Expanding Applications in College Admissions*

Wei-Cheng Chen

Department of Economics, National Chung Cheng University

Yi-Yi Chen

Department of Economics, Feng Chia University

Yi-Cheng Kao**

Department of Business Administration, Chung Yuan Christian University

Abstract

In college admissions, a standardized test typically consists of main tests and subject tests. Main tests are necessary but subject tests are usually optional. Students taking subject tests must be a subset of those who take main tests. If a college only requires students to take main tests, called using the expanding strategy, it will benefit from a size effect but suffer from a mismatch effect. The size effect comes from an expanding pool with more applicants of high caliber. The mismatch effect is due to the absence of measuring applicants' capacity in specific subjects. Our analysis shows that, the expanding strategy is used if the size effect dominates the mismatch effect. Moreover, when colleges have similar levels of prestige, the mismatch effect can be eliminated by a limiting strategy such as early decision programs or simultaneous exams. As a result, combining the expanding strategy and the limiting strategy can yield an efficient and stable matching.

JEL classification: C78; D82; I23

Keywords: early decision; limiting applications; simultaneous screening

---

* This paper is prepared for presenting in the 14th Meeting of Society for Social Choice and the 18th Annual SAET Conference.

** Corresponding author. Department of Business Administration, Chung Yuan Christian University, No. 200, Chung Pei Road, Chung Li District, Taoyuan City, 32023, Taiwan (R.O.C). E-mail: kao@cycu.edu.tw. Support from the Ministry of Science and Technology via Grant MOST 105-2410-H-033-004 is gratefully acknowledged.

1. Introduction

In most college admissions, students are required to take a standardized test.[1] In addition, a standardized test typically has two categories of tests: main tests and subject tests, such as SAT I and SAT II for the decentralized college admissions in the US.[2] In the SAT, main tests are required but subject tests are optional. Similarly, the 2014 policy reform for the centralized college admissions in China also starts allowing colleges to decide the combination of subject tests.[3] In both cases, students taking subject tests must be a subset of those who take main tests. If a college only requires students to take main tests, it actually expands its applicant pool with a cost that students' capacity in specific subjects is not measured by the tests. In this paper, we consider a college admissions problem in which colleges could attract desired students by expanding the applicant pool via testing policy, called the expanding strategy, and might eliminate the aforementioned cost by restricting applicants' choices, called the limiting strategy. Under certain conditions, using the two strategies together can yield an efficient and stable matching.

We further illustrate the problem in Section 2 by cases in Taiwan since Taiwan contains both centralized and decentralized mechanisms in the college admissions. The main message in Section 2 is that colleges using the expanding strategy on average attract students of higher caliber in a standardized test. This raises a question of why the expanding strategy is not generally adopted in Taiwan. Similarly, in the US, we also observe two different cases that Yale uses the expanding strategy of requiring only SAT I (main tests) but MIT requires both SAT I and SAT II (subject tests) in their 2018 testing policies.[4]

---

[1] In some cases like *gaokao* in China (Chen and Kesten, 2017) and YGS/LYS in Turkey (Balinski and Sönmez 1999), colleges admit students only rely on students' preference lists and their scores in the standardized tests.

[2] In *gaokao*, main tests are math, Chinese, and foreign language, and subject tests are physics, chemistry, biology, geography, history, and politics (Chen and Kesten, 2017).

[3] See the document at http://www.gov.cn/zhengce/content/2014-09/04/content_9065.htm

[4] Yale's testing policy in 2018 states "SAT Subject Tests are recommended but not required. Applicants who do not take SAT Subject Tests will not be disadvantaged in the application process." (https://admissions.yale.edu/standardized-testing) In contrast, MIT's testing policy

Our explanation is as follows. If a college uses the expanding strategy, it will benefit from a size effect but suffer from a mismatch effect. The size effect comes from an expanding pool with more applicants of high caliber because all students have to take the main tests. The mismatch effect is due to the absence of measuring applicants' capacity in specific subjects. Science colleges like MIT may care about admitted students' capacity in science subjects and the mismatch effect is crucial for those colleges. In contrast, colleges like Yale may emphasize a general high level of applicants' caliber in the main tests and hence the mismatch effect is minor for them. Our analysis shows that the expanding strategy is used if the size effect dominates the mismatch effect. Moreover, in a decentralized college admissions, when colleges have similar levels of prestige, the mismatch effect can be eliminated by using the limiting strategy such as early decision or simultaneous exams. This is because students only can apply to one college under the limiting strategy and this restriction plays a role resembling that of subject tests. For example, students who are good at science subjects would apply to a science college although the subject tests actually are not required by the college.

The limiting strategy is commonly used by a second-ranked college in practice because it may attract some students of high caliber who could have been admitted to the best college if their application choice is not restricted. The limiting strategy is observed in entrance examinations in many Asian countries (Avery, Lee, and Roth, 2014; Chen and Kao, 2014 and 2016; Kao and Lin, 2017) and is also observed in early decision programs in the US (Avery, Fairbanks, and Zeckhauser, 2003; Lee, 2009; Avery and Levin, 2010; Kim, 2010).[5] In particular, an experimental study of Chen, Chen, and Kao (2017) demonstrate that a second-ranked college could benefit from using the limiting strategy under certain conditions, such as sufficient levels of its

---

in 2018 states "We require two SAT Subject Tests: one in math (level 1 or 2), and one in science (physics, chemistry, or biology e/m)." (http://mitadmissions.org/apply/freshman/tests)

[5] In the limiting strategy, colleges compete for students *ex ante* by constructing a desired applicant pool. On the other hand, given an applicant pool, Chade, Lewis, and Smith (2014) and Che and Koh (2016) investigate college competition *ex post* by setting different admission standards.

prestige and the uncertainty about admission outcomes; however, the best college would suffer from losing desired students in this situation.

In our model, both the best and second-ranked colleges can gain from using the limiting strategy. Indeed, if both colleges use a combination of the limiting strategy and the expanding strategy, the matching outcome can be efficient and stable in a decentralized college admissions.[6] The reason is as follows. There are two types of students in our model: good and ordinary students. All students are divided into two groups: the science-stream students and the humanities-stream students. A test is an imperfect device to screen students' caliber and inefficiency occurs when ordinary students perform better than good students. This inefficiency can be reduced by using the expanding strategy because there are more good students in a larger pool of applications. However, such a reduction can create an unstable matching due to the mismatch effect, e.g., a good humanities-stream student is admitted by a science college. As long as the mismatch effect can be eliminated by using the limiting strategy, we obtain an efficient and stable matching. In other words, both colleges can admit their desired good students.

The rest of the paper is organized as follows. Section 2 illustrates the problem by Taiwanese cases. Section 3 presents the base model. Section 4 solves the equilibrium in the centralized and decentralized mechanisms. Section 5 discusses the stability and the efficiency about matching outcomes. Section 6 extends the base model. Section 7 concludes.

2. College Admissions in Taiwan

In Taiwan, there are two major mechanisms in college admissions: a centralized mechanism called the examination channel and a decentralized mechanism called the application channel (Li, Lee, and Lien, 2016; Luoh, 2016). In the centralized mechanism, students have to take a standardized test by the college entrance examination center (CEEC). After CEEC reports the test scores, students submit their preference

[6] Note that the expanding strategy can be used in a centralized mechanism as well as a decentralized mechanism, but the limiting strategy can only be used in a decentralized mechanism.

lists of departments and departments enroll students via the deferred acceptance algorithm of Gale and Shapley (1962). To lighten the terminology, we use college to denote a department in a college or in a university throughout the paper.

The decentralized mechanism consists of two steps. In the first step, colleges screen students based on application documents as well as scores from another standardized test by CEEC. In the second step, the invited students have to attend colleges for interviewing or other individual exams implemented by the colleges. If two colleges strategically decide to have the same exam date in the second step, students only can apply to one of them. Since this strategy actually limits the applicant pool, we call it the limiting strategy.

In the two major mechanisms in Taiwan, colleges have to decide the combination of required subject tests and the relative weights of those subjects in a standardized test. The combination can be changed every year. For example, in 2010, the Department of Finance at Tamkang University had changed the combination from main tests (Chinese, English, and math B) and subject tests (history and geography) to only main tests. This action potentially expands the applicant pool because the science stream students in high schools who do not take the history and geography tests now can apply to the department. On the other hand, the humanities stream students who would take those subject tests still can apply to the department. Thus, we call this action, only requiring students to take main tests, the expanding strategy.

The expanding strategy is commonly used by the departments in the fields of humanities and social-science. Figure 1 shows the weighted average scores and the sum of subject weights for Taiwanese departments in the centralized mechanism during the periods from 2006 to 2013. The red-solid points are departments using the expanding strategy and others are departments requiring both main tests and subject tests. High values of weighted average scores imply that students enrolled by those departments have high scores in the test.[7] According to the pattern in Figure 1,

[7] For example, a department only requires main tests and sets the relative weights of 1, 1.5, and 1.5 for Chinese, English, and math B, respectively. After the admission process, CEEC only announces the threshold of the weighted sum of scores for students enrolled by that department. If the threshold is 360, then the weighted average scores is 90 (=360/4).

departments on average seem to attract better students in the test by using the expanding strategy. This raises a question of why the expanding strategy is not generally adopted in Taiwan. We answer this question in the following analysis.

3. Model

Four students apply to two colleges. Students' caliber has two types: good and ordinary. Students in high school are divided into two groups, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$, which are respectively the science-stream students and the humanities-stream students. x_1 and y_1 are the good-type students and x_2 and y_2 are the ordinary-type students. Colleges cannot observe students' types directly and they admit students according to entrance tests.

3.1 Tests

There are three entrance tests: physics, history, and math, denoted by $\{P, H, M\}$, respectively. The science-stream students take the tests of physics and math and the humanities-stream students take the tests of history and math. Hence we call M and $\{P, H\}$ the major test and the subject tests, respectively.[8]

The entrance tests are imperfect devices to screen students' types. There are two states of nature $\Omega = \{\omega_1, w_2\}$ with the probabilities of $p(\Omega) = 1$, $p(\omega_1) = \pi$, $p(\omega_2) = 1 - \pi$, and $\pi \in (0.5, 1)$. Let $T = \{t_1, t_2, t_3, t_4\}$ be the test scores such that $t_1 > t_2 > t_3 > t_4$. An entrance test is a random mapping from students to scores $\{X, Y\} \to T$. In the physics test, we have

$$(x_1, x_2) \to (t_1, t_2) \text{ in } \omega_1;$$
$$(x_1, x_2) \to (t_2, t_1) \text{ in } \omega_2.$$

In the history test, we have

[8] In practice, the main test consists of more than one subject. For example, in China, main tests are math, Chinese, and foreign language, and subject tests are physics, chemistry, biology, geography, history, and politics (Chen and Kesten, 2017). If a science college requires main test and subject tests, the combination should be the three main tests plus the physics, chemistry, and biology tests. To simplify notations, we use M to denote the main tests and use P and H to denote the combination of the main tests plus the subject tests for science-stream and humanities-stream students, respectively.

$$(y_1, y_2) \to (t_1, t_2) \text{ in } \omega_1;$$
$$(y_1, y_2) \to (t_2, t_1) \text{ in } \omega_2.$$

In the math test, we have

$$(x_1, x_2, y_1, y_2) \to (t_1, t_3, t_2, t_4) \text{ in } \omega_1;$$
$$(x_1, x_2, y_1, y_2) \to (t_2, t_4, t_1, t_3) \text{ in } \omega_2.$$

Thus, in the ω_1 state students' types are correctly measured, i.e., good-type students have higher scores.[9] Note that given students' type, the science-stream students have higher scores than the humanities-stream students in the math test when ω_1 is the realized state.

3.2 Preferences

There are two colleges $\{A, B\}$ with corresponding prestige of a and b; and each has a capacity of one seat, denoted by $q_A = q_B = 1$. A is the best college and we assume $\mu a > b > 0$ to simplify the analysis.[10] Moreover, in the base model, we assume that A is a science college and it chooses either the physics test or the math test to screen students; B is a humanities college and it chooses either the history test or the math test. Let $(\tilde{A}, \tilde{B})$ be the profile of colleges' strategy in choosing the test. Since all students take the math test, we call $\tilde{A} = M$ and $\tilde{B} = M$ the expanding strategy.

Students strictly prefer A to B and their preference relation over colleges $\{\succ_i\}_{i \in \{X,Y\}}$ can be represented by an utility function of

$$u_i(\cdot) = \begin{cases} a \text{ if student } i \text{ is enrolled by } A \\ b \text{ if student } i \text{ is enrolled by } B \\ 0 \text{ if student } i \text{ is not enrolled by any department} \end{cases}.$$

[9] The correlation between states of tests may have two interpretations. First, since those tests are taken in two days, the realized states are the same. Second, since those tests are taken at different time, the realized states can be different, i.e., the outcomes of scores between tests are independent. For simplicity, we consider the first interpretation in this paper, but the main result still holds under the second one.

[10] This assumption can avoid the problem of multiple equilibria encountered in Chen and Kao (2014) when $b \to a$. The main result in this paper still holds with more complicated analysis for split cases if we consider a more general assumption of $a > b$.

Colleges' preference relation over students $\{R_j\}_{j\in\{A,B\}}$ can be represented by an utility function of

$$u_j(\cdot) = v_i - \beta_j I$$

if x_i or y_i is assigned to college j, and $u_j(\varnothing) = 0$, for $i \in \{1,2\}$ and $j \in \{A,B\}$. We assume $2v_2 > v_1 > v_2 > 0$. $I \in \{0,1\}$ is an indicator variable to capture the mismatch effect in which $I = 1$ means a humanities-stream (science-stream) student is enrolled by the science (humanities) colleges; $I = 0$ means a humanities-stream (science-stream) student is enrolled by the humanities (science) college. We assume $v_2 > \beta_A > \beta_B \geq 0$. That is, given the same level of v_i, the science (humanities) college prefers the science-stream (humanities-stream) students and the mismatch effect is larger in the science (humanities) college. For example, $x_1 R_A y_1$ because $u_A(x_1) = v_1$ is larger than $u_A(y_1) = v_1 - \beta_A$. To simplify the analysis, we assume $\beta_B = 0$ in the base model.

Finally, a matching is a function from students to colleges $\mu : \{X,Y\} \to \{A,B,\varnothing\}$. The image of null set $\varnothing$ means the cases in which students are unassigned. The matching mechanism is the deferred acceptance algorithm of Gale and Shapley (1962).

4. Equilibrium

In this section, we compute the Nash equilibrium under two different mechanisms in the college admissions: a centralized mechanism and a decentralized mechanism.

4.1 Centralized Mechanism

In the centralized mechanism, students with different streams take their corresponding tests and then colleges admit students according to scores of the required tests. Table 1 summarizes the matching outcomes with different strategies in different states. For example, $(\tilde{A},\tilde{B}) = (M,M)$ is the case that the expanding strategy is used by both colleges, and x_1 and y_1 respectively has the highest score in ω_1 and ω_2. Thus, in this example, A admits x_1 in ω_1 and admits y_1 in ω_2, and B admits y_1 in ω_1 and admits x_1 in ω_2.

Substituting matching outcomes of Table 1 into the utility function of colleges yields the expected payoffs of colleges under different strategies in Table 2. For example, when $(\tilde{A},\tilde{B})=(M,M)$, $E(u_A)=\pi v_1+(1-\pi)(v_1-\beta_A)$ because A has the probability of π to admit x_1 and has the probability of $1-\pi$ to admit y_1.

According to Table 2, $\tilde{B}=M$ is the dominant strategy for college B. Hence the equilibrium depends on the expected utility of college A under $(\tilde{A},\tilde{B})=(P,M)$ and $(\tilde{A},\tilde{B})=(M,M)$. If $\pi v_1+(1-\pi)(v_1-\beta_A)>\pi v_1+(1-\pi)v_2$, the expanding strategy $(\tilde{A},\tilde{B})=(M,M)$ is used by both colleges and the condition can be reduced to $\beta_A<(v_1-v_2)$. The equilibrium in the centralized mechanism is summarized in the following proposition.

Proposition 1. *In the centralized mechanism, the equilibrium is* $(\tilde{A},\tilde{B})=(M,M)$ *if* $\beta_A\in(0,(v_1-v_2))$, *and* $(\tilde{A},\tilde{B})=(P,M)$ *if* $\beta_A\in((v_1-v_2),v_2)$.

The equilibrium in the centralized mechanism depends on the values of the mismatch effect, β_A. If the mismatch effect is minor, both colleges will use the expanding strategy. If the mismatch effect is large, the science college cares much about students' capacity in science subjects and hence chooses the physics test.

4.2 Decentralized Mechanism

In the decentralized mechanism, in addition to choosing the tests, colleges also have to decide their exam dates $\tilde{d}_j\in\{d_1,d_2,\cdots,d_n\}$ for $j\in\{A,B\}$. Let $\tilde{d}_A\neq\tilde{d}_B$ denote the case that colleges choose different dates and $\tilde{d}_A=\tilde{d}_B$ denote the case that colleges choose the same date. In the case of $\tilde{d}_A=\tilde{d}_B$, students only can apply to one of the colleges, and we call colleges' decision in this case the limiting strategy. In addition, let $(\tilde{x}_1,\tilde{x}_2,\tilde{y}_1,\tilde{y}_2)$ be the profile of students' application strategy in the decentralized mechanism. For example, $(\tilde{x}_1,\tilde{x}_2,\tilde{y}_1,\tilde{y}_2)=(A,A,B,B)$ means the science-stream (humanities-stream) students apply to the science (humanities) college. We simply assume that students will apply to A when applying to A or B are indifferent.

Table 3 summarizes the expected payoffs of colleges in the decentralized mechanism when colleges use $\tilde{d}_A\neq\tilde{d}_B$. Note that in this case all students can apply to both colleges, and the matching outcome is the same as that in Table 1, the centralized matching. Thus, the expected payoffs in Table 3 are equal to that in Table 2. The equilibrium is hence the same as Proposition 1 when both colleges use $\tilde{d}_A\neq\tilde{d}_B$.

Table 4 summarizes the matching outcome in the decentralized mechanism when colleges use $\tilde{d}_A = \tilde{d}_B$. In the case of $(\tilde{A}, \tilde{B}) = (M, H)$ and $(\tilde{A}, \tilde{B}) = (M, M)$, good students play an application game in Figure 2. If $b < (1-\mu)a$, the Nash equilibrium is $(\tilde{x}_1, \tilde{y}_1) = (A, A)$ and if $b > (1-\mu)a$, the Nash equilibrium is $(\tilde{x}_1, \tilde{y}_1) = (A, B)$. Substituting matching outcomes of Table 4 into the utility function of colleges yields the expected payoffs of colleges in Table 5 when $\tilde{d}_A = \tilde{d}_B$ is used.

Comparing the expected payoffs of colleges in Tables 3 and 5, we solve the Nash equilibrium as in the following proposition.

Proposition 2. *In the decentralized mechanism, when* $b < (1-\mu)a$, *colleges will use* $\tilde{d}_A \neq \tilde{d}_B$ *and the equilibrium profile of tests is* $(\tilde{A}, \tilde{B}) = (M, M)$ *if* $\beta_A \in (0, (v_1 - v_2))$, *and* $(\tilde{A}, \tilde{B}) = (P, M)$ *if* $\beta_A \in ((v_1 - v_2), v_2)$. *When* $b > (1-\mu)a$, *colleges will use* $\tilde{d}_A = \tilde{d}_B$ *and the equilibrium profile of tests is* $(\tilde{A}, \tilde{B}) = (M, M)$ *and the equilibrium profile of students' application strategy is* $(\tilde{x}_1, \tilde{x}_2, \tilde{y}_1, \tilde{y}_2) = (A, A, B, A)$.

5. Stability and Efficiency

The efficiency and stability are defined as follows. A matching μ is efficient if it is not Pareto dominated by another matching; that is, there is no matching λ such that $\lambda(i) \succ_i \mu(i)$ or $\lambda^{-1}(j) R_j \mu^{-1}(j)$ for some student $i \in \{X, Y\}$ or some college $j \in \{A, B\}$ without harming others' welfare. A matching μ is stable if there is no matching λ such that $\lambda(i) \succ_i \mu(i)$ and $i R_j i'$ for $j = \lambda(i)$, $i \notin \{\mu^{-1}(j)\}$, and $i' \in \{\mu^{-1}(j)\}$ for some students $i \in \{X, Y\}$ and colleges $j \in \{A, B\}$.

According to Proposition 2, when college B's prestige is higher than the threshold of $(1-\mu)a$, $\tilde{d}_A = \tilde{d}_B$ is used by the two colleges and the math test is also chosen by the two colleges, i.e., colleges use both the limiting strategy and the expanding strategy. Note that the matching in this case is always $(x_1, x_2, y_1, y_2) \to (A, \varnothing, B, \varnothing)$ in ω_1 and ω_2. That is, the humanities (science) college can admit the good humanities-stream (science-stream) student for sure. Since both colleges admit their desired students and there is no seat left, this matching outcome is efficient and stable.

In particular, the mismatch effect can be eliminated by using the limiting strategy. This is because students only can apply to one college under the limiting strategy and this restriction plays a role resembling that of subject tests. In fact, Proposition 2 shows that, when $\tilde{d}_A = \tilde{d}_B$ is used, x_1 and y_1 actually apply to A and B, respectively.

6. Extension

We now consider a general case in which many students apply to colleges A and B. Their preferences are the same as the utility functions in Section 3. Students' caliber also has two types: good and ordinary. Let G and N be index sets with $|\mathrm{G}| = m$ and $|\mathrm{N}| = n$, where $|\mathrm{G}|$ and $|\mathrm{N}|$ respectively denote the number of elements in G and N, and we assume $m \le n$. The capacities of colleges are $q_A \in [1, m]$ and $q_B \in [1, m]$, respectively. Students in high school are divided into two groups, $X = \{x_{1i}, x_{2j}\}_{i\in\mathrm{M}, j\in\mathrm{N}}$ and $Y = \{y_{1i}, y_{2j}\}_{i\in\mathrm{M}, j\in\mathrm{N}}$, which are respectively the science-stream students and the humanities-stream students. For all $i \in \mathrm{M}$ and $j \in \mathrm{N}$, $\{x_{1i}, y_{1i}\}$ are the good-type students and $\{x_{2j}, y_{2j}\}$ are the ordinary-type students.

The entrance tests are imperfect devices to screen students' types. There are $(m+1)\times(n+1)$ states of nature $\{\omega_{lk}\}_{l\in\{0,\mathrm{M}\}, k\in\{0,\mathrm{N}\}}$ with probabilities of $p(\omega_{lk}) = p_{lk}$ and $\sum_{l\in\{0,\mathrm{M}\}, k\in\{0,\mathrm{N}\}} p_{lk} = 1$. Let $T = \{t_i\}_{i\in\{1,2,3,4\}}$ be the test scores such that $t_i > t_j$ for all $i < j$. An entrance test is a random mapping from students to score $g: \{X, Y\} \to T$. In the physics test, we have $X \to \{t_1, t_2\}$ such that

$$|\{x_{1i} : i \in \mathrm{M} \text{ and } g(x_{1i}) = t_1\}| = l$$
$$\text{and } |\{x_{2j} : j \in \mathrm{N} \text{ and } g(x_{2j}) = t_1\}| = m - l$$

in the states of $\{\omega_{lk}\}_{k\in\{0,\mathrm{N}\}}$ with $\sum_{k\in\{0,\mathrm{N}\}} p_{lk} = C_l^{\lambda m} C_{m-l}^{n} / C_m^{\lambda m+n}$ for $l \in \{0, \mathrm{M}\}$, where $C_l^{\lambda m}$ is a binomial coefficient and $\lambda > 1$ is a natural number to represent the validity of the test, called the validity coefficient.[11] Note that $\sum_{k\in\{0,\mathrm{N}\}} p_{lk} \to 1$ when $l = m$ and $\lambda \to \infty$, which means all good-type students have t_1 almost surely in the test. In the history test, we have $Y \to \{t_1, t_2\}$ such that

$$|\{y_{1i} : i \in \mathrm{M} \text{ and } g(y_{1i}) = t_1\}| = l$$

[11] In fact, $\sum_{k\in\{0,\mathrm{N}\}} p_{lk} = C_l^{\lambda m} C_{m-l}^{n} / C_m^{\lambda m+n}$ is a generalized hypergeometric distribution with mean $m\overline{p}$ and variance $m\overline{p}(1-\overline{p})(\lambda m + n - m)/(\lambda m + n - 1)$, where $\overline{p} = \lambda m/(\lambda m + n)$. When $\lambda = 1$, it is reduced to a simple hypergeometric distribution, which means each good-type or ordinary-type student has an equal probability to have t_1 in the test. In such a case, the test result provides no information about students' types. We thus assume $\lambda > 1$ to avoid this trivial case.

$$\text{and } |\{y_{2j} : j \in \mathrm{N} \text{ and } g(y_{2j}) = t_1\}| = m - l$$

in the states of $\{\omega_{lk}\}_{k\in\{0,\mathrm{N}\}}$ with $\sum_{k\in\{0,\mathrm{N}\}} p_{lk} = C_l^{\lambda m} C_{m-l}^{n} / C_m^{\lambda m+n}$ for $l \in \{0, \mathrm{M}\}$. In the math test, we have

$$\{x_{1i}, y_{1i}\}_{i\in\mathrm{M}} \to \{t_1, t_2\}$$
$$\text{and } \{x_{2j}, y_{2j}\}_{j\in\mathrm{N}} \to \{t_3, t_4\}$$

such that

$$|\{x_{1i} : i \in \mathrm{M} \text{ and } g(x_{1i}) = t_1\}| = l$$
$$\text{and } |\{y_{1i} : i \in \mathrm{M} \text{ and } g(y_{1i}) = t_1\}| = m - l$$

in the states of $\{\omega_{lk}\}_{k\in\{0,\mathrm{N}\}}$ with $\sum_{k\in\{0,\mathrm{N}\}} p_{lk} = C_l^{\lambda m} C_{m-l}^{m} / C_m^{\lambda m+m}$ for $l \in \{0, \mathrm{M}\}$, and

$$|\{x_{2j} : j \in \mathrm{N} \text{ and } g(x_{2j}) = t_3\}| = k$$
$$\text{and } |\{y_{2j} : j \in \mathrm{N} \text{ and } g(y_{2j}) = t_3\}| = n - k$$

in the states of $\{\omega_{lk}\}_{l\in\{0,\mathrm{M}\}}$ with $\sum_{l\in\{0,\mathrm{M}\}} p_{lk} = C_k^{\lambda n} C_{n-k}^{n} / C_n^{\lambda n+n}$ for $k \in \{0, \mathrm{N}\}$. In addition, we have

$$p_{lk} = \frac{C_l^{\lambda m} C_{m-l}^{m}}{C_m^{\lambda m+m}} \times \frac{C_k^{\lambda n} C_{n-k}^{n}}{C_n^{\lambda n+n}}$$

in the math test. Note that $p_{lk} \to 1$ when $l = m$, $k = n$, and $\lambda \to \infty$, which means $g(x_{1i}) = t_1$ and $g(x_{2j}) = t_3$ almost surely for all $i \in \mathrm{M}$ and $j \in \mathrm{N}$ in the math test.

Proposition 3. *In the centralized mechanism, the equilibrium in the general case is* $(\tilde{A}, \tilde{B}) = (M, M)$ *if* $\beta_A \in (0, (v_1 - v_2)(1 + \lambda - \lambda / \delta))$, *and* $(\tilde{A}, \tilde{B}) = (P, M)$ *if* $\beta_A \in ((v_1 - v_2)(1 + \lambda - \lambda / \delta), v_2)$, *where* $\delta = (\lambda + n / m)(1 - \lambda / (\lambda + 1))$.

Proof:

Let $\hat{x}_{1z}$ and $\hat{y}_{1z}$ be random variables that respectively represent the numbers of the humanities-stream and the science-stream good-type students having t_1 in a z test, for $z \in \{P, M, H\}$. In the centralized mechanism, $\tilde{B} = M$ is the dominant strategy for college B. We show this under two cases. First,
$E[u_B \mid (\tilde{A}, \tilde{B}) = (P, M)] > E[u_B \mid (\tilde{A}, \tilde{B}) = (P, H)]$ because

$$E[\hat{x}_{1M}] = m[\lambda / (\lambda + 1)],$$
$$E[\hat{y}_{1M}] = m[1 - \lambda / (\lambda + 1)], \text{ and}$$
$$E[\hat{y}_{1H}] = m[\lambda m / (\lambda m + n)];$$

and hence we have

$$E[u_B \mid (\tilde{A},\tilde{B})=(P,M)]=q_B v_1 \quad \text{and}$$
$$E[u_B \mid (\tilde{A},\tilde{B})=(P,H)]=q_B[\lambda m/(\lambda m+n)]v_1+q_B[1-\lambda m/(\lambda m+n)]v_2\,,$$

which is strictly smaller than $q_B v_1$. Second, $E[u_B \mid (\tilde{A},\tilde{B})=(M,M)]=q_B v_1$ because of $q_A\in[1,m]$ and $q_B\in[1,m]$; and $E[u_B \mid (\tilde{A},\tilde{B})=(M,H)]<E[u_B \mid (\tilde{A},\tilde{B})=(P,H)]$, these imply

$$E[u_B \mid (\tilde{A},\tilde{B})=(M,M)]>E[u_B \mid (\tilde{A},\tilde{B})=(M,H)].$$

Given $\tilde{B}=M$, we have

$$E[u_A \mid (\tilde{A},\tilde{B})=(M,M)]=q_A[\lambda/(\lambda+1)]v_1+q_A[1-\lambda/(\lambda+1)](v_1-\beta_A) \quad \text{and}$$
$$E[u_A \mid (\tilde{A},\tilde{B})=(P,M)]=q_A[\lambda m/(\lambda m+n)]v_1+q_A[1-\lambda m/(\lambda m+n)]v_2\,.$$

Let $\overline{\beta}=(v_1-v_2)(1+\lambda-\lambda/\delta)$, where $\delta=(\lambda+n/m)(1-\lambda/(\lambda+1))$. When $\beta_A=\overline{\beta}$, we have $E[u_A \mid (\tilde{A},\tilde{B})=(M,M)]=E[u_A \mid (\tilde{A},\tilde{B})=(P,M)]$. Thus, $\tilde{A}=M$ is used by A if $\beta_A<\overline{\beta}$. Q.E.D.

Note that the condition in Proposition 3 is equal to that in Proposition 1 if $m=n$. That is, $(v_1-v_2)(1+\lambda-\lambda/\delta)=(v_1-v_2)$ when $m=n$.

7. Concluding Remarks

In some labor markets, firms compete for desired workers by offering early contracts in order to restrict workers' choices (Li and Rosen, 1998; Li and Suen, 2000 and 2004; Suen, 2000) and this phenomenon is extensively studied and called unraveling in the literature (Sönmez, 1999; Ünver, 2001, Kagel and Roth, 2000; Ünver, 2005; Haruvy, Roth, and Ünver, 2006; Niederle and Roth, 2009; Niederle, Roth, and Ünver, 2013). Inefficiency occurs in those markets because information about workers is not fully revealed, and this inefficiency could be reduced by a centralized clearinghouse (Roth and Xing, 1994). A successful clearinghouse produces a stable matching (Niederle, McKinney, and Roth, 2005) and expands the market scope (Niederle and Roth, 2003). In our analysis, a parallel result is found in a decentralized college admissions such that an efficient and stable matching could be obtained by limiting applicants' choices in an expanded pool of applications.

References

Avery, C., A. Fairbanks, and R. Zeckhauser, (2003). *The Early Admissions Game: Joining the Elite*, Cambridge, MA: Harvard University Press.

Avery, C. and J. Levin, (2010). "Early Admissions at Selective Colleges," *American Economic Review* 100, 2125-2156.

Avery, C., S. Lee, and A. E. Roth, (2014). "College Admissions as Non-Price Competition: The Case of South Korea," *NBER Working Paper*, No. 20774.

Balinski, M. and T. Sönmez., (1999). "A Table of Two Mechanisms: Student Placement," *Journal of Economic Theory*, 84, 73-94.

Chade, H., G. Lewis, and L. Smith, (2014). "Student Portfolios and the College Admissions Problem," *Review of Economic Studies* 81, 971-1002.

Che, Y. K., and Y. Koh, (2016). "Decentralized College Admissions," *Journal of Political Economy*, 124(5), 1295-1338.

Chen, W-C., and Y-C. Kao, (2014). "Simultaneous Screening and College Admissions," *Economics Letters* 122, 296-298.

Chen, W-C. and Y-C. Kao, (2016). "Limiting Applications in College Admissions and Evidence from Conflicting Examinations," working paper, available at SSRN 2512712.

Chen, W-C., Y-Y. Chen, and Y-C. Kao, (2017). "Limited Choice in College Admissions: An Experimental Study," *Games and Economic Behavior*, forthcoming.

Chen, Y. and O. Kesten, (2016). "Chinese College Admissions and School Choice Reforms: A Theoretical Analysis," *Journal of Political Economy*, 125(1), 99-139.

Gale, D. and L. S. Shapley, (1962). "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9-15.

Haruvy, E., A. E. Roth, and M. U. Ünver, (2006). "The Dynamics of Law Clerk Matching: An Experimental and Computational Investigation of Proposals for Reform of the Market," *Journal of Economic Dynamics and Control* 30.3, 457-486.

Kagel, J. H., and A. E. Roth, (2000). "The Dynamics of Reorganization in Matching Markets: A Laboratory Experiment Motivated by a Natural Experiment," *The Quarterly Journal of Economics* 115.1, 201-235.

Kao, Y-C and C-H Lin, (2017). "Enrollment Game: Conflicting Entrance Examinations of Graduate Colleges in Taiwan," *Taiwan Economic Review* 45, 495-513.

Kim, M., (2010). "Early Decision and Financial Aid Competition among Need-Blind Colleges and Universities," *Journal of Public Economics* 94, 410-420.

Lee, S-H., (2009). "Jumping the Curse: Early Contracting with Private Information in University Admissions," *International Economic Review* 50, 1-38.

Li, H., and S. Rosen, (1998). "Unraveling in Matching Markets," *American Economic Review* 88, 371-387.

Li, H., and W. Suen, (2000). "Risk Sharing, Sorting, and Early Contracting," *Journal of Political Economy* 108, 1058-1091.

Li, H., and W. Suen, (2004). "Self-Fulfilling Early-Contracting Rush," *International Economic Review* 45, 301-324.

Li, H-C, W-C Lee, and H-M Lien, (2016). "Investigating the Equality of Two-Track College Recruitment—Evidence from Administrative Data of National Chengchi University," *Academia Economic Papers*, forthcoming.

Luoh, M-C, (2016). "Who are NTU Students? (2001-2014)—the Effects of Multi-Channel Admission Program," *Taiwan Economic Review*, forthcoming.

Niederle, M., C. N. McKinney, and A. E. Roth, (2005). "The Collapse of a Medical Labor Clearinghouse (and Why Such Failures Are Rare)," *American Economic Review* 95.2, 878-889.

Niederle, M., and A. E. Roth, (2003). "Unraveling Reduces Mobility in a Labor Market: Gastroenterology with and without a Centralized Match," *Journal of Political Economy* 111.6, 1342-1352.

Niederle, M., and A. E. Roth, (2009). "Market Culture: How Rules Governing Exploding Offers Affect Market Performance," *American Economic Journal: Microeconomics* 1.2, 199-219.

Niederle, M., A. E. Roth, and M. U. Ünver, (2013). "Unraveling Results from Comparable Demand and Supply: An Experimental Investigation," *Games* 4, 243-282.

Roth, A. E., and X. Xing, (1994). "Jumping the Gun: Imperfections and Institutions Related to the Timing of Market Transactions," *American Economic Review* 84, 992-1044.

Sönmez, T., (1999). "Can Pre-arranged Matches be Avoided in Two-sided Matching Markets?" *Journal of Economic Theory* 86.1, 148-156.

Suen, W., (2000). "A Competitive Theory of Equilibrium and Disequilibrium Unravelling in Two-Sided Matching," *Rand Journal of Economics* 31, 101–20.

Ünver, M. U., (2001). "Backward Unraveling over Time: The Evolution of Strategic Behavior in the Entry Level British Medical Labor Markets," *Journal of Economic Dynamics and Control* 25.6, 1039-1080.

Ünver, M. U., (2005). "On the Survival of Some Unstable Two-sided Matching Mechanisms," *International Journal of Game Theory* 33.2, 239-254.

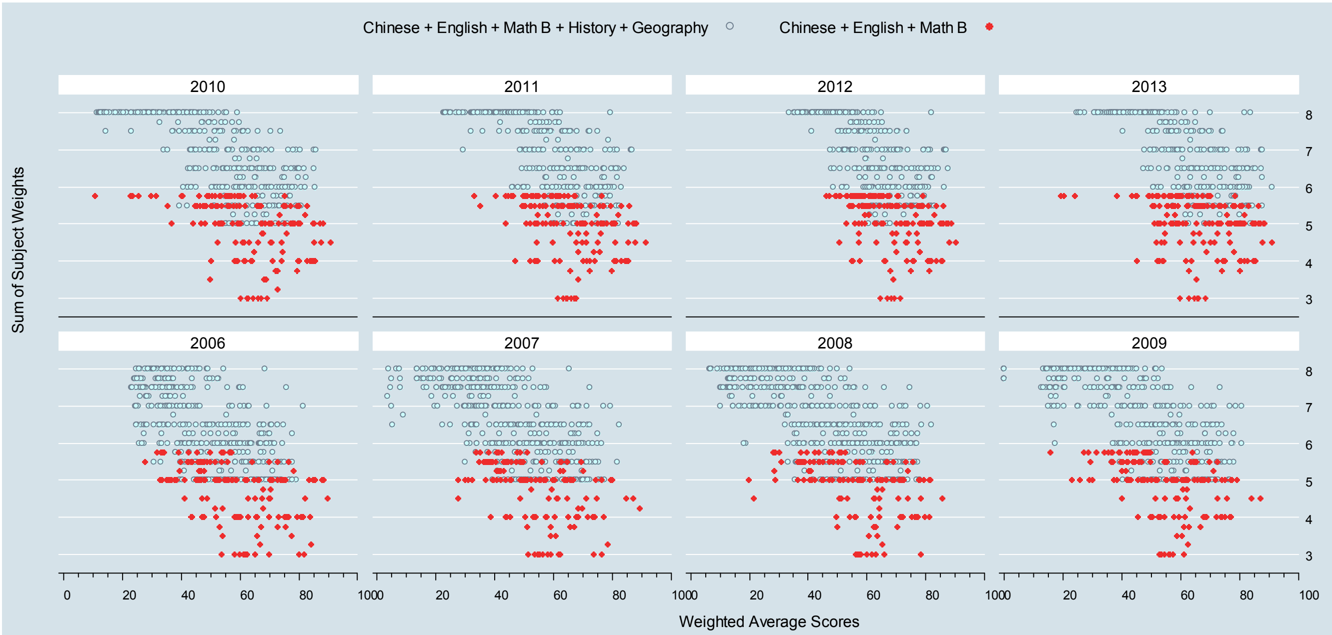

Figure 1. Distribution of student scores under different testing policies

		y_1	
		A	B
x_1	A	$\mu a, (1-\mu)a$	a, b
	B	b, a	$\mu b, (1-\mu)b$

Figure 2. Game of students when $(\tilde{A}, \tilde{B}) = (M, H)$ or $(\tilde{A}, \tilde{B}) = (M, M)$

Table 1. Matching outcomes with different strategies in the centralized mechanism

	ω_1	ω_2
$(\tilde{A},\tilde{B})=(P,H)$	$(x_1,x_2,y_1,y_2)\to(A,\varnothing,B,\varnothing)$	$(x_1,x_2,y_1,y_2)\to(\varnothing,A,\varnothing,B)$
$(\tilde{A},\tilde{B})=(P,M)$	$(x_1,x_2,y_1,y_2)\to(A,\varnothing,B,\varnothing)$	$(x_1,x_2,y_1,y_2)\to(\varnothing,A,B,\varnothing)$
$(\tilde{A},\tilde{B})=(M,H)$	$(x_1,x_2,y_1,y_2)\to(A,\varnothing,B,\varnothing)$	$(x_1,x_2,y_1,y_2)\to(\varnothing,\varnothing,A,B)$
$(\tilde{A},\tilde{B})=(M,M)$	$(x_1,x_2,y_1,y_2)\to(A,\varnothing,B,\varnothing)$	$(x_1,x_2,y_1,y_2)\to(B,\varnothing,A,\varnothing)$

Table 2. Expected payoffs of colleges in the centralized mechanism

	$E(u_A)$	$E(u_B)$
$(\tilde{A},\tilde{B})=(P,H)$	$\pi v_1+(1-\pi)v_2$	$\pi v_1+(1-\pi)v_2$
$(\tilde{A},\tilde{B})=(P,M)$	$\pi v_1+(1-\pi)v_2$	v_1
$(\tilde{A},\tilde{B})=(M,H)$	$\pi v_1+(1-\pi)(v_1-\beta_A)$	$\pi v_1+(1-\pi)v_2$
$(\tilde{A},\tilde{B})=(M,M)$	$\pi v_1+(1-\pi)(v_1-\beta_A)$	v_1

Table 3. Expected payoffs of colleges when $\tilde{d}_A \neq \tilde{d}_B$

	$E(u_A)$	$E(u_B)$
$(\tilde{A}, \tilde{B}) = (P, H)$	$\pi v_1 + (1-\pi) v_2$	$\pi v_1 + (1-\pi) v_2$
$(\tilde{A}, \tilde{B}) = (P, M)$	$\pi v_1 + (1-\pi) v_2$	v_1
$(\tilde{A}, \tilde{B}) = (M, H)$	$\pi v_1 + (1-\pi)(v_1 - \beta_A)$	$\pi v_1 + (1-\pi) v_2$
$(\tilde{A}, \tilde{B}) = (M, M)$	$\pi v_1 + (1-\pi)(v_1 - \beta_A)$	v_1

Table 4. Matching outcomes when $\tilde{d}_A = \tilde{d}_B$

	$(\tilde{x}_1, \tilde{x}_2, \tilde{y}_1, \tilde{y}_2)$	ω_1	ω_2
$(\tilde{A}, \tilde{B}) = (P, H)$	(A, A, B, B)	$(x_1, x_2, y_1, y_2) \to (A, \varnothing, B, \varnothing)$	$(x_1, x_2, y_1, y_2) \to (\varnothing, A, \varnothing, B)$
$(\tilde{A}, \tilde{B}) = (P, M)$	(A, A, B, A)	$(x_1, x_2, y_1, y_2) \to (A, \varnothing, B, \varnothing)$	$(x_1, x_2, y_1, y_2) \to (\varnothing, A, B, \varnothing)$
$(\tilde{A}, \tilde{B}) = (M, H)$	$\begin{cases}(A, A, A, B) \\ (A, A, B, B)\end{cases}$	$\begin{cases}(x_1, x_2, y_1, y_2) \to (A, \varnothing, \varnothing, B) \\ (x_1, x_2, y_1, y_2) \to (A, \varnothing, B, \varnothing)\end{cases}$	$\begin{cases}(x_1, x_2, y_1, y_2) \to (\varnothing, \varnothing, A, B) \\ (x_1, x_2, y_1, y_2) \to (\varnothing, A, \varnothing, B)\end{cases}$
$(\tilde{A}, \tilde{B}) = (M, M)$	$\begin{cases}(A, B, A, B) \\ (A, A, B, A)\end{cases}$	$\begin{cases}(x_1, x_2, y_1, y_2) \to (A, B, \varnothing, \varnothing) \\ (x_1, x_2, y_1, y_2) \to (A, \varnothing, B, \varnothing)\end{cases}$	$\begin{cases}(x_1, x_2, y_1, y_2) \to (\varnothing, \varnothing, A, B) \\ (x_1, x_2, y_1, y_2) \to (A, \varnothing, B, \varnothing)\end{cases}$

Table 5. Expected payoffs of colleges when $\tilde{d}_A = \tilde{d}_B$

	$(\tilde{x}_1, \tilde{x}_2, \tilde{y}_1, \tilde{y}_2)$	$E(u_A)$	$E(u_B)$
$(\tilde{A}, \tilde{B}) = (P, H)$	(A, A, B, B)	$\pi v_1 + (1-\pi)v_2$	$\pi v_1 + (1-\pi)v_2$
$(\tilde{A}, \tilde{B}) = (P, M)$	(A, A, B, A)	$\pi v_1 + (1-\pi)v_2$	v_1
$(\tilde{A}, \tilde{B}) = (M, H)$	$\begin{cases}(A, A, A, B)\\(A, A, B, B)\end{cases}$	$\begin{cases}\pi v_1 + (1-\pi)(v_1 - \beta_A)\\ v_1\end{cases}$	$\begin{cases}v_2\\ \pi v_1 + (1-\pi)v_2\end{cases}$
$(\tilde{A}, \tilde{B}) = (M, M)$	$\begin{cases}(A, B, A, B)\\(A, A, B, A)\end{cases}$	$\begin{cases}\pi v_1 + (1-\pi)(v_1 - \beta_A)\\ v_1\end{cases}$	$\begin{cases}v_2\\ v_1\end{cases}$