Non-Cognitive Skills, Parenting Time, and Growth*

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March 2018

Abstract

We analyze the role of parenting time and material investment in children and public spending on education in an overlapping-generations model of growth with human capital. In the model, human capital has cognitive and non-cognitive skill components. Cognitive skills are formed by education and material investment by parents, whereas parenting time develops children's non-cognitive skills. In this framework, we demonstrate that an increase in government subsidies to parents' investment in children's human capital funded by an increase in either labour or capital income tax can have a negative effect on growth. We also show that the negative effect of labour income tax on growth is much stronger when the share of non-cognitive skill in human capital is low relative to the share of cognitive skill. The opposite holds for the negative effect of growth from capital income tax or material investment subsidies. In other words, the distortionary effect of labour income tax on growth is stronger in an economy where cognitive skills are more important for labour productivity than non-cognitive skills. The opposite is true for the distortionary effect on growth of the capital income tax.

Keywords: Skill formation technology, time allocation, human capital, over-

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lapping generations
JEL classification: E71, H31, J24

1 Introduction

The most valuable of all capital is that invested in human beings.

-Alfred Marshall (1890)

There has been a dramatic proliferation of research concerns with personality traits (i.e. non-cognitive skills), seeing Cunha and Heckman (2007), Borghans et al. (2008), DellaVigna (2009), Cunha et al. (2010), Almlund et al. (2011), and Heckman and Mosso (2014). Unlike cognitive skills, there is no schooling which allows to learn and evaluate non-cognitive skills. Instead, psychologists argue that Parenting Time contributes to develop non-cognitive skills (Hirschi and Stark (1969); Baumrind (1971, 1991); Maccoby and Martin (1983); Coleman (1988); Collins et al. (1995); Holmbeck et al. (1995); Bernal and Keane (2010); Del Boca et al. (2014); Lee and Seshadri (2014)). In addition, skill attainment at one stage of the life cycle raises skill attainment at later stages of the life cycle (self-productivity), and early investment facilitates the productivity of later investment (complementarity), seeing Cunha et al. (2006). Zhu and Vural (2013) provides a theoretical model in which altruism parents pass their human capital on in two ways: material investment and parenting time investment. According to those theoretical and empirical studies, exploring the importance of non-cognitive skills in the production of human capital is imperative. To our best knowledge, the close study is documented by Casarico and Sommacal (2012), they suggest that omission of parenting time from the technology of skill formation can bias the results related to the impact of labor income taxation on growth. But no further theoretical explanations of non-cognitive skills are provided in the extant literature of economic growth.

For the purpose that exploring the incorporating parenting time into production of human capital by developing non-cognitive skills, this paper undertakes the role of parenting time, material investment in children, and tax policies in a three-period overlapping generations endogenous growth model to provide a theoretical explanation. Agents live for three periods: childhood, parenthood, and retirement. In the first period, the child receives formal care and informal care. Formal care contains the public spending on education and material investment, whereas informal care involves parenting time. In the second period, the agent determines how much to invest material investment in children and saving, and how much time to devote to parenting time and labour. In the third period, the agent retires and consumes all income. One of the novelties of this model is found in how it embeds non-cognitive skills in that of production of human capital. More specifically, in this model, the agent allocates time into parenting time for higher stock of next generation's human capital or into labour supply for higher consumption in old age. However, parenting time brings the agent with utility. The agent has to fine the balance between parenting time and consumption in old age.

We find that the overall effects of labour income taxation, capital income taxation, and subsidies on material investments in children rely on the specific values of the parameters. The choice of parenting time devoted to children is only determined by altruism and patience. Furthermore, the quality of the parenting time, public spending on education, and material investments in children have direct impacts on growth, as they directly enters the production of human capital. Indeed, we posit that cognitive skills and non-cognitive skills are complements in the production of human capital. However, using a richer than is possible in a theoretical analysis but means one can rely on numerical analysis to generate results.

This paper constructs comparative static analysis based on benchmark parametrization. The simulation results indicates that increases in labour income tax rate, capital income tax rate, and subsidies on material investment can lead to a reduction in growth rate respectively.

In addition, at the chosen parameter values, the simulation results of tax reform show that labour income taxation has significant impact on material investment in children, consumption in old age, public spending on education, real wage rate, and interest rate. In contrast, the simulation results of tax reform also find that capital income taxation and subsidies on material investment in children have ambiguous impacts on variables.

The simulation results of sensitivity analysis indicate that the quality of parenting time and the marginal productivity between cognitive skills and non-cognitive skills lower the impacts of labour income taxation, capital income capital, and subsidies on material investment in children on growth. Moreover, higher share of public spending on education in the production of human capital with cognitive skills could result in that labour income taxation and subsidies on material investment in children are beneficial to growth.

The key component of our analysis suggests that there are two distinctive features to improve the production of human capital. The first one is that informal care; that is, parenting time enters the production of human capital by developing non-cognitive skills. The other one is public spending on education and material investments in children both make formal care entering the production of human capital with cognitive skills. The informal care and formal care both contribute to the production of human capital.

This paper is structured as following. In section 2, this paper introduce the main methodology; overlapping generations model incorporates non-cognitive skills with parenting time with altruism into an endogenous growth model. Section 3 presents benchmark parameterization. After that, in section 4, the results of tax analysis including comparative static analysis, tax reform, and sensitivity analysis are reported. Finally, this paper give a conclusion in section 5.

2 The model

Allais (1947), Samuelson (1958) and Diamond (1965) are the pioneers of the concept of overlapping generations model (OLG). Azariadis (1993) use this setup as the main workhorse macroeconomic model. De La Croix and Michel (2002) give a three-period-lived example. The model we present here analyzes the roles of parenting time, material investment in children, public spending on education and tax policies in a three-period overlapping generations model.

We assume the population is constant and normalize it to N=1. This implies population growth rate is zero. We also normalizes the total time equals to 1.

2.1 Human capital production function

In line with the theory of human capital theory in Romer (1986) and Lucas (1988), there is no role of physical capital in the formulation of human capital. Note that human capital does not necessarily require externalities across individuals of same generation.

The production of human capital with the technology of cognitive skills considers public spending on education G_t and material investment in children C_t. Therefore, the following equation describes the technology of human capital with cognitive skills:

$$H_c = B_t {G_t}^{\omega} {C_t}^{1-\omega} \tag{1}$$

where H_c is the production of human capital with cognitive skills. The variable B_t is the exogenous total factor productive (TFP) variable. Parameter

ω determines the relative importance of public spending on education and material investments in children. This Cobb-Douglas production function captures the complementarity between public spending on education and material investments in children.

The technology of non-cognitive skills formation has been documented in Cameron and Heckman (2001), Carneiro and Heckman (2003), Cunha et al. (2006), and Cunha and Heckman (2007), in which, their model involves parental investment to promote child's learning process and the accumulation of human capital by developing non-cognitive skills. Heckman and Mosso (2014) considers that the investment of parents on child and the accumulation of human capital by developing non-cognitive skills over generations. This section employs this setting. So the production of human capital with non-cognitive skills is given by

$$H_{nc} = D_t {\phi_t}^{\gamma} H_t \tag{2}$$

where H_{nc} is informal care. D_t is the efficient factor of the production of human capital with non-cognitive skills. Parameter γ represents the quality of parenting time. H_t is a vector of human capital by developing cognitive skills and non-cognitive skills at generation t (Cameron and Heckman (2001); Carneiro and Heckman (2003); Cunha et al. (2006)). Variable ϕ_t indicates parenting time.

This paper models the production of human capital in the next period depending positively on currently available human capital with cognitive skills and non-cognitive skills. Therefore, the production of human capital is described as the following equation:

$$H_{t+1} = {H_c}^{1-\upsilon} {H_{nc}}^{\upsilon} \tag{3}$$

where parameter υ indicates the relative importance of formal care and informal care in the production of human capital. This Cobb-Douglas function describes the idea that cognitive skills and non-cognitive skills both matter to the production of human capital.

Cunha and Heckman (2007) argues that non-cognitive skill can prove dynamic complementarity and self-productivity. *Dynamic complementarity* implies skills produced at one stage raise the productivity of investment at subsequent stages. Dynamic complementarity arises when $\frac{\partial^2 H_{t+1}}{\partial H_t \partial \phi_t} > 0$. Heckman and Mosso (2014) finds that complementarity tends to increase over the life cycle, so does inter-generation. This implies that compensatory investments tend to be less effective over following generations. The learning technology satisfies the Inada-like condition. But new skills will come in non-cognitive skills accumulation along with new investment strategies.

Self-productivity means when higher stocks of skills in one period create higher stocks of skills in the next period. Self-productivity arises when $\frac{H_{t+1}}{H_t} > 0$. Complementarity coupled with self-productivity leads to the important concept of dynamic complementarity introduced in Cunha and Heckman (2007). Dynamic complementarity also suggests that parenting resources at early ages can have lasting lifetime consequences that have impact on later ages.

2.2 Production

This production function satisfies neoclassical assumptions, including perfectly competitive market. In line with Azariadis and Drazen (1990) and Akabayashi (2006), the production function applying the trade-off between labour supply and parenting time is the following form:

$$Y_t = AK_t{}^{\alpha}[(1-\phi_t)H_t]^{1-\alpha} \tag{4}$$

where Y_t denotes the goods output. A is an exogenous productivity parameter 'efficiency units', K_t is the stock of physical capital and H_t is the human capital by developing non-cognitive skills. Parameter α and $1-\alpha$ are the elasticity of production with respect to physical capital and human capital, respectively. $1-\phi_t$ determines the time spent on supplying labor to the market.

2.3 Household

This paper adapts the canonical overlapping generations model of agent lives for three periods: childhood, adulthood, and retirement. According to Acemoglu (2008), log preference ensures that income and substitute effect exactly cancel each other out, so that changes in the interest rate have no effect on the saving rate, so does the capital-labor ratio of economy. Parents are assumed to be the decision-maker in the household. The children passively accepts material investment.

Following Andreoni (1989) and Taylor and Irwin (2000), this section applies altruism in the agent's utility. Along with parenting time, the material investment in children, and consumption in old age also bring the agent with utility. This section also assumes that parents do not drive utility their from consumption in adult age to focus on the analysis the interaction between material investment in children and parenting time. Therefore, the utility function is given by

$$U_t = (1-\eta)lnC_t + \beta lnX_{t+1} + \eta ln\phi_t \tag{5}$$

where X_{t+1} is the consumption in old age. The parameter β is the psychology discount factor (i.e. patience). In line with the setting in De La Croix and Michel (2002), parameter η represents the strength of altruism on material investment and parenting time, and is greater than zero. Equation (5) indicates that parents derive utility from material investment in children, the consumption in old age, and parenting time.

Now we turn our attention to the budget constraints of household. In the first period, the agent is young and only has consumption, which is material investments provided by parents. In the second period, the agent is endowed with one unit of labor and supplies to firms inelastically. In time t, The income is from the real wage w_t times human capital H_t. The agent allocates the labor income into the material investment in children and saving, which is invested in the firms. In the third period, when the agent is old, receiving income is from saving of previous period. This saving is inelastically supplied to representative firm which pay $r_{t+1}S_t$ to the agent when agent is retired. Therefore, the budget constraints for the agent are

$$(1-\theta_c)C_t + S_t = (1-\tau_L)(1-\phi_t)w_tH_t \tag{6}$$

$$X_{t+1} = [1 + (1-\tau_k)r_{t+1}]S_t \tag{7}$$

where parameter θ_c is the subsidies on the material investment in children. Parameter τ_L is the labour income tax rate. Parameter τ_K is the capital income tax rate. S_t denotes saving. Equation (6) indicates the budget constraints during adulthood: the taxed labour income can be allocated to material investment in children and savings. Equation (7) is the budget constraints during retirement: the income of the second period, which is given by savings plus the interests earned on them, facing capital income taxation, goes entirely to consumption. There is no 'accident of birth'. There is no role for initial financial wealth, parental income in determining the optimal level of investment because parent can borrow freely in the market to finance the wealth to maximize the level of investment (Heckman and Mosso (2014)). In addition, this section considers 'the problem of parents'. This minimum level of material and time investment are necessary for the child to attend college (Cunha (2013)). However, due to our interest in analyzing parenting time in endogenous growth model, we assume bequests play no role in this model (see Castelló-Climent and Doménech (2008)).

2.4 Government

The government aims to maximize social welfare by implementing taxes and subsidies properly. Following Caucutt and Lochner (2012) to set up subsidies to material investment in children, the government budget constraint at t is

$$G_t + \theta_c C_t = \tau_L H_t w_t (1 - \phi_t) + \tau_K r_t K_t \tag{8}$$

This subsection takes parameters τ_L, τ_K, and θ_c as the exogenous policy variables.

2.5 The optimization problem

2.5.1 Firm's optimization problem

The firm determines the demand of physical capital and human capital by maximizing its profit with given factor prices of wage and rent, which are determined under competitive market:

$$\max_{K_t, H_t} \pi_t = A K_t^{\alpha} [(1 - \phi_t) H_t]^{1-\alpha} - w_t (1 - \phi_t) H_t - r_t K_t \tag{9}$$

where w_t represents real wage rate, r_t is the rent rate of physical capital. This problem shows the firm sells its goods and pays the rental rate of physical capital and real wage of human capital.

Rental rate of physical capital is given by

$$r_t = A \alpha K_t^{\alpha-1} [(1 - \phi_t) H_t]^{1-\alpha} \tag{10}$$

Real wage rate of human capital is given by

$$w_t = A(1 - \alpha) K_t^{\alpha} [(1 - \phi_t) H_t]^{-\alpha} \tag{11}$$

Equation (10) states that interest rate equals to the marginal productivity of capital. Equation (11) requires that the wage per efficiency units is equal to the marginal productivity of aggregate labour in efficiency units.

Since the current physical capital stock K_t is fully depreciating at the end of the current period, which means $K_{t+1} = I_t$, where I_t is the aggregate investment. The equality states that physical capital available in next period *t+1* equals the saving from the current period. Aggregate investment equals aggregate saving. So the equilibrium on the capital market yields

$$K_{t+1} = I_t = S_t \tag{12}$$

2.5.2 Agent's optimization problem

The agent seeks to maximize the utility (5) under the constraints in (6) and (7). One can form a Lagrange function as follows:

$$\begin{aligned}\mathcal{L}_t =&(1-\eta)lnC_t + \beta lnX_{t+1} + \eta ln\phi_t \\ &+ \lambda_t\Big\{(1-\tau_L)(1-\phi_t)w_tH_t - (1-\theta_c)C_t - \frac{X_{t+1}}{[1+(1-\tau_K)r_{t+1}]}\Big\}\end{aligned} \tag{13}$$

where λ_t is the shadow price of physical capital.

First-order conditions for an interior solution respect to C_t, X_{t+1}, ϕ_t, and λ_t are

$$C_t = \frac{1-\eta}{\lambda_t(1-\theta_c)} \tag{14}$$

$$X_{t+1} = \frac{\beta[1+(1-\tau_K)r_{t+1}]}{\lambda_t} \tag{15}$$

$$\phi_t = \frac{\eta}{\lambda_t[(1-\tau_L)H_tw_t]} \tag{16}$$

$$0 = (1-\tau_L)(1-\phi_t)w_tH_t - (1-\theta_c)C_t - \frac{X_{t+1}}{[1+(1-\tau_K)r_{t+1}]} \tag{17}$$

Equation (14) give us that the equilibrium allocation of material investment in children. Equation (15) indicates that the optimal consumption in old age. Equation (16) reflects the optimum choices of parenting time. (16) finds that parenting time is only depends on patience and the strength of altruism, it also shows that the strength of altruism is beneficial to the parenting time, while patience reduces parenting time.

Substituting (14), (15), and (16) into (17), one obtain

$$\lambda_t = \frac{1+\beta}{(1-\tau_L)} H_t w_t \tag{18}$$

Incorporating equation (18), equation (14), (15), and (16) can be rewritten as:

$$C_t = \frac{(1-\eta)(1-\tau_L)}{(1+\beta)(1-\theta_c)} H_t w_t \tag{19}$$

$$X_{t+1} = \frac{\beta[1+(1-\tau_K)r_{t+1}](1-\tau_L)}{1+\beta} H_t w_t \tag{20}$$

$$\phi_t = \frac{\eta}{1+\beta} \tag{21}$$

Equation (19) is useful for informing how θ_c affect material investment in children. Equation (19) also confirms that SES affects the material investment in children directly (Hout and Dohan (1996)). Equation (20) indicates that the optimal consumption in old age. (20) also shows that how τ_L, τ_K, θ_c, and altruism, η, affect the consumption in old age.

One use (10) and (11) to rewrite (19) as

$$C_t = cY_t \tag{22}$$

where $c \equiv (1-\alpha)(1-\eta)(1-\tau_L)/(1-\eta+\beta)(1-\theta_c)$.

Plugging rental rate of physical capital, real wage rate of human capital, the equilibrium, (19), (20), and (21), one can observe

$$G_t = gY_t \tag{23}$$

where $g \equiv \alpha\tau_K + (1-\alpha)(\tau_L - \theta_c(1-\eta)(1-\tau_L)/(1-\eta+\beta)(1-\theta_c))$.

Together with $K_{t+1} = S_t$, (10), and (11), plugging (20) into (7), one have

$$K_{t+1} = \frac{\beta}{1+\beta} A_t(1-\alpha)(1-\tau_L)(1-\phi_t)^{-\alpha} K_t^{\alpha} H_t^{1-\alpha} \tag{24}$$

Using (18), (19) and (21), $K_{t+1} = S_t$, one can observe (3) as

$$H_{t+1} = \sigma_t(\frac{K_t}{H_t})^{\alpha(1-\upsilon)} \tag{25}$$

where $\sigma_t \equiv [A_t B_t (1-\phi_t)^{1-\alpha} g^{\omega} c^{1-\omega}]^{1-\upsilon} D_t^{\upsilon} \phi_t^{\gamma\upsilon}$.

One now define the variables in intensive form. Let $k_t \equiv K_t/H_t$, the ration of physical capital to human capital, subjecting to real wage rate and rental rate of physical capital, One obtain

$$k_{t+1} = \psi_t {k_t}^{\alpha \upsilon} \tag{26}$$

where $\psi_t \equiv A_t \beta(1-\alpha)(1-\tau_L)/\sigma_t(1+\beta)(1-\phi_t)^{\alpha}$

2.6 Steady State

In steady state, the transformation variable remains at same level. This yields $k_{t+1} = k_t = k^*$. It is straightforward to verify that there exists a unique steady state. Solving equation (24) for k^* yields

$$k^* = \psi^{\frac{1}{1-\alpha\upsilon}} \tag{27}$$

The left-hand side of the equation is exactly the ratio of physical capital to human capital in the intensive form where K_{t+1} has to increase as H_{t+1} increase, thus k^* is fixed in equilibrium.

2.7 Growth Rate

It is straightforward to verify ρ_H grows at the same rate via (23), so one can define $\rho_K = \rho_H = \rho^*$. Then, Substituting (25) into (22), the growth rate takes the following form:

$$\begin{aligned} 1+\rho^* = \\ \Big[\frac{A\beta(1-\alpha)}{1+\beta(1-\frac{\eta}{1-\beta})^{\alpha}}\Big]^{\frac{\alpha(1-\upsilon)}{1-\alpha\upsilon}} \Big[D^{\gamma}(\frac{\eta}{1+\beta})^{\gamma\upsilon}\big[AB(1-\frac{\eta}{1+\beta})^{1-\alpha}\big]^{1-\upsilon}\Big]^{\frac{1-\alpha}{1-\alpha}\upsilon} \\ (1-\tau_L)^{\frac{\alpha(1-\upsilon)}{1-\alpha\upsilon}}\Big\{\Big[\alpha\tau_K + (1-\alpha)(\tau_L - \frac{\theta_c(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta_c)}\Big]^{\omega}\Big[\frac{(1-\alpha)(1-\eta)(1-\tau_L)}{(1-\eta+\beta)(1-\theta_c)}\Big]^{1-\omega}\Big\}^{\frac{(1-\alpha)(1-\upsilon)}{1-\alpha\upsilon}} \end{aligned} \tag{28}$$

This equation finds that ρ^* only has ambiguous relationship with policies variables.

(28) can be rewritten as

$$1+\rho^* = \xi(A, B, D)\varrho(\tau_K, \tau_L, \theta_c)$$

This implies that growth rate contains two parts. The first part has three efficient factors from goods production and human capital production. The other part has policy parameters. The following sections are going to use benchmark parameterization to present the comparative static in this section.

3 Benchmark Parameterization

Here, a key finding is that the reasonable parameterizations. This feature enables the model to reproduce broad features of the parenting time allocation, public spending on education material investment in children, and policies variables.

According to Dhont and Heylen (2008), the average tax rate on labour income and capital income in the US are 0.347 and 0.393, respectively. Hence, the initial tax rates on income are set to be $\tau_K = 0.393$ and $\tau_L = 0.347$. In line with empirical evidences, the parameter α is the capital share in goods production and is set to 1/3 to match the empirical counterpart. Given that β does not influence qualitative features of the model that we are interested in, we choose a value that is standard in the literature, $\beta = 0.99$, following De La Croix and Doepke (2003). Literature also suggests β is 0.5 for one generation is 25years, we perform sensitivity analysis.

Table 1: Parametrization and policy variables of benchmark

Labour income tax rate	τ_L	0.347
Capital income tax rate	τ_K	0.393
Capital share of production	α	0.33
Discounter factor	β	0.99
Quality of parenting time	γ	0.893
Strength of altruism	η	0.271
subsidies on material investment in children	θ_c	0.1
Weight of non-cognitive skills	υ	0.5
Weight of public spending on education	ω	0.18
TFP in the goods production	A_t	4.4658;
TFP in the production of human capital with cognitive skills	B_t	2.5987
TFP in the production of human capital with non-cognitive skills	D_t	2.5987

For parameter γ, which is the share of parenting time in human capital formulation. Zhu and Vural (2013) points out that the share of parenting time on informal care human capital accumulation is 0.893. For the strength of altruism, Craig (2005) and Milkie et al. (2004) suggest that the parenting time is 12 percent, which implies that η is 0.2713, as we discuss after equation (16). This is consistent with De La Croix and Doepke (2003).

With regard to parameter υ, which is the relative importance of formal care and informal care in the production of human capital. to the best of our knowledge, no database provides information on υ. However, following Heckman (2006), non-cognitive skills development and cognitive skills development are equally important in explaining a variety of aspects of social and economic life. Therefore, the parameter υ is selected to be 0.5. We perform a sensitivity analysis in the interval (0,1).

Parameter θ_c determines the subsidies on material investment in children. Caucutt and Lochner (2012) uses data from the Children of the NLSY to point out the parameter θ_c is around 10 percent. Hence θ_c is chosen to be 0.1. We choose the parameter ω, elasticity of public expenditures on education ω, is chosen to be 0.18 as documented in Annabi et al. (2011).

Moreover, the initial output in the foods sector, Y_t is normalized into 1, so that all other economic variables can be easily presented as a fraction of Y_t. The productivity level A, B_c and B_{nc} are scale parameters and are set to be 1.

In addition, we present the calibrations of the ratios of physical capital, material investment in children, consumption in old age, real wage rate, interest rate, public spending on education to human capital. The benchmark parameterization and calibration are given in table 1.

4 Tax analysis

This section first conducts a comparative static analysis to examine the impacts of taxes on material investments in children, consumption in old age, efficient physical capital and growth rate in steady state value. Next, it analyzes a revenue-neutral tax reform regarding taxes on labour income and capital income, and subsidies on material investment in children. Moreover, this section also proposes a sensitivity analysis to test the robustness of the results.

4.1 Comparative static analysis

To investigate the impacts of τ_L, τ_K, and θ_c on the variables in steady state, one can perform comparative static analysis based on the steady state solutions. According to (28), τ_L affects ρ^* through the following channel:

$$\frac{1}{1+\rho}\frac{\partial \rho^*}{\partial \tau_L} = -\frac{\alpha(1-\upsilon)}{(1-\alpha\upsilon)(1-\tau_L)} - \frac{(1-\alpha)^2(1-\upsilon)(1-\omega)(1-\eta)}{c(1-\alpha\upsilon)(1-\eta+\beta)(1-\theta_c)} + \frac{\omega(1-\upsilon)(1-\alpha)^2}{g(1-\alpha\upsilon)}\Big[1+\frac{\theta_c(1-\eta)}{(1-\eta+\beta)(1-\theta_c)}\Big] \tag{29}$$

Equation (28) indicates that the affect of τ_L on ρ^* depends on the last term, which is

Referring to (28), the negative relationship between τ_K and ρ^* can be disentangled into the following form:

$$\frac{1}{1+\rho}\frac{\partial \rho^*}{\partial \tau_K} = \frac{\alpha\omega(1-\upsilon)(1-\alpha)}{g(1-\alpha\upsilon)} \tag{30}$$

According to (28), the change in θ_c should affect g^* directly:

$$\frac{1}{1+\rho}\frac{\partial \rho^*}{\partial \theta_c} = \frac{(1-\alpha)^2(1-\upsilon)(1-\eta)(1-\tau_L)}{(1-\alpha\upsilon)(1-\eta+\beta)(1-\theta_c)^2}\Big(\frac{1-\omega}{c}-\frac{\omega}{g}\Big) \tag{31}$$

Equation (31) finds that the affect of θ_c on g^* depends on the last term, which is $(1-\omega)(1-\theta_c)\Delta - \omega(1-\alpha)(1-\tau_L)$.

Equation (25) indicates that the changes in τ_L should affect k^* directly:

$$\frac{\partial k^*}{\partial \tau_L} = \frac{1}{1-\alpha\upsilon}\Big\{\frac{A^\upsilon\beta(1+\beta+\eta)^{\upsilon(\alpha+\gamma-1)}[(1-\tau_L)(1-\alpha)]^{\upsilon+\omega-\upsilon\omega}}{{B_c}^{1-\upsilon}{B_{nc}}^{\upsilon}\Delta^{\omega(1-\upsilon)}\eta^{\gamma\upsilon}(1+\beta)^{\alpha\upsilon}(1-\theta_c)^{-1+\upsilon+\omega-\upsilon\omega}}\Big\}^{\frac{1}{1-\alpha\upsilon}} \Big\{-\frac{\upsilon+\omega-\upsilon\omega}{1-\tau_L} - \frac{\omega(1-\upsilon)}{\Delta}\big[(1-\alpha)(1+\beta)+\frac{\theta_c(1-\alpha)}{1-\theta_c}\big]\Big\} < 0 \tag{32}$$

Equation (32) shows that τ_L has negative effect on k^*.

The negative relationship between τ_K and k^* can be disentangled into the following form:

$$\frac{\partial k^*}{\partial \tau_K} = \frac{1}{1-\alpha\upsilon}\Big\{\frac{A^\upsilon\beta(1+\beta+\eta)^{\upsilon(\alpha+\gamma-1)}[(1-\tau_L)(1-\alpha)]^{\upsilon+\omega-\upsilon\omega}}{{B_c}^{1-\upsilon}{B_{nc}}^{\upsilon}\Delta^{\omega(1-\upsilon)}\eta^{\gamma\upsilon}(1+\beta)^{\alpha\upsilon}(1-\theta_c)^{-1+\upsilon+\omega-\upsilon\omega}}\Big\}^{\frac{1}{1-\alpha\upsilon}} \Big\langle\frac{\Delta}{\alpha\omega(1-\upsilon)(1+\beta)}\Big\rangle > 0 \tag{33}$$

According to (25), the change in θ_c should affect k^* directly:

$$\frac{\partial k^*}{\partial \theta_c} = \frac{1}{1-\alpha\upsilon}\left\{\frac{A^{\upsilon}\beta(1+\beta+\eta)^{\upsilon(\alpha+\gamma-1)}[(1-\tau_L)(1-\alpha)]^{\upsilon+\omega-\upsilon\omega}}{{B_c}^{1-\upsilon}{B_{nc}}^{\upsilon}\Delta^{\omega(1-\upsilon)}\eta^{\gamma\upsilon}(1+\beta)^{\alpha\upsilon}(1-\theta_c)^{-1+\upsilon+\omega-\upsilon\omega}}\right\}^{\frac{1}{1-\alpha\upsilon}}$$
$$\left\langle\frac{\Delta(1-\theta_c)^2}{(1-\upsilon)\big[(1-\omega)(1-\theta_c)\Delta-\omega(1-\alpha)(1-\tau_L)\big]}\right\rangle \tag{34}$$

Equation (34) indicates that the affect of θ_c on k^* depends on the last term, which is $(1-\omega)(1-\theta_c)\Delta-\omega(1-\alpha)(1-\tau_L)$.

Using numerical simulation, we can observe how parameters affect material investment in children, the consumption in old age, and parenting time. With baseline benchmark value, table 2 summarizes the outcomes of numerical simulation of decentralized economy. The parenting time only depends on parameter β and parameter η, so we do not report parenting time in table 2.

Table 2: Numerical simulation of decentralized economy

	τ_L	τ_K	θ_c
c^*	+	+	-
x^*	-	+	+
k^*	-	+	-
g^*	-	-	-

Intuitively, θ_c benefices c^*. However, as shown in table 2, it is surprising that θ_c lowers c^*. The intuition is that due to revenue-neutral taxation, rising θ_c implies rising tax rate. Hence the agent has to reduce the disposable income on material investment in children. On the other hand, increasing labour income tax rate or capital income tax rate means there is a increase on subsidies on material investment in children.

4.2 Social welfare

This subsection focuses on analysis of social welfare via revenue-neutral tax reforms. This derivation starts from the simplification of government budget (8).

$$G_t + \theta_c C_t = \tau_L H_t w_t (1-\phi_t) + \tau_K r_t K_t$$

Due to the complex interraltionship between variables, changes in taxes and subsidies could result in ambiguous effects in government budget and complicate the tax reform analysis. To solve this problem, this paper apply the benchmark calibrated parameters listed in table 1. With assumption of fixed government expenditure, the paper analyzes 5 cases of tax reform in the economy as followed;

Case 1. A increase in θ_c is funded by a increase 10 percents (i.e. τ_K changes from 0.393 to 0.4323).

Case 2. a 10 percent increase of τ_L (i.e. τ_L changes from 0.347 to 0.3817)funds a increase in θ_c.

Case 3. g is increased by a 10 percent increases in τ_K.

Case 4. a increase in g is funded by a 10 percent increases in τ_L

Case 5. a increase in θ_c is funded by increases g.

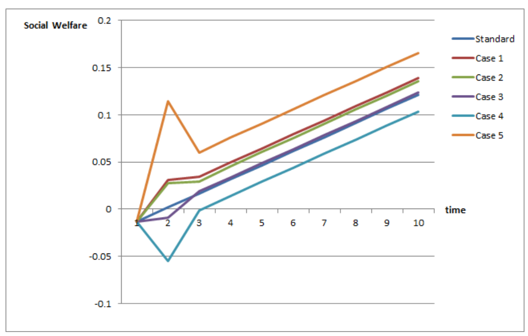

Figure 1: social welfare

this economy subsidies to material investment in children by parents are welfare-improving, even when funded by higher taxes. At the same time, the benefits of government spending on education do not outweigh the welfare cost of higher taxes. Thus, the best policy change is to fund the subsidies to parents by cuts in education spending.

Also, raising tax on capital income is less harmful than raising tax on labour income. This is consistent with the model prediction that the growth rate is positively related to the capital income tax rate.

4.3 Sensitivity analysis

To test the robustness of the results, this subsection performs a sensitivity analysis on the quality of parenting time γ, the share of the human capital with non-cognitive skills υ, the share of public spending on education ω, patience β and strength of altruism η.

Table 4 studies the effects of a 10 percent reduction namely in labour income taxation, capital income taxation, and subsidies on material investment in children. We find that a reduction in τ_L and θ_c both increase ρ^* and a reduction in τ_K always overstates the positive impact on ρ^*. The elasticity of ρ^* to τ_L is -0.1252. This implies following a 10 percent reduction in τ_L leads to ρ^* increases by percent. The elasticity of ρ^* to τ_K is 0.0703. 10 percent reduction in τ_K results in 7 percent increase in ρ^*. In addition, table 2 shows that the elasticity of ρ^* to θ_c is 0.3234. It means ρ^* decreases 3.2 percent by lowers 10 percent in θ_c.

Table 3: Elasticities of growth rate to labour income tax rate, capital income tax rate, and subsidies on material investment in children: benchmark case

	$\partial g^*/\partial\tau_L$	$\partial g^*/\partial\tau_K$	$\partial g^*/\partial\theta_c$
$\beta = 0.5$, $\gamma = 0.893$, $\eta = 0.271$, $\upsilon = 0.5$, $\omega = 0.18$	-0.6712	0.0703	0.3234

We now discuss how our conclusions of the previous subsection change when we depart from the benchmark of γ, υ, and ω. We examine different values of γ, υ, and ω in order to understand how the results differ for taxes variables on g^*. Table 5 presents the results of sensitivity analysis. We vary each parameter at a time, setting all the others according to the procedure used in the benchmark case.

The parameter γ determines the efficiency of parenting time on production of human capital with non-cognitive skills, we consider values in the interval (0,1), where $\gamma > 1$ does not change the conclusion. we consider two cases, namely, 0.5 and 0.1. The higher γ is , the smaller impacts of τ_L, τ_K, and θ_c on g^*. For examples, when $\gamma = 0.1$, a 10 percent reduction in τ_L, the elasticity of g^* to τ_L is -0.246. When γ is 0.5, a 10 percent reduction in τ_L leads to 1.7 percent increase in g^*.

The parameter υ clearly plays a role in our analysis because it determines the relative marginal productivity of cognitive skills versus non-cognitive skills in the production of human capital. For υ, w we consider two extremely

Table 4: Elasticities of growth rate to labour income tax rate, capital income tax rate, and subsidies on material investment in children: sensitivity analysis

	$\partial g^*/\partial \tau_L$	$\partial g^*/\partial \tau_K$	$\partial g^*/\partial \theta_c$
$\upsilon = 0.1$	-1.0227	0.1084	0.4929
$\upsilon = 0.9$	-0.1563	0.0166	0.0753
$\omega = 0.5$	-0.1978	0.1938	0.0883
$\omega = 0.9$	0.3775	0.3488	-0.1976
$\beta = 0.5$, $\eta = 0.2045$	-0.6509	0.0717	0.2935

cases, 0.1 and 0.9, respectively. The first case indicates the the production of human capital with cognitive skills is more efficient than the production of human capital with non-cognitive skills. The first case shows that a 10 percent reduction in τ_L results in 2.2 percent increase in g^*. The first case also shows that reducing 10 percent in τ_K leads to 9.5 percent increase in g^*. Following a 10 percent reduction in θ_c, there is 2.1 percent increase in g^*. In contrast, when υ is 0.9, human capital with non-cognitive skills dominates in production of human capital, and thus we find that higher υ reduces the impacts of τ_L and θ_c on g^* and overstates the impact of τ_K on g^*.

For ω, which affects elasticity of substitution between public spending on education and material investment in children, we consider values in the interval (0, 1). This higher ω is, higher efficiency of public spending on education. We take $\omega = 0.5$ and 0.9 as our examples. It is worth noting that when ω is greater than 0.64, a increase in τ_L and θ_c is beneficial to g^*. Literature also suggests parameter β is 0.5 for one generation is 25 years, it leads to the parameter η is 0.2045. With the chosen values, a 10 percent reduction of θ_c has more 0.3 percent positive influence on g^* than benchmark case.

5 The Planning Problem

5.1 Environment

In building on prior literature, there is dynamic inefficient issue in a decentralized economy (Acemoglu (2008)). With limited life-time, individuals choose the equilibrium allocation of material investment in children, the consumption in old generation, and the parental time.

In the centralized version of the model, we assume that there exists a long-lived and far-sighted central planner in this economy. It is therefore possible to improve the welfare of one agent without diminishing the welfare of another agent. This results in the First Welfare Theorem (Arrow (1951) and Debreu (1954)). We consider a central planner who chooses the allocation of output in order to maximize the present discounted value of current and future generations. In this economy, social planner looks at exact time period t, and considers the whole generations (see De La Croix and Michel (2002)). Assuming that the central planner's discount factor is β, the social welfare function takes the following form

$$\sum_{t=0}^{\infty} \beta_t[(1-\eta)lnC_t + lnX_t + \eta ln\phi_t]$$

subject to the resource constraints

$$\begin{aligned} K_{t+1} &= F(K_t) - C_t - X_t - G_t \\ H_{t+1} &= (B_t {G_t}^{\omega} {C_t}^{1-\omega})^{1-\upsilon}(D_t {\phi_t}^{\gamma} H_t)^{\upsilon} \end{aligned}$$

5.2 General Equilibrium

Writing social planner problem in Lagrange form to yield

$$\begin{aligned} \mathcal{L}_t = &\sum_{t=0}^{\infty} \beta_t[(1-\eta)lnC_t + lnX_t + \eta ln\phi_t] \\ &+ \beta q_{t+1}[A_t {K_t}^{\alpha}[H_t L_t]^{1-\alpha} - C_t - X_t - G_t - K_{t+1}] \\ &+ \beta \mu_{t+1}[(B_t {G_t}^{\omega} {C_t}^{1-\omega})^{1-\upsilon}(D_t {\phi_t}^{\gamma} - H_{t+1}] \end{aligned}$$

Optimality leads to the maximum of $\mathcal{L}_t$ which respect to C_t, D_t, K_t, and H_t. Due to the methodology in McKenzie (1986) and De La Croix and Michel (2002), $\mathcal{L}_t$ is equal to the sum of the current utilities and the increase in the shadow value of the capital stock: $\beta q_{t+1} K_{t+1} - q_t K_t$ and $\beta \mu_{t+1} H_{t+1} - \mu_t H_t$, i.e.,

$$\begin{aligned} \mathcal{L}_t = &(1-\eta)lnC_t + lnX_t + \eta ln\phi_t \\ &+ \beta q_{t+1}[A{K_t}^{\alpha}[H_t L_t]^{1-\alpha} - C_t - X_t - G_t] - q_t K_t \\ &+ \beta \mu_{t+1}[(B_t {G_t}^{\omega} {C_t}^{1-\omega})^{1-\upsilon}(D_t {\phi_t}^{\gamma} H_t)^{\upsilon}] - \mu_t H_t \end{aligned} \tag{35}$$

First-order conditions for an interior solution (assuming it exists):

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = \frac{1-\eta}{C_t} - \beta q_{t+1} + \beta \mu_{t+1} \frac{H_{t+1}(1-\upsilon)(1-\omega)}{C_t} = 0 \tag{36}$$

$$\frac{\partial \mathcal{L}_t}{\partial X_t} = \frac{1}{X_t} - \beta q_{t+1} = 0 \tag{37}$$

$$\frac{\partial \mathcal{L}_t}{\partial \phi_t} = \frac{\eta}{\phi_t} - \beta q_{t+1}(1-\alpha)\frac{F(K_t)}{1-\phi_t} + \beta \mu_{t+1} \upsilon \gamma \frac{H_{t+1}}{t} = 0 \tag{38}$$

$$\frac{\partial \mathcal{L}_t}{\partial G_t} = -\beta q_{t+1} + \beta \mu_{t+1}(1-\upsilon)\omega \frac{H_{t+1}}{G_t} = 0 \tag{39}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = \beta q_{t+1} F'(K_t) - q_t = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}_t}{\partial H_t} = \beta q_{t+1}(1-\alpha)\frac{Y_t}{H_t} \beta \mu_{t+1} \upsilon \frac{H_{t+1}}{H_t} - \mu_t = 0 \tag{41}$$

Where β is the planner's discount factor, or social discount factor. When utilities are bounded, the assumption that β is smaller than 1 ensures that objective function is finite (i.e. $\sum_{t=0}^{\infty} \beta_t < \infty$). These conditions are necessary and sufficient for optimally of the constant path starting at k, as this path satisfies the transversality condition. This condition is indeed verified with constant quantities, since we assume $\beta < 1$.

Equation (39) is the optimal allocation of material investment in children. Equation (40) indicates consumption of old generation equal to the next period of shadow price times discount factor, describing the optimal allocation of old generation. Combing equation (39) and (40), the intuition is straightforward. In stationary equilibria, a switch of one unit a switch of one unit of consumption of an agent from youth generation to old age is not equivalent to removing one unit of consumption from each of the young agents in the living generation and giving the total amount to the contemporary older generation. Equation (41) reveals that the marginal utility of material investment in children corrected to parenting time is equalized to the marginal utility of consumption of the old generation. Note that, contrary to the standard Diamond (1965) model, this planner's first-order condition does not respect the first-order condition the individual chooses for himself in a decentralized economy. Equations (42) and (43) are the resource constraint of physical capital and human capital of economy, respectively.

Using (42), we obtain

$$q_{t+1} = \frac{q_t K_t}{\alpha\beta F(K_t)} \tag{42}$$

where $F'(K_t) = \alpha F(K_t)/K_t$ Substituting (44) into (41) to obtain

$$\mu_{t+1} = \Big[\frac{q_t K_t(1-\alpha)}{\alpha(1-\phi_t)} - \frac{\eta}{\phi_t}\Big]\frac{\phi_t}{\beta\upsilon\gamma H_{t+1}} \tag{43}$$

One can substitute (44) and (45) back into equation (39) to yield

$$C_t = \alpha F(K_t)\Big[\frac{(1-\eta)}{q_t K_t} - \frac{\eta(1-\upsilon)(1-\omega)}{q_t K_t \upsilon\gamma} + \frac{\phi_t}{1-\phi_t}\frac{(1-\alpha)(1-\upsilon)(1-\omega)}{\alpha\upsilon\gamma}\Big] \tag{44}$$

Substituting (44) into (40), on can observe

$$X_t = \frac{\alpha F(K_t)}{q_t K_t} \tag{45}$$

Plugging () and (), one can obtain

$$G_t = F(K_t)\Big[\frac{\phi_t}{1-\phi_t}\frac{\omega(1-\alpha)(1-\upsilon)}{\alpha\upsilon\gamma} - \frac{\alpha\eta\omega(1-\upsilon)}{q_t K_t \upsilon\gamma}\Big] \tag{46}$$

Substituting (46) and (47) into resource constrains to obtain

$$\begin{aligned} K_{t+1} =& F(K_t) \\ &- \alpha F(K_t)\Big[\frac{(1-\eta)}{q_t K_t} - \frac{\eta(1-\upsilon)(1-\omega)}{q_t K_t \upsilon\gamma} + \frac{\phi_t}{1-\phi_t}\frac{(1-\alpha)(1-\upsilon)(1-\omega)}{\alpha\upsilon\gamma}\Big] \\ &- \frac{\alpha F(K_t)}{q_t K_t} - F(K_t)\Big[\frac{\phi_t}{1-\phi_t}\frac{\omega(1-\alpha)(1-\upsilon)}{\alpha\upsilon\gamma} - \frac{\alpha\eta\omega(1-\upsilon)}{q_t K_t \upsilon\gamma}\Big] \end{aligned} \tag{47}$$

By multiplying the two (44) and (48) term by term, one have

$$q_{t+1}K_{t+1} = q_t K_t\Big[\frac{(1-\phi_t)\upsilon\gamma - \phi_t(1-\alpha)(1-\upsilon)}{\alpha\beta\upsilon\gamma(1-\phi_t)}\Big] + \frac{\beta(1-\upsilon) - \upsilon\gamma(2-\eta)}{\beta\upsilon\gamma}$$

since $F'(K_t) = \alpha F(K_t)/K_t$. Thus, $q_t K_t$ is solution to a linear dynamic equation, and the general solution of this equation is

$$q_t K_t = \frac{\alpha(1-\phi_t[\eta(1-\upsilon) - \upsilon\gamma(2-\eta)])}{(\alpha\beta-1)(1-\phi_t)\upsilon\gamma + \phi_t(1-\alpha)(1-\upsilon)} + \varepsilon\Big[\frac{(1-\phi_t)\upsilon\gamma - \phi_t(1-\alpha)(1-\upsilon)}{\alpha\beta\upsilon\gamma(1-\phi_t)}\Big]^t$$

with ε is a real constant. There is a unique solution that verifies the transversality condition $\lim_{t\to\infty}\beta^t q_t K_t = 0$: the constant solution (see De La Croix and Michel (2002)). The transversality condition states that the limit of the actual shadow value of the capital stock is equal to zero. Therefore, we have

$$q_t K_t = \frac{\alpha(1-\phi_t[\eta(1-\upsilon)-\upsilon\gamma(2-\eta)])}{(\alpha\beta-1)(1-\phi_t)\upsilon\gamma+\phi_t(1-\alpha)(1-\upsilon)} \tag{48}$$

Following pp. 31-32 of De La Croix and Michel (2002), since $q_t K_t = \textit{constant}$ satisfy the transversality condition, the solution for equation (12) should be $\mu_t H_t = \textit{constant}$:

$$\mu_t H_t = \mu_{t+1}H_{t+1} = q_t K_t \frac{(1-\alpha)}{\alpha(1-\beta\upsilon)} \tag{49}$$

Finally, we can use equation (41) to compute the allocation of parenting time in equilibrium:

$$\frac{\eta}{\phi_t} = q_t K_t\Big[\frac{1-\alpha}{\alpha}\frac{1}{1-\phi_t} - \frac{\beta\upsilon\gamma}{\alpha(1-\beta\upsilon)}\frac{1}{\phi_t}\Big] \tag{50}$$

Using $q_{t+1}Y_t = q_t K_t/\alpha\beta$ and (51), and rearranging (52) to obtain

$$\phi_t = \frac{\eta\upsilon\gamma(\alpha\beta-1)(1-\beta\gamma)+\beta\upsilon\gamma(1-\alpha)[\eta(1-\upsilon)-\upsilon\gamma(2-\eta)]}{\upsilon\gamma(1-\alpha)(2-\eta)[\beta\gamma(1+\upsilon)-1]-\eta\upsilon\gamma[(\alpha\beta-1)(1-\beta\gamma)+\beta(1-\alpha)(1-\upsilon)]} \tag{51}$$

The results of taking partial differential of (53) on φ, η, and γ are ambiguous. The numerical solution is provided in the later section.

Substituting (49) and (53) back into (49) and (47), we obtain

$$\begin{aligned} C_t =& F(K_t)\big[\gamma\upsilon(\alpha\beta-1)+(1-\alpha)(1-\upsilon)\frac{\phi_t}{1-\phi_t}\big]\frac{1-\eta-\frac{\eta(1-\upsilon)(1-\omega)}{\upsilon\gamma}}{\eta(1-\upsilon)-\upsilon\gamma(2-\eta)} \\ &+ F(K_t)\frac{\phi_t}{1-\phi_t}\frac{(1-\alpha(1-\upsilon)(1-\omega)}{\upsilon\gamma} \end{aligned} \tag{52}$$

where the first term is greater than zero. This is the necessary and sufficient condition ensuring that the material investment in children is strictly non-negative.

$$X_t = F(K_t)\frac{\gamma\upsilon(\alpha\beta-1)+(1-\alpha)(1-\upsilon)\frac{\phi_t}{1-\phi_t}}{\eta(1-\upsilon)-\upsilon\gamma(2-\eta)} \tag{53}$$

where () is greater than zero. This is the necessary and sufficient condition ensuring that the consumption of old generation is strictly non-negative.

$$G_t = F(K_t)\frac{(1-\upsilon)\omega}{\eta(1-\upsilon)-\upsilon\gamma(2-\eta)}\{\eta(1-\alpha\beta)-\frac{\phi_t}{1-\phi_t}(1-\alpha)(2-\eta)\} \quad (54)$$

$$K_{t+1} = F(K_t)\alpha\beta \quad (55)$$

5.3 steady state

In steady state,

$$k^* = \Delta^{\frac{1}{1-\alpha\upsilon}} \quad (56)$$

where $\Delta = K_{t+1}/H_{t+1}$

5.4 Growth Rate

Substituting (58) into (48), the growth rate in centralized economy takes the following form,

$$1+\varrho = \alpha\beta A_t(\frac{K_t}{H_t})^{\alpha-1}(1-\phi_t)^{1-\alpha} \quad (57)$$

Evaluating these conditions and the restrictions of the problem in the steady state, we have system that defines β, for a given planner's discount rate. The theoretical results in the previous and previous sections highlight two channels through which parenting time and material investment in children in the accumulation of human capital. First, incorporating parental time into the formulation of human capital leads to the accumulation of human capital by developing non-cognitive skills, and since the production function for non-cognitive skills is concave, non-cognitive skills raise future average non-cognitive skill. Second, parenting time in utility function implies individuals will increase parenting time. This decision compresses working time. This leads to lower income, results in lower consumption in old generation. Therefore, the trade-off between substitution effect and income effect is worth to discuss. The question arises which effect is more important, and how large the effects are quantitatively. To answer this question, we simulate our model and provide numerical simulations of the evolution of material investment in children, the consumption in old generation, and the parental time. Moreover, it is complex to compute the signs of the influence of φ, η, and γ on material investment in children, the consumption of old generation, and the parental time. Hence the next section will simulate those parameters in decentralized and centralized economy.

5.5 Benchmark Parameterization: centranlized economy

6 Conclusion

While the relationship between cognitive skills and growth as well as growth and education have been well explored, little effort has been made to understand the relationship between non-cognitive skills and growth. In this paper, we identify new channels through labour income taxation, capital income taxation, and subsidies on material investment in children affect growth. If parenting time influences the technology of non-cognitive skills formation, and the skills formation is a input of production of human capital, then changes in taxation also affect human capital accumulation through their impacts on material investment and time investment choices.

We show that the equilibrium allocation of parenting time depends on patience and the strength of altruism. We also find a increase in labour income taxation, capital income taxation, and subsidies on material investment in children results in a reduction in growth rate respectively. It is also found that significant impacts of labour income taxation. On the other hand, the impacts of capital income taxation and subsidies on material investment in children are ambiguous.

The parameter which determines the quality of parenting time is very important. The lower it is, the less favorable is the growth impacts of taxes cut. Casarico and Sommacal (2012) also points out the quality of parenting time matters for the human capital accumulation process and for the effects of taxation on growth. Moreover, the parameter which affects the marginal productivity between cognitive skills and non-cognitive skills plays a role in our analysis. The simulation result also suggests that the lower it is, the higher impacts of taxes cut on growth. In addition, higher share of public spending on education in the production of human capital with cognitive skills could result in that labour income taxation and subsidies on material investment in children are beneficial to growth.

This is the first step toward the application of non-cognitive skills via parenting time into the accumulation of human capital. This paper explains how parenting time with impure altruism affect the accumulation of human capital. A greater knowledge of the mechanisms behind learning is crucial for the design of more effective policies and interventions. Successful interventions alter parental behavior. Understanding why this happens, how non-cognitive skills can be incentivized, and through which channels non-cognitive skills influences child development are crucial tasks for the future studies of personality in economic growth literature.

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