Ideological Contests with Quality Investments

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Abstract

We consider an asymmetric contest in which policy-motivated players make proposals with both a spatial location and a "quality" component, which must be generated at an up-front cost. Potential applications include elections, legislative policymaking, lobbying, expertise acquisition in bureaucracies, and judicial opinion writing. We show that equilibria are essentially unique and in two-dimensional mixed strategies, and provide an elegant analytical characterization of strategies, outcomes, and payoffs that yields many results. In equilibrium, more extreme proposals are both higher quality and strictly better for the decisionmaker. A more ideologically extreme *or* able player is more likely to make a proposal, and makes more extreme proposals, but is also more likely to win due to their proposals' substantially higher quality. Simultaneously, her opponent is harmed and moderates her own proposals. When the players are highly asymmetric one almost always wins, but the decisionmaker may nevertheless strongly benefit from the potential for competition.

1 Introduction

In many political environments, ideologically-motivated actors try to increase the appeal of their policy proposals by working to increase their appeal along non-ideological dimensions. For example, within legislatures, lawmakers as well as interest group lobbies invest effort drafting detailed legislative text for bills that not only further their ideological aims, but are also well designed; e.g., clearly written, cost-efficient, and with predictable consequences (Hirsch and Shotts (2012); Hitt, Volden and Wiseman (2017)). Within the executive branch, competing bureaucrats exercise "informal authority" by strategically investing in expertise at implementing *specific* policies and not others, in order to incentivize their political superiors to choose those policies (Ting (2011); Turner (2017)). And on multi-member courts, competing justices expend considerable effort to craft judicial opinions supporting their ideological positions that are also clear, persuasive, and implementable by lower courts in order to gain their colleagues' votes (Lax and Cameron (2007)).

In this paper, we develop a simple model to understand the implications of this common strategic dynamic for political competition, policy development, and policy outcomes. In the model, two policy-motivated actors simultaneously make *proposals* consisting of both an ideological "spatial" dimension over which they disagree, and a common value "quality" dimension that must be produced at an up-front cost. A decisionmaker with preferences over both ideology and quality then selects one of the two proposals (or another alternative from a set of exogenous outside options), and the game ends. The decisionmaker cannot commit ex-ante to which proposal she will choose as a function of the strategies, nor can she go back to the proposers and request additional investments or alternative policies. The model is thus particularly well-suited suited to political decisionmaking, although it may be applied to any organizational setting that shares the above properties.

While the model is exceedingly simple to formulate, it nevertheless has several interesting properties. First, the resulting "contest" has an "all pay" component (Siegel (2009)) – players decide how much effort to exert before knowing what their opponent has proposed. Consequently, equilibria are in mixed strategies. Second, it is multidimensional – players gain support for their proposals through a mixture of ideological concessions and up-front quality investments, with more of one reducing the need for the other. Third, it is asymmetric – players may differ both in their ability at generating quality, and in their intrinsic ideological extremism. (Unlike the standard all-pay contest, differences in costs and "valuations" are not isomorphic.)

Fourth, the players are purely policy-motivated – they care only about the ideology and quality of the chosen proposal, rather than whose proposal is chosen. This creates two additional complexities in the contest structure. First, ideological concessions are "winner pay," in that the cost of making a concession is only suffered if the proposal actually wins (Che and Gale (2003)). Second and more importantly, there is a "second order rank order spillover" (Baye, Kovenock and Vries (2012); Chowdhury and Sheremeta (2010)); the strategy of the (first ranked) winner has a direct effect on the utility of the (second ranked) loser, because she must live under the winner's policy.

We first show that equilibrium strategies are characterized by univariate atomless probability distributions over the decisionmaker's utility with common convex support, combined with simple functions associating each utility with a unique combination of ideology and quality. The distributions are characterized by an intuitive coupled system of differential equations. While the basic form of the conditions is unsurprising, the proofs are complicated by the second-order spillover, which rules out the use of techniques that exploit exogenously fixed payoffs from losing (Siegel (2009)).

We next derive a unique analytical solution to the system of differential equations, thereby fully characterizing equilibrium strategies and payoffs in the general asymmetric case. This yields a variety of substantive results and comparative statics, a subset of which we highlight.

First, participation in the contest is generically asymmetric; one proposer always makes a positive-quality proposal, while the other sits out with strictly positive probability. Second, more ideologically extreme proposals in the support of a player's strategy are both higher quality and strictly better for the decisionmaker. The reason is that ideological concessions are effectively "winner pay," and therefore more expensive to make on proposals that are more likely to win. (See also Hirsch and Shotts (2015)). An additional implication is that competition fails to generate "convergence" to the decisionmaker's ideal policy in a strong sense – the decisionmaker is always presented with, and always chooses, a proposal whose ideology differs from his own. This property emerges naturally from the proposers' attempts to exploit quality investments to gain support for ideological outcomes they prefer, and occurs even though contest outcomes are deterministic (see Munster (2006)).

Third, more able *or* extreme proposers dominate the contest and are more likely to win by virtue of their greater motivation and ability. Thus, extreme preferences – far from a liability – are an asset in the contest because they produce a greater willingness to invest in quality to realize ideological gains.

Fourth, a proposer becoming more extreme *or* more able benefits the DM – both a greater *willingness* or *ability* to invest in quality are sufficiently channeled into productive investments that the DM benefits in expectation. This is true even though unilaterally greater extremism or ability by one proposer *also* causes that proposer to make more ideologically extreme proposals, which harms her competitor and also induces the competitor to moderate.

Lastly and in contrast to the standard two-player all pay contest, *asymmetries* between the proposers may actually benefit the DM if they result from one proposers' greater extremism or ability. We show that if the proposers begin with symmetric extremism and costs, and this symmetry is broken by one proposer becoming more extreme or able, then the DM is made unambiguously better off. This is true even though the weaker proposer becomes less likely to participate in the contest. Moreover, in the limiting case of one proposer becoming arbitrarily extreme or able, the probability her competitor participates approaches 0 – the stronger player thus has the appearance of being a monopoly agenda setter. But, the DM's utility is bounded away from what it would be in a true monopoly (Hirsch and Shotts (2017)). In fact, if one proposer becomes arbitrarily extreme, the decisionmaker becomes *unboundedly better* off.

1.1 Related Literature

The model is a direct generalization of Hirsch and Shotts (2015), extending that symmetric model to consider arbitrary asymmetries in both the proposers' preferences and abilities. Other minor differences are that this paper allows for different potential outside options for the decisionmaker (including no outside option, as in a two-candidate election), and also varying weights on quality vs. ideology. These papers in turn relate to several literatures.

The first is a growing literature in political science that analyzes political decisionmaking when experts can make costly investments to endow specific policy proposals with "quality" that is valued by all players. One microfoundation for quality is a reduction in the variance of spatial policy outcomes when ideological players are risk-averse. This conceptualization of expertise generates very different strategic incentives than models built on Crawford and Sobel (1982), where experts acquire knowledge of an unknown state of the word that uniformly shifts all players' ideal policies (See Hirsch and Shotts (2012)). In models with policy-specific quality, experts "attempt to exploit their monopoly power over investments to compel decision makers to accept policies that promote their interests" (Hirsch and Shotts (2015)), an effect that is akin to "real authority" in Aghion and Tirole (1997) and expertise in "complex" environments in Callander (2008).[1] Such models have been used to analyze decisionmaking in a variety of institutional settings, including interbranch bargaining (Londregan (2000)), bureaucratic expertise (Ting (2011); Turner (2017)), lobbying and influence (Hirsch and Shotts (2017)), policymaking in legislatures (Hitt, Volden and Wiseman (2017)), and judicial opinion writing (Lax and Cameron (2007)). Like us, the latter two works consider competition (the former briefly), but assume that the proposers move in a previously-determined sequence.

The second is a large literature spanning across political science and economics analyzing applied models of two-player contests in which strategies have both an "all pay" and "spatial" component (Ashworth and Bueno de Mesquita (2009); Epstein and Nitzan (2004); Herrera, Levine and Martinelli (2008); Munster (2006); Serra (2010); Wiseman (2006); Zakharov (2009)).[2] With few exceptions (Konrad (2000)) such models have been used to study electoral contests or lobbying contests. The models are diverse in their assumptions, differing in terms of their sequencing (e.g. spatial component first and all-pay component second, all-pay first and spatial second, or one proposer first and the other second), the proposers' motivations (winning, policy-motivated, a mixture), and the proposers' knowledge of the decisionmaker's preferences (modelled abstractly with a contest success function (CSF), modelled explicitly with uncertainty, or deterministic and known). Our model has two key differences with this collection of works. First, the all pay component is a common value investment in quality, and therefore intrinsically valued by the proposers as well as the decisionmaker. Second, all choices – both spatial and all pay, and by both proposers– are made simultaneously.

The third is the broader literature on contests with an all-pay component, studied in a wide variety of permutations. The standard contest with complete information and asymmetric valuations and costs is fully examined by Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996). Political applications include Baye, Kovenock and Vries (1993) (who study a decisionmaker that can restrict participation), Che and Gale (1998) (who augment the model with spending caps), and Meirowitz (2008) (who studies both spending caps and "head starts"). Our model differs from the standard contest in a variety of other ways. The simultaneous choice of policy and quality generates a multidimensional contest with an

[1]Callander (2008) specifically models an unknown state of the world known only to an expert as the realized path of a Brownian motion mapping policies to outcomes; policy-specific quality may be thought of as the limiting case in which the variance tends to infinity.

[2]Spatial contests defined as such are also connected to $n \geq 3$ player contests with identity-dependent externalities, which could result from participants with spatial preferences who are constrained to propose their own ideal points (see Esteban and Ray (1999); Linster (1993), and Klose and Kovenock (2015)).

all-pay component, similar to the multidimensional contest studied by Che and Gale (2003) in which firms choose the quality and price of an innovation (see also Siegel (2009)). In addition, the policy-motivation of the proposers generates a rank order spillover, as analyzed by Baye, Kovenock and Vries (2012) in deterministic contests and Chowdhury and Sheremeta (2010) with a CSF (both in symmetric contests). To our knowledge, ours is the first model to combine asymmetries and rank order spillovers, rank order spillovers and multidimensional strategies, or the three properties together.

The paper proceeds as follows. Section 2 introduces the model. Section 3 develops concepts and notation, and presents some general results. Section 4 uses an intermediate result to analytically characterize the likelihood each player makes a proposal, the probabilities of victory, the conditions under which there is a dominant player, and the equilibrium distributions over ideologies. Section 5 fully characterizes equilibrium strategies analytically, and uses this to derive payoffs and related results. Section 6 concludes. Proofs that are straightforward or yield important intuition are in the main text; the remainder are in the Appendix.

2 The Model

Two proposers (Left and Right) make competing proposals for consideration by a decision-maker (DM). A proposal (γ, q) consists of a unidimensional *ideology* $\gamma \in \mathbb{R}$ and a level of *quality* $q \in [0, \infty) = \mathbb{R}^+$; quality must be produced at an up-front cost. All three players care only about the characteristics of the proposal ultimately chosen, rather than *whose* proposal is chosen. Utility over proposals takes the form

$$U_i(\gamma, q) = \left(\frac{1}{\lambda}\right) q - (\gamma - X_i)^2,$$

where X_i is player $i's$ ideological ideal point, and $\frac{1}{\lambda}$ is the weight all players place on quality. The proposers' ideal points are on either side of the decisionmaker ($X_L < X_D < X_R$).

The game proceeds as follows. First, the proposers simultaneously choose proposals (γ_i, q_i). Making a proposal with quality q_i costs $c_i(q_i) = a_i q_i$ up-front, where $a_i > \frac{1}{\lambda}$. Second, the DM chooses one of the two proposals or something else from an exogenous set of outside options $\mathbb{O}$, where $\mathbb{O}$ may contain the DM's ideal point with no quality $(0, 0)$ and/or proposals that are strictly worse (and can be empty).

3 Preliminary Analysis

The game is a multidimensional contest (Che and Gale (2003)) in which the scoring rule applied to "bids" (γ, q) is just the DM's utility $U_D(\gamma, q) = \left(\frac{1}{\lambda}\right) q - (X_D - \gamma)^2$. To facilitate the analysis we thus reparameterize proposals (γ, q) to be expressed in terms of (s, y), where $y = \gamma - X_D$ is the (signed) distance of a proposal's ideology from the DM's ideal, and $s = \left(\frac{1}{\lambda}\right) q - y^2$ is the DM's utility for a proposal, or its *score*. The implied quality of a proposal (s, y) is then $q = \lambda(s + y^2)$. Using this we re-express the proposers' utility and cost functions in terms of (s, y). Note that the decisionmaker's ideal point with 0-quality has exactly 0 score, and is the most competitive "free" proposal to make.

Definition 1.

1. *Player i's utility for proposal (s, y) is*

$$V_i(s, y) = U_i\left(y + X_D, \lambda\left(s + y^2\right)\right) = -x_i^2 + s + 2x_i y$$

where $x_i = X_i - X_D$ is the (signed) distance of i's ideal from the DM.

2. *Proposer i's cost to make proposal (s, y) is*

$$c_i(\lambda\left(s + y^2\right)) = a_i \cdot \lambda\left(s + y^2\right) = \alpha_i\left(s + y^2\right)$$

where $\alpha_i = a_i\lambda$ is i's weighted marginal cost of generating quality.

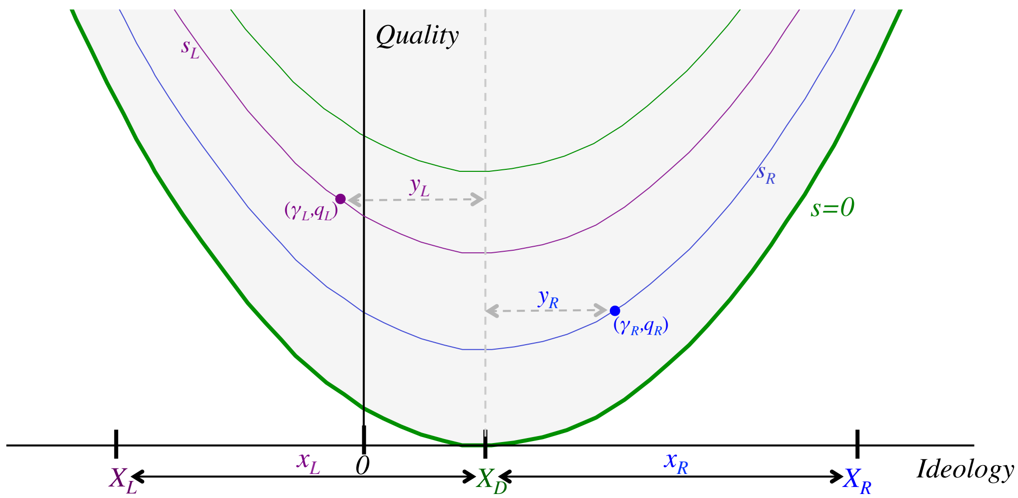
Figure 1 depicts the game in ideology-quality space. The DM's indifference curves, i.e., the proposals with equal score, are depicted by green lines. Definition 1 reparameterizes proposals into score and ideological distance (henceforth just ideology) (s, y), and the five primitives (X_i, a_i, λ) into four parameters (x_i, α_i) describing the proposers' ideal ideological distance from the DM $x_i = X_i - X_D$ (henceforth just ideal ideology) and weighted marginal costs of generating quality $\alpha_i = a_i\lambda$ (henceforth just costs).

3.1 Necessary and Sufficient Equilibrium Conditions

In the reparameterized game, a proposer's pure strategy (s_i, y_i) is a two-dimensional element of $\mathbb{B} \equiv \{(s, y) \in R^2 \mid s + y^2 \geq 0\}$. A mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$, and let $F_i(s)$ denote the CDF over scores induced by i's mixed strategy σ_i.[3]

[3]For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.

Figure 1: Setup of the Policy Contest



We now derive necessary and sufficient equilibrium conditions in a series of four lemmas. Let $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ denote i's expected utility for making proposal (s_i, y_i) with $s_i \geq 0$ if a tie would be broken in her favor. Clearly this is i's expected utility from making a proposal at any $s_i > 0$ where $-i$ has no atom, and i can always achieve utility arbitrarily close to $\bar{\Pi}_i(s_i, y_i; \sigma_{-i})$ by making $\varepsilon-$higher score proposals. Now $\bar{\Pi}_i(s_i, y_i; \sigma_{-i}) =$

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int\limits_{s_{-i} > s_i} V_i(s_{-i}, y_{-i})\, d\sigma_{-i}. \qquad (1)$$

The first term is the up-front cost of generating the proposal's quality. The second term is the probability i's proposal is selected, times her utility for it. The third term is i's utility should she lose, which requires integrating over all her opponent's proposals with higher score than s_i. Taking the derivative with respect to y_i yields the first Lemma.

Lemma 1. *At any score $s_i > 0$ where $F_{-i}(\cdot)$ has no atom, the proposal $(s_i, y_i^*(s_i))$, where $y_i^*(s_i) = F_{-i}(s_i) \cdot \frac{x_i}{\alpha_i}$, is the strictly best score-s_i proposal.*

Proof: Straightforward. QED

Lemma 1 states that at almost every score $s_i > 0$, proposer i's unique best combination of ideology and quality to generate that score is just a weighted average of the proposer's and DM's ideal ideologies $\frac{x_i}{\alpha_i}$, multiplied by the probability $F_{-i}(s_i)$ that i's opponent makes a lower-score proposal. Note that i's optimal ideology does not depend directly on her opponent

$-i$'s ideologies, since a proposal's ideology (holding score fixed) only matters conditional on winning. The optimal ideology also depends on the exact score s_i only *indirectly* through probability $F_{-i}(s_i)$ the proposal wins the contest, since i's utility conditional on winning is additively separable in score and ideology.

The second lemma establishes that at least one of the proposers is always *active*, in the sense of making a proposal with strictly positive score (all positive-score proposals are positive-quality, but the reverse is not necessarily true). Intuitively, this holds because the proposers wish to move policy in opposite directions from the DM, and can beat negative-score proposals for "free" by proposing the DM's ideal with 0-quality.

Lemma 2. *In equilibrium $F_k(0) > 0$ for at most one $k \in \{L, R\}$.*

Proof: Suppose not, so $F_i(0) > 0$ $\forall i$ in some equilibrium. Let U_i^* denote proposer i's equilibrium utility, which can be achieved by mixing according to her strategy conditional on making score-$s \leq 0$ proposal. Let $\bar{y}^0$ denote the expected ideological outcome and $\bar{s}^0$ the expected score outcome conditional on both sides making score ≤ 0 proposals. Since $x_L < 0 < x_R$, we have $V_k(\bar{s}^0, \bar{y}^0) \leq V_k(0, 0)$ for at least one k, which implies k has a profitable deviation since $U_k^* \leq \bar{\Pi}_k(0, 0; \sigma_{-k}) < \bar{\Pi}_k(0, y_k^*(0); \sigma_{-k})$ (since $F_{-k}(0) > 0$). QED

The third Lemma establishes that in equilibrium there is 0 probability of a tie at a strictly positive score. The absence of score ties is an intuitive consequence of exactly opposing ideological interests and the fact that generating quality is "all pay" – at least one proposer will find it in her interests to invest up-front in a bit more quality to break the tie, and make an ideological proposal that is weakly better than the expected outcome from a tie.

Lemma 3. *In equilibrium there is 0-probability of a tie at scores $s > 0$.*

Proof: Suppose not, so each proposer's strategy generates an atom of size $p_i^s > 0$ at some $s > 0$. Proposer i achieves her equilibrium utility U_i^* by mixing according to her strategy conditional on a score-s proposal. Let $\bar{y}^s$ denote the expected ideological outcome conditional on both sides making score-s proposals; then $V_k(s, \bar{y}^s) \leq V_k(s, 0)$ for at least one k, who has a profitable deviation. If k's proposal at score s is $(s, 0)$, then $U_k^* \leq \bar{\Pi}_k(s, 0; \sigma_{-k}) < \bar{\Pi}_k(s, y_k^*(s); \sigma_{-k})$ (since $F_{-k}(s) > 0$). If k sometimes proposes something else, then $U_k^* < \left(1 - \frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k(s, E[y_k|s]; \sigma_{-k}) + \left(\frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k(s, 0; \sigma_{-k})$, which is k's utility if she were to instead propose $(s, 0)$ with probability $\frac{p_{-k}}{F_{-k}(s)}$, and the expected ideology $E[y_k|s]$ of her strategy at score s with the remaining probability (and always win ties). QED

Lemmas 1 – 3 jointly imply that in equilibrium, proposer i can compute her expected utility as if her opponent only makes proposals of the form $\left(s_{-i}, y^*_{-i}\left(s_{-i}\right)\right)$. The utility from making *any* proposal (s_i, y_i) with $s_i > 0$ where $-i$ has no atom (or a tie would be broken in i's favor) is therefore $\bar{\Pi}^*_i\left(s_i, y_i; F\right) =$

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}\left(s_i\right) \cdot V_i\left(s_i, y_i\right) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y^*_{-i}\left(s_{-i}\right)\right) dF_{-i}. \tag{2}$$

Proposer i's utility from making the *best* proposal with score s_i is $\bar{\Pi}^*_i\left(s_i, y^*_i\left(s_i\right); F\right)$, which we henceforth denote $\bar{\Pi}^*_i\left(s_i; F\right)$.

Fourth and finally, we establish that equilibrium score CDFs must satisfy the following natural properties arising from the all pay component of the contest.

Lemma 4. *The support of the equilibrium score CDFs over $\mathbb{R}^+$ is common, convex, and includes* 0.

Proof: We first argue $\hat{s} > 0$ in support of $F_i \to F_{-i}\left(s\right) < F_{-i}\left(\hat{s}\right)\ \forall s < \hat{s}$. Suppose not; so $\exists s < \hat{s}$ where $-i$ has no atom and $F_{-i}\left(s\right) = F_{-i}\left(\hat{s}\right)$. Then $\bar{\Pi}_i\left(\hat{s}, y_i; F\right) - \bar{\Pi}_i\left(s, y_i; F\right) = -\left(\alpha_i - F_{-i}\left(\hat{s}\right)\right) \cdot \left(\hat{s} - s\right) < 0$, implying i's best score-s proposal is strictly better than her best score-$\hat{s}$ proposal, a contradiction. We now argue this yields the desired properties. First, an $\hat{s} > 0$ in i's support but not $-i$ implies $\exists\ \delta > 0$ s.t. $F_{-i}\left(s - \delta\right) = F_{-i}\left(s\right)$. Next, if the common support were not convex or did not include 0, then there would $\exists\,\hat{s} > 0$ in the common support s.t. neither proposer has support immediately below, so $F_i\left(s\right) < F_i\left(\hat{s}\right)\ \forall i$, $s < \hat{s}$ would imply both proposers have atoms at $\hat{s}$, a contradiction. QED

We conclude by combining the preceding lemmas to state a preliminary characterization of all equilibria in the form of necessary and sufficient conditions.

Proposition 1. *Necessary conditions for SPNE are as follows:*

1. ***(Ideological Optimality)*** *With probability* 1, *proposals are either*
   - *(a) negative score* $s_i \leq 0$ *and* 0*-quality* $\left(s_i + y_i^2 = 0\right)$
   - *(b) positive score* $s_i > 0$ *with ideology* $y_i = y^*_i\left(s_i\right) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}\left(s_i\right).$
2. ***(Score Optimality)*** *The profile of score CDFs* $\left(F_i, F_{-i}\right)$ *satisfy the following boundary conditions and differential equations.*

- ***(Boundary Conditions)*** *$F_k(0) > 0$ for at most one proposer k, and there $\exists \bar{s} > 0$ such that $\lim_{s\to\bar{s}} \{F_i(s)\} = 1\ \forall i$.*
- ***(Differential Equations)*** *For all i and $s \in [0, \bar{s}]$,*

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left(y_i^*(s) - y_{-i}^*(s)\right)$$

The above and $F_i(s) = 0\ \forall i, s < 0$ are sufficient for equilibrium.

Proof: Appendix

3.2 Preliminary Observations about Equilibria

Proposition 1 implies that all equilibria have a simple form. At least one proposer (henceforth labelled $-k$) is always active – thus, competition not only strictly benefits the DM in expectation, but with probability 1. The other proposer (henceforth labelled k) may also always be active ($F_k(0) = 0$), or be inactive with strictly positive probability ($F_k(0) > 0$). Inactivity may manifest as proposing the DM's ideal point with no quality $(0,0)$, or as "position-taking" with more distant 0-quality proposals that always lose ($s_k < 0$ and $s_k + y_k^2 = 0$). However, any equilibrium exhibiting the latter is payoff-equivalent to one exhibiting the former; we thus focus on the former for comparative statics.[4] When either proposer i is active, she mixes smoothly over the ideologically-optimal proposals $(s, y_i^*(s)) = \left(s, \frac{x_i}{\alpha_i} F_{-i}(s)\right)$ with scores in a common mixing interval $[0, \bar{s}]$ according to the CDF $F_i(s)$.[5]

The differential equations characterizing the equilibrium score CDFs arise intuitively from the proposers' indifference condition over $[0, \bar{s}]$. The left hand side is $i's$ net marginal cost of making a higher-score proposal, given a fixed probability $F_{-i}(s)$ of winning the contest; the proposer pays marginal cost $\alpha_i > 1$ for sure, but with probability $F_{-i}(s)$ her proposal is chosen and she enjoys a marginal benefit of 1 (because she values quality). The right hand side represents $i's$ marginal ideological benefit of increasing her score. Doing so increases by $f_{-i}(s)$ the probability that her proposal wins, which changes the ideological outcome from her opponent's optimal ideology $y_{-i}^*(s)$ at score s to her own optimal ideology $y_i^*(s)$.

[4]Profiles with "position-taking" are equilibria if the position-taking does not invite a deviation by $-k$ to negative scores; whether this is the case depends on k's score-CDF below 0 and the DM's outside options $\mathbb{O}$. When $(0,0) \in \mathbb{O}$ the necessary conditions are also sufficient.

[5]Technically, the proposition does not state that the support interval is also bounded ($\bar{s} < \infty$), but this is later shown indirectly through the analytical equilibrium derivation.

An interesting property of equilibria (see also Hirsch and Shotts (2015)) is the positive association between a proposal's distance from the DM and its score, implied by $y_i^*(s) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s)$. That is, in equilibrium more distant proposals are not merely higher-quality than closer ones – they are also strictly better for the DM, implying that their additional quality *overcompensates* him for his ideological losses. Intuitively, a proposal giving the DM greater utility will be paired with a more-extreme ideology because it has a higher chance of being chosen, so the proposer is more willing to pay the sure costs of producing quality for the uncertain benefits of ideological change.

Figure 2 depicts a typical equilibrium involving symmetrically located proposers ($-x_L = x_R$) but a cost advantage for the right proposer ($\alpha_R < \alpha_L$). The top panel depicts proposers' score CDFs. The right proposer is always active due to her cost advantage ($F_R(0) = 0$), whereas the left proposer is sometimes inactive ($F_L(0) > 0$). The right proposer's proposals are better for the DM in a first-order stochastic sense; we later show this is a general property of symmetric ideologies and asymmetric costs.

The bottom panel depicts the ideological locations and qualities of the equilibrium proposals – that is, a parametric plot of $\left(y_i^*(s), s + (y_i^*(s))^2\right)$ for $s \in [0, \bar{s}]$. The support of i's ideological proposals extends out to $\frac{x_i}{\alpha_i}$, which is i's optimal ideology to propose absent competition. The right proposer exploits her cost advantage to make more ideologically-extreme proposals at every score. Her proposals are also first-order stochastically more extreme. We later show this is also a general feature of symmetric ideologies paired with asymmetric costs.

4 Activity, Strength, Dominance, and Ideology

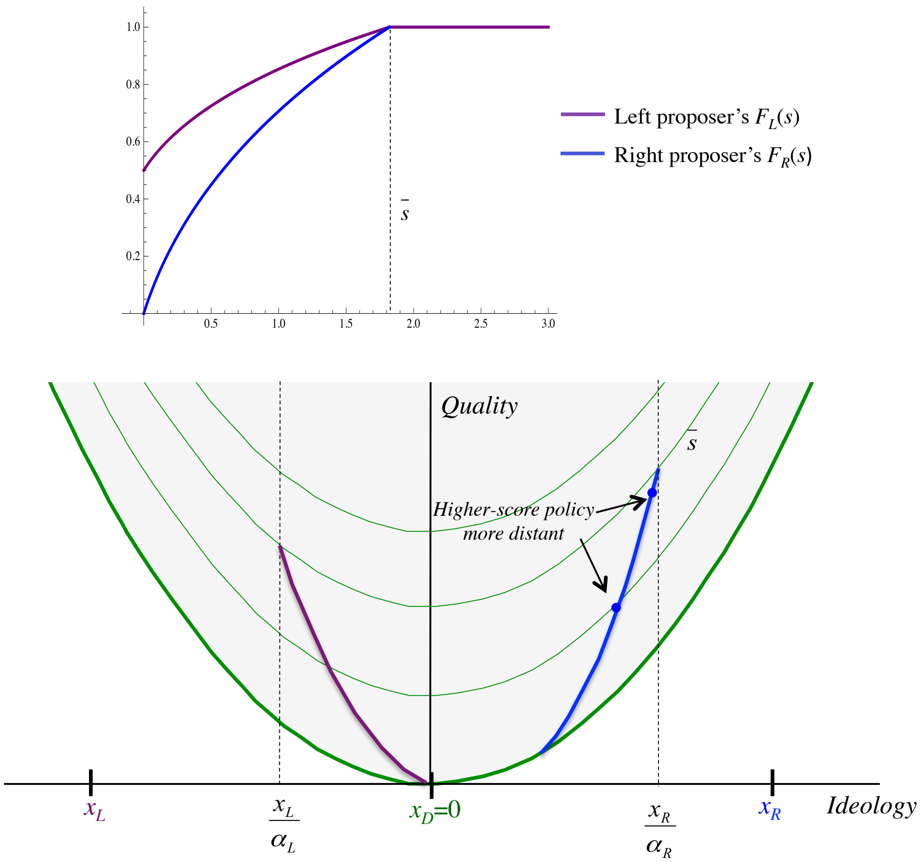
We begin by deriving properties of equilibrium that do not require a complete characterization of the equilibrium score CDFs (these are fully characterized in Section 5). To do so we use the following simple result describing the equilibrium *relationship* between the proposers' score CDFs.

Lemma 5. *In any SPNE,* $\epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s)) \; \forall s \geq 0$, *where*

$$\epsilon_i(p) = \int_p^1 \frac{|x_i|}{\alpha_i - q} dq = |x_i| \log\left(\frac{\alpha_i - p}{\alpha_i - 1}\right)$$

Proof: Rearranging the differential equation in score optimality yields $\frac{f_{-i}(s)\cdot|x_i|}{\alpha_i - F_{-i}(s)} = \frac{f_i(s)\cdot|x_{-i}|}{\alpha_{-i} - F_i(s)} \; \forall s \in [0, \bar{s}] \to \int_s^{\bar{s}} \frac{f_{-i}(s)\cdot|x_i|}{\alpha_i - F_{-i}(s)} ds = \int_s^{\bar{s}} \frac{f_i(s)\cdot|x_{-i}|}{\alpha_{-i} - F_i(s)} ds \; \forall s \in [0, \bar{s}]$; a change of variables

Figure 2: Equilibrium Score CDFs and Policies ($x_L = -x_R$, $\alpha_L > \alpha_R$)



and the boundary condition $F_i(\bar{s}) = 1$ yields $\int_s^{\bar{s}} \frac{f_{-i}(s) \cdot |x_i|}{\alpha_i - F_{-i}(s)} ds = \int_{F_{-i}(s)}^{1} \frac{|x_i|}{\alpha_i - q} dq = \epsilon_i(F_{-i}(s))$. The relationship holds trivially for $s > \bar{s}$. QED

We refer to the property in Lemma 5 as the *engagement equality.* To see why, observe that the decreasing function $\epsilon_i(p)$ captures i's relative willingness to deviate from a proposal winning with probability p to one that wins for sure (since the marginal ideological benefit of moving policy in her direction is $|x_i|$, and the net marginal cost of increasing score on a proposal winning the contest with probability q is $\alpha_i - q$). We call this function i's *engagement at probability* p. The engagement equality $\epsilon_i(F_{-i}(s)) = \epsilon_{-i}(F_i(s))$ states that at every score $s \geq 0$ both proposers must be equally engaged given the resulting probabilities of winning the contest, and therefore equally willing to deviate to the maximum score $\bar{s}$. It is easily verified that $\epsilon_i(1) = 0$ $\forall i$ and $\epsilon_i(p)$ is strictly increasing in $|x_i|$ and decreasing in α_i $\forall p \in [0, 1)$.

Usefully, the engagement equality implies a simple functional relationship between the players' score CDFs that must hold in equilibrium regardless of their exact values. Letting

$$p_i(\epsilon) = \alpha_i - (\alpha_i - 1)\, e^{\frac{\epsilon}{|x_i|}}$$

denote the inverse of $\epsilon_i(p)$ (which is decreasing in p, increasing in $|x_i|$, and decreasing in α_i) equilibrium requires that $F_i(s) = p_{-i}(\epsilon_i(F_{-i}(s)))\ \forall s \in [0, \bar{s}]$.

4.1 Activity and Strength

We first use the engagement equality to derive the identity of the sometimes-inactive proposer k and the probability $F_k(0)$ that she is sometimes inactive, as well as perform comparative statics on $F_k(0)$.

Proposition 2. *In equilibrium* $k \in \arg\min_i \{\epsilon_i(0)\}$ *and*

$$F_k(0) = p_{-k}(\epsilon_k(0)) = \alpha_{-k} - (\alpha_{-k} - 1)\left(\frac{\alpha_k}{\alpha_k - 1}\right)^{\left|\frac{x_k}{x_{-k}}\right|}.$$

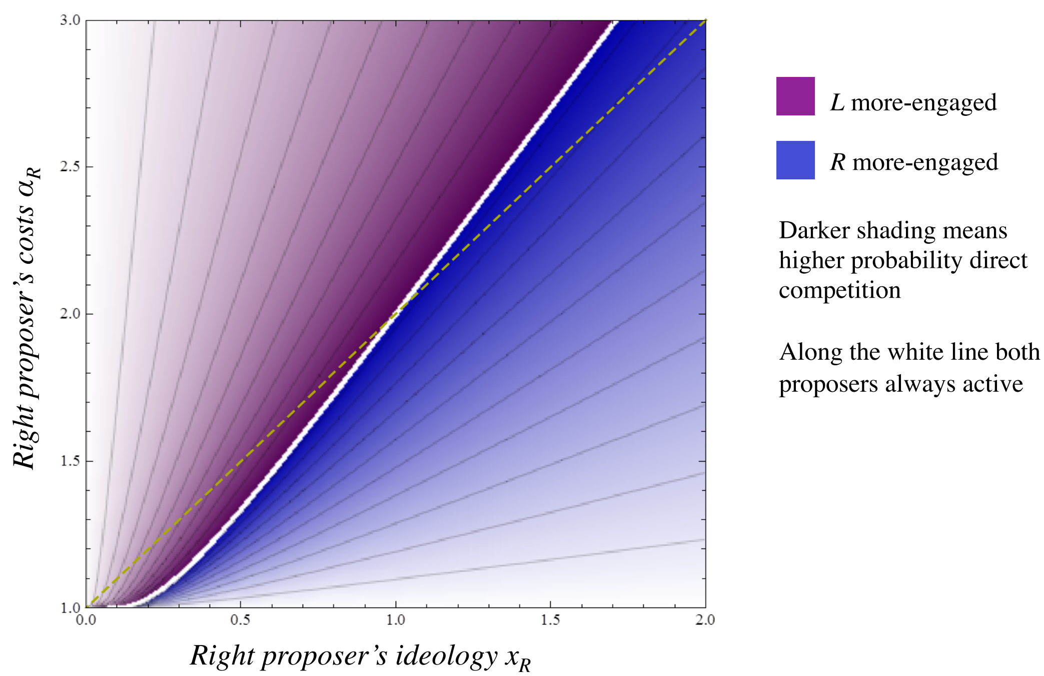
The probability k is inactive $F_k(0)$ is decreasing in her distance from the DM $|x_k|$ and her opponent's quality costs α_{-k}, and increasing in her opponent's distance from the DM $|x_{-k}|$ and her own quality costs α_k. In addition, $\lim_{|x_k|\to 0}\{F_k(0)\} = \lim_{|x_{-k}|\to\infty}\{F_k(0)\} = \lim_{\alpha_k\to\infty}\{F_k(0)\} = \lim_{\alpha_{-k}\to 1}\{F_k(0)\} = 1$.

Proof: Suppose $\epsilon_k(0) < \epsilon_{-k}(0)$; then $F_k(0) = 0$ and the engagement equality would imply $F_{-k}(0) < 0$, a contradiction. Since $F_i(0) = 0$ for some i we must have $F_{-k}(0) = 0$ and $F_k(0) = p_{-k}(\epsilon_k(0)) > 0$. Comparative statics and limit statements follow from previous observations on $\epsilon_i(\cdot)$ and $p_i(\cdot)$. QED

The sometimes-inactive proposer is thus the one with the lowest engagement at probability 0 – that is, who is least willing to participate in the contest entirely. Figure 3 is a contour plot of the probability that the sometimes-inactive proposer enters the contest as a function of the ideology x_R and costs α_R of the right proposer, holding parameters of the left proposer fixed. In the purple region the right proposer is sometimes inactive; increases in her ideological extremism $|x_R|$ or decreases in her costs α_R promote greater parity in the contest and an increase in the right proposer's activity. In the blue region the left proposer is sometimes inactive, and increases in the right proposer's extremism or decreases in her costs promote imbalance and a decrease in the left proposer's activity. Along the white curve both proposers are always active.

The pattern of activity thus resembles two-player all pay contests without spillovers. Observable competition is a function of *parity* in the motivations and abilities of the participants. In addition, the probability one player stays out approaches 1 as the *imbalance* between them becomes arbitrarily extreme (Baye, Kovenock and de Vries (1996); Hillman and Riley (1989)). In the present contest, this occurs if the sometimes-inactive proposer's ideal x_k approaches the DM's (so she loses her ideological motive to participate in the contest), her marginal cost of producing quality α_k becomes arbitrarily high, her opponent's ideal x_{-k} moves arbitrarily far from the DM (so he becomes arbitrarily motivated), or her opponent's marginal cost of quality α_{-k} approaches its intrinsic value of 1.

Figure 3: Effect of R's Parameters on Probability of Direct Competition ($x_L = -1$, $\alpha_L = 2$)



We next use the engagement equality to derive the players' probabilities of victory.

Proposition 3. *In equilibrium the probability proposer k loses the contest is*

$$\int_0^1 p_{-k}\left(\epsilon_k\left(p\right)\right)dp = \int_0^1 \left(\alpha_{-k} - \left(\alpha_{-k}-1\right)\left(\frac{\alpha_k - p}{\alpha_k - 1}\right)^{\left|\frac{x_k}{x_{-k}}\right|}\right)dp$$

which is decreasing in her distance from the DM $|x_k|$ and her opponent's quality costs α_{-k}, and increasing in her opponent's distance from the DM $|x_{-k}|$ and her own quality costs α_k.

Proof: The probability k loses the contest is $\int_0^{\bar{s}} f_{-k}(s) F_k(s)\, ds$; applying the engagement equality this is $\int_0^{\bar{s}} p_{-k}(\epsilon_k(F_{-k}(s))) f_{-k}(s)\, ds$, and applying a change of variables of $F_{-k}(s)$ for p (recalling $F_{-k}(0) = 0$) yields the result. QED

The probability k loses thus obeys the same comparative statics as her probability of inactivity. Somewhat paradoxically, she becomes *less* likely to win when her preferences are closer to the DM or her opponent's are more distant. More intuitively, she becomes more likely to win if she is more able or her opponent less able.

4.2 Dominance

In the standard two-player all-pay contest with asymmetries in valuations and costs, there is always an unambiguously weaker player, who makes bids that are first-order stochastically worse for the DM (and by implication less likely to win). In the present contest, in contrast, there may be no unambiguously weaker player in this sense.

Proposition 4. *Proposer i is dominated ($F_{-i}(s) < F_i(s)\ \forall s \in (0, \bar{s})$) i.f.f. she is less engaged at every probability p ($\epsilon_i(p) < \epsilon_{-i}(p)\ \forall p \in (0,1)$). Equivalently, she is dominated i.f.f. both $\int_0^1 \frac{|x_i|}{\alpha_i - q} dq \leq \int_0^1 \frac{|x_{-i}|}{\alpha_{-i} - q} dq$ and $\frac{|x_i|}{\alpha_i - 1} \leq \frac{|x_{-i}|}{\alpha_{-i} - 1}$, where the latter condition is stronger than the former i.f.f. i has a cost advantage.*

Proof: Appendix

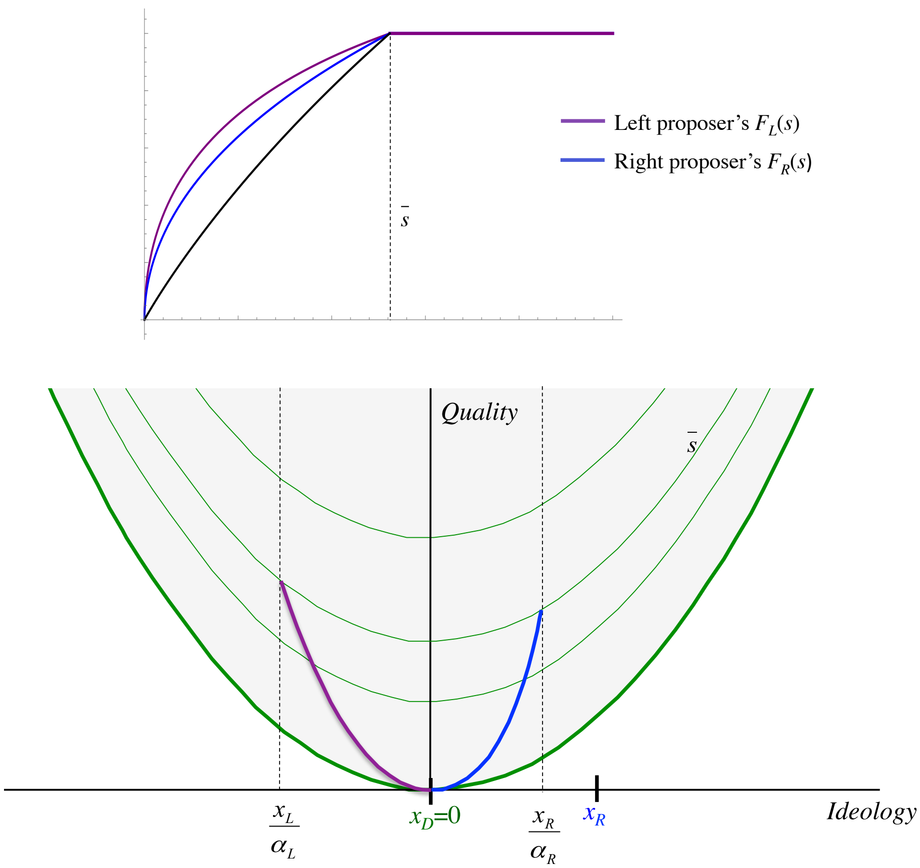
Clearly, a proposer k who is *both* less extreme ($|x_k| \leq |x_{-k}|$) and less able ($\alpha_k \geq \alpha_{-k}$) (with one strict) satisfies both conditions and is therefore dominated. This implies that a proposer who is more ideologically *distant* from the DM actually dominates the contest (provided she is no less able).

However, when one proposer is more extreme while the other is more able, then lower engagement at probability 0 is necessary but not sufficient for the more able proposer to be dominated. The reason is that the proposers intrinsically value their quality investments. This causes the *effective* marginal cost quality to decrease for higher score proposals, since such proposals are more likely to chosen, and their quality more likely to be enjoyed. Consequently, if proposer k is sometimes inactive despite a cost advantage (because her opponent

$-k$ is much more extreme), then although she may be less willing to invest in quality *on average*, she may still be more willing to invest in quality on higher-score proposals.

To see this concretely, consider two proposers (L, R) who are equally engaged at probability 0 ($\epsilon_L(0) = \epsilon_R(0)$) because L is a less able "extremist" while R is a more able "moderate" ($|x_L| > |x_R|$ and $\alpha_L > \alpha_R$). The two panels of Figure 4 depict such an equilibrium. The moderate strictly dominates the contest and is strictly more likely to win even though the proposers both always participate. If the example is then perturbed to make R slightly less able, then she will sometimes be inactive (and is therefore no longer dominant), but remains strictly more likely to win the contest (by continuity) and is not dominated either.

Figure 4: Equilibrium with Equal Engagement ($x_R < -x_L$, $\alpha_L > \alpha_R$)



4.3 Ideology

Lastly, the engagement equality directly yields simple expressions for the probability distribution over the *ideology* of each player's proposals.

Proposition 5. *Let $G_i(y) = \Pr(|y_i| \leq |y|)$ denote the probability that i's proposal is closer to the DM than y. Then*

$$G_i(y) = p_{-i}\left(\epsilon_i\left(\frac{y}{x_i/\alpha_i}\right)\right) = \alpha_{-i} - (\alpha_{-i} - 1)\left(\frac{x_i - y}{x_i - x_i/\alpha_i}\right)^{\left|\frac{x_i}{x_{-i}}\right|},$$

which is first-order stochastically increasing in i's distance from the DM $|x_i|$, decreasing in her costs α_i, decreasing in her opponent's distance from the DM $|x_{-i}|$, and increasing in her opponent's costs α_i.

Proof: Proposer i's ideology at score s is $y_i^*(s) = \frac{x_i}{\alpha_i} F_{-i}(s)$ (from ideological optimality), so $F_{-i}(s_i^*(y)) = \frac{y}{x_i/\alpha_i}$ where $s_i^*(y)$ is the inverse of $y_i^*(s)$. That is, the probability $-i$ makes a proposal with score $\leq s_i^*(y)$ is $\frac{y}{x_i/\alpha_i}$. Now the probability $G(y)$ that i makes a proposal closer to the DM than y is $F_i(s_i^*(y))$, which is $= p_{-i}(\epsilon_i(F_{-i}(s_i^*(y)))) = p_{-i}\left(\epsilon_i\left(\frac{y}{x_i/\alpha_i}\right)\right)$ from the engagement equality. Comparative statics are straightforward. QED

Thus, when a proposer i becomes more ideologically extreme (higher $|x_i|$) or able (lower α_i), she reacts by making first-order stochasically more extreme proposals. In the former case she is more motivated to exploit quality to realize ideological gains, and in the latter case she is better able to do so. Combined with the previous results, this means that a more extreme (but no less able) proposer *or* a more able (but no more extreme) proposer maintains her dominance in the contest *despite* the greater ideological extremism of her proposals.

How does proposer i's opponent $-i$ react to i becoming more extreme or more able? By *moderating* the ideological extremism of her own proposals (first-order stochasically) to improve her chances of winning the contest. A more extreme or able i endogenously increases the ideological stakes of the contest with her more-extreme proposals, thereby increasing the importance of victory to $-i$ and the value of moderating her proposal in order to win.

5 Payoffs

We complete the analysis by calculating payoffs. This requires first characterizing score CDFs $F_i(s)$ satisfying Proposition 1, which are shown constructively to be unique.

Proposition 6. *The unique score CDFs over $s \geq 0$ satisfying Proposition 1 are $F_i(s) = p_{-i}(\epsilon(s))\ \forall i$, where $\epsilon(s)$ is the inverse of*

$$s(\epsilon) = 2\int_{\epsilon}^{\epsilon_k(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\hat{\epsilon})\, d\hat{\epsilon}.$$

The inverse score CDFs are $s_i(F_i) = s(\epsilon_{-i}(F_i))\ \forall i$. The function $s(\epsilon)$ is strictly increasing in x_i and strictly decreasing in α_i $\forall \epsilon \in [0, \epsilon_k(0))$, and the maximum score is $\bar{s} = s(0)$.

Proof: From the engagement equality $\epsilon_i(F_{-i}(s)) = \int_{F_{-i}(s)}^{1} \frac{|x_i|}{\alpha_i - q} dq = \epsilon(s)\ \forall i, s$ for some $\epsilon(s)$. We characterize the unique $\epsilon(s)$ implying score CDFs $F_i(s) = p_{-i}(\epsilon(s))$ and optimal ideologies $y_i(s) = \frac{x_i}{\alpha_i} p_i(\epsilon(s))$ that satisfy score optimality. First observe that $\epsilon'(s) = f_i(s)\,\epsilon'_{-i}(F_i(s)) = -\frac{f_{-i}(s)|x_i|}{\alpha_i - F_{-i}(s)}$. Next the differential equations may be rewritten as $\frac{\alpha_i - F_{-i}(s)}{f_{-i}(s) \cdot |x_i|} = 2\sum_j y_j(s)$. Substituting the preceding observations into both sides yields $\frac{1}{\epsilon'(s)} = -2\sum_j \frac{x_j}{\alpha_j} p_j(\epsilon(s))$, and rewriting in terms of the inverse $s(\epsilon)$ yields $s'(\epsilon) = -2\sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon)$. Lastly $\epsilon_k(F_{-k}(s)) = \epsilon(s)$ and $F_{-k}(0) = 0$ imply the boundary condition $s(\epsilon_k(0)) = 0$ so $s(\epsilon) = \int_{\epsilon}^{\epsilon_k(0)} -s'(\hat{\epsilon})\, d\hat{\epsilon} = 2\int_{\epsilon}^{\epsilon_k(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\hat{\epsilon})\, d\hat{\epsilon}$. Now $s(\epsilon)$ is straightforwardly increasing in $|x_i|$ and decreasing in α_i given previous observations about $p_j(\hat{\epsilon})$. QED

An interesting implication of the equilibrium characterization is that the maximum score $\bar{s}$ changes continuously with the parameters of both proposers, even when one is dominant. This contrasts with the standard 2-player all pay contest, where the mixing interval is unaffected by the parameters of the stronger player.

The preceding characterization transparently yields the following comparative statics.

Corollary 1. *Increasing a proposer's extremism $|x_i|$ or decreasing her costs α_i first-order stochastically increases her own score CDF, but has ambiguous effects on her opponent's score CDF.*

To see that the effect of a proposer's parameters on her opponent's score CDF is necessarily ambiguous, suppose that the always-active proposer $-k$ becomes even more extreme or able. Then her opponent k becomes less likely to be active, but also the range of scores $[0, \bar{s}]$ over which she mixes *when* she is active increases. She thus has a higher probability of making very high-score proposals, even while she is simultaneously less likely to enter the contest.

5.1 Proposer Payoffs

Using Proposition 6, the proposers' equilibrium payoffs are as follows.

Proposition 7. *Proposer i's equilibrium utility is $\Pi_i^*(\bar{s}; F^*) = -\left(1 - \frac{1}{\alpha_i}\right) x_i^2 - (\alpha_i - 1)\,\bar{s}$, which is decreasing in her own costs α_i as well as either players' extremism $|x_j|\,\forall j$, and increasing in her opponent's costs α_{-i}.*

Proof: Appendix

A proposer's equilibrium utility has two components. The first $-\left(1 - \frac{1}{\alpha_i}\right) x_i^2$ is her utility if she could make proposals as a "monopolist" (and the DM's outside option included $(0, 0)$). The second $-(\alpha_i - 1)\,\bar{s}$ is the cost generated by competition, which forces her to make proposals that leave the DM strictly better off than the best "free" proposal $(0, 0)$ in order to maintain influence. This competition cost is increasing in i's marginal cost α_i of generating quality (holding $\bar{s}$ fixed) as well as the maximum score $\bar{s}$, which in turn is increasing in both proposers' ideological extremism and decreasing in their costs everywhere in the parameter space.

A proposer is thus strictly harmed when her competitor becomes more extreme. She is also harmed when her competitor becomes more able, despite the fact that ability is channeled into common value investments. In the model these investments are exploited to gain ideological influence. Both effects are distinct from all pay contests without spillovers (Siegel (2009)), where the equilibrium utility of the "sometimes inactive" player is pinned at her fixed value for losing, and is unaffected by changes in her opponent's parameters unless they influence who is more active.

A proposer also worse off when her *own* preference become more distant from the decision-maker. This occurs two reasons; (1) it becomes more expensive to make equally-competitive proposals that are the same distance from her own ideal, and (2) the intensity of competition increases. Finally, a proposer is worse off when her costs of producing quality increase – even though there is a countervailing effect of reducing the intensity of competition (and indeed, the competition cost $(\alpha_i - 1)\bar{s}$ alone is not generically monotonic in α_i).

5.2 DM Payoffs

Lastly, again using Proposition 6 the DM's equilibrium utility and the proposers' average scores (which bound the DM's utility from below) are as follows.

Proposition 8. *The DM's equilibrium utility is* $U^*_{DM} = \int_{\epsilon_k(0)}^{0} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} \left(\prod_j p_j(\epsilon) \right) d\epsilon =$

$$2 \int_0^{\epsilon_k(0)} \left(1 - \prod_j p_j(\epsilon) \right) \cdot \left(\sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \right) d\epsilon$$

Proposer i's average score is $E[s_i] = \int_{\epsilon_k(0)}^{0} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} (p_{-i}(\epsilon)) \, d\epsilon =$

$$2 \int_0^{\epsilon_k(0)} (1 - p_{-i}(\epsilon)) \cdot \left(\sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \right) d\epsilon$$

Proof: $F_i(s) F_{-i}(s)$ is the CDF of $\max\{s_i, s_{-i}\}$ so the DM's utility is $\int_0^{\bar{s}} s \cdot \frac{\partial}{\partial s} \left(\prod_j F_j(s) \right) ds =$ $\int_0^{\bar{s}} s \cdot \frac{\partial}{\partial s} \left(\prod_j p_j(\epsilon(s)) \right) ds$. A change of variables from s to ϵ yields the first expression and integration by parts and rearranging yields the second. A nearly identical series of steps yields i's average score. QED

Direct comparative statics on the DM's utility U^*_{DM} are not straightforward because changing a proposer's parameters has mixed effects on her opponent's score CDF. However, the fact that unilaterally greater extremism and ability always strictly increases the maximum score $\bar{s}$ suggests that *imbalance* between the participants may be beneficial to the DM if it results from one proposer's greater motivation or ability. This is quite different from the standard complete-information all pay contest, where imbalance reduces the participation of the weaker player and makes the DM worse off ((Baye, Kovenock and de Vries (1996); Hillman and Riley (1989))). The following local comparative statics result verifies this intuition.

Proposition 9. *When the proposers are symmetric ($|x_i| = |x_{-i}|$ and $\alpha_i = \alpha_{-i}$), the DM's utility is locally increasing eithers' extremism or ability.*

Proof: Appendix

The DM thus strictly benefits locally if symmetry between the players is broken by one becoming more extreme or able – even though the other also becomes less active.

Lastly, the effect of asymmetry on the DM's utility may be examined globally by considering his utility in the limit as the contest becomes very imbalanced.

Proposition 10. *The DM's utility exhibits the following limiting behavior*

$$0 = \lim_{\alpha_i \to \infty} U^*_{DM} = \lim_{x_i \to 0} U^*_{DM} < \lim_{\alpha_i \to 1} U^*_{DM} < \lim_{x_i \to \infty} U^*_{DM} = \infty$$

Proof: Observe that $E[s_{-k}] \leq U^*_{DM} \leq \bar{s}$. For the first two limiting statements it is easily verified that $\bar{s} \to 0$ as $\alpha_k \to \infty$ or $x_k \to 0$. For the third limiting statement it is easily verified that $\lim_{\alpha_{-k} \to 1} E[s_{-k}]$ and $\lim_{\alpha_{-k} \to 1} \bar{s}$ are strictly positive and finite. For the fourth limiting statement observe that $E[s_{-k}] \geq \frac{|x_{-k}|}{\alpha_{-k}} p_{-k}(\epsilon_k(0)) \cdot 2 \int_0^{\epsilon_k(0)} (1 - p_k(\epsilon))\, d\epsilon$ which $\to \infty$ as $|x_{-k}| \to \infty$ since the first term $\to \infty$ and the remaining terms are non-decreasing. QED

If an extreme imbalance is the result of one proposer's incompetence *or* ideological moderation, the DM's utility approaches 0, her utility if $-i$ were a "monopolist" (and the DM's outside options included $(0, 0)$). (Proposer $-i$'s utility also approaches her utility if she were a monopolist). The DM thus never benefits from having one proposer be an ideological "ally" vs. someone more extreme – an ally is unmotivated to invest in quality because her ideological interests are already represented by the DM (see also Hirsch and Shotts (2015)).

However, if extreme imbalance is the result of one proposer's greater ability to produce quality (specifically, if her marginal cost of producing quality approaches its intrinsic value), then the DM's utility is bounded away from 0. In this case the DM strictly benefits from the potential for competition, even though actual competition is almost never observed (since $F_{-i}(0) = F_k(0)$ approaches 1).

Finally and most strikingly, unilateral ideological extremism benefits the decisionmaker in a strong sense; the DM can achieve arbitrarily high utility with a proposer whose preferences are sufficiently *distant* from her own. As one proposer becomes increasingly unilaterally extreme, her proposals also become increasingly extreme, but the DM becomes increasingly better off due to the proposals' greater quality. At the same time, the extremist's opponent becomes increasingly unlikely to enter the contest, moderate in her proposals, and (unboundedly) worse off.

6 Discussion and Conclusion

This paper considers a contest among ideologically-motivated players. Two proposers have differing ideological objectives from a decisionmaker and each other. They may make costly up-front investments to increase the value of their proposals to all players, termed *quality* (valence in the political science literature), and may have differential ability at doing so. In the resulting equilibrium, the proposers exploit a mixture of quality investments and ideological concessions to gain the support of the decisionmaker. The model is a generalization of Hirsch and Shotts (2015) that allows for asymmetric proposers, differential weight between quality and ideology, and different potential outside options for the decisionmaker (including no outside option as in a two-candidate election).

Applications of the model in political science and political economy are widespread and include elections, lawmaking, lobbying, bureaucratic policy implementation, and judicial opinion writing. More generally, the model may be applied to organizational choice when there are a mixture of competing and common interests, freedom among several individuals or groups to make proposals, and a unitary decisionmaker lacking the ability to write ex-ante contracts and facing a deadline.

Although the model is exceedingly simple to formulate, it results in a contest with complexities that are difficult to analyze jointly. It is complete information, multidimensional, and asymmetric. Strategies have both "all-pay" and "winner-pay" components. Most importantly, it features a "second-order rank over spillover" – the strategy of the (first ranked) winner exerts a direct effect on the utility of the (second ranked) loser, because she must live under the winner's policy. We show that the game yields a unique equilibrium in continuous mixed strategies with an elegant analytical characterization. The solution methods employed can be used to solve similar models in which players compete to have their preferred ideological policies enacted by exerting costly up-front effort (for example, if effort represented campaign advertising in an election that only influenced the voters).[6]

Result from the model are counterintuitive, and key themes are two-fold: (i) unilaterally greater extremism benefits the DM because the resulting motivation is partially channelled into productive investment; even though the resulting policies are also more extreme as a byproduct, and the other proposer is harmed; (ii) greater ability at common-value investments has similar equilibrium effects to greater ideological extremism, because such investments are exploited for ideological gains.

Formal results are numerous and spread throughout the paper; we provide a self-contained review here. Unless otherwise noted, comparative statics statements are in a first-order stochastic sense. First, the (essentially) unique equilibrium is in continuous mixed strategies; each proposer mixes over a curve of proposals with different ideologies and qualities. Second, in equilibrium more ideologically-extreme proposals in the support are strictly better for the

[6]Solving similar models computationally is straightforward; analytic solutions are elusive in the asymmetric case.

DM than more moderate ones. Third, one proposer is always active in the sense of making a positive-quality proposal – she is the one with the highest value of $\int_0^1 \frac{|x_i|}{\alpha_i - q} dq$. Fourth, a proposer is dominant in the sense of making first-order stochastically better proposals for the DM i.f.f. she has the higher value of both $\int_0^1 \frac{|x_i|}{\alpha_i - q} dq$ and $\frac{|x_i|}{\alpha_i - 1}$. Fifth, comparative statics are as follows.

Comparative Statics. *Increasing proposer i's extremism* $|x_i|$*:*

- *either increases her own probability of being active (if she is sometimes-inactive), or decreases her opponent's probability of being active*
- *increases her probability of winning*
- *maintains her dominance if she was already dominant*
- *makes her proposals more extreme but also better for the DM*
- *makes her opponent's proposals more moderate, and neither better nor worse unambiguously for the DM*
- *decreases both her own and her opponent's utility*

Decreasing proposer i's costs α_i *yields all the same aforementioned effects, except that it increases rather than decreases proposer i's utility*

Sixth, the DM benefits locally when symmetry between the proposers is broken because one becomes unilaterally more extreme or able. Lastly, if a proposer becomes arbitrarily extreme or able, the probability of observing direct competition approaches zero, but the DM's utility is bounded away from what it would be absent competition. In fact, as one proposer becomes unboundedly extreme the DM's utility becomes unboundedly large.

Our analysis suggests several avenues for future work. One is to expand the number of possible participants in the contest. This is natural in settings where proposals are made by individuals rather than teams, or when proposals may come from varied sources (e.g., interest groups as well as legislators developing potential legislation). It is straightforward to show that when costs are common, there always exists an equilibrium in which the two most ideologically-extreme potential proposers play their equilibrium strategies in the present model, while the other potential proposers are inactive. (Hirsch and Shotts (2015) contains the weaker result for the symmetric model).

A second is to allow the proposers to also gain the decisionmaker's support using targeted benefits (pork, as in vote buying models) in addition to collective benefits (as in our model), and analyze when they use targeted vs. common benefits.

A third is to study institutional design (again without the ability to write ex-ante contracts). Primitives of the model potentially under control of an institutional designer include the marginal costs of generating quality (using subsidies), the ideology of the proposers (by choosing organizational membership), the ideology of the decisionmaker (through an appointment or collective choice procedure), and other constrained aspects of the "scoring rule" (e.g. providing a "head start" to one proposer).

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7 Appendix

Proof of Proposition 1

(*Score Optimality*) A score $\hat{s} > 0$ in the common support implies $[0, \hat{s}]$ in the common support (by Lemma 4) implying $\lim_{s \to \hat{s}^-} \left\{ \bar{\Pi}_i(s; F) \right\} \geq U_i^*$. Equilibrium also requires $\bar{\Pi}_i(s; F) \leq U_i^*$ $\forall s$ so $\bar{\Pi}_i(s; F) = U_i^*$ $\forall s \in [0, \bar{s}]$, further implying the F's are absolutely continuous over $(0, \infty)$ (given our initial assumptions), and therefore $\frac{\partial}{\partial s}(\Pi_i^*(s; F)) = 0$ for almost all $s \in [0, \bar{s}]$. This straightforwardly yields the differential equations for score optimality, with the boundary conditions implied by Lemma 4. (*Ideological Optimality*) At most one proposer (k) makes ≤ 0-score proposals with positive probability, so $F_{-k}(0) = 0$. Such proposals lose for sure and never influence a tie, and therefore must be 0-quality with probability 1, yielding property (a). Atomless score CDFs $\forall s > 0$ implies $(s, y_i^*(s))$ is the strictly best score-s proposal (by Lemma 1), yielding property (b). (*Sufficiency*) Necessary conditions imply all $(s, y_i^*(s))$ with $s \in (0, \bar{s}]$ yield a constant U_i^*. $F_{-k}(0) = 0$ implies k's strictly best score-0 proposal is $(0, y_k^*(0)) = (0, 0)$ and yields $\bar{\Pi}_k(0; F)$, and $F_k(s) = 0$ for $s < 0$ implies k has a size $F_k(0)$ atom here. Thus both proposers' mixed strategies yield U_i^*, and neither can profitably deviate to $s \in (0, \bar{s}]$. To see neither can profitably deviate to $s > \bar{s}$, observe $\Pi_i^*(s; F) - \Pi_i^*(\bar{s}; F) = -(\alpha_i - 1)(s - \bar{s}) < 0$. To see k cannot profitably deviate to $s_k \leq 0$, $F_{-k}(0) = 0$ implies such proposals lose and never influence a tie, and so yield utility $\leq U_k^*$. To see $-k$ cannot profitably deviate to $s_{-k} \leq 0$, observe all such proposals result in either $(0, y_{-k})$ or $(0, 0)$ when $s_k \leq 0$ (since the DM's other choices are $(0, 0)$ and $\mathbb{O}$), and thus yield utility $\leq \max\left\{\bar{\Pi}_{-k}(0, 0; F), \bar{\Pi}_{-k}(0, y_{-k}; F)\right\}$ which is $\leq U_{-k}^*$. QED

Proof of Proposition 4

Lemma 5 and the engagement function $\epsilon_i(p)$ strictly decreasing when $p \in [0, 1)$ immediately implies $sign(\epsilon_{-k}(F_{-k}(s)) - \epsilon_k(F_{-k}(s))) = sign(F_k(s) - F_{-k}(s))$ $\forall s \in [0, \bar{s})$, which straightforwardly yields the first statement. Now let $\delta(p) = \epsilon_{-k}(p) - \epsilon_k(p)$, so $\delta(0) \geq 0 = \delta(1)$. We argue $\delta'(1) \leq 0$ is necessary and sufficient. For necessity, $\delta'(1) > 0 = \delta(1) \to \delta(p) < 0$ in a neighborhood below 1. For sufficiency, it is easily verified that $\delta'(p) = \frac{|x_k|}{\alpha_k - p} - \frac{|x_{-k}|}{\alpha_{-k} - p}$ crosses 0 at most once when the proposers are asymmetric; thus $\delta(0) \geq 0 = \delta(1) \geq \delta'(0)$ implies $\delta(p)$ strictly quasi-concave over $[0, 1]$ and $\delta(p) > \min\{\delta(0), \delta(1)\} \geq 0$ for $p \in (0, 1)$.

We last argue $\delta(0) \geq 0$ and $\alpha_k > \alpha_{-k} \to \delta'(1) < 0$. Observe that $\alpha_k < \alpha_k$ and $\delta'(0) = \frac{x_k}{\alpha_k} - \frac{x_{-k}}{\alpha_{-k}} \leq 0 \to \delta'(1) = \frac{|x_k|}{\alpha_k}\left(\frac{1}{1 - 1/\alpha_k}\right) - \frac{|x_{-k}|}{\alpha_{-k}}\left(\frac{1}{1 - 1/\alpha_{-k}}\right) < 0$. If $\delta'(0) \leq 0$ we are done; if $\delta'(0) > 0$ then $\delta'(1) \geq 0 \rightarrow \delta'(p) > 0\ \forall p \in [0,1) \rightarrow \delta(1) > 0$, a contradiction. QED

Proof of Proposition 7

A proposer's equilibrium utility is straightforward since $(\bar{s}, y_i^*(\bar{s}))$ is in the support of their strategy and wins for sure. Comparative statics of a proposer's i's parameters on her opponent $-i$'s utility, as well as of x_i on her own utility, follow immediately from previously-shown statics on $\bar{s} = s(\epsilon)$. Taking the derivative with respect to α_i, substituting in $\frac{\partial}{\partial \alpha_i}\left(\frac{p_i(\epsilon)}{\alpha_i}\right) = \frac{x_i p_i'(\epsilon)}{(\alpha_i - 1)\alpha_i^2}$, $\frac{\partial \epsilon_k(0)}{\partial \alpha_k} = -\frac{|x_k|}{\alpha_k(\alpha_k - 1)}$, $-\frac{p_i'(\epsilon) x_i}{\alpha_i - p_i(\epsilon)} = 1$, performing a change of variables, and rearranging the expression yields $-\mathbf{1}_{i=k} \cdot 2\int_0^{\epsilon_k(0)} \frac{|x_{-k}|}{\alpha_{-k}}\left(p_{-k}(\epsilon) - \left(\alpha_k \log\left(\frac{\alpha_k}{\alpha_k - 1}\right)\right)^{-1} p_{-k}(\epsilon_k(0))\right) d\epsilon$ $-\left(\frac{|x_i|}{\alpha_i}\right)^2 \left(1 + 2\int_{p_i(\epsilon_k(0))}^1 \left(\frac{\alpha_i p}{\alpha_i - p} - 1\right)\right) dp$. The first term is negative since $p_{-k}(\epsilon) > p_{-k}(\epsilon_k(0))$ for $\epsilon < \epsilon_k(0)$ and $\frac{1}{\alpha_k} < \int_0^1 \frac{1}{\alpha_k - p} dp = \log\left(\frac{\alpha_k}{\alpha_k - 1}\right)$. The second term is also negative since $1 + 2\int_{p_i(\epsilon_k(0))}^1 \left(\frac{\alpha_i p}{\alpha_i - p} - 1\right) > (1 - p_i(\epsilon_k(0))) + 2\int_{p_i(\epsilon_k(0))}^1 \left(\frac{\alpha_i p}{\alpha_i} - 1\right) = \int_{p_i(\epsilon_k(0))}^1 (2p - 1)\, dp \geq 0$. QED

Proof of Proposition 9

First differentiating the DM's utility U_{DM}^* with respect to $|x_{-k}|$ and applying symmetry yields $\frac{2}{\alpha}\int_0^{\epsilon(0)} \left(\left(1 - 3(p(\epsilon))^2\right) \cdot x\frac{\partial p(\epsilon)}{\partial x} + \left(1 - (p(\epsilon))^2\right) p(\epsilon)\right) d\epsilon$ which is transparently $\geq 2\frac{x}{\alpha}\int_0^{\epsilon(0)} \left(1 - 3(p(\epsilon))^2\right) \frac{\partial p(\epsilon)}{\partial x} d\epsilon$. Now substituting $\frac{\partial p(\epsilon)}{\partial x} = -\log\left(\frac{\alpha - p(\epsilon)}{\alpha - 1}\right) p'(\epsilon)$ and a change of variables yields $2\frac{x}{\alpha}\int_0^1 (1 - 3p^2) \log\left(\frac{\alpha - p}{\alpha - 1}\right) dp = 2\frac{x}{\alpha}\int_0^1 \left(\frac{p - p^3}{\alpha - p}\right) dp > 0$. Next differentiating U_{DM}^* w.r.t. α_{-k} and applying symmetry yields $2x\int_0^{\epsilon(0)} \left(\left(1 - (p(\epsilon))^2\right) \frac{\partial}{\partial \alpha}\left(\frac{p(\epsilon)}{\alpha}\right) - \frac{2}{\alpha}(p(\epsilon))^2 \frac{\partial p(\epsilon)}{\partial \alpha}\right) d\epsilon$. Substituting $\frac{\partial}{\partial \alpha}\left(\frac{p(\epsilon)}{\alpha}\right) = \frac{x}{(\alpha - 1)\alpha^2} p'(\epsilon)$, $\frac{\partial p(\epsilon)}{\partial \alpha} = -\left(\frac{1 - p(\epsilon)}{\alpha - 1}\right)$, $-\frac{p'(\epsilon) x}{\alpha - p(\epsilon)} = 1$, rearranging, and a change of variables yields $\frac{2x^2}{(\alpha - 1)\alpha^2}\int_0^1 \left(2p^2\left(\frac{\alpha - \alpha p}{\alpha - p}\right) - (1 - p^2)\right) dp < 0$. QED