Strategic Experimentation with Asymmetric Information*

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Abstract

I study strategic experimentation, with one player initially being better informed about the state of nature than the other. Players are otherwise symmetric, and observe past experimentation decisions and outcomes. I construct an equilibrium in which a mutual encouragement effect arises: as the public information becomes discouraging, the informed player's high effort continuously brings in good news, encouraging the uninformed player to experiment; in return, the uninformed player's experimentation pattern yields an increasing reward, encouraging the informed player to experiment. Due to this effect, players' total effort can increase over time, and the uninformed player may grow increasingly optimistic, despite the discouraging public information. Moreover, creating information asymmetry improves total welfare if the informed player's initial signal is sufficiently precise.

1 Introduction

Experimentation is an important mechanism through which agents discover new ideas and learn their value, thereby promoting technological change, and driving economic growth.[1] In many environments, agents learn both from their own and from others' experiments.

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[1] Endogenous technological change is a key driver of economic growth, as argued by endogenous growth theory (Romer, 1990; Aghion and Howitt, 1992). On the role of experimentation in the discovery and selection of new ideas, see Romer (1994, page 12), Nelson and Winter (1994, Chapter 11).

For example, farmers learn from their own and others' experiences whether a new fertilizer improves yield (Foster and Rosenzweig, 1995; Conley and Udry, 2010); physicians learn through their own and others' prescriptions the efficacy of a new drug after it is approved by the FDA (Coleman, Katz and Menzel, 1957; Iyengar, Van den Bulte and Valente, 2011); firms in a strategic alliance learn from one another whether their newly developed product has a high demand.

The information generated from experimentation is a public good. A free-riding problem naturally arises: agents experiment less than if they acted cooperatively. This free-riding problem has been studied by Bolton and Harris (1999), Keller, Rady and Cripps (2005), and formed the basis of a large literature.[2]

I introduce initial information asymmetry to these environments. My motivation is twofold. First, information asymmetry is empirically relevant: some agent (a well educated farmer, a specialist physician, the designer of a new product) may have better information about the value of experimentation initially. The action of such a "leader" may signal useful information thereby influencing the incentive of the less-informed "follower." Second, inducing information asymmetry — for instance, by hiding information from some agents — can be a tool to curb free riding.

This agenda motivates the following questions. How does initial asymmetric information affect agents' experimentation behavior? Does it mitigate or exacerbate free-riding? Can it be welfare-improving to induce information asymmetry in an otherwise symmetric environment?

The central contribution of this paper is to show that initial information asymmetry qualitatively changes agents' experimentation behavior — unlike in the symmetric information setting, agents can increase experimentation even after a history of unsuccessful experiments (that is, experiments that do not lead to any breakthrough). The key mechanism is a novel mutual encouragement effect: a better-informed player — the leader — signals good news through persistent high (experimentation) effort, encouraging an uninformed player — the follower — to experiment; the follower follows his lead, and increases her effort over time, encouraging the leader to persevere. Thanks to this mutual encouragement effect, inducing information asymmetry mitigates free-riding and can improve total welfare.

This paper builds on the two-player version of the exponential-bandit model (Keller, Rady and Cripps, 2005). At each point in time, each player must divide a unit of resource between a safe project with known payoffs and a risky project of unknown quality. Learning is conclusive: only good risky projects deliver payoffs (breakthroughs), governed by a Poisson process. Players observe past experimentation decisions and payoffs. I add one source of information asymmetry: at date 0, one player, called the informed player (he), privately observes a binary noisy signal. He thus becomes either an *optimistic* type whose posterior belief is higher than the uninformed player's (she), or a *pessimistic* type.

With asymmetric information, a public history carries two components of information.

[2]The free-riding problem has also been well documented empirically; Foster and Rosenzweig (1995) find that during the adoption of high-yielding seed varieties associated with the Green Revolution in India, farmers do not fully incorporate the village returns to learning in making adoption decisions.

The first component is the information generated from the experimentation technology, represented by the informed player's beliefs. This component is "passive," as it depends only on the public history. The second component is the private information that the informed player reveals to the uninformed player, represented by the uninformed player's belief about the informed player being the optimistic type, called his reputation. This component is "strategic," as it depends also on the informed player's strategies.

I construct a Markov perfect equilibrium (MPE) using these two components of information as state variables. During a gradual revelation phase of this equilibrium, the pessimistic type mixes between mimicking the optimistic type's high effort and revealing himself such that, as long as he keeps mimicking, his reputation gradually increases. This rising reputation induced by the informed player's persistent high effort (the strategic component) counterbalances the pessimism induced by the absence of a breakthrough (the passive component), and encourages the uninformed player to increase her effort over time.[3]

This rising effort dynamics of the uninformed player occurs when the pessimistic type's belief is such that neither player would experiment if his signal were public. Intuitively, during the gradual revelation phase, the pessimistic type has to be indifferent between mimicking the optimistic type so as to be willing to convince the uninformed player to exert effort, and revealing himself, thereby inducing both players to stop experimentation. The marginal value of both players' efforts to the pessimistic type is dropping over time due to the absence of a breakthrough; for him to be indifferent, the uninformed player has to accelerate her information production to reward the pessimistic type's persistence.

The joint behavior pattern during the gradual revelation phase — the informed player maintaining high effort and the uninformed player increasing her effort despite the absence of a breakthrough — admits the following intuitive interpretation. Leaders motivates followers through role modeling: a leader articulates an appealing vision, which may or may not be reachable; however, as the leader sees further and more accurately than his follower,[4] his putting in long hours during setbacks gradually convinces the follower of his optimism about the vision, and hence motivates the follower to work harder. That leaders enhance followers' commitment to their visions through role modeling is a recurring theme in both modern leadership theories[5] and leadership guidelines in popular management books.[6]

The constructed MPE exists if the initial signal of the informed player is precise enough, and the fraction of the pessimistic type is not too low. If the prior belief is not too low, then the distribution of the equilibrium paths of the constructed MPE is unique among the MPEs that satisfy (1) players playing the symmetric MPE after the informed player reveals

[3]The uninformed player may even become increasingly optimistic about the risky project before a breakthrough occurs, another novel qualitative impact of information asymmetry.

[4]See for instance, page 2 of March and Weil (2009).

[5]Examples are charismatic leadership theory, transformational leadership theory (Bass and Bass, 2009; Yukl, 2010), and authentic leadership theory (Gardner et al., 2005; Avolio and Gardner, 2005).

[6]For instance, Yukl (2010) gives the following guidelines "for leaders seeking to inspire followers and enhance their self-confidence and commitments to the mission": articulate a clear and appealing vision; explain how the vision can be attained; act confident and optimistic; express confidence in followers; use dramatic, symbolic actions to emphasize key values; and lead by example (role modeling). See page 290–293.

his type (on path); (2) a criterion in the spirit of D1.

The mutual encouragement effect leads to interesting welfare implications. Suppose a social planner (she), who cares about total welfare, observes the informed player's private signal. Would she hide it from the uninformed player? I find that, if the signal is precise enough, she would hide it. Intuitively, in such cases, the optimistic type, being still optimistic before revealing his type, has little to learn, and thus suffers little from asymmetric information. The pessimistic type and the uninformed player, on the other hand, benefit significantly from the mutual encouragement effect. As a result, asymmetric information improves total welfare.

Drawing from this welfare implication, a policy maker aiming at promoting new technology adoption may find it desirable to target certain individuals first by giving them relevant information or training. Companies promoting new experience goods might find it profitable to target some consumers, say early adopters, or experts; indeed, pharmaceutical companies spend huge amounts of money targeting marketing activities at "opinion leaders," for instance, by giving them detailed information about their new drugs, a process called detailing (Nair, Manchanda and Bhatia, 2010).

2 Literature Review

Strategic experimentation in a (non-competitive) team environment was first introduced by Bolton and Harris (1999), in a two-armed Brownian bandit model. They analyze how an encouragement effect — a player's future effort encourages another player to experiment now — interacts with the free-riding effect and shapes (Markov) equilibrium experimentation strategies. Keller, Rady and Cripps (2005) propose the exponential bandit model to analyze the experimentation problem. Notably, they find that in (Markov) equilibrium, there is no encouragement effect, in the sense that players acquire the same amount of information in total as a single player does.[7] In both papers, players are symmetrically informed. This paper shows that, by introducing initial asymmetric information, a new mutual encouragement effect arises, leading to qualitatively different behavioral and belief dynamics.[8]

This paper is closely related to the recent experimentation literature that explores private learning. Bonatti and Hörner (2011) study moral hazard in teams within an exponential bandit framework. They find that players procrastinate (in the unique symmetric equilibrium) and that perfect monitoring on actions exacerbates the procrastination problem (in the symmetric MPE). This paper points out one advantage of perfect monitoring: signaling

[7]This encouragement effect does occur in MPEs with infinite switching. However, an MPE with infinite switching fails to be a limit equilibrium of discrete-time games as the length of a period shrinks to 0.

Note that the encouragement effect defined by Keller, Rady and Cripps (2005) differs from that defined by Bolton and Harris (1999): the former is an equilibrium property and the latter concerns best responses. This paper follows the former definition.

[8]In Bolton and Harris (1999), higher future efforts by other players increase a player's *continuation value*, thereby encouraging the player to experiment to collect the continuation value. In my paper, a higher reputation of the informed player increases both the uninformed player's instantaneous payoff and her continuation value, thereby encouraging her to experiment.

by the informed player can push both players to work harder than under symmetric information. Building on Bonatti and Hörner (2011), Guo and Roesler (2016) study a collaboration problem with hidden experimentation efforts but with public and irreversible exit decisions. In their model, players may privately learn the quality of their joint project over time if it is bad. As there are payoff externalities, a player who learns that the project is bad still stays in the game, delaying the abandonment of socially inefficient projects. Their paper is the closest to my paper in that both papers study signaling in experimentation problems. However, signaling plays different roles. In my paper, signaling is through experimentation effort and pushes the players to acquire more useful information. In their paper, signaling is through maintaining in the game and makes players pursue bad projects for too long.

Private learning is also examined in environment where experimentation decisions are observable but players may privately learn the quality of risky projects over time. Heidhues, Rady and Strack (2015) analyze a discrete-time version of the exponential bandit model but with private payoffs and cheap talk. They find that if the common prior is sufficiently optimistic, then the cooperative solution can be implemented (by reverting to an equilibrium which is outcome equivalent with the symmetric MPE under symmetric information once a deviation occurs). Das (2017) studies a winner-takes-all contest where efforts and payoffs are public but players may privately learns the true state over time if it is good. In both papers, equilibrium effort strategies are cutoff strategies. Therefore, the joint behavior pattern in this paper does not occur in theirs.

The question of whether hiding information from some agent(s) can improve welfare is related to information design on experimentation. Halac, Kartik and Liu (2017) study information design in contests for experimentation, in which both experimentation decisions and experimentation outcomes are private. They find that a "hidden equal-sharing" contest can outperform a "public winner-takes-all contest." The mechanisms are different. In their paper, signaling does not play a role. In my paper, it is a driving force behind the mutual encouragement effect.

More broadly, that asymmetric information may improve total welfare also relates to the leadership literature. Hermalin (1998) and Komai, Stegeman and Hermalin (2007) analyze a static model of moral hazard in team, in which the leader who knows the state of the world signals to the followers the value of their joint project by working hard, thereby partially overcoming the free-riding problem. Different from them, this paper focuses on a dynamic model, aiming at explaining the dynamic provision of informational public goods, which cannot be analyzed in their static setup. Moreover, the welfare implications are different: when the informed player knows the risky project is good but does not know it is bad, creating information asymmetry improves welfare in my setup (with purely informational externalities), whereas it may not be so in their setup (with purely payoff externalities).

Finally, in this paper, ignorance gives the uninformed player some commitment power to reward the informed player's good behavior and to punish his bad behavior. This point is similar to (Crémer, 1995) in the contracting literature and to (Carrillo and Mariotti, 2000) in the self-control literature. Crémer (1995) finds that, a principal unable to commit not to renegotiate a contract is better off with an arm's length relationship because it gives her commitment power to punish an agent's poor performance. Carrillo and Mariotti (2000) find that a time-inconsistent decision maker may forgo free useful information, in fear that her future selves would not be able to commit to the optimal consumption plan she makes today, after observing the information.

3 The Model

Time is continuous, indexed by $t \in [0, \infty)$. There are two players. Each player is endowed with one unit of a divisible resource per unit of time, and must divide it between a safe project and a risky project. A safe project delivers a known return; the return of a risky project depends on its quality θ, unknown and common to both players, with $\theta = g$ referring to a good project, and $\theta = b$ to a bad one. If a player allocates a fraction $a_t \in [0, 1]$ of resource to the risky project over a time interval $[t, t+dt]$, and hence $(1-a_t)$ to the safe project, then the player receives $(1 - a_t)sdt$ from the safe project, and a lump-sum payoff h with probability $a_t\lambda\mathbb{1}_{\{\theta=g\}}dt$ from the risky project, where $\lambda > 0$. That is, a bad risky project delivers zero payoffs whereas a good risky project delivers lump-sum payoffs, called breakthroughs, that arrive at a Poisson rate. Learning is thus conclusive: a single breakthrough perfectly reveals good quality. At any time t, players observe all past experimentation decisions and payoffs.[9] Both players prefer a good risky project to a safe project, and a safe project to a bad risky project: $\lambda h > s > 0$. They discount future payoffs with a common discount rate $r > 0$.

Initially, players assign a common prior probability q_0 on the risky projects being good. At time 0, one player, called the informed player (player I, he), receives a favorable signal s_+ with probability ρ_θ, and an unfavorable signal s_- with probability $1 - \rho_\theta$. The favorable signal s_+ is more likely to occur to a good risky project than to a bad risky project: $1 > \rho_g > \rho_b \geq 0$. By Bayes' rule, after receiving signal s_+, I adjusts his belief upward to some q_0^+, strictly higher than the uninformed player's (player U, she) posterior q_0, thereby becoming an *optimistic* type; otherwise, he adjusts his belief downward to some $q_0^- < q_0$, thereby becoming a *pessimistic* type. The parameters q_0, ρ_g, and ρ_b are common knowledge.[10] The initial information asymmetry is the only divergence from the two-player version of the canonical exponential-bandit model of Keller, Rady and Cripps (2005).

Remark. [A joint project interpretation.] Because a breakthrough publicly reveals good quality and there is no payoff externality, the game essentially ends once a breakthrough occurs. It then becomes a dominant strategy for a player to use the risky project forever, bringing a discounted payoff $\lambda h/r$. Note also that the player who receives the first breakthrough enjoys an additional payoff h relative to the other player. Therefore, the model admits the following joint project interpretation: instead of working on two risky projects of

[9] Specifically, if we use (a_s^I, a_s^U) to denote player I's and player U's efforts taken at time s, then right before players take actions at time t, they observe the effort history before t, $(a_s^I, a_s^U)_{s<t}$, and the payoff history before t.

[10] Such initial information asymmetry arises for instance if I is an incumbent, U is a new entrant who is not sure of how long I has been experimenting before time 0.

the same quality, the two players work on one joint risky project; a breakthrough occurs to the risky project with the same probability as in our model, bringing a lump-sum payoff $\lambda h/r$ to each player, and an additional intrinsic satisfaction h to the first player who experiences the breakthrough; the project is completed once a breakthrough arrives.

3.1 The cooperative solution

If players act cooperatively to maximize their joint surplus, the informed player would reveal his signal truthfully to the uninformed player.[11] Therefore, from time 0 on, both players would share a common posterior belief q_t, which continuously decreases over time as long as players experiment and no breakthrough has arrived. Both players adopt a cutoff strategy: experimenting if q_t is higher than a cutoff $q_2^* \in (0, 1)$, defined by

$$r(\lambda q_2^* h - s) + 2\lambda q_2^*(\lambda h - s) = 0, \tag{1}$$

and stopping otherwise. On the left-hand side, the first term is the flow marginal benefit of experimentation at the cooperative cutoff q_2^* (relative to the safe return; in the sequel, all return is relative to the safe return if not mentioned), and the second term the marginal option value of information to both players. Equation (1) says at the optimal cutoff, the total marginal benefit of experimentation is 0 (a smooth pasting condition).

For future use, we also introduce the single-player solution, a similar cutoff strategy with cutoff q_1^*, where q_1^* is determined by equation (1) with the number 2 being replaced by 1. Since the option value of information to two players is twice as much as that to a single player (at the same belief) whereas the flow benefit is the same, a two-player team in the cooperative solution acquire more information than does a single player: $q_2^* < q_1^*$. Intuitively, the more valuable the information, the more information player(s) should acquire.

4 Beliefs and the Equilibrium Concept

Following the experimentation literature with symmetric information (for instance, Bolton and Harris, 1999; Keller, Rady and Cripps, 2005), we focus on Markov perfect equilibrium (MPE). Different from them, there is no single state variable for the solution concept, because players do not share a common posterior belief. Now a public history carries two components of information. The first is the information obtained from the experimentation technology, depending only on the public history, independent of players' equilibrium strategies, hence is called "passive." This component of information can be represented by how the informed player updates his beliefs. The other component is the informed player's private information revealed through his actions, depending also on his equilibrium strategies and hence is called "strategic." This component of information can be represented by how the

[11] He can do so by playing some action for an infinitesimal amount of time. An alternative way to implement the cooperative solution is for the uninformed player to match the action of the informed player, and for the informed player to stop experimenting once his belief reaches the cutoff q_2^* specified below.

uninformed player updates her belief about the informed player being the optimistic type. Based on this observation, we define state variables. Strategies, belief systems, and equilibria are defined afterward.

4.1 The state variables

The passive component—the background belief. Consider an outsider who knows the model except that he mistakenly believes that neither player has observed the initial signal of the informed player's. Assume that he starts with the same prior belief $p_0 \equiv q_0$ and observes the same public histories as our players do. Denote his posterior belief at time t by p_t, and call it the *background belief.*[12]

Of course, this background belief differs from I's posterior belief. But if after a public history, the outsider is told of I's private signal, he would then adjust his belief to exactly I's. That is, after a public history, if the background belief is p, type s_+'s posterior belief must be $\boldsymbol{q}^+(p)$ given by Bayes rule,

$$\boldsymbol{q}^+(p) = \frac{p\rho_g}{p\rho_g + (1-p)\rho_b}, \tag{2}$$

and type s_-'s must be $\boldsymbol{q}^-(p)$ given by

$$\boldsymbol{q}^-(p) = \frac{p(1-\rho_g)}{p(1-\rho_g) + (1-p)(1-\rho_b)}. \tag{3}$$

Equations (2) and (3) imply that the background belief p and the signals of the informed player, s_- and s_+, are sufficient to track the informed player's posterior beliefs. To track U's belief about the risky project, we still need the strategic component of information.

The strategic component—I's reputation: the probability U assigns to I being type s_+, denoted by μ. Together with the background belief, I's reputation determines U's posterior belief about the risky project by

$$\boldsymbol{q}^U(p,\mu) \equiv \mu\boldsymbol{q}^+(p) + (1-\mu)\boldsymbol{q}^-(p), \tag{4}$$

and hence directly affects U's instantaneous payoff. As U's incentives are affected by I's strategies, which are type dependent, U's belief about I's types is necessary to compute U's continuation payoffs.

In sum, the background belief, p, and I's reputation, μ, are sufficient to represent the two components of information and are thus used as state variables.

To reduce the burden of notation, denote the expected arrival rate of breakthroughs per unit of effort for type s_+, type s_-, and U at state (p,μ), by $\lambda^{I+}(p)$, $\lambda^{I-}(p)$, and $\lambda^U(p,\mu)$, respectively, which are equal to the corresponding posterior beliefs about the risky project multiplied by the arrival rate of breakthroughs of a good risky project λ.

[12] Appendix A gives a formal definition of the background belief.

Remark. Singling out this passive background belief from a public history is not only technically convenient, but also empirically relevant. We interpret the outsider that we introduce to define the background belief as an econometrician who mistakenly believes that players are symmetrically informed. He therefore misspecifies the true asymmetric information model as a symmetric information model. We will discuss the empirical consequences of such a misspecification later.

4.2 Strategies and belief systems

Players' strategies are Markov. A pure strategy for U is a mapping from the state space into the effort space, $a^U : [0,1]^2 \to [0,1]$, with $a^U(p,\mu)$ denoting U's effort level at state (p,μ). Type s_+'s and type s_-'s pure strategies are similarly defined and denoted by a^{I+} and a^{I-} respectively. We are interested in equilibria in which both U and type s_+ play pure strategies, and type s_- plays a pure strategy after his type is revealed (on path). In such equilibria, a mixed strategy for type s_- is a mixture over his pure strategies, and can be defined based on Aumann (1964).[13] Abusing notations, we still use a^{I-} to denote the pessimistic type's strategy.

A belief system is denoted by $\mu(s_+|\cdot)$, which associates to each public history, a probability that U assigns to I being type s_+. We require the belief system to satisfies Bayes' rule whenever possible.

4.3 Equilibrium

Given a Markov strategy profile (a^{I-}, a^{I+}, a^U) and a belief system $\mu(s_+|\cdot)$, the expected average payoff to type s_l, $l \in \{+,-\}$, at time 0 is

$$E^l_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}\left[\int_0^\infty re^{-rt}\left(\left(1-a_t^{Il}\right)s + a_t^{Il}\lambda h\theta\right)dt\right],$$

which is equal to

$$E^l_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}\left[\int_0^\infty re^{-rt}\left(\left(1-a_t^{Il}\right)s + a_t^{Il}\lambda^{Il}(p_t)h\right)dt\right], \tag{5}$$

by the law of iterated expectations, where $E^l_{(a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}$ refers to type s_-'s expectation under the probability distribution induced by the strategy profile (a^{I-}, a^{I+}, a^U) and the belief system $\mu(s_+|\cdot)$.

[13] Specifically, let a^{I+} denote a pure strategy for the optimistic type, and $a^{I-}(\cdot,0)$ a pure strategy for the pessimistic type after he reveals his type. A mixed strategy for the type s_- is implemented as follows: at the start of the game, type s_- draws a number from the uniform distributed on $[0,1]$; if r is realized, then type s_- plays a^{I+} as long as the background belief is strictly higher than $\hat{p}(r)$, and plays $a^{I-}(\cdot,0)$ otherwise, where $\hat{p}:[0,1]\to[0,1]$ is a decreasing function that assigns to each realization from the uniform distribution a cutoff background belief at which type s_- reveals himself.

Similarly, the expected average payoff of player U at time 0 is

$$E_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}\left[\int_0^\infty re^{-rt}\left(\left(1-a_t^U\right)s+a_t^U\lambda^U(p_t,\mu_t)h\right)dt\right],$$

where $E_{a^{I-},a^{I+},a^U,\mu(s_+|\cdot)}$ refers to U's expectation under the probability distribution induced by the strategy profile (a^{I-},a^{I+},a^U) and the belief system $\mu(s_+|\cdot)$.

A strategy profile (a^{I-},a^{I+},a^U) and a belief system $\mu(s_+|\cdot)$ is an MPE if given the other player's strategy and the belief system, a player finds it optimal to play her equilibrium strategy, and if the belief system satisfies Bayes rule whenever possible.

4.4 The evolution of the state variables

Given an action path $(a_t^I,a_t^U)_{t\geq 0}$ (on or off the equilibrium path), before a breakthrough occurs, the background belief process $(p_t)_{t\geq 0}$ evolves according to

$$dp_t=-p_t(1-p_t)(a_t^I+a_t^U)\lambda dt, \tag{6}$$

by Bayes' rule.[14]

The evolution of a reputation process $(\mu_t)_{t\geq 0}$ depends on the equilibrium prescription. Fix a candidate equilibrium $(a^{I+},a^{I-},a^U;\mu(s_+|\cdot))$. We focus on how μ_t evolves along the path such that *no breakthrough has occurred and I has been taking type s_+'s (prescribed) effort.*[15]

If the equilibrium involves pooling from time 0 to time T, then U's belief about the risky project coincides with the background belief over the time interval $[0,T]$. As a result, I's reputation μ_t at background belief p_t for $t\in[0,T]$ is equal to

$$\boldsymbol{\mu}^o(p_t)\equiv p_t\rho_g+(1-p_t)\rho_b, \tag{7}$$

where the mapping $\boldsymbol{\mu}^o:[0,1]\to[0,1]$ is called a *pooling path.* Since $p_0=q_0$, $\boldsymbol{\mu}^o(q_0)$ is I's reputation at time 0. Note that during pooling, I's reputation μ_t decreases as p_t decreases over time. Intuitively, since signal s_+ is more likely to occur to a good risky project, as U becomes more and more pessimistic about the risky project being good, so does she about s_+ having occurred.

Once the equilibrium diverges from pooling, I's reputation μ_t would differ from $\boldsymbol{\mu}^o(p_t)$. A candidate equilibrium $(a^{I+},a^{I-},a^U;\mu(s_+|\cdot))$ induces a distribution over the set of histories

[14] To see this, suppose at background belief p_t, players take efforts (a_t^I,a_t^U) during a dt duration of time. Then in the absence of a breakthrough, the background belief at $t+dt$, p_{t+dt}, satisfies

$$p_{t+dt}=\frac{p_t(1-(a_t^I+a_t^U)\lambda dt)}{p_t(1-(a_t^I+a_t^U)\lambda dt)+(1-p_t)}$$

by Bayes rule. Therefore, the belief change in this time interval, $dp_t\equiv p_{t+dt}-p_t$, is given by equation (6).

[15] Once I takes an action different from type s_+'s on the equilibrium path, then his reputation will stay at 0; once a breakthrough occurs, his reputation ceases to matter.

such that no breakthrough has occurred before type s_-'s revelation, which consists of histories such that either "type s_- has revealed himself (by stopping mimicking type s_+)" or "type s_- has not revealed himself and no breakthrough has arrived." This distribution further induces a cumulative distribution function (CDF) $Y : [0, \infty) \to [0, 1]$ over the time at which type s_- reveals himself, where Y_t denotes the probability that type s_- has revealed himself before or at time t conditional on no breakthrough having occurred before his revelation. [16]

By Bayes' rule, I's reputation μ_t satisfies

$$\mu_t = \frac{\boldsymbol{\mu}^o(p_t)}{\boldsymbol{\mu}^o(p_t) + [1 - \boldsymbol{\mu}^o(p_t)]\,(1 - Y_t)}. \tag{8}$$

Written in its differential form, the reputation process $(\mu_t)_{t \geq 0}$ evolves according to

$$\frac{d\mu_t}{\mu_t(1-\mu_t)} = \frac{d\boldsymbol{\mu}^o(p_t)}{\boldsymbol{\mu}^o(p_t)\,(1 - \boldsymbol{\mu}^o\,(p_t))} + \frac{dY_t}{1 - Y_t}, \tag{9}$$

where $\frac{dY_t}{1-Y_t}$ denotes the probability that type s_- reveals himself during the time interval $[t, t+dt)$, conditional on *no breakthrough having occurred and I having been taking type s_+'s effort.*

4.5 The continuation game under after revelation

I focus on the equilibria such that after I's type is revealed (on path), players play the unique symmetric MPE under symmetric information as the continuation equilibrium. Keller, Rady and Cripps (2005) characterize this equilibrium: players exert effort 1 when they are sufficiently optimistic (when their *posterior belief* is above a threshold q^S); they exert effort 0 when they are sufficiently pessimistic (when their posterior belief is below the single-player cutoff q_1^*); and they exert interior effort when in between. Denote this MPE by $a^S : [0, 1] \to [0, 1]$ whose argument is players' true *posterior belief.*

Two features will change qualitatively in the asymmetric information framework. First, there is no encouragement effect: players acquire the same amount of information as a single

[16] There are three types of time-t public histories (on path): (1) histories such that a breakthrough has occurred and I has been taking type s_+'s (prescribed) action, (2) the history such that no breakthrough has occurred and I has been taking type s_+'s action, and (3) histories such that I has stopped taking type s_+'s action. Fix the candidate equilibrium. Conditional on a history being either the second or the third type, if I is the optimistic type then the second type history occurs with probability 1, whereas if I is the pessimistic type then the second type history occurs with probability $1 - Y_t$ and the third type occurs with probability Y_t.

We now show how Y_t is determined by the candidate equilibrium. According to type s_-'s mixed strategy induced by the randomization device in Footnote 13, if the random number r is such that p_t is below the threshold $\hat{p}(r)$, then type s_- must stop mimicking type s_+ before or at t, resulting in a time-t history of the third type; otherwise, if r is such that p_t is strictly above $\hat{p}(r)$, then type s_- must continue mimicking type s_+ at least until t (because the background beliefs before time t is higher than p_t, which is strictly higher than $\hat{p}(r)$), resulting in a time-t history is of the second type. Therefore, Y_t by definition is equal to the probability that r is below $\hat{p}^{-1}(p_t)$, which is equal to $\hat{p}^{-1}(p_t)$ (as r is uniformly distributed).

player does, as they stop experimentation at the single-player cutoff belief q_1^*. Second, effort monotonically decreases over time in the absence of a breakthrough.

Figure 1 translates this equilibrium in the language of the background belief. The dashed curve corresponds to the continuation equilibrium following type s_-'s revelation, where the cutoff p_1^{S-} is such that players are willing to switch from effort 1 to interior efforts: $\boldsymbol{q}^-(p_1^{S-}) = q^S$, and p_1^{*-} is such that players' posterior belief is at the single-player cutoff: $\boldsymbol{q}^-(p_1^{*-}) = q_1^*$. The solid curve corresponds to the continuation equilibrium following type s_+'s revelation, with the two cutoffs similarly defined. To avoid redundancy, whenever no confusion arises, we call p_1^{*-} type s_-'s single-player cutoff (background belief), $p_2^{*-} \equiv (\boldsymbol{q}^-)^{-1}(q_2^*)$ his cooperative cutoff, and p^{S+} type s_+'s switching cutoff.

4.6 The odds ratio

A crucial factor driving the momentum of the mutual encouragement effect is the belief difference between I's two types, naturally measured by $\frac{\boldsymbol{q}^+(p)}{1-\boldsymbol{q}^+(p)} / \frac{\boldsymbol{q}^-(p)}{1-\boldsymbol{q}^-(p)}$. By Bayes rule (equations (2) and (3)), this ratio is also equal to an *odds ratio* O defined by

$$O \equiv \frac{\rho_g/(1-\rho_g)}{\rho_b/(1-\rho_b)}, \tag{10}$$

the ratio of the odds of signal s_+ occurring to a good project to the odds of it occurring to a bad project. Therefore, the odds ratio O measures both the belief difference between I's two types and the informativeness of I's private signal.

The following assumption greatly eases the exposition of the mutual encouragement effect. Section 8 discusses what happens if this assumption does not hold.

Assumption 1. *The odds ratio O is greater than or equal to $O^S \equiv \frac{q^S}{1-q^S} / \frac{q_2^*}{1-q_2^*}$.*

Under Assumption 1, when type s_-'s posterior belief is at the cooperative cutoff q_2^*, type s_+'s would be weakly greater than the switching cutoff q^S (after the same public history). It means the belief difference between I's two types is large, so that after the players with public information s_- find it optimal to stop experimenting when playing cooperatively, the players with public information s_+ still experiment with full resource for at least some time (when playing the symmetric MPE). The parameters in Figure 1 satisfy this assumption, because at the background belief p_2^{*-} (type s_-'s cooperative cutoff), players with public information s_+ are still willing to exert effort 1.

5 MPE with Gradual Revelation

This section constructs the MPE of interest. We first highlight its main structure and elaborate the equilibrium behavior dynamics and belief dynamics. Detailed equilibrium construction is postponed to the last subsection and equilibrium uniqueness to the Section 8.

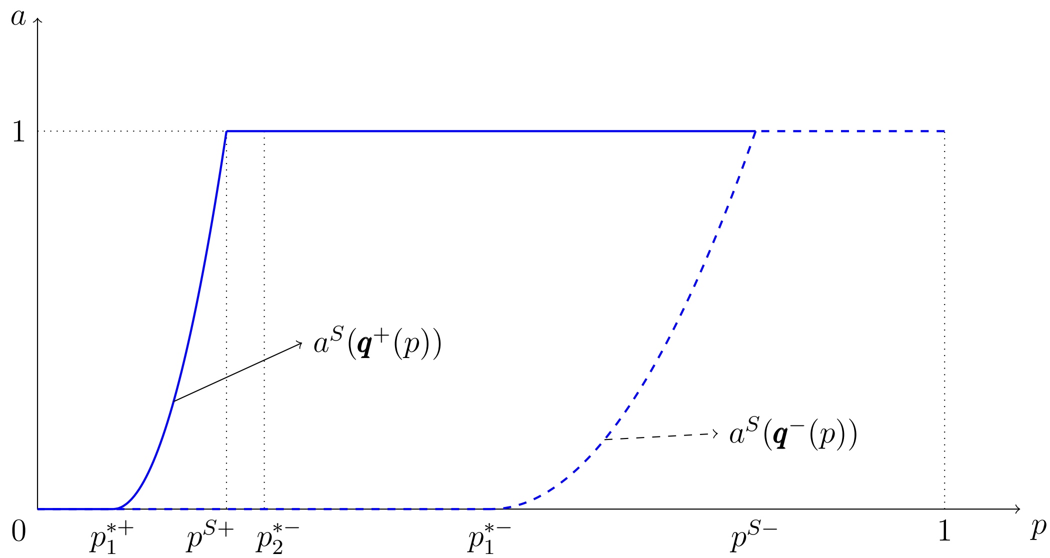

Figure 1: The symmetric MPE as functions of the background belief

Recall that we focus on equilibria such that, after I's type is revealed (on path), the players play the symmetric MPE a^S (defined in Section §4.5). In the sequel, by type s_- revealing himself, we mean that he plays this equilibrium strategy, and immediately after this, U follows suit. All equilibrium descriptions are conditioned on no breakthrough having occurred.

The equilibrium has three phases.

1. When type s_- is sufficiently optimistic — over background beliefs $(p_{gr}, 1)$, p_{gr} to be determined — the equilibrium involves *pooling*, during which, both players exert effort 1. As a result, I's reputation gradually decreases over time (along the pooling path $\boldsymbol{\mu}^o$). The eroding reputation path is illustrated by the dash-dot line over the interval $(p_{gr}, 1]$ in Figure 2: as time passes by, I's reputation decreases along this line from right to left until p reaches p_{gr}.

2. When type s_-'s belief is intermediate — over background beliefs $(p_2^{*-}, p_{gr}]$ — the equilibrium involves *gradual revelation*, during which, type s_+ still exerts effort 1, whereas type s_- mixes between mimicking type s_+ and revealing himself, such that as long as he keeps mimicking, his reputation gradually increases, along a *gradual revelation path* $\hat{\boldsymbol{\mu}} : [p_2^{*-}, p_{gr}] \to [0, 1]$. This rising reputation path is illustrated by the solid curve in Figure 2: as time passes by, I's reputation increases along this line from right to left.

3. When type s_- is sufficiently pessimistic — over background beliefs $(0, p_2^{*-}]$ — the equilibrium involves *separation*, during which, type s_+ plays the symmetric MPE strategy under symmetric information s_+, whereas type s_- stops experimenting immediately. Referring to Figure 2, if I stops experimentation, the state variables jump on the line

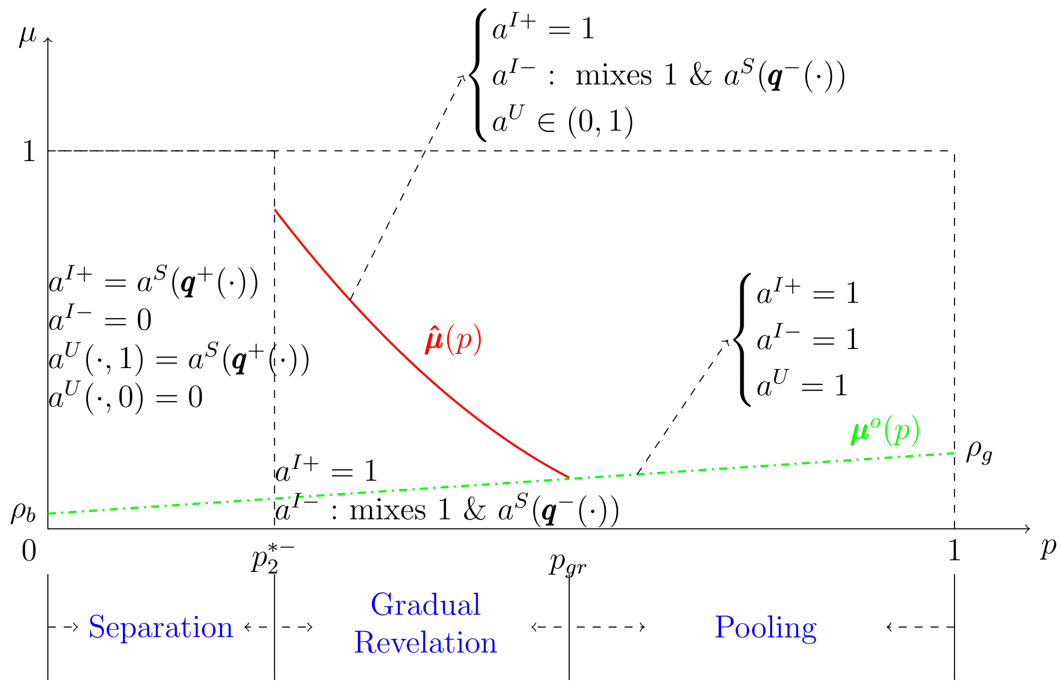

Figure 2: An MPE with gradual revelation

$\mu = 0$ and then cease to move; otherwise, the state variables jump on the line $\mu = 1$ and move along it from right to left until experimentation ends.[17]

Call this equilibrium an *MPE with gradual revelation*. Figure 3 illustrates how phase transitions occur. If the prior belief q_0 (which, recall, is equal to p_0) lies in the pooling region $(p_{gr}, 1)$, say at the closed circle on the solid part of $\boldsymbol{\mu}^o$, then the equilibrium begins with the pooling phase, during which, the state variables move from right to left along the pooling path $\boldsymbol{\mu}^o$ until the background belief reaches p_{gr}. After this, the gradual revelation begins, during which, the state variable move along the gradual revelation path $\hat{\boldsymbol{\mu}}$ until the background belief reaches p_2^{*-}. After this follows the separation phase. The solid arrowed curve illustrates how the state variables evolve over time, conditional on no breakthrough and I having been playing type s_+'s effort.

If the prior belief q_0 lies in the gradual revelation region $(p_2^{*-}, p_{gr}]$, say at the open circle on the dashed part of $\boldsymbol{\mu}^o$, then type s_- reveals with some probability such that upon non-revealing, the state variables immediately jump up on the curve $\hat{\boldsymbol{\mu}}$. The gradual revelation phase then begins and the equilibrium dynamics is the same as in the previous case. The dashed arrowed curve illustrates how the state variables evolve over time conditional on no breakthrough and I having been playing type s_+'s effort.

We are ready to present the first main result of the paper — the qualitative features of the behavior dynamics and belief dynamics.

[17] Of course, the state variables stop moving when the background belief reaches p_1^{*+}, below which, even type s_+ stops experimenting.

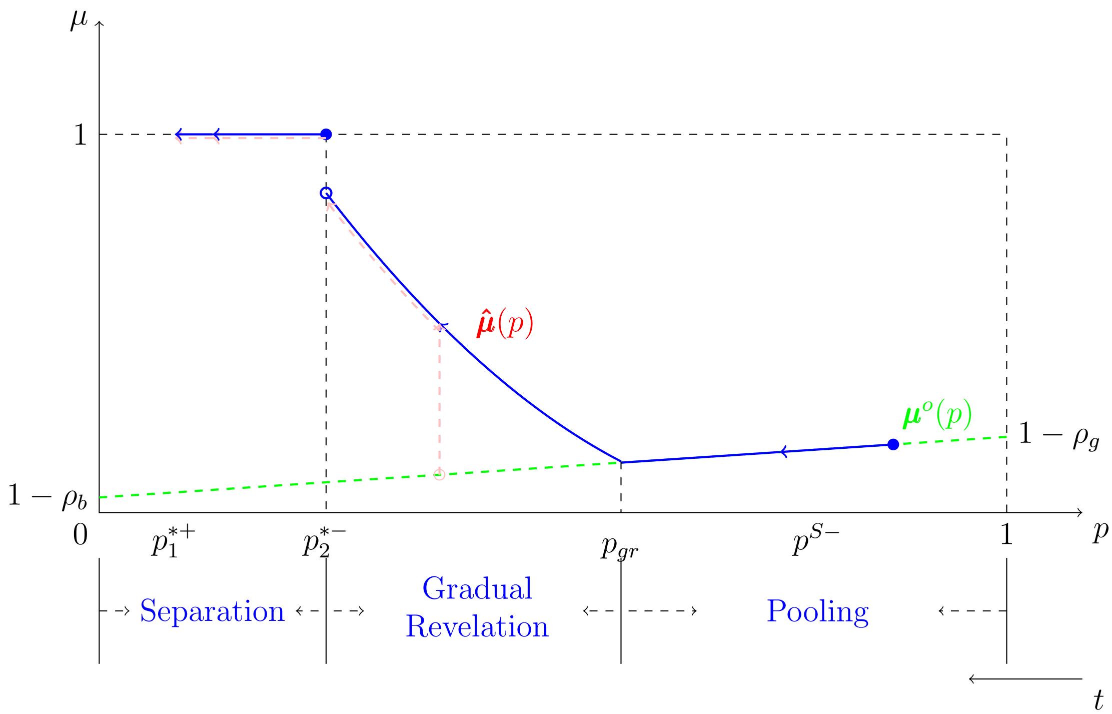

Figure 3: An MPE with gradual revelation: two paths of the state variables

Proposition 1. *During the gradual revelation phase of the MPE with gradual revelation, as long as no breakthrough has arrived and I has been playing type s_+'s effort, then over time,*

1. *U's effort gradually increases, when the background belief is between type s_-'s cooperative cutoff p_2^{*-} and his single-player cutoff p_1^{*-};*
2. *I's reputation gradually rises;*
3. *U's belief about the risky project is either increasing or U-shaped, if the informativeness of I's initial signal is intermediate (that is, if the odds ratio O is not too high but still satisfies Assumption 1)*

We have illustrated the rising reputation in Figure 3. In Figure 4, the arrowed curve displays the uninformed player's effort path conditional on her facing an optimistic type: U increases her effort over time when the background belief is between p_1^{*-} and p_2^{*-}. We postpone discussing U's decreasing effort (over time) during the gradual revelation phase until Section 5.4.1.

Figure 5 and Figure 6 contrast two distinct paths of U's belief about the risky project's quality. In Figure 5, U's belief decreases over time before the separation phase occurs; this typically occurs in an environment with a high odds ratio. In Figure 6, U's belief is U-shaped before the separation phase occurs; this typically occurs in an environment with an intermediate odds ratio.

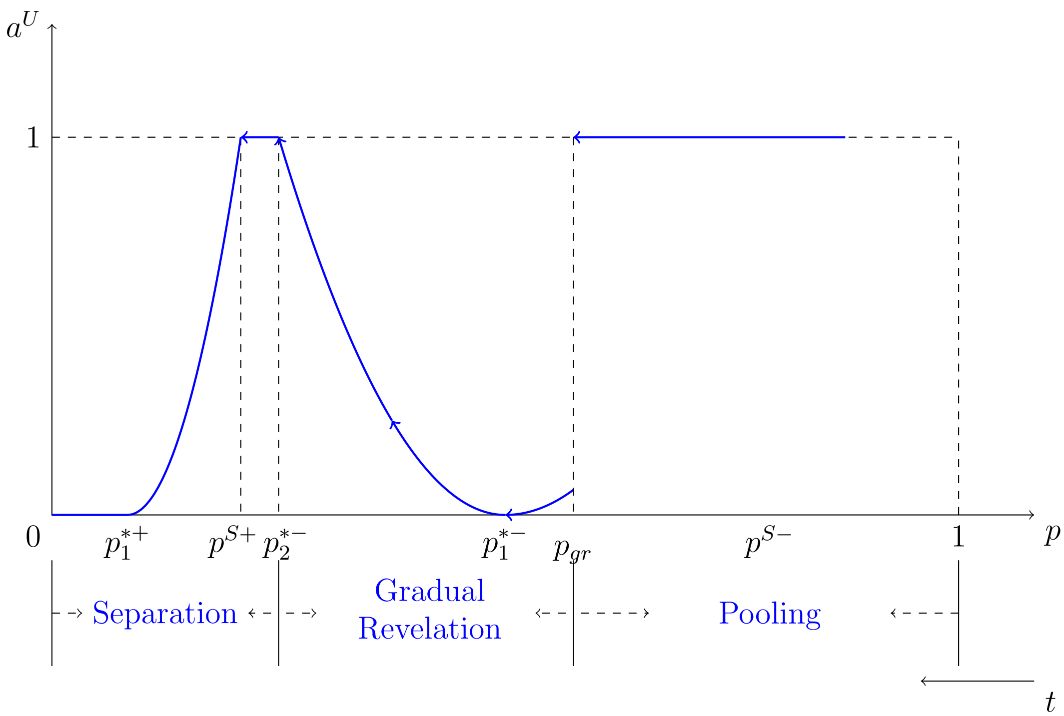

Figure 4: The uninformed player's effort

Compared with the symmetric MPE under symmetric information, two features of the current equilibrium stand in sharp contrast.

First, the uninformed player can *increase effort*, and *become more optimistic* about the risky project over time, despite the absence of a breakthrough. She does so because I's high effort continually brings in good news, compensating for the absence of a breakthrough, and encouraging her to experiment.

Second, the pessimistic type *experiments beyond the single-player cutoff*, until the cooperative cutoff, with positive probability. He does so because the uninformed player responds to his hard work by also working hard, thereby producing more information over time, encouraging him to experiment at beliefs he would not if were he alone or were his signal public.

We therefore have identified a *mutual encouragement effect*: I's rising reputation compensates the dropping background belief, encouraging U to experiment; U's increasing effort compensates type s_-'s growing pessimism, encouraging him to persevere. Driven by this effect, the joint behavior pattern — the informed player keeps exerting high effort and the uninformed player increases effort, despite the absence of a breakthrough — does not occur in any MPE that is a limit MPE of the symmetric information discrete-time games.[18] This

[18] To be specific, it does not occur in any MPE that is a limit MPE of a discretization of the continuous-time experimentation game. Hörner, Klein and Rady (2014) (in Lemma 1) show that in any perfect Bayesian equilibrium (hence MPE) of such discrete time game, players do not experiment when their posterior is below

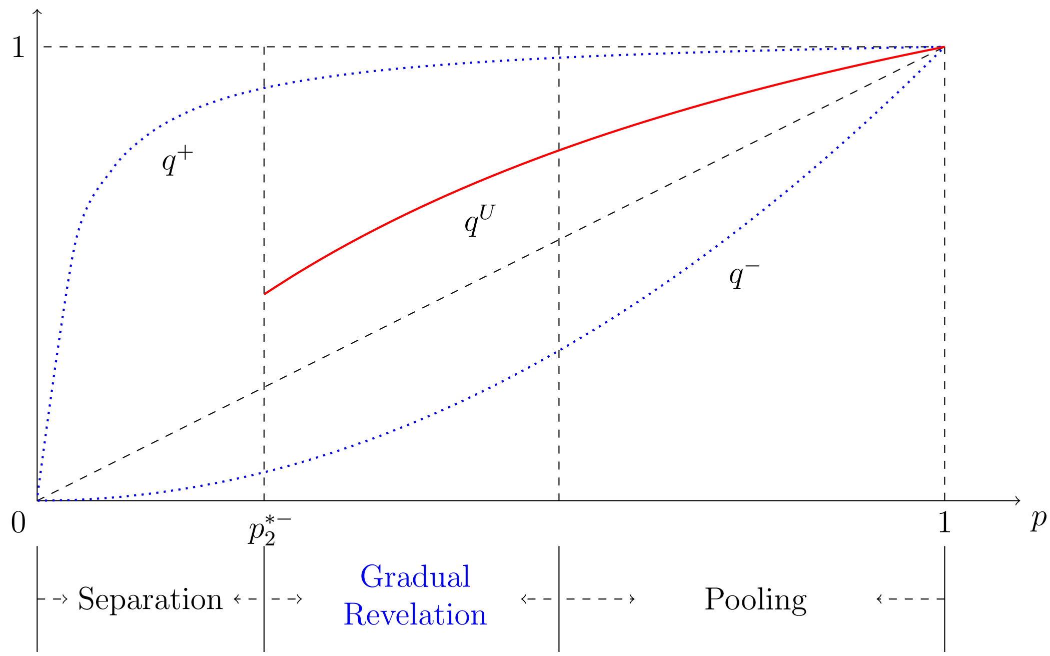

Figure 5: U's growing pessimism before separation (a large odds ratio)

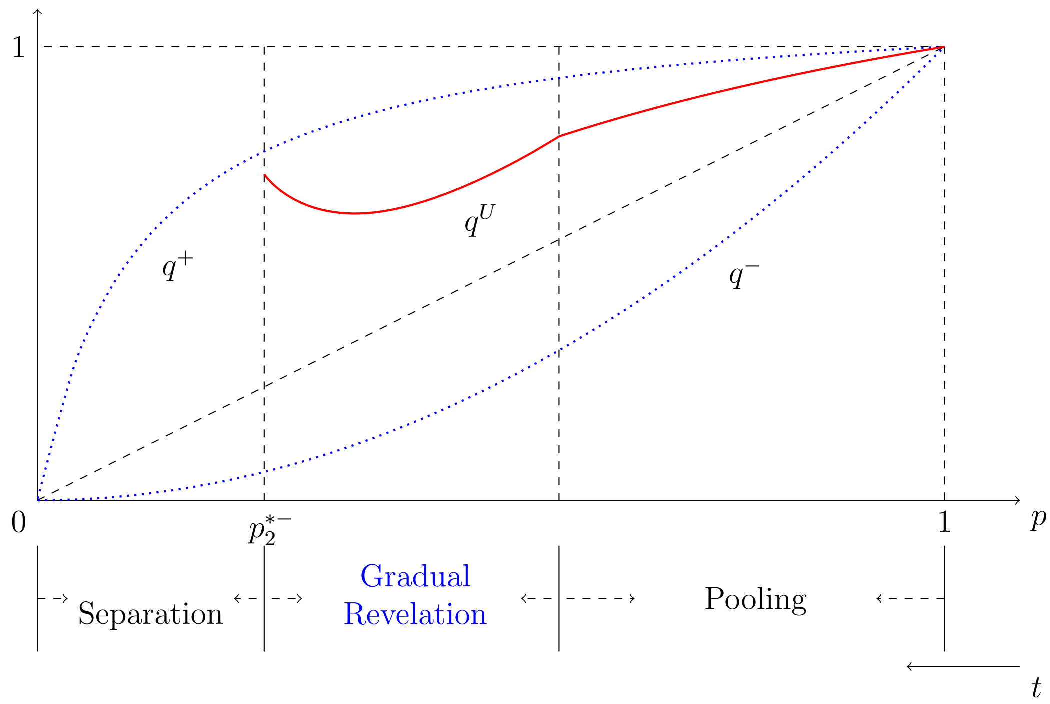

Figure 6: U's growing optimism before separation (an intermediate odds ratio)

pattern leads to qualitatively different empirical predictions, which we will discuss in Section 7.

The above discussion highlights two requirements for the mutual encouragement effect to arise. First, it is able to counterbalance the deterioration of the background belief. This is guaranteed by Assumption 1, which ensures that a perfect reputation brings in sufficiently good news to encourage U to experiment (fixing the continuation equilibrium). Second, it is needed to counterbalance the deterioration of the background belief, which occurs if the fraction of type s_+ is not too high, guaranteed by:

Assumption 2. *Signal s_+ is not too likely to occur: $\rho_g \leq \frac{s}{(r+\lambda)h+\lambda h-s}$.*

With these two assumptions, the mutual encouragement effect can arise, so does the constructed MPE:

Proposition 2. *Under Assumption 1 and 2, an MPE with gradual revelation exists.*

If Assumption 2 is not satisfied, then the gradual revelation phase can be empty, that is, the pooling phase lasts until the background belief reaches p_2^{*-}.

We now elaborate on the intuitions behind Proposition 1.

5.1 The informed player's rising reputation

During the gradual revelation phase, I's rising reputation counterbalances the declining background belief, maintaining U's indifference about experimentation, thereby incentivizing her to take interior effort. To see this, consider the following two main elements that drive U's experimentation incentives.[19]

1. U's instantaneous marginal benefit of experimentation, which depends only on her belief about the risky project. The higher her belief, the higher her willingness to experiment.

2. U's continuation marginal benefit of experimentation, which depends on her expected continuation value. The higher her expected continuation value, the higher her incentive to speed up experimentation so as to enjoy it earlier.

Suppose instead I's reputation does not increase in the absence of a breakthrough. Then as time passes by, U becomes more pessimistic and hence her instantaneous marginal benefit decreases. Moreover, with both of her ex post continuation values decreasing, together with I's dropping reputation, so is her expected continuation value. Consequently, if at some point in time she is indifferent about experimentation, she would strictly prefer not to experiment afterward.

the single-player cutoff. Using this result, we can show that in any limit MPE, total effort cannot strictly decrease in players' posterior.

[19] I's current effort also affects U's experimentation incentive, due to the strategic substitutability of players' current effort decisions, as in the symmetric information game. This element is absent here because I's current effort is 1 with probability 1.

Therefore, for U to be indifferent, I's reputation must rise over time. Indeed, during this phase, the incentive-enhancing effect of I's rising reputation (driven by the strategic component) exactly balances out the incentive-dampening effect of the deteriorating background belief (driven by the passive component), maintaining U's willingness to take the effort in Figure 4 — in particular, to increase her effort when the background belief is between (type s_-'s single-player cutoff) p_1^{*-} and (type s_-'s cooperative cutoff) p_2^{*-}.

5.2 The uninformed player's increasing effort

When the background belief is between the pessimistic type's single-player cutoff p_1^{*-} and his cooperative cutoff p_2^{*-}, U's increasing effort compensates his growing pessimism, keeping him indifferent between mimicking type s_+ (by continuing experimenting) and revealing himself (by stopping experimenting). As a result, he is willing both to experiment beyond his single-player cutoff, and to stop so that mimicking type s_+ indeed continually carries encouraging news.

Specifically, by revealing himself, he induces both players to stop experimenting as the background belief is below his single-player cutoff; he thereby receives zero relative to the safe return. By continuing mimicking type s_+ for a dt duration of time, he receives an instantaneous benefit, $r\left(\lambda^{I-}(p)\,h - s\right)dt$, which is decreasing with the absence of a breakthrough, and a continuation benefit, an upward jump of his continuation value in case a breakthrough arrives, $\lambda h - s$, with probability $\left(1 + a^U\right)\lambda^{I-}(p)\,dt$.[20] For him to be indifferent, the two options must give him the same payoff. That is, U's effort must satisfy

$$a^U\left(p, \hat{\boldsymbol{\mu}}(p)\right) = \frac{r(s - \lambda^{I-}(p)\,h)}{\lambda^{I-}(p)\,(\lambda h - s)} - 1, \text{ for } p \in [p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]. \tag{11}$$

which increases over time, as the background belief deteriorates. Intuitively, since effort becomes less and less valuable to type s_- due to the absence of a breakthrough, U must produce more information — that is, to increase her effort — to reward type s_-'s perseverance.

We thus call this sub-phase of the gradual revelation phase the "rewarding sub-phase." Note that in Figure 4 that $a^U = 0$ at $p = p_1^{*-}$. This is because, p_1^{*-} being type s_-'s single-player cutoff, U does not need to provide any extra reward for him to experiment. Note also that $a^U = 1$ at $p = p_2^{*-}$. This is because, p_2^{*-} being his cooperative cutoff, U needs to respond one for one to type s_-'s (reputation-building) effort, so that type s_- indirectly internalizes the social benefit of his effort.[21] The gradual revelation phase ends at p_2^{*-} because U reaches the budget limit that she can reward I's hard working. U's effort in the other sub-phase of the gradual revelation phase, that is, when p is in (p_1^{*-}, p_{gr}), is left to the final subsection.

[20] In case no breakthrough arrives, type s_-'s continuation value stays at the safe return s and hence he receives no continuation benefit after this event.

[21] Conditional on signal s_- being realized, since players are symmetric, I's benefit from (the information produced by) U's effort is exactly equal to U's benefit from I's effort. Therefore, by rewarding I's effort with (the same amount of) U's effort, it is as if adding to I's incentive U's benefit from I's effort, thereby making I internalize the social benefit of his effort.

5.3 U's growing optimism about the risky project

How much encouraging information should the informed player reveal to the uninformed player so as to maintain her experimentation incentive? It depends on how informative I's private signal is:

Lemma 1. *There exists $\tilde{O} \in (O^S, \infty)$ such that, during the pooling phase and the gradual revelation phase of the MPE constructed, the uninformed player's belief about the risky project's quality*

1. *strictly decreases over time, if the odds ratio is sufficiently high, that is, if $O \in [\tilde{O}, \infty)$;*
2. *is U-shaped — it first decreases over time, and then after reaching some point in the gradual revelation region, it begins to increase — if odds ratio is intermediate, that is, if $O \in [O^S, \tilde{O})$.*

U's growing optimism in Proposition 1 follows from the second case. We here give an intuition for why near the end of the gradual revelation phase, U becomes increasingly pessimistic over time if I's private signal is sufficiently informative ($O \in [\tilde{O}, \infty)$), and increasingly optimistic if it is intermediately informative ($O \in [O^S, \tilde{O})$).

For ease of illustration, we decompose U's marginal benefit of experimentation (see Section §5.1) into the following three parts: (1) her instantaneous marginal benefit, (2) her option value of the information generated from the experimentation technology — the passive component, and (3) her option value of the information revealed by I's action — the strategic component. The first two parts, combined together, depend only on U's belief about the risky project's quality q^U and changes in the same direction as q^U changes. The third part depends on the spread of the informed player's private information, measured by the spread of the informed player's beliefs q^- and q^+; the bigger spread, the more useful I's private information to U, and hence the higher her experimentation incentive.

A drop in q^- widens the spread between q^- and q^+. This means that if μ were to change in such a way that the uninformed player's belief were to increase, then the uninformed player's continuation marginal benefit of experimentation must also increase. On the contrary, a drop in q^+ shrinks the spread between q^- and q^+. This means that if μ were to change in such a way that the uninformed player's belief were to decrease, then her continuation marginal benefit of experimentation must also decrease.[22]

When the odds ratio is sufficiently large, during the gradual revelation phase, the optimistic type's belief q^+ is close to 1 and hence barely decreases over time (by Bayes rule). As a result, the effect of the dropping q^- (due to the lack of a breakthrough) dominates. From the above analysis, if U's belief about the risky project does not decrease over time, then her total marginal benefit of experimentation would strictly increase, and consequently she would not be indifferent about experimentation, which could not occur in an MPE with gradual revelation.

[22] Appendix C.3.3 gives a detailed illustration.

When the odds ratio is intermediate (greater than and close to a^S), U is willing to experiment only if I is sufficiently likely to be type s_+, that is, only if I's reputation μ is close to 1.[23] As a result, the dropping q^- ceases to matter as its impact is weighted by $1-\mu$, and the effect of the dropping q^+ dominates. If U's belief about the risky project does not increase over time, then her total marginal benefit of experimentation would be strictly decreasing, which could not occur in an MPE with gradual revelation.

We have completed the (sketch of) proof of Proposition 1.

5.4 Detailed equilibrium construction

This subsection studies the following questions. First, what necessary conditions should U's effort a^U during the gradual revelation phase and the gradual revelation path $\hat{\boldsymbol{\mu}}$ satisfy if the MPE with gradual revelation is an equilibrium? Second, if a^U and $\hat{\boldsymbol{\mu}}$ indeed satisfy these conditions, is the MPE with gradual revelation indeed an equilibrium? To reduce the burden of notations, we omit the arguments $(p, \hat{\boldsymbol{\mu}}(p))$ of the continuation value functions W^{I+}, W^{I-}, and W^U, and of the pure effort strategies a^{I+} and a^U, whenever no confusion arises.

5.4.1 Necessary conditions for equilibrium construction

(1) U's effort function a^U for $p \in (p_1^{*-}, p_{gr})$. At any state $(p, \hat{\boldsymbol{\mu}}(p))$ during gradual revelation, type s_- faces two options: mimicking type s_+ (by exerting effort a^{I+}) and revealing himself.

If he mimics, he will receive continuation value $W^{I-}(p, \hat{\boldsymbol{\mu}}(p))$, which satisfies the Hamilton–Jacobi–Bellman (HJB) equation:

$$\begin{aligned} r\left(W^{I-} - s\right) &= a^{I+}\left[r\left(\lambda^{I-}(p)\, h - s\right) - \lambda p\,(1-p)\,\frac{dW^{I-}}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}\right)\right] \\ &\quad + a^U\left[-\lambda p\,(1-p)\,\frac{dW^{I-}}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}\right)\right]. \end{aligned} \tag{12}$$

This equation says, type s_-'s flow continuation value — the left-hand side, must be equal to the sum of his instantaneous benefit $r\left[a^{I+}\left(\lambda^{I-}(p)\, h - s\right)\right]$ and the value of information. The latter consists of two parts: in case a breakthrough arrives, occurring at a rate of $\left(a^{I+} + a^U\right)\lambda^{I-}(p)$, his continuation value increases by $(\lambda h - W^{I-})$; in case no breakthrough arrives, his continuation value changes at a rate of $\frac{dW^{I-}}{dp}\frac{dp_t}{dt} = -\frac{dW^{I-}}{dp}\left(a^{I+} + a^U\right)\lambda p\,(1-p)$.

If he does not mimic, then players will play the symmetric MPE (with signal s_- public), whereby type s_- receives a continuation value $w^S\left(\boldsymbol{q}^-(p)\right)$. Note that a player is indifferent between experimenting and not experimenting when the background belief is in $(p_1^{*-}, p_{gr}]$ (because $p_{gr} < p^{S-}$), meaning that type s_- would obtain the same continuation value by

[23] See the discussion of Lemma 3 in the last subsection for further explanation.

exerting effort a^{I+} instead of a^S given that U plays a^S. Therefore, $w^S(\boldsymbol{q}^-(p))$ satisfies (omitting the argument $\boldsymbol{q}^-(p)$):

$$
\begin{aligned}
r\left(w^S - s\right) &= a^{I+}\left[r\left(\lambda^{I-}(p)\,h - s\right) - \lambda p(1-p)\frac{dw^S}{dp} + \lambda^{I-}(p)\left(\lambda h - w^S\right)\right] \\
&\quad + a^S\left[-\lambda p(1-p)\frac{dw^S}{dp} + \lambda^{I-}(p)\left(\lambda h - w^S\right)\right]. \qquad (13)
\end{aligned}
$$

Since type s_- is indifferent between these two options, we have $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = w^S(\boldsymbol{q}^-(p))$. This equality, together with equations (12) and (13), and the fact that information is valuable (that is, the terms in the square brackets on the second line of equation (12) is positive), imply that

$$a^U(p, \hat{\boldsymbol{\mu}}(p)) = a^S\left(\boldsymbol{q}^-(p)\right), \quad p \in (p_1^{*-}, p_{gr}].$$

That is, U exerts the same level of effort whether the informed player continues exerting high effort a^{I+} or not. Intuitively, since type s_- is willing to take type s_+'s effort a^{I+} even after losing his reputation, mimicking type s_+ is costless and hence he should not be rewarded for doing so. We thus call $(p_1^{*-}, p_{gr}]$ the *non-responding* region of the gradual revelation phase.

The non-responding region is empty if $p_{gr} \leq p_1^{*-}$. For a given odds ratio, if the informed player is likely to be type s_+, that is, if ρ_b is high, then U is willing to experiment even at low background beliefs, implying a short gradual revelation phase and hence a low p_{gr}, and consequently, an empty non-responding region. It can be shown that for each odds ratio satisfying Assumption 1, there is a threshold of ρ_b below which, the non-responding region exists, and above which, it does not.

Lemma 2 summarizes U's effort during the non-responding region and the rewarding region.

Lemma 2. *If the MPE with gradual revelation is an equilibrium, then along the gradual revelation path* $\hat{\boldsymbol{\mu}}$,

- *over the rewarding region* $(p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]$, *$U$'s effort satisfies equation* (11), *and hence is strictly increasing over time.*
- *over the non-responding region* $(\min\{p_1^{*-}, p_{gr}\}, p_{gr}]$ *(if nonempty), U's effort equals the symmetric MPE effort under public information s_-, and hence is strictly decreasing over time.*

Referring to Figure 4 again, U's effort is decreasing over time over the non-responding region (p_1^{*-}, p_{gr}),[24] and increasing over the rewarding region (p_2^{*-}, p_1^{*-}).

[24]Note although U reduces her effort, and hence remains indifferent about experimentation in both this non-responding region of the asymmetric information game and in the symmetric information benchmark, she does so out of different reasons: in the former, she is indifferent because I's rising reputation compensates the absence of breakthrough , whereas in the latter, I's dropping effort compensates the absence of a breakthrough (due to the substitutability of players' current effort decisions).

(2) The gradual revelation path $\hat{\boldsymbol{\mu}}$. According to Lemma 2, U's effort is interior, meaning that she is indifferent about experimentation during the gradual revelation phase. This incentive condition pins down $\hat{\boldsymbol{\mu}}$. To show this, we need to analyze the value of information to U, in particular, the rate at which I reveals his private information to U.

Equation (9) links to each (differentiable) gradual revelation path $\hat{\boldsymbol{\mu}}$ a CDF Y over the times at which type s_- stops mimicking type s_+'s effort, conditional on either no breakthrough having occurred or type s_- having revealed himself. Specifically, suppose the time-t state (p_t, μ_t) is $(p, \hat{\boldsymbol{\mu}}(p))$, and that during the time interval $[t, t+dt]$, I's effort is a^{I+} and U's is a. Then between time t and $t+dt$, type s_- reveals himself at a *rate* of

$$\begin{aligned}\frac{dY_t/dt}{1-Y_t} &= \left(\frac{\hat{\boldsymbol{\mu}}_p(p)}{\hat{\boldsymbol{\mu}}(p)(1-\hat{\boldsymbol{\mu}}(p))} - \frac{\boldsymbol{\mu}^o_p(p)}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))}\right)\frac{dp_t}{dt} \\ &= \left(\frac{\boldsymbol{\mu}^o_p(p)}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))} - \frac{\hat{\boldsymbol{\mu}}_p(p)}{\hat{\boldsymbol{\mu}}(p)(1-\hat{\boldsymbol{\mu}}(p))}\right)(a^{I+}+a)p(1-p)\lambda, \end{aligned} \tag{14}$$

where $\hat{\boldsymbol{\mu}}_p$ denotes the derivative of $\hat{\boldsymbol{\mu}}$. Note that U's effort a affects type s_-'s revealing rate: the higher her effort, the quicker negative information (i.e., no breakthrough) accumulates, and hence the higher rate at which type s_- needs to reveal himself so that non-revealing brings encouraging information fast enough.

The value of information to U consists of three parts:

- in case a breakthrough arrives, which occurs at a rate of $(a^{I+}+a)\lambda^U$, U's continuation value jumps by $(\lambda h - W^U)$;
- in case no breakthrough arrives and I continues exerting effort a^{I+}, U's continuation value changes at a rate of $\frac{dW^U}{dp}\frac{dp_t}{dt}$;
- in case no breakthrough arrives and I stops exerting effort a^{I+}, which occurs at a rate of $(1-\mu_t)\frac{dY_t/dt}{1-Y_t}$, U's continuation value drops by $|W^U(p,0) - W^U(p,\hat{\boldsymbol{\mu}}(p))|$.

Summing up and applying equation (14), the value of information to U is thus $(a^{I+} + a)A(p,\hat{\boldsymbol{\mu}}(p))$, where $A(p,\hat{\boldsymbol{\mu}}(p))$ denotes U's continuation marginal benefit of experimentation (omitting the arguments $(p,\hat{\boldsymbol{\mu}}(p))$):

$$\begin{aligned} A \equiv\ & (1-\hat{\boldsymbol{\mu}}(p))\left(\frac{\boldsymbol{\mu}^o_p(p)}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))} - \frac{\hat{\boldsymbol{\mu}}_p(p)}{\hat{\boldsymbol{\mu}}(p)(1-\hat{\boldsymbol{\mu}}(p))}\right)p(1-p)\lambda\left(W^U(p,0)-W^U\right) \\ & -\lambda p(1-p)\frac{dW^U}{dp} + \lambda^U\left(\lambda h - W^U\right) \end{aligned} \tag{15}$$

Using the fact that type s_+'s effort is 1 during gradual revelation, U's continuation value function W^U satisfies the HJB equation:

$$r\left(W^U - s\right) = \max_{a\in[0,1]} a\left[r\left(\lambda^U h - s\right) + A\right] + A. \tag{16}$$

For U to be indifferent, her marginal benefit must be 0:

$$r\left(\lambda^U h - s\right) + A = 0. \tag{17}$$

Equations (16) and (17) imply that U's continuation value function is given by

$$W^U - s \;=\; s - \lambda^U h. \tag{18}$$

U's indifference condition (17) and her continuation value function (18) give an ODE that the gradual revelation path $\hat{\boldsymbol{\mu}}$ must satisfy:

$$\hat{\boldsymbol{\mu}}_p(p) = g(p, \hat{\boldsymbol{\mu}}(p)), \qquad p \in (p_2^{*-}, p_{gr}), \tag{19}$$

where the formula of g is given in Appendix C.1.3 due to its complexity.

The following lemma characterizes the gradual revelation path $\hat{\boldsymbol{\mu}}$.

Lemma 3. *If the MPE with gradual revelation is an equilibrium, then the gradual revelation path* $\hat{\boldsymbol{\mu}}$ *is the unique solution to the first order ODE problem defined by equation* (19)*, with the initial value condition*

$$\hat{\boldsymbol{\mu}}(p) = \frac{s - \lambda^{I-}(p)\, h}{w^S\left(\boldsymbol{q}^+(p)\right) - s + \lambda^{I+}(p)\, h - \lambda^{I-}(p)\, h}, \qquad p = p_2^{*-}, \tag{20}$$

and the boundary p_{gr} *being the smallest* p *satisfying*

$$\hat{\boldsymbol{\mu}}\left(p_{gr}\right) = \boldsymbol{\mu}^o(p_{gr}). \tag{21}$$

The initial value condition comes from the value matching condition of W^U at p_2^{*-}:

$$s - \lambda^U h = \hat{\boldsymbol{\mu}}(p)\left(w^S\left(\boldsymbol{q}^+(p)\right) - s\right), \qquad p = p_2^{*-}, \tag{22}$$

where the left-hand side is U's continuation value (18) at the end of gradual revelation p_2^{*-} and the right-hand side U's expected continuation value at the beginning of separation: with probability $\hat{\boldsymbol{\mu}}(p_2^{*-})$, U faces type s_+ and hence achieves a continuation value $w^S\left(\boldsymbol{q}^+\left(p_2^{*-}\right)\right) - s$, and with the complementary probability, U faces type s_- and experimentation ends.

Finally, the gradual revelation path $\hat{\boldsymbol{\mu}}$ must lie above the pooling path $\boldsymbol{\mu}^o$ and intersects at the right boundary of gradual revelation p_{gr}. This condition pins down p_{gr} as in the proposition.

5.4.2 Sufficient conditions for equilibrium construction and existence

The previous subsection shows that players' (on-path) behaviors in an MPE with gradual revelation are necessarily characterized by Lemma 2 and Lemma 3. This subsection shows that these are also sufficient conditions for equilibrium existence. Proposition 2 follows from Lemma 4.

Lemma 4. *Under Assumptions 1 and 2, an MPE with gradual revelation can be sustained as an equilibrium, if during gradual revelation, the uninformed player's effort is as in Lemma 2, and the gradual revelation path* $\hat{\boldsymbol{\mu}}$ *is the unique solution to the ODE problem defined by* (19)*,* (20)*, and* (21)*.*

For example, an MPE with gradual revelation characterized by Lemma 2 and Lemma 3 is an equilibrium, if the belief system and type s_+'s strategy are specified as follows.

(1) The belief system. Low effort completely depletes reputation: if I takes an effort strictly lower than type s_+'s equilibrium effort, he will be taken as type s_-. The belief updating rule for $a^I = a^{I+}(p, \mu)$ is pinned down by Bayes' rule.

(2) Type s_+'s strategy. Type s_+ plays the symmetric MPE strategy under symmetric information as long as his reputation is strictly positive; otherwise he plays the single-player solution. With Assumption 1, the strategy implies he always exerts effort 1 before the separation phase occurs, hence is consistent with our equilibrium prescription.

6 Welfare Analysis

Does inducing information asymmetry, by hiding information from one player, improves welfare? To answer this question, I compare players' ex ante total welfare at the common prior belief q_0 (which, recall, is equal to the initial background belief p_0) in the MPE with gradual revelation:

$$W^U(q_0, \boldsymbol{\mu}^o(q_0)) + \boldsymbol{\mu}^o(q_0) W^{I+}(q_0, \boldsymbol{\mu}^o(q_0)) + (1 - \boldsymbol{\mu}^o(q_0)) W^{I-}(q_0, \boldsymbol{\mu}^o(q_0)), \tag{23}$$

and that in the symmetric MPE of the symmetric information game in which the informed player's private information is made public:

$$2\boldsymbol{\mu}^o(q_0) w^S(\boldsymbol{q}^+(q_0)) + 2(1 - \boldsymbol{\mu}^o(q_0)) w^S(\boldsymbol{q}^-(q_0)). \tag{24}$$

Asymmetric information is said to *improve* welfare if the former is greater than the latter, and *deteriorate* welfare if the former is smaller than the latter.

Thanks to the mutual encouragement effect, asymmetric information creates a benefit: in case I holds signal s_-, players experiment more than in the symmetric information benchmark. Asymmetric information may also incur a cost: in case I holds signal s_+, then during the gradual revelation phase, U experiments less than in the symmetric information benchmark. However, if the informed player's private signal is informative enough, the benefit outweighs the cost, as stated in the following proposition.

Proposition 3. *If the odds ratio is high enough, that is,* $O \in [1 + 2\frac{\lambda}{r}, \infty)$*, then asymmetric information improves welfare, and strictly so if the common prior belief* q_0 *in the gradual revelation region or the pooling region* $(p_2^{*-}, 1)$*.*

Interested readers may refer to Proposition 4 at the end of this section for a detailed welfare characterization when the odds ratio is intermediate. Asymmetric information does not affect welfare if the prior belief q_0 lies in the separation region. This is why in Proposition 3 asymmetric information strictly improves welfare only if q_0 is not in the separation region.

We now elaborate on the intuition behind Proposition 3. Compared with symmetric information, in the asymmetric information game, type s_+ exerts the same level of effort, type s_- more effort; U exerts less effort than in the symmetric MPE when s_+ is public, and more when s_- is public. At the interim stage (right after I learns his type),

(i) type s_+ suffers from asymmetric information because he does not learn as much as he learns from U's experimentation in the symmetric benchmark due to U's lower effort, except when $\rho_b = 0$. When $\rho_b = 0$ (or, a is infinity), type s_+ knows that the risky project is good and hence does not need to learn from U.

(ii) Type s_- (weakly) benefits from asymmetric information because he always has the option to reveal himself by exerting some low effort, whereby he guarantees himself the same payoff as in the symmetric information benchmark.

(iii) U benefits from asymmetric information. U always has the option of matching her effort to I's. Doing so, in case I holds signal s_+, both players would experiment as in the symmetric information setup with s_+ public, whereby U achieves the same ex post continuation value as under symmetric information. In case I holds signal s_-, both players would experiment more than in the symmetric information setup with s_- public, but still less than in the cooperative solution; as a result, U achieves a strictly greater continuation value than under symmetric information. Therefore, under asymmetric information, by taking this effort-matching option, U can guarantee herself a higher interim value (in the MPE with gradual revelation) than in the symmetric benchmark.

Type s_+'s loss from asymmetric information is decreasing in the odds ratio. Intuitively, the greater the belief difference between I's types, the more optimistic type s_+ is during the gradual revelation region, and hence the less he needs to learn from U's experimentation, consequently the less he suffers from asymmetric information. Type s_- and U's gain from asymmetric information is increasing in the odds ratio. Intuitively, the greater the belief difference, the less type s_- needs to stop experimenting to compensate U during the gradual revelation region, and hence the higher probability that the two players continue experimenting over time, implying a greater welfare gain.

At one extreme when the odds ratio O is infinity (that is, if $\rho_b = 0$), type s_+ does not suffer from asymmetric information; asymmetric information thus makes a Pareto improvement. At the other extreme when the odds ratio is equal to O^S, U is willing to experiment at the end of the gradual revelation phase only if she believes I is very likely to be type s_+; as a result, type s_+'s loss dominates, and asymmetric information deteriorates welfare (at least for background belief close to p_2^{*-}). By continuity and the monotonicity of the ex post gains and losses in the odds ratio, there exists a threshold such that, asymmetric information improves ex ante total welfare universally if the odds ratio is above the threshold, and does not if the odds ratio is below the threshold (and if the fraction of the pessimistic type is not too low so that the gradual revelation phase is not empty).

We are thus done with the main message. We now discuss the welfare impact of asymmetric information when the informativeness of the informed player's private signal is intermediate, that is, when $O \in [O^S, 1 + 2\frac{\lambda}{r})$. If the fraction type s_- is not too low so that the gradual revelation phase is not empty, then the pooling phase lasts until the background belief hits the pessimistic type's cooperative cutoff p_2^{*-}. Asymmetric information does not incur any cost and hence makes a Pareto improvement. We now focus on the parameter region where the gradual revelation phase is not empty, which is guaranteed by Assumption 2:

Proposition 4. *Assume that the odds ratio is intermediate: $O \in [O^S, 1 + 2\frac{\lambda}{r})$, and that Assumption 2 is satisfied. Then*

1. *either asymmetric information deteriorates welfare, and strictly so if the common prior q_0 lies in the gradual revelation region or the pooling region $(p_2^{*-}, 1)$;*
2. *or there exists $\tilde{p} \in (p_2^{*-}, p^{S-}]$, such that asymmetric information deteriorates welfare if the common prior q_0 is in $(0, \tilde{p})$, and improves it if q_0 is in $(\tilde{p}, 1)$.*

A sufficient and necessary condition for the second case to occur is ρ_b being either sufficiently low or sufficiently high. Intuitively, when ρ_b is sufficiently low, then there is a large fraction of type s_-, implying the two players' expected gain (occurring only in case I holds signal s_-) is large, relative to their expected loss (occurring only in case I holds signal s_+); when ρ_b is sufficiently high, then there is a small fraction of type s_-, implying a short gradual revelation phase, and hence a small expected loss of the two players (occurring during gradual revelation), relative to the their expected gain (occurring during pooling).

7 Empirical Implications

- On testing "learning from others' experimentation." That players can increase experimentation over time despite the absence of a breakthrough during the gradual revelation phase has important implications for empirical work. Take the example of farmers learning about whether a new seed improves yield. An econometrician unaware of the information asymmetry in the environment may reject "farmers learning from neighbors' experimentation," if he or she finds that farmers' land allocation to the new seed does not positively correlate with neighbors' yield. Such a rejection can be incorrect if information asymmetry is relevant.

  This paper implies that, one can amend the previous test by focusing on positive news of the yield, that is, by testing whether farmers allocate larger land to the new seed in reaction to neighbors' yield increase. This is because, with asymmetric information, although players react to negative news (the absence of a breakthrough) in an ambiguous way, they do react to positive news by experimenting more.

- Divergent learning dynamics of two identical groups of players who receive exactly the same information. Suppose the non-responding region of the gradual revelation phase is not empty ($p_{gr} > p_1^{*-}$) and the prior q_0 is not too low ($q_0 > p_1^{*-}$). Then, conditional on the informed player being the pessimistic type and the risky project being bad, with positive probability the pessimistic type reveals himself at a late time (after the background belief reaches p_1^{*-}), after which, experimentation ends immediately; and with positive probability the pessimistic type reveals himself at an early time (before the background belief reaches p_1^{*-}) and players play the symmetric MPE, in which, free-riding is so severe that they never abandon the bad projects in finite time (it can be shown that players' effort is so low that their posterior belief does not reach the single-player cutoff in finite time).

  Therefore, combining the joint-project interpretation (see page 6), the MPE with gradual revelation predicts that two identical groups of players receiving the same information can exhibit divergent learning dynamics. In one group, the leader leads by example for a long time; as a result, learning is fast, and the joint project is completed or abandoned in finite time. In another group, the leader leads by example for a short time; as a result, players free ride, projects are highly inertial with little learning, and bad projects are not abandoned in finite time. That failing projects of strategic alliances are highly inertial are well documented in the management literature (for instance, Doz, 1996).

- On testing strategic experimentation in labs: It is also possible to test the result of this paper in experiments, by replacing an incumbent player with a new player. Suppose two symmetric players have stopped experimenting. The model in this paper predicts that, replacing one player by a third player would have no impact if the third player observes the whole experimentation history (due to symmetric information). However, it would restart experimentation if the third player can only observe the part of the experimentation history after he enters (due to asymmetric information).

- Further empirical predictions. This paper predicts that experienced players experiment more than inexperienced players do and that their experimentation behavior is less sensitive to unfavorable news[25] or other players' experimentation behavior. These predictions are in line with the empirical findings of Bandiera and Rasul (2006), and Conley and Udry (2010). While such predictions are also compatible with models in which players are myopic and experienced players have more precise information, the following prediction distinguishes these models from the current model: the experimentation behavior of a player with an inexperienced neighbor is less sensitive to negative news than that of a player with a neighbor with similar experienced.

[25]In an exponential bandit model, "no news" is unfavorable news.

8 Multiplicity of Equilibria

8.1 Equilibrium Uniqueness

Not surprisingly, the asymmetric information game has multiple MPEs, due to the arbitrariness of assigning off-equilibrium beliefs in a signaling game, and to the multiplicity (asymmetric) MPEs even under symmetric information (caused by the strategic substitutability of current effort decisions). To select sensible equilibria, I focus on MPEs that survive a criterion in the spirit of the D1 criterion, and that players play the symmetric MPE under symmetric information after the informed player reveals his type on the equilibrium path. For simplicity, call the former restriction D1, and the latter restriction SMPE.

Even so, multiplicity is unavoidable when the belief difference between the two types (or, the odds ratio) is small: during a gradual revelation phase, type s_+'s effort can be lower than 1 and indeterminant, even if he strictly prefers to experiment had he no concern of his reputation. In such cases, type s_+ does not deviate to effort 1 to signal his type, in fear that doings so would make U to free ride more. However, if the belief difference is large enough, type s_+ does not benefit much from U's effort, then D1 can play a role.

Claim 1. *There exists $\bar{O}$, such that if the odds ratio O is greater than $\bar{O}$ and the prior belief q_0 is not low (above p_2^{*-}), then the distribution of the equilibrium path of any MPE satisfying D1 and SMPE coincides with that of the MPE with gradual revelation in Section 5.*

The proof is in Appendix D.1. The intuition behind this result is that, under Assumption **??**, during a gradual revelation phase, if there is no reputation concerns, then type s_+ strictly prefers to experiment, whereas type s_- either strictly prefers not to experiment (over the rewarding region) or is indifferent (over the non-responding region). Therefore, the reason that type s_+ might choose effort lower than 1 in some MPE must be that effort 1 leads to a continuation equilibrium in which, U free-rides in the future. To rule out such possibilities, for a given MPE, we first identify the smallest p (before separation), say $\hat{p}$, around which type s_+'s equilibrium effort is below 1; we then show that the set of reputations (that is, continuation equilibria) making type s_+ strictly benefits from deviating to effort 1 is strictly larger than the set of reputations (that is, continuation equilibria) making type s_- weakly benefits from such a deviation. By D1, whenever I deviates to effort 1 over $[\hat{p}, \hat{p} + \epsilon]$, he should be taken as type s_+. If $\hat{p}$ is not too low, guaranteed by a high odds ratio, type s_+ indeed has a profitable deviation; hence the MPE under consideration cannot survive the D1 criterion.

8.2 MPEs without Assumption 1

If Assumption 1 does not hold, then there are multiple MPEs satisfying D1, SMPE, and close to the MPE with gradual revelation constructed in Section 5. Typically, such an MPE has an additional pooling phase, which occurs between its gradual revelation phase and its separation phase.[26] That is, an MPE has four phases: pooling when p is high; gradual revelation when p is moderately high; pooling when p is moderately low;[27] and separation when p is low.

I focus on MPEs with gradual revelation because it delivers new insights, and qualitatively different behavior and belief dynamics. Moreover, it is useful to construct other equilibria. It represents one extreme where, U's experimentation incentive is maintained through the encouraging private information gradually revealed by the informed player. In another extreme, U's experimentation incentive can be maintained by the informed player gradually reducing his effort (because current effort decisions are strategic substitutes), which can indeed occur in equilibrium if the odds ratio is small. Between these two extremes, it is possible to construct hybrid MPEs, in which, U's experimentation incentive is maintained by the two forces combined together.

[26]I still call the region where both players stop experimenting (that is, for the background beliefs in $[0, p_1^{*+}]$) a separation region, because type s_+ could exert a positive effort at $t = 0$ (right after learning his type) and reveal his type. Such an action is costless in continuous time.

[27]This new pooling phase occurs over a subset of $[\max\{p_2^{*-}, p_1^{*+}\}, p^{S+})$. When Assumption 1 does not hold, a gradual revelation phase may not last until p reaches $\max\{p_2^{*-}, p_1^{*+}\}$. This is because, for gradual revelation to occur, the uninformed player's effort must be interior and strictly lower than the optimistic type's effort; the former implies that both U's effort and the optimistic type's are lower than the symmetric MPE effort under symmetric information s_+, meaning that the optimistic type has strict incentive to experiment. The MPE (with the above-mentioned feature) may not satisfy D1, because the optimistic type has a higher incentive than the pessimistic type to deviate and show that he is indeed a optimistic type, so as to induce both players to work harder.

A Background Belief

This section defines the background belief formally.

Denote $\Omega \equiv \{0,1\} \times \{s_+, s_-\} \times \Omega_N$, where Ω_N is the set of point process paths. Similarly denote $\Omega^t \equiv \{0,1\} \times \{s_+, s_-\} \times \Omega_N^t$, where Ω_N^t is the set of point process paths till time t. Let $\sigma(\{0,1\} \times \{s_+, s_-\})$ be the power set of $\{0,1\} \times \{s_+, s_-\}$, $(\mathcal{F}_t^N)_t$ the filtration generated by the point process N. Define $(\mathcal{F}_t)_t \equiv \sigma(\{0,1\} \times \{s_+, s_-\}) \otimes (\mathcal{F}_t^N)_t$ and $\mathcal{F} \equiv \mathcal{F}_\infty$. For a given prior p_0, an effort path $a \equiv (a_t^I, a_t^U)_{t \geq 0}$ induces a distribution P_{a,p_0} over the filtered space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0})$, satisfying for each $(\theta, s_l, N^t) \in \Omega^t$ where N^t denoting the experimentation outcome history such that no breakthrough has arrived till time t,[28]

$$\begin{aligned} P_{a,p_0}(\theta, s_l, N^t) &= P_{a,p_0}(s_l, N^t|\theta) P_{a,p_0}(\theta) \\ &= P_{a,p_0}(s_l|\theta) P_{a,p_0}(N^t|\theta) P_{a,p_0}(\theta), \end{aligned}$$

where the second inequality is due to the fact that given e and conditional on θ, s_l and N^t are independently distributed. Note that $P_{a,p_0}(\theta) = p_0$, $P_{a,p_0}(s_l|\theta)$ does not depend on e and p_0, and $P_{a,p_0}(N^t|\theta)$ does not depend on p_0.

Given P_{a,p_0}, the distribution of θ conditional on (s_l, N^t), if $\sum_\theta P_{a,p_0}(s_l|\theta) P_{a,p_0}(N^t|\theta) P_{a,p_0}(\theta) > 0$, is

$$P_{a,p_0}(\theta|s_l, N^t) = \frac{P_{a,p_0}(s_l|\theta) P_{a,p_0}(N^t|\theta) P_{a,p_0}(\theta)}{\sum_\theta P_{a,p_0}(s_l|\theta) P_{a,p_0}(N^t|\theta) P_{a,p_0}(\theta)} \tag{25}$$

In the asymmetric information game, $P_{a,p_0}(\theta|s_l, N^t)$ is the probability that type s_l assigns on the risky project's quality being θ, after he observes effort history and and that no breakthrough has occurred up to time t.

(1) We now show that for a given effort path a, this posterior does not depend on whether he observes s_l before N^t or after.

Dividing both the numerator and denominator of the right-hand side of equation (25) by $\sum_{\tilde{\theta}} P_{a,p_0}(s_l|\tilde{\theta}) P_{a,p_0}(\tilde{\theta})$, we have

$$\begin{aligned} P_{a,p_0}(\theta|s_l, N^t) &= \frac{P_{a,p_0}(N^t|\theta)[P_{a,p_0}(s_l|\theta) P_{a,p_0}(\theta) / \sum_{\tilde{\theta}} P_{a,p_0}(s_l|\tilde{\theta}) P_{a,p_0}(\tilde{\theta})]}{\sum_\theta P_{a,p_0}(N^t|\theta)[P_{a,p_0}(s_l|\theta) P_{a,p_0}(\theta) / \sum_{\tilde{\theta}} P_{a,p_0}(s_l|\tilde{\theta}) P_{a,p_0}(\tilde{\theta})]} \\ &= \frac{P_{a,p_0}(N^t|\theta) P_{a,p_0}(\theta|s_l)}{\sum_\theta P_{a,p_0}(N^t|\theta) P_{a,p_0}(\theta|s_l)} \end{aligned}$$

The second equality is by Bayes rule. This equality can be interpreted as follows: after observing the signal s_l, a player (or an outsider) with prior p_0 updates his or her prior to $P_{a,p_0}(\theta|s_l)$ (which is independent of a); then the player observes a path of effort up to time t, a^t, an experimentation result history up to time t, N^t, and he or she updates belief according to Bayes rule, using $P_{a,p_0}(\theta|s_l)$ the new "prior." This is how the informed player in the asymmetric information game updates his belief.

[28] That is, N^t refers to the constant function $0^{[0,t]}$.

Similarly, dividing both the numerator and denominator of the right-hand side of equation (25) by $\sum_{\tilde{\theta}} P_{a,p_0}(N^t|\tilde{\theta})P_{a,p_0}(\tilde{\theta})$, we have

$$\begin{aligned} P_{a,p_0}(\theta|s_l,N^t) &= \frac{P_{a,p_0}(s_l|\theta)[P_{a,p_0}(N^t|\theta)P_{a,p_0}(\theta)/\sum_{\tilde{\theta}} P_{a,p_0}(N^t|\tilde{\theta})P_{a,p_0}(\tilde{\theta})]}{\sum_{\theta} P_{a,p_0}(s_l|\theta)[P_{a,p_0}(N^t|\theta)P_{a,p_0}(\theta)/\sum_{\tilde{\theta}} P_{a,p_0}(N^t|\tilde{\theta})P_{a,p_0}(\tilde{\theta})]} \\ &= \frac{P_{a,p_0}(s_l|\theta)P_{a,p_0}(\theta|N^t)}{\sum_{\theta} P_{a,p_0}(s_l|\theta)P_{a,p_0}(\theta|N^t)}. \end{aligned} \tag{26}$$

This equality can be interpreted as follows: after observing a path of effort up to time t, a^t, an experimentation result history up to time t, N^t, a player (or an outsider) with prior p_0 updates his or her prior to $P_{a,p_0}(\theta|N^t)$; then the player observes the noisy signal s_l, and he or she updates belief according to Bayes rule, using $P_{a,p_0}(\theta|N^t)$ the new "prior."

(2) In the asymmetric information game, any two public histories that lead to the same posterior of type s_- must also lead to the same posterior of type s_+. This is because on the right-hand side of equation (26), $P_{a,p_0}(s_l|\theta)$ is independent of a and p_0, if the left-hand side when s_l replaced by s_- (which represents type s_-'s posterior) is equal to some q^-, then there is a unique value of $P_{a,p_0}(\theta|N^t)$ satisfying equation (26), denoted as p, and hence a unique value of the left-hand side of equation (26) when s_l replaced by s_+ (which represents type s_+'s posterior), denoted as q^+. That is, one variable, be it q^-, q^+, or p, is sufficient to represent the posteriors of the two types of the informed player. This paper uses p, that is, $P_{a,p_0}(\theta|N^t)$, and call it "background belief."

B Some Best Responses

B.1 U's best response to a pooling strategy profile of I

The following lemma analyzes U's best response, when both types of I are prescribed to exert effort 1 over an interval of the back ground beliefs, say $[\underline{p},\bar{p}]$, in the asymmetric information game. Let $\boldsymbol{\mu}(p)$ denote I's reputation at the background belief $p \in [\underline{p},\bar{p}]$, determined by Bayes rule (equation (8) with Y_t being a constant).

Lemma 5. *Assume over some interval $[\underline{p},\bar{p}]$ both types of I are prescribed to exert effort 1 (by a strategy profile of I). Let $\tilde{a}^U : [\underline{p},\bar{p}] \times [0,1] \to [0,1]$ be U's best response, and $\tilde{W}^U : [\underline{p},\bar{p}] \times [0,1] \to \mathbb{R}$ her corresponding continuation value function. Then*

1. *U finds it optimal to use corner solutions, that is, either to exert effort 1, or not to experiment. At any point of p at which U switches actions, U's continuation value satisfies $\tilde{W}^U(p,\boldsymbol{\mu}) - s = s - \lambda^U(p,\boldsymbol{\mu})h$.*
2. *If $\tilde{W}^U(p,\boldsymbol{\mu}) - s > s - \lambda^U(p,\boldsymbol{\mu})h$, then $\tilde{a}^U(p,\boldsymbol{\mu}) = 1$; If $\tilde{W}^U(p,\boldsymbol{\mu}) - s < s - \lambda^U(p,\boldsymbol{\mu})h$, then $\tilde{a}^U(p,\boldsymbol{\mu}) = 0$; If $\tilde{W}^U(p,\boldsymbol{\mu}) - s = s - \lambda^U(p,\boldsymbol{\mu})h$, then $\tilde{a}^U(p,\boldsymbol{\mu}) \in [0,1]$.*
3. *If moreover U's value function satisfies the boundary condition $W^U(\underline{p},\boldsymbol{\mu}) - s = s - \lambda^U(\underline{p},\boldsymbol{\mu})h$, then she finds it optimal to adopt a cutoff strategy: to exert effort 1 if $p \geq p^*$, and not to experiment otherwise, for some $p^* \in [\underline{p},\bar{p}]$.*

4. *If on top of the boundary condition in point 3, at $p = \underline{p}$ is also satisfied*

$$r\left(\lambda^U(p,\boldsymbol{\mu})h - s\right) + \lambda p(1-p)\frac{d\lambda^U(p,\boldsymbol{\mu})}{dp}h + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - s - \left(s - \lambda^U(p,\boldsymbol{\mu})h\right)\right) \geq 0 \quad (27)$$

then U finds it optimal to exert effort 1 over $[\underline{p},\bar{p}]$.

Proof. Point 1. Given I's strategy profile, U's value function $\tilde{W}^U$ satisfies the following HJB equation, for $p \in (\underline{p},\bar{p})$,

$$\begin{aligned} r\tilde{W}^U(p,\boldsymbol{\mu}) &= \max_{a\in[0,1]} a\left[r\left(\lambda^U(p,\boldsymbol{\mu})h - s\right) - \lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right)\right] \\ &\quad + \left[-\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right)\right] + rs \end{aligned}$$

At any state $(p,\boldsymbol{\mu})$ where U is indifferent between experimenting and not experimenting, we have

$$r\left(s - \lambda^U(p,\boldsymbol{\mu})h\right) = -\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right) \quad (28)$$

and consequently the HJB equation of $\tilde{W}^U$ reduces to

$$r\tilde{W}^U(p,\boldsymbol{\mu}) - rs = -\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right)$$

The above equations imply that

$$\tilde{W}^U(p,\boldsymbol{\mu}) - s = s - \lambda^U(p,\boldsymbol{\mu}). \quad (29)$$

Since there is no subinterval of $[\underline{p},\bar{p}]$ over which $\tilde{W}^U$ satisfy equation (28) and (29) simultaneously, there is no subinterval of $[\underline{p},\bar{p}]$ over which player U strictly prefers an interior level of experimentation.

Point 2 is obvious with the above analysis.

Point 3. We will show that $\tilde{W}^U(p,\boldsymbol{\mu})$ intersects with $s + s - \lambda^U(p,\boldsymbol{\mu})$ at most once for $p \in (\underline{p},\bar{p})$. Then combining *Point 2*, we obtain *Point 3*.

To show the former, it is sufficient to show that if there is some $\tilde{p} \in [\underline{p},\bar{p})$ such that $\tilde{W}^U(\tilde{p},\boldsymbol{\mu}) = s + s - \lambda^U(\tilde{p},\boldsymbol{\mu})$, and $\frac{d\tilde{W}^U(\tilde{p}+,\boldsymbol{\mu}(\tilde{p}+))}{dp} > -\frac{d\lambda^U\left(\underline{p},\boldsymbol{\mu}(\underline{p})\right)}{dp}$, then $\tilde{W}^U(p,\boldsymbol{\mu}) - s = s - \lambda^U(p,\boldsymbol{\mu})$ for all $p \in (\tilde{p},\bar{p})$. Suppose by contradiction that there is some $\check{p} \in (\tilde{p},\bar{p})$ such that $\tilde{W}^U(\check{p},\boldsymbol{\mu}) - s = s - \lambda^U(\check{p},\boldsymbol{\mu})$, then we must have $\frac{d\tilde{W}^U(\check{p}-,\boldsymbol{\mu}(\check{p}))}{dp} < \frac{d\tilde{W}^U(\tilde{p}+,\boldsymbol{\mu}(\tilde{p}+))}{dp}$, and $\tilde{W}^U(\check{p},\boldsymbol{\mu}) < \tilde{W}^U(\tilde{p},\boldsymbol{\mu})$ since $s - \lambda^U(p,\boldsymbol{\mu})$ strictly decreases in p. But these inequalities imply that equation (28) cannot be satisfied at both $\tilde{p}$ and $\check{p}$. A contradiction.

Point 4. Condition (27), together with equation (28) and $W^U(\underline{p},\boldsymbol{\mu}) - s = s - \lambda^U(\underline{p},\boldsymbol{\mu})$, implies that $\frac{d\tilde{W}^U\left(\underline{p}+,\boldsymbol{\mu}(\underline{p}+)\right)}{dp} > -\frac{d\lambda^U\left(\underline{p},\boldsymbol{\mu}(\underline{p})\right)}{dp}$, hence by the argument employed to prove *Point 3*, we conclude that $\tilde{a}^U(p,\boldsymbol{\mu}) = 1$ over $(\underline{p},\bar{p}]$.

□

B.2 Best response in the symmetric information game

In the symmetric information game, a counterpart of Lemma 5, with $\boldsymbol{\mu}$ replaced by 0, is valid. More generally, if one player exerts a constant effort over some interval of background belief, then the other player finds it optimal to use corner solutions over this interval, with at most two cutoffs. The proof is similar and hence omitted.

C Equilibrium Construction and Analysis

C.1 Characterization of the gradual revelation phase

C.1.1 U's strategy (proof of the second case of Lemma 2)

We now derive U's strategy during the non-responding region of the gradual revelation phase (that is, for $p \in (p_1^{*-}, p_{gr})$, if nonempty), assuming that the equilibrium with gradual revelation is an equilibrium.

Proof. First, $W^{I-}(p,0)$ satisfies the same HJB equation as equation (12), with the arguments $(p, \hat{\boldsymbol{\mu}}(p))$ in all functions replaced by $(p,0)$. Since for $p \in (p_1^{*-}, p_{gr})$, $a^{I-}(p,0) = a^S(\boldsymbol{q}^-(p)) \in (0,1)$, the terms in equation (12) that are directly affected by type s_-'s effort must be 0:

$$\left[r\left(\lambda^{I-}(p)\,h - s\right) - \lambda p\,(1-p)\,\frac{dW^{I-}(p,0)}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}(p,0)\right)\right] = 0.$$

As $\lambda^{I-}(p)\,h - s < 0$ for $p \in (p_1^{*-}, p_{gr})$, we also have

$$\left[-\lambda p\,(1-p)\,\frac{dW^{I-}(p,0)}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}(p,0)\right)\right] > 0.$$

During the gradual revelation phase, type s_- is indifferent between revealing and not revealing, and hence $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = W^{I-}(p,0)$. This equality, together with both $W^{I-}(p, \hat{\boldsymbol{\mu}}(p))$ and $W^{I-}(p,0)$ satisfying the HJB equation (12), implies that $a^U(p, \hat{\boldsymbol{\mu}}(p)) = a^U(p,0)$ over (p_1^{*-}, p_{gr}). Since by construction $a^U(p,0) = a^S(\boldsymbol{q}^-(p))$, we have $a^U(p, \hat{\boldsymbol{\mu}}(p)) = a^S(\boldsymbol{q}^-(p))$. Therefore, the uninformed player's effort is decreasing over time during the non-rewarding region. □

C.1.2 U's HJB equation and experimentation incentive

We here derive heuristically U's value function during the gradual revelation phase, equation (16).

Proof. Suppose at time-t state $(p_t, \hat{\boldsymbol{\mu}}(p_t))$, player U considers the following strategy: experimenting with resource $\tilde{a}$ during the time interval $[t, t+dt)$, experimenting with her equilibrium effort $a^U(p_t, \hat{\boldsymbol{\mu}}(p_t))$ during the time interval $[t+dt, t+2dt)$ if player I does not reveal over $[t, t+dt)$, and playing according to the candidate equilibrium strategy at other states.

The flow continuation value of doing so, $r(W^U(p, \hat{\boldsymbol{\mu}}) - s) \cdot 2dt$, should equal the right-hand side of equation (16): the expected instantaneous payoff in the $2dt$ duration of time,

$$r\left(\tilde{a} + a^U(p, \hat{\boldsymbol{\mu}})\right)\left(\lambda^U(p, \hat{\boldsymbol{\mu}})h - s\right)dt,$$

plus the value of information,

$$E[W(p_{t+2dt}, \hat{\boldsymbol{\mu}}(p_{t+2dt})) - W(p_t, \hat{\boldsymbol{\mu}}(p_t))],$$

which can be further decomposed into three parts. The first two parts resembles that in equation (12), that is, the change in her continuation value in case no breakthrough arrives and type s_- does not reveal his type in $[t, t+2dt)$, and the change in her continuation value in case a breakthrough arrives in $[t, t+2dt)$, multiplied by the probability of each event respectively. The third part comes from the possibility that type s_- reveals his type in $[t, t+2dt)$ in the absence of a breakthrough, an event that would cause her continuation value to reduce by an amount $|W^U(p, 0) - W^U(p, \hat{\boldsymbol{\mu}})|$.

It is crucial to note that type s_-'s revealing rate over $[t, t+dt)$, denoted as y, does not depend on U's effort level $\tilde{a}$ over $[t, t+dt)$, while type s_-'s revealing rate over $[t, t+2dt)$, denoted as $\tilde{y}$, does. We now analyze the latter effect. Conditional on type s_- not revealing his type in the interval $[t, t+dt)$, the state at $t+dt$ before players move, will evolve to

$$\left(p_t - (1+\tilde{a})\lambda p(1-p)dt, \hat{\boldsymbol{\mu}}(p_t) - \hat{\boldsymbol{\mu}}_p(1+\tilde{a})\lambda p_t(1-p_t)dt\right),$$

which is below the curve $\hat{\boldsymbol{\mu}}$. Therefore, at time $t+dt$, according to the equilibrium prescription, type s_- will reveal with a probability such that the action of non-revealing will push the state up to the curve $\hat{\boldsymbol{\mu}}$ again, implying that the new state at $t+2dt$ will be

$$\left(p_t - \left(1+\tilde{a}+1+a^U(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t(1-p_t)dt, \hat{\boldsymbol{\mu}}\left(p_t - \left(1+\tilde{a}+1+a^U(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t(1-p_t)dt\right)\right).$$

Ignoring higher order terms of dt, the amount of adjustment in state variable in this $2dt$ duration of time equals to

$$\left(-\left(1+\tilde{a}+1+a^U(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t(1-p_t)dt, -\hat{\boldsymbol{\mu}}_p\left(1+\tilde{a}+1+a^U(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t(1-p_t)dt\right).$$

Employing equation (14), the amount of adjustment implies that in this $2dt$ duration of time, type s_- would reveal his type with probability

$$(y+\tilde{y})dt = \left(\phi(p, \mu) - \frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\right)\left(1+\tilde{a}+1+a^U(p_t, \hat{\boldsymbol{\mu}})\right)\frac{p(1-p)\lambda}{(1-\hat{\boldsymbol{\mu}})}dt.$$

Rearranging, we obtain

$$\begin{aligned} 2r(W^U(p, \hat{\boldsymbol{\mu}}) - s) &= \max_{\tilde{a}\in[0,1]} \tilde{a}\left[r\left(\lambda^U(p, \hat{\boldsymbol{\mu}})h - s\right) + A(p, \hat{\boldsymbol{\mu}})\right] + A(p, \hat{\mu}) \\ &\quad + a^U(p, \hat{\boldsymbol{\mu}})\left[r\left(\lambda^U(p, \hat{\boldsymbol{\mu}})h - s\right) + A(p, \hat{\mu})\right] + A(p, \hat{\boldsymbol{\mu}}) \end{aligned}$$

This equation is the same with equation (16), since $a^U(p,\hat{\mu})$ is a solution to the maximization problem.

During gradual revelation phase, U's equilibrium effort is interior (except at $p = p_1^{*-}$). Therefore, her IC condition (17) must hold.

This IC condition, together with the HJB equation (16), implies that player U's value function also satisfies equation (18). □

C.1.3 The gradual revelation path $\hat{\boldsymbol{\mu}}$ (proof of Lemma 3)

Formula of g.

For simplicity, define two functions B and C by

$$\begin{aligned} B(p,\mu) &= \left(s-\lambda^U(p,\mu)\,h\right)-\left(s-\lambda^{I-}(p)\,h\right)a^{I-}(p,0)\,, \\ C(p,\mu) &= r\left(\lambda^U(p,\mu)\,h-s\right)+\lambda p\,(1-p)\,\lambda_p^U(p,\mu)\,h+\lambda^U(p,\mu)\left(\lambda h-s-\left(s-\lambda^U(p,\mu)\,h\right)\right), \end{aligned}$$

where B is U's continuation value drop caused by type s_-'s revelation, and C could be interpreted as U's marginal benefit from experimentation excluding the part obtained from the private information revealed by I. Then, using U's indifference condition (17) and her value function (18), we have

$$\begin{aligned} \hat{\boldsymbol{\mu}}_p &= g(p,\hat{\boldsymbol{\mu}}) \\ &\equiv -\left(\frac{C(p,\hat{\boldsymbol{\mu}})}{\lambda p\,(1-p)\,B(p,0)}-\phi(p,\hat{\boldsymbol{\mu}})\,\frac{B(p,\hat{\boldsymbol{\mu}})}{B(p,0)}\right)\hat{\boldsymbol{\mu}},\quad p\in\left(p_2^{*-},p_{gr}\right) \end{aligned} \tag{30}$$

where

$$\begin{aligned} \phi(p,\mu) &\equiv \frac{(1-\mu)\,(\rho_g-\rho_b)}{\boldsymbol{\mu}^o(p)\,(1-\boldsymbol{\mu}^o(p))} \\ &= \frac{(1-\mu)\left(\lambda^{I+}(p)-\lambda^{I-}(p)\right)}{\lambda p\,(1-p)} \\ &= \frac{\lambda^{I+}(p)-\lambda^U(p,\mu)}{\lambda p\,(1-p)}. \end{aligned} \tag{31}$$

Before proving Lemma 3, we derive some preliminary results. We first derive some convenient formula for for $\hat{\boldsymbol{\mu}}_p/\hat{\boldsymbol{\mu}}$ and for $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp$, which will be used in this section and the following sections. In Lemma 6, we show that $\hat{\boldsymbol{\mu}}$ is a strictly decreasing function. Lemma 7 shows that the ODE problem defined by equations (19)-(21) has a unique solution.

We now establish equalities (32) to (35):

$$\begin{aligned} -\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}} &= \frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})\,h/dp}{B(p,\hat{\boldsymbol{\mu}})}+\frac{1}{\lambda p\,(1-p)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})\,h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right. \\ &\quad \left.+\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I+}(p)\right] \end{aligned} \tag{32}$$

$$-\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp} = \frac{\lambda^U_\mu \hat{\boldsymbol{\mu}} B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right.$$
$$\left.+\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda+\lambda^{I-}(p)-\frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b}\lambda+\lambda^{I-}(p)\right)\right] \tag{33}$$
$$= \frac{\lambda^U_\mu \hat{\boldsymbol{\mu}} B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right.$$
$$\left.+\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda+\lambda^{I-}(p)-\frac{\left(\lambda-\lambda^{I-}(p)\right)\lambda^{I-}(p)}{\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I-}(p)}\right] \tag{34}$$
$$= \frac{\lambda^U_\mu \hat{\boldsymbol{\mu}} B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right.$$
$$\left.-\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I-}(p)}\right]. \tag{35}$$

Combining equations (19) and (31), at p such that $\hat{\boldsymbol{\mu}}(p)\neq 0$, we have

$$-\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}} = \frac{C(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}-\phi(p,\hat{\boldsymbol{\mu}})\frac{B(p,\hat{\boldsymbol{\mu}})}{B(p,0)}$$
$$= \frac{\lambda^U_p(p,\hat{\boldsymbol{\mu}})h}{B(p,0)}+\frac{B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right.$$
$$\left.+\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I+}(p)\right] \tag{36}$$

Applying equalities $d\lambda^U/dp = \lambda^U_p + \lambda^U_\mu \hat{\boldsymbol{\mu}}_p$ and $B(p,0) = B(p,\hat{\boldsymbol{\mu}}) + \lambda^U_\mu \hat{\boldsymbol{\mu}} h$, we arrive at equation (32).

We now derive an explicit formula of $d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp$. Before this, we need an explicit form of $\frac{\lambda^U_p}{\lambda^U_\mu \hat{\boldsymbol{\mu}}}$.

By the definition of $\lambda^{I+}(p)$ and $\lambda^{I-}(p)$, we have

$$\lambda^{I+}(p)-\lambda^{I-}(p) = \frac{\lambda p(1-p)\left(\frac{\rho_b}{\rho_g}-\frac{\rho_b}{\rho_g}\right)}{\left(p+(1-p)\frac{\rho_b}{\rho_g}\right)\left(p+(1-p)\frac{\rho_b}{\rho_g}\right)}$$

Expanding λ_p^U, λ_μ^U, and apply the above equation, we have

$$
\begin{aligned}
\frac{\lambda_p^U}{\lambda_\mu^U \hat{\boldsymbol{\mu}}} &= \frac{\lambda_p^U}{\hat{\boldsymbol{\mu}}\left(\lambda^{I+}(p) - \lambda^{I-}(p)\right)} \\
&= \frac{\rho_g(1-\rho_g)}{p(1-p)(\rho_g-\rho_b)}\left[\frac{p+(1-p)\frac{\rho_b}{\rho_g}}{p+(1-p)\frac{\rho_b}{\rho_g}}\frac{\rho_b}{\rho_g} + \left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\frac{p+(1-p)\frac{\rho_b}{\rho_g}}{p+(1-p)\frac{\rho_b}{\rho_g}}\frac{\rho_b}{\rho_g}\right] \\
&= \frac{\lambda}{\lambda p(1-p)}\left[\left(\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b} - \frac{\lambda^{I+}(p)}{\lambda}\right) + \left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\left(\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b} + \frac{\lambda^{I-}(p)}{\lambda}\right)\right] \\
&= \frac{\lambda}{\lambda p(1-p)}\left[\left(1 - \frac{\lambda^{I+}(p)}{\lambda}\right) + \frac{1}{\hat{\boldsymbol{\mu}}}\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b} + \left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\frac{\lambda^{I-}(p)}{\lambda}\right] \qquad (37)
\end{aligned}
$$

Subtracting $\frac{\lambda_p^U}{\lambda_\mu^U \hat{\boldsymbol{\mu}}} + \frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp}{B(p,\hat{\boldsymbol{\mu}})}$ from both sides of equation (32), and using equation (37), we have

$$
\begin{aligned}
& -\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp}\frac{B(p,0)}{\lambda_\mu^U \hat{\boldsymbol{\mu}} B(p,\hat{\boldsymbol{\mu}})} \\
= \; & \frac{1}{\lambda p(1-p)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h - s\right) + \lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - s - s + \lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})} + \lambda^U(p,\hat{\boldsymbol{\mu}}) - \lambda^{I+}(p)\right. \\
& \left. -\lambda + \lambda^{I+}(p) + \lambda^{I-}(p) - \frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b}\lambda + \lambda^{I-}(p)\right)\right]
\end{aligned}
$$

Equation (33) follows from this equation immediately. Using the following equality, which is obtained after some algebra,

$$
\frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_b(1-\rho_g)}{\rho_g-\rho_b}\lambda + \lambda^{I-}(p)\right) = \frac{\left(\lambda - \lambda^{I-}(p)\right)\lambda^{I-}(p)}{\lambda^U(p,\hat{\boldsymbol{\mu}}) - \lambda^{I-}(p)},
$$

we obtain equality (34). Rearranging terms, we have equality (35).

Before proving existence of solution to the ODE problem (19)-(21), we will show that $\hat{\boldsymbol{\mu}}$ satisfying equation (19) has some a priori bound (if we impose some conditions that are necessary for the candidate equilibrium to be an equilibrium). Lemma 6 is a useful step towards this. Also, note that $B(p^{S-},0) = 0$ (since $a^S \circ \boldsymbol{q}^-(p^{S-}) = 1$), and $B(p,0) > 0$ for $p \in [p_2^{*-}, p^{S-})$, hence we will treat the point p^{S-} with care.

Lemma 6. *Let $\alpha \in (p_2^{*-}, p^{S-})$ and $\hat{\boldsymbol{\mu}}|_{[p_2^{*-},\alpha)}$ be a solution to the ODE problem defined by (19) restricted over $[p_2^{*-}, \alpha]$ and the initial condition (20). If $\hat{\boldsymbol{\mu}} \in (0,1)$ and $B(p,\hat{\boldsymbol{\mu}}) > 0$ over (p_2^{*-}, α), then $\hat{\boldsymbol{\mu}}_p < 0$ over (p_2^{*-}, α).*

Proof. Define

$$
D(p,\hat{\boldsymbol{\mu}}) \equiv \frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h - s\right) + \lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - s - s + \lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})} + \lambda^U(p,\hat{\boldsymbol{\mu}}) - \lambda. \qquad (38)
$$

Note that $d\lambda^U(p, \hat{\boldsymbol{\mu}})/dp \leq 0$, implies $\hat{\boldsymbol{\mu}}_p < 0$. Therefore, if $D(p, \hat{\boldsymbol{\mu}}) > 0$ on a Gradual Revelation path, then Lemma 6 would follow. We now show $D(p, \hat{\boldsymbol{\mu}}) > 0$.

Observe that, (i) if $d\lambda^U(p, \hat{\boldsymbol{\mu}})/dp \leq 0$, then $D(p, \hat{\boldsymbol{\mu}}) > 0$, from equality (34) and the definition of D; (ii) if $d\lambda^U(p, \hat{\boldsymbol{\mu}})/dp > 0$, then $B(p, \hat{\boldsymbol{\mu}}(p))$ strictly decreases as p increases, and hence $D(p, \hat{\boldsymbol{\mu}})$ strictly increases as p increases. Therefore, if we show $D(p, \hat{\boldsymbol{\mu}}) > 0$ at $p = p_2^{*-}$, then by continuity of D and the above two observations, we have $D(p, \hat{\boldsymbol{\mu}}) > 0$ for $p \in [p_2^{*-}, p_{gr})$.

From now till the end of this proof, if not mentioned, p is fixed at p_2^{*-}. Using the initial condition (20), at $p = p_2^{*-}$, we have

$$\frac{\lambda^U(p, \hat{\boldsymbol{\mu}})(\lambda h - s)}{s - \lambda^U(p, \hat{\boldsymbol{\mu}})h} = \frac{s\left(\lambda^{I+}(p) - \lambda^{I-}(p)\right) + \lambda^{I-}(p)\left(w^S(\boldsymbol{q}+(p)) - s\right)}{(s - \lambda^{I-}(p)h)(w^S(\boldsymbol{q}+(p)) - s)}(\lambda h - s)$$

Applying the definition of D, and the fact that $a^S \circ \boldsymbol{q}^-(p) = 0$ at $p = p_2^{*-}$, we obtain

$$D(p, \hat{\boldsymbol{\mu}}) + \lambda - \lambda^{I+}(p) = \frac{1}{s - \lambda^{I-}(p)h}\left[s\left(\lambda^{I+}(p) - \lambda^{I-}(p)\right)\frac{\lambda h - w^S(\boldsymbol{q}+(p))}{w^S(\boldsymbol{q}+(p)) - s} + \lambda^{I-}(p)\left(\lambda^{I+}(p) - \lambda h\right)\right]$$

Employing the definition of p_2^{*-} (equation (1)) and of λ^{I-}, the above equality becomes

$$D(p, \hat{\boldsymbol{\mu}}) + \lambda - \lambda^{I+}(p) = \frac{-w^S(\boldsymbol{q}+(p))\left(r + 2\lambda^{I+}(p)\right) + (2\lambda + r)\lambda^{I+}(p)h}{2(w^S(\boldsymbol{q}+(p)) - s)} \tag{39}$$

By Assumption 1, $p_2^{*-} \geq p^{S+}$, we have $a^S(\boldsymbol{q}+(p)) = 1$ at $p = p_2^{*-}$. Recall w^S is the continuation value function corresponding to the symmetric MPE under symmetric information, which is a function of the *true posterior*, rather than the *background belief*, that is, $w^S(\boldsymbol{q}^+(p)) = W^{S+}(p)$ (with $\boldsymbol{q}^+$ defined in equation (3)).

Since in the symmetric MPE in the symmetric information setup, both players experiment with full resource if their common posterior is above q^S, w^S satisfies the following HJB equation for $\boldsymbol{q}^+(p) > q^S$,

$$rw^S\left(\boldsymbol{q}^+\right) - rs = r\left(\lambda\boldsymbol{q}^+h - s\right) - 2\lambda\boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right)w_q^S\left(\boldsymbol{q}^+\right) + 2\lambda\boldsymbol{q}^+\left(\lambda h - w^S\left(\boldsymbol{q}^+\right)\right),$$

where the argument of $\boldsymbol{q}^+$ is omitted.

Rearranging terms, we have

$$-w^S\left(\boldsymbol{q}^+\right)\left(r + 2\lambda\boldsymbol{q}^+\right) + (2\lambda + r)\lambda\boldsymbol{q}^+h = 2\lambda\boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right)w_q^S\left(\boldsymbol{q}^+\right). \tag{40}$$

With this equation, equation (39) becomes

$$D(p, \hat{\boldsymbol{\mu}}) + \lambda - \lambda^{I+}(p) = \frac{\lambda\boldsymbol{q}^+(1 - \boldsymbol{q}^+)w_q^S(\boldsymbol{q}^+)}{(w^S(\boldsymbol{q}^+) - s)}. \tag{41}$$

Therefore,

$$\begin{aligned}
D\left(p,\hat{\boldsymbol{\mu}}\right) &= \frac{\lambda\boldsymbol{q}^{+}\left(1-\boldsymbol{q}^{+}\right)w_q^S\left(\boldsymbol{q}^{+}\right)}{\left(w^S\left(\boldsymbol{q}^{+}\right)-s\right)}-\left(1-\boldsymbol{q}^{+}\right)\lambda \\
&= \left(1-\boldsymbol{q}^{+}\right)\lambda\frac{\boldsymbol{q}^{+}w_q^S\left(\boldsymbol{q}^{+}\right)-\left(w^S\left(\boldsymbol{q}^{+}\right)-s\right)}{\left(w^S\left(\boldsymbol{q}^{+}\right)-s\right)} \qquad (42)\\
&> \left(1-\boldsymbol{q}^{+}\right)\lambda\frac{\left(\boldsymbol{q}^{+}-q_1^*\right)w_q^S\left(\boldsymbol{q}^{+}\right)-\left(w^S\left(\boldsymbol{q}^{+}\right)-s\right)}{\left(w^S\left(\boldsymbol{q}^{+}\right)-s\right)} \\
&> 0 \qquad (43)
\end{aligned}$$

The second-to-last inequality is due to $w_q^S(\boldsymbol{q}^{+})>0$ at $p=p_2^{*-}$; the last inequality is due to the convexity of w^S over $[q_1^*,1]$, and that $w^S\left(q_1^*\right)=s$. □

Lemma 7. *If $\hat{\boldsymbol{\mu}}(p_2^{*-})$ defined by (20) is such that $\hat{\boldsymbol{\mu}}(p_2^{*-})<\boldsymbol{\mu}^o(p_2^{*-})$, then the ODE problem defined by (19)-(21) has a unique solution $\hat{\boldsymbol{\mu}}$. Moreover, the right boundary p_{gr} is in (p_2^{*-},p^{S-}).*

Proof. Let $\epsilon\in(0,p^{S-})$ be such that

$$s-\lambda^U\left(p^{S-}-\epsilon,\boldsymbol{\mu}^o\left(p^{S-}-\epsilon\right)\right)h=W^{S-}\left(p^{S-}-\epsilon\right)-s.$$

Such an ϵ exists because both sides of the above equation are continuous in ϵ, the left-hand side is strictly greater than the right-hand side at $\epsilon=p^{S-}$, and strictly smaller than the latter at $\epsilon=0$. As $\epsilon>0$, we have $a^{I-}(p,0)<1$ over $[p_2^{*-},p^{S-}-\epsilon]$,[29] and hence $B\left(p,0\right)$ is bounded for $p\in[p_2^{*-},p^{S-}-\epsilon]$. $C(p,\hat{\boldsymbol{\mu}})$, $\phi(p,\hat{\boldsymbol{\mu}})$, and $B(p,\hat{\boldsymbol{\mu}})$ are also bounded for $p\in[p_2^{*-},p^{S-}-\epsilon]$ and $\hat{\boldsymbol{\mu}}\in[0,1]$. To restrict $\hat{\boldsymbol{\mu}}$ to take values in $[0,1]$, define

$$\chi(\mu)=\begin{cases}\mu, & \text{if } \mu\in[0,1];\\ 0, & \text{if } \mu<0;\\ 1, & \text{if } \mu>1.\end{cases}$$

Existence. Consider the following initial value problem

$$\hat{\boldsymbol{\mu}}_p=-\left(\frac{C\left(p,\chi\left(\hat{\boldsymbol{\mu}}\right)\right)}{\lambda p\left(1-p\right)B\left(p,0\right)}-\phi\left(p,\chi\left(\hat{\boldsymbol{\mu}}\right)\right)\frac{B\left(p,\chi\left(\hat{\boldsymbol{\mu}}\right)\right)}{B\left(p,0\right)}\right)\chi\left(\hat{\boldsymbol{\mu}}\right), \qquad p\in(p_2^{*-},p^{S-}-\epsilon), \qquad (44)$$

with the initial condition (20). $\hat{\boldsymbol{\mu}}_p$ as a function of $(p,\hat{\boldsymbol{\mu}})$, defined by equation (44), is bounded, and Lipschitz continuous. According to standard theorems (for example, Picard–Lindelöf theorem), this initial value problem has a unique solution, denoted as as $\hat{\boldsymbol{\mu}}$ (with an abuse of notation). Let $p_{gr}\equiv\inf\{p\in[p_2^{*-},p^{S-}-\epsilon]\mid\hat{\boldsymbol{\mu}}(p)>\boldsymbol{\mu}^o(p)\}$.

We will show at the end of this proof that $\hat{\boldsymbol{\mu}}$ satisfies

Claim 2. *Over the interval (p_2^{*-},p_{gr}), we have $\hat{\boldsymbol{\mu}}\in(0,1)$, and $B\left(p,\hat{\boldsymbol{\mu}}\right)>0$.*

[29] Recall that over $[p_2^{*-},p^{S-}]$, $a^{I-}\left(p,0\right)\equiv a^S\circ\boldsymbol{q}^{-}(p)$, and is strictly increasing from 0 to 1.

Claim 3. $\hat{\boldsymbol{\mu}}\left(p^{S-}-\epsilon\right)<\boldsymbol{\mu}^{o}\left(p^{S-}-\epsilon\right)$.

Claim 2 says that $\hat{\boldsymbol{\mu}}$, the unique solution to ODE (44) and (20), satisfies ODE (19) over $[p_2^{*-}, p_{gr}]$. Claim 3, together with $\hat{\boldsymbol{\mu}}\left(p_2^{*-}\right)>\boldsymbol{\mu}^{o}\left(p_2^{*-}\right)$ and the continuity of $\hat{\boldsymbol{\mu}}$ and $\boldsymbol{\mu}^o$, implies that $p_{gr} \in (p_2^{*-}, p^{S-}-\epsilon)$.

Therefore, $\hat{\boldsymbol{\mu}}$ restricted over $[p_2^{*-}, p_{gr}]$ is a solution to the ODE problem (19)-(21).

Uniqueness. $\hat{\boldsymbol{\mu}}$ restricted over $[p_2^{*-}, p_{gr}]$ is the unique solution to this ODE problem. Suppose this ODE problem has another solution $\check{\boldsymbol{\mu}}$, and let $\check{p} \in [p_2^{*-}, p_{gr}]$ be the infimum of p such that $\check{\boldsymbol{\mu}}$ differs from $\hat{\boldsymbol{\mu}}$. Then we have $\check{\boldsymbol{\mu}}(\check{p})=\hat{\boldsymbol{\mu}}(\check{p}) \in (0,1)$, and $\check{\boldsymbol{\mu}} \in (0,1)$ over $[\check{p}, \check{p}+\eta]$ for some small η. Hence $\check{\boldsymbol{\mu}}|_{[}\check{p}, \check{p}+\eta]$ is a solution to the ODE problem (44) restricted over $[\check{p}, \check{p}+\eta]$ with the initial value given by $\check{\boldsymbol{\mu}}(\check{p})=\hat{\boldsymbol{\mu}}(\check{p})$, which contradicted with the latter having a unique solution.

We now prove the two claims above.

Proof of Claim 3. Suppose $\hat{\boldsymbol{\mu}}\left(p^{S-}-\epsilon\right) \geq \boldsymbol{\mu}^{o}\left(p^{S-}-\epsilon\right)$, then $W^U(p^{S-}-\epsilon, \hat{\boldsymbol{\mu}}(p^{S-}-\epsilon))$ defined by equation (18) is smaller than $W^{S-}(p^{S-}-\epsilon)$ by our choice of ϵ, then using the equation that $B(p, \hat{\boldsymbol{\mu}})=W^U(p, \hat{\boldsymbol{\mu}})-W^{S-}(p)$, we have $B(p^{S-}-\epsilon, \hat{\boldsymbol{\mu}}(p^{S-}-\epsilon)) \leq 0$. This contradicts with Claim 2. □

Proof of Claim 2. Suppose by negation that there is some $p \in [p_2^{*-}, p_{gr}]$ such that $B\left(p, \hat{\boldsymbol{\mu}}\right) \leq 0$; denote the smallest p satisfying this inequality as $\tilde{p}$. Since $B\left(p, \hat{\boldsymbol{\mu}}\right)>0$ at $p=p_2^{*-}$ and B is continuous, we have $\tilde{p}>p_2^{*-}$, and $B\left(p, \hat{\boldsymbol{\mu}}\right)>0$ for $p \in [p_2^{*-}, \tilde{p})$.

$\hat{\boldsymbol{\mu}}$ is strictly decreasing over $[p_2^{*-}, \tilde{p})$. Because otherwise, there would exist a $\tilde{\tilde{p}} \in (p_2^{*-}, \tilde{p})$ such that $\hat{\boldsymbol{\mu}}_p(\tilde{\tilde{p}})=0$,[30] implying that $\hat{\boldsymbol{\mu}}|_{[p_2^{*-}, \tilde{\tilde{p}})}$ satisfies ODE (19) when restricted over $[p_2^{*-}, \tilde{\tilde{p}})$, and the initial condition (20), and yet it violates Lemma 6, a contradiction. Therefore, $\chi(\hat{\boldsymbol{\mu}})=\hat{\boldsymbol{\mu}}$ for $p \in [p_2^{*-}, \tilde{p}]$, hence $\hat{\boldsymbol{\mu}}$ also satisfies ODE (19). But the function W^U defined by $s+s-\lambda^U(\hat{\boldsymbol{\mu}}, p)h$ (that is, equation (18)) would not satisfy equation (17) (because the left-hand side>0 for p sufficiently close to $\tilde{p}$), which contradicts with $\hat{\boldsymbol{\mu}}$ satisfying ODE (19). □

□

Proof of Lemma 3. First, $\hat{\boldsymbol{\mu}}$ is continuous over (p_2^{*-}, p_{gr}). Suppose by negation that there is some $\tilde{p} \in (p_2^{*-}, p_{gr})$ at which $\hat{\boldsymbol{\mu}}$ is discontinuous, that is, $\hat{\boldsymbol{\mu}}(\tilde{p}+)<\hat{\boldsymbol{\mu}}(\tilde{p}-)$.[31] Since over a small right neighborhood of $\tilde{p}$, player U is indifferent between experimenting and not experimenting, we must have $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))=s+s-\lambda^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))h$, by equation (18). Similarly, we have $W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))=s+s-\lambda^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))h$. Since λ^U strictly increases in its second argument, these two inequalities imply that $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))>W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))$. But this contradicts with the fact that $W^U \geq s$, as $W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))$ is the average between $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))$ and s.

Similarly, $\hat{\boldsymbol{\mu}}$ is continuous at p_{gr}. The difference between this case and the previous case is that, over a small right neighborhood of p_{gr}, player U strictly prefers to experiment,

[30] $\tilde{\tilde{p}}>p_2^{*-}$ because $\hat{\boldsymbol{\mu}}(p_2^{*-})<0$.

[31] Note in our candidate equilibrium, $\hat{\boldsymbol{\mu}}$ is discontinuous if and only if type s_- reveals with a lump-sum probability, hence at any p, $\hat{\boldsymbol{\mu}}$ can only jump downward.

and type s_- strictly prefers not to reveal. Therefore, by Point 2 of Lemma 5, we have $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) \geq s + s - \lambda^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))h$. Continuity of $\hat{\boldsymbol{\mu}}$ at p_{gr} follows the same logic as in the previous case.

Finally, we show that $\hat{\boldsymbol{\mu}}$ does not have singular continuous part. Let $\tilde{\boldsymbol{\mu}}$ be the solution to the ODE problem (19)-(21), and let $\tilde{W}^U$ be player U's continuation value function corresponding to the equilibrium associated with Gradual Revelation path $\tilde{\boldsymbol{\mu}}$.[32] Suppose by negation there is another Gradual Revelation path $\check{\boldsymbol{\mu}}$ (corresponding to another equilibrium which takes the same feature with the candidate equilibrium) that satisfies equation (19) almost everywhere, equations (20), and (21), and that $\tilde{\boldsymbol{\mu}}(p')$ differs from $\check{\boldsymbol{\mu}}(p')$ for some $p' \in (p_2^{*-}, p_{gr})$. Without loss of generality, suppose $\tilde{\boldsymbol{\mu}}(p') > \check{\boldsymbol{\mu}}(p')$. Let p'' be the largest p's such that $p \leq p'$ and that $\tilde{\boldsymbol{\mu}}(p) \geq \check{\boldsymbol{\mu}}(p)$. Existence of p'' is due to the continuity of $\tilde{\boldsymbol{\mu}}$ and $\check{\boldsymbol{\mu}}$, and that $\tilde{\boldsymbol{\mu}}(p_2^{*-}) = \check{\boldsymbol{\mu}}(p_2^{*-})$ (from the initial condition (20)). By the definition of p'', we have $\limsup_{\epsilon \downarrow 0} \frac{\check{\boldsymbol{\mu}}(p''+\epsilon)-\check{\boldsymbol{\mu}}(p'')}{\epsilon} < \tilde{\boldsymbol{\mu}}_p(p'')$. Let $\check{W}^U$ be U's continuation value function of the equilibrium with Gradual Revelation path $\check{\boldsymbol{\mu}}$. Then $\tilde{\boldsymbol{\mu}}(p) > \check{\boldsymbol{\mu}}(p)$ over (p'', p') implies that $\check{W}^U(p, \check{\boldsymbol{\mu}}) > \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$. We now show that $\tilde{\boldsymbol{\mu}}(p) > \check{\boldsymbol{\mu}}(p)$ over (p'', p') and that $\tilde{\boldsymbol{\mu}}(p'') = \check{\boldsymbol{\mu}}(p'')$ imply that $\check{W}^U(p, \check{\boldsymbol{\mu}}) < \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$ for $p \in (p'', p'' + \epsilon_1)$, if ϵ_1 small enough. A contradiction.

Fix a small $\epsilon > 0$, we change the strategies of type s_- and of player U in the equilibrium associated with Gradual Revelation path $\check{\boldsymbol{\mu}}$ as follows: starting at $(p'' + \epsilon, \check{\boldsymbol{\mu}}(p'' + \epsilon))$, type s_- does not reveal his type as long as background belief is in $(p'', p'' + \epsilon]$; at the background belief p'', he reveals with a lump-sum probability such that his reputation jumps to $\check{\boldsymbol{\mu}}(p'')$; at all other states, he plays his equilibrium strategy associated with Gradual Revelation path $\check{\boldsymbol{\mu}}$. U plays a best response to I's new strategy (, which is 0 effort). Denote U's continuation value corresponding to this new strategy profile as $\check{\check{W}}^U$, which will be written as simply a function of the background p, for ease of notation. As type s_- works harder in this new strategy profile, we have $\check{\check{W}}^U \geq \check{W}^U$ at $p = p'' + \epsilon$.[33] At $p = p''$, according to the new prescription, type s_- will reveal with probability $\frac{\check{Y}(0)-\check{Y}(\epsilon)}{1-\check{Y}(\epsilon)}$,[34]where $\check{Y}$ solves

$$\check{\boldsymbol{\mu}}(p'' + \eta) = \frac{\boldsymbol{\mu}^o(p'' + \eta)}{\boldsymbol{\mu}^o(p'' + \eta) + [1 - \boldsymbol{\mu}^o(p'' + \eta)]\left(1 - \check{Y}(\eta)\right)}, \tag{45}$$

for $\eta = 0, \epsilon$. That is, $\check{Y}(\epsilon)$ is type s_-'s cumulative revelation probability giving him a reputation $\check{\boldsymbol{\mu}}(p'' + \epsilon)$, when the background belief is $p'' + \epsilon$. Similarly we can define $\check{Y}(0)$, $\tilde{Y}(\epsilon)$, and $\tilde{Y}(0)$.

Since according to the new strategy of type s_-, he will reveal with a lump-sum probability at p'', we have

$$\check{\check{W}}^U(p''+) = \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)} s + (1 - \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}) \check{W}^U(p'', \check{\boldsymbol{\mu}}(p'')). \tag{46}$$

[32]Need result from the Verification section showing that this is indeed an equilibrium.

[33] That is, $\check{\check{W}}^U(p'' + \epsilon) \geq \check{W}^U(p'' + \epsilon, \check{\boldsymbol{\mu}}(p'' + \epsilon))$.

[34] Note with type s_-'s new revealing strategy, $\check{Y}(\epsilon)$ will be a constant when background belief is in $(p'', p'' + \epsilon]$.

Let t_ϵ denote the time it takes for the background belief to drop from $p'' + \epsilon$ to p''. By using equation (46), and the fact that U's best response to I's pooling strategy over $(p'', p'' + \epsilon)$ is 0 effort, we have

$$\check{\check{W}}^U(p'' + \epsilon) = (rs + \lambda^U \lambda h)t_\epsilon + (1 - rt_\epsilon - \lambda^U t_\epsilon)(\frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}s + (1 - \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)})\check{W}^U(p''))\tag{47}$$

$(rs + \lambda^U \lambda h)t_\epsilon$ is the "flow" value of both players' effort to player U in the t_ϵ duration of time: as U does not experiment, she receives rst_ϵ from her own arm; player I experiments with full resource, hence to player U, good news will arrive with probability $\lambda^U t_\epsilon$, leading to a discounted value λh.

Now coming back to the equilibrium with Gradual Revelation Path $\tilde{\boldsymbol{\mu}}$. Since U is indifferent between experimenting and not experimenting as long as no revealing and $p \in (p'', p''+\epsilon)$, not experimenting is an optimal strategy for U. Hence we have

$$\tilde{W}^U(p'' + \epsilon) = (rs + \lambda^U \lambda h)t_\epsilon + (1 - rt_\epsilon - \lambda^U t_\epsilon)(\frac{\tilde{Y}(0) - \tilde{Y}(\epsilon)}{1 - \tilde{Y}(\epsilon)}s + (1 - \frac{\tilde{Y}(0) - \tilde{Y}(\epsilon)}{1 - \tilde{Y}(\epsilon)})\tilde{W}^U(p'') + o(t_\epsilon)\tag{48}$$

where we use the fact that during t_ϵ duration of time, type s_-'s expected effort differs from 1 with probability $\frac{\tilde{Y}(0)-\tilde{Y}(\epsilon)}{1-\tilde{Y}(\epsilon)}$, which is of order t_ϵ (since by assumption $\tilde{\boldsymbol{\mu}}$ is absolutely continuous), causing a difference in U's flow payoff during this t_ϵ time of second order of t_ϵ.

Since $\tilde{\boldsymbol{\mu}}(p) > \check{\boldsymbol{\mu}}(p)$ over (p'', p'), and $\tilde{\boldsymbol{\mu}}(p'') = \check{\boldsymbol{\mu}}(p'')$, we have $\tilde{Y}(0) = \check{Y}(0)$, and $\tilde{Y}(\epsilon) > \check{Y}(\epsilon)$, hence

$$\frac{\tilde{Y}(0) - \tilde{Y}(\epsilon)}{1 - \tilde{Y}(\epsilon)} < \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}.$$

Recall we also have $\check{W}^U(p, \check{\boldsymbol{\mu}}) = \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$ at $p = p''$. Therefore, by equations (47) and (48), we have $\check{\check{W}}^U(p'' + \epsilon) < \tilde{W}^U(p'' + \epsilon)$. As $\check{\check{W}}^U(p'' + \epsilon) \geq \check{W}^U(p'' + \epsilon)$, we also have $\check{W}^U(p'' + \epsilon) < \tilde{W}^U(p'' + \epsilon)$.

Since this inequality holds for all small $\epsilon > 0$, we conclude that $\check{W}^U(p, \check{\boldsymbol{\mu}}) < \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$ for $p \in (p'', p'' + \epsilon_1)$, if ϵ_1 small enough.

□

C.2 Verification (proof of Lemma 4)

As the argument for type s_+'s incentive compatibility only depends on whether the stage is separation or not, we first analyze his incentive. We then check type s_-'s and player U's incentive stage by stage, backwardly.

At the separation region ($p \leq p_2^{*-}$), if the state is $(p, 1)$, type s_+ has no incentive to deviate because both him and the uninformed player play the symmetric MPE with public information s_+. If the state is $(p, 0)$, then the uninformed player does not experiment, hence it is optimal for type s_+ to adopt the individually optimal solution.

Before separation ($p > p_2^{*-}$), type s_+'s incentive to follow the prescribed equilibrium strategy is implied by the following lemma, since in the prescribed equilibrium, there is no $p > p_1^{*-}$ such that $a^{I-}(p, 0) = 0$ and hence no $p > p_1^{*-}$ such that $a^{I+}(p, 0) = 0$.

Lemma 8. *Let a strategy profile (a^{I+}, a^{I-}, a^U) and a belief system satisfy the following conditions: (i) a^{I+} is pure; (ii) before separation, for any $p > p_1^{*-}$ such that $W^{I+}(p,0) = W^{I+}(p,\mu)$ and $a^{I+}(p,0) = 0$, we have $a^{I+}(p,\mu) = 0$; (iii) a^U increases in μ; (iv) the belief system satisfies (**??**). Then at any state (p,μ) that is not separation, type s_+ has no incentive to deviate to effort lower than $a^{I+}(p,\mu)$.*

Proof. Let a strategy profile and a belief system satisfy the conditions in Lemma 8. Before separation, as long as good news does not arrive and type s_- plays a^{I+}, the informed agent's reputation at t, when the time-t background belief is at p_t, will be $\tilde{\boldsymbol{\mu}}(p_t)$ for some function $\tilde{\boldsymbol{\mu}}$.

Suppose Lemma 8 does not hold. That is, there is some p' such that type s_+ finds it optimal to deviate to a lower effort than equilibrium effort. According to condition (iv), immediately after this deviation type s_+'s reputation will be 0, hence the highest continuation value he can get is $W^{I+}(p',0)$, which is obtained from type s_+ playing a best response to $a^U(\cdot,0)$. Therefore, at p', $W^{I+}(p',0) > W^{I+}(p',\tilde{\boldsymbol{\mu}}(p'))$. Since $W^{I+}(p,0) \le W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))$ at the background belief where separation occurs, and both $W^{I+}(p,0)$ and $W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))$ are continuous in p, there exists some $p'' \in (0,p')$ (before separation) such that $W^{I+}(p'',0) = W^{I+}(p'',\tilde{\boldsymbol{\mu}}(p''))$, and that $\frac{dW^{I+}(p''+,0)}{dp} > \frac{dW^{I+}(p''+,\tilde{\boldsymbol{\mu}}(p''+))}{dp}$.

Due to condition (iv), that is, a reputation once lost is lost forever, $W^{I+}(p,0)$ satisfies the following HJB equation:

$$\begin{aligned} & rW^{I+}(p,0) \\ = \; & \max_{a\in[0,1]} a\left[r\left(\lambda^{I+}(p)h - s\right) - \lambda p(1-p)\frac{dW^{I+}(p,0)}{dp} + \lambda^{I+}(p)\left(\lambda h - W^{I+}(p,0)\right)\right] \\ & + a^U(p,0)\left[-\lambda p(1-p)\frac{dW^{I+}(p,0)}{dp} + \lambda^{I+}(p)\left(\lambda h - W^{I+}(p,0)\right)\right] + rs \end{aligned}$$

$W^{I+}(p,\tilde{\boldsymbol{\mu}})$ satisfies the following HJB equation:

$$\begin{aligned} & rW^{I+}(p,\tilde{\boldsymbol{\mu}}(p)) \\ = \; & a^{I+}(p,\tilde{\boldsymbol{\mu}}(p))\left[r\left(\lambda^{I+}(p)h - s\right) - \lambda p(1-p)\frac{dW^{I+}(p,\tilde{\boldsymbol{\mu}}(p))}{dp} + \lambda^{I+}(p)\left(\lambda h - W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))\right)\right] \\ & + a^U(p,\tilde{\boldsymbol{\mu}}(p))\left[-\lambda p(1-p)\frac{dW^{I+}(p,\tilde{\boldsymbol{\mu}}(p))}{dp} + \lambda^{I+}(p)\left(\lambda h - W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))\right)\right] + rs \end{aligned}$$

If $a^{I+}(p'',0) > 0$, then by $W^{I+}(p'',0) = W^{I+}(p'',\tilde{\boldsymbol{\mu}}(p''))$, and that $\frac{dW^{I+}(p''+,0)}{dp} > \frac{dW^{I+}(p''+,\tilde{\boldsymbol{\mu}}(p''+))}{dp}$, we must have $a^{I+}(p'',\tilde{\boldsymbol{\mu}}) = 1$. But these inequalities, together with condition (iii), contradict with the two HJB equations above. Therefore, $a^{I+}(p'',0) = 0$, which implies $a^{I+}(p'',\tilde{\boldsymbol{\mu}}) = 0$ by condition (ii). But again, the inequalities $W^{I+}(p'',0) = W^{I+}(p'',\tilde{\boldsymbol{\mu}}(p''))$, that $\frac{dW^{I+}(p''+,0)}{dp} > \frac{dW^{I+}(p''+,\tilde{\boldsymbol{\mu}}(p''+))}{dp}$, and condition (iii), contradict with the two HJB equations above. □

We now check type s_-'s and U's incentive to deviate.

(1) separation ($p \leq p_2^{*-}$). The nontrivial sub-case is when $p_1^{*+} < p \leq p_2^{*-}$, and $\mu = \boldsymbol{\mu}^o(p)$.

Type s_- has no incentive to deviate because, given the updating rule and that player U will choose the same action as he does (because they will play the symmetric MPE corresponding to the information revealed by player I's action), the tradeoff of working or not he faces is exactly the same with that a two-player team with public information s_- faces: in both cases, working has the same current flow payoff and twice the "capital gain" as working alone.Since a two-player team finds it optimal to stop if $p < p_2^{*-}$, so does player I of type s_-.

The uninformed player has no incentive to deviate because the informed player already perfectly reveals his type and both play the symmetric MPE in the symmetric information game thereafter.

(2) Gradual revelation ($p_2^{*-} < p < p_{gr}$).

The state is along $\hat{\boldsymbol{\mu}}$. Note that given player U's equilibrium strategy and the belief system, whatever deviation that player I plans to employ, the state will be on $\hat{\boldsymbol{\mu}}$ as long as player I hasn't revealed himself. Likewise, given player I's equilibrium strategy, whatever deviation that player U uses, the state will be on $\hat{\boldsymbol{\mu}}$ as long as player I hasn't revealed himself.

We have shown in the previous step that type s_-'s value at (p_2^{*-}, μ) is at most s. Then in the interval $[p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]$, given the construction of f, type s_- has no incentive to deviate, and that $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = s$. In the interval $[\min\{p_1^{*-}, p_{gr}\}, p_{gr}]$ (if nonempty), player U's effort along $\hat{\boldsymbol{\mu}}$ is the same as in the symmetric MPE with public information s_-, since type s_- is indifferent between experimenting and not experimenting in the symmetric MPE, he is also indifferent between experimenting (and hence not revealing) and $a^S(\boldsymbol{q}^-(p)) \in (0,1)$ (and hence revealing) along $\hat{\boldsymbol{\mu}}$ in the asymmetric information game.

That the uninformed player has no incentive to deviate along $\hat{\boldsymbol{\mu}}$ follows from the following two lemmas.

Lemma 9. *Suppose in the candidate equilibrium constructed in Section §**??**, during gradual revelation, the uninformed player's effort is as in equation* (**??**)*, and that type s_-'s revealing rate y is such that, the associated revealing path $\hat{\boldsymbol{\mu}}$ is a solution to the ODE problem defined by* (19)*,* (20)*, and* (21)*. The player U's value function during gradual revelation is given by equation* (18)*.*

Proof. By our equilibrium construction, trivially, $W^U(p_2^{*-}, \hat{\boldsymbol{\mu}}(p_2^{*-}))$ satisfies equation (18). Now suppose that there is some $p' \in (p_2^{*-}, p_{gr})$ such that equation (18) does not hold at p. Without loss of generality, assume $W^U(p', \hat{\boldsymbol{\mu}}) - s > s - \lambda^U(p', \hat{\boldsymbol{\mu}})\, h$. Then there must exist an interval of p, subset of (p_2^{*-}, p_{gr}), over which $W^U(p, \hat{\boldsymbol{\mu}}) - s > s - \lambda^U(p, \hat{\boldsymbol{\mu}})\, h$, and $\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp} > \frac{-\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}$. As W^U satisfies the HJB equation (16), these two inequalities imply that $W^U(p, \hat{\boldsymbol{\mu}}) - s < s - \lambda^U(p, \hat{\boldsymbol{\mu}})\, h$ for all p in this interval. A contradiction. □

Along $\hat{\boldsymbol{\mu}}$, the revealing strategy of type s_- guarantees the local incentive of player U, that is, if $\hat{\boldsymbol{\mu}}$ is a solution to the ODE problem defined by (19), (20), and (21), and if W^U is given by equation (18) (by Lemma 9), then player U's local incentive condition (17) is satisfied. The following lemma shows that player U does not have a profitable global deviation.

Lemma 10. *Suppose the informed player's strategy and the belief system is as in Section §5, and that type s_-'s revealing rate y is such that, the associated revealing path $\hat{\boldsymbol{\mu}}$ is a solution to the ODE problem defined by* (19), (20), *and* (21), *then the uninformed player has no incentive to deviate.*

Proof. Suppose player U has some profitable deviation $\tilde{a}$ starting at some state $(p', \hat{\boldsymbol{\mu}}(p'))$, $p' \in (p_2^{*-}, p_{gr})$, that gives her continuation value $\tilde{W}^U(p', \hat{\boldsymbol{\mu}}(p')) > W^U(p', \hat{\boldsymbol{\mu}}(p'))$. $\tilde{W}^U$ satisfies the HJB equation (16) with a^U replaced by her deviating action. Since $\tilde{W}^U \leq W^U$ at $(p_2^{*-}, \hat{\boldsymbol{\mu}}(p_2^{*-}))$, there must exist some $p'' \in (p_2^{*-}, p')$ such that $\tilde{W}^U(p'', \hat{\boldsymbol{\mu}}(p'')) > W^U(p'', \hat{\boldsymbol{\mu}}(p''))$, and $\frac{d\tilde{W}^U(p'', \hat{\boldsymbol{\mu}}(p''))}{dp} > \frac{dW^U(p'', \hat{\boldsymbol{\mu}}(p''))}{dp}$, but these inequalities, together with the HJB equation (16), imply that $\tilde{W}^U(p'', \hat{\boldsymbol{\mu}}(p'')) < W^U(p'', \hat{\boldsymbol{\mu}}(p''))$. A contradiction. □

The state is along $\boldsymbol{\mu}^o$. If type s_- deviates, whether the deviation starts today or not, he would either reveals himself, or the state variable jumps on the curve $\hat{\boldsymbol{\mu}}$; in both cases, type s_- at most obtains continuation value $W^{S-}(p)$, which is his value of following the equilibrium strategy. Therefore, he has no incentive to deviate.

(3) pooling ($p > p_{gr}$). Type s_- has no incentive to deviate: on the one hand, player U works harder when type s_- has not revealed himself than when he has revealed already; on the other, one optimal continuation strategy of type s_- after revelation, that is, experimenting with full resource for $p > p_{gr}$, is the same with his equilibrium strategy of not revealing; since type s_- benefits from player U's effort and he will have the same continuation value at p_{gr} whether he deviates now or not, he strictly prefers not to deviate.

The uninformed player has no incentive to deviate because according to Point 4 in Lemma 5, if player U's continuation value at p_{gr} is as specified by equation (18), and that $C(p_{gr}, \hat{\boldsymbol{\mu}}(p_{gr})) > 0$, which is shown by step 1 in Lemma 7, then player U finds it optimal to experiment with full resource for $p > p_{gr}$. (Here we replace the $\underline{p}$ in Lemma 5 by p_{gr}.)

C.3 Dynamics

C.3.1 U's belief about the risky project (proof of Proposition 1)

Proposition 1 is a result of Lemma 11 and Lemma 12.

Lemma 11. *There exists $\tilde{O} \in (1, \infty)$, such that, if $O > \tilde{O}$, then $d\lambda^U(p, \hat{\boldsymbol{\mu}}(p))/dp|_{p=p_2^{*-}} > 0$; if $O < \tilde{O}$, then $d\lambda^U(p, \hat{\boldsymbol{\mu}}(p))/dp|_{p=p_2^{*-}} < 0$.*

Proof. Let $\boldsymbol{q}_2^{*+} : [1,\infty) \to [0,1]$ be defined by $\boldsymbol{q}_2^{*+}(O) = \frac{1}{1+(\frac{1}{q_2^*}-1)\frac{1}{O}}$, for $O \in [1,\infty)$, where $\boldsymbol{q}_2^{*+}(O)$ refers to type s_+'s posterior about the risky project, when type s_-'s posterior is q_2^* and the odd ratio is O. Let $\hat{\boldsymbol{\mu}}^*(O)$ denote the initial value of $\hat{\boldsymbol{\mu}}$ at p_2^{*-}, implied by condition (20), when the odd ratio is a. (The notations $\boldsymbol{q}_2^{*+}$ and $\hat{\boldsymbol{\mu}}^*$ are only used in this proof.)

Using equation (33), (38) (the definition of function D) and (42), we have, $d\lambda^U(p+, \hat{\boldsymbol{\mu}}(p+))/dp|_{p=p_2^{*-}} < 0$ if and only if at $p = p_2^{*-}$,

$$\lambda\left(1-\boldsymbol{q}_2^{*+}(O)\right)\frac{\left[w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)\boldsymbol{q}_2^{*+}(O)-\left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s\right)\right]}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s}$$
$$-\frac{\lambda}{\hat{\boldsymbol{\mu}}^*(O)}\frac{\rho_g(1-\rho_b)}{\rho_g-\rho_b}-\left(\frac{1}{\hat{\boldsymbol{\mu}}^*(O)}-1\right)\lambda^{I-}(p) \;>\; 0. \tag{49}$$

Define the following functions:

$$\hat{D}_1(q) \equiv \lambda(1-q)\frac{\left(W_q^S(q)\,q-\left(w^S(q)-s\right)\right)}{w^S(q)-s}, \tag{50}$$

$$\hat{D}_2(O) \equiv -\frac{\lambda}{\hat{\boldsymbol{\mu}}^*(O)}\frac{1}{O-1}-\left(\frac{1}{\hat{\boldsymbol{\mu}}^*(O)}-1\right)\lambda q_2^*. \tag{51}$$

Then the left-hand side of inequality (49) equals to

$$\hat{D}(O) \equiv \hat{D}_1\left(\boldsymbol{q}_2^{*+}(O)\right)+\hat{D}_2(O). \tag{52}$$

We now show that there is a unique $\tilde{O} \in (1,\infty)$, such that $\hat{D}(O) > 0$ for $O < \tilde{O}$, and $\hat{D}(O) < 0$ for $O > \tilde{O}$, which completes the proof of Lemma 11. The former statement follows directly from the two claims below:

Claim 4. *$\hat{D}(O)$ strictly decreases in O.*

Claim 5. *$\hat{D}\left(O^S\right) > 0$ for the O^S such that $\boldsymbol{q}_2^{*+}\left(O^S\right) = q^S$ (or equivalently, $p_2^{*-} = p^{S+}$); and* $\lim_{O\to\infty}\hat{D}(O) < 0$.

Recall that we construct equilibrium for $p_2^{*-} \geq p^{S+}$, which is equivalent with $O \in [O^S,\infty)$. By continuity of $\hat{D}$, Lemma 5 says that there is a threshold $\tilde{O}$ such that $\hat{D}(O) > 0$ for $O \in (O^S,\tilde{O})$, and $\hat{D}(O) < 0$ for $O \in (\tilde{O},\infty)$.

Proof of Claim 4. If we show $\hat{D}_1(q)$ strictly decreases in q, and $\hat{D}_2(O)$ strictly decreases in O, then, since $\boldsymbol{q}_2^{*+}(O)$ strictly increases in O, we would have $\hat{D}(O)$ strictly decreases in O.

(i) $\hat{D}_1(q)$ strictly decreases in q.

Replacing w_q^S in the expression of $\hat{D}_1$ by equation (40), we have

$$\hat{D}_1(q) = \lambda\left(\frac{q\left(\left(\frac{r}{2\lambda}+1\right)\lambda h-s\right)-\frac{r}{2\lambda}s}{w^S(q)-s}-\left(\frac{r}{2\lambda}+1\right)\right).$$

Taking derivative with respect to q and rearranging terms, we have

$$\begin{aligned}
\frac{d\hat{D}_1(q)}{dq} &= \lambda \frac{\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)\left(w^S(q) - s - q w_q^S(q)\right) + w_q^S(q)\frac{r}{2\lambda}s}{\left(w^S(q)-s\right)^2} \\
&= \lambda \frac{\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)\left(w^S(q) - s - (q - q_2^*)\, w_q^S(q)\right)}{\left(w^S(q)-s\right)^2} \\
&< 0,
\end{aligned}$$

where the second inequality uses equality $\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right) q_2^* = \frac{r}{2\lambda}s$; the third uses the convexity of w^S and that $w^S(q_2^*) = s$.

(ii) $\hat{D}_2(O)$ strictly decreases in O.

Recall the definition of $\hat{D}_2$, we have

$$\hat{D}_2(O) = -\frac{\lambda}{\hat{\boldsymbol{\mu}}^*(O)} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{O}{O-1} + \lambda q_2^*$$

Taking derivative with respect to O, and using the value of $\hat{\boldsymbol{\mu}}^*(O)$ (by equation (20)), we have

$$\begin{aligned}
& -\frac{1}{\lambda}\frac{d\hat{D}_2(O)}{dO} \\
= \; & \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) + \lambda h \boldsymbol{q}_2^{*+}(O) - \left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s\right) - \lambda h\left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right)}{s - \lambda q_2^* h} \frac{q_2^*}{\left(\boldsymbol{q}_2^{*+}(O)\right)^2} \frac{O}{O-1} \frac{d\boldsymbol{q}_2^{*+}(C}{dO} \\
& - \frac{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right)}{s - \lambda q_2^* h} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{1}{(O-1)^2} \\
= \; & \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) - \left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s\right) + \lambda h q_2^*}{s - \lambda q_2^* h} \frac{1}{O(O-1)} (1 - q_2^*) \\
& - \frac{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right)}{s - \lambda q_2^* h} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{1}{(O-1)^2} \\
= \; & \frac{(q - q_2^*)(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) - \left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s\right) + q_2^* w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) + \lambda h q_2^*}{s - \lambda q_2^* h} \frac{1}{O(O-1)} (1 - q_2^*) \\
& - \frac{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right)}{s - \lambda q_2^* h} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{1}{(O-1)^2}
\end{aligned}$$

Define $\hat{D}_3 \equiv \left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right) w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) - \left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s\right)$. Since $\frac{O-1}{O}(1 - q_2^*) = 1 -$

$\frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)}$ and $\hat{D}_3 > 0$, we have

$$\begin{aligned}
& -\frac{1}{\lambda}\frac{d\hat{D}_2(O)}{dO} \\
= \quad & \frac{\hat{D}_3 \boldsymbol{q}_2^{*+}(O)/q_2^* + \boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) + \lambda h \boldsymbol{q}_2^{*+}(O)}{s - \lambda q_2^* h} \frac{1}{O(O-1)} \frac{q_2^*(1-q_2^*)}{\boldsymbol{q}_2^{*+}(O)} \\
& -\frac{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right)}{s - \lambda q_2^* h} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{1}{(O-1)^2} \\
= \quad & \frac{1}{s - \lambda q_2^* h} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{1}{(O-1)^2}\left[\left(\hat{D}_3 \frac{\boldsymbol{q}_2^{*+}(O)}{q_2^*} + \left(\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right) + \lambda h \boldsymbol{q}_2^{*+}(O)\right)\right)\left(1 - \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)}\right)\right. \\
& \left. - \left(w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(O) - q_2^*\right)\right)\right] \\
= \quad & \frac{1}{s - \lambda q_2^* h} \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)} \frac{1}{(O-1)^2}\left[\hat{D}_3 \frac{\boldsymbol{q}_2^{*+}(O)}{q_2^*}\left(1 - \frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)}\right) + \hat{D}_3\right] \\
= \quad & \frac{1}{s - \lambda q_2^* h} \frac{1}{(O-1)^2} \hat{D}_3 \\
> \quad & 0
\end{aligned}$$

□

Proof of Claim 5. As $O \to \infty$, we have $\boldsymbol{q}_2^{*+}(O) \to 1$, and $\lim_{O\to\infty} \hat{\boldsymbol{\mu}}^*(O) \in (0,1)$ by the initial condition. Therefore, as $O \to \infty$, we have $\hat{D}_1 \to 0$, $\hat{D}_2 \to -\left(\lim_{O\to\infty} \frac{1}{\hat{\boldsymbol{\mu}}^*(O)} - 1\right)\lambda q_2^* < 0$.

If O is such that $\boldsymbol{q}_2^{*+}(O) = q^S$, then we have $\hat{\boldsymbol{\mu}}^*(O) = 1$ by the initial condition. Therefore

$$\begin{aligned}
\frac{1}{\lambda}\hat{D}\left(\boldsymbol{q}_2^{*+}(O)\right) &= \frac{1}{\lambda}\hat{D}_1\left(\boldsymbol{q}_2^{*+}(O)\right) - \frac{1}{O-1} \\
&= \left(1 - \boldsymbol{q}_2^{*+}(O)\right)\frac{\hat{D}_3}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s} + \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s}\frac{1 - \boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)} q_2^* - \frac{1}{O-1}
\end{aligned} \tag{53}$$

If we show that

$$\frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s}\frac{1 - \boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)} q_2^* - \frac{1}{O-1} > \frac{1}{O}\left[\frac{1}{O-1} - \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s}\frac{1 - \boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)} q_2^*\right], \tag{54}$$

then we have $\frac{\boldsymbol{q}_2^{*+}(O) w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right) - s}\frac{1-\boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)} q_2^* - \frac{1}{O-1} > 0$. This inequality, together with $\hat{D}_3 > 0$ and equation (53), imply $\hat{D} > 0$.

We now show inequality (54). First, by the definition of $\boldsymbol{q}_2^{*+}$, we have $\frac{1-\boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)}q_2^* = \frac{1}{O}\left(1-q_2^*\right)$. Then,

$$\begin{aligned}
&\frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)}q_2^* - \frac{1}{O-1} \\
= & \left(\frac{\left(\boldsymbol{q}_2^{*+}(O)-q_2^*\right) w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s} + \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(O)}\right)\frac{1}{O}\left(1-q_2^*\right) - \frac{1}{O-1} \\
= & \left(\frac{\left(\boldsymbol{q}_2^{*+}(O)-q_2^*\right) w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s} + \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)}q_2^*\right)\frac{1}{O} - \frac{1}{O-1} \\
> & \left[\frac{1}{O-1} - \frac{\boldsymbol{q}_2^{*+}(O)\, w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(O)}{\boldsymbol{q}_2^{*+}(O)}q_2^*\right]\frac{1}{O}.
\end{aligned}$$

The last inequality uses $\frac{\left(\boldsymbol{q}_2^{*+}(O)-q_2^*\right)w_q^S\left(\boldsymbol{q}_2^{*+}(O)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(O)\right)-s} > 1$, and $\frac{1}{O}-\frac{1}{O-1}=\frac{1}{O(O-1)}$. □

□

Lemma 12. *For any $p \in \left(p_2^{*-}, p_{gr}\right)$ such that $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}=0$, we have $\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp^2}<0$.*

Proof. First, simple algebra gives us equality $B(p,0) = B(p,\hat{\boldsymbol{\mu}}) + \hat{\boldsymbol{\mu}}\lambda_\mu^U(p,\hat{\boldsymbol{\mu}})$, and $\hat{\boldsymbol{\mu}}\lambda_\mu^U(p,\hat{\boldsymbol{\mu}}) = \lambda^U(p,\hat{\boldsymbol{\mu}}) - \lambda^{I-}(p)$. Recall that $\boldsymbol{q}^-(p)$ is defined as type s_-'s posterior about the risky project when the background belief is p, and that $\lambda^{I-}(p) = \boldsymbol{q}^-(p)\,\lambda$.

Suppose there exists some $p' \in \left(p_2^{*-}, p_{gr}\right)$ such that $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}=0$. Taking derivative on both sides of equality (35) with respect to p, at the $p=p'$, and applying $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}=0$, we have

$$\begin{aligned}
&-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0) \\
= & \frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\Bigg[-\lambda\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - s - s + \lambda^U(p,\hat{\boldsymbol{\mu}})h\right)\right) \\
&-\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})\right)\frac{dB(p,\hat{\boldsymbol{\mu}})}{d\boldsymbol{q}^-}\Bigg]
\end{aligned}$$

Since $B(p,\hat{\boldsymbol{\mu}}) = s + s - \lambda^U(p,\hat{\boldsymbol{\mu}}) - w^S(\boldsymbol{q}^-)$, and $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}=0$ at $p=p'$, we have at $p=p'$,

$$\begin{aligned}
&-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0) \\
= & \frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\Bigg[-\lambda\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - w^S\left(\boldsymbol{q}^-\right) - B(p,\hat{\boldsymbol{\mu}})\right)\right) \\
&+\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\lambda\boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)w_q^S\left(\boldsymbol{q}^-\right)\Bigg]
\end{aligned}$$

If $p' \in [p_2^{*-}, p_1^{*-}]$, then $w^S(\boldsymbol{q}^-(p')) = s$ and hence $w_q^S(\boldsymbol{q}^-(p')) = 0$. Apply inequality (43) and the definition of function D, we have $-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}|_{p=p'} < 0$.

If $p' \in (p_1^{*-}, p_{gr}]$, (which is possible only if $(p_1^{*-}, p_{gr}]$ is nonempty,) then by the fact that $a^S(\boldsymbol{q}^-(p')) \in (0,1)$ (that is, in the symmetric MPE under symmetric information, a player is indifferent between experimenting and not experimenting, given that the other player plays the MPE strategy), we have

$$\lambda\boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)w_q^S\left(\boldsymbol{q}^-\right) = r\left(\lambda\boldsymbol{q}^- h - s\right) + \lambda\boldsymbol{q}^-\left(\lambda h - W^S\left(\boldsymbol{q}^-\right)\right).$$

Using this equation to get rid of w^S and w_q^S, we have

$$\begin{aligned}
& -\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0) \\
= & \frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\left[\left(\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})}{\lambda\boldsymbol{q}^-}-1\right)\lambda rs + \lambda\lambda^U(p,\hat{\boldsymbol{\mu}})B(p,\hat{\boldsymbol{\mu}})\right. \\
& \left. + \frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\left(r\left(\lambda\boldsymbol{q}^- h - s\right) + \lambda\boldsymbol{q}^-\left(\lambda h - w^S\left(\boldsymbol{q}^-\right)\right)\right)\right] \\
= & \frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\left[-\frac{\lambda - \lambda^U(p,\hat{\boldsymbol{\mu}})}{1-\boldsymbol{q}^-}\left(\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})}{\lambda\boldsymbol{q}^-}-1\right)rs\right] \qquad (55) \\
< & \ 0. \qquad (56)
\end{aligned}$$

The last inequality uses the fact that the right-hand side of equation (35) equals to 0 when $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp = 0$ (and some algebra).[35] □

C.3.2 The revealing rate of type s_-

Lemma 13. *In the gradual revelation phase, for $p \in (p_2^{*-}, p_1^{*-})$ such that $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp < 0$, the revealing rate of the informed player $(1-\hat{\boldsymbol{\mu}})y(p,\hat{\boldsymbol{\mu}})$ strictly decreases in p.*

[35] To obtain equality (55), notice that

$$\begin{aligned}
& \frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\left(r\left(\lambda\boldsymbol{q}^- h - s\right) + \lambda\boldsymbol{q}^-\left(\lambda h - w^S\left(\boldsymbol{q}^-\right)\right)\right) \\
= & \frac{\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{(1-\boldsymbol{q}^-)}\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h - s\right) + \lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - w^S\left(\boldsymbol{q}^-\right)\right)\right) + \frac{\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)^2}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}rs
\end{aligned}$$

Using the fact that the right-hand side of equation (35) equals to 0 when $d\lambda^U/dp = 0$, the right-hand side of the equation above equals to

$$-\lambda\lambda^U(p,\hat{\boldsymbol{\mu}})B(p,\hat{\boldsymbol{\mu}}) + \frac{\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)^2}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}rs.$$

The equality (55) follows.

Proof. Applying equation (14) and U's indifference condition (17), we have, in gradual revelation phase,

$$\begin{aligned} & \frac{(1-\hat{\boldsymbol{\mu}})\, y\,(p,\hat{\boldsymbol{\mu}})\, B\,(p,\hat{\boldsymbol{\mu}})}{a^{I+}\,(p,\hat{\boldsymbol{\mu}})+a^{U}\,(p,\hat{\boldsymbol{\mu}})} \\ = \;& r\left(\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\, h-s\right)+\lambda p\,(1-p)\, h d\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\,/dp+\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-\left(s-\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\, h\right)\right) \end{aligned} \tag{57}$$

Using equation (35) to replace $d\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\,/dp$, we have

$$\begin{aligned} & \frac{(1-\hat{\boldsymbol{\mu}})\, y\,(p,\hat{\boldsymbol{\mu}})\, B\,(p,\hat{\boldsymbol{\mu}})}{a^{I+}\,(p,\hat{\boldsymbol{\mu}})+a^{U}\,(p,\hat{\boldsymbol{\mu}})} \\ = \;& \left(r\left(\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\, h-s\right)+2\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\,(\lambda h-s)\right)\frac{B\,(p,\hat{\boldsymbol{\mu}})}{B\,(p,0)} \end{aligned} \tag{58}$$

Using equality $a^{I+}\,(p,\hat{\boldsymbol{\mu}})=1$ and rearranging terms, we have

$$(1-\hat{\boldsymbol{\mu}})\, y\,(p,\hat{\boldsymbol{\mu}})\, B\,(p,0) \;=\; \left(1+a^{U}\,(p,\hat{\boldsymbol{\mu}})\right)\left(r\left(\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\, h-s\right)+2\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\,(\lambda h-s)\right)$$

For $p\in(p_2^{*-},p_1^{*-})$, we have $a^{U}=f(p)=\frac{r(s-\lambda^{I-}(p)h)}{\lambda^{I-}(p)(\lambda h-s)}-1$; also, $B(p,0)=s-\lambda^{I-}\,(p)\, h$. Therefore,

$$(1-\hat{\boldsymbol{\mu}})\, y\,(p,\hat{\boldsymbol{\mu}}) \;=\; \frac{r\left(r\left(\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\, h-s\right)+2\lambda^{U}\,(p,\hat{\boldsymbol{\mu}})\,(\lambda h-s)\right)}{\lambda^{I-}\,(p)\,(\lambda h-s)} \tag{59}$$

From this equation, if $d\lambda^{U}/dp\leq 0$, then the left-hand side of equation (59), the revealing intensity, strictly decreases as p increases. □

C.3.3 U's growing pessimism or growing optimism right before separation

We here give a more detailed argument than in the main text. We do this in three steps.

Step 1. *U's continuation MB of experimentation equals her flow continuation value.* U's continuation value comes from both players' efforts, with I's effort contributing only to the continuation value whereas her own effort also to the flow value:

$$\underbrace{r(W^{U}-s)}_{\text{flow continuation value}} \;=\; a^{U}[\underbrace{r(\lambda^{U}h-s)}_{\text{flow MB}}+\text{continuation MB}]+a^{I}[\text{continuation MB}].$$

Since I takes effort 1 with probability 1 at any state of the gradual revelation phase, and U's total MB is 0 due to her indifference about experimentation, we have

$$\underbrace{r(W^{U}-s)}_{\text{flow continuation value}} \;=\; \text{continuation MB}.$$

Step 2. *U's continuation value can be approximated (up to first order) by her expected continuation value if I's type were public and both players played the symmetric MPE.*[36] Since U is indifferent about experimenting and not experimenting during the gradual revelation phase, we assume that she takes effort 1 during this phase, that is, she matches her effort with the informed player's effort. Under this alternative strategy, in case I is of type s_+, she receives the same payoff as in the symmetric MPE (under symmetric information s_+). In case I is of type s_-, both players equally share the effort load, which is higher than the single-player solution and lower than the cooperative solution; hence each player's continuation value is between the continuation value corresponding to the symmetric MPE solution and to the cooperative solution; since both the continuation value and its derivative with respect to q^- coincides under the symmetric MPE solution and the cooperative solution, each player's continuation value can be approximated by the symmetric MPE solution, in case I is of type s_-.

Step 3. *A drop in q^- reduces U's flow MB relatively more than it reduces U's continuation MB, whereas a drop in q^+ has the reverse effect.*

U's flow MB is the expectation of her ex post flow MBs, $r(\lambda q^+ h - s)$ and $r(\lambda q^- h - s)$, which are linear in her ex post posteriors, (that is, I's posteriors). This means, a mean-preserving spread (henceforth, MPS) of U's belief profile does not change her flow MB.

U's continuation MB, as we will show later, equals her flow continuation value $r(W^U - s)$, and can be approximated by the weighted average of her flow continuation values in the symmetric information benchmark, $r(w^S(q^+) - s)$ and $r(w^S(q^-) - s)$, near the end of the gradual revelation phase. That is, $W^U - s = \mu(w^S(q^+) - s) + (1-\mu)(w^S(q^-) - s)$. Different from the flow MB function, w^S is convex (from KRC-2005), meaning an MPS of U's belief profile increases her continuation MB. Intuitively, the bigger gap between the ex post posteriors q^+ and q^-, the more precise players' information will be after separation, hence the lower chance they will use the inferior project, leading to a higher future value to U, and consequently, a higher incentive for U to accelerate experimentation so as to reap this future value earlier.

A reduction in q^- widens the spread between q^+ and q^- whereas a reduction in q^+ narrows it, resulting in distinct evolution patterns of U's flow MB and her continuation MB, if her total MB were to stay constant. For example, the adjustment — q^+ stays constant, q^- drops by dq, and μ rises by $d\mu$ to keep U's belief about the risky project unchanged — creates an MPS of U's belief profile, whereby it increases her continuation MB without affecting her flow MB. See Figure 7 for an illustration, in which, q^U denotes U's (interim) posterior about the risky project, and $\hat{q}^- = q^- - dq$ type s_-'s posterior after the adjustment; the rise in her continuation MB due to this MPS is represented by the upward pointing arrow. This implies, to keep her total MB constant, I's reputation μ needs to adjust back partially, so that the newly dropping flow MB neutralizes the rising continuation MB. On the contrary, the adjustment — q^- keeps constant, q^+ drops by dq, and μ rises by $d\mu$ to keep U's belief unchanged — makes U's old belief profile an MPS of this new one, whereby it decreases her continuation MB, preserving her flow MB. See Figure 8 for an illustration, in which,

[36] That is, U's continuation MB can be approximated by $\mu w^S(\boldsymbol{q}^+(p)) + (1-\mu) w^S(\boldsymbol{q}^-(p))$.

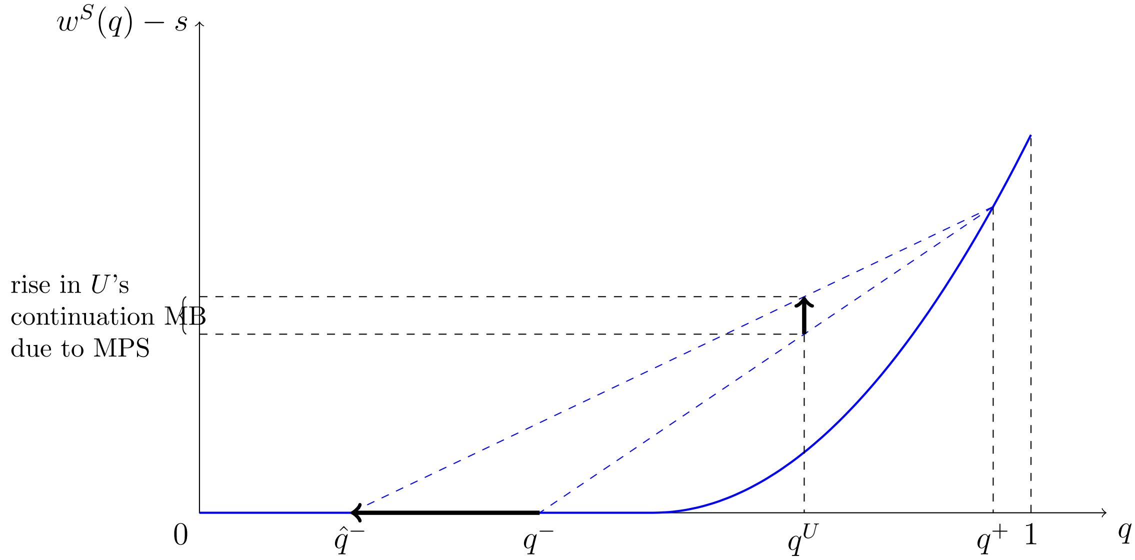

Figure 7: The rise in U's continuation MB due to a mean preserving spread

$\hat{q}^+ = q^+ - dq$ denotes type s_+'s posterior after the adjustment; the drop in U's continuation MB due to this MPS is represented by the downward pointing arrow. Thus, to keep her total MB constant, μ needs to rise further, so that the newly rising flow MB counterpoises the dropping continuation MB.

C.4 Welfare analysis

C.4.1 Proof of Proposition 1

Depending on the parameters ρ_b and ρ_g, both subcases of case 2 can happen: for instance, consider $a < \hat{a}$ but sufficiently close to $\hat{a}$. If (ρ_b, ρ_g) are sufficiently low, player I will start with a high reputation, implying a short gradual revelation phase, that is, p_{gr} will be close to p_2^{*-}. Hence $\tilde{p}_a = p_{gr}$, and the first subcase of case 2 occurs. If (ρ_b, ρ_g) are sufficiently high, player I will start with a low reputation, implying a long gradual revelation phase, that is, p_{gr} will be far from p_2^{*-}. Hence $\tilde{p}_a < p_{gr}$, and the second subcase of case 2 occurs.

To prove Lemma 1, we first derive an HJB equation for ΔW (in Claim 6), and then show that whenever $\Delta W = 0$ for some $\tilde{p} \in (p_2^{*-}, p_{gr})$, we must have $\Delta W > 0$ for all $p \in (\tilde{p}, p_{gr}]$ (in Claim 7).

Claim 6. *During gradual revelation phase, ΔW satisfies HJB equation* (**??**).

We first prove this HJB equation holds and then offers an interpretation of it.

Proof of Claim 6. Rewrite the HJB equations of W^U, W^{I+}, W^{S+}, and W^{S-} in the following

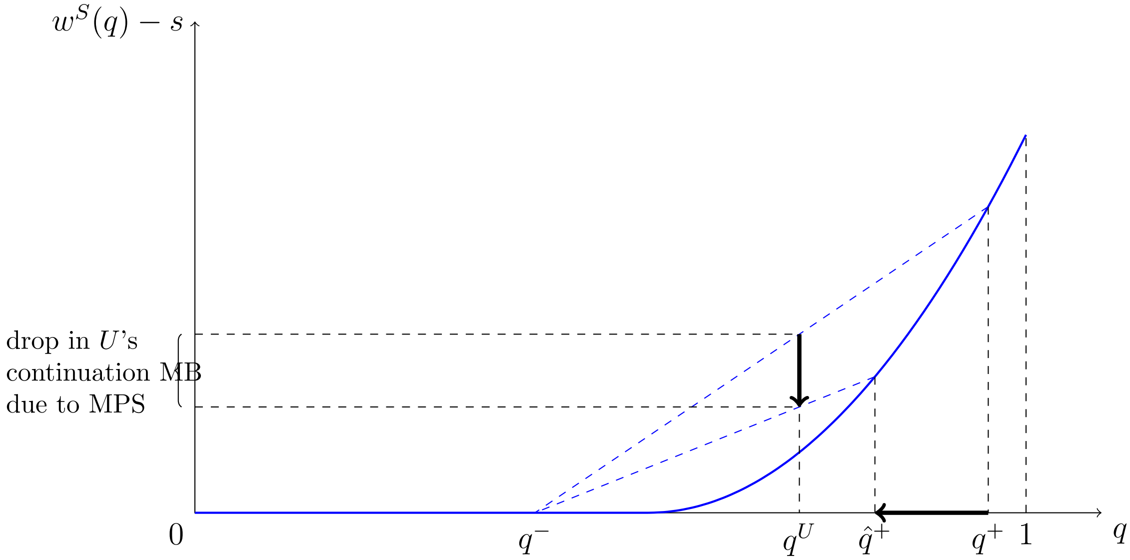

Figure 8: The drop in U's continuation MB due to a mean preserving contraction

way:

$$\begin{aligned} & r\left(W^U(p,\hat{\boldsymbol{\mu}})-s\right) \\ = \; & a^U(p,\hat{\boldsymbol{\mu}})\,r\left[\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right]-(1-\hat{\boldsymbol{\mu}})\,y\left(W^U(p,\hat{\boldsymbol{\mu}})-W^U(p,\hat{\boldsymbol{\mu}})\right) \\ & +\left(1+a^U(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp}+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-\left(W^U(p,\hat{\boldsymbol{\mu}})-s\right)\right)\right]. \end{aligned} \tag{60}$$

This is the same with HJB equation (16), except that we collect the two usual optional value parts together: the change in value if no revealing and no good news arrives, and the change in value if no revealing and good news arrives.

$$\begin{aligned} & r\left(W^{I+}(p,\hat{\boldsymbol{\mu}})-s\right) \\ = \; & r\left[\lambda^{I+}(p)\,h-s\right] \\ & +\left(1+a^U(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^{I+}(p,\hat{\boldsymbol{\mu}})}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(W^{I+}(p,\hat{\boldsymbol{\mu}})-s\right)\right)\right]. \end{aligned} \tag{61}$$

$$
\begin{aligned}
& r\left(w^{S}\left(\boldsymbol{q}+(p)\right)-s\right) \\
= \; & r\left[\lambda^{I+}(p)\,h-s\right] \\
& +\left(1+a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[-\lambda p\left(1-p\right)\frac{dw^{S}\left(\boldsymbol{q}+(p)\right)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}+(p)\right)-s\right)\right)\right] \\
& +\left(1-a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[-\lambda p\left(1-p\right)\frac{dw^{S}\left(\boldsymbol{q}+(p)\right)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}+(p)\right)-s\right)\right)\right] \\
= \; & r\left[\lambda^{I+}(p)\,h-s\right] \\
& +\left(1+a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[-\lambda p\left(1-p\right)\frac{dw^{S}\left(\boldsymbol{q}+(p)\right)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}+(p)\right)-s\right)\right)\right] \\
& +\frac{\left(1-a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)}{2}\left[w^{S}\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]. \qquad (62)
\end{aligned}
$$

The first equality is due to the fact that $p > p^{S+}$, and hence both players experiment with full resource in the symmetric MPE of the symmetric information game. In the second equality, we replace the optional value by $\left[w^{S}\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]/2$, which is obtained from the first equality. The term

$$
\left(1-a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[w^{S}\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]
$$

can be interpreted as the welfare loss to both players in case player I is type s_{+}, caused by the lack of effort of player U (in the equilibrium of asymmetric information game, compared with the symmetric MPE in the symmetric information game).

$$
\begin{aligned}
& r\left(w^{S}\left(\boldsymbol{q}-(p)\right)-s\right) \\
= \; & r\left[\lambda^{I-}(p)\,h-s\right] \\
& +\left(1+a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[-\lambda p\left(1-p\right)\frac{dw^{S}\left(\boldsymbol{q}-(p)\right)}{dp}+\lambda^{I-}(p)\left(\lambda h-s-\left(w^{S}\left(\boldsymbol{q}-(p)\right)-s\right)\right)\right]. \qquad (63)
\end{aligned}
$$

Writing W^{S-} in this way, we are saying that, in the symmetric information game with public signal s_{-}, U's payoff is obtained by her experimenting with full resource during $p \in (p_2^{*-}, p_{gr})$ and her teammate experimenting with resource $a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)$. (Note the role switching between the players.) We may call this a pseudo-equilibrium (as her teammate does not find it optimal to play like this). The reason we interpret W^{S-} in this way is because, the sum of effort will be the same in the equilibrium constructed for the asymmetric information game, and in this pseudo-equilibrium we just specified in the symmetric information game with public signal s_{-}. This means that in the asymmetric information game, player U enjoys an additional flow payoff $r\left(1-a^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left(s-\lambda^{I-}(p)\,h\right)$ due to effort saving in case I has signal s_{-}, compared with the symmetric information case.

We now combine these four HJB equations to derive an HJB equation of ΔW.

From equation (23), and replacing $\boldsymbol{\mu}^o$ by $\hat{\boldsymbol{\mu}}$, we have

$$\begin{aligned}
\frac{d\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)}{dp} &= \left(\frac{dW^U\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}-2\hat{\boldsymbol{\mu}}\frac{dw^S\left(\boldsymbol{q}+(p)\right)}{dp}-\left(1-\hat{\boldsymbol{\mu}}\right)\frac{dw^S\left(\boldsymbol{q}-(p)\right)}{dp}\right)\\
&\quad+\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\hat{\boldsymbol{\mu}}\left(W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)-2w^S\left(\boldsymbol{q}+(p)\right)+w^S\left(\boldsymbol{q}-(p)\right)\right) \qquad (64)\\
&= \left(\frac{dW^U\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}-2\hat{\boldsymbol{\mu}}\frac{dw^S\left(\boldsymbol{q}+(p)\right)}{dp}-\left(1-\hat{\boldsymbol{\mu}}\right)\frac{dw^S\left(\boldsymbol{q}-(p)\right)}{dp}\right)\\
&\quad+\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\left(\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)-W^U\left(p,\hat{\boldsymbol{\mu}}\right)+w^S\left(\boldsymbol{q}-(p)\right)\right) \qquad (65)
\end{aligned}$$

Using equation (**??**), the four HJB equations (60) to (63), we obtain an HJB equation for $\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)$, with a term

$$\left(\frac{dW^U\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}-2\hat{\boldsymbol{\mu}}\frac{dw^S\left(\boldsymbol{q}+(p)\right)}{dp}-\left(1-\hat{\boldsymbol{\mu}}\right)\frac{dw^S\left(\boldsymbol{q}-(p)\right)}{dp}\right).$$

We then apply equalities (64) and (65) to get rid of this term, and apply equations (14) and (31) to get rid of $\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}$. Finally, rearranging terms, we would obtain the HJB equation (**??**). □

We now interpret the HJB equation of ΔW. To derive a tractable HJB equation of ΔW, we manipulate the value functions (W^{S-} in particular), so that they have a common component $1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)$ in the optional values. After this manipulation, the effort levels can be interpreted as: in the equilibrium constructed for the asymmetric information game, total effort is $1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)$, and U's effort is $a^U\left(p,\hat{\boldsymbol{\mu}}\right)$; in the symmetric information game with public information s_+, total effort is 2, and U's effort is 1; in the symmetric information game with public information s_-, total effort is $1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)$, and U's effort is 1. Therefore, in the asymmetric information equilibrium, compared with the symmetric information benchmark, total effort is reduced by $1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)$ in case player I has signal s_-, which causes a reduction in the sum of optional value $\left(1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[w^S\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]$ (see derivation of HJB equation W^{S+} in the proof above); also, in the asymmetric information equilibrium, U's effort is reduced by $1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)$ (for both cases of signals), saving experimentation cost $\left(1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\left[s-\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h\right]$. Theses two terms are the first line on the right-hand side of equation (**??**), resembling the "flow payoff" in a usual HJB equation. The second line on the right-hand side of equation (**??**), the usual optional value of keeping asymmetric information, is easy to explain: in case good news does not arrive and type s_- does not reveal, ΔW changes by $\frac{d\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)}{dp}dp$; with probability $\left(1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)dt$, good news arrives, and ΔW jumps to 0; with probability $\left(1-\hat{\boldsymbol{\mu}}\right)y\left(p,\hat{\boldsymbol{\mu}}\right)dt$, type s_- reveals his type, and and ΔW jumps to 0 also. In this interpretation, it is important to notice that in the definition of ΔW, type s_-'s welfare gain or loss is cancel out, hence such manipulation does not affect type s_-'s welfare gain or loss.

Rearranging terms, we have

$$\begin{aligned}
& \left(1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda p\left(1-p\right)\frac{d\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)}{dp} \\
= \; & -\left[r+\left(1+a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)+\left(1-\hat{\boldsymbol{\mu}}\right)y\left(p,\hat{\boldsymbol{\mu}}\right)\right]\Delta W\left(p,\hat{\boldsymbol{\mu}}\right) \\
& -\left(1-a^U\left(p,\hat{\boldsymbol{\mu}}\right)\right)r\left[\hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)+\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right]. \quad (66)
\end{aligned}$$

We will show that

Claim 7. *During gradual revelation phase, if there is some $\tilde{p}$ such that $\Delta W\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)=0$, and $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp}\geq 0$, then $\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)>0$ for all $p\in\left(\tilde{p},p_{gr}\right]$.*

Proof of Claim 7. At $\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)=0$, by using $W^U\left(p,\hat{\boldsymbol{\mu}}\right)=s+s-\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h$, we also have

$$\begin{aligned}
& -\left[\hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)+\lambda^U\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right] \\
= \; & \hat{\boldsymbol{\mu}}\left(w^S\left(\boldsymbol{q}+(p)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)+\left(1-\hat{\boldsymbol{\mu}}\right)\left(w^S\left(\boldsymbol{q}-(p)\right)-s\right) \quad (67)
\end{aligned}$$

To prove Claim 7, it is sufficient to show that if the right-hand side of equation (67) is positive at some $\tilde{p}$, then it is positive for all $p\in\left(\tilde{p},p_{gr}\right]$.

First, $d\left(w^S\left(\boldsymbol{q}+(p)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)/dp>0$. This is because, $w^S\left(\boldsymbol{q}+(p)\right)$ and $W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)$, when taken as functions of type s^+'s posterior $\boldsymbol{q}^+(p)$, have derivative w.r.t $\boldsymbol{q}^+(p)$ in $[0,\lambda h]$. Hence the derivative of $w^S\left(\boldsymbol{q}+(p)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)$, when taken as function $\boldsymbol{q}^+(p)$, w.r.t. $\boldsymbol{q}^+(p)$, is in $[-\lambda h,\lambda h]$. Since the derivative of $\lambda^{I+}\left(p\right)h$, when taken as function $\boldsymbol{q}^+(p)$, w.r.t. $\boldsymbol{q}^+(p)$, is λh, we have that $\left(w^S\left(\boldsymbol{q}+(p)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)$, when taken as function of $\boldsymbol{q}^+(p)$, has positive derivative w.r.t. $\boldsymbol{q}^+(p)$. As $\boldsymbol{q}^+(p)$ strictly increases in p during gradual revelation phase, we have $d\left(w^S\left(\boldsymbol{q}+(p)\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)/dp>0$

Second, the right-hand side of equation (67) is strictly increasing in p whenever it is 0. This is because if at p where it is 0, we have $\left(w^S\left(\boldsymbol{q}+(p)\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)\leq 0$. As $\hat{\boldsymbol{\mu}}$ strictly decreases in p by Proposition 1, and W^{S-} strictly increases in p, the right-hand side of equation (67) is strictly increasing at such p's. □

Therefore, if type s_+'s true posterior is above the myopic threshold, then asymmetric information improves welfare. The intuition is the following. In the gradual revelation phase, right before separation, U's gain (per unit of time) from asymmetric information in case I holding signal s_-, due to the team's high effort, is

$$ra^U(\lambda^{I-}h-s)+(1+a^U)(\lambda h-s),$$

which, by applying the formula for a^U, equals

$$r(1-a^U)(s-\lambda^{I-}h).$$

The team's loss (per unit of time) from asymmetric information in case I holding signal s_+, caused by U's low effort, is

$$r(1-a^U)[(\lambda^{I-}h-s)+(W^{S+}-s-(\lambda^{I+}h-s)),$$

where the first part in the square bracket is U's forgone flow benefit per unit saved effort by U, and the second part is the team's forgone optional value per unit saved effort by U.

Hence the team's net gain from asymmetric information right before separation, equals

$$r(1-a^U)[(s-\lambda^U h)-\mu(W^{S+}-s-(\lambda^{I+}h-s)),$$

which is the difference between U's expected flow gain from her saved effort and the loss of the team's optional value in case of s_+ due to U's saved effort. During gradual revelation phase, U's expected flow gain from experimentation also equals the required return $r(W^U-s)$, which, right before separation, comes only from the required return in case the risky project being good weighted by its probability, $r\mu(W^{S+}-s)$.[37] Therefore, the team's net gain per unit of U's saved effort from asymmetric information equals difference between the required return in case of s_+ and the optional value, which is the flow gain from experimentation in case of s_+ weighted by its probability μ:

$$r\mu(\lambda^{I+}h-s).$$

C.4.2 Welfare analysis in the pooling phase

Once we know the sign of $\Delta W(p,\hat{\boldsymbol{\mu}})$ at p_{gr}, we would know the sign of $\Delta W(p,\hat{\boldsymbol{\mu}})$ at the pooling Phase, that is, for $p\in(p_{gr},1)$, because the two have the same sign. The intuition is simple; since during pooling Phase, both players experiment in the same manner as in the symmetric MPE of the symmetric information game, hence the material payoff collected during the pooling Phase is the same as in the symmetric MPE of the symmetric information game (for the corresponding posteriors), implying that whether asymmetric information improves welfare depends solely on the continuation value of $\Delta W(p,\hat{\boldsymbol{\mu}})$ at p_{gr}.

From Lemma 1 and the above argument, we arrive at the following corollary:

Corollary 1. *There exists $\hat{O}\in(O^S,\infty)$ such that*

1. *If $O\geq\hat{O}$, then $\Delta W(p,\hat{\boldsymbol{\mu}})>0$ over $(p_2^{*-},1)$.*
2. *If $O<\hat{O}$, and*
   (a) *if there exists $\tilde{p}_O\in(p_2^{*-},p_{gr})$ such that $\Delta W(\tilde{p}_O,\hat{\boldsymbol{\mu}})=0$, then $\Delta W(p,\hat{\boldsymbol{\mu}})<0$ over $(p_2^{*-},\tilde{p}_O)$, and $\Delta W(p,\hat{\boldsymbol{\mu}})>0$ over $(\tilde{p}_O,1)$;*
   (b) *otherwise, $\Delta W(p,\hat{\boldsymbol{\mu}})<0$ over $(p_2^{*-},1)$.*

D Other Equilibria

D.1 Proof of Claim 1

Following the discussion right above Claim 1, we have, if $a>\bar{a}$, then I's continuation value at p_2^{*-} is his continuation value in the symmetric MPE with his private information

[37] This is obtained from the initial condition (20).

being public: $W^{I+}(p,\mu) = w^S(\boldsymbol{q}^+(p))$, and $W^{I-}(p,\mu) = w^S(\boldsymbol{q}^-(p))$ $(= s)$, at $p = p_2^{*-}$, and $\mu > 0$.

Let $\hat{\boldsymbol{\mu}} : (p_2^{*-}, p_{gr}) \to [0,1]$ be the gradual revelation path in the constructed MPE. And denote U's effort during the rewarding region of the gradual revelation path as $f(p)$. Note that, given an MPE that coincides with the constructed MPE over background beliefs $[p_2^{*-}, p]$, if I's reputation at p is fixed, then type s_+ strictly prefers to experiment over $[p, p+dp]$, whereas type s_- strictly prefers not to so over the rewarding region or over the non-responding region if the uninformed player's effort is strictly higher than $a^S(\boldsymbol{q}^-)$; over $[p, p+dp]$, the uninformed player strictly prefers to experiment for $\mu > \hat{\boldsymbol{\mu}}$, and be willing to experiment at $\mu < \hat{\boldsymbol{\mu}}$ only if the informed player's equilibrium effort strictly lower than 1. [Recall again that players' current efforts are strategic substitutes; the uninformed player is willing to experiment at lower beliefs only if the informed player's effort is lower, before separating].

We will use backward induction to show that, in any MPE, if the equilibrium strategies over $[p_2^{*-}, p]$ is the same as in the constructed MPE, then the equilibrium strategy over $[p, p+dp]$ in the former MPE will be the same as in the constructed MPE.

Proof. Note that in the constructed MPE, $\hat{\boldsymbol{\mu}}$ is the borderline such that, if I's effort is 1, then U strictly prefers to experiment if the state is above it, and strictly prefers not to if the state is below it, and is indifferent if the state is on this curve. Hence below this curve, U is willing to experiment only if I's effort is lower than 1.

Consider an MPE of the asymmetric information game, and denote the equilibrium effort strategies of the uninformed player and of type s_+ as $\tilde{a}^U$, $\tilde{a}^{I+}$. Let $\tilde{p}$ be the infimum over $[p_2^{*-}, p_{gr})$ such that equilibrium strategies differ from the constructed MPE.

(1) If there is some $\mu < \hat{\boldsymbol{\mu}}$ such that the equilibrium strategies differ from the constructed MPE over $[\tilde{p}, \tilde{p}+dp]$, then let $\tilde{\mu}$ be such that the averaged uninformed player's effort is the lowest over $[\tilde{p}, \tilde{p}+dp]$. For type s_- to mimic type s_+, we must have either, $\tilde{a}^U/\tilde{a}^{I+} \geq f(p)$ over the rewarding region, or $\tilde{a}^U \geq a^S(\boldsymbol{q}^-(p))$ over the non-responding region, both requiring $\tilde{a}^{I+} < 1$ (otherwise U would strictly prefer not to experiment, given that μ is low). If I takes action $\tilde{a}^{I+}$, then at $\tilde{p}$, he will end up at state $(\tilde{p}, \hat{\boldsymbol{\mu}})$ (since the equilibrium over $[p_2^{*-}, \tilde{p}]$ coincides with the constructed MPE). Now consider I deviating to effort 1 at reputation $\tilde{\mu}$ and over the interval $(\tilde{p}, \tilde{p}+dp)$. Type s_+ strictly benefits from such a deviation as long as he does not get a perfect bad reputation: he obviously gains if his reputation reaches above $\hat{\boldsymbol{\mu}}$; if he gets a reputation below $\hat{\boldsymbol{\mu}}$ (but still positive), then at $\tilde{p}$ his reputation immediately jumps upward to $\hat{\boldsymbol{\mu}}$ after taking his equilibrium strategy $\tilde{a}^{I+}(=1)$, hence he also benefits. But type s_- strictly loses if his reputation does not change (hence jumps upward to $\hat{\boldsymbol{\mu}}$ after taking action $\tilde{a}^{I+}(=1)$ at $\tilde{p}$). That is, the set of reputation making type s_+ strictly benefit from such a deviation is strictly larger than the set of reputation making type s_- weakly benefit from it. Therefore, by D1, after such a deviation, I should receive a perfect reputation; but this suggests that type s_- strictly prefers to deviate to effort 1 over $(\tilde{p}, \tilde{p}+dp)$ at reputation $\tilde{\mu}$.

(2) If the equilibrium strategies differ from the constructed MPE over $[\tilde{p}, \tilde{p}+dp]$, only at reputations $\mu \geq \hat{\boldsymbol{\mu}}$, which is possible only if $\tilde{a}^{I+} < 1$ and $\tilde{a}^U = 1$. Consider I deviating to effort 1 at such a reputation μ, over the interval $(\hat{p}, \hat{p}+dp)$. Then type s_+ strictly benefits

from such a deviation as long as he receives a reputation weakly above μ, whereas type s_- strictly loses if he gets a reputation weakly below μ; therefore, the set of reputation making type s_+ strictly benefit from such a deviation is strictly larger than the set of reputation making type s_- weakly benefit from it. By D1, after such a deviation, I should receive a perfect reputation; but this suggests that type s_- strictly prefers to deviate to effort 1 over $(\tilde{p}, \tilde{p} + dp)$ at reputation μ.

Therefore, any MPE satisfying D1 should coincide with the constructed MPE over $[p_2^{*-}, p_{gr}]$ (the gradual revelation region); that such MPE coincides with the constructed MPE over the pooling region follows similar steps. □

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