, it suffices to show there exists a $(\sigma,\phi)\in[0,1]^2$ such that equation (8) holds, i.e., $Q_L(\sigma,\phi)=v_L$ where the second argument states that all other sellers also trade with intensity σ. Note that by varying σ from 0 to 1, Q_L ranges from $[(1-\delta)c_L+\delta v_L,(1-\delta)c_L+\delta v_H]$. By continuity of Q_L and Assumption 2, the intermediate value theorem gives the result.

Let us denote by $\pi_i(\mathrm{z}_i;\sigma)$ the posterior belief following news realization $\mathbf{z}_{-i}$ in an equilibrium with trading probability σ. Consider the following two candidate equilibria. The first candidate is an equilibrium in which the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(0);\sigma)=\bar{\pi}$, and the second candidate equilibrium is when the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(N);\sigma)=\bar{\pi}$. Although there can be other equilibria as well, we do not focus on them. We will now show that these two equilibria coexist when λ and δ are large enough.

1. $\underline{\pi_i(\mathbf{z}(0);\sigma) = \bar{\pi}}$. Note that there is at most one such equilibrium since the trading intensity σ in this category is fully pinned down by the requirement that $\pi_i(\mathbf{z}(0);\sigma) = \bar{\pi}$. Let x be the value of σ such that $\pi_i(\mathbf{z}(0);x) = \bar{\pi}$. As ϕ varies from 0 and 1, $Q_L(x,\phi)$ varies continuously from $(1-\delta)c_L + \delta v_L$ to $(1-\delta)c_L + \delta(\rho_L^i(\mathbf{z}(0))c_H + (1-\rho_L^i(\mathbf{z}(0)))v_L)$ where $\rho_L^i(\mathbf{z}(0)) > 0$. Hence, there exists a $\bar{\delta}_\lambda < 1$, such that $Q_L(x,1) = v_L$. Clearly, this equilibrium exists if $\delta > \bar{\delta}_\lambda$. Moreover, it is straightforward to show that $\inf_\lambda \rho_L^i(\mathbf{z}(0)) > 0$. Hence, this equilibrium exists if δ is larger than $\bar{\delta} \equiv \sup_{\lambda\in(1-\pi,1)} \bar{\delta}_\lambda < 1$.

2. $\underline{\pi_i(\mathbf{z}(N);\sigma) = \bar{\pi}}$. Note that there is at most one such equilibrium since the trading intensity σ is fully pinned down by the requirement that $\pi_i(\mathbf{z}(N);\sigma) = \bar{\pi}$. Let y be the value of σ such that $\pi_i(\mathbf{z}(N);y) = \bar{\pi}$. As ϕ varies from 0 to 1, Q_L varies continuously from

$$(1-\delta)c_L + \delta\left(\rho_L^i(\mathbf{z}(N))v_L + \sum_{\mathbf{z}_{-i}\neq\mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i})V(\pi_i(\mathbf{z}_{-i};y))\right)$$

to

$$(1-\delta)c_L + \delta\left(\rho_L^i(\mathbf{z}(N))c_H + \sum_{\mathbf{z}_{-i}\neq\mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i})V(\pi_i(\mathbf{z}_{-i};y))\right).$$

Moreover, it is straightforward to show that $\lim_{\lambda\to 1}\rho_L^i(\mathbf{z}(N)) = 1$. Therefore, it follows that the range of Q_L converges to the interval $((1-\delta)c_L + \delta v_L, (1-\delta)c_L + \delta c_H]$ as λ goes to 1. By Assumption 2, v_L is in this interval. This establishes the existence of the threshold $\bar{\lambda}_\delta$ such that this equilibrium exists whenever $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$.

Thus, we conclude that multiple equilibria exist when $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$. ■

Proof of Lemma 1. Uniqueness of equilibrium follows from the fact that $Q_L^{i,fict} = (1-\delta)c_L + \delta v_H > v_L$ when $\sigma_i = 1$, and because $Q_L^{i,fict}$ is monotonically increasing in σ_i, and in ϕ_i when buyer mixing is part of an equilibrium. Hence, the unique equilibrium must feature no trade if $Q_L^{i,fict}(0,0) \geq v_L$. Finally, it is straightforward to check that $Q_L^{i,fict}(0,0) \geq v_L$ holds if and only if $\lambda \geq \bar{\lambda}$ and $\delta \geq \bar{\delta}_\lambda$. ■

For the proof of Theorem 1, it will be useful to reference the following lemma, which is straightforward to verify so the proof is omitted. Let $\pi_i(s;\sigma)$ denote the buyers' posterior belief about seller i following a rejection, conditional on observing that the state is s. Then, for $s \in \{l,h\}$, we have:

$$\pi_i(s;\sigma) = \frac{\pi_\sigma \cdot \mathbb{P}(S=s|\theta_i=H)}{\pi_\sigma \cdot \mathbb{P}(S=s|\theta_i=H) + (1-\pi_\sigma)\cdot\mathbb{P}(S=s|\theta_i=L)},$$

where as before π_σ is the interim belief.

Lemma A.1 *Given a sequence $\{\sigma_N\}_{N=1}^{\infty}$ of trading probabilities corresponding to a sequence of equilibria along which information aggregates, we also have convergence of posteriors: $\pi_i(\mathbf{z}_{-i};\sigma_N) \to^p \pi_i(S;\sigma_N)$ as $N \to \infty$.*

Proof of Theorem 1. Part (i). Suppose to the contrary that $(\star)$ holds with strict inequality, but that information aggregation obtains. Recall that in equilibrium, for any N, we must have:

$$v_L = Q_L^i(\sigma_N, \phi_i) = (1-\delta)\, c_L + \delta \sum_{\mathbf{z}_{-i}} \rho_L^i\left(\mathbf{z}_{-i}\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right),$$

where

$$\begin{aligned}\sum_{\mathbf{z}_{-i}} \rho_L^i\left(\mathbf{z}_{-i}\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right) &= \sum_{s=l,h} \mathbb{P}\left(S=s|\theta_i=L\right) \sum_{\mathbf{z}_{-i}} \mathbb{P}\left(\mathbf{z}_{-i}|S=s\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right) \\ &> \lambda \cdot v_L + (1-\lambda) \cdot \sum_{\mathbf{z}_{-i}} \mathbb{P}\left(\mathbf{z}_{-i}|S=h\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right),\end{aligned}$$

because in equilibrium we must have $F_L\left(\pi_i\left(\mathbf{z}(0);\sigma_N\right), \phi_i\right) > v_L$.

Since by Lemma A.1, $\pi_i\left(\mathbf{z}_{-i};\sigma_N\right) \to^p \pi_i\left(h;\sigma_N\right)$ when the state is h, and because $(\star)$ holding strictly implies that $\pi_i\left(h;\sigma_N\right) > \pi_i\left(h;0\right) > \bar{\pi}$, we have that for a given $\epsilon > 0$, if N is large enough, then:

$$\sum_{\mathbf{z}_{-i}} \mathbb{P}\left(\mathbf{z}_{-i}|S=h\right) \cdot F_L\left(\pi_i\left(\mathbf{z}_{-i};\sigma_N\right), \phi_i\right) > V\left(\pi_i\left(h;\sigma_N\right)\right) - \epsilon.$$

Therefore, we conclude that for sufficiently large N:

$$\begin{aligned}v_L = Q_L^i\left(\sigma_N, \phi_i\right) &> (1-\delta)\, c_L + \delta \cdot \left(\lambda \cdot v_L + (1-\lambda) \cdot V\left(\pi_i\left(h;\sigma_N\right)\right)\right) - \delta \cdot (1-\lambda) \cdot \epsilon \\ &> (1-\delta)\, c_L + \delta \cdot \left(\lambda \cdot v_L + (1-\lambda) \cdot V\left(\pi_i\left(h;0\right)\right)\right) - \delta \cdot (1-\lambda) \cdot \epsilon.\end{aligned}$$

Since ϵ was arbitrary, it must be that:

$$v_L \geq (1-\delta)\, c_L + \delta \cdot \left(\lambda \cdot v_L + (1-\lambda) \cdot V\left(\pi_i\left(h;0\right)\right)\right),$$

which violates $(\star)$ holding with strict inequality, a contradiction.
Part (ii). If $(\star)$ does not hold, then in the fictitious economy, the unique equilibrium trading probability in the first period must satisfy $\sigma^* > 0$. We next construct an equilibrium sequence $\{\sigma_N\}$ of the actual economy such that the sequence is uniformly bounded away from zero, which

then implies that information aggregates along this sequence. First, consider a sequence $\{\widehat{\sigma}_N\}$, not necessarily an equilibrium one, such that $\widehat{\sigma}_N = \widehat{\sigma} \in (0, \sigma^*)$, i.e., this is a sequence of constant trading probabilities that are positive but strictly below σ^*. Along such a sequence, information clearly aggregates and, by Lemma A.1, $\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N) \to^p \pi_i(S, \widehat{\sigma}_N)$. Therefore, combined with the fact that $\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N) = \pi_i(\mathbf{z}_{-i}, \widehat{\sigma}) < \pi_i(\mathbf{z}_{-i}, \sigma^*)$, there exists an N^* such that for $N > N^*$, we have:

$$\mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N), \phi_i)\} < \mathbb{E}_L^{fict}\{F_L(\pi_i(S, \sigma^*))\} = \frac{v_L - (1-\delta)\cdot c_L}{\delta},$$

where the last equality holds since $\sigma^* > 0$ implies that, in the fictitious economy, the low type must be indifferent to trading at $t = 1$ and delaying trade to $t = 2$. The correspondence $\mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i}, \sigma), \cdot)\}$ is upper hemicontinuous in σ for each N, and has a maximal value of v_H that is strictly greater than $\mathbb{E}_L^{fict}\{F_L(\pi_i(S, \sigma^*))\}$. Hence, for each $N > N^*$, we can find a σ_N such that $\sigma_N \geq \widehat{\sigma}_N > 0$ and $\mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i}, \sigma_N), \phi_i)\} = \frac{v_L-(1-\delta)\cdot c_L}{\delta}$. This delivers the desired equilibrium sequence $\{\sigma_N\}$ along which information aggregates. ■

Proof of Theorem 2. We establish the conditions for the coexistence of aggregating and non-aggregating equilibria. To do so, we first show that if $\lambda > \bar{\lambda}$, there exists a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist if $\delta > \delta_2(\lambda)$. Second, we show that for λ large enough $\delta_2(\lambda) < \bar{\delta}_\lambda$. Therefore, both non-aggregating and aggregating equilibria exist if $\delta \in (\delta_2(\lambda), \bar{\delta}_\lambda)$, since $(\star)$ is violated (see Theorem 1).

Consider a candidate sequence of equilibria with trading probabilities $\{\sigma_N\}$, such that $\sigma_N = \kappa_N \cdot N^{-1}$ and:

$$\pi_i\left(\mathbf{z}(0); \kappa_N \cdot N^{-1}\right) = \bar{\pi} \tag{19}$$

Solving (19) for κ_N and taking the limit as $N \to \infty$ gives $\kappa_N \to \kappa$ where

$$\kappa \equiv \frac{1}{\lambda - \frac{(1-\lambda)(1-\pi)}{\pi}} \cdot \log\left(\frac{\lambda - \left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}\right) \cdot \frac{(1-\lambda)(1-\pi)}{\pi}}{\left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}\right) \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) - (1-\lambda)}\right) \in (0, \infty). \tag{20}$$

Seller i expects to receive an offer of v_L in all events other than $\mathbf{z}(0)$ and an expected offer $\phi_i c_H + (1-\phi_i)v_L$ for some $\phi_i \in [0,1]$ in the event $\mathbf{z}(0)$. Therefore, the sequence of trading probabilities defined above constitutes an equilibrium if δ is sufficiently high and the probability of the event $\mathbf{z}(0)$ conditional on the seller's type being low is bounded away from zero. To establish the latter, note that:

$$\begin{aligned}\mathbb{P}(\mathbf{z}(0)|\theta_i = L) &= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot (1 - \sigma_N \cdot \mathbb{P}(\theta_i = L|S = s))^N \\ &= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot \left(1 - \kappa_N \cdot N^{-1} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N \\ &\to \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L|S=s)} > 0,\end{aligned}$$

where the last limit as $N \to \infty$ follows from Lemma A.2. In these equilibria, information fails to aggregate because as a result $\mathbb{P}(\mathbf{z}(0)|S = s)$ is bounded away from zero in both states of nature (see Lemma A.4). Thus, for each $\lambda > \bar{\lambda}$, we have established the existence of a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist whenever $\delta > \delta_2(\lambda)$. Finally, from (20) we have that:

$$\lim_{\lambda \to 1} \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L|S=s)} = \frac{1 - \bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1 - \pi} \in (0, 1),$$

and hence $\lim_{\lambda \to 1} \delta_2(\lambda) < 1$. Letting $\widehat{\delta} = \lim_{\lambda \to 1} \delta_2(\lambda)$ and noting that $\lim_{\lambda \to 1} \bar{\delta}_\lambda = 1$ implies the result.

Next, we establish that when $\lambda < \bar{\lambda}$ or δ is sufficiently small, then only aggregating equilibria exist. First, suppose that $\lambda < \bar{\lambda}$ and assume to the contrary that information aggregation fails along some sequence of equilibria, and pick a subsequence of equilibria with $\sigma_N \to 0$ as N goes to ∞ (See Lemma A.3 for the existence of such a subsequence). But note that for each N, we have $\pi_i(\mathbf{z}_{-i}, \sigma_N) \le \pi_i(h, \sigma_N)$, i.e., the posterior beliefs must be weakly lower than if the state were revealed to be high. Since $\pi_i(h, \sigma_N)$ is continuous in σ_N, and since $\lambda < \bar{\lambda}$ implies that $\pi_i(h, 0) < \bar{\pi}$, it follows that for N large enough all posterior beliefs are strictly below $\bar{\pi}$. But then for N large, $Q_L^i < v_L$ and therefore $\sigma_N = 1$, contradicting Property 3.

Second, consider $\hat{\delta}$ defined by $v_L = (1 - \hat{\delta})c_L + \hat{\delta}V(\pi)$, and assume that $\delta < \hat{\delta}$ (Note that Assumption 2 can still be satisfied since $V(\pi) < c_H$). Suppose to the contrary that information aggregation fails along a sequence of equilibria, and again pick a subsequence of equilibria with $\sigma_N \to 0$ as N goes to ∞. By continuity, we must also have that $\pi_{\sigma_N}^{Int} \to \pi$ along this subsequence. But, note that for each N along this subsequence, it must be that:

$$\begin{aligned}v_L = Q_L^i(\sigma_N, \phi_i) &= (1 - \delta)c_L + \delta \mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i}, \sigma_N), \phi_i)\} \\ &\le (1 - \delta)c_L + \delta \mathbb{E}_L\{V(\pi_i(\mathbf{z}_{-i}, \sigma_N))\} \\ &\le (1 - \delta)c_L + \delta V(\pi_{\sigma_N}^{Int}).\end{aligned}$$

where the first inequality follows immediately from (6) and the second from the fact that V is linear function and $\pi_i(\mathbf{z}_{-i}, \sigma_N)$ is a supermartingale conditional on $\theta_i = L$. Since $\delta < \hat{\delta}$ and $V(\pi^{Int}_{\sigma_N}) \to V(\pi)$, the last expression is lower than v_L for N large enough, a contradiction. $\blacksquare$

Proof of Proposition 2. In Progress... Consider a sequence of equilibria along which information aggregates and let $\{\sigma_N\}$ denote the corresponding sequence of trading probabilities. By Theorem 1, it must be that either ($\star$) holds with equality or it is violated, i.e. $Q^{i,fict}_L = v_L$ and $\sigma^{fict} \geq 0$. If $\{\sigma_N\}$ (or any subsequence of it) were strictly above σ^{fict} and bounded away from it, then it is straightforward to show that, for N large enough, due to the convergence of posteriors (see Lemma A.1) we have $Q^i_{L,N} > Q^{i,fict}_L = v_L$ and therefore $\sigma_N = 0$, which contradicts Property 4. On the other hand, if σ_N were strictly below σ^{fict} and bounded away from it, then again, for N large enough, we would have $Q^i_{L,N} < Q^{i,fict}_L = v_L$ and therefore $\sigma_N = 1$, which contradicts Property 3. Hence, σ_N converges to σ^{fict}.

It is clear that the distribution of Y_N in state S becomes degenerate at $\sigma^{fict} Prob\,(\theta^i = L|S = s)$. Finally, in any equilibrium, we have $Q^i_{L,N} = v_L = Q^{i,fict}_L$ and the convergence of posteriors also implies that $Q^i_{H,N} \to Q^{i,fict}_H$, which establishes the welfare result. $\blacksquare$

Proof of Proposition 3. The bounds on the equilibrium trading probabilities follow from Lemma A.3. The convergence of distributions follows by the Poisson Limit Theorem. The proof of the welfare result to be completed. $\blacksquare$

Lemma A.2 *Let $\{\alpha_x\}$ be any non-negative sequence of real numbers such that $\alpha_x \to \alpha$ as $x \to \infty$ where $\alpha \in (0, 1)$. Then $\left(\frac{x-\alpha_x}{x}\right)^x \to e^{-\alpha}$ as $x \to \infty$.*

Proof. Assume that for any $\gamma \in (0, 1)$, $\left(\frac{x-\gamma}{x}\right)^x \to e^{-\gamma}$ as $x \to \infty$. Then, given $\epsilon > 0$ so that $\epsilon < \alpha < 1 - \epsilon$, if x is large enough then $|\alpha_x - \alpha| < \epsilon$, $\left(\frac{x-\alpha-\epsilon}{x}\right)^x \geq e^{-\alpha-\epsilon} - \epsilon$, and $\left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon$. This in turn implies that:

$$e^{-\alpha-\epsilon} - \epsilon \leq \left(\frac{x - \alpha - \epsilon}{x}\right)^x \leq \left(\frac{x - \alpha_x}{x}\right)^x \leq \left(\frac{x - \alpha + \epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon.$$

Since ϵ is arbitrary, we conclude that $\left(\frac{x-\alpha_x}{x}\right)^x \to e^{-\alpha}$ as $x \to \infty$. Next, we prove the supposition that for any $\gamma \in (0, 1)$, $\left(\frac{x-\gamma}{x}\right)^x \to e^{-\gamma}$ as $x \to \infty$. Note that $\left(\frac{x-\gamma}{x}\right)^x = e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)}$ and by L'Hospital's rule:

$$\lim_{x\to\infty} x \cdot \log\left(\frac{x - \gamma}{x}\right) = \lim_{x\to\infty} \frac{\log\left(\frac{x-\gamma}{x}\right)}{x^{-1}} = -\lim_{x\to\infty} \frac{\gamma \cdot x}{x - \gamma} = -\gamma.$$

By continuity, $\lim_{x\to\infty} e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)} = e^{-\gamma}$. $\blacksquare$

Lemma A.3 *Suppose that there is a sequence of equilibria* $\{\sigma_N\}$ *along which information aggregation fails. Then there exist a subsequence of equilibria with trading probabilities* $\{\sigma_{N_m}\}$ *such that for some* $0 < \underline{\kappa} < \bar{\kappa} < \infty$*, we have* $\underline{\kappa} < \sigma_{N_m} N_m < \bar{\kappa}$ *for all* m*.*

Proof. Suppose for contradiction that for all subsequences with trading probabilities $\{\sigma_{N_m}\}$ we have $\lim_{m\to\infty} \sigma_{N_m} N_m = \infty$. Let X_i denote the indicator that takes value of 1 if seller i has traded in the first period. Define $Y_{N_m} = N_m^{-1} \cdot \sum_{i=1}^{N_m} X_i$ be the fraction of sellers who have traded in the first period, and note that conditional on the state being s, Y_{N_m} has a mean p_{s,N_m} and variance $N_m^{-1} \cdot p_{s,N_m} \cdot (1 - p_{s,N_m})$, where recall that $p_{s,N_m} = \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = s)$. Since $p_{l,N_m} > p_{h,N_m}$,

$$\mathbb{P}\left(Y_{N_m} \geq \frac{p_{h,N_m} + p_{l,N_m}}{2}|S = h\right) = \mathbb{P}\left(Y_{N_m} - p_{h,N_m} \geq \frac{p_{l,N_m} - p_{h,N_m}}{2}|S = h\right)$$
$$\leq \mathbb{P}\left((Y_{N_m} - p_{h,N_m})^2 \geq \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 |S = h\right)$$

And by Markov's inequality:

$$\mathbb{P}\left((Y_{N_m} - p_{h,N_m})^2 \geq \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 |S = h\right) \leq \frac{\mathbb{E}\left\{(Y_{N_m} - p_{h,N_m})^2 |S = h\right\}}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2}$$
$$= \frac{N_m^{-1} \cdot p_{h,N_m} \cdot (1 - p_{h,N_m})}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2}$$
$$= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = h) - \sigma_{N_m}^2 \cdot \mathbb{P}(\theta_i = L|S = h)^2}{N_m \cdot \sigma_{N_m}^2 \cdot (\mathbb{P}(\theta_i = L|S = l) - \mathbb{P}(\theta_i = L|S = h))^2}$$

which by our assumption tends to 0 as $m \to \infty$. By a similar reasoning, we have that:

$$\begin{aligned}
\mathbb{P}\left(Y_{N_m} < \frac{p_{h,N_m} + p_{l,N_m}}{2} | S = l\right) &= \mathbb{P}\left(p_{l,N_m} - Y_{N_m} > \frac{p_{l,N_m} - p_{h,N_m}}{2} | S = l\right) \\
&\leq \mathbb{P}\left(\left(p_{l,N_m} - Y_{N_m}\right)^2 > \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 | S = l\right) \\
&\leq \frac{\mathbb{E}\left\{\left(Y_{N_m} - p_{l,N_m}\right)^2 | S = l\right\}}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\
&= \frac{N_m^{-1} \cdot p_{l,N_m} \cdot (1 - p_{l,N_m})}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\
&= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = l\right) - \sigma_{N_m}^2 \cdot \mathbb{P}\left(\theta_i = L | S = l\right)^2}{N_m \cdot \sigma_{N_m}^2 \cdot \left(\mathbb{P}\left(\theta_i = L | S = l\right) - \mathbb{P}\left(\theta_i = L | S = h\right)\right)^2}
\end{aligned}$$

which again tends to 0 as $m \to \infty$. Combining these two observations, we conclude that information about the state must aggregate along all subsequences, a contradiction.

Next, suppose for contradiction that for all subsequences with trading probabilities $\{\sigma_{N_m}\}$ we have that $\lim_{m \to \infty} \sigma_{N_m} N_m = 0$. Then, given any $\epsilon > 0$ and m large enough, we have:

$$\left(1 - \sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = s\right)\right)^{N_m} = \left(\frac{N_m - \sigma_{N_m} \cdot N_m \cdot \mathbb{P}\left(\theta_i = L | S = s\right)}{N_m}\right)^{N_m} \geq \left(\frac{N_m - \epsilon}{N_m}\right)^{N_m}$$

for $s \in \{l, h\}$, where the last expression converges to $e^{-\epsilon}$ by Lemma A.2. Since ϵ is arbitrary, $\left(1 - \sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = s\right)\right)^{N_m}$ goes to 1 as $m \to \infty$. Hence, we have that for $\theta \in \{L, H\}$:

$$\mathbb{P}\left(Y_{N_m} = 0 | \theta_i = \theta\right) = \sum_{s=l,h} \mathbb{P}\left(S = s | \theta_i = \theta\right) \cdot \left(1 - \sigma_{N_m} \cdot \mathbb{P}\left(\theta_i = L | S = s\right)\right)^{N_m} \to 1.$$

Now, consider the posterior belief about the seller conditional on event that no seller has traded. For any m, since the low type must expect offers above v_L with positive probability and since $\mathbf{z}(0)$ is the best possible news, it must be that:

$$\begin{aligned}
&\pi_i(\mathbf{z}(0), \sigma_{N_m}) \geq \bar{\pi} \\
&\qquad \iff \frac{\pi_{\sigma_{N_m}} \cdot \mathbb{P}(Y_{N_m} = 0 | \theta_i = H)}{\pi_{\sigma_{N_m}} \cdot \mathbb{P}(Y_{N_m} = 0 | \theta_i = H) + (1 - \pi_{\sigma_{N_m}}) \cdot \mathbb{P}(Y_{N_m} = 0 | \theta_i = L)} \geq \bar{\pi}.
\end{aligned}$$

But note that, since $\sigma_{N_m} \to 0$ and $\pi_{\sigma_{N_m}}$ is continuous, the left-hand side converges to $\pi < \bar{\pi}$, a contradiction. ■

Lemma A.4 *Consider a sequence of equilibria with trading probabilities $\{\sigma_N\}$ such that $\sigma_N N <$*

$\bar{\kappa}$ for some $\bar{\kappa} < \infty$. Then $\mathbb{P}(Y_N = 0|S = s)$ is bounded away from zero, uniformly over N, for $s \in \{l, h\}$.

Proof. We have that $\mathbb{P}(Y_N = 0|S = s) = (1 - p_{s,N})^N$ for $s \in \{l, h\}$. By assumption, $p_{s,N} \leq N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)$. Therefore,

$$\mathbb{P}(Y_N = 0|S = s) \geq \left(1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N$$

and by Lemma A.2, $\lim_{N\to\infty} \left(1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N = e^{-\bar{\kappa}\cdot\mathbb{P}(\theta_i = L|S=s)} > 0$. ■

B Proofs for Section 4

To be completed.

C Infinite Horizon Model

In this section, we extend our aggregation results to a setting with an infinite number of trading opportunities $t \in \{1, 2, ...\}$. Intuitively, one might expect that with more trading periods there are more opportunities to learn from trading behavior and hence more information will be revealed. However, there is a countervailing force; there are more opportunities for (strategic) sellers to signal through delay. It turns out that two factors essentially cancel each other out.

Besides allowing for an infinite number of trading opportunities, the model and the information structure is identical to the one presented in Section 2. The only additional notation we will require is the public history at (the end of) date t, which we denote by $\mathbf{z}^t = \{\mathbf{z}_1, ..., \mathbf{z}_t\}$, consists of the history of all the trades that have taken place at dates prior to and including t. Note that $\mathbf{z}^t$ also corresponds to buyers' information set prior to making offers in date $t + 1$.

Characterizing the set of all possible equilibria in the infinite horizon model is more difficult because the space of relevant histories is a complex object. In principle, the path of play can depend on sellers' beliefs about the quality of other sellers' assets, the distribution of buyers' beliefs about the quality of each seller's asset, the buyers' and the sellers' beliefs about the aggregate state, as well as the number of assets remaining on the market. Nevertheless, we are able to obtain sharp predictions regarding the information aggregation properties of the set of equilibria.

In order to illustrate these findings, we must generalize our notion of information aggregation. Let $\pi_{t,N}^{State}$ denote the buyers' posterior belief that the state is high, conditional on having observed the trading history, $\mathbf{z}^t$, in an economy with $N + 1$ sellers.

Definition 5 *There is* ***information aggregation at date t*** *along a given sequence of equilibria if* $\pi^{State}_{t,N} \rightarrow^p 1_{\{S=h\}}$ *as* $N \rightarrow \infty$.

We say that *information aggregates* along a given sequence if there exists a $t < \infty$ such that information aggregates at date t. Otherwise, we say that *information aggregation fails.*

The following theorem shows that, with an infinite trading horizon, $(\star)$ is indeed necessary and sufficient to rule out aggregating equilibria.

Theorem 3 *Consider the infinite horizon model.*

(i) If $(\star)$ *holds with strict inequality, then information aggregation fails along any sequence of equilibria.*

(ii) If $(\star)$ *does not hold, then there exists a sequence of equilibria along which information aggregates.*

(iii) There exists a $\widehat{\delta} < 1$ *such that whenever* $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ *and* λ *is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails.*

The proof hinges on arguments similar to those used in the two-period economy. For (i), we show that the earliest date in which information about the state is supposed to aggregate is similar to the first period in a two-period economy. That is, suppose that information aggregates at some date τ but not before. Because $(\star)$ holds, the option value of waiting for the state to be revealed is sufficiently high to make sellers strictly prefer to delay trade at date τ. But if sellers do not trade in date τ, then no information is revealed, which means that τ cannot possibly be the earliest date of aggregation.

In order to establish (ii) and (iii), we construct a class of equilibria that essentially share the information aggregation properties of the two-period economy. A feature of this class is that once the belief about the seller weakly exceeds $\bar{\pi}$, all future bids are pooling. When $(\star)$ does not hold (i.e., $\delta < \bar{\delta}_\lambda$ or $\lambda < \bar{\lambda}$), we show that such equilibria exist and that there is an equilibrium sequence within this class along which information aggregates. Then, following arguments similar to those for the proof of Theorem 2, we show that under the conditions stated in (iii), there also exists another sequence of equilibria (still within the class) in which aggregation fails.

C.1 Proof of Theorem 3

We establish parts (i)-(iii) of Theorem 3 separately.

Proof of Theorem 3, part (i). We proceed by contradiction and suppose to the contrary that there is some finite date t at which information aggregates. In particular, suppose that information has not aggregated before t, but it aggregates at t. Consider seller i who trades with probability in $(0,1)$ at t. We know that the number of such sellers must grow to ∞ with N, since otherwise there would be insufficient information learned at t. Without loss of generality assume that all sellers trade with probability in $(0,1)$ at t. By the skimming property, the bid for this seller's asset must be v_L, which the high type rejects whereas the low type accepts with some probability $\sigma_{i,N} \in (0,1)$.

Let $Q^{i,N}_{L,t}$ denote the low type seller i's continuation value from rejecting a bid v_L at time t. Define

$$\bar{Q}^N_t \equiv (1-\delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot v_L + (1-\lambda_{L,t}) \cdot V(\pi_i(h;0))\right), \tag{21}$$

where (i) $\lambda_{L,t} = \mathbb{P}_t(S = l | \theta_i = L)$ is the posterior belief that the state is l conditional on trading history up to period t and the seller's type being L, and (ii) $\pi_i(h;0)$ is the posterior belief about the seller i conditional on the state being h. In Lemma C.1, we show that:

$$\lim_{N\to\infty} \mathbb{P}_t\left(Q^{i,N}_{L,t} \geq \bar{Q}^N_t\right) = 1, \tag{22}$$

i.e., $\bar{Q}^N_t$ provides a lower bound on the low type's continuation value. Next, we use this result to show that with probability bounded away from zero in both states of nature, if N is large enough, then $Q^{i,N}_{L,t} > v_L$. This immediately implies that the low types strictly prefer to delay trade at t, contradicting aggregation and thus establishing our result.

Since $(\star)$ holds, $\bar{Q}^N_1 > v_L$. Thus, information aggregation must fail in the first period. In Lemma C.2, we show that failure of information aggregation at t implies that the probability of the event that no seller trades in that period must be bounded away from zero, uniformly over N, in both states of nature. Because this event is 'good' news about the state, then following it in the first period, we have $\lambda_{L,2} < \lambda_{L,1}$ and, thus, $\bar{Q}^N_2 > v_L$. But then again information aggregation must fail in the second period and, therefore, the probability that no seller trades in the second period must remain bounded away from zero in both states of nature. Repeating this argument until period t, we can construct a history that occurs with probability bounded away from zero in both states of nature, in which $\bar{Q}^N_t > v_L$, as was stated above. ■

Proof of Theorem 3, part (ii). The proof is by construction. Consider a candidate equilibrium in which for any period t, the following properties hold:

(i) If $\pi_{i,t} < \bar{\pi}$, then the bid is v_L, which the low type accepts w.p. $\sigma_t \in [0,1)$ whereas the high type rejects w.p.1.

(ii) If $\pi_{i,t} > \bar{\pi}$, then the bid is $V(\pi_{i,t})$ and both types accept it w.p.1.

(iii) If $\pi_{i,t} = \bar{\pi}$, then the bid is $V(\pi_{i,t})$ w.p. ϕ_t (and both types accept it w.p.1) and is v_L w.p. $1 - \phi_t$ (and both types reject it).

The only off-equilibrium path event in a candidate satisfying (i)-(iii) is a rejection when $\pi_{i,t} > \bar{\pi}$, in which case the interim belief as given by Bayes rule is not well defined. For such cases, we specify $\pi_{\sigma_i,t} = \pi_{i,t}$ (i.e., unexpected rejections are attributed to random trembles).[8]

We will now verify that an equilibrium satisfying (i)-(iii) exists (with off-path beliefs as specified immediately above). To do so, consider any history and let N_t ($\mathcal{N}_t$) denote the number (set) of sellers who have not yet traded at the beginning of period t. Notice that the seller's value function under the proposed equilibrium is the same as in (5) and (6), where (π_i, ϕ_i) is replaced by $(\pi_{i,t}, \phi_t)$. By symmetry of the candidate, $\pi_{i,t} = \pi_{j,t} = \pi_t$ for all $j \neq i \in \mathcal{N}_t$. We now show that there exists (σ_t, ϕ_{t+1}) such that profitable deviations do not exist:

(i) Suppose that $\pi_t < \bar{\pi}$. Since continuation values in period $t+1$ are the same as in the two-period model, we know by Proposition 1, that for any N_t, there exists at least one (σ_t, ϕ_{t+1}) pair such that the low types' continuation value is exactly v_L. Hence, a low-type seller is willing to mix. Clearly, a high-type seller strictly prefers to reject.

(ii) Suppose that $\pi_t > \bar{\pi}$ and seller i rejects, Since all other seller accept w.p.1. there is no information revealed by other sellers and therefore (given the off-path specification above) $\pi_{i,t+1} = \pi_t$. Therefore, rejecting the offer leads to a payoff of $(1-\delta)c_\theta + \delta F_\theta(\pi_t) < V(\pi_t)$.

(iii) Suppose that $\pi_t = \bar{\pi}$. If trade does not occur, buyers in period $t+1$ attribute all rejections to a low offer made by buyers in period t. Hence, if the seller rejects, $\pi_{i,t+1} = \pi_{i,t}$ and by the same argument as in (ii), such a deviation is not profitable for the seller.

That buyers do not have a profitable deviation from the candidate follows a similar reasoning to the argument for Property 1 in the two-period model. Belief consistency is by construction. Thus, there exists an equilibrium of the candidate form.

Notice that, by construction, the second period payoff to a type-θ seller is the same as in the two-period model. Therefore, if ($\star$) does not hold then following the same argument as in the proof of Theorem 1, we can construct a sequence of first-period trading probabilities $\{\sigma_{N,1}\}$ that are uniformly bounded away from zero, which ensures information aggregates in the first period. ■

Proof of Theorem 3, part (iii). From Theorem 3, part (ii), when ($\star$) does not hold, there is a sequence of equilibria along which information aggregates. In the class of equilibria

[8]The precise specification of off-path beliefs is not crucial for the construction, any $\pi_{\sigma_i,t} \leq \min\{g(\pi_{i,t}), 1\}$ will suffice, where $g(\pi_{i,t}) > \pi_{i,t}$ is such that $(1-\delta)c_H + \delta V(g(\pi_{i,t})) = V(\pi_{i,t})$ and ensures that a high type cannot profitably deviate from rejecting.

constructed in the proof of Theorem 3 (ii), equilibrium play in the first play coincides with the equilibrium play of all equilibria in the two period economy of Section 3. As a result, under the same conditions as in Theorem 2 (namely, that $\delta \in (\hat{\delta}, \bar{\delta}_\lambda)$), there exists a sequence of equilibria along which information aggregation fails in the first period. Furthermore, by construction of this sequence of equilibria (see proof of Theorem 2), (i) the probability of the event that no seller trades in the first period remains bounded away from zero in both states of nature and (ii) the posterior belief about the seller in the second period following this event is equal to $\bar{\pi}$. But then, following this event, by construction no additional information about the state is revealed through trade. ■

In what follows, we prove the two lemmas used in the proof of Theorem 3, part (i).

Lemma C.1 *Suppose that* ($\star$) *holds, and information aggregates in period* t *but not before. Then* $\lim_{N\to\infty} \mathbb{P}_t \left(Q_{L,t}^{i,N} \geq \bar{Q}_t^N \right) = 1$.

Proof. The low type's continuation value from rejecting bid v_L at date t is:

$$\begin{aligned} Q_{L,t}^{i,N} &= (1-\delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = l \right\} + (1 - \lambda_{L,t}) \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = h \right\} \right) \\ &> (1-\delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot v_L + (1 - \lambda_{L,t}) \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = h \right\} \right), \end{aligned}$$

where $\mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = s \right\}$ denotes the low type's expected payoff conditional on history up to t and the state being s. For the inequality, we used the fact that the payoffs at $t+1$ must be strictly above v_L with positive probability, since otherwise no seller would be willing to delay trade to $t+1$. We next show that, for any $\varepsilon > 0$,

$$\lim_{N\to\infty} \mathbb{P}_t \left(F_{L,t+1}^{i,N} \geq V\left(\pi_i(h;0)\right) - \varepsilon | S = h \right) = 1, \tag{23}$$

which, since ε is arbitrary, will establish the result.

Suppose that the state is h and let T be the smallest number of periods such that:

$$(1-\delta^T) \cdot c_H + \delta^T \cdot v_H < V(\pi_i(h;0)),$$

which is finite since ($\star$) implies $V\left(\pi_i\left(h;0\right)\right) > c_H$. Since information aggregates at t (by hypothesis), we can choose N large enough so that (w.p. close to 1) the agents' belief that the state is h is close to 1 in the periods $t+1$ through $t+1+T$. Let us consider histories in which this is the case. If we show that (w.p. going to 1 as N goes to ∞) the bid at $t+1$ is pooling and both seller types accept the bid, then we are done.

Suppose to the contrary that for any N, there is strictly positive probability (bounded away from zero) that the bid is not pooling at $t+1$. There are two cases to consider at t. First, it could be that, with probability bounded away from zero, the buyers make a bid that is rejected by both types. Second, it could be that, with probability bounded away from zero, the bid is v_L and the low types accept it with positive probability.

The first case is straightforward to rule out, since otherwise the buyers could profitably deviate and attract both seller types to trade at date t. For the second case, note that also at $t+2$, with probability bounded away from zero, the bid v_L must be made and accepted by the low type with some probability. Otherwise, if the pooling bid were made instead (w.p. close to 1), the low type would not be willing to trade at $t+1$ (Assumption 2). We can repeat this argument until and including period T and construct sub-histories that occur with probability bounded away from zero, in which the buyers make a bid v_L which is accepted with positive probability by the low types in periods $t+1$ through $t+1+T$.

Let Ω_τ denote the set of sub-histories at $\tau \in \{t+1, ...t+1+T\}$ in which the bid is v_L in periods $t+1$ through τ, and let ω_τ denote an element of Ω_τ. For $\tau' > \tau$, let $\Omega_{\tau'}|\omega_\tau$ denote the sub-histories in $\Omega_{\tau'}$ that have ω_τ as a predecessor. Now, for any $\omega_\tau \in \Omega_\tau$, in order for buyers not to be able to attract the high type at τ, it must be that:

$$V\left(\pi_{i,\tau}(\omega_\tau)\right) \leq Q^{i,N}_{H,\tau}(\omega_\tau), \tag{24}$$

i.e., the high type would weakly prefer to reject a pooling offer and get his continuation value. The high type's continuation value in turn satisfies:

$$Q^{i,N}_{H,\tau}(\omega_\tau) \leq (1-\delta)\cdot c_H + \delta \cdot \max\left\{\mathbb{E}_H\{Q^{i,N}_{H,\tau+1}|\{\Omega_{\tau+1}|\omega_\tau\}\}, \mathbb{E}_H\{V(\pi_{i,\tau+1})|\{\Omega_{\tau+1}|\omega_\tau\}^c\}\right\}.$$

Since the beliefs that the state is h are arbitrarily close to 1 in all periods $\tau \in \{t+1, t+1+T\}$, the posterior beliefs about the seller are arbitrarily close to each other in any such period τ. Hence, combining with (24), for any $\varepsilon > 0$, we can choose N large enough so that:

$$Q^{i,N}_{H,\tau}(\omega_\tau) \leq (1-\delta)\cdot c_H + \delta \cdot \mathbb{E}_H\{Q^{i,N}_{H,\tau+1}|\{\Omega_{\tau+1}|\omega_\tau\}\} + \varepsilon$$

for all $\tau \in \{t+1, t+1+T\}$, which implies that:

$$Q^{i,N}_{H,t+1}(\omega_{t+1}) \leq \left(1-\delta^T\right)\cdot c_H + \delta^T \cdot \mathbb{E}_H\{Q^{i,N}_{H,t+1+T}|\{\Omega_{t+1+T}|\omega_{\tau+1}\}\} + \hat{\varepsilon},$$

where $\hat{\varepsilon}$ can be made small by choosing ε small. Since the value to the seller in any period cannot exceed v_H, then with probability approaching 1 as N goes to ∞,

$$V(\pi_i(h;0)) - \hat{\varepsilon} \leq V\left(\pi_{i,t+1}(\omega_{t+1})\right) \leq \left(1-\delta^T\right)\cdot c_H + \delta^T \cdot v_H + \hat{\varepsilon},$$

which, since $\hat{\varepsilon}$ is arbitrary, contradicts our choice of T. ■

Lemma C.2 *Suppose that information aggregation fails at t along a sequence of equilibria with time-t trading probabilities $\{\sigma_{i,N}\}$. Then there is a subsequence of equilibria along which the probability that no seller trades at t remains bounded away from zero, in both states of nature.*

Proof. Assume that there is a subsequence of equilibria with trading probabilities $\{\sigma_{N_m}\}$ with $\sum_{i=1}^{N_m} \sigma_{i,N_m} < \bar{\kappa} < \infty$ for some $\bar{\kappa} > 0$ and all m. Note that $1 - \sigma_{i,N_m} \cdot P(\theta_i = L|S = s) \geq e^{-\sigma_{i,N_m} \cdot K}$ for any K satisfying $1 - P(\theta_i = L|S = l) \geq e^{-K}$. But for any such K, we have:

$$
\begin{aligned}
P(\text{no seller trades at } t|S = s) &= \Pi_{i=1}^{N_m} \left(1 - \sigma_{i,N_m} \cdot P(\theta_i = L|S = s)\right) \\
&\geq \Pi_{i=1}^{N_m} e^{-\sigma_{i,N_m} \cdot K} \\
&= e^{-K \cdot \sum_{i=1}^{N_m} \sigma_{i,N_m}} \\
&\geq e^{-K \cdot \bar{\kappa}} > 0,
\end{aligned}
$$

which establishes the result.

We are left to prove the assertion that there is a subsequence $\{\sigma_{N_m}\}$ with $\sum_{i=1}^{N_m} \sigma_{i,N_m} < \bar{\kappa} < \infty$ for some $\bar{\kappa} > 0$ and all m. Suppose to the contrary that for all subsequences $\lim_{m \to \infty} \sum_{i=1}^{N_m} \sigma_{i,N_m} = \infty$. Let $X_i \in \{0, 1\}$ denote the indicator that seller i has traded and $Y_{N_m} = N_m^{-1} \sum_{i=1}^{N_m} X_i$ denote the fraction of sellers who have traded. Let $p_{i,N_m}(s) = \sigma_{i,N_m} \cdot \mathbb{P}(\theta_i = L|S = s)$ and note that:

$$
\mu_{N_m}(s) \equiv \mathbb{E}\{Y_{N_m}|S = s\} = N_m^{-1} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(s),
$$

and

$$
\nu_{N_m}(s) \equiv \mathbb{E}\left\{\left(Y_{N_m} - \mu_{N_m}(s)\right)^2 |S = s\right\} = N_m^{-2} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(s) \cdot \left(1 - p_{i,N_m}(s)\right).
$$

Since $\mu_{N_m}(l) > \mu_{N_m}(h)$,

$$
\begin{aligned}
P\left(Y_{N_m} \geq \frac{\mu_{N_m}(h) + \mu_N(l)}{2}|S = h\right) &= P\left(Y_{N_m} - \mu_{N_m}(h) \geq \frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}|S = h\right) \\
&\leq P\left(\left(Y_{N_m} - \mu_{N_m}(h)\right)^2 \geq \left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2 |S = h\right).
\end{aligned}
$$

And, by Markov's inequality:

$$P\left(\left(Y_{N_m}-\mu_{N_m}(h)\right)^2 \geq \left(\frac{\mu_{N_m}(l)-\mu_{N_m}(h)}{2}\right)^2 |S=h\right) \leq \frac{\nu_{N_m}(h)}{\left(\frac{\mu_{N_m}(l)-\mu_{N_m}(h)}{2}\right)^2}$$

$$= \frac{N_m^{-2}\cdot\sum_{i=1}^{N_m} p_{i,N_m}(h)\cdot\left(1-p_{i,N_m}(h)\right)}{\left(N_m^{-1}\cdot\sum_{i=1}^{N_m}\frac{p_{i,N_m}(l)-p_{i,N_m}(h)}{2}\right)^2}$$

$$= 4\cdot\frac{\sum_{i=1}^{N_m}\sigma_{i,N_m}\cdot P\left(\theta_i=L|S=h\right)-\sum_{i=1}^{N_m}\sigma_{i,N_m}^2\cdot P\left(\theta_i=L|S=h\right)^2}{\left(\sum_{i=1}^{N_m}\sigma_{i,N_m}\right)^2\cdot\left(P\left(\theta_i=L|S=l\right)-P\left(\theta_i=L|S=h\right)\right)^2},$$

which by our assumption tends to 0 as m goes to ∞. By a similar reasoning, we have that:

$$\mathbb{P}\left(Y_{N_m} < N_m^{-1}\cdot\sum_{i=1}^{N_m}\frac{p_{i,N_m}(l)+p_{i,N_m}(h)}{2}|S=l\right)\to 0,$$

as m goes to ∞. Combining these two observations, we conclude that information about the state must aggregate along all subsequences, a contradiction. ■