Mobility with private information and privacy suppression*

Saara Hämäläinen† and Vaiva Petrikaitė‡

April 8, 2018

Abstract

We consider a problem of matching guests with suitable hosts in a dynamic, directed search market in which a visitor's private taste and plans are subject to change. Guests learn about hosts by visiting them personally, which reveals whether the destination merits a repeat visit. Hosts prefer to target guests with high willingness to pay but, assuming full privacy, cannot tell whether they should rely on previous visitors or hold a sale to attract new visitors.

We find that guests' private learning reduces matching frictions by sustaining longer visits to particularly fitting destinations. The strength of this effect depends on competition intensity. We also discover that a ban of tracking and targeting technologies may reduce consumer surplus. Specifically, access to visitor data enables their earlier hosts to respond more rapidly to demand changes, which can intensify competition and put downward pressure on prices.

Keywords: *Mobility, Private information, Privacy suppression, Directed search, Experience goods, Changing tastes.*

JEL-codes: D82, D83, L11.

---

*We thank CK Chi, Jan Eeckhout, Mats Godenhielm, Klaus Kultti, Sandro Shelegia, Makoto Watanabe, Yi Zheng, and the audiences in IAE/CSIC (Barcelona), HECER (Helsinki), ACE (Turku), IIOC 2017, SaM 2017, the 8^{th} Consumer Search and Switching Workshop, and the FEA XL Annual Meeting for their invaluable comments. V. Petrikaitė acknowledges the financial support of the Ministry of Economy and Competitiveness of Spain (Grants No. ECO2014-59959-P and No. ECO2015-74328-JIN (AEI/FEDER/UE)) and 2016 FBBVA grant "Innovación e información en la Economía Digital". S. Hämäläinen acknowledges the financial support of the OP Group research foundation. This paper has been previously circulated under the title "privacy and changing tastes in directed search".

†University of Helsinki and HECER. Email: saara.hamalainen@helsinki.fi.

‡Instituto de Análisis Económico and Barcelona GSE. Email: asvaiva@gmail.com.

1 Introduction

For better or worse, people are clearly on the move these days.[1] Increased mobility promises great gains to workers and tourists, their host firms and cities but also constitutes a complex global matching problem. Mobility frictions manifest, for example, in seasonal unemployment in agricultural Yuma (Arizona) and touristic congestion in Barcelona (Spain). Higher prices in crowded areas and lower property values in declining areas are a frequent concern for local residents.

In this paper we study regional matching problems with free mobility and learning. Nowadays, people have become more mobile and therefore have more experience of different places. There is also easier access to various sources of coordinating information on the internet. We provide an analysis for understanding how well markets can work under these new circumstances. Compared to previous work, we find that information transmission reduces matching frictions greatly.

We employ a so-called directed search approach (e.g., Peters (1984, 1991), Moen (1997), Julien et al. (2000), and Burdett et al. (2001)) to capture the essence of regional matching problems. These models study how markets price congestion externalities and coordination frictions when people make individually optimal mobility decisions in decentralized markets without any central planning.[2] The main twist in this paper is that local products and services, which hosts are providing to their guests, are horizontally differentiated experience goods. Consumer-good-specific match values are continuously (uniformly) distributed and, to account for varying preferences in hosting destinations, independent between time periods.

After a visit, guests learn their updated match values with their most recent hosts automatically at the beginning of the following period. Search is therefore directed not only by public prices but also by privately acquired match value information. The idea is that while staying in a certain place consumers have had time to personally see everything that is relevant to their well-being. So if they are researchers, they have met the local faculty and participated in local seminars. If they are surfers, they have tested the winds and waves. If they are movie-goers, they have checked out the cinemas. Immediately afterwards, they should thus know whether they would prefer to visit the place repeatedly or stay somewhere else.[3]

Guests visit their previous hosts again if their newest match values with them are sufficiently high (above a certain cut-off). In our model, familiarity thus generates a linkage between history and the future. Due to matching frictions, prices depend on how the market trades off hosts' availability

[1] The numbers of immigrant and non-immigrant visa issuances in the US were 617 752 and 10 381 491, respectively, in 2016 (source: US Department of State, Bureau of Consular Affairs) whereas a total of 4.7 million immigrated to an EU-28 country and 2.8 million emigrated from an EU-28 country in 2015 (source: Eurostat). Additionally, according to UNESCO, the US hosts annually about 0.9 million university students and, according to Eurostat, around 1.48 million tertiary level students were studying in 2014 in different EU countries than where they received their secondary education. The compilation of statistics by the World Tourism Organization also shows that, for example, the US received 156 245 overnight visitors in 2015 whereas China received 133 820 and Spain alone 109 764. The number of first time asylum applications was 1 259 955 in the EU in 2016 (source: Eurostat).

[2] See the recent excellent survey by Wright et al. (2017).

[3] Because the faculty, movies or beaches change, this may not be the case very long after the visit, and we thus assume that memory decays in a period.

for suitability (how the cut-off adjusts). Clarifying these tradeoffs is one of our main contributions.

This paper starts by asking (i) to what extent prices and previously acquired information can solve regional matching problems and then discusses (ii) how additional coordination (e.g., tracking recent visitors) and alternative mechanisms (e.g., second price auctions) can help in this. We characterize the stationary Markov equilibrium with and without visitor tracking; when tracking is enabled, hosts can observe their previous guests' new match values. Although we show that learning is generally welfare improving it also introduces some additional coordination problems. These problems are removed by giving hosts access to consumer-specific information for targeting the right visitor group. Thus, only the equilibrium without privacy is (constrained) efficient. Auctions help partially.

As an example of consumer tracking, the most popular travel site of 2017 Booking.com is built on a platform that collects information about the pages its visitors have browsed, what they have shared on social media, their reviews and their pictures, etc. These data can then be combined by routine algorithms with personal information for the purpose of direct marketing and targeted advertizing via display ads or emails.[4,5] In our model, tracking and targeting increases matching efficiency by informing hosts about their visitors' new visit plans and preferences. The ability to target prices to previous visitors invites price discrimination, which (i) enhances market efficiency but (ii) empowers surplus extraction.[6] However, we find that due to the general equilibrium effects of better market coordination, which put downward pressure on prices, privacy loss can also benefit visitors.

What is notable about learning in general is that long, repeat visits to particularly nice destinations render prices, match values and trade probabilities positively correlated. The realized match surplus can thus exceed that of perfectly coordinated markets with random matches, which has earlier served as a typically unattainable benchmark in the literature. This paper also obtains novel results concerning match value cut-offs that trigger a mobility decision; we demonstrate that the mobility of people never ceases (cut-off is interior for all discount factors) and booming locations (low cut-off: less screening) and busting locations (high cut-off: more screening) respond differently to competition and consumer tracking. Screening is generally too intensive with private information because hosts only care about their own price and trade probability and fail to account for positive externalities that matching with their previous guests imposes on others. Privacy suppression removes the problem by allowing hosts to internalize these externalities.

Our findings require us to apply a model which incorporates general equilibrium effects and endogenous surplus division. Directed search models feature both desirable properties. We use standard modeling assumptions from the literature. It is assumed that each host (city or firm) has limited capacity of accommodation or vacancies available. Here a host can accommodate only one person per period without congestion.[7] The exact number of people that fit into a destination is

[4]For a more complex selling mechanism, BookingSuite provides a revenue management toolbox for tailoring prices to demand over the booking horizon.

[5]As for labor-based mobility, human resources management uses various communication channels to keep updated on key persons' job satisfaction.

[6]The ability to attract the same visitor multiple times resembles commodity bundling in Adams and Yellen (1976), which also discusses these tradeoffs.

[7]Clearly, most providers of local goods and services operate under limited capacity; exceeding the limits lowers the value of the commodity.

inconsequential.[8] Before choosing a location, visitors observe the price (salary) level of each host and form expectations about the prevailing congestion. Thus, because of negative externalities that lower the value of staying in a particularly popular destination, better terms of trade attract more visitors to a host but many would still prefer a less congested host.

The literature concentrates on coordination frictions modeled by focusing on symmetric visit strategies. This means that, in a large decentralized market populated by strangers, consumers visit equally often destinations that have no observable differences from their view point. As a consequence, because anonymous consumers cannot agree who visits whom beforehand, some of the hosts might have too many visitors while others might receive too few. Our paper continues with this usual assumption. However, here visitors can also condition their strategies on recently learned match value information. Access to information about some hosts – while others remain unknown – appears to be crucial for mobility. Many people have experience about particular cities as workers and tourists and hence either avoid them or visit them again. In this paper, we show how this privately observed information helps in matching the right people and places. Accordingly, we are able to make progress in understanding better coordinating forces that arise in steady state setups with learning.[9] Most directed search models are either static or do not make use of the rich strategic possibilities that open up in dynamic markets.[10]

At the beginning of each period, our market is populated by two kinds of guests and hosts: *popular* hosts were recently visited by a guest who is now *informed* about his or her match value with the host. Knowing this, popular hosts increase their prices after a visit. A higher price level discourages uninformed visitors from choosing a popular host. This can help its previous visitor who then faces less congestion. It is therefore possible to achieve higher trading probabilities between informed guests and their hosts than unpopular hosts and uninformed guests.

In particular, in booming markets where competition is softer, we find that popular hosts insure themselves against no-show of previous consumers by also targeting some new visitors (they target a "mass market"). In busting markets where competition is harder, we can show this would be too expensive and popular hosts instead concentrate on attracting only the most motivated previous visitors (they target a "niche market"). Intuition is that demand from informed visitors is more (less) inelastic for highest (lowest) match values than the demand from new guests, which becomes more elastic with stronger competition. Monopolistic competition requires keeping the elasticity fixed.

This entails that prices have both a sorting and screening role (e.g., Eeckhout and Kircher, 2010, 2011): First, only the most satisfied informed visitors make repeat visits to the same host (screening). Second, either all or most of those who visit a new host are channeled to unpopular hosts (sorting). Moreover, the extent of sorting and the intensity of screening are determined endogenously: booming markets feature only impartial sorting and less intensive screening. Without visitor privacy, sorting is perfect and screening less intensive, which is also consistent with efficiency.

[8]For discussion and extensions to a larger number of products, see Burdett et al. (2001), Lester (2010) and Godenhielm and Kultti (2015).

[9]Our model retains the spirit of a directed search approach in that we make no use of complicated punishment strategies and we let only payoff-relevant features direct search decisions.

[10]Camera and Kim (2016) observe, for example, that if hosts apply trigger strategies for punishment, it is possible to sustain a continuum of collusive outcomes in dynamic directed search.

Our results are related to literature on directed search, consumer privacy, and switching costs. Directed search has recently developed to incorporate more heterogeneity (see Guerrieri et al. (2010), Watanabe (2010), Menzio and Trachter (2015), Albrecht et al. (2016), Chang (2017), etc.) into the originally simpler setting. We contribute to this research program by analyzing the effects of learning and consumer tracking and investigating (i) how this new information alleviates the mobility problem and (ii) how market conditions affect the prevalence of prolonged matching.[11]

Shi (2016) also studies endogenous customer relationships and their effects on price dynamics.[12] However, customer relationships in Shi (2016) are sustained by priority treatment for previous customers and here they arise only from price differences and prior information.[13] The models also have other noteworthy differences. In his case, the match value between a consumer and a product is persistent and either low (consumer visits the same firm) or high (consumer visits some other firm). In our case, this match value is distributed continuously and the cut-off which tells whether the guest visits the same host again is determined endogenously.

Coles and Eeckhout (2000) find that introducing visitor heterogeneity results in perfectly directed search and coordinated visit strategies; sellers post direct mechanisms in the spirit of Vickrey-Clarke-Groves that induce efficient choices. Instead, consumer search in our paper never becomes perfectly directed because low match value consumers visit unrelated hosts. Moreover, individual hosts cannot perfectly separate because there are only two kinds of hosts in the market. In Coles and Eeckhout (2000) every consumer is informed about her relative position in the market as in assignment literature whereas we study the usual independent private values case.

Moraga-González and Watanabe (2016) consider directed, sequential search for a satisfactory match. They observe that efficiency requires consumers to put higher emphasis on product availability than product suitability. Their paper does not study dynamic market interactions and learning about products from earlier purchasing experience. Delacroix and Shi (2013) explore the signaling potential of posted prices. They allow goods to have an experience component but their focus is on the role of higher prices in signaling product quality. Here we have no vertical product differences but higher prices essentially signal lower availability to uninformed visitors.

Acquisti and Varian (2005) and Taylor (2004) find that consumer privacy may increase welfare by encouraging low valuation consumers to purchase earlier. Both papers analyze two-period models with horizontally differentiated products in which firms set the same price for all consumers with consumer privacy but can apply price discrimination when the history of consumer purchases is observed.[14] Without privacy, firms charge such high prices in the first period that only high valuation consumers are willing to pay for them. The price is lower in the second period and thus the previously excluded low valuation consumers postpone buying until that stage. This does not arise in our steady-

[11]Relatedly, Keisuke et al. (2014) consider directed labor search with moving costs, which give rise to inefficiency. Here the source of inefficiencies is asymmetric information. Li and Weng (2017) study efficient learning about constant match quality with directed on-the-job search.

[12]The introduction of match values to the final paper, Shi (2016), is contemporary to this work. The paper which existed when we started to present our findings in 2016, Shi (2013), has no match values. Priority treatment is then the only way to sustain repeat purchases.

[13]The effects of customer relationships are also studied in the recent work by Gourio and Rudanko (2014).

[14]Acquisti and Varian (2005) analyzes a monopoly model whereas Taylor (2004) studies a duopoly market.

state setup.[15] Rather, here gains from consumer privacy suppression come from (i) reduced search frictions, (ii) improved match quality, and (iii) competitive general equilibrium effects. While the latter mechanism is new, the former ones have some resemblance to earlier results.[16,17] As discussed by Leahy (2011), Campbell (1997), Varian (2009), and Wang and Petrison (1993) consumers may tolerate privacy intrusion if it helps firms to better address their individual needs.

The paper has connections to switching costs literature as well.[18] Informed visitors are reluctant to switch to new hosts that offer them a lower trade probability and, perhaps, a smaller expected match value. Most of these switching costs are thus endogenous and they originate from differences between the prices that target old and new consumers. Shy and Stenbacka (2015) study switching costs and find that welfare and consumer surplus increase with more protection. Weaker privacy allows for more aggressive poaching (lower prices for new consumers) but helps price discrimination (higher prices for old consumers). In our work the former effect can be dominant but in their paper the latter effect is always stronger. The main distinction from switching costs literature is that in our model price differences arise, ultimately, because of limited capacity. We have many small hosts. Furthermore, a consumer's match values change from period to period, which prevents hosts from harvesting informed consumers permanently. Still, the typical pricing pattern with switching costs features bargains for new consumers and ripoffs for old consumers much like here.[19,20]

The rest of this paper is organized as follows. Section 2 describes our modeling assumptions. We characterize the market with consumer privacy in Section 3 and the market without consumer privacy in Section 4. In Section 5, we compare different privacy regimes and, in Section 6, we consider second price auctions. Section 7 provides a concluding discussion. The proofs and additional derivations are in the Appendix.

2 Model

We consider a large decentralized market in infinite discrete time $t \in \mathbb{Z}$. There are N^h *hosts* (firms, cities) and N^g *guests* (consumers, visitors) both of whom are infinitely lived. The number of hosts and the number of guests are large $N^g \to \infty$ and $N^h \to \infty$ but their ratio $\nu = N^h/N^g$ is strictly

[15] As discussed by Cabral (2016), a downside of two period models relative to steady state setups is the start-period-final-period effects that can make the setup too clean.

[16] Our work can be viewed to formalize the hypothesis of Stigler (1980) and Posner (1981), that privacy may prevent the realization of matching benefits (Hui and Png, 2006).

[17] Lauermann (2012) notices further that welfare effects of consumer privacy can be reversed when bilateral trade is embedded in a dynamic setting. In a one-shot game, consumer privacy leads to market failure, whereas in a dynamic setting, the market is almost efficient without privacy but otherwise inefficient. In his model, privacy benefits consumers because they obtain no surplus without privacy.

[18] See Farrell and Klemperer (2007) for a review of the literature.

[19] There is a slight twist here if we look at values and not prices: in a separating equilibrium the market value of new consumers and old consumers is the same, whereas in a semi-pooling equilibrium the market value is greater for old consumers despite their paying higher prices. Since there is no true switching cost or consumer holdup problem, old consumers cannot obtain less than new consumers.

[20] For example, Cabral (2016), Fudenberg and Tirole (2000) and Chen (1997) allow firms to charge different prices to old and new consumers, with higher prices for old consumers.

positive and finite.[21] All market participants are equally patient and their common discount factor equals $\delta \in (0,1)$. For concreteness, we later refer to guests by feminine pronouns and to hosts by masculine pronouns.

Differences in hosts' availability and suitability

We model congestion by assuming that hosts are small and they have limited capacity. Each host can comfortably accommodate exactly one guest per period. Their identical costs of production are normalized to zero per unit.

Some environments suit a visitor's taste and plans better than others. The utility of a consumer i for visiting a location j equals the difference $u_{ij} - p_j$, where u_{ij} is a consumer-and-location specific match value and p_j stands for the price.[22] These match values are distributed uniformly over the unit interval $[0,1]$ and are independent across consumers, locations and periods.

Accommodation involves the provision of local goods like hotel rooms in a particular surrounding environment. Every guest needs only one unit of accommodation at each time and she is restricted to visiting only one host in a period.

Moreover, we make the standard assumption that consumers cannot coordinate their visit strategies with each other. Therefore, visitors use symmetric mixed strategies and visit with the same probabilities all equal looking hosts. To be specific, consumers can condition their visit strategies only on public prices and their private match values but not, say, on hosts' names. As usual, this could lead to either local excess demand or supply because either fewer than one or more than one consumer might visit the same host.

Technically, we model limited capacity by making the simplifying assumption that a host is matched with one of the guests at random. Only one of the guests is thereby provided the opportunity to enjoy local goods and experiment with them. For the prospect of future visits, this implies that only this one guest may visit that host again whereas the others almost never come back. The other guests could either be so disturbed by congestion that they would never consider returning or would not get a clear idea about the place.[23]

We have now described our model in terms of matching visitors with suitable holiday resorts. However, if we want to interpret this as a labor market model, the price could stand for the local purchasing power of a host's salary offer. Hosts could be, for example, different IT departments or laboratories in which visitors (IT specialists or researchers) could consider working.

Variations in guests' information and preferences

Hence, there are two types of hosts and two types of guests in the market each period t. We refer to them as type 0 and type 1 depending on whether they traded in period $t-1$. Type 1 consumers are *informed* consumers. They matched with some hosts in period $t-1$ and can thus infer their new match value with those hosts in period t without necessarily visiting them. Type 0 consumers are *uninformed* consumers. They did not succeed in matching with any host in period $t-1$ and are thereby now unaware of their match value with any host in period t. Likewise, all hosts who were

[21] See Burdett et al. (2001) or the survey by Wright et al. (2017).
[22] The option of not matching and trading gives visitors no utility.
[23] Without learning, visitors also come back with probability zero.

visited in period $t-1$ are of type 1 in period t whereas a host who did not receive a visit in period $t-1$ becomes or remains a host of type 0 in period t. Type 1 hosts are called popular (or related) and type 0 hosts unpopular (or unrelated). Note that popularity is information that any host is able to observe directly from his own sales accounts. To keep the analysis tractable, hosts' types become common knowledge at the beginning of each time period. For simplicity, it is further assumed that consumers have a finite memory, which lasts for only one period. That is, if an informed consumer matches with some host in period t, she learns her newest match value with her most recent host in period $t+1$ but not with any other hosts she may have visited earlier.[24] We hence refer to an informed consumer who knows her match value with host j in time t also as *j's informed/previous/related consumer.*

The assumption that consumers' match values are independently distributed over time provides a relatively simple framework for addressing the question of how local hosts are affected by variation in consumers' preferences and thus the risk of losing demand. A family may spend three memorable vacations in a certain spa resort in Corsica but then want to switch to another holiday destination. Foreign visitors may work some time on a certain project abroad but then prefer to return home or switch to another project elsewhere. Without a working communication channel, these changes often come as a surprise to local hosts.

We abstract from possible preference persistence for several reasons. First, the analysis is simpler with changing tastes than constant tastes. Fixed preferences give rise to additional learning dynamics because then also hosts start to learn about the preferences of their recent guests. In this paper, the focus is on other aspects of mobility. Second, in the long-run mobility is mostly driven by changing tastes because in steady states coordination problems disappear with constant tastes. Such mobility dynamics are not particularly interesting. With constant tastes we thus need to add new exogenous elements to sustain mobility while changing tastes present a natural explanation to equilibrium mobility in themselves.

Furthermore, regarding the welfare effects of learning and privacy, both assumptions about preferences work qualitatively quite alike. We therefore do not lose much either by using our simpler setting and abstracting from possible correlation of consumer preferences.[25,26] A model like this where consumer tastes change also provides insights into setups where different buyer cohorts enter and leave the market, become mobile and then settle down. A newly drawn match value could be interpreted, say, as a replacement of a satiated consumer by a non-satiated consumer with word-of-mouth communication between the cohorts.

Timing, trading and privacy

We focus on stationary Markov equilibria in type-symmetric strategies.[27] To summarize, our game has the following extensive form in each period:

[24] The underlying idea is that tastes change rapidly and observed information becomes quickly obsolete.

[25] Kovbasyuk and Spagnolo (2017) allow the reputation of a host to last more than one period, which is equivalent to letting the consumer observe her match value with a host for more than one period.

[26] Caminal and Vives (1996) and Caminal and Vives (1999) study correlation of consumers' match values by allowing consumers to infer the quality of experience goods from past market shares and prices.

[27] The type of a host is either popular or unpopular, $j = 0, 1$, and the type of a guest is either informed or uninformed, $i = 0, 1$, and for the informed it includes also the most recent match value, u_{ij}.

1. Nature draws new match values between all possible host-guest pairs. Informed visitors (of type $i = 1$) learn their current match values with their latest hosts (of type $j = 1$). Otherwise, all visitors remain ignorant of their newly drawn match values.

2. Hosts choose their prices simultaneously and they are published immediately afterwards. Type $j = 0$ hosts post p_0 and type $j = 1$ hosts post p_1.

3. Consumers choose which hosts to visit. They visit with equal probabilities all hosts who offer them the same price p_j and the same expected or observed match value u_{ij}.

4. Hosts match and trade randomly with one of the guests who visits them: the host obtains profit p_j and the guest derives utility $u_{ij} - p_j$.

This game assumes full consumer privacy. Match values that informed visitors observe in the first stage remain their private information. We study later the game without privacy. In that case, both the informed visitor and her last host observe the visitor's new match value. After acquiring this information, the host can also offer his previous guest a special price.[28]

2.1 Visit probabilities and queue lengths

We turn to analyzing market dynamics. The fraction of visitors who succeed to consume local goods in period t is denoted by ρ_t. There are hence $\rho_t N^g$ pairs of hosts and guests who match with each other in period t. Also, this entails that there are $(1 - \rho_t) N^g$ uninformed consumers and $\rho_t N^g$ informed consumers in period $t+1$. In the following analysis, ρ_t acts as an endogenous state variable. Its evolution dynamics are derived in Section 2.2. In stationary Markov equilibria, which we focus on, $\rho_t = \rho_{t+1} = \rho$.[29]

Every period visitors decide where to go. We denote the probability that a visitor of type i approaches a particular host of type j by $\theta_{ij} = \theta_{ij}[u_{ij}]$, where u_{ij} captures the visitor's prior information about the (expected or observed) match value with this host. The value this visit generates for the consumer is denoted by $v_{ij} = v_{ij}[u_{ij}]$. The maximum expected utility that a consumer of type i can obtain in the market is denoted by $V_i = V_i[u_{ij}]$. In the literature, this is called market utility.[30,31]

The optimality of visitor strategies requires that the following inequalities (1) hold with complementary slackness for all $i = 1, 2$ and for all $j = 1, 2$.

$$\theta_{ij}[u_{ij}] \geq 0 \text{ and } v_{ij}[u_{ij}] \leq V_i[u_{ij}]. \tag{1}$$

First, consider a consumer of type 0. This consumer has no recent match history on which her visit strategies could be conditioned, i.e., $u_{ij} = 1/2$. Hence, she visits all hosts that have the same

[28]In booming markets, targeting is implicit (no other consumer would accept the special price) and, in busting markets, targeting is explicit (some other consumer might accept the special price).

[29]To prevent confusion with multiple indexes, we remove the period-specific index unless it is necessary.

[30]V_i is consumer i's indirect utility of pursuing her optimal visit strategy.

[31]For simplicity, we emphasize the dependence on u_{ij} only in this section.

type and the same price with equal probabilities. Because the visiting probabilities of a consumer add up to one, the following condition must hold in equilibrium.

$$\theta_{00}\left[1/2\right]\left(N^{h}-\rho N^{g}\right)+\theta_{01}\left[1/2\right]\rho N^{g}=1. \tag{2}$$

Consider a visitor of type 1 next. In contrast to the previous case, the visitor now has recent match history and has thereby just learned her new match value with the host she was matched with last period. This match value u_{ij} is generally different from the average match value 1/2 that the consumer can expect to obtain at some random host.

As a consequence, if the recently observed match value is sufficiently high, the consumer has an incentive to visit again the same host with probability one, whereas if u_{ij} is low enough all things considered, the informed consumer rather visits some other host and thereby behaves like uninformed consumers. Optimally, the decision follows a cut-off strategy:

$$\theta_{11(r)}\left[u_{11(r)}\right]=\begin{cases}1, & \text{if } u_{11(r)}\geq a,\\ 0, & \text{if } u_{11(r)}<a.\end{cases}$$

Above, $u_{11(r)}$ denotes the match value with the most recent host $1(r)$ and a denotes the cut-off match value such that, if $u_{11(r)}\geq a$, the visitor goes back to this host.[32] Otherwise, the consumer visits some other host and applies similar contact strategies with them as any of the uninformed consumers, i.e., $\theta_{01}\left[1/2\right]=\theta_{11}\left[1/2\right]$ and $\theta_{10}\left[1/2\right]=\theta_{00}\left[1/2\right]$. In this case, an almost identical adding up constraint applies to consumers of type 1 as type 0.[33]

$$\theta_{10}\left[1/2\right]\left(N^{h}-\rho N^{g}\right)+\theta_{11}\left[1/2\right]\left(\rho N^{g}-1\right)=1. \tag{3}$$

We can now proceed to derive the expected measures of new (i.e., uninformed or informed but unrelated) consumers who approach some host they did not visit in the most recent time period. We denote by q_{ij} the so called queue length of new consumers of type $i\in\{0,1\}$ at a host of type $j\in\{0,1\}$. Intuitively, this is just the number (measure) of new consumers of type i who visit hosts of type j over the number (measure) of these hosts.[34]

The equilibrium queue length of new consumers at a host of type 0 satisfies the following

$$q_{0}:=\underbrace{N^{g}\left(1-\rho\right)\theta_{00}\left[1/2\right]}_{q_{00}}+\underbrace{N^{g}\rho a\theta_{10}\left[1/2\right]}_{q_{10}}, \tag{4}$$

whereas the queue length of new consumers at a host of type 1 can be derived as follows

$$q_{1}:=\underbrace{N^{g}\left(1-\rho\right)\theta_{01}\left[1/2\right]}_{q_{01}}+\underbrace{\left(N^{g}\rho-1\right)a\theta_{11}\left[1/2\right]}_{q_{11}}. \tag{5}$$

[32]The optimal match value threshold a is determined in the equilibrium.

[33]Note that $u_{i1(r)}\neq 1/2$ almost surely and, if $\theta_{11(r)}\left[u_{i1(r)}\right]=1$ with host $1(r)$, then $\theta_{1,j(-r)}\left[1/2\right]=0$ with other hosts $j(-r)$.

[34]Technically, it is the Radon-Nikodym derivative, in other words, a change of measure. See Eeckhout and Kircher (2010).

In both expressions, the first summand represents the number of uninformed consumers and the second summand represents the number of informed but unrelated consumers, whose match values u_{ij} are less than a with their previous hosts.

By joining the adding up constraints (2) and (3) with the definitions of queue lengths (4) and (5) we obtain the following equilibrium condition that relates the queue lengths q_0 and q_1, cut-off a, and state variable ρ to market tightness ν:

$$\rho(1-a) + (\nu - \rho)q_0 + \rho q_1 = 1 \tag{6}$$

This shows where the average consumer is: $\rho(1-a)$ is the probability that she visits the same host. Otherwise, the consumer is either in queue q_0 or in queue q_1 (contacting one of the $(\nu - \rho)N^g$ type 0 hosts or one of the $\rho(N^g - 1)$ type 1 hosts).

2.2 Trade probabilities and state dynamics

A host always trades when he is visited by a consumer. By following standard arguments, we observe that a host of type 0 is not visited by any consumer with probability e^{-q_0}. Thus, a host of type 0 trades with probability $1 - e^{-q_0}$.

Similarly, a host of type 1 trades with probability $1 - e^{-q_1}a$, where e^{-q_1} is the probability that no unrelated consumer comes to visit this host, and a is the probability that the host's previous consumer has a match value below a.

Because hosts must treat equally all consumers who come to them, we know that if n consumers approach a host, all the consumers have the same probability $1/n$ of getting matched. Therefore, the probability of trading for an unrelated consumer with a host of type 0 is $\left(1 - e^{-q_0}\right)/q_0$. Likewise, the probability of trading for the related consumer with the host of type 1 is $\left(1 - e^{-q_1}\right)/q_1$.

Trading probabilities with a host of type 1 are a bit different for other visiting consumers, however, because the presence of the host's related informed consumer reduces their chances of trading. A host of type 1 is visited by his related, informed consumer with probability $1 - a$. We thus obtain that the trade probability of any other unrelated, uninformed consumer with this host is[35]

$$\frac{1 - e^{-q_1}}{q_1} - \frac{1 - e^{-q_1} - q_1 e^{-q_1}}{q_1^2}\left(1 - a\right).$$

To shorten the expressions of payoffs, we use the following notation for trade probabilities and

[35] The probability of matching with the host is the same for both the previous visitor and new uninformed visitors but their trade probabilities differ because new visitors compete both with other new visitors and the previous visitor. The details are in the Appendix. The idea is that, once a guest has chosen to visit a certain host of type j, $\frac{q_j^n}{n!}e^{-q_j}$ denotes the probability that n competing new consumers also come to visit it. As we know well, in a continuum market like this, new visits follow a Poisson model whereas, in a finite market, new visits would follow a Binomial model.

auxiliary variables:

$$\beta_j^h(q_j) := 1 - e^{-q_j},\ \text{for } j \in \{0,1\},$$
$$\beta_j^g(q_j) := \frac{1 - e^{-q_j}}{q_j},\ \text{for } j \in \{0,1\},$$
$$\alpha_1^g(q_j) := \frac{1 - e^{-q_1} - q_1 e^{-q_1}}{q_1^2}.$$

The number of guests of type 1 (the number of hosts of type 1) in period $t+1$ depends on how many hosts have traded in period t. The number of type 0 hosts who traded in period t is $(1 - e^{-q_0})\left(N^h - \rho_t N^g\right)$ and the number of type 1 hosts who traded in period t is $(1 - e^{-q_1} a)\,\rho_t N^g$. The sum of these gives us the number of informed, related consumers in period $t+1$

$$\rho_{t+1} N^g = \left(1 - e^{-q_0}\right)\left(N^h - \rho_t N^g\right) + \left(1 - e^{-q_1} a\right)\rho_t N^g. \tag{7}$$

By dividing equation (7) by N^g, we obtain equation (8)

$$\rho_{t+1} = \left(1 - e^{-q_0}\right)(\nu - \rho_t) + \left(1 - e^{-q_1} a\right)\rho_t, \tag{8}$$

which immediately gives us the steady-state value of ρ

$$\rho = \frac{1 - e^{-q_0}}{1 - e^{-q_0} + e^{-q_1} a}\nu. \tag{9}$$

Note that, because ρ denotes the share of informed visitors, it can be regarded as a measure of market information.[36] Successful trading increases market information: ρ is decreasing in trading failure e^{-q_0} (with type 0 hosts) and $e^{-q_1} a$ (with type 1 hosts).

3 Market equilibrium with full privacy

We next move on to characterize the stationary Markov equilibrium for the case with $\delta = 0$ (only history-related linkages between periods). This choice keeps the exposition clear and simple and keeps it comparable with the standard case in the literature where the market is static. Our dynamic modeling approach is nevertheless crucial here because it enables us to endogenize prior information that arises from recent visits. The supplementary material that contains the case with $\delta \in (0,1)$ (also future-related linkages between periods) is relegated to the Online Appendix.[37]

Figures 1b and 1a describe the feasible equilibrium patterns with privacy. A popular host of type 1 charges either (i) a higher price that is attractive only to his previous consumer, who visits the host

[36] Because ρ depends on history, our model has endogenous types. As in Peters (2016), a trading stage follows a previous stage in which consumers have explored some offers.

[37] We comment on the slight differences that arise when there are not only information linkages but also continuation value linkages between periods later in the main text.

only if she has a sufficiently high match value, $u_{ij} \geq a$, or (ii) a lower price that is attractive also to new unrelated consumers, whose visits serve as a partial insurance against a low match value, $u_{ij} < a$. We call the first case *a separating equilibrium* and the second case *a semi-pooling equilibrium*. An unpopular host of type 0 has no recent visit history and thus attracts only new visitors.

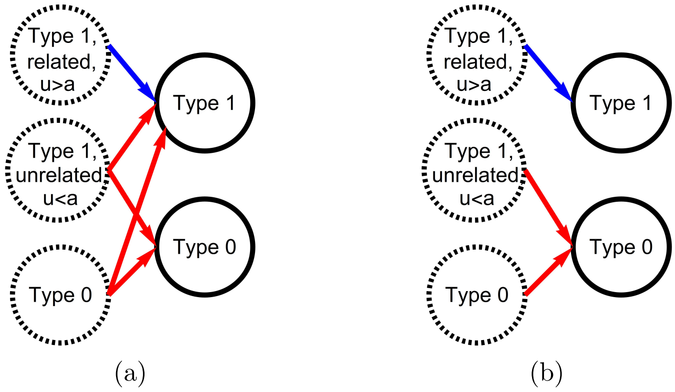


Figure 1: Semi-pooling equilibrium (a) and separating equilibrium (b) with consumer privacy. Dashed and solid circles represent guests and hosts, respectively.

We proceed by defining equilibrium conditions and characterize the circumstances where different equilibria arise. We then compare welfare in each case to markets without learning and other closely related literature benchmarks.

3.1 Equilibrium definition

The values that a consumer derives from visiting, respectively, an unrelated host of type 0, an unrelated host of type 1, or her related host $1(r)$ with a known match value $u_{11(r)}$ are shown below:

$$\begin{aligned} v_{00} &= \beta_0^g \left(1/2 - p_0\right), \\ v_{01} &= \left(\beta_1^g - \alpha_1^g(1-a)\right)\left(1/2 - p_1\right), \text{ and} \\ v_{11(r)} &= \beta_1^g \left(u_{11(r)} - p_1\right). \end{aligned}$$

Since hosts are horizontally differentiated, a consumer selects her next host considering *both* the match value with this host (net of the price) *and* the level of congestion and the probability of trade.

Consumers maximize expected utility. In constructing an equilibrium, it is therefore necessary that consumers have no profitable deviations from their postulated visit strategies. The standard way to formalize this condition is to express the problem of a host as an optimization problem constrained by market utility V_0. To make sure that new consumers remain indifferent between different places, the queue length q_0 that a type 0 host attracts by posting p_0 is determined by[38]

$$v_{00} = \beta_0^g \left(1/2 - p_0\right) = V_0. \tag{10}$$

[38]For the foundations of this market utility approach, see Wright et al. (2017).

The intuitive idea is that a host must provide his visitors at least as much utility as they obtain at maximum by visiting some other host. To attract more visitors, a host should generally have either a lower price or provide his visitor a more suitable or less congested environment. The outside option of visitors is captured by market utility V_0, which is endogenous. However, because the impact on the larger market is negligible, it can be taken as given in a host's decision problem.

Specifically, if a host sets a lower price than other hosts, his trade probability keeps increasing until his guests are indifferent between approaching him and some other host. It increases until the marginal visitor gets V_0. Nevertheless, as consumers trade off lower prices for increasing congestion, a price cut never attracts all consumers to one host who can only accommodate one of the guests. The demand from a host is (piecewise) continuous and decreasing in price.[39]

The main difference between the two feasible equilibrium patterns which arise in our model is that the queue length q_1 for a type 1 host is zero in a separating equilibrium and positive in a semi-pooling equilibrium. It is determined jointly with equilibrium cut-off a. That is, if a type 1 host is visited by some new consumers ($q_1 > 0$), his match value cut-off for previous visitors a and his queue length of unrelated visitors q_1 must jointly satisfy the following equations

$$v_{1j} = \beta_1^g (a - p_1) = V_0, \tag{11}$$
$$v_{01} = (\beta_1^g - \alpha_1^g(1-a))(1/2 - p_1) = V_0. \tag{12}$$

On the other hand, if a type 1 host is only visited by previous consumers ($q_1 = 0$), the cut-off a is determined by

$$v_{1j} = a - p_1 = V_0. \tag{13}$$
$$v_{01} = (1 - 1/2(1-a))(1/2 - p_1) \leq V_0. \tag{14}$$

The last inequality is sufficient to guarantee that unrelated consumers have no incentive to approach a type 1 host.[40]

The problem of a type 0 host can now be written simply as

$$J_0 = \max_{p_0} \quad (1 - e^{-q_0})p_0 \tag{15}$$
$$\text{s.t.} \quad \beta_0^g (1/2 - p_0) = V_0.$$

[39] As a and q_1 are determined jointly, a continuity break can occur at the point where we move from (separating) cases where $a > 1/2$ and $q_1 = 0$ to (semi-pooling) cases where $a < 1/2$ and $q_1 > 0$.

[40] If it is not beneficial for an unrelated consumer to visit this host when there are no other unrelated consumers, i.e., with $q_1 = 0$, it cannot be beneficial with more congestion either, i.e. with $q_1 > 0$.

The problem of a type 1 host can be expressed similarly as

$$
\begin{aligned}
J_1 = & \max_{p_1} \quad (1 - e^{-q_1} a) p_1 && (16)\\
\text{s.t.} \quad & \beta_1^g (a - p_1) = V_0, && (17)\\
\text{and} \quad & (\beta_1^g - \alpha_1^g(1-a)) (1/2 - p_1) = V_0, \text{ if } q_1 > 0, && (18)\\
\text{or} \quad & a - p_1 = V_0, \\
\text{and} \quad & (1 - 1/2(1-a)) (1/2 - p_1) \leq V_0, \text{ if } q_1 = 0.
\end{aligned}
$$

Both hosts face a tradeoff between the expected demand (trade probability $1 - e^{-q_0}$ or $1 - e^{-q_1} a$) and the price. However, the demand from popular hosts is less elastic at a given price because a type 1 host has a related consumer but a type 0 host does not. Popularity therefore increases prices.

The constraint set of the type 0 host's problem defines a one-to-one relationship between the price p_0 and the queue length q_0: congestion determines the price. To simplify the problem, we can thus think that, instead of choosing the price, the type 0 host selects directly the queue length q_0.

$$
J_0 = \max_{q_0} \quad \beta_0^h \left(\frac{1}{2} - \frac{V_0}{\beta_0^g} \right). \tag{19}
$$

The constraint set of the type 1 host is more complicated. His price p_1 may not define a unique cut-off a and a unique queue length q_1. To render the problem tractable, we hence apply the following selection argument: when there are many pairs (a, q_1) consistent with a given price p_1, we assume that the host can select the profit maximizing (a, q_1). With this assumption, we can rewrite the problem of the type 1 host simply as follows.

$$
\begin{aligned}
J_1 = & \max_{a, q_1} \quad (1 - e^{-q_1} a) \left(a - \frac{V_0}{\beta_1^g} \right) && (20)\\
\text{s.t.} \quad & (\beta_1^g - \alpha_1^g(1-a)) \left(1/2 - a + \frac{V_0}{\beta_1^g} \right) = V_0, \text{ if } q_1 > 0, \\
\text{or} \quad & (1 - 1/2(1-a)) (1/2 - a + V_0) \leq V_0, \text{ if } q_1 = 0.
\end{aligned}
$$

Lemma 1 highlights an important consequence of optimal directed search.

Lemma 1 *If $a \geq 1/2$, then $q_1 = 0$ or, equivalently, if $q_1 > 0$, then $a < 1/2$.*

Proof. By (17) and (18), a necessary condition for $q_1 > 0$ is that the following equality is satisfied:

$$
1/2 - a = V_0 \left(\frac{1}{\beta_1^g - \alpha_1^g(1-a)} - \frac{1}{\beta_1^g} \right). \tag{21}
$$

Hence, there is no solution (a, q_1) to (20) where $a \geq 1/2$ and $q_1 > 0$ because $\beta_1^g - \alpha_1^g(1-a) < \beta_1^g$. □

Lemma 1 shows that hosts of type 1 can assume two different marketing strategies; They can either target a niche market or a mass market. When a popular host targets a niche market, he sells only to very selective informed consumers, $a \geq 1/2$, and he does not sell to uninformed consumers,

$q_1 = 0$[41], whereas, when the host targets a mass market, he additionally sells to some uninformed consumers, $q_1 \geq 0$, and also to less selective informed consumers, $a < 1/2$. A niche market strategy features intensive screening in the sense that only the visitors that have above average match values visit the same host again. There is also perfect sorting because hosts cannot attract any new visitors using niche market strategy. A mass market strategy features instead more relaxed screening because visitors with lower than average match values also repeatedly visit their last host. The sorting of new visitors is then impartial in that unrelated visitors approach both types of hosts. To use colorful language, the objective of a niche market strategy seems to be to "squeeze every last penny from the devoted visitors" and the objective of a mass market strategy is seemingly to "flood the house with numerous ignorant tourists". A niche market strategy corresponds to the separating equilibrium (Fig. 1b) and a mass market strategy corresponds to the semi-pooling equilibrium (Fig. 1a). Which of these strategies is more profitable depends on market conditions, particularly, on the intensity of competition, ν. We study these cases next one by one. The equilibrium is defined as follows.

Definition 1 *A Stationary Markov equilibrium (SMPE) with consumer privacy is determined by a tuple (q_0, q_1, a, ρ, V_0) where the following conditions hold:*

1. *Strategies are optimal given beliefs: $q_0, q_1, a,$ and V_0 solve (19) and (20).*
2. *Beliefs are consistent with strategies: $q_0, q_1, a,$ and ρ satisfy (6) and (9).*

Note that we have five unknowns and an equilibrium is thus mostly pinned down by the following five equations: the first-order conditions for q_0, q_1, and a, the adding-up condition (6) that relates them to ρ, and the steady-state condition (9) for ρ.

3.2 Separating equilibrium

In a separating equilibrium, the first-order conditions of type 0 and type 1 hosts' problems are expressed, respectively, as

$$\frac{e^{-q_0}}{2} - V_0 = 0, \tag{22}$$

$$2a - 1 - V_0 = 0. \tag{23}$$

This pins down the values of V_0 and a as functions of q_0, which captures the intensity of competition for new visitors

$$V_0 = \frac{e^{-q_0}}{2} < \frac{1}{2}, \tag{24}$$

$$a = \frac{1 + e^{-q_0}/2}{2} > \frac{1}{2}. \tag{25}$$

The existence of a separating equilibrium requires that the market is sufficiently competitive and has aggregate excess supply.

[41]Lemma 1 together with the fact that $V_0 \leq 1/2$ implies that the second constraint in (20) is always satisfied with a strict inequality: after moving V_0 to the left-hand-side of the equation, we obtain $\frac{1}{2}(1/2 - a)(1 + a) + \frac{1}{2}V_0(a - 1) < 0$.

Proposition 1 *There exists a unique SMPE with consumer privacy if $\nu > 2$. The equilibrium is separating: $q_0 < 1$, $q_1 = 0$ and $a > 1/2$. The hosts of type 0 set p_0 and the hosts of type 1 set p_1 such that $p_1 > p_0$.*

The intuition for this result is that, when ν is getting smaller, there are fewer competing hosts. It may therefore become attractive for a host of type 1 to deviate to a significantly lower price. A large enough price cut also attracts to the host new visitors in addition to the previous visitor. The higher the value of ν, however, the fewer additional consumers are attracted by a lower price. With sufficiently many competitors and high enough values of ν, the deviation is not profitable.

As usual, the trade surplus created in association with type 0 hosts is given by the probability of trade $1 - e^{-q_0}$ times the expected trade surplus $1/2$. The host's share of the trade surplus is

$$p_0 = \frac{1}{2} - \frac{V_0}{1 - e^{-q_0}} q_0 = \frac{1}{2}\left(1 - \frac{e^{-q_0}}{1 - e^{-q_0}} q_0\right), \tag{26}$$

whereas the consumer's share is given by

$$1/2 - p_0 = \frac{1}{2} - \frac{1}{2}\left(1 - \frac{e^{-q_0}}{1 - e^{-q_0}} q_0\right) = \frac{1}{2}\frac{e^{-q_0}}{1 - e^{-q_0}} q_0, \tag{27}$$

which is decreasing in queue length q_0.

The trade surplus created in association with type 1 hosts is given by the probability of trade $1 - a$ times the expected trade surplus $E[a] = (1 + a)/2$. The host's share of trade surplus is

$$p_1 = a - V_0 = a - \frac{e^{-q_0}}{2}, \tag{28}$$

whereas the consumer's expected share is given by

$$(1 + a)/2 - p_1 = (1 - a)/2 + \frac{e^{-q_0}}{2} = \frac{1}{2}\left(1 - a + e^{-q_0}\right), \tag{29}$$

which is decreasing in the queue q_0 and the cut-off a.

Note that more intensive screening in terms of a higher cut-off a reduces the likelihood of trading but increases the hosts' surplus share and decreases the guests' surplus share. Hosts thus benefit from maintaining relatively high screening intensity. Both host types also obtain higher surplus with higher queue length q_0 in the more competitive "submarket for new visitors", which determines a base-line competition level for the whole market. By contrast, guests are always better off if there are fewer competing uninformed visitors and less intensive screening of previous visitors.

3.3 Semi-pooling equilibrium

The separating equilibrium, where hosts of type 1 focus on their informed visitors, fails to exist for some parameter values, especially for low values of ν. If there are relatively few hosts and thus little competition between them, type 0 hosts have rather long queues. New consumers are thus less reluctant to pay higher prices to type 1 hosts who might also have previous visitors. These popular hosts can therefore significantly decrease their no trade risk by slightly lowering their prices, which attracts to them both previous informed visitors and new uninformed visitors. An equilibrium where popular hosts trade with different consumer types may arise. We next characterize this equilibrium and study the conditions for its existence.

With a mass market strategy, the problem of a host of type 0 is similar to what we described previously. The type 0 host has thus the same first-order condition as before, which is given by (22). Meanwhile, a host of type 1 must now satisfy two visitor optimality constraints. One is the optimality condition (17) for the marginal consumer with match value a and the other one is the optimality condition (18) for uninformed consumers: both must obtain the same market utility. Together, the associated optimality conditions give (21). This equation allows us to derive a as a function of q_1 and then plug this into J_1. The problem of host 1 thus boils down to choosing the optimal q_1. The first-order condition of a type 1 host is given by

$$e^{-q_1}\left(a - \frac{\partial a}{\partial q_1}\right)\left(a - \frac{V_0}{\beta_1^g}\right) + \left(1 - e^{-q_1}a\right)\left(\frac{\partial a}{\partial q_1} - \frac{V_0 \alpha_1^g}{\left(\beta_1^g\right)^2}\right) = 0, \tag{30}$$

where we have used the fact that $\partial \beta_1^g / \partial q_1 = -\alpha_1^g$.

The existence of a semi-pooling equilibrium requires that the market is not particularly competitive and has aggregate excess demand.

Proposition 2 *There exists a unique SMPE with consumer privacy if* $\nu < 1/2$. *The equilibrium is semi-pooling:* $q_0 > 1$, $q_1 > 0$ *and* $a < 1/2$. *The hosts of type* 0 *set* p_0 *and the hosts of type* 1 *set* p_1 *such that* $p_1 > p_0$.

In a semi-pooling equilibrium, hosts of type 1 are visited by fewer new consumers than hosts of type 0 ($q_1 < q_0$), but because of low cut-offs $a < 1/2$ for previous visitors, their trade probabilities are nevertheless higher, $1 - e^{-q_1}a > 1 - e^{-q_0}$, and their prices are higher, $p_1 > p_0$ (Figure 2). However, the differences are quite small because of very high congestion, $\nu < 1/2$.[42] This implies that the probability of trade with a type 1 host is not much higher for the previous consumer than for new consumers who approach this popular host.[43]

When an informed visitor manages to trade with her host, their trading still generates a higher expected match value of $E\left[u_{ij}\right] \approx 3/4$ than trading with uninformed visitors. But with visitor privacy this occurs very rarely and does not affect market welfare much. Because of limited capacity, differences between hosts therefore vanish as competition between them becomes sufficiently mild. Ultimately, all hosts offer almost the same price and the same trade probability and the informed consumer thus applies the cut-off $a \approx 1/2$ for $\nu \approx 0$.

3.4 Mixed equilibrium and whole equilibrium set

We have now characterized the unique equilibrium with full visitor privacy for parameters $\nu > 2$ and $\nu < 1/2$. Our numerical analysis suggests further that a unique equilibrium exists also between $\nu = 2$

[42] As seen from Figures 4b and 5b, the differences are larger for $\nu > 2$.
[43] For $\nu < 1/2$, $q_1 > 1.25$ and $a > 0.45$ which gives $\alpha_1^g(1-a) < 0.15$.

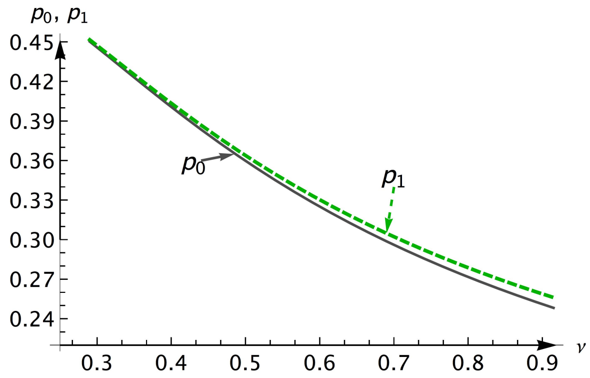


Figure 2: Prices in a semi-pooling equilibrium.

and $\nu = 1/2$. Moreover, the equilibrium set features attractive comparative statics with respect to competition intensity ν.

Remark 1 *Let us denote by* $\underline{\nu} \approx 0.938$ *and* $\overline{\nu} \approx 1.280$ *the values of* ν *separating different equilibrium types and let* $w \approx 0.148$. *With this notation, our numerical and analytical findings show that a unique equilibrium exists for all* ν *as follows:*

1. *A unique pure equilibrium with a mass market strategy exists for all* $\nu \leq \underline{\nu}$. *These equilibria feature a low market utility,* $V_0 \leq w$.

2. *A unique mixed equilibrium exist for all* $\nu \in (\underline{\nu}, \overline{\nu})$. *These equilibria feature a constant market utility,* $V_0 = w$.

3. *A unique pure equilibrium with a niche market strategy exists for all* $\nu \geq \overline{\nu}$. *These equilibria feature a high market utility,* $V_0 \geq w$.

We sketch a proof below and provide additional derivations in the Online Appendix. We can numerically show that the highest ν for which a pure equilibrium with a mass market strategy exists is $\nu = \underline{\nu}$ and the lowest ν for which a pure equilibrium with a niche market strategy exists is $\nu = \overline{\nu}$. Both equilibria have $V_0 = w$ and thus the same $q_0^m = q_0^n$. In the former equilibrium, hosts of type 1 post a lower price that attracts a positive queue $q_1^m > 0$ and determines a below-average cutoff $a^m < 1/2$. In the latter equilibrium, hosts of type 1 post a higher price that attracts no queue $q_1^n = 0$ and determines an above-average cutoff $a^n > 1/2$.

This more intensive screening under a niche market strategy entails that more informed consumers return to the market looking for a new host, which lowers market utility V_0. Particularly, although V_0 is increasing in ν with both potential market strategies, we observe that the same market utility $V_0 = w$ is reached for a lower $\nu = \underline{\nu}$ if hosts are restricted to using a mass market strategy and for a higher $\nu = \overline{\nu}$ if hosts are restricted to using a niche market strategy.[44] This w has a special role here because we find that hosts prefer a mass market strategy when $V_0 \leq w$ and a niche market strategy when $V_0 \geq w$.[45] Together these findings rule out profitable deviations to a niche market strategy for $\nu < \underline{\nu}$ and to a mass market strategy for $\nu > \overline{\nu}$ and guarantee equilibrium uniqueness.

[44]See Fig. C1 (a) in the Online Appendix.
[45]See Fig. C1 (b) in the Online Appendix.

Knowing the boundary equilibria at $\nu = \underline{\nu}$ and $\nu = \overline{\nu}$, we can also easily construct mixed equilibria for the intermediate values $\nu \in (\underline{\nu}, \overline{\nu})$ where a fraction $\mu > 0$ of hosts apply the mass market strategy with the same (a^m, q_0^m, q_1^m) as for $\nu = \underline{\nu}$ and a fraction $1 - \mu > 0$ of hosts apply the niche market strategy with the same (a^n, q_0^n, q_1^n) as for $\nu = \overline{\nu}$. Because $q_0^m = q_0^n$, this sustains a constant $V_0 = e^{-q_0^m}/2 = e^{-q_0^n}/2 = w$ such that hosts are indeed indifferent between their mass market strategy and their niche market strategy. So proving existence requires us only to show that for all $\nu \in [\underline{\nu}, \overline{\nu}]$ there exists a unique (ρ, μ) which solves the (modified) steady-state and adding-up conditions. Further analysis suggests that this mixed equilibrium pattern is unique.

3.5 Efficiency comparison

We next consider welfare in the equilibria that we have described in Sections 3.2 and 3.3.

Welfare effects of learning: niche market strategy

To determine the effect of consumer learning on welfare under a niche market strategy, we compare welfare (i) in the benchmark equilibrium without learning and (ii) in the separating equilibrium with consumer learning. The homogenous benchmark equilibrium is as in Peters (1991) and arises as the unique equilibrium outcome if we remove our key assumption that consumers can direct their search based on prior information.[46]

In that case, all consumers are uninformed in every period. The expected match value with every host is therefore 1/2. As a result, the equilibrium queue length of any one host is $q_0' = N^g/N^h = 1/\nu$ because, without previous information, all consumers randomize uniformly over all hosts. Welfare per consumer is given by $W' = \left(1 - e^{-q_0'}\right)\nu/2$.

Consider now the separating equilibrium with consumer learning as described in Proposition 1. In this equilibrium all transactions with type 0 hosts produce the expected joint surplus of 1/2. Meanwhile, all transactions with type 1 hosts generate the expected joint surplus of $(1 + a)/2$. Thus, welfare per consumer in a steady state solves

$$\begin{aligned} W &= \left(1 - e^{-q_0}\right)(\nu - \rho)\frac{1}{2} + \rho(1 - a)\frac{1 + a}{2} \\ &= \left(1 - e^{-q_0}\right)\left(\frac{1 - e^{-q_0} + a}{1 - e^{-q_0}}\rho - \rho\right)\frac{1}{2} + \rho(1 - a)\frac{1 + a}{2} \\ &= \left(1 + a - a^2\right)\frac{\rho}{2}. \end{aligned}$$

Generally, there are two different effects of consumer learning on welfare in markets where hosts apply a niche market strategy. On the one hand, given that hosts of type 1 essentially just wait for their previous consumers to visit them again, some of them inevitably wait in vain because their related consumers draw low match values, $u_j < a$. This *coordination failure* postpones trading and has therefore a negative welfare effect. Both the hosts and their potential new consumers would benefit from better coordination.

[46] The underlying assumption in earlier papers is that consumers have no information differences because earlier visits do not transmit any relevant information about current match values.

Moreover, informed consumers who draw high match values, $u_j > a > 1/2$, have a higher probability of trade than the rest of the visitors because their hosts are not inviting other consumers. In other words, there is *positive correlation* between a host's suitability and availability: guests who value their hosts dearly enough trade with them with certainty, without congestion. The impact on welfare is positive because the expected match values of uninformed consumers are $1/2$ and those of informed consumers are $\frac{1+a}{2}$.

In addition, learning and coordination failure *changes competition* for new consumers by affecting $q_0 = \frac{1-\rho(1-a)}{\nu-\rho}$ and $\rho = \frac{1-e^{-q_0}}{1-e^{-q_0}+a}\nu$. This competition pins down market utility $V_0 = e^{-q_0}/2$. Due to the coordination problems, the submarket for new visitors has a higher queue length q_0. Because informed guests return there in the same period they draw low match values, whereas hosts return there in the next period. In isolation, this tends to relax competition. Learning also supports long-term matching and screening. Both affect competition through ρ and a, respectively. First, long-term relationships keep on average $\rho(1-a)$ guests and ρ hosts away from the market for new visits. For $\nu > 1$ and $a = 0$, a higher ρ tends to raise q_0. Second, more intensive screening drives a larger wedge between the numbers of related guests $\rho(1-a)$ and hosts ρ. In isolation, a higher a therefore tends to raise q_0. However, general effects of ρ and a on competition and congestion are ambiguous and they may depend on the precise $a > 1/2$ and $\nu > 2$.

To sum up, whether learning improves welfare depends on which of these effects is strongest. Observe that W is higher than W' if

$$\left(1+a-a^2\right)\frac{\rho}{2} > \left(1-e^{-\frac{1}{\nu}}\right)\frac{\nu}{2} \tag{31}$$

The definitions of q_0 and ρ (see Equations (47) and (48) in the proof of Proposition 1), yield the following equivalent condition for $W > W'$

$$\begin{aligned}
&\rho\left(1+a-a^2\right) > \nu\left(1-e^{-\frac{1}{\nu}}\right)\\
&\nu\frac{1-e^{-q_0}}{1-e^{-q_0}+a} > \nu\frac{1-e^{-\frac{1}{\nu}}}{1+a-a^2}\\
&-a^2 + \underbrace{\frac{\left(1-e^{-q_0}\right)-\left(1-e^{-\frac{1}{\nu}}\right)}{1-e^{-q_0}}}_{=:b}a + \underbrace{e^{-\frac{1}{\nu}}}_{=:c} > 0
\end{aligned} \tag{32}$$

We prove in the Appendix that (i) c (the no trade probability of a type 0 host in markets without learning) is necessarily so high and (ii) b (the learning induced change in the trade probability of a type 0 host) is necessarily so low that (31) is satisfied for any cut-off $a \in (1/2, 3/4)$, which is the relevant value range for cut-offs under a niche market strategy by Equation (25).

Thus, positive effects of consumer learning are stronger than the negative effects: learning improves welfare. Generally, this finding also illustrates how introducing taste heterogeneity can alleviate frictions in a decentralized market. Learning about match values facilitates coordination; previously observed information becomes a natural coordinating device.

Welfare effects of learning: mass market strategy

To analyze the effect of consumer learning under a mass market strategy, we compute market welfare in a steady-state

$$W = \left(1 - e^{-q_0}\right) \frac{1}{2} (\nu - \rho) + \frac{1}{2} q_1 \rho \left(\beta_1^g - \alpha_1^g (1 - a)\right) + \rho (1 - a) \beta_1^g \frac{1 + a}{2} \tag{33}$$

The first summand in (33) is the expected joint trading surplus with a host of type 0. The second summand is the expected joint surplus obtained when uninformed consumers trade with hosts of type 1. The third summand is the expected joint surplus generated when informed consumers trade with their previous hosts.

There are two opposite effects of learning on market welfare. First, because popular and unpopular hosts offer different prices, $p_0 \neq p_1$, search for a new host is no longer random across all hosts. This search distorting effect tends to lower trading surplus for new hosts who offer the same expected match value 1/2. Random search is known to minimize trading frictions in uncoordinated market setups with homogenous products. The effect is captured jointly by the two first terms of (33).

Second, learning obviously reduces trading frictions for informed consumers. It allows these guests to direct their search immediately to their previous hosts if other hosts seem unattractive either because of higher congestion or lower suitability. This idea is captured by the last term of (33). It shows that hosts of type 1 derive higher trading surplus with learning, because they can trade with their informed related consumer with a positive probability yielding $(1 + a)/2 > 1/2$.

To sum up, we find that the positive search directing effect for informed consumers is stronger than the negative search distorting effect for new consumers, which leads to higher total welfare with learning (Figure 3a).

Welfare effects of learning: market efficiency and perfect coordination

We have found that learning improves matching efficiency. This is so for all levels of competition intensity we analyze, irrespective of equilibrium type (separating or semi-pooling). However, neither of these equilibria is fully efficient.

Proposition 3 *Equilibrium with learning and privacy is inefficient.*

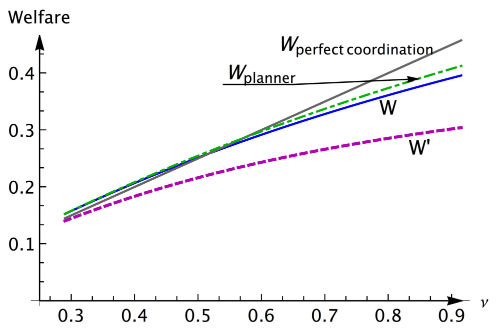
To show this result, we solve the problem of the social planner who maximizes trade surplus subject to identical coordination frictions as hosts and guests. The planner cannot violate the equal treatment condition for different market participants of the same type. It is thus assumed that the planner can freely choose q_0 and q_1 to set the screening intensity for informed visitors and sort new visitors between different hosts.[47]

Considering maximal joint surplus in each case, we observe market outcome can be improved (Figure 3). The intuition behind this result is that the social planner can better exploit the fact that an average trade surplus for an informed consumer is higher than 1/2 for any $a > 0$. The planner would therefore optimally reduce the cut-off a below the level generated by market forces. Because

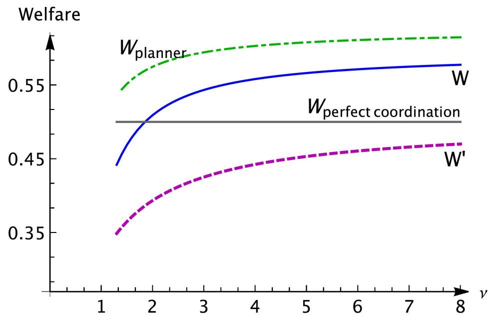
[47] The cut-off is determined by consumer self-sorting. The details are in the Online Appendix.

hosts have local monopoly power, they have instead an incentive to intensify screening excessively. This has a direct effect on their profit.

Note that, if there are few hosts ($\nu < 1$), relaxed screening (lower a) raises congestion at hosts of type 1. This decreases q_1 and increases q_0. As a result, we can conclude that prices are too low in the semi-pooling equilibrium. Meanwhile, if there are many hosts ($\nu > 1$), lower a directly reduces q_0 because the optimal q_1 equals zero. Hence, prices are too high in the separating equilibrium compared to the social optimum.



(a)



(b)

Figure 3: Welfare in markets with learning (W) and without learning (W') compared to the social planner's solution (with learning) and perfectly coordinated markets (without learning).

We also notice that learning can increase market welfare even above the level that would be generated if all agents on the shorter market side were matched one for one with agents on the longer market side: this would yield the total trade surplus of $\max\{\nu, 1\}/2$ (per visitor). As shown in Figure 3, this upper bound is never attained in markets without learning. Because learning reduces trading for low $u_{i,j}$, we can beat this benchmark for very low values of ν (relatively few hosts trade almost surely for above average match values) and also for higher values of ν (relatively few guests trade almost surely for above average match values). This makes it clear that learning provides welfare gains that go much beyond simple coordination.

4 Market equilibrium without privacy

In this section, we characterize the equilibrium without consumer privacy. Specifically, we assume that at the beginning of each period a host of type 1 can observe the identity and the match value of his previous visitor.[48] Nevertheless, the host still knows nothing about other visitors. The host can thus either choose to make his previous visitor a take-it-or-leave-it price offer or make a general price offer that targets new visitors. In other words, we are studying the welfare consequences of consumer

[48] A more sophisticated case of consumer tracking would be to allow the host to observe an imperfect but informative signal s about his previous visitor's match value u. Then, the host would make an offer $\text{argmax}_{\hat{p}_k} Pr(u > \hat{p}_k + \hat{V}_0|s)\hat{p}_k$ that targets his visitor if this gives more profit than $\hat{J}_0$ from targeting new visitors; assuming that values and signals are positively correlated, the price $\hat{p}_k(s)$ would be higher for a higher signal s. This entails that imperfect signals reduce coordination frictions between a host and his previous visitor but do not totally remove them like our perfect signals do.

tracking and targeting in the booking industry. To distinguish this case from the setting in Section 3, all the variables relating to markets without visitor privacy have a wedge ($\wedge$) above them.

First, consider a host of type 1 that we here denote by k. The immediate effect of suppressed privacy is that the host can make his informed visitor such a price offer that his previous consumer is indifferent between accepting the offer and going to some other host. In other words, hosts of type 1 need not leave their previous consumers any information rents unlike in Section 3. This implies that market utilities equal $\hat{V}_1 = \hat{V}_0$ for all visitor types.

Thus, the price offer $\hat{p}_k$ to the previous informed consumer is

$$\hat{p}_k = u_{11(k)} - \hat{V}_0,$$

and the popular host can hence earn the certain profit of $\hat{J}_k = \hat{p}_k$.

Sometimes the previous visitor may not receive a special price offer. Because some guests may not like the host very much, host k targets his previous informed consumer only if her willingness to pay for a continuation visit is sufficiently large. We denote the lowest value of $u_{11(k)}$ for which an informed consumer obtains the special price offer by $\hat{a}$. The highest price that host k may charge equals $1 - \hat{V}_0$ whereas the smallest price is equal to $\hat{a} - \hat{V}_0$.

The value of $\hat{a}$ has to be such that host k is indifferent between charging $\hat{p}_a$ and $\hat{p}_0$. This corresponds to the optimal choice of a consumer segment. By charging $\hat{p}_a$, the host attracts only his informed consumer whereas, by charging $\hat{p}_0$, the host only attracts uninformed consumers. From this, we obtain the following indifference condition for the cut-off $\hat{a}$

$$\hat{a} - \hat{V}_0 = \frac{1}{2}\left(1 - e^{-\hat{q}_0} - \hat{q}_0 e^{-\hat{q}_0}\right). \tag{34}$$

The right-hand-side (RHS) of this equation denotes the profit $\hat{J}_0$ from targeting new consumers.

To derive the cut-off $\hat{a}$, note that the problem of a host of type 0, who remains completely uniformed, has not changed from Section 3. The first-order condition of his profit maximization problem is still given by $\hat{V}_0 = e^{-\hat{q}_0}/2$. By combining this condition with (34), we can see that host k applies a cut-off, which lies below the average match value

$$\hat{a} = \frac{1 - \hat{q}_0 e^{-\hat{q}_0}}{2} < \frac{1}{2}. \tag{35}$$

Suppression of privacy therefore limits screening, which supports long term customer relationships.

In equilibrium, there are $N^h - N^g \hat{\rho}(1 - \hat{a})$ hosts charging price $\hat{p}_0$ and $N^g(1 - \hat{\rho} + \hat{\rho}\hat{a})$ consumers randomizing uniformly over them. Thus, the queue length at each of these hosts is

$$\hat{q}_0 = \frac{1 - \hat{\rho}(1 - \hat{a})}{\nu - \hat{\rho}(1 - \hat{a})}. \tag{36}$$

In a steady state, the evolution equation of ρ is

$$\hat{\rho} = (\nu - \hat{\rho}(1 - \hat{a}))\left(1 - e^{-\hat{q}_0}\right) + \hat{\rho}(1 - \hat{a}). \tag{37}$$

This with the adding-up condition (36) yields

$$\hat{a} = \frac{(1-\nu)\left(1-e^{-\hat{q}_0}\right)}{\hat{q}_0\nu - \nu\left(1-e^{-\hat{q}_0}\right) - e^{-\hat{q}_0}}. \tag{38}$$

Because match values are uniformly distributed, the average price offer to previous visitors is

$$E[\hat{p}_1] = \frac{1+\hat{a}}{2} - \hat{V}_0.$$

With this background information, we can show that the following unique equilibrium arises.

Proposition 4 *There exists a unique SMPE in the market without consumer privacy. The equilibrium is separating:* $\hat{q}_1 = 0$ *and* $\hat{a} < 1/2$. *The hosts of type* 0 *set a fixed price* $\hat{p}_0$ *and the hosts of type* 1 *apply perfect price discrimination to high match value consumers such that both* $\hat{p}_1 < \hat{p}_0$ *and* $\hat{p}_1 > \hat{p}_0$ *are possible.*

This equilibrium pattern demonstrates that when hosts have sufficient information on target groups the choice of a visitor segment becomes straightforward. First off, the possibility of tracking eliminates coordination problems that arise with privacy because hosts are unable to stay tuned into their visitor's plans. Further, the possibility of targeting removes the need to attract simultaneously previous and new visitors to offer insurance if earlier guests fail to show up. Even more importantly, additional information also allows popular hosts to internalize the externalities, that their pricing choices cause to their previous visitors' matching probability. We prove in the next section that the equilibrium without privacy is efficient.

5 Comparison of equilibria with and without privacy

In this section, we compare the previously derived equilibria with and without consumer privacy from Section 3 and 4, respectively. One of the main conclusions of this comparison is that tracking and targeting not only improves welfare but may also benefit consumers.

Consumer surplus is determined by market utility $V_0 = e^{q_0}/2$, which depends on how strongly unpopular firms are competing for new consumers; the intensity of competition is captured by q_0. With privacy, this queue length is obtained directly from Eq. (39)

$$q_0 = \frac{1-\rho(1-a)}{\nu-\rho} - \frac{\rho}{\nu-\rho}q_1. \tag{39}$$

However, without privacy, it changes to

$$\hat{q}_0 = \frac{1-\hat{\rho}(1-\hat{a})}{\nu-\hat{\rho}(1-\hat{a})} \tag{40}$$

because popular hosts no longer target (i) previous visitors who have low match values and (ii) previous and new visitors simultaneously.

Comparing q_0 and $\hat{q}_0$ suggests that privacy suppression has four general equilibrium effects on consumer surplus through V_0 alone: one pro-competitive, one anti-competitive, and two that could work in either direction. First, because tracking enables hosts to respond immediately to a low demand realization,[49] there will be more hosts in the market competing for new consumers. With privacy the number of hosts who charge the lower price, p_0, is $\nu - \rho$ (per consumer) whereas, without privacy, it is $\nu - \hat{\rho}(1 - \hat{a})$ (per consumer). By eliminating unnecessary coordination frictions, visitor tracking thus reduces the queue length q_0, which tends to have a pro-competitive effect.

Second, targeting removes sorting to different "submarkets" because all new consumers visit unpopular hosts and no one approaches popular hosts anymore. In booming markets, this means that a number of new consumers reallocates themselves from $\hat{q}_1$ to $\hat{q}_0$, which tends to increase the queue length, $\hat{q}_0$. Taken alone, this weaker sorting reduces market utility $\hat{V}_0$.

Finally, average trading probability ρ increases without privacy. Whether the effect strengthens competition depends on market tightness. If there are more guests, $\nu < 1$, $\hat{q}_0$ is increasing in $\hat{\rho}$ (less competition without privacy) whereas, if there are more hosts, $\nu > 1$, $\hat{q}_0$ is decreasing in $\hat{\rho}$ (more competition without privacy). Lower screening intensity $\hat{a}$ has similar effects.

5.1 Prices

We can show that the equilibrium features more trading and less screening without consumer privacy, which implies $\hat{\rho} > \rho$ and $a > 1/2 > \hat{a}$. This entails that aggregate demand conditions, ν, determine the effect of privacy on market prices. When there are more hosts, $\nu > 1$, queue lengths q_0 become shorter when type 1 hosts can observe the match values of their previous visitors,

$$q_0 > \frac{1 - \rho\,(1 - a)}{\nu - \rho\,(1 - a)} > \frac{1 - \hat{\rho}\,(1 - \hat{a})}{\nu - \hat{\rho} + \hat{\rho}\hat{a}} = \hat{q}_0.$$

The opposite holds when there are more guests, $\nu < 1$.

We study first a market with higher $\nu > 1$. In that case, shorter queues foster tougher competition: $p_0 > \hat{p}_0$ and $V_0 < \hat{V}_0$. Furthermore, hosts that charge $\hat{p}_1$ are disciplined by hosts that charge $\hat{p}_0$. Without privacy all hosts leave all guests the same larger consumer surplus $\hat{V}_0$. Thus, an informed consumer whose match value equals a pays less without privacy.

Obviously, an informed consumer who has the highest match value $u_{11(r)} = 1$ is still worse off under perfect price discrimination.[50] Nevertheless, considering that $a > 1/2$ is above average with privacy and $\hat{a} < 1/2$ is below average without privacy for $\nu > 1$, informed consumers will pay less on the average in the equilibrium without consumer privacy.[51, 52]

This does not hold with fewer hosts $\nu < 1$. In that case, $p_0 < \hat{p}_0$ implies $V_0 > \hat{V}_0$. We can thus conclude that popular hosts charge higher prices on average: $E\left[\hat{p}_1\right] > p_1$. To summarize, weaker

[49] By posting $\hat{p}_k$ for $u_{11(k)} \geq \hat{a}$ and posting $\hat{p}_0$ for $u_{11(k)} < \hat{a}$.

[50] There should thus exist some value $u_{ij} \in (a, 1)$ at which the previous visitor is indifferent between paying p_0 and paying $\hat{p}_1$.

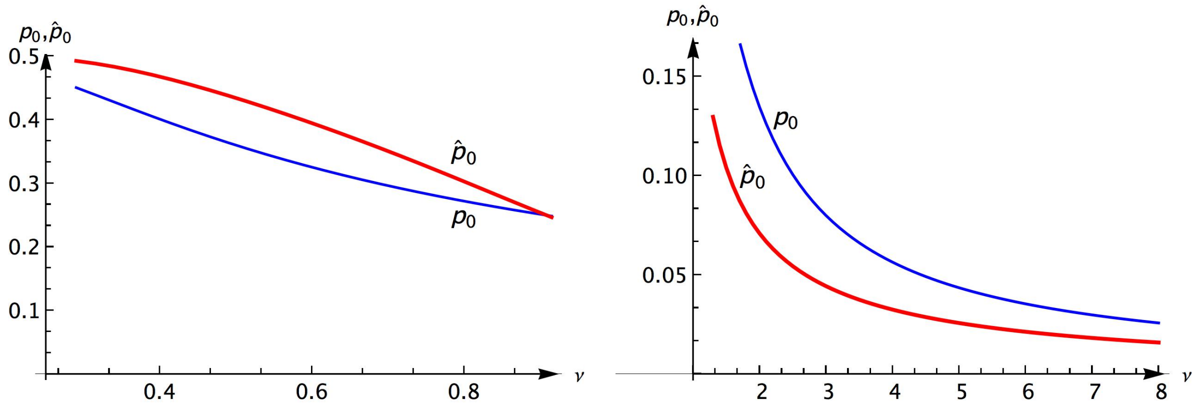
[51] This result resembles the outcome in oligopolistic markets with "best-response asymmetry" where hosts may set different prices in weak and strong markets (see Armstrong (2006)). Different from that setting, our results arise due to hosts' capacity constraints instead of negative correlation of match values across products.

[52] Lester (2011) studies a market where a fraction of consumers is uninformed about prices. He finds that more information can either increase or decrease prices. Here some consumers are informed about their private match values. Increasing this fraction ρ can also both increase and decrease prices, albeit for different reasons.

consumer privacy has a polarising effect on prices: it reduces them further when competition is sufficiently hard, and raises them when competition is sufficiently mild:

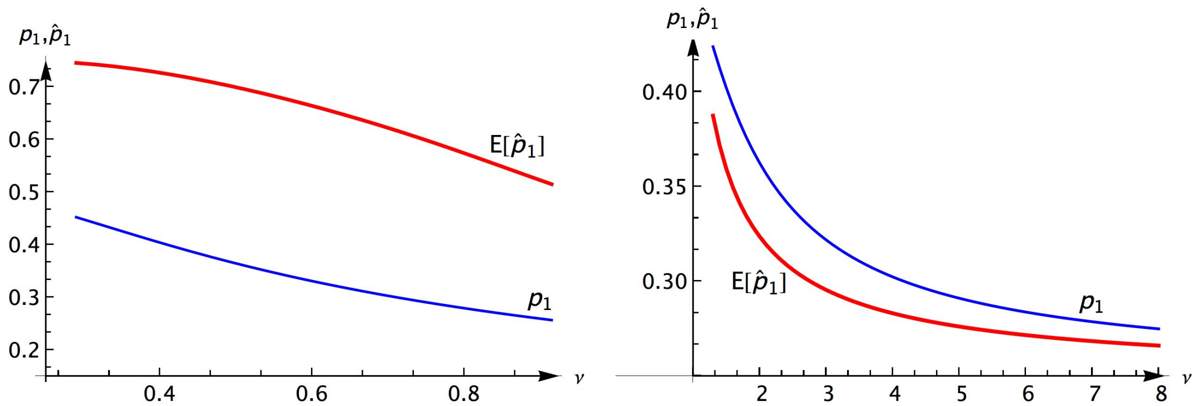
Proposition 5 *If $\nu > 2$, prices are lower without privacy: $\hat{p}_0 < p_0$ and $E[\hat{p}_1] < p_1$ whereas, if $\nu < 1/2$, prices are higher without privacy: $p_0 < \hat{p}_0$ and $E[\hat{p}_1] > p_1$.*

In a cyclical market, where competition intensity is different, say, in the summer and winter booking seasons, this suggests that tracking and targeting technologies may benefit certain consumer segments but not all. Similarly, access to information about recent demographic trends (e.g., use of online/survey/register data to support planning at a city or firm level) might affect differently expanding and declining areas.



(a) Comparison to semi-pooling (b) Comparison to separating

Figure 4: Prices p_0 with and without privacy.



(a) Comparison to semi-pooling (b) Comparison to separating

Figure 5: Prices p_1 with and without privacy.

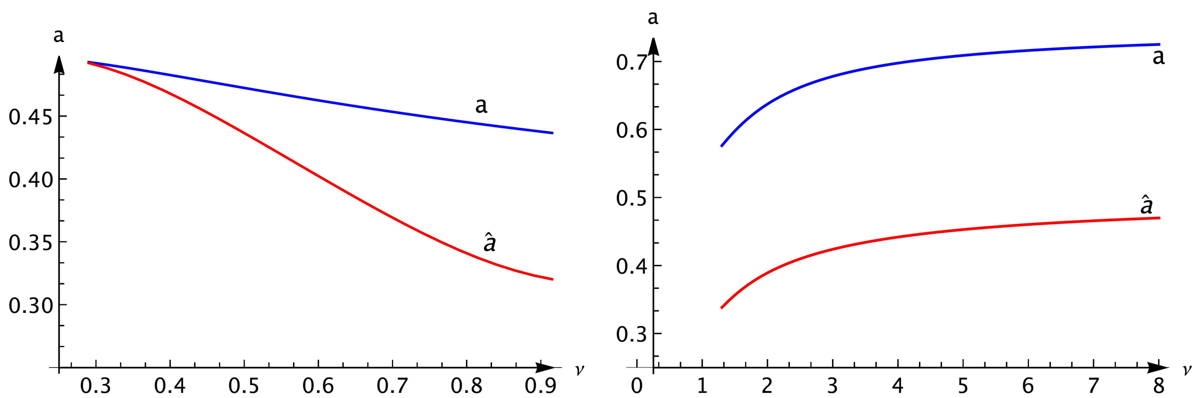
5.2 Cut-offs

The relationship between the cut-off a and competition ν is determined by two conflicting incentives: *screening motive* to keep prices high and *insurance motive* to keep demand high. As shown by Lemma

1, a host can either use a niche market strategy ($a > 1/2$ and $q_1 = 0$) or a mass market strategy ($a < 1/2$ and $q_1 > 0$). The advantage of a niche market strategy is that it allows the host to extract more surplus from his high match value guests. The disadvantage is that the host has more uncertain demand because his previous consumers are then less likely to return. A mass market strategy does not allow for strong screening of informed consumers but, by targeting different consumer segments, it offers some demand insurance in case the host loses his previous visitor.

Whether screening or insurance motive is stronger with visitor privacy depends on competition intensity, which modifies the tradeoff of prices and demand. For a fixed price $p_1 \in (0, 1/2)$ the maximal demand that a host can attract decreases in the market utility V_0. When visitors have a high market utility $V_0 \approx 1/2$, p_1 is attractive to only the most motivated informed consumers with high match values but, when visitors have low outside option $V_0 \approx 0$, new visitors and informed visitors with low match values are also attracted by this same p_1. With additional market competition, higher demand requires large price sacrifices from a host, which are otherwise not necessary. The host has therefore an incentive to expand his demand aggressively for lower $\nu < 1$ and V_0 but dispose of some of his demand for higher $\nu > 1$ and V_0.

Specifically, when there are more hosts, competition among hosts of type 0 becomes stronger and p_0 is therefore decreasing in ν. Thus, a firm of type 1 must also leave more surplus to his visitors. There are two different ways to achieve this: the host can either lower his price to maintain his previous demand (similar a and q_1) or keep the same price but focus on visitors with higher match values (higher a and lower q_1). The former strategy is attractive when it is cheap to attract new consumers but becomes less so when there are many hosts competing for the average visitor. Figure 6 compares cut-offs with and without privacy.



(a) Comparison to semi-pooling (b) Comparison to separating

Figure 6: cut-offs a with and without privacy.

Note that the demand from previous visitors with observed match value $a = 1/2$ is less elastic than that from new visitors with expected match value $u_{ij} = 1/2$. However, the demand from previous visitors with very low match values is the most elastic and the demand from previous visitors with very high match values the most inelastic. All demands become more elastic with competition but the demand from previous visitors with very high u_{ij} reacts the least and the demand from informed consumers with very low u_{ij} reacts the most. To satisfy the standard Lerner formula[53] under different market conditions, a host will thus target only highest match value visitors if competition is hard and lowest and average match value visitors if competition is mild. This optimality condition requires that the price elasticity of demand remains one for all ν.

To see how competition affects the elasticity of (inverse) demand, recall that $p_1 = a - V_0/\beta_1^g$ in the semi-pooling equilibrium and $p_1 = a - V_0$ in the separating equilibrium. For simplicity, consider some constant p_0 and a that are kept the same in both equilibria. Then, if ν increases, q_0 goes down, which raises V_0. In a separating equilibrium, p_1 decreases at a rate -1 and, in a semi-pooling equilibrium, it decreases at a rate $-1/\beta_1^g < -1$. Also, q_1 decreases in ν which further reduces $\partial\left(-V_0/\beta_1^g\right)/\partial\nu$. This shows that the price p_1 is more sensitive to aggregate conditions ν when the number of hosts is small. Regarding a, note that, since p_1 and q_1 decrease rapidly with $\nu < 1$, it becomes more attractive to visit the same host and a thus decreases but, since p_1 decreases more slowly with $\nu > 1$, other hosts start to look relatively more attractive and a thus increases.

5.3 Welfare

We have seen that with visitor privacy the equilibrium is inefficient and generally features too intensive screening. To compare this result with market performance without privacy, we next consider the problem of a planner in this case. The planner is subject to the same matching frictions as others and cannot handle differently individuals of the same type. The idea is to look for maximum welfare in markets with similar coordination frictions that we have so far assumed. In doing so, we thus abstract from prices that only transfer surplus from one side of the market to the other, and instead let the planner choose the sorting between q_0^* and q_1^* and the appropriate screening intensity a^*. We denote all variables in the planner's solution by an asterisk ($*$).

The planner observes the type of each visitor, including the match values of informed visitors, and thereafter determines (i) whether an informed visitor with a particular match value should approach the same host or some other host (this is given by the cut-off a^*) and (ii) whether a visitor who approaches some new host visits in random one of the popular hosts or unpopular hosts (this is given by the queues q_0^* and q_1^*). Without privacy, a host k of type $1(k)$ is counted in q_1 if $u_{11(k)} \geq a^*$ and in q_0 if $u_{11(k)} < a^*$. However, the problem can be simplified significantly from this because the planner does not gain anything by directing both an informed visitor and a new consumer to the same host. Similar to markets, the planner thus chooses $q_1^* = 0$.[54]

The problem of the planner can now be expressed as the problem of choosing the optimal cut-off a_t in period t given some prior information in the market, ρ

$$W_t = \max_{a_t} \left(1 - e^{-q_{0t}(a_t)}\right)\left(\nu - \rho_{t-1}\left(1 - a_t\right)\right)\frac{1}{2} + \rho_{t-1}\frac{1 - a_t^2}{2},$$

[53] $\max p(D - MC)$ implies $\frac{p-MC}{p} = -\frac{D(p)}{D'(p)}\frac{1}{p} = -\frac{1}{\epsilon}$, which gives $-\frac{1}{\epsilon} = 1$ assuming $MC = 0$.

[54] Note that the planner's problem is defined in a different way with and without privacy. In the earlier section, the planner chooses (q_0, q_1, a) without knowing the consumers' match values. The cut-off a must thus satisfy the visitor's incentive compatibility constraint. In the current section, the planner chooses (q_0, q_1, a) knowing the consumers' match values.

where ρ_{t-1} needs to satisfy a similar condition to (37) and the adding up constraint gives $q_{0t} = (1 - \rho_{t-1}(1 - a_t)) / (\nu - \rho_{t-1}(1 - a_t))$. The first-order condition of the planner's problem in a steady-state is given by

$$\frac{\rho^*}{2}\left(1 - a^* - q_0^* e^{-q_0^*}\right) = 0. \tag{41}$$

We can now compare the first-order conditions in the planner solution and in the market solution (35) with the respective steady-state conditions. Both sets of equations pin down the same cut-off.[55]

Proposition 6 *The equilibrium without consumer privacy is socially efficient: the social planner sets the same cut-off as markets* $a^* = \hat{a}$.

Corollary *Market welfare is higher without consumer privacy for all considered parameter values* $\nu < 1/2$ *(for* $q_0 > 1/2$ *and* $q_1 > 1$*) and* $\nu > 2$.

As discussed, consumer privacy may either increase or decrease consumer surplus. This is because some consumers are better off whereas other consumers are worse off without privacy. The total effect hence depends on the sizes of these different consumer groups. Let us first study higher values of ν for which hosts apply a niche market strategy with consumer privacy. If consumer privacy is suppressed, all new consumers who pay $\hat{p}_0$ obtain more surplus, although there are now fewer of them, and informed consumers with match values in the middle range $u \in [\hat{a}, a]$ also receive more surplus. However, informed consumers whose match values u are closer to 1 pay higher prices and gain less surplus. Summing these effects, we observe that the positive effects dominate when ν is small (below $\nu \approx 4$) and the negative effects dominate when $\nu > 2$ is large (above $\nu \approx 4$) because the difference in the intensity of competition $\hat{V}_0 - V_0$ is decreasing in ν. By contrast, for low values of ν for which hosts apply a mass market strategy with consumer privacy, suppression of privacy reduces consumer surplus by alleviating competition: $\hat{V}_0 < V_0$ (Figure 7a).[56] Figure 7b describes these patterns.

6 Market equilibrium with second price auctions

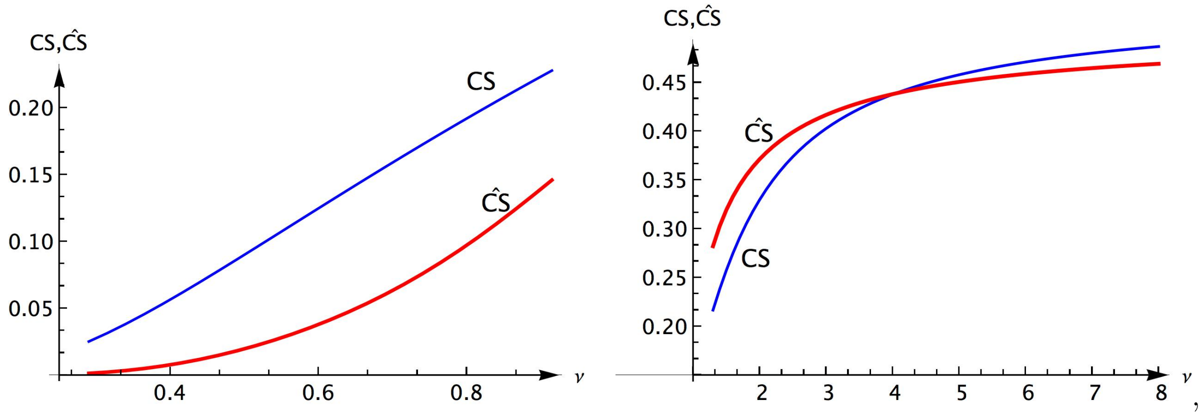
In this section, we consider another way of learning the previous visitor's match value, namely, auction like booking or selling mechanisms which automatically reveal more private information than posted prices. The exact timing of the auction setup is as follows. First, hosts post their reservation prices. Then, guests decide which hosts to visit. After the arrival of consumers at a host, visitors observe how many of them have come and participate in the auction.

We denote the reservation prices chosen by hosts of types 0 and 1 by r_0 and r_1 respectively. The maximization problem of a host of type 0 thereby becomes

$$J_0 = \max_{r_0} \quad q_0 e^{-q_0} r_0 + \left(1 - e^{-q_0} - q_0 e^{-q_0}\right)\frac{1}{2}. \tag{42}$$

[55]However, we find that $a^* > \hat{a}$ if $\delta > 0$ because hosts then invest excessively in customer relationships that allow them to extract more profit from guests (Prop. 4 in the Online Appendix of this paper).

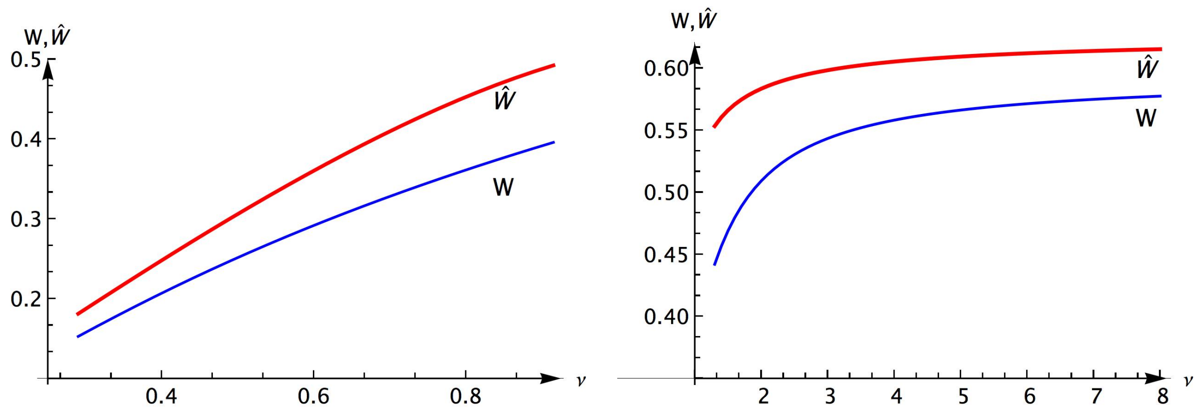
[56]If δ is positive such that the semi-pooling equilibrium exists for higher values of ν, for which $\hat{V}_0 > V_0$, the ranking of consumer surplus can be the opposite (Fig. 23 in the Online Appendix of this paper).



(a) Comparison to semi-pooling

(b) Comparison to separating

Figure 7: Consumer surplus comparison with and without consumer privacy.



(a) Comparison to semi-pooling

(b) Comparison to separating

Figure 8: Market welfare comparison with and without consumer privacy.

If only one visitor arrives, the host gets the reservation price r_0 but, if more than one visitor contacts him, he obtains the expected valuation $1/2$ for a new host.

As before, a host of type 0 must take into account visitor choices. A guest must obtain at least the market value V_0 in order to visit the host. A visitor receives positive surplus only if she is the only visitor and $r_0 < 1/2$. Otherwise, the winner of the auction pays $1/2$, which leaves her without surplus. Hence, the constraint coming from a visitor side is

$$e^{-q_0}\left(\frac{1}{2} - r_0\right) = V_0. \tag{43}$$

After taking this constrain into account, we obtain that the host maximizes the following problem

$$\max_{q_0} \quad \left(1 - e^{-q_0}\right)\left(\frac{1}{2} - \frac{V_0}{\beta_0^c}\right).$$

The first-order condition of this problem is the same as with posted prices: $e^{-q_0} = 2V_0$ in Sections 3.2 and 3.3. As is typical for homogenous or ex ante similar-looking products, the optimal reservation price r_0 thus equals zero.[57]

Somewhat similarly as with posted prices, the profit of a host of type 1 depends on the reservation price r_1, which determines whether the host attracts only his previous visitor (if $u_{ij} \geq a$) or also some new unrelated visitors. Consider first a higher r_1 such that a host of type 1 only attracts his previous visitor. Then, popular hosts maximize the following profit expression

$$J_1 = \max_{r_1} (1-a) r_1,$$

subject to the optimality constraint for the informed consumer: $a - r_1 = V_0$. Again, this problem is identical to the one with posted prices in Section 3.2. The same first-order condition applies for optimal r_1 as for optimal p_1 in a separating equilibrium: $1 - 2a + V_0 = 0$. We can thus concludes that $r_1 = p_1$ and $r_0 = 0$ in this candidate equilibrium with second price auctions.

Note that the optimal cut-offs and queue lengths at both types of hosts are also the same as those with posted prices in a separating equilibrium. Total market welfare is thus the same and surplus division is unchanged and still given by Equations (26), (27), (28) and (29). Posted prices and second price auctions are thus equivalent[58] with and without learning and the analysis of beneficial learning effects applies for both.

Nevertheless, the range of ν for which this equilibrium type exists may not be the same with auctions because of different welfare and surplus division when $q_1 > 0$. To analyze this case, suppose next that r_1 is so low that a host of type 1 attracts both his previous visitor and some new consumers. To attract a positive $q_1 > 0$, it must be that $r_1 < 1/2$ and $a < 1/2$.[59] Then, a host of type 1 maximizes the following profit expression

$$\begin{aligned} J_1 = \max_{r_1} & \left((1-a)e^{-q_1} + a q_1 e^{-q_1}\right) r_1 + q_1 e^{-q_1} \left(\frac{1}{2} - a\right)\left(\frac{1}{4} + \frac{a}{2}\right) \\ & + \left(1 - e^{-q_1}(1-a) - a e^{-q_1} q_1 - q_1 e^{-q_1}\left(\frac{1}{2} - a\right) - e^{-q_1} a\right)\frac{1}{2} \end{aligned} \tag{44}$$

The first summand in (44) denotes the revenue of the host if he is visited by only one consumer: then the host earns r_1. Instead, if the host is visited by one new consumer and his previous consumer, and the match value of the informed consumer is less than or equal to $1/2$, then the host earns this match value, which in expectation is $\frac{1}{1/2-a}\int_a^{1/2} u du = 1/4 + a/2$. This is captured by the second summand in (44). Otherwise (if there are more than one visitor whose match value is either higher or equal to $1/2$), the host always earns $1/2$, which gives the last summand in (44).

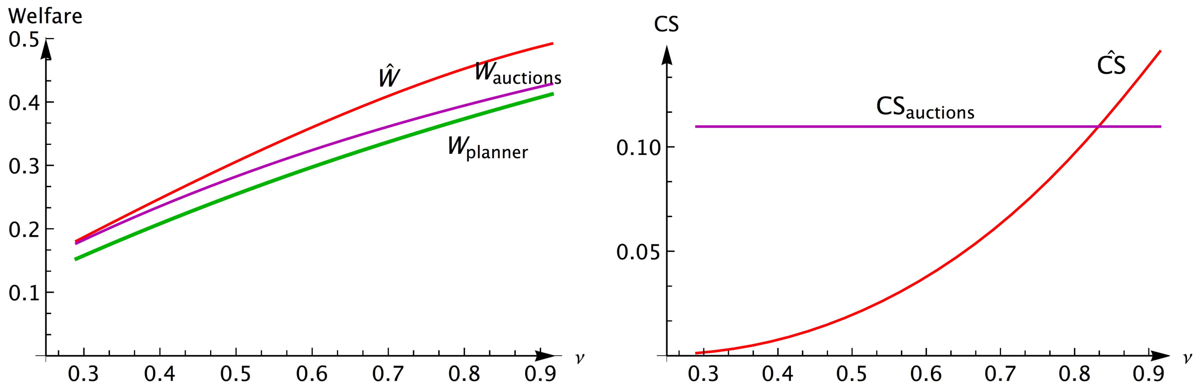
Thus, popular hosts maximize (44) subject to following two constraints:

$$e^{-q_1}(a - r_1) = V_0 \tag{45}$$

[57]See the discussion in Eeckhout and Kircher (2010).

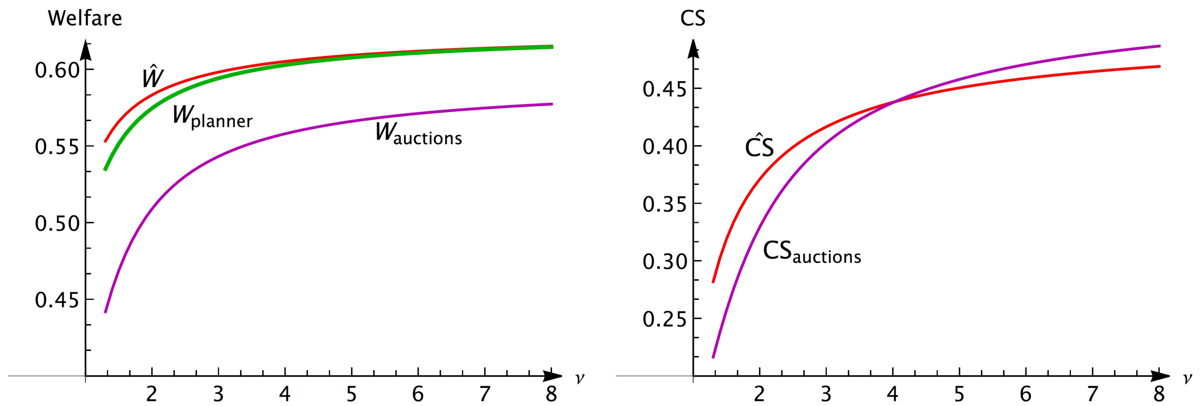
[58]This equivalence is demonstrated by Kultti (1999).

[59]The setting with $r_1 < 1/2$ and $a \geq 1/2$ is infeasible because there exist no $a \geq 1/2$ and $r_1 < 1/2$ that satisfy the indifference constraints of an informed and an uninformed consumer simultaneously.



(a) Welfare (b) Consumer surplus

Figure 9: Comparison of welfare W and consumer surplus CS in the planner solution with privacy, in auctions, and with tracking



(a) Welfare (b) Consumer surplus

Figure 10: Comparison of welfare W and consumer surplus CS in the planner solution with privacy, in auctions, and with tracking

and

$$ae^{-q_1}\left(\frac{1}{2}-r_1\right)+e^{-q_1}\left(\frac{1}{2}-a\right)\left(\frac{1}{2}-\frac{1}{4}-\frac{a}{2}\right)=V_0. \tag{46}$$

When $q_1 > 0$, an informed consumer with a match value a obtains surplus $a - r_1$ if no other consumer comes, and obtains nothing if there is at least one uninformed consumer. This gives constraint (45). A new consumer obtains positive surplus only if she is either the only visitor (the first summand on the LHS in (46)) or the only uninformed visitor against an informed visitor with a match value less than $1/2$ (the second summand in the LHS of (46)).

Numerical analysis shows immediately that the basic equilibrium pattern is similar with auctions and posted prices: if there are fewer hosts ($\nu < 1$), $a < 1/2$ and $q_1 > 0$ whereas, if there are fewer guests ($\nu > 1$), $a > 1/2$ and $q_1 = 0$; mixing between higher r_1 (and $q_1 = 0$) and lower r_1 (and $q_1 > 0$) occurs around $\nu = 1$. If $q_1 > 0$, we can show that the cut-off $a < 1/2$ is lower than the corresponding cut-off in the semi-pooling equilibrium with prices and, if $q_1 = 0$, we know already that the cut-off

$a > 1/2$ is equal to the corresponding cut-off in the separating equilibrium with prices.[60]

The underlying intuition is similar to before: market utility V_0 increases with ν and popular hosts will thus look for less elastic demand with additional competition. When $\nu > 2$, hosts only target marginal informed consumers with $a > 1/2$ but, when $\nu < 1/2$ is low, they also target marginal informed consumers with $a < 1/2$ and some new uninformed consumers. However, as we move below the average match value cut-off $a = 1/2$, which occurs at about the same point as with prices (around $\nu \approx 1$), things become slightly different with auctions.

Namely, relative to posted prices, the demand from a marginal previous visitor is more responsive to q_1 (because she never wins uninformed consumers $u_{ij} = 1/2$) and the demand from new consumers is less responsive to reductions in a (because they always win informed visitors with $u_{ij} < 1/2$). This is so because auctions allocate local goods efficiently to the highest match value visitor whereas posted prices allocate them randomly. To maintain constant demand elasticities, as required by the Lerner pricing formula, a host will thus choose lower a and lower q_1 for a given V_0.

Interestingly, we find that relative to the equilibrium without privacy, auctions encourage hosts to overinvest in relationships. This helps them to coordinate when they do not have visitor data.

Proposition 7 *Relative to equilibrium with consumer tracking, auctions feature too low cut-offs if $\nu < 1/2$ ($a_{spa} < a^\star$) and too high cut-offs if $\nu > 2$ ($a_{spa} > a^\star$).*

This entails that aggregate surplus remains smaller with auctions than with tracking (Figure 9a). Regarding consumer welfare, however, tracking might sometimes still be less attractive (Figure 9b).[61]

7 Concluding discussion

We consider learning and visitor tracking in a dynamic, directed search market with horizontally differentiated experience goods and taste variation. All equilibria with learning feature implicit consumer relationships between a host and his previous visitor, which alleviates coordination frictions. This looks like habit formation but does not rely on stable tastes. Our model helps in understanding how (i) public price information, (ii) private taste information and (iii) the revelation of taste information to concerned hosts affect market welfare. It establishes the importance of privately observed information flows. Tracking and targeting technologies allow hosts to stay tuned into their guests' future plans. This can intensify competition and reduce prices in markets with some aggregate excess supply.

We discuss our results in the context of (i) mobility of tourists and developments in the online booking industry, and (i) mobility of urban labor and competition between metropolitan economies. Gyourko et al. (2013) have data on so called superstar cities. They find that the main drivers of price dynamics in urban areas are (i) the fixed short run capacity of housing and services and (ii) the demand for the unique local living environment. These are also the main ingredients in our steady state setup.[62] Because the capacity of a given place cannot be expanded instantly, prices must absorb

[60] The details are in the Appendix.

[61] Initially, we presumed they might work similarly. Both are ways to reveal private information to hosts.

[62] An extensive literature on spatial sorting analyzes the effects of observable and unobservable skills on

the changes in demand for a particular location.[63] Beyond travel, our results extend to consumer markets with logistics frictions where people do not travel to local products but local products travel to people.

Our findings contribute to the ongoing discussion about the need for additional regulation on online behavioral advertizing, that is, the profiling and targeting of consumers based on their preferences as inferred from their internet browsing or purchase history.[64] We find that the relation between consumer privacy and consumer surplus is not straightforward. In particular, collected private data may result in some additional consumer surplus transfer from consumers to hosts. However, in some cases (it depends on market tightness), an increase in total surplus due to lower search frictions is shared between both sides of the market. Applied to a single large market like the US or the EU, this entails that universal rulings on consumer tacking can have unforeseen general equilibrium effects on the rate of matching consumers to suitable products and the aggregate price level. Specifically, we demonstrate in this paper that looser privacy regulation allows hosts to respond more rapidly to demand changes, which could strengthen competition (for new consumers) and put downward pressure on prices (for all consumers). As conservative policy advice, this suggests that it may not be desirable to go all the way in the protection of consumer privacy for online markets.

The problems of privacy suppression in online markets have been extensively documented (Acquisti et al., 2016). The key issues relate to firms' increased ability to engage in price discrimination with efficient computation facilities. It is well known that, in a text book setup, price discrimination is harmful because it boosts firms' profits at the expense of consumers. Our paper shows a notable exception to this rule. In our model, privacy suppression results in *perfect* price discrimination. However, although this equalizes the surplus across different consumers, it does not necessarily impair the average consumer's welfare. That is, we describe a mechanism that allows consumers also profit from the improved matching efficiency associated with price discrimination.

Our paper contributes additionally to the understanding of efficiency in mobility problems, which is still deficient. For instance, the concept of matching efficiency differs in *stochastic frontier analysis*[65], and in *directed search models* that generate an endogenous matching function (Acemoglu and Shimer (1999)). In discussing the relative merits of matching models, Stevens (2007) points out that the problem with the former approach is that is has only weak micro-foundations whereas the

location choices. For references, see especially Eeckhout et al. (2014). Our contribution is to highlight the role of idiosyncratic taste differences on matching efficiency and mobility decisions. Also, the focus of our paper is more on short-term visits on business or vacation than on long-term choices of permanent residence on which the spatial sorting literature concentrates.

[63]Our model can be compared to a standard Roy model where earnings prospects in a country are $log(w) = \alpha + rs + \epsilon$, where w gives the wage, r denotes returns to observable skills s and ϵ captures the effects of unobserved skills. Roy models have been applied to study selection of immigrants based on observable and unobservable characteristics (e.g., Borjas et al. (2015)). Our setup features negative selection for match values: because private tastes direct mobility, consumers with lowest match values move away.

[64]In the US, the regulation is based on the *Do Not Track Me Online Act* (2011) and the *Kerry/McCain Commercial Privacy Bill of Rights Act* (2011) and the self-regulation of the Network Advertising Alliance (NAA). In the EU, the legislative framework for the single market is provided by the *ePrivacy Directive* (2009), possibly later modified by the recent European Commission (EC) *Proposal for a Regulation on Privacy and Electronic Communications* (2017). See Dwoskin (2015), Kaye (2015), and Pociask (2016).

[65]See Destefanis and Fonseca (2007), Fahr and Sunde (2006), Ibourk et al. (2004), and Ilmakunnas and Pesola (2003) (2003).

problem with the latter approach is that it seems to lead to unreasonably high coordination frictions. We attempt to circumvent both problems here. This is done by introducing learning dynamics into an otherwise typical directed search framework. Trade history endows a party with a piece of prior information about the other side, after which the previously similar-looking counter-parties no longer look the same. This makes it easier to match the right people with the right places and, in our big data extension where both sides view the same information, avoids the costly delay of investing in uninterested trading partners. Additional information may also have beneficial effects on market competition. In the future, we would like to study this in a setup with network effects.

References

Acemoglu, D. and R. Shimer (1999). Holdups and efficiency with search frictions. *International Economic Review 40*(4), 827–49.

Acquisti, A., C. Taylor, and L. Wagman (2016). The economics of privacy. *Journal of Economic Literature 54*(2), 442–92.

Acquisti, A. and H. R. Varian (2005). Conditioning prices on purchase history. *Marketing Science 24*(3), 367–381.

Adams, W. J. and J. L. Yellen (1976). Commodity bundling and the burden of monopoly. *The Quarterly Journal of Economics 90*(3), 475–498.

Albrecht, J., P. A. Gautier, and S. Vroman (2016). Directed search in the housing market. *Review of Economic Dynamics 19*, 218–231.

Armstrong, M. (2006). Recent developments in the economics of price discrimination. In R. Blundell, W. Newey, and T. Persson (Eds.), *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress: volume II*, pp. 97–141. Cambridge University Press: Cambridge, UK.

Borjas, G. J., I. Kauppinen, and P. Poutvaara (2015). Self-selection of emigrants: Theory and evidence on stochastic dominance in observable and unobservable characteristics. Working paper.

Burdett, K., S. Shi, and R. Wright (2001). Pricing and matching with frictions. *Journal of Political Economy 109*(5), 1060–1085.

Cabral, L. (2016). Dynamic pricing in customer markets with switching costs. *Review of Economic Dynamics 20*, 43–62.

Camera, G. and J. Kim (2016). Dynamic directed search. *Economic Theory 62*(1), 131–154.

Caminal, R. and X. Vives (1996). Why market shares matter: An information-based theory. *The RAND Journal of Economics 27*(2), 221–239.

Caminal, R. and X. Vives (1999). Price dynamics and consumer learning. *Journal of Economics & Management Strategy 8*(1), 95–131.

Campbell, A. J. (1997). Relationship marketing in consumer markets: A comparison of managerial and consumer attitudes about information privacy. *Journal of Direct Marketing 11*(3), 44–57.

Chang, B. (2017). Adverse selection and liquidity distortion. *Review of Economic Studies 85*(1), 275–306.

Chen, Y. (1997). Paying customers to switch. *Journal of Economics & Management Strategy 6*(4), 877–897.

Coles, M. G. and J. Eeckhout (2000). Heterogeneity as a coordination device. Working paper.

Delacroix, A. and S. Shi (2013). Pricing and signaling with frictions. *Journal of Economic Theory 148*(4), 1301–1332.

Destefanis, S. and R. Fonseca (2007). Matching efficiency and labour market reform in italy: A macroeconometric assessment. *Labour 21*(1), 57–84.

Dwoskin, E. (2015, Dec. 16). Eu data-privacy law raises daunting prospects for u.s. companies. The WSJ, http://www.wsj.com/articles/eu-data-privacy-law-raises-daunting-prospects-for-u-s-companies-1450306033. Website accessed: Apr. 2016.

Eeckhout, J. and P. Kircher (2010). Sorting and decentralized price competition. *Econometrica 78*(2), 539–574.

Eeckhout, J. and P. Kircher (2011). Identifying sorting: In theory. *The Review of Economic Studies 78*(3), 872–906.

Eeckhout, J., R. Pinheiro, and K. Schmidheiny (2014). Spatial sorting. *Journal of Political Economy 122*(3), 554–620.

Fahr, R. and U. Sunde (2006). Regional dependencies in job creation: An efficiency analysis for western germany. *Applied Economics 38*(10), 1193–1206.

Farrell, J. and P. Klemperer (2007). Chapter 31 coordination and lock-in: Competition with switching costs and network effects. In M. Armstrong and R. Porter (Eds.), *Handbook of Industrial Organization*, Volume 3 of *Handbook of Industrial Organization*, pp. 1967–2072. Elsevier.

Fudenberg, D. and J. Tirole (2000). Customer poaching and brand switching. *The RAND Journal of Economics 31*(4), 634–657.

Godenhielm, M. and K. Kultti (2015). Directed search with endogenous capacity. *The BE Journal of Theoretical Economics 15*(2), 211–249.

Gourio, F. and L. Rudanko (2014). Customer capital. *The Review of Economic Studies 81*(3), 1102–1136.

Guerrieri, V., R. Shimer, and R. Wright (2010). Adverse selection in competitive search equilbirium. *Econometrica 78*(6), 1823–1862.

Gyourko, J., C. Mayer, and T. Sinai (2013). Superstar cities. *American Economic Journal: Economic Policy 5*(4), 167–99.

Hui, K.-L. and I. P. L. Png (2006). Economics of privacy. In T. Hendershott (Ed.), *Economics and Information System*, Volume 1, Chapter 9, pp. 471–493. Emeral Group Publishing, Ltd.

Ibourk, A., B. Maillard, S. Perelman, and H. R. Sneessens (2004). Aggregate matching efficiency: A stochastic production frontier approach france 1990–1994. *Empirica 31*(1), 1–25.

Ilmakunnas, P. and H. Pesola (2003). Regional labour market matching functions and efficiency analysis. *Labour 17*(3), 413–437.

Julien, B., J. Kennes, and I. King (2000). Bidding for labor. *Review of Economic Dynamics 4*(3), 619–649.

Kaye, K. (2015, Dec 18). New european privacy rules mean more complexity for data-hungry marketers. `http://adage.com/article/datadriven-marketing/eu-privacy-rules-complexity-data-marketers/301854/`. Website Accessed Apr., 2016.

Keisuke, K., N. Kentaro, and S. Yasuhiro (2014). Competitive search with moving costs. Working paper.

Kovbasyuk, S. and G. Spagnolo (2017). Memory and markets. Working paper.

Kultti, K. (1999). Equivalence of auctions and posted prices. *Games and Economic behavior 27*(1), 106–113.

Lauermann, S. (2012). Asymmetric information in bilateral trade and in markets: An inversion result. *Journal of Economic Theory 147*(5), 1969–1997.

Leahy, R. (2011). Relationships in fast moving consumer goods markets: The consumers' perspective. *European Journal of Marketing 45*(4), 651–672.

Lester, B. (2010). Directed search with multi-vacancy firms. *Journal of Economic Theory 145*(6), 2108–2132.

Lester, B. (2011). Information and prices with capacity constraints. *American Economic Review 101*(4), 1591–1600.

Li, F. and X. Weng (2017). Efficient learning and job turnover in the labor market. *International Economic Review 58*(3), 727–750.

Menzio, G. and N. Trachter (2015). Equilibrium price dispersion with sequential search. *Journal of Economic Theory 160*, 188–215.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of political Economy 105*(2), 385–411.

Moraga-González, J. L. and M. Watanabe (2016). Competitive sequential search equilibrium. Working paper.

Peters, M. (1984). Bertrand equilibrium with capacity constraints and restricted mobility. *Econometrica 52*(5), 1117–1127.

Peters, M. (1991). Ex ante price offers in matching games non-steady states. *Econometrica 59*(5), 1425–1454.

Pociask, S. (2016, Mar. 31). Consumer privacy protections must work for everyone. `http://www.forbes.com/sites/stevepociask/2016/03/31/consumer-privacy-protections-must-work-for-everyone/\#5f6f5c7834a5`. Website Accessed Jul. 2016.

Posner, R. A. (1981). The economics of privacy. *The American Economic Review 71*(2), 405–409.

Shi, S. (2013). Customer relationship and sales. Working paper.

Shi, S. (2016). Customer relationship and sales. *Journal of Economic Theory 166*, 483–516.

Shy, O. and R. Stenbacka (2015). Customer privacy and competition. *Journal of Economics & Management Strategy 25*(3), 539–562.

Stevens, M. (2007). New microfoundations for the aggregate matching function. *International Economic Review 48*(3), 847–868.

Stigler, G. J. (1980). An introduction to privacy in economics and politics. *The Journal of Legal Studies 9*(4), 623–644.

Taylor, C. R. (2004). Consumer privacy and the market for customer information. *The RAND Journal of Economics 35*(4), 631–650.

Varian, H. R. (2009). Economic aspects of personal privacy. In W. H. Lehr and L. Pupillo (Eds.), *Internet policy and economics*, pp. 101–109. Springer.

Wang, P. and L. A. Petrison (1993). Direct marketing activities and personal privacy: A consumer survey. *Journal of Direct Marketing 7*(1), 7–19.

Watanabe, M. (2010). A model of merchants. *Journal of Economic Theory 145*(5), 1865–1889.

Wright, R., P. Kircher, B. Jullen, and V. Guerrieri (2017). Directed search: A guided tour. Working Paper.

Appendix

Derivation of consumers' trading probabilities. We derive these probabilities by following the methodology of Shi (2013).

The probability that a consumer of type 0 trades with a host of type 0 is $g_0(1)$, where

$$g_0(x) = \sum_{i=0}^{(1-\rho)N^g-1} \sum_{j=0}^{\rho N^g} \frac{C^i_{(1-\rho)N^g-1}(1-\theta_{00})^{(1-\rho)N^g-1-i}(x\theta_{00})^i}{i+j+1} \times \left[C^j_{\rho N^g}(1-a\theta_{10})^{\rho N^g-j}(xa\theta_{10})^j\right]$$

Then,

$$\begin{aligned}\frac{\partial xg(x)}{\partial x} &= \sum_{i=0}^{(1-\rho)N^g-1} C^i_{(1-\rho)N^g-1}(1-\theta_{00})^{(1-\rho)N^g-1-i}(x\theta_{00})^i \times \sum_{j=0}^{\rho N^g} C^j_{\rho N^g}(1-a\theta_{10})^{\rho N^g-j}(xa\theta_{10})^j \\ &= (1-\theta_{00}+x\theta_{00})^{(1-\rho)N^g-1}(1-a\theta_{10}+xa\theta_{10})^{\rho N^g}\end{aligned}$$

Because $xg_0(x)$ and $g_0(x)$ are bounded for all $x \in (0,1)$, they are also integrable there. Then

$$xg_0(x) = \int_0^x (1-(1-y)\theta_{00})^{(1-\rho)N^g-1}(1-(1-y)a\theta_{10})^{\rho N^g}\,dy$$

By using the definition of queue lengths we obtain

$$xg_0(x) = \int_0^x \left(1-(1-y)\frac{q_{00}}{(1-\rho)N^g}\right)^{(1-\rho)N^g-1}\left(1-(1-y)\frac{q_{10}}{\rho N^g}\right)^{\rho N^g} dy$$

Finally, we take the limit $N^g \to \infty$ and obtain

$$xg_0(x) = \int_0^x e^{-q_{00}(1-y)-q_{10}(1-y)}dy = \int_0^x e^{-q_0(1-y)}dy = \frac{e^{q_0(x-1)}-e^{-q_0}}{q_0}$$

After we set $x=1$, we obtain the probability of trade for a consumer with a host of type 0 that equals $(1-e^{-q_0})/q_0$.

Now we derive the probability of a consumer of type 0 to trade with a host of type 1. We denote this probability by $g_1(1)$. This probability is

$$\begin{aligned}g_1(x) = &\sum_{i=0}^{(1-\rho)N^g-1} \sum_{j=0}^{\rho N^g-1} \frac{(1-a)xC^i_{(1-\rho)N^g-1}(1-\theta_{01})^{(1-\rho)N^g-1-i}(x\theta_{01})^i}{i+j+2} \times \left[C^j_{\rho N^g-1}(1-a\theta_{10})^{\rho N^g-j}(xa\theta_{10})^j\right] \\ &+ \sum_{i=0}^{(1-\rho)N^g-1} \sum_{j=0}^{\rho N^g-1} \frac{aC^i_{(1-\rho)N^g-1}(1-\theta_{10})^{(1-\rho)N^g-1-i}(x\theta_{10})^i}{i+j+1} \times \left[C^j_{\rho N^g-1}(1-a\theta_{10})^{\rho N^g-j}(xa\theta_{10})^j\right]\end{aligned}$$

We take the derivative of $xg(x)$ with respect to x.

$$\begin{aligned}
\frac{\partial x g_1(x)}{\partial x} &= (1-a)\, x \left(\sum_{i=0}^{(1-\rho)N^g-1} C^i_{(1-\rho)N^g-1} (1-\theta_{10})^{(1-\rho)N^g-1-i} (x\theta_{10})^i \right. \\
&\quad \left. \times \sum_{j=0}^{\rho N^g-1} C^j_{\rho N^g-1} (1-a\theta_{10})^{\rho N^g-j} (xa\theta_{10})^j \right) \\
&\quad + a \left(\sum_{i=0}^{(1-\rho)N^g-1} C^i_{(1-\rho)N^g-1} (1-\theta_{10})^{(1-\rho)N^g-1-i} (x\theta_{10})^i \right. \\
&\quad \left. \times \sum_{j=0}^{\rho N^g-1} C^j_{\rho N^g-1} (1-a\theta_{10})^{\rho N^g-j} (xa\theta_{10})^j \right) \\
&= (1-a)\, x \left(1-\theta_{10}(1-x)\right)^{N^g(1-\rho)-1} \left(1-a\theta_{10}(1-x)\right)^{\rho N^g-1} \\
&\quad + a \left(1-\theta_{10}(1-x)\right)^{N^g(1-\rho)-1} \left(1-a\theta_{10}(1-x)\right)^{\rho N^g-1}
\end{aligned}$$

By following the same substitution of probabilities with the queue lengths we obtain

$$xg_1(x) = \int_0^x (a + y(1-a)) \left(1 - (1-y)\frac{q_{10}}{(1-\rho)N^g}\right)^{(1-\rho)N^g-1} \times \left(1-(1-y)\frac{q_{10}}{\rho N^g - 1}\right)^{\rho N^g - 1} dy.$$

After taking the limit $N^g \to \infty$ and using q_1, we obtain

$$\begin{aligned}
xg_1(x) &= \int_0^x (a + y(1-a))\, e^{-q_1(1-y)} dy \\
&= a\frac{-e^{-q_1} + e^{-q_1(1-x)}}{q_1} + (1-a)\frac{e^{-q_1} + e^{-q_1(1-x)}(q_1 x - 1)}{q_1^2}
\end{aligned}$$

and

$$g_1(1) = a\frac{1-e^{-q_1}}{q_1} + (1-a)\frac{e^{-q_1} - 1 + q_1}{q_1^2} = \frac{1-e^{-q_1}}{q_1} + (1-a)\frac{e^{-q_1}(1+q_1) - 1}{q_1^2}.$$

The probability for an informed consumer to trade with her host equals $\left(1-e^{-q1}\right)/q_1$.

□

Proof of Proposition 1

Assuming that $q_1 = 0$ (6) and (9), results in

$$\begin{aligned}
q_0 &= \frac{N^g(1-\rho) + N^g \rho a}{N^h - \rho N^g} = \frac{1-\rho(1-a)}{\nu - \rho}, \\
\rho &= \frac{1-\nu q_0}{1-a-q_0},
\end{aligned} \tag{47}$$

and

$$
\begin{aligned}
\rho N^g &= \left(1-e^{-q_0}\right)\left(N^h-\rho N^g\right)+\rho N^g\left(1-a\right), \\
\rho &= \nu \frac{1-e^{-q_0}}{1-e^{-q_0}+a}.
\end{aligned} \tag{48}
$$

By joining (47) and (48), we obtain

$$
\begin{aligned}
\rho &= \frac{1-\nu q_0}{1-a-q_0} = \nu \frac{1-e^{-q_0}}{1-e^{-q_0}+a}, \\
a &= \frac{1}{1+\xi}, \text{ where } \xi = \frac{2-e^{-q_0}-\nu q_0}{(\nu-1)(1-e^{-q_0})}.
\end{aligned} \tag{49}
$$

Together with the first order conditions (22) and (23), this implies that the following two conditions, which define a as a function of q_0 and the parameter ν, must hold in a separating equilibrium with consumer privacy:

$$
a = \frac{2+e^{-q_0}}{4}, \tag{50}
$$

$$
a = \frac{1}{1+\xi}, \tag{51}
$$

where

$$
\xi = \frac{2-e^{-q_0}-\nu q_0}{(\nu-1)(1-e^{-q_0})}.
$$

We split the further proof of the proposition into three parts. In the first part we prove that there is a unique fixed point for the system of equations (51) and (50). In the second part, we specify the values of ν that make large downwards deviations of a host of type 1 unprofitable. In the third part, we discuss about the shape of J_1 and demonstrate that the payoff function is well-behaved.

Part I. Proving the existence of a fixed-point. We denote the the left-hand-side (LHS) of (50) by a_1. We observe that the RHS of the equation is decreasing in q_0. a_1 equals 3/4 if $q_0 = 0$ and $\lim_{q_0 \to \infty} a_1 = 1/2$. Thus the solution a is in between 1/2 and 3/4.

Next, we denote the LHS of (51) by a_2. The denominator of a_2 equals zero if $q_0 = q_a = \left(1+\nu+\nu W\left(-e^{-1-1/\nu}\right)\right)/\nu$, where W is a Lambert-W function. The point q_a is where $a_2(q_0)$ is discontinuous.

The derivative of the RHS of the equation with respect to q_0 has the same sign as

$$
\begin{aligned}
-\left(\nu-1\right)\left(1-e^{-q_0}\right)^2 \frac{\partial \xi}{\partial q_0} &= -\left(1-e^{-q_0}\right)\left(e^{-q_0}-\nu\right)+e^{-q_0}\left(2-e^{-q_0}-\nu q_0\right) \\
&= e^{-q_0}-\nu\left(e^{-q_0}-1+e^{-q_0}q_0\right).
\end{aligned} \tag{52}
$$

Because $e^{-q_0}-1+e^{-q_0}q_0<0$, we conclude that (52) is positive and a_2 is increasing in q_0.

We observe that $\lim_{q_0 \to 0} a_2 = 0$, and $\lim_{q_0 \to q_0} a_2 = \infty$. Thus, it crosses with a_1 once, which guarantees the uniqueness of the solution.

Part II. Ruling out large deviations. Suppose that a host of type 1 deviates to a lower price p_1^d to attract uninformed consumers for one period. After the deviation, the host returns to the equilibrium path and it is common knowledge. In this case, the informed related consumer of the deviant trades with the host with probability β_d^c, and the consumer is indifferent if $u = a^d$ that is given by

$$\beta_d^g \left(a^d - p_1^d \right) = V_0.$$

Since an uninformed consumer visits the deviant and trades with probability $-\alpha_d^g \left(1 - a^d\right) + \beta_d^g$, she must be indifferent between visiting the host and any other seller, which gives

$$p_1^d = \frac{1}{2} - \frac{V_0}{\beta_d^g - \left(1 - a^d\right) \alpha_d^g}.$$

By merging the last two equations we obtain

$$\beta_d^g \left(a^d - \frac{1}{2} + \frac{V_0}{\beta_d^g - \left(1 - a^d\right) \alpha_d^g} \right) = V_0, \tag{53}$$

which requires $a^d < 1/2$.

The profit of the deviant is

$$J_1^d = \left(1 - a^d e^{-q_d}\right) p_1^d = \left(1 - a^d e^{-q_d}\right) \left(\frac{1}{2} - \frac{V_0}{\beta_d^g - \alpha_d^g \left(1 - a^d\right)} \right).$$

We use the first-order condition of a host of type 0 and obtain that $V_0 = e^{-q_0}/2$. Thus, we have

$$J_1^d = \left(1 - a^d e^{-q_d}\right) \frac{1}{2} \left(1 - \frac{e^{-q_0}}{\beta_d^g - \alpha_d^g \left(1 - a^d\right)} \right).$$

The profit of a host of type 1 in equilibrium equals

$$J_1 = (1 - a)(a - V_0) = (1 - a)(1 - a),$$

where the second equality has been obtained by using the first-order condition of a host of type 1.

Then the deviation is not profitable if

$$\left(1 - a^d e^{-q_d}\right) \frac{1}{2} \left(1 - \frac{e^{-q_0}}{\beta_d^g - \alpha_d^g \left(1 - a^d\right)} \right) - (1 - a)^2 < 0. \tag{54}$$

We replace a with $\left(2+e^{-q_0}\right)/4$ and take the derivative with respect to q_0. This derivative is

$$\left(1-a^d e^{-q_d}\right)\frac{1}{2}\frac{e^{-q_0}}{\beta_d^g-\alpha_d^g\left(1-a^d\right)}-2\left(1-\frac{2+e^{-q_0}}{4}\right)\frac{e^{-q_0}}{4}=$$
$$\frac{e^{-q_0}}{2}\left(\frac{1-a^d e^{-q_d}}{\beta_d^g-\alpha_d^g\left(1-a^d\right)}-\frac{1}{2}+\frac{e^{-q_0}}{4}\right).$$

We observe that $\frac{1-a^d e^{-q_d}}{\beta_d^g-\alpha_d^g\left(1-a^d\right)}$ is decreasing in a^d and $\frac{1-e^{-q_d}/2}{\beta_d^g-\alpha_d^g/2}$ is increasing in q_d. Therefore,

$$\frac{1-a^d e^{-q_d}}{\beta_d^g-\alpha_d^g\left(1-a^d\right)}-\frac{1}{2}+\frac{e^{-q_0}}{4}\geq\lim_{q_d\to 0}\frac{1-e^{-q_d}/2}{\beta_d^g-\alpha_d^g/2}-\frac{1}{2}+\frac{e^{-q_0}}{4}=\frac{2}{3}-\frac{1}{2}+\frac{e^{-q_0}}{4}>0.$$

Thus, (54) is less than

$$\left(1-a^d e^{-q_d}\right)\frac{1}{2}\left(1-\frac{e^{-q_b}}{\alpha_d^g\left(1-a^d\right)+\beta_d^g}\right)-\left(\frac{1}{2}-\frac{e^{-q_b}}{4}\right)^2, \tag{55}$$

where $q_b=\left(7-\nu+3\nu W\left(\frac{e^{\frac{1}{3}-\frac{7}{3\nu}}(\nu-4)}{3\nu}\right)\right)/(3\nu)$ is obtained by setting $a=3/4$ in (51).

The probability of trade $\alpha_d\left(1-a^d\right)+\beta_d^c$ is smaller if q^d is higher, Thus (55) is less than after taking $\lim_{q_d\to 0}\left(\alpha_d^g\left(1-a^d\right)+\beta_d^g\right)=\left(1+a^d\right)/2$, which gives

$$\left(1-a^d e^{-q_d}\right)\frac{1}{2}\left(1-\frac{2e^{-q_b}}{1+a^d}\right)-\left(\frac{1}{2}-\frac{e^{-q_b}}{4}\right)^2<\left(1-a^d e^{-q_b}\right)\frac{1}{2}\left(1-\frac{2e^{-q_b}}{1+a^d}\right)-\left(\frac{1}{2}-\frac{e^{-q_b}}{4}\right)^2 \tag{56}$$

The derivative of the RHS of (56) with respect to a^d is

$$\frac{e^{-q_b}}{2\left(1+a^d\right)^2}\left(2\left(e^{-q_b}+1\right)-\left(1+a^d\right)^2\right),$$

and the RHS of (56) attains its maximum when

$$a^d=\sqrt{2\left(1+e^{-q_b}\right)}-1.$$

We plug this value of a^d in the RHS of (56) and obtain

$$\frac{15e^{-2q_b}}{16}+\frac{3e^{-q_b}}{4}-\frac{\sqrt{2}e^{-2q_b}}{\sqrt{e^{-q_b}+1}}-\frac{\sqrt{2}e^{-q_b}}{\sqrt{e^{-q_b}+1}}+\frac{1}{4} \tag{57}$$

The expression (57) is negative if $q_b<0.7$. This implies that ν must be greater than than 2.

Part III. The shape of J_1. We take a host of type 1 that we label host k. Further, we take a set of p_k and plot J_{1k} for fixed values of V_0. The result is given in Figure 11. The payoff function has two peaks. The right peak is at a symmetric separating equilibrium $p_k=p_1$. If p_k sufficiently decreases, then we obtain a kink, which is followed by the left peak. At the left peak the deviation profit of host k is maximized and $q_k=q_d>0$. While running numerical simulations, we obtained that if $V_0>0.148$, then the right peak is higher. Thus, no large downward deviation is profitable.

However, if V_0 is less than this value, then host k prefers to deviate to a smaller price. The value $V_0 = 0.148$ corresponds to a unique $\nu = 1.2798$. Hence, if $\nu > 1.2798$ the equilibrium in the market is the described separating equilibrium. $\square$

Proof that learning improves welfare in the separating equilibrium under $\nu > 2$. In the separating equilibrium, by (50) and (51) there is a one-to-one relationship between ν and q_0

$$\begin{aligned}
\nu &= \frac{1-e^{-q_0}+a}{\left(1-e^{-q_0}\right)\left(1-a-q_0\right)+q_0\left(1-e^{-q_0}+a\right)} \\
&= \frac{1-e^{-q_0}+a}{\left(1-e^{-q_0}\right)\left(1-a\right)+q_0 a} = \frac{1-e^{-q_0}+\frac{2+e^{-q_0}}{4}}{\left(1-e^{-q_0}\right)\left(1-\frac{2+e^{-q_0}}{4}\right)+q_0\frac{2+e^{-q_0}}{4}} \\
&= \frac{3\left(2-e^{-q_0}\right)}{\left(1-e^{-q_0}\right)\left(2-e^{-q_0}\right)+q_0\left(2+e^{-q_0}\right)}
\end{aligned}$$

We plug this value of ν in (32) and set $a = \left(2+e^{-q_0}\right)/4$. As a result, (32) becomes a function of q_0. The obtained function is positive for any $q_0 < 0.7$ $\square$

Proof of Proposition 2 We distinguish three parts in the proof. In the first part, we prove the existence of a unique fixed point. In the second part, we rule out large deviations of hosts of type 1. In the third part, we discuss the concavity of the payoff of a host of type 1.

Part I. Proving the existence of a fixed-point. In the proof, we focus on $q_1 < q_0$. The indifference condition (21) can be rewritten as

$$a^2\alpha_1^g - a\left(\frac{3\alpha_1^g}{2} - \beta_1^g + \frac{V_0\alpha_1^g}{\beta_1^g}\right) + \frac{V_0\alpha_1^g}{\beta_1^g} + \frac{1}{2}\alpha_1^g - \frac{\beta_1^g}{2} = 0. \tag{58}$$

The LHS of (58) is a convex second-degree polynomial of a. If $a = 0$, then the LHS of the equation is negative because

$$\begin{aligned}
\frac{V_0\alpha_1^g}{\beta_1^g} - \frac{1}{2}\left(\frac{e^{-q_1}\left(q_1+1\right)-1+q_1-e^{-q_1}q_1}{q_1^2}\right) &= \frac{V_0\alpha_1^g}{\beta_1^g} - \frac{1}{2}\left(\frac{e^{-q_1}-1+q_1}{q_1^2}\right) < \\
&\frac{e^{-q_1}\left(1-e^{-q_1}\left(1+q_1\right)\right)}{2\left(1-e^{-q_1}\right)q_1} - \frac{1}{2}\left(\frac{e^{-q_1}-1+q_1}{q_1^2}\right) < 0.
\end{aligned}$$

Thus, it is the larger one of the two roots of the second order equation that is of our interest. The root lies below $1/2$ because

$$\frac{\alpha_1^g}{4} - \frac{3\alpha_1^g}{4} + \frac{\beta_1^g}{2} - \frac{V_0\alpha_1^g}{2\beta_1^g} + \frac{V_0\alpha_1^g}{\beta_1^g} + \frac{1}{2}\alpha_1^g - \frac{\beta_1^g}{2} = \frac{V_0\alpha_1^g}{2\beta_1^g} > 0.$$

More specifically, this root equals

$$\begin{aligned}
a &= \frac{\frac{3}{2}\alpha_1^g - \beta_1^g + \frac{V_0\alpha_1^g}{\beta_1^g} + \sqrt{D}}{2\alpha_1^g} = \frac{3\alpha_1^g\beta_1^g - 2\left(\beta_1^g\right)^2 + 2V_0\alpha_1^g + 2\beta_1^g\sqrt{D}}{4\alpha_1^g\beta_1^g} \\
&= \frac{3}{4} - \frac{\beta_1^g}{2\alpha_1^g} + \frac{V_0}{2\beta_1^g} + \frac{\sqrt{D}}{2\alpha_1^g},
\end{aligned} \tag{59}$$

where the discriminant is given by

$$D=\left(-\frac{3\alpha_1^g}{2}+\beta_1^g-\frac{V_0\alpha_1^g}{\beta_1^g}\right)^2+4\alpha_1^g\left(-\frac{V_0\alpha_1^g}{\beta_1^g}-\frac{1}{2}\alpha_1^g+\frac{\beta_1^g}{2}\right).$$

We also observe that the functions β_1^g/α_1^g and $1/\beta_1^g$ are continuous in q_1. Hence, we continue by studying the properties of $D/\left(4\left(\alpha_1^g\right)^2\right)$, which equals

$$\frac{1}{4\left(\alpha_1^g\right)^2}\left(-\frac{3\alpha_1^g}{2}+\beta_1^g-\frac{V_0\alpha_1^g}{\beta_1^g}\right)^2+\frac{4\alpha_1^g}{4\left(\alpha_1^g\right)^2}\left(-\frac{V_0\alpha_1^g}{\beta_1^g}-\frac{1}{2}\alpha_1^g+\frac{\beta_1^g}{2}\right)= \\ \frac{1}{4}\left(-\frac{3}{2}+\frac{\beta_1^g}{\alpha_1^g}-\frac{V_0}{\beta_1^g}\right)^2+\left(-\frac{V_0}{\beta_1^g}-\frac{1}{2}+\frac{\beta_1^g}{2\alpha_1^g}\right) \tag{60}$$

Clearly, the expression (60) is continuous in q_1. In addition, the derivative of (60) with respect to V_0 is negative:

$$-\frac{1}{2\beta_1^g}\left(\frac{\beta_1^g}{\alpha_1^g}+\frac{1}{2}-\frac{V_0}{\beta_1^g}\right)<-\frac{1}{2\beta_1^g}\left(\frac{\left(1-e^{-q_1}\right)q_1}{1-e^{-q_1}-e^{-q_1}q_1}+\frac{1}{2}-\frac{e^{-q_1}q_1}{2\left(1-e^{-q_1}\right)}\right)<0.$$

We set $V_0/\beta_1^g=e^{-q_1}/\left(2\beta_1^g\right)$ in (60), and obtain a positive expression

$$\frac{1}{4}\left(-\frac{3}{2}+\frac{\beta_1^g}{\alpha_1^g}-\frac{e^{-q_1}}{2\beta_1^g}\right)^2+\left(-\frac{e^{-q_1}}{2\beta_1^g}-\frac{1}{2}+\frac{\beta_1^g}{2\alpha_1^g}\right)>0.$$

As a result, we conclude that (59) is a continuous function in q_1. In what follows, we study the behavior of the implicit relationship $q_0^1\left(q_1\right)$ that is obtained by merging the first-order conditions of both host types and the indifference condition (58). We analyze how $q_0^1\left(q_1\right)$ behaves in the q_1q_0-plane, where q_1 is hence on the horizontal axis. Particularly, we focus on the area above 45−degree line where $q_0>q_1$.

The derivative $\partial a/\partial q_1$ exists for all $q_1<q_0$. Thus, there is a continuous implicit relationship $q_0^1\left(q_1\right)$ that is defined by (30). To proceed, we next take the limit of the LHS of (30) as $q_1\to 0$, which gives us

$$\frac{14V_0^2-37V_0+9+\left(3-17V_0\right)\sqrt{4V_0^2-20V_0+9}}{24\sqrt{4V_0^2-20V_0+9}}=0. \tag{61}$$

The equation (61) has a unique fixed point $V_0=0.218731$, which gives $q_0=0.826764$.

On the other hand, if we set $q_1=q_0$, we obtain a negative expression that approaches zero as q_0 approaches infinity. Thus, the implicit relationship $q_0^1\left(q_1\right)$ never crosses 45-degree line although it approaches it asymptotically as $q_0\to\infty$.

Next, we study the steady state conditions

$$\frac{\left(1-e^{-q_0}\right)\left(\nu+q_1\nu-1\right)}{e^{-q_1}\left(1-\nu q_0\right)+\nu\left(1-e^{-q_0}\right)}-a=0, \tag{62}$$

where a is given by (59). This defines an implicit relationship $q_0^2\left(q_1\right)$, which is not continuous. In particular, this relation is not defined for

$$e^{-q_1} = \frac{\nu\left(1-e^{-q_0}\right)}{1-\nu q_0}. \tag{63}$$

The RHS of (63) is increasing in q_0. After we set $q_0 = q_1$ and move the terms on the RHS to the LHS, we obtain the following inequality

$$\begin{aligned} e^{-q_1} - \frac{\nu\left(1-e^{-q_1}\right)}{1-\nu q_1} &> e^{-q_1} - \lim_{\nu\to\infty} \frac{\nu\left(1-e^{-q_1}\right)}{1-\nu q_1} \\ &= e^{-q_1} + \frac{1-e^{-q_1}}{q_1} > 0. \end{aligned}$$

As a result, the implicit relationship $q_0^2\left(q_1\right)$ is continuous if $q_0 > q_1$.

Thus, we again set $q_1 = 0$ and obtain that the LHS of (62) equals

$$\frac{\left(1-e^{-q_0}\right)(\nu-1)}{1-\nu q_0+\nu\left(1-e^{-q_0}\right)} - \frac{1}{4}\left(\sqrt{4V_0^2-20V_0+9}+2V_0-1\right) = 0. \tag{64}$$

We solve (62) for ν

$$\nu = \frac{ae^{-q_1}+1-e^{-q_0}}{\left(1-e^{-q_0}\right)\left(1+q_1-a\right)+ae^{-q_1}q_0}. \tag{65}$$

Then we plug the value of a from (59), set $V_0 = e^{-q_0}/2$ and take $q_1 \to 0$. This gives ν as a decreasing function of q_0. When q_0 approaches 0.826764, the value of ν is less than 1.447.
and set $q_1 = q_0$ to obtain

$$\nu = \frac{ae^{-q_0}+1-e^{-q_0}}{\left(1-e^{-q_0}\right)\left(1+q_0-a\right)+ae^{-q_0}q_0}. \tag{66}$$

The LHS of (66) is a decreasing function in q_0 and there is a value of q_0 that solves (66) for any $\nu < 1.447$. As a result, the implicit relationship $q_0^2\left(q_1\right)$, necessarily crosses 45-degree line and thus the other implicit relationship $q_0^1\left(q_1\right)$. Consequently, we conclude that the fixed point exists.

Part II. Ruling out large upward deviations. Suppose that a host of type 1 deviates to a very high price such that no uninformed consumer visits him, and the informed related consumer is indifferent between visiting the deviant and going to any other host. Then the deviant host chooses a^d to maximize

$$\left(1-a^d\right)\left(a^d-V_0\right).$$

Now, the profit-maximizing a^d equals $\left(1+V_0\right)/2$ and the deviation profit is given by

$$J^d = \left(1-\frac{1+V_0}{2}\right)\left(\frac{1+V_0}{2}-V_0\right) = \frac{\left(1-V_0\right)^2}{4}.$$

The deviation is therefore unprofitable if $J^d < J_1$. The equilibrium profits are instead

$$J_1 = \left(1-ae^{-q_1}\right)\left(a-\frac{V_0}{\beta_1^g}\right)$$

We next replace the cut-off a with (59) and take the derivative of J_1 with respect to V_0

$$\frac{\partial a}{\partial V_0}\left(1-e^{-q_1}\left(2a-\frac{V_0}{\beta_1^g}\right)\right)-\frac{1-ae^{-q_1}}{\beta_1^g},$$

where

$$\frac{\partial a}{\partial V_0}=\frac{1}{2\beta_1^g}\left(1-\frac{\left(\frac{\alpha_1^g}{2}+\beta_1^g-\frac{V_0\alpha_1^g}{\beta_1^g}\right)}{\sqrt{\left(-\frac{3\alpha_1^g}{2}+\beta_1^g-\frac{V_0\alpha_1^g}{\beta_1^g}\right)^2+4\alpha_1^g\left(-\frac{V_0\alpha_1^g}{\beta_1^g}-\frac{1}{2}\alpha_1^g+\frac{\beta_1^g}{2}\right)}}\right).$$

We then observe that

$$\left(\frac{\alpha_1^g}{2}+\beta_1^g-\frac{V_0\alpha_1^g}{\beta_1^g}\right)^2-\left(-\frac{3\alpha_1^g}{2}+\beta_1^g-\frac{V_0\alpha_1}{\beta_1^g}\right)^2-4\alpha_1^g\left(-\frac{V_0\alpha_1^g}{\beta_1^g}-\frac{1}{2}\alpha_1+\frac{\beta_1^g}{2}\right)=2\alpha_1\beta_1^g>0$$

Therefore, cut-off a is decreasing in V_0, and profit J_1 is decreasing in V_0. The derivative of J_1-J^d with respect to V_0 is given by

$$\frac{\partial a}{\partial V_0}\left(1-e^{-q_1}\left(2a-\frac{V_0}{\beta_1^g}\right)\right)-\frac{1-ae^{-q_1}}{\beta_1^g}+\frac{1-V_0}{2}.$$

We note that $a\leq 1/2$ and find that $-\frac{1-ae^{-q_1}}{\beta_1^g}+\frac{1-V_0}{2}$ is decreasing in V_0. By setting $V_0=0$ in $-\frac{1-ae^{-q_1}}{\beta_1^g}+\frac{1-V_0}{2}$, we obtain a negative function of q_1

$$-\frac{1-a|_{V_0=0}\,e^{-q_1}}{\beta_1^g}+\frac{1}{2}<0.$$

Hence, either the difference J_1-J^d is decreasing in V_0 or is increasing in q_0. In any case, the difference is always larger than what it is at $q_0=q_1$. Once we make the substitution $q_0=q_1$, we obtain J_1-J^d as a function of q_1. This function is positive if $q_1\geq 1.45103$.

When does this hold? If we set $q_1=1.455$ in (30), we get the corresponding value of $q_0=2.16922$. Also, if we set $q_1=1.46$ in (65) we find that the RHS of the equation is decreasing in $q_0\geq 1.455$. To be sure about the existence of equilibrium, we take $q_0>2.16922$ in (65) which gives us $\nu\leq 0.499222$.

Part III. The shape of J_1. We again take a host k that is of type 1 and plot his payoff for any p_k for different V_0 (Figure 11). The payoff has two peaks. The left peak corresponds to the symmetric semi-pooling equilibrium ($p_k=p_1$ and $q_k=q_1>0$). If the host increases p_k, then q_k decreases until it becomes equal zero (this gives a kink in J_k). If the host increases his price further, then only an informed related consumer of the seller buys, provided her match value is sufficiently high. The right peak corresponds to the deviation profit of host k. If V_0 is small, then the left peak is higher, which renders a deviation to a very high price unprofitable. However, if $V_0>0.148$, then the right peak is higher, which makes the semi-pooling equilibrium unsustainable. Hence, if $V_0<0.148$ or $\nu<0.938$ the equilibrium in the market is the specified semi-pooling equilibrium.

□

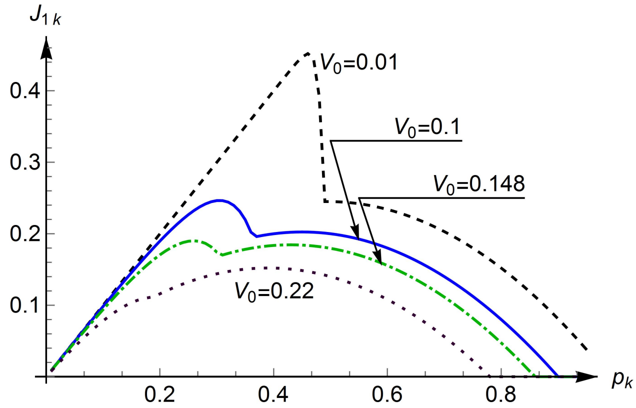


Figure 11: The payoff of host k

Proof of Proposition 3.

Consider first higher values of ν such that the equilibrium is separating. There the cut-off a is more than 1/2. By using the fist-order condition of the social planner (see the problem of the social planner in Part B in the Online appendix) we obtain, that the planner would set[66]

$$a^p = \frac{e^{-q_0^p}}{2},$$

Because $e^{-q_0^p} \leq 1$, $a^p \leq 1/2$. Hence, the equilibrium is inefficient.

Next, consider small values of ν such that the equilibrium is semi-pooling. In this case, the social planner picks such values of q_0 and q_1 that the probabilities of trade of an uninformed consumer are the same across the types of hosts, i.e.

$$\beta_1^{g,p} - \alpha_1^{g,p} (1 - a^p) = \beta_0^{g,p}.$$

Meanwhile in the separating equilibrium we have

$$\beta_1^g - \alpha_1^g (1 - a) \left(\frac{1}{2} - p_1 \right) = \beta_0^g \left(\frac{1}{2} - p_0 \right)$$

Because $p_1 \neq p_0$, the probabilities of trade are not equal, which means the the planner would prefer different queue lengths. As a result, we conclude that the equilibrium is inefficient in this case too. □

Proof of Proposition 4 To prove the existence of the equilibrium, we must show that there exists $\hat{q}_0 > 0$ such that it solves the first-order condition of a host of type 0, given the value of $\hat{a}$.

The equation (38) defines the relationship between $\hat{a}$ and $\hat{q}_0$. We denote this relationship by $\hat{a}_1$. The equation (35) gives the second relationship between $\hat{a}$ and $\hat{q}_0$. We denote this relationship by $\hat{a}_2$. In what follows we show that $\hat{a}_1$ and $\hat{a}_2$ cross in the plane $\hat{q}_0 \times \hat{a}$, where $\hat{q}_0 \in \mathbb{R}_+$ and $\hat{a} \in [0, 1/2]$.

[66]The superscript p shows that the variable is coming from the problem of the planner.

The derivative of $\hat{a}_1$ with respect to $\hat{q}_0$ is

$$\frac{\partial \hat{a}_1}{\partial \hat{q}_0} = \frac{(1-\nu)\, e^{-\hat{q}_0} \left(\hat{q}_0 \nu - \nu \left(1-e^{-\hat{q}_0}\right) - e^{-\hat{q}_0}\right) - (1-\nu)\left(1-e^{-\hat{q}_0}\right)\left(\nu - \nu e^{-\hat{q}_0} + e^{-\hat{q}_0}\right)}{\left[\hat{q}_0 \nu - \nu \left(1-e^{-\hat{q}_0}\right) - e^{-\hat{q}_0}\right]^2}$$
$$= \frac{(1-\nu)\left(\nu\left(\hat{q}_0 e^{-\hat{q}_0} - 1\right) + e^{-\hat{q}_0}(\nu-1)\right)}{\left[\hat{q}_0 \nu - \nu \left(1-e^{-\hat{q}_0}\right) - e^{-\hat{q}_0}\right]^2}.$$

Take $\nu > 1$. Then, $1-\nu < 0$ and the expression in the second parentheses of the numerator is decreasing in ν. If $\nu = 1$, the latter expression is negative. A a result, $\hat{a}_1$ is increasing in $\hat{q}_0$ when $\nu > 1$. If $\hat{q}_0$ approaches zero, then $\hat{a}_1$ equals zero. The highest value of $\hat{q}_0$ is such that the denominator of $\hat{a}_1$ equals zero, which is $1 + W\left(\frac{1-\nu}{e\nu}\right)$. When $\hat{q}_0$ approaches this value, $\hat{a}_1$ approaches infinity.

If $\nu < 1$, then $1-\nu > 0$ and the second expression in the numerator is negative (it is decreasing in ν, and it is negative when $\nu = 0$). Thus, the whole numerator is negative. As a result $\hat{a}_1$ is decreasing in $\hat{q}_0$. When $\hat{q}_0$ approaches $1 + W\left(\frac{1-\nu}{e\nu}\right)$ from the right, then $\hat{a}_1$ approaches infinity. If $\hat{q}_0 < 1 + W\left(\frac{1-\nu}{e\nu}\right)$, then $\hat{a}_1 < 0$. Also, $\lim_{\hat{q}_0 \to \infty} \hat{a}_1 = 0$.

If $\nu > 1$, then $1 + W\left(\frac{1-\nu}{e\nu}\right) < 1$. Thus, we consider $\hat{q}_0 < 1$ and obtain that the relationship $\hat{a}_2$ is decreasing in $\hat{q}_0$. When $\hat{q}_0 = 0$, then $\hat{a}_2 = 1/2$. If $\hat{q}_0$ approaches one, then $\hat{a}_2 = \left(1-e^{-1}\right)/2 < 1/2$. As a result, we conclude that if $\nu > 1$, then there is a unique crossing point of $\hat{a}_1$ and $\hat{a}_2$.

If $\nu < 1$, then $1 + W\left(\frac{1-\nu}{e\nu}\right) > 1$. Hence, we consider $\hat{q}_0 > 1$ and obtain that the relationship $\hat{a}_2$ is increasing in $\hat{q}_0$. We have that $\lim_{\hat{q}_0 \to \infty} \hat{a}_2 = 1/2$. As a result, we obtain that there is a unique crossing point of $\hat{a}_1$ and $\hat{a}_2$ in this instance.

If $\nu \to 1$, then $1 + W\left(\frac{1-\nu}{e\nu}\right) = 1$, $\hat{q}_0 = 1$ and $\hat{a} = \left(1-e^{-1}\right)/2$. □

Proof of Proposition 5. *The comparison of prices of type* 0 *hosts.* Take high values of ν for which the equilibrium with consumer privacy is separating. We take a steady state with a, ρ, q_0 and q_1 and make the following unexpected change: we force all informed consumers to visit other hosts when their match values with familiar products are less than a, hosts of type 1 must charge p_0 if their informed visitors draw $u < a$, and hosts of type 1 whose informed related consumers draw $u \geq a$ trade with their related guests.

After this change, the number of consumers who visit hosts charging p_0 does not change. However, the number of hosts charging p_0 increases by ρa. Thus, the queue length at a host charging p_0 decreases to $(1-\rho(1-a))/(\nu-\rho+\rho a)$.[67]

The number of consumers who trade changes by

$$\rho(1-a) + (1-\rho(1-a)) \frac{1-e^{-\frac{1-\rho(1-a)}{\nu-\rho(1-a)}}}{\frac{1-\rho(1-a)}{\nu-\rho(1-a)}} - (1-\rho(1-a)) \frac{1-e^{-\frac{1-\rho(1-a)}{\nu-\rho}}}{\frac{1-\rho(1-a)}{\nu-\rho}} - \rho(1-a) =$$
$$(1-\rho(1-a)) \left(\frac{1-e^{-\frac{1-\rho(1-a)}{\nu-\rho(1-a)}}}{\frac{1-\rho(1-a)}{\nu-\rho(1-a)}} - \frac{1-e^{-\frac{1-\rho(1-a)}{\nu-\rho}}}{\frac{1-\rho(1-a)}{\nu-\rho}} \right).$$

The probability of trade for a visitor who contacts a host charging p_0 is decreasing in q_0. As

[67]Because $\nu > 1$, the queue length decreases after adding more hosts. If $\nu < 1$, then the shift of some hosts of type 1 would make the queues longer.

a result, we conclude that there are more consumers who trade after this unexpected change. In a steady state without consumer privacy, $a < \hat{a}$. Therefore, there are fewer informed consumers who visit hosts charging p_0 As a result there is even more trade than after the unexpected change and $\hat{\rho} > \rho$.

The ratio $(1 - \rho(1-a)) / (\nu - \rho + \rho a)$ is decreasing in $\rho(1-a)$. Because $a > 1/2 > \hat{a}$ and $\rho < \hat{\rho}$, we obtain that

$$q_0 > \frac{1 - \rho(1-a)}{\nu - \rho(1-a)} > \frac{1 - \hat{\rho}(1-\hat{a})}{\nu - \hat{\rho} + \hat{\rho}\hat{a}} = \hat{q}_0.$$

The result $q_0 > \hat{q}_0$ implies tougher price competition which gives $p_0 > \hat{p}_0$ and $V_0 < \hat{V}_0$.

Further, we take the steady state in a semi-pooling equilibrium and small ν and make the same unexpected change. The number of visitors per host charging p_0 increases to $(1 - \rho(1-a)) / (\nu - \rho(1-a))$. Because $\nu < 1$, this ratio is increasing in ρ and it is decreasing in a.

There are more hosts that trade after this change because the hosts of type 1 whose informed consumers draw $u \geq a$ trade with probability one as before. The rest of the hosts of type 1 and the hosts of type 0 trade more often because the queues at them become longer.

We have that $\hat{a} < a$. Therefore, there is even more trade without consumer privacy and $\hat{\rho} > \rho$. Therefore, we have that

$$q_0 < \frac{1 - \rho(1-a)}{\nu - \rho(1-a)} < \frac{1 - \hat{\rho}(1-\hat{a})}{\nu - \hat{\rho}(1-\hat{a})} = \hat{q}_0.$$

Longer queues imply relaxed competition, which gives $p_0 < \hat{p}_0$ and $V_0 > \hat{V}_0$.

The comparison of prices of type 1 *hosts and large* ν. We take large values of ν such that the equilibrium with consumer privacy is separating. We use the fact that $V_0 = e^{-q_0}/2$ and obtain

$$p_1 = a - \frac{e^{-q_0}}{2} = \frac{2 - e^{-q_0}}{4}.$$

Similarly, we obtain

$$E[\hat{p}_1] = \frac{1 + \hat{a} - e^{-\hat{q}_0}}{2} = \frac{3 - \hat{q}_0 e^{-\hat{q}_0} - 2e^{-\hat{q}_0}}{4}.$$

We observe that p_1 is increasing in q_0 and $E[\hat{p}_1]$ is increasing in $\hat{q}_0$. As a result, the difference between p_1 and $E[\hat{p}_1]$ is more than after setting the smallest q_0 and the greatest $\hat{q}_0$.

From the proof of Proposition 1 we know that $a \geq 1/2$, which implies that $q_0 > q_z = \left(3 - \nu + \nu W\left(\frac{e^{1-\frac{3}{\nu}}(\nu-2)}{\nu}\right)\right)/\nu$, which has been obtained by setting $a = 1/2$ in (51). Additionally, we know that $\hat{a} \leq 1/2$, which implies that $\hat{q}_0 \geq \hat{q}_z = \left(2 - \nu + \nu W\left(\frac{e^{1-\frac{2}{\nu}}(\nu-1)}{\nu}\right)\right)/\nu$. The latter value has been obtained by setting $a = 1/2$ in (38).

We plot $-1 - e^{-q_z} + \hat{q}_z e^{-\hat{q}_z} + 2e^{-\hat{q}_z}$ for $\nu > 1$ and obtain that the difference is positive for $\nu > 11/10$.

The comparison of prices of type 1 *hosts and small* ν. Next, we take small values of ν such that the equilibrium with consumer privacy is semi-pooling.

Because $a > \hat{a}$ and $\hat{V}_0 < V_0$, we obtain that

$$\begin{aligned} E[\hat{p}_1] - p_1 &= \frac{1+\hat{a}}{2} - \hat{V}_0 - a + \frac{V_0}{\beta_1^g} \geq \\ &\geq \frac{1}{2} + \frac{\hat{V}_0}{\beta_1^g}(1-\beta_1^g) - a. \end{aligned}$$

From the proof of Proposition 2 Equation (59) we have that

$$\frac{\partial a}{\partial V_0} = \frac{1}{2\beta_1^g}\left(1 - \frac{\alpha_1^g + 2\beta_1^g - 2\frac{V_0\alpha_1^g}{\beta_1^g}}{2\sqrt{D}}\right) < 0,$$

where the inequality has been obtained because

$$\left(\alpha_1^g + 2\beta_1^g - 2\frac{V_0\alpha_1^g}{\beta_1^g}\right)^2 - 4D = 8\alpha_1^g\beta_1^g \geq 0.$$

As a result,

$$\begin{aligned} \frac{1}{2} + \frac{\hat{V}_0}{\beta_1^g}(1-\beta_1^g) - a &\geq \\ \frac{1}{2} + \frac{\hat{V}_0}{\beta_1^g}(1-\beta_1^g) - \frac{3}{4} + \frac{\beta_1^g}{2\alpha_1^g} - \frac{\hat{V}_0}{2\beta_1^g} - \frac{\sqrt{D|_{V_0=\hat{V}_0}}}{2\alpha_1^g} &\geq \\ \left(\frac{1}{2} + \frac{\hat{V}_0}{\beta_1^g}(1-\beta_1^g) - \frac{3}{4} + \frac{\beta_1^g}{2\alpha_1^g} - \frac{\hat{V}_0}{2\beta_1^g} - \frac{\sqrt{D|_{V_0=\hat{V}_0}}}{2\alpha_1^g}\right)_{\hat{V}_0=0} &= \\ \frac{\beta_1^g}{2\alpha_1^g} - \frac{1}{4} - \frac{\sqrt{\left(\beta_1^g - \frac{3\alpha_1^g}{2}\right)^2 + 2\alpha_1^g(\beta_1^g - \alpha_1^g)}}{2\alpha_1^g} &= \\ -\frac{\left(\sqrt{(e^{-q_1} - e^{-q_1}q_1 - 1 + 2q_1)^2} - 2q_1 + q_1 e^{-q_1} + 1 - e^{-q_1}\right)}{4(1 - e^{-q_1}(1+q_1))} & \end{aligned} \tag{67}$$

We take $q_1 > 1$ because the semi-pooling equilibrium with consumer privacy exists for $q_1 > 1.45$. Then we have that $e^{-q_1} - e^{-q_1}q_1 - 1 + 2q_1 > 0$. Therefore the last line of (67) can be rewritten as

$$-\frac{(e^{-q_1} - e^{-q_1}q_1 - 1 + 2q_1 - 2q_1 + q_1 e^{-q_1} + 1 - e^{-q_1})}{4(1 - e^{-q_1}(1+q_1))} = 0.$$

Thus, we conclude that the first line of (67) is positive, which gives $E[\hat{p}_1] > p_1$. □

The proof that $a > \hat{a}$ with a semi-pooling equilibrium The indifference conditions imply that a satisfies

$$\beta_1^g\left(a - \frac{1}{2} + \frac{V_0}{\beta_1^g - \alpha_1^c(1-a)}\right) = V_0.$$

The fact that $\hat{V}_0 < V_0$ implies that

$$a > \frac{1}{2} + \frac{\hat{V}_0}{\beta_1^g} - \frac{\hat{V}_0}{\beta_1^g - \alpha_1^c (1-a)} = \hat{a} + \hat{V}_0 \hat{q}_0 - \frac{\hat{V}_0 \alpha_1^c (1-a)}{\beta_1^g \left(\beta_1^g - \alpha_1^c (1-a)\right)}, \tag{68}$$

where the equality follows from (35). The fraction $\frac{\hat{V}_0 \alpha_1^c (1-a)}{\beta_1^g \left(\beta_1^g - \alpha_1^c (1-a)\right)}$ is decreasing in a. Thus,

$$\hat{V}_0 \hat{q}_0 - \frac{\hat{V}_0 \alpha_1^g (1-a)}{\beta_1^g \left(\beta_1^g - \alpha_1^g (1-a)\right)} > \hat{V}_0 \hat{q}_0 - \frac{\hat{V}_0 \alpha_1^g / 2}{\beta_1^g \left(\beta_1^g - \alpha_1^g / 2\right)} > \hat{V}_0 \left(q_1 - \frac{\alpha_1^g / 2}{\beta_1^g \left(\beta_1^g - \alpha_1^g / 2\right)} \right) > 0. \tag{69}$$

and $a > \hat{a}$. □

The analysis for second price auctions

From the constraint (45), we obtain $r_1 = a - V_0 / e^{-q_1}$. We plug this value into (46) and obtain equation

$$a e^{-q_1} \left(\frac{1}{2} - a + \frac{V_0}{e^{-q_1}} \right) + \frac{1}{2} e^{-q_1} \left(\frac{1}{2} - a \right)^2 = V_0,$$

which can be rewritten as

$$-\frac{1}{2} a^2 e^{-q_1} + V_0 a + \frac{1}{8} e^{-q_1} - V_0 = 0. \tag{70}$$

The LHS of (70) is a second degree concave polynomial in a (its second derivative with respect to a equals $-e^{-q_1} \leq 0$). If $a = 0$, then the LHS of (70) is negative: $e^{-q_1}/8 - e^{-q_0}/2 < 0$ and the first derivative of the expression with respect to a at this point is positive $V_0 \geq 0$. Hence, one root of the polynomial is negative, and the relevant root is the higher one, which is

$$\begin{aligned} a(q_1) &= \frac{-V_0 - \sqrt{V_0^2 + 2e^{-q_1} \left(\frac{e^{-q_1}}{8} - V_0 \right)}}{-e^{-q_1}} \\ &= \frac{V_0 + \sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}}}{e^{-q_1}}. \end{aligned}$$

Then the derivative of $a(q_1)$ is

$$\begin{aligned} \frac{\partial a(q_1)}{\partial q_1} &= \frac{\frac{e^{-q_1}\left(-\frac{1}{2} e^{-2q_1} + 2V_0 e^{-q_1}\right)}{2\sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}}} + e^{-q_1} \left(V_0 + \sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}} \right)}{e^{-2q_1}} \\ &= \frac{-\frac{1}{2} e^{-2q_1} + 2V_0 e^{-q_1} + 2V_0^2 + \frac{1}{2} e^{-2q_1} - 4V_0 e^{-q_1} + 2V_0 \sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}}}{2e^{-q_1} \sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}}} \\ &= \frac{-V_0 e^{-q_1} + V_0^2 + V_0 \sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}}}{e^{-q_1} \sqrt{V_0^2 + \frac{1}{4} e^{-2q_1} - 2V_0 e^{-q_1}}} \\ &= \frac{-V_0 e^{-q_1} + V_0^2 + V_0 \left(a e^{-q_1} - V_0 \right)}{e^{-q_1} \left(a e^{-q_1} - V_0 \right)} = \frac{V_0 (a - 1)}{a e^{-q_1} - V_0}., \end{aligned}$$

We set $r_1 = a - V_0/e^{-q_1}$, and the maximization problem of a firm of type 1 becomes

$$\max_{q_1} \left(1 - a + aq_1\right)\left(ae^{-q_1} - V_0\right) + \frac{1}{2}q_1 e^{-q_1}\left(\frac{1}{4} - a^2\right) + \frac{1}{2}\left(1 - e^{-q_1} - \frac{1}{2}q_1 e^{-q_1}\right),$$

which gives the first-order condition

$$\begin{aligned} a\left(ae^{-q_1} - V_0\right) - ae^{-q_1}\left(1 - a + aq_1\right) + \frac{1}{2}\left(1 - q_1\right)e^{-q_1}\left(\frac{1}{4} - a^2\right) + \\ \frac{1}{2}e^{-q_1} - \frac{1}{4}e^{-q_1}\left(1 - q_1\right) + \left(\left(q_1 - 1\right)\left(ae^{-q_1} - V_0\right) + e^{-q_1}\left(1 - a + aq_1\right) - q_1 e^{-q_1} a\right)\frac{\partial a}{\partial q_1} = \\ a\left(2ae^{-q_1} - V_0 - e^{-q_1} - ae^{-q_1}q_1\right) + \frac{1}{2}e^{-q_1}\left(\frac{3}{4} - a^2\right) + \frac{1}{2}q_1 e^{-q_1}\left(\frac{1}{4} + a^2\right) + \\ \left(e^{-q_1}\left(1 - 2a + q_1 a\right) - V_0\left(q_1 - 1\right)\right)\frac{\partial a\left(q_1\right)}{\partial q_1} = 0. \end{aligned}$$

Next, we plug in the value of $\partial a\left(q_1\right)/\partial q_1$, and substitute V_0/e^{-q_1} with $\frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1-a}$ (it comes from (70)). This gives

$$\begin{aligned} ae^{-q_1}\left(2a - \frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1 - a} - 1 - aq_1\right) + \frac{1}{2}e^{-q_1}\left(\frac{3}{4} - a^2\right) + \frac{1}{2}q_1 e^{-q_1}\left(\frac{1}{4} + a^2\right) + \\ e^{-q_1}\left(1 - 2a + q_1 a - \left(q_1 - 1\right)\frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1 - a}\right)\frac{\frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1-a}\left(a - 1\right)}{a - \frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1-a}} = \\ e^{-q_1}\frac{16a^5 - 96a^3 + 128a^2 - 69a + 12}{8\left(1 - a\right)\left(4a^2 - 8a + 1\right)} = 0 \end{aligned}$$

which implies that a does not depend on ν.

The solution to $16a^5 - 96a^3 + 128a^2 - 69a + 12 = 0$ is less than $1/3$.

Because

$$r_1 = a - V_0/e^{-q_1} = \frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1 - a},$$

the reservation price r_1 does not depend on the number of firms too.

The equilibrium values of q_1 and q_0 are determined by equation $\frac{\frac{1}{2}\left(\frac{1}{4} - a^2\right)}{1-a} = a - \frac{e^{-q_0}}{2e^{-q_1}}$ and the equation that comes from the steady state condition and the definition of ρ:

$$\nu = \frac{ae^{-q_1} + 1 - e^{-q_0}}{\left(1 - e^{-q_0}\right)\left(1 - a + q_1\right) + ae^{-q_1}q_0}.$$

The value of a that solves $16a^5 - 96a^3 + 128a^2 - 69a + 12 = 0$ is less than a^* because the polynomial $16a^5 - 96a^3 + 128a^2 - 69a + 12$ is decreasing in a and

$$16\left(1 - q_0^* e^{-q_0^*}\right)^5 - 96\left(1 - q_0^* e^{-q_0^*}\right)^3 + 128\left(1 - q_0^* e^{-q_0^*}\right)^2 - 69\left(1 - q_0^* e^{-q_0^*}\right) + 12 < 0.$$

$\square$