Equilibrium Coalitional Behavior*

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October 27, 2017

Abstract

I develop two related solution concepts, equilibrium coalitional behavior and credible equilibrium coalitional behavior, which capture foresight and impose the requirement that each coalition in a sequence of coalitional moves chooses optimally among all its available options. The model does not require, but may use, the apparatus of a dynamic process or a protocol that specifies the negotiation procedure underlying coalition formation. Therefore, it forms a bridge between the non-cooperative and the cooperative approaches to foresight.

JEL classification: C70; C71; C72; D71

1 Introduction

The paper contributes to the literature on farsighted coalition formation. I define a cooperative domain, *extended coalitional games*, and two related solution concepts on this domain, *equilibrium coalitional behavior* (ECB) and *credible equilibrium coalitional behavior* (CECB), that capture foresight.

Extended Coalitional Games

Let N be a set of players and Z be a set of nodes. A_z denotes the set of all possible coalitional actions available at node z, where an action is a triple (z, z', S) that denotes the possibility of coalition S to move the game from node z to node z'. At some nodes, it might not be feasible for any coalition to take an action. This is represented as the particular action $(z, z, \emptyset)$ that denotes remaining at node z. A path is a sequence of actions. A terminal path is a path that is either infinite or that ends with 'no action'.

An *extended coalitional game* is defined as $\Gamma = \{N, Z, \{A_z\}_{z\in Z}, \{\succsim_i\}_{i\in N}\}$, where the preferences are defined over the set of terminal paths.

*I am particularly grateful to Rajiv Vohra for continuous support and guidance throughout the preparation of this work. I would also like to thank Pedro Dal Bo, Mark Dean, Geoffroy de Clippel, Jack Fanning, Licun Xue, Roberto Serrano, seminar participants at X. Winter Workshop at Koç University and 22nd Coalition Theory Workshop for helpful comments and suggestions.

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An *extended coalitional game* is a direct generalization of an extensive form game of perfect information, but it is most closely related to the domain of *the abstract game* (see, for example, Chwe (1994), Rosenthal (1972) and Xue (1998)). There are two differences between an abstract game and an extended coalitional game, these are:

1) In an abstract game utilities are defined over the nodes, whereas in an extended coalitional game they are defined over the paths. This is the major difference between the two domains. Nevertheless, given an abstract game, the utility a player gets from a path can be appropriately defined based on the approach one wants to use. There are two main approaches in the literature: the static approach and the dynamic approach. The former assumes that players only care about the final outcome the negotiations lead to, hence the utility of a path should correspond to the utility of the final node on the path. The latter approach assumes that players discount the utilities with a discount factor δ, hence the utility of a path should correspond to the discounted utility of the nodes along the path (see Section 4.2 for a formal analysis).[1]

The point is that, defining the utilities over the paths cause no loss in generality. This is because, the approaches on the abstract game also implicitly use utilities over paths derived from the utilities on the nodes.

2) In an abstract game, it is always possible to take 'no action' in each node, whereas in an extended coalitional game the modeler is free to choose whether to include 'no action' as a possible action. This is a minor difference that is self-evident.

(Credible) Equilibrium Coalitional Behavior

A *coalitional behavior* is a complete plan of action defined on the extended coalitional game. That is, it assigns a unique action to each node of an extended coalitional game. Thereby, it also assigns a 'path of play' (terminal path) to each node of an extended coalitional game.

A coalition $S \subseteq N$ can deviate from a coalitional behavior by refusing to take some of the prescribed actions, for this S needs to have a non-empty intersection with the coalitions taking these actions. Instead, S can take actions unspecified by the coalitional behavior, for this S needs to contain the coalitions that have the power to take these actions. A deviation is profitable if every $i \in S$ prefers the resulting path of play to the initial path of play at each node where an action changes.

An ECB is simply a coalitional behavior that is immune to profitable deviations.

A profitable deviation is credible if the deviation is not going to be followed by other deviations, in a sense that will be made clear later on. A CECB is a coalitional behavior that is immune to profitable and credible deviations.

1.1 Contribution

The literature on farsighted coalition formation has developed extensively in recent years. Earlier solution concepts have suffered from what is known as the problem of maximality (see Ray and Vohra (2014)), where players used to form unreasonable expectations instead of taking the best course of action available to them.

[1] There are other approaches. For instance, some authors define a non-cooperative game from the abstract game and define their concepts over this non-cooperative game. Notable examples include Herings et al. (2004) and Granot and Hanany (2016). This approach is discussed in Section 4.2.1.

(C)ECB directly overcomes this problem by imposing consistent expectations. This is not new as several other solution concepts have overcome this problem similarly before (see Dutta and Vohra (2016), Konishi and Ray (2003) and Ray and Vohra (2014)). Nevertheless, there are two ways in which (C)ECB supports this claim further:

Backward Induction: Intuitively, one might expect a farsighted solution concept to incorporate considerations similar to backward induction in a cooperative setting. Nevertheless, in the absence of the structure of an extensive form, it is not clear how to incorporate such considerations. *Extended coalitional games* have this structure in certain contexts and indeed we see that (C)ECB can be found through backward induction whenever it is possible.

Noncooperative Justification: The problem of maximality is not present in non-cooperative solution concepts such as the subgame perfect equilibrium. Hence, if it is possible to develop an intuitive non-cooperative game from an extended coalitional game, whose subgame perfect equilibrium outcomes coincide with the cooperative solution concept, then this is a good indication that the solution concept we have started with incorporates the notion of maximality. With (C)ECB, this exercise can be done under certain conditions on the domain.

Secondly, (C)ECB directly looks at possible profitable deviations to see if a plan of action is stable. Although many other solution concepts also require immunity from deviations, (C)ECB diverges in one important aspect: it does not restrict attention to 'one-step deviations'.

One might surmise that restriction to one-step deviations is without loss of generality. But depending on the situation and how the utilities are defined, this might not be the case. Indeed, there are situations, where one cannot replicate a profitable deviation by a one-step deviation under (C)ECB. Mainly because, a deviation to a 'cycle' (or an infinite path) might be the only profitable deviation available.

Hence, the issue is also related to the issue of cycles. The static solution concepts never prescribe cycles. As the utilities are defined over the paths, (C)ECB allows cycles to be incorporated as in the dynamic approach. But, even the dynamic solution concepts such as the EPCF (see Konishi and Ray (2003) and Ray and Vohra (2014)) ignore cycles to some extent by restricting attention to one-step deviations. In some situations this assumption rules out (possibly profitable) deviations to cycles (see the example in Figure 5 in Section 4.2). This is no longer the case with (C)ECB.

Finally, and perhaps most importantly, the literature on farsighted coalition formation is very fragmented and (C)ECB shows that it is possible to connect the different approaches to foresight.

(C)ECB is a versatile solution concept that might be used to study a variety of domains existing in the literature. The versatility of (C)ECB is a result of the generality of its domain, *extended coalitional games*.

In an extended coalitional game utilities of the players are defined over the paths. This allows one the flexibility to define the solution concept as a static concept, where players only care about the final outcome the negotiations lead to, and as a dynamic concept, where players also care about the outcomes along the paths through which an agreement is reached.

In this way (C)ECB is able to bridge these two different strands of the literature on farsighted coalition formation. Indeed, ECB can be directly related to solution concepts in both approaches: SREFS (see Dutta and Vohra (2016)) in the static approach and EPCF (see Konishi and Ray (2003)) in the dynamic approach.[2]

The versatility of the extended coalitional games also allow us to study certain noncooperative games with (C)ECB. In particular, extensive form games of perfect information are extended coalitional games and (C)ECB is closely related to the most popular solution concept on this domain: the subgame perfect equilibrium. In finite extensive form games of perfect information (C)ECB reduces down to subgame perfect equilibrium and ECB refines subgame perfect equilibrium in infinite horizon games.

The variety of the approaches and solution concepts used to study farsighted coalition formation is one of the problems of the literature: "as one surveys the landscape of this area of research, the first feature that attracts attention is the fragmented nature of the literature." (Ray and Vohra (2014)).

Hence, it would be useful to have a framework that could connect the different approaches to foresight. By allowing a translation of the assumptions of the different approaches into its own structure, extended coalitional games provide this framework to a certain extent.

Indeed, ECB shows that the developments in the literature have made it possible to achieve some level of unification between both the non-cooperative and cooperative strands of literature and within the cooperative approach. The evidence to this is the relationships ECB engenders with solution concepts in all three approaches discussed above. This is especially important when one considers how fragmented the literature on farsighted coalition formation is.

1.2 Outline of the Results

A CECB exists in finite extended coalitional games. In finite acyclic games[3] both ECB and CECB satisfy the *one step deviation property* and hence can be found through a simple backward induction algorithm.

ECB and CECB can be related to the myopic core and the stable set in an indirect way. In particular, a simple characterization of CECB can be obtained by recursively finding stable sets (with the profitable deviation relationship) in finite acyclic games. The same can be done for the ECB with the core. Hence, in finite acyclic games, one can view each solution concept as incorporating foresight through bringing recursivity to the myopic notions of the core and the stable set.

On the noncooperative side, one can find a simple bargaining game defined over an extended coalitional game whose stationary subgame perfect equilibrium is related to (C)ECB. In particular, in games that satisfy a condition called strong acyclicity, each ECB can be supported as a stationary subgame perfect equilibrium of the bargaining game. Furthermore, if we impose further conditions, then we can obtain full implementation of both ECB and CECB through the stationary subgame perfect equilibrium of the bargaining game.

One attractive feature of (C)ECB is that it can be applied to general classes of games that are used extensively in the literature. In Section 4, I choose three such domains: the extensive form games, the abstract game and characteristic function games. I demonstrate

[2]CECB is also related to SREFS (albeit not as strongly as ECB), but not to EPCF.

[3]A game is acyclic if whenever nodes $z \neq z'$ is reachable from z' through some sequence of actions, then z' is not reachable from z

that (C)ECB is related to attractive solution concepts in these domains: subgame perfect equilibrium in extensive form games, SREFS and EPCF in the abstract game and the core in characteristic function games.

I start with the description of the domain and the solution concept in Section 2. Section 3 analyzes the properties of (C)ECB, Section 4 studies (C)ECB and compares it to solution concepts in extensive form games, characteristic function games and the abstract game. Section 5 concludes with the literature review. All proofs are in the Appendix.

2 Preliminaries

2.1 Extended Coalitional Games

An extended coalitional game is defined as $\Gamma = \{N, Z, \{A_z\}_{z\in Z}, \{\succeq_i\}_{i\in N}\}$, where N is the finite set of players, Z is the set of nodes, A_z is the set of actions available at node $z \in Z$ and $\succeq_i$ is the preference relation of player $i \in N$ over the set of terminal paths, which will be defined shortly.

An action is a triple (z, z', S), where the first entry $z \in Z$ denotes the node at which the action can be taken, the second entry $z' \in Z$ denotes the node to which the action is leading to and the third entry denotes the coalition $S \subseteq N$ that can take the action. This coalition is called the *initiator* of the action. At some nodes, it might be possible for no coalition to take an action. This is represented as the particular action $(z, z, \emptyset)$, that denotes the possibility of remaining at node z.

A_z is required to be non-empty. If there is 'no action' available at node z, then $A_z = (z, z, \emptyset)$.

A path is a sequence of actions $\{a_k\}_{k=1,\ldots,K} = \{(z_k, z_{k+1}, S_k)\}_{k=1,\ldots,K}$, where K might be infinite. A path $\{a_k\}_{k=1,\ldots,K}$ is terminal if it is infinite or if $a_K = (z_K, z_K, \emptyset)$. Let $\mathcal{P}$ denote the set of all terminal paths. $\succeq_i$ denotes the preference relation of $i \in N$ on $\mathcal{P}$.

2.2 Equilibrium Coalitional Behavior

A coalitional behavior is a complete plan of action. It prescribes a unique action to each node of an extended coalitional game. Let A denote the set of all actions, i.e. $A = \cup_{z\in Z} A_z$.

Definition 1. *Coalitional Behavior*

A coalitional behavior is a mapping $\phi : Z \to A$, where $\phi(z) \in A_z$ for all $z \in Z$.

In turn, each coalitional behavior also prescribes a unique terminal path at each node of an extended coalitional game, this path is called the path of play. Let $\mathcal{P}_z$ denote the set of terminal paths that start at node z.

Definition 2. *Path of play*

Given a coalitional behavior ϕ, a path of play is a mapping $\sigma : Z \to \mathcal{P}$ such that for each $z \in Z$, $\sigma(z) = \{(z_k, z_{k+1}, S_k)\}_{k=1,..,K} = \{\phi(z_k)\}_{k=1,..,K} \in \mathcal{P}_z$, where $z_1 = z$.

To define our solution concept, we need the definition of a coalitional deviation from a prescribed plan of action. Intuitively, no coalition can be forced to take an action. Hence, each coalition S might refuse to take a prescribed action $\phi(z)$ if S has a nonempty intersection with the initiators of $\phi(z)$. Instead, S can take an action (z, z', T), which it has the power to implement, i.e., where S contains the initiators of this action. A deviation by S is profitable if at every node at which an action changes, everybody in S prefers the new path of play to the initially prescribed path of play.

Definition 3. *Coalitional Deviation*

$S \subseteq N$ has a deviation from a coalitional behavior ϕ to a coalitional behavior ϕ' if for every $z \in Z$ such that $\phi(z) \neq \phi'(z)$ we have

- *If $\phi(z) = (z, z', T)$, where $T \neq \emptyset$, then $S \cap T \neq \emptyset$ (If an action specified by ϕ is not taken, then S has a member who can refuse to take this action)*
- *If $\phi'(z) = (z, z', T)$ then $S \supseteq T$ (If an action not specified by ϕ is taken, then S should be able to induce this action)*

We say that the deviation by S is profitable if for every $z \in Z$ such that $\phi(z) \neq \phi'(z)$, we have $\sigma'(z) \succ_i \sigma(z)$ for all $i \in S$.[4]

An ECB is simply a coalitional behavior immune to profitable deviations.

Definition 4. *Equilibrium Coalitional Behavior*

A coalitional behavior ϕ is an ECB if there does not exist a profitable coalitional deviation from ϕ.

Note the simplicity of the definition, which is in contrast to most other farsighted solution concepts. The typical approach in the literature is to look at a set of outcomes or paths that has some 'stability' properties, whereas ECB directly looks at profitable deviations from plans of actions. Even though defined with no reference to stability, we will see that ECB still embodies a notion of stability.

In contrast to this simplicity, ECB is also a powerful concept in the sense that it is related to a wide variety of solution concepts from different strands of the literature, such as subgame perfect equilibrium, the core, SREFS and EPCF. However, ECB has issues regarding existence. CECB gets over this problem by also considering the credibility of a profitable deviation.

2.3 Credible Equilibrium Coalitional Behavior

The subgame at $z \in Z$, denoted by $\Gamma(z)$, is the game that includes only those nodes that are reachable from z and the actions between these nodes. The utilities are defined the same way as in the original game. We say that a subgame is nontrivial, if it includes (strictly) more than one node.

A game is a basic game if every non-trivial subgame of the game is the game itself. An example of a basic game is a tree of length one, another example would be a stand-alone cycle. We will first define the notion of credibility for a basic game and then extend it to any extended coalitional game.

The idea is that a profitable deviation should only be credible if it is not going to be followed by further deviations. To incorporate this idea, I define a credible set of coalitional behaviors as a set of coalitional behaviors such that any profitable deviation from a coalitional behavior in the set might be followed by a further profitable deviation

[4] σ' and σ are the path of plays corresponding to ϕ' and ϕ, respectively.

back into the credible set. Hence, any profitable deviation from a credible coalitional behavior is avoided by a further deviation to a credible coalitional behavior. One should note the similarity of the definition to the definition of the consistent set of Chwe (1994).

Let Φ denote the set of all coalitional behaviors. For $\phi, \phi' \in \Phi$, we say that ϕ dominates ϕ', $\phi >_D \phi'$, if there exists a profitable deviation from ϕ' to ϕ.

Definition 5. *In a basic game, a set of coalitional behaviors $Y \subseteq \Phi$ is credible if $\phi \in Y$ if and only if for all $\phi' \in \Phi$ such that $\phi' >_D \phi$ there exists $\phi^* \in Y$ with $\phi^* >_D \phi'$.*

There might be multiple credible sets, but the lemma below establishes that there always exists a unique credible set that contains all of the other credible sets. This set is called the *largest credible set* (LCRS), again following the largest consistent set of Chwe (1994).

Lemma 1. *In any basic game, there uniquely exists a Y such that Y is credible and (Y' credible $\implies Y' \subseteq Y$). Y is called the LCRS.*

In a basic game, we say that a coalitional behavior is credible if it is included in the LCRS. Similarly, we say that a profitable deviation from ϕ to ϕ' is credible if the deviation cannot be followed by a profitable deviation to a credible coalitional behavior.

Definition 6. *Credible Deviation in a Basic Game*

In a basic game, a profitable deviation from ϕ to ϕ' is credible if there does not exist a profitable deviation from ϕ' to a credible coalitional behavior.

It is easy to see that a coalitional behavior ϕ is immune to profitable and credible deviations if and only if ϕ is in the LCRS. Now, to obtain the definition of CECB we need to extend the definition of a credible deviation from a basic game to any extended coalitional game.

Once a coalition S deviates from a coalitional behavior ϕ' to a coalitional behavior ϕ, they would expect the game to continue as in ϕ at any proper subgame $\Gamma(z)$. That is, players might assume that any action leading to a proper subgame is going to be followed by $\sigma(z)$.[5] I will define a reduced game based on this idea.

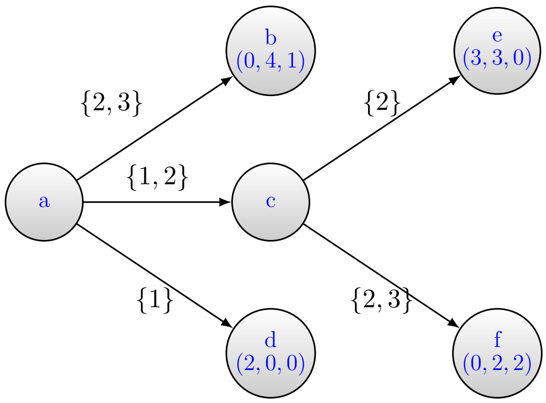
Given a coalitional behavior ϕ, the game obtained by removing $\Gamma(z)$ from Γ with respect to ϕ, denoted by $\Gamma \setminus \Gamma_\phi(z)$, is the game obtained through Γ by removing every action in $\Gamma(z)$, removing every node in $\Gamma(z)$ except for z, assigning $A_z = (z, z, \emptyset)$ and setting the utility of each terminal path p that ends at z in the new game equal to the utility of $(p, \sigma(z))$ in the original game. The utility of all other terminal paths remain the same.

A proper subgame is maximal if it is not (strictly) included in any other proper subgame. The reduced game at node z^* given ϕ, denoted by $\Gamma(z^*, \phi)$, is obtained from $\Gamma(z^*)$ through removing all maximal proper subgames of $\Gamma(z^*)$ with respect to ϕ. See Figure 1 for an example.

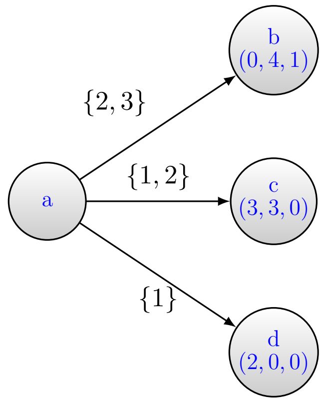
If the graph of the game is a tree then each reduced game is going to be a tree of length one. Whereas if the game includes cycles then the reduced game at a node on a cycle will include all the cycles this node belongs to. Also note that every reduced game is necessarily a basic game.

We are going to check the credibility of a deviation at each reduced game, the idea being that if the players expect the game to be continued as in ϕ then at any reduced game their deviation should be credible.

[5] σ corresponds to the path of play induced by ϕ.



(a) The Game



(b) Reduced game at node a given ϕ_2, where $\phi_2(c) = (c, e, \{2\})$

Figure 1: An example

To formalize this idea we need an additional definition. The deviation from ϕ' to ϕ restricted to the reduced game $\Gamma(z, \phi)$ is the deviation in $\Gamma(z, \phi)$ from the coalitional behavior that matches with ϕ' in the non-terminal nodes included in $\Gamma(z, \phi)$ to the coalitional behavior that matches with ϕ in the non-terminal nodes included in $\Gamma(z, \phi)$.

Definition 7. *Credible Deviation*

A profitable deviation from ϕ' to ϕ is credible if for each $z \in Z$ for which $\phi(z) \neq \phi'(z)$, the deviation restricted to $\Gamma(z, \phi)$ is credible in the reduced game $\Gamma(z, \phi)$.

Definition 8. *Credible ECB*

A coalitional behavior ϕ is a CECB if it is immune to profitable and credible deviations.

2.4 An Example

Consider the game represented in Figure 1a). There are three players, 1, 2 and 3. There are six nodes, four of which is terminal. The utility of a terminal path is the utility of the node the path terminates in.

Consider the coalitional behavior ϕ_1 that assigns $\phi_1(a) = (a, d, \{1\})$ and $\phi_1(c) = (c, f, \{2, 3\})$. This is not an ECB, because the coalition $\{1, 2\}$ has a profitable deviation in which player 1 blocks the action $(a, d, \{1\})$, instead the coalition $\{1, 2\}$ takes the action $(a, c, \{1, 2\})$ and player 2 blocks the action $(c, f, \{2, 3\})$ at node c, instead takes the action $(c, e, \{2\})$. Let us call the resulting coalitional behavior ϕ_2, where $\phi_2(a) = (a, c, \{1, 2\})$ and $\phi_2(c) = (c, e, \{2\})$.

But now, the coalition $\{2, 3\}$ has a profitable deviation from ϕ_2 in which player 2 blocks the action $(a, c, \{1, 2\})$ and instead the coalition $\{2, 3\}$ takes the action $(a, b, \{2, 3\})$. Let us call the resulting coalitional behavior ϕ_3, where $\phi_3(a) = (a, b, \{2, 3\})$ and $\phi_3(c) = (c, e, \{2\})$. It is easy to see that there is no profitable deviation from ϕ_3, and indeed this is the unique ECB of this game.

CECB incorporates the idea that the initial deviation from ϕ_1 is not credible since it is going to be followed by another deviation. To see this formally, consider the reduced game at node a given ϕ_2, which is depicted in Figure 1b). The unique LCRS of this game is $\{(a, b, \{2, 3\}), (a, d, \{1\})\}$. Note that $(a, d, \{1\})$ is in the LCRS, because any profitable deviation will be followed by another profitable deviation to an action in the LCRS. Hence the initial profitable deviation is not credible.

Nevertheless, there exists a profitable and credible deviation from ϕ_1 in which player 2 blocks the action $(c, f, \{2, 3\})$ and instead takes the action $(c, e, \{2\})$. Note that the reduced game at node c is the subgame that starts at node c which has a sigleton LCRS composed of $(c, e, \{2\})$, hence the deviation is credible. Let us call the resulting coalitional behavior ϕ_4, where $\phi_4(a) = (a, d, \{1\})$ and $\phi_4(c) = (c, e, \{2\})$.

ϕ_4 is a CECB and since ϕ_3 is an ECB, it is also a CECB. One can easily check that these are all the CECBs of this game.

3 Properties

3.1 One Step Deviation Property and Backward Induction

We say that a deviation is a one-step deviation if every action involved in the deviation stems from the same node. We say that ECB satisfies the one-step deviation property if the existence of a profitable deviation implies the existence of a profitable one-step deviation. Similarly, CECB satisfies the one-step deviation property if the existence of a credible and profitable deviation implies the existence of a one-step credible and profitable deviation. An acyclic game is a game that does not contain cycles.[6]

Proposition 1. *In finite acyclic games, both ECB and CECB satisfy the one-step deviation property.*

In such games any deviation leads to finite paths and hence in any deviation there exists a node at which the action changes, but the action at each node reachable from this node stays the same. The deviation in which only the action at this node changes would be one-step, (credible) and profitable.

The one step deviation property no longer holds in infinite horizon games or games with cycles. The reason is that, under a (C)ECB a deviation is profitable only if it improves the payoffs of the deviators at every node at which an action changes. Hence, in infinite games, for a deviation to be profitable a coalition might need to promise to change its actions at infinitely many places. An example is available in Section 4.1.

A nice implication of this result is that both ECB and CECB can be found through backward induction in finite acyclic games. Let Γ be a finite acyclic game. For any $z \in Z$ let $\Gamma(z)$ denote the subgame starting at z and let $l(\Gamma(z))$ be the length of this game (the length of the longest terminal path).

Backward induction works as follows. For any z for which $l(\Gamma(z)) = 1$, find a (C)ECB for $\Gamma(z)$, let the path of play at z be denoted by $p(z)$. Suppose the (C)ECBs of every subgame of length $l < k$ is found, take any z with $l(\Gamma(z)) = k$. Let $\hat{\Gamma}(z)$ denote the reduced basic game in which the set of nodes are only those nodes that are directly reachable from z and the utility of taking an action (z, z', S) is given by the utility of $\{(z, z', S), p(z')\}$. Find the (C)ECB of this reduced basic game and let $p(z)$ denote the resulting path of play at z. Continue this way until all of the nodes are exhausted.

The above algorithm finds all of the (C)ECBs of any finite acyclic game. Note that existence at any point of the algorithm is not an issue for the CECB, because as we will see below, any finite game has a CECB. Whereas for ECB, existence might be an issue even for finite acyclic games.

[6]Formally, a game is acyclic if whenever node $z \neq z'$ is reachable from z' through some sequence of actions, then z' is not reachable from z.

3.2 Existence

A desirable feature of the CECB is that it always exists in finite games.

Proposition 2. *CECB exists in any finite extended coalitional game.*

By Lemma 1, we already know that an LCRS exists in finite basic games. In the proof, I show that the LCRS necessarily satisfies the external stability property when the game is finite, hence LCRS is non-empty in any finite basic game. This proves existence of CECB in basic games. Finally, in any finite game, in a way similar to the backward induction algorithm above, one can find the CECBs through recursively finding CECBs of basic games, hence the result follows. See the Appendix for details.

3.3 Recursive Stability

ECB and CECB is related to the myopic core and the stable set in an indirect way. In particular, in finite acyclic games ECB can be found through recursively finding the cores of each reduced game. For the CECB, the same is true with the stable set. Hence, in acyclic games, ECB and CECB incorporate foresight through bringing recursivity to the myopic notions of the core and the stable set, respectively. I will start the analysis with CECB.

Let Γ be an acyclic basic game with root z. Note that, an acyclic basic game necessarily contains a root and each coalitional behavior is just an action.

In an acyclic basic game, an E-stable set is simply a stable set (see Von Neumann and Morgenstern (1944)) with the relation $>_D$.

Definition 9. *E-stable Sets*

Let Γ be an acyclic basic game with root z. The set $V \subseteq A_z$ is E-stable if

- *(Internal Stability) For any $b \in V$, there does not exist $a \in V$ with $a >_D b$.*
- *(External Stability) For any $b \notin V$, there exists $a \in V$ with $a >_D b$.*

Consider the following algorithm for finite acyclic games, which finds the E-stable sets of reduced games recursively and assigns a stable action to each node. For any z for which $l(\Gamma(z)) = 1$, find an E-stable set for $\Gamma(z)$, choose some stable action a and let $\phi(z) = a$. Suppose an action is assigned for every subgame of length $l < k$, take any z with $l(\Gamma(z)) = k$. Let $\hat{\Gamma}(z)$ denote the reduced basic game in which the set of nodes are only those nodes that are directly reachable from z and the utility of taking an action (z, z', S) is given by the utility of $\{(z, z', S), \sigma(z')\}$. Find the E-stable set of this reduced basic game and choose some stable action a, let $\phi(z) = a$. Continue until all the nodes are exhausted. Let us call this algorithm Alg_e.

The following proposition establishes that in finite acyclic games, by recursively finding E-stable sets, we can find a CECB of the game.

Proposition 3. *Let Γ be a finite acyclic game. If ϕ is obtained through Alg_e then ϕ is a CECB.*

Since CECB can be found through backward induction, it will be sufficient to show that in an acyclic basic game any E-stable set is indeed a credible set. If any action is in the E-stable set then a deviation is necessarily to the outside of the E-stable set and can be followed by a deviation inside the set. And if an action is outside the E-stable set then there is a deviation to an action inside the set, by internal stability this deviation cannot be followed by a deviation inside the set. Hence, an E-stable set is indeed credible. See the Appendix for the details of the proof.

Note that an E-stable set might not exist, but a CECB always exists in finite games. This implies that the other direction cannot hold. Now, I will show that in a class of games, Alg_e completely characterizes the CECBs of a finite game.

We say that a game is *strongly acyclic* if the game is acyclic and the dominance relation $>_D$ defined on Φ is acyclic.

Proposition 4. *Let Γ be a finite strongly acyclic game. Then ϕ is a CECB of Γ if and only if ϕ can be obtained through Alg_e.*

To show this, it is sufficient to show that in a strongly acyclic basic game LCRS is a stable set. From the proof of Proposition 2, we already know that the LCRS satisfies the external stability property. A similar argument can be used to show that when the dominance relation contains no cycles, then LCRS also satisfies the internal stability property. See the Appendix for details.

For the analysis of ECB and its relation to the core, first we define the E-Core for basic acyclic games.

Definition 10. *E-Core*

Let Γ be an acyclic basic game with root z. The E-Core, C, of Γ is defined as:

$$C = \{a \in A_z |\ \nexists b \in A_z \text{ with } b >_D a\}$$

Consider the following algorithm, which finds the E-Cores of reduced games recursively and assigns a stable action to each node. For any z for which $l(\Gamma(z)) = 1$, find an E-Core for $\Gamma(z)$, choose some action a in the core and let $\phi(z) = a$. Suppose an action is assigned for every subgame of length $l < k$, take any z with $l(\Gamma(z)) = k$. Let $\hat{\Gamma}(z)$ denote the reduced basic game in which the set of nodes are only those nodes that are directly reachable from z and and the utility of taking an action (z, z', S) is given by the utility of $\{(z, z', S), \sigma(z')\}$. Find the E-Core of this reduced basic game and choose some stable action a, let $\phi(z) = a$. Continue until all the nodes are exhausted. Let us call this algorithm Alg_c.

The following proposition establishes that in finite acyclic games, ϕ is an ECB if and only if it can be found through Alg_c. Note that unlike in CECB, in this proposition there is no need for strong acyclicity.

Proposition 5. *Let Γ be a finite acyclic game. Then ϕ is an ECB of Γ if and only if ϕ can be obtained through Alg_c.*

To see this, one can simply observe that Alg_c is equivalent to the backward induction algorithm discussed in Section 2.1.

3.4 A Noncooperative Game

Stationary subgame perfect equilibrium of a simple bargaining game defined over an extended coalitional game can be associated with the (C)ECBs of the underlying game, which provides further evidence that (C)ECB embodies the idea of maximality.

The extensive form game will be defined for extended coalitional games in a tree structure, but it is easy to see that the results can be generalized to any acyclic extended coalitional game. Furthermore, we will assume that the extended coalitional game satisfies the following property: if $(z, z, \emptyset) \in A_z$ then $A_z = (z, z, \emptyset)$, i.e. if 'no action' is available at node z then node z is a terminal node.

We will start with the description of the extensive form. An order of players for each node is given. The game starts at the root z_0.

1. The first player in the exogenously given order becomes the proposer.
2. The proposer proposes an action available at the node.
3. Each player active at that action sequentially accepts or rejects the proposal according to the given order.
4. If everyone accepts, the action is taken, the game moves onto the corresponding node, go back to 1) if it is not a terminal node. If it is terminal, the payoffs are realized.
5. If someone rejects, then he becomes the new proposer, go back to 2).

If the negotiations come to an end, i.e. if a terminal node is reached, then each player gets the utility corresponding to the utility of the terminal path the game visits. Ongoing negotiations provide the worst utility.

The first result shows that any ECB can be supported as a stationary subgame perfect equilibrium in finite strongly acyclic games. The proposition provides further evidence that the predictions of ECB are indeed sensible, where sensibility is checked against the stationary subgame perfect equilibrium of a simple bargaining game.

Proposition 6. *In finite strongly acyclic games, any ECB ϕ can be supported as a stationary subgame perfect equilibrium outcome for some order of players.*

Both of the solution concepts can be found through backward induction, so one can restrict attention to trees of length 1. Suppose (z, x, S) is an ECB for such a tree. Suppose the order of players is such that someone in S comes first. Consider the stationary (incomplete) strategy in which everybody in S offers (z, x, S) and accepts this offer. Note that for any action $(z, y, T) \neq (z, x, S)$ either $(z, x, S) \succ_S (z, y, T)$ or there exists $i \in T$ with $(z, x, S) \succeq_i (z, y, T)$. In the former case, everyone in S would like to prevent the action, in the latter case, there exists $i \in T$, who would be willing to reject the action for the sake of (z, x, S). This gives a rough idea as to how the given incomplete strategies can be completed in a way that supports (z, x, S) as the stationary subgame perfect equilibrium outcome, where strong acyclicity is needed to complete the described strategy. See the Appendix for the details of the proof.

One might wonder if there exists a condition that would relate the stationary subgame perfect equilibrium to the (C)ECB in a stronger way. It turns out that indeed there is. The condition is somewhat strong, but it provides us with full implementation of both ECB and CECB.

For a coalitional behavior ϕ, let $\mathcal{I}(\phi)$ denote the set of individuals that become active at some point according to ϕ, i.e. $\mathcal{I}(\phi) = \{i \in N |$ for some $z \in Z, i \in \mathcal{I}(\phi(z))\}$, where $\mathcal{I}(\phi(z))$ is the initiators of $\phi(z)$.

Definition 11. *A coalition S is potent if there exists a coalitional behavior ϕ^* such that*

- $\mathcal{I}(\phi^*) = S$ *and* $\mathcal{I}(\phi) \cap S \neq \emptyset$ *for all* $\phi \neq \phi^*$.

- *For any* $\phi \in \Phi$, *either* $\phi^* \succ_S \phi$ *or* $\phi^* \sim_S \phi$.

The potent coalition has the ability to impose their most preferred coalitional behavior. Furthermore, for any coalitional behavior, the potent coalition has a member that is needed to impose that coalitional behavior.

Proposition 7. *Suppose* Γ *is a finite acyclic game that contains a potent coalition. Then* ϕ *is a stationary subgame perfect equilibrium outcome if and only if* ϕ *is a CECB if and only if* ϕ *is an ECB.*

The proposition states that the existence of a potent coalition makes certain predictions highly salient, that all three solution concepts predict these coalitional behaviors.

Since the predictions of all three solution concepts can be found recursively, the above proposition can also be applied recursively. Namely, one can show that when applying backward induction each reduced game has a potent coalition. Then the proposition above still applies even if the whole game does not have a potent coalition. That is, in such games we still have equivalence between stationary subgame perfect equilibrium, ECB and CECB. A good example is the domain of extensive form games of perfect information. In this domain, in general no potent coalition would exist. But, it is easy to see that in the process of backward induction each reduced game definitely contains a potent coalition. Hence, the theorem applies on this domain.

4 Classes of Games

In this section we will look at applications to general classes of games that are being extensively used in the economics literature. We will start by analyzing the (C)ECBs of *extensive form games*, which will be followed by the *abstract game* and the *characteristic function games*.

4.1 Extensive Form Games

It is easy to see that an extensive form game of perfect information is an extended coalitional game.

Definition 12. *Extensive Form Games of Perfect Information*

An extended coalitional game Γ *is an extensive form game if the graph of* Γ *is a tree and for all* $z \in Z$

- *Either* $A_z = \{(z, z, \emptyset)\}$ *(i.e. z is a terminal node) or for all* $(z, z', S) \in A_z$ *we have* $S = \{i\}$ *for the same* $i \in N$ *(i.e. only one individual is active at each node).*

- *For* $p', p'' \in \mathcal{P}_z$ *and* $i \in N$, $p' \succeq_i p''$ *if and only if* $(p, p') \succeq_i (p, p'')$ *for any path p that ends at z.*

The first condition basically states that a single individual is active at each node. The second condition is the usual consistency condition on the payoffs that is inherent in extensive form games: namely if p' and p'' are two terminal paths that start at z, then an individual prefers p' to p'' if and only if she prefers (p, p') to (p, p'') for any path p ending at z.

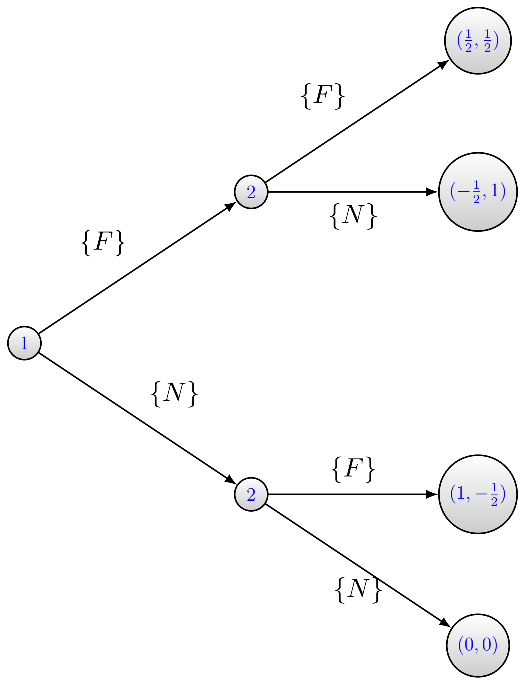


Figure 2: Favor Exchange

In finite extensive form games, since both subgame perfect equilibrium and (C)ECB satisfy the one-step deviation property, it is easy to show that subgame perfect equilibrium and (C)ECB are equivalent. However, the equivalence breaks down in infinite games. But, one can easily establish that every ECB is going to be a subgame perfect equilibrium.

Proposition 8. *In any finite extensive form game, ϕ is an ECB if and only if ϕ is a CECB if and only if ϕ is a subgame perfect equilibrium. Furthermore, if ϕ is an ECB for any extensive form game then ϕ is a subgame perfect equilibrium.*

The reason why equivalence breaks down in infinite games is that unlike subgame perfect equilibrium, the one step deviation property of (C)ECB is not inherited in infinite horizon games that are continuous at infinity, such as games with discounting. In games that are continuous at infinity, under the subgame perfect equilibrium any profitable infinite deviation can be replaced with a profitable finite deviation. This is done by truncating the deviation after some period T. As the payoffs get less and less important in future periods the resulting deviation is still profitable. But under a (C)ECB this may not be the case, because whatever T we truncate the deviation at, (C)ECB also imposes the restriction that at period $T-1$ the deviation should be profitable.

For instance, consider the *favor exchange* game in Figure 2. Here, player 1 can do a favor for player 2 or not. Following player 1's decision player 2 decides whether to do a favor for player 1 or not. A player that receives a favor gets a utility of 1 and a player who provides a favor incurs a cost of $\frac{1}{2}$.

There is a unique subgame perfect equilibrium of this game, which specifies that each player chooses to provide 'no favor' at each node of the game. By Proposition 8, this is also the unique (C)ECB of the game.

Suppose that the players repeatedly play the favor exchange game. At each period they get the payoffs of the favor exchange game played in that period and the payoffs are discounted with a common discount factor $\delta \in (0, 1)$.

There is still a subgame perfect equilibrium of this game in which each player chooses $\{N\}$ in each period. But, this subgame perfect equilibrium is not a (C)ECB for δ big enough. This is because there is a profitable deviation by $S = \{1, 2\}$ in which the players change to playing $\{F\}$ as long as no $\{N\}$ is chosen. The deviation is profitable since at every node at which S changes its action, both of the players are better off. Furthermore, the deviation is credible, because if any player chooses to deviate from the newly prescribed action then he will be punished by the perpetual play of $\{N\}$.

This example also shows that unlike subgame perfect equilibrium, under (C)ECB the one step deviation property no longer holds in infinite horizon extensive form games with discounting. The above deviation cannot be replaced by a finite deviation, because in any finite deviation, the last player to deviate would have no incentive to keep his promise.

4.2 The Abstract Game

An abstract game is defined as $\Gamma = \{N, Z, \{v_i\}_{i\in N}, \{\xrightarrow{S}\}_{S\subseteq N, S\neq\emptyset}\}$ (see, for example, Chwe (1994), Rosenthal (1972) and Xue (1998)). Where N is the set of players, Z is the set of states, $\{v_i\}$ is player i's utility function defined on the set of states and $\{\xrightarrow{S}\}_{S\subseteq N, S\neq\emptyset}$ are effectiveness relations defined on Z. The effectiveness relation $\{\xrightarrow{S}\}$ describes what coalition S can do at every state, i.e. $a \xrightarrow{S} b$ for $a, b \in Z$ if and only if when a is the status quo coalition S can change state a with state b.

All of the ingredients of an abstract game, except for the utilities, admit an obvious translation to the extended coalitional game. Set of players N and the set of nodes Z will be the same. The set of actions available at any node will be given by the effectiveness relation, but also note that in an abstract game it is possible for 'no action' to be taken at every node. Hence, for each $z \in Z$, A_z will be defined as: $(z, x, S) \in A_z$ if and only if $((x = z \text{ and } S = \emptyset) \text{ or } (z \xrightarrow{S} x \text{ for } S \neq \emptyset))$.

By changing the way we define the utilities over the paths, (C)ECB is able to mimic the way different solution concepts analyze the abstract game. Main approaches can be divided into two:

Static Approach: Some solution concepts such as LCS, FSS (see Chwe (1994)), OSSB, CSSB (see Xue (1998)), REFS and SREFS (see Dutta and Vohra (2016)) assume that players only care about the final outcome the path leads to. This might be easily represented with the following assumption on preferences:

For any $p \in \mathcal{P}$, let $\mathcal{T}(p)$ denote the node the path terminates in if p is finite, otherwise let $\mathcal{T}(p) = \emptyset$. For all $i \in N$ and $p \in \mathcal{P}$, we set $u_i(p) = v_i(\mathcal{T}(p))$ if p is finite, otherwise $u_i(p) = -\infty$. I will call the (C)ECB under this assumption, the *static (C)ECB*.[7]

Dynamic Approach: Some solution concepts such as the EPCF (see Konishi and Ray (2003)) assume that there is a discount factor δ and players discount the utilities among a path. This might be easily represented with the following assumption on preferences:

For $p = \{z_k\}_{k=1,..,K} \in \mathcal{P}$, let $u_i(p) = \sum_{k=0,..,K} \delta^k v_i(z_k) + \sum_{k=K+1,...} \delta^k v_i(z_K)$. I will call the (C)ECB under this assumption, the *dynamic (C)ECB*.

Early approaches to study farsighted coalition formation suffered from what Ray and Vohra (2014) calls the problem of maximality. This refers to the observation that instead of considering the best course of action, players used to form extreme expectations based

[7]Assigning the lowest utility to infinite paths cause no loss in generality as the solution concepts in the static approach never prescribe cycles or infinite paths.

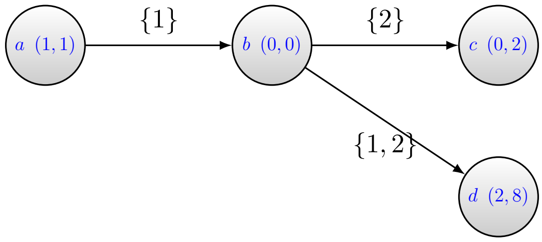


Figure 3: LCS differs from ECB (Xue (1998))

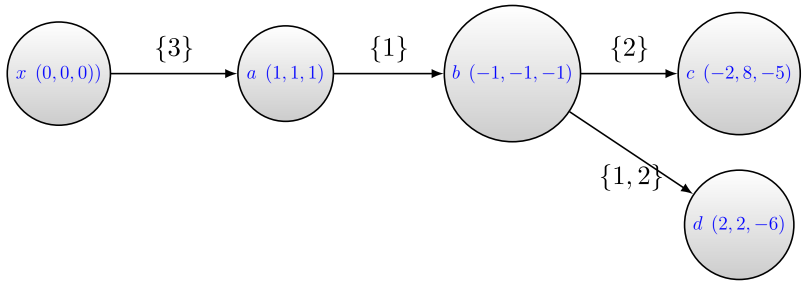


Figure 4: FSS differs from ECB

on optimism (such as FSS, OSSB) or pessimism (such as LCS and CSSB).

The example in Figure 3 taken from Xue (1998) demonstrates this for the LCS. In this example, the LCS is $\{a, c, d\}$. According to LCS, a is stable because under the assumptions of the LCS players are pessimistic and player 1 is afraid that player 2 will move to state c following state b. It is easy to see that (C)ECB makes the 'correct' prediction in this game.[8]

Similarly, the example in Figure 4 demonstrates the problem for the FSS. The reasonable prediction and the unique (C)ECB ϕ of this game specifies $\phi(b) = (b, c, 2)$, $\phi(a) = (a, a, \emptyset)$ and $\phi(x) = (x, a, 3)$. However, under the FSS a is not stable, because 1 optimistically and unreasonably believes that 2 will move to d with him, but this in turn makes x stable under the unique FSS. Hence, $FSS = \{x, c, d\}$.

Nevertheless, some recent solution concepts such as the REFS, SREFS and EPCF have managed to solve this issue by considering consistent expectation just like (C)ECB does.

One important way in which (C)ECB differs from other solution concepts on the domain of the abstract game is that all these solution concepts restrict attention to 'one-step deviations', whereas (C)ECB allows for any arbitrary deviation.

This does not seem to be a problem in static solution concepts such as the REFS and SREFS, however the same cannot be said of the dynamic solution concepts such as the EPCF.

To see this, consider the example in Figure 5, assume that the utilities are defined as discounted utilities over paths. At node b, player 1 moves to a and settles for a payoff of 1. Similarly, player 2 moves from c to d and settles for a payoff of 1. There is no profitable one-step deviation from this coalitional behavior for δ big enough. But, there is a profitable deviation by the coalition $\{1, 2\}$ to the cycle. Hence, for δ big enough the depicted coalitional behavior is not a (C)ECB, but it is an EPCF, although the profitable deviation to the cycle would increase the payoff of every individual.

As has been demonstrated, one advantage of (C)ECB is its versatility. By changing

[8]The unique (C)ECB ϕ specifies $\phi(a) = (a, b, \{1\})$ and $\phi(b) = (b, d, \{1, 2\})$.

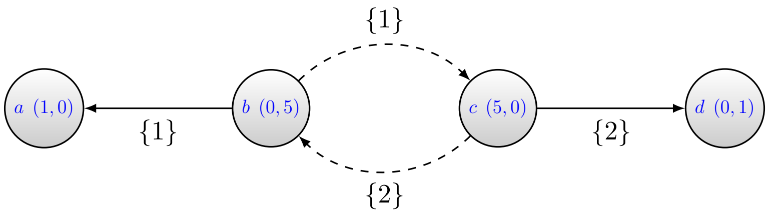


Figure 5: Profitable Deviation to a Cycle

the way we define the utilities over the paths (C)ECB is able to mimic the way different solution concepts analyze the abstract game.

Indeed, one can easily show that ECB is related to a solution concept both in the static approach (SREFS) and the dynamic approach (EPCF). When one also includes the relationship between (C)ECB and SPE, one obtains a solution concept that is able to bridge the noncooperative approach with the static and the dynamic approaches to foresight. Now, we will show this formally.

Definition 13. *Indirect Dominance*

$x \in Z$ indirectly dominates $y \in Z$ ($x \gg y$) if there exists $x_0, x_1, ...x_n \in Z$ and $S_0, S_1, S_2, ..., S_{n-1}$ such that $x_0 = y$, $x_n = x$, $(x_i, x_{i+1}, S_i) \in A_{x_i}$ and $v_j(x_n) > v_j(x_i)$ for all $j \in S_i$, for all $i = 0, ..n-1$.

The solution concepts in the static approach are defined using the indirect dominance relation which uses strict preference. Whereas under the (C)ECB coalitions might take some actions even if they are indifferent. We will always see a difference between these concepts and the (C)ECB due to the different ways with which they treat indifference. I do not view this difference as essential. Therefore, not to bump on it again and again, from now on I will assume that the environment satisfies *no indifference*.

Definition 14. *No Indifference*

An abstract game Γ satisfies no indifference if for all $i \in N$ and $z, z' \in Z$, where $z \neq z'$, we have $v_i(z) \neq v_i(z')$.

Using coalitional behavior rather than expectations we can restate the stability concept used in SREFS in the language of our framework. Let $S(\phi)$ denote the set of stable nodes in coalitional behavior ϕ, i.e. $S(\phi) = \{z \in Z | \phi(z) = (z, z, \emptyset)\}$.

Definition 15. *SREFS (Dutta and Vohra (2016))*

A set $V \subseteq Z$ is an SREFS if there exists an acyclic coalitional behavior ϕ such that $S(\phi) = V$ and

(IS) *If $x \in V$ then there does not exist $y \in Z$ and $S \subseteq N$ such that $(x, y, S) \in A_x$ and $v_i(\mathcal{T}(\sigma(y))) > v_i(x)$ for all $i \in S$.*

(ES) *If $x \notin V$ then $\sigma(x)$ is an indirect dominance path.*

(M) *If $x \notin V$ and if T is the initiator at x then there does not exist $y \in Z$ and $F \subseteq N$ with $T \cap F \neq \emptyset$ and $(x, y, F) \in A_x$ such that $v_i(\mathcal{T}(\sigma(y))) > v_i(\mathcal{T}(\sigma(x)))$ for all $i \in F$.*

The first and second conditions are interpreted as internal and external stability conditions with respect to the expectation. Whereas the third condition requires optimality of the move at any node x, where optimality is conditioned on a one-step deviation.

It turns out that under the weak assumption that actions are monotonic, in the sense that whenever a coalition S is able to take a certain action then any coalition T containing S can also take this action, ECB is equivalent to SREFS.

Definition 16. *Monotonicity of Actions*

We say that an extended coalitional game satisfies monotonicity of actions if whenever $(z, z', S) \in A_z$ for some $z \in Z$ we also have that $(z, z', T) \in A_z$ for all $T \supseteq S$.

Proposition 9. *Let Γ be an abstract game that satisfies no indifference and monotonicity of actions. Then,*

- *If V is an SREFS and ϕ is the coalitional behavior that supports it then ϕ is a static ECB, and hence a static CECB.*
- *If ϕ is a static ECB then $S(\phi)$ is an SREFS supported by ϕ.*

This establishes that (C)ECB (when defined as a static concept) is closely related to a solution concept in the static approach and in turn it also means that static (C)ECB's predictions also satisfy the internal and external stability properties imposed by this solution concept. On the side of SREFS, this result shows that the conditions of internal and external stability with the one-step deviation condition rules out every possible profitable deviation, even those that involve multiple actions.

EPCF also uses consistent expectations just like SREFS and (C)ECB. The difference of EPCF and SREFS (apart from one being a static and the other being a dynamic concept) is that the former is directly defined as an expectation (a coalitional behavior) that is immune to certain deviations. Using coalitional behavior, we can restate the definition of EPCF under *no indifference* in the language of our framework.[9]

Definition 17. *EPCF*

A (deterministic) EPCF[10] *is a coalitional behavior ϕ such that for all $x \in Z$*

- *if $\phi(x) = (x, y, S)$, where $y \neq x$ then $\sigma(y) \succ_S \sigma(x)$ and there does not exist z with $(x, z, S) \in A_x$ and $\sigma(z) \succ_S \sigma(y)$.*
- *if x is such that there exists $y \in Z$ and $S \subseteq N$ with $(x, y, S) \in A_x$ and $\sigma(y) \succ_S \sigma(x)$ then $\phi(x) \neq (x, x, \emptyset)$*

where for any $z, z' \in Z$, $\sigma(z) \succ_i \sigma'(z)$ if and only if the discounted utility of the former is greater.

[9]The assumption of *no indifference* is needed here for an altogether different reason. In particular in the original definition of EPCF, Konishi and Ray allow a coalition to move at state x even if it is indifferent between moving or staying at x. But they do not allow this if there exists a coalition that can move at x and that would strictly improve by taking an action at x. Whereas under a (C)ECB this is also allowed.

[10]Konishi and Ray (2003)'s EPCF can also be stochastic, as mixing is not included in the definition of an ECB I restrict attention to deterministic EPCFs.

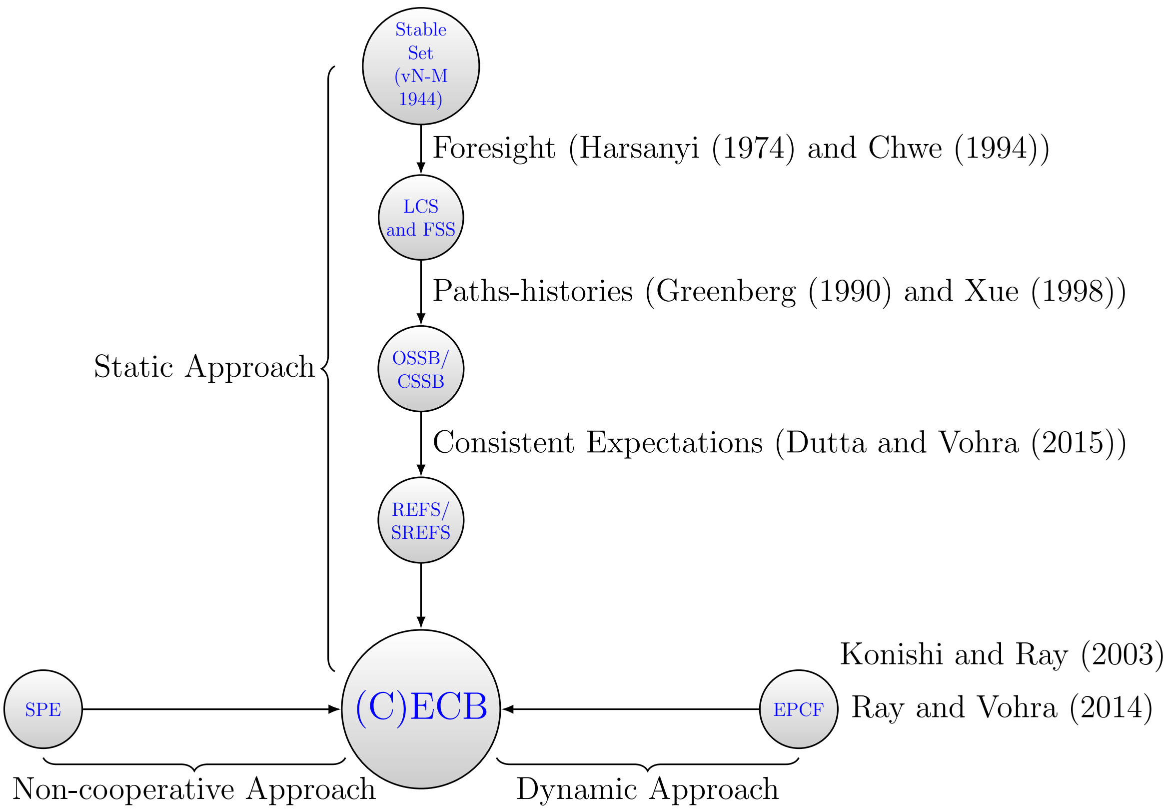


Figure 6: An Overview

The definition is similar to the definition of dynamic (C)ECB with the major difference being that this definition does not consider deviations that involve multiple actions.[11] With the 'dynamic' assumption on the utilities, dynamic (C)ECB does not satisfy the one step deviation property. This means that the restriction to one-step deviations in the above definition is with loss of generality. In particular, there might exist a deviation to a cycle that might make the deviators better off (see Figure 5). Which also implies that there will be EPCFs that are not ECBs (again see Figure 5 for an example).

Finally, it is easy to establish that every dynamic ECB is an EPCF.

Proposition 10. *If ϕ is a dynamic ECB then it is an EPCF, but an EPCF may not be a dynamic ECB.*

When one adds to these, the relationship of ECB to subgame perfect equilibrium, one gets a solution concept that is directly or indirectly related to a wide area of the literature on farsighted coalition formation, see Figure 6.

Hence, I believe that one major contribution of this paper is that it shows that the developments in the literature have made it possible to achieve some level of unification between both the noncooperative and cooperative strands of the literature and within the cooperative approach to farsighted coalition formation.

4.2.1 Other Approaches on the Abstract Game

Some solution concepts such as the one studied in Herings et al. (2004) and Granot and Hanany (2016), propose to define solution concepts on the abstract game by directly

[11] Another difference is that the definition of the deviations are in general weaker under EPCF.

defining a noncooperative game from the abstract game. Specifically, Herings et al. (2004) take the abstract game as the primitive and define a multistage game associated with the abstract game, then they define on appropriate notion of rationalizability on this multi stage game. In Granot and Hanany (2016), the evolution of play resulting from deviations is modeled as an extensive form game.

Although fruitful, this approach is fundamentally different than the approach I am taking with the (C)ECB and the other approaches discussed so far, where the solution concept is directly defined over the domain with no use of individual beliefs or strategies. In this sense, (C)ECB and the solution concepts of the dynamic and static approach are firmly situated as 'cooperative solution concepts', where cooperation is not explained through the help of a non-cooperative game. Whereas in Herings et al. (2004) and Granot and Hanany (2016) this is not the case.

These approaches should be seen as complementary and it is beyond the scope of this work to compare the utility of (C)ECB over these solution concepts.

4.3 Characteristic Function Games

A characteristic function game is a pair (N, V), where N is the finite set of players and for each coalition $S \subseteq N$, $V(S) \subseteq \mathbb{R}^S$ denotes the set of payoff vectors achievable by coalition S. A coalition structure P is a partition of N. A state z is a pair (x, P) where P is a coalition structure and x is an allocation that satisfies $x_S \in V(S)$ for all $S \in P$. Let Z denote the set of all states.The core is probably the most well-known solution concept defined for characteristic function games. It is the set of states that no coalition can improve upon.

Definition 18. *The core of the game (N, V) is defined as*

$$C(N, V) = \{(x, P) \in Z| \text{ there does not exist } S \subseteq N \text{ and } y_S \in V(S) \text{ such that } y_S > x_S{}^{12}\}$$

I am going to study characteristic function games in line with the farsighted coalition formation literature which assumes that a deviation by a coalition might be followed by any deviation by any other coalition and the players are farsighted in the sense that they care about the final allocation the negotiations lead to.

Basically, I will be translating the abstract game used in Ray and Vohra (2015) to an extended coalitional game. For any $(x, P) \in Z$, Ray and Vohra (2015) require the set of actions to satisfy the three conditions below. (Also see Konishi and Ray (2003) and Koczy and Lauwers (2004), who use similar conditions.)

1. $((x, P), (x, P), \emptyset) \in A_{(x,P)}$
2. If $((x, P), (y, P'), S) \in A_{(x,P)}$ then $y_S \in v(S)$ and if $T \in P$ is such that $S \cap T = \emptyset$ then $T \in P'$ and $x_T = y_T$
3. For all $(x, P) \in Z$, $T \subseteq N$ and $z_T \in V(T)$ such that either $z_T \neq x_T$ or $T \notin P$, there exists $((x, P), (y, P'), T) \in A_{(x,P)}$ such that $T \in P'$ and $y_T = z_T$.

The first condition states that it is possible to stay in every state. The second condition requires that when a coalition deviates from an outcome, it has to get something

[12] $y_S > x_S$ if $y_i > x_i$ for all $i \in S$.

feasible for itself and it cannot dictate the payoffs and structures of the coalitions that are unrelated to it. Finally, the third condition states that if a payoff z_T is feasible for a coalition T ($z_T \in V(T)$), then T should be able to get z_T or if a coalition T has not formed then T should be able to form..

Hence, an extended coalitional game that corresponds to a characteristic function game is $\Gamma = \{N, Z, \{A_z\}_{z\in Z}, \{u_i\}_{i\in N}\}$, where N is the set of players, Z corresponds to all states, $\{A_z\}_{z\in Z}$ is any set of actions that satisfy the restrictions above, and the utilities correspond to the static approach, i.e. for all $i \in N$ and $p \in \mathcal{P}$ we set $u_i(p) = v_i(\mathcal{T}(p))$ if p is finite, otherwise $u_i(p) = -\infty$, where $v_i((x, P)) = x_i$.

CECB is not a very satisfactory concept for characteristic function games for a particular set of reasons: A) Characteristic function games are basic games, hence backward induction cannot be applied even at the basic game level. This makes the solution concept too permissive on characteristic function games. B) Because of the multitude of coalitional behaviors possible in such games, it is very hard to find the set of CECBs in such games. For these reasons, I will only be analyzing the ECB in this class.

The proposition below establishes that if under an ECB, the path of play from every node terminates at the same state, then this state is in the core. Furthermore, if a state is in the core, then there exists an ECB such that the path of play from every node terminates at this core state. That is, ECBs with a single prediction completely characterize the core.

Proposition 11.

- *If $(x^*, P^*) \in C(N, V)$, then there exists an ECB ϕ such that $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ for every $(x, P) \in Z$.*
- *If ϕ is an ECB such that $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ for every $(x, P) \in Z$ then $(x^*, P^*) \in C(N, V)$.*

The proposition establishes that a farsighted solution concept completely characterizes a well-known myopic concept, the core. The result also complements other results in the literature that are close in spirit to this result. Other results that show that the core incorporates foresight include Ray (1989), Diamantoudi and Xue (2003), Konishi and Ray (2003), Mauleon, Vannetelbosch and Vergote (2011) and Ray and Vohra (2015).[13] Ray (1989) shows that core is immune to nested objections. The results in Diamantoudi and Xue (2003), Mauleon, Vannetelbosch and Vergote (2011) and Ray and Vohra (2015) concern the farsighted stable set. Whereas Konishi and Ray (2003)'s result is related to the EPCF.

Proposition 11 implies that if a characteristic function game has an empty core, then we cannot find an ECB for that game with a single stable outcome. But this does not mean that an ECB does not exist in such a game. An example is available from the author upon request.

5 Literature Review

In this section I will briefly review the different strands of the literature on farsighted coalition formation. Inevitably, the review is incomplete, for extensive reviews see Mari-

[13] Green(1974), Feldman(1974), Koczy and Lauwers (2004) and Sengupta and Sengupta(1996) show how myopic objections lead to the core in specific environments. Although these papers are also related, they are fundamentally different from the current paper, since players are not assumed to be farsighted.

otti and Xue (2003), Ray (2008) and Ray and Vohra (2014).

The quest to incorporate foresight into the static cooperative solution concepts goes back at least to Harsanyi (1974), who criticized von Neumann and Morgenstern's stable set (1944) for being myopic. Chwe (1994) formalized Harsanyi's criticism and developed the solution concepts of the farsighted stable set and the largest consistent set. These are set valued concepts in the tradition of von Neumann and Morgenstern's stable set, which use the indirect dominance relation instead of the direct dominance relation the stable set uses.[14]

Xue (1998) argued that this approach is not entirely satisfactory as farsighted players should not only consider the final states their actions lead to, but they should also consider how these states are reached. By using Greenberg (1990)'s framework Xue (1998) proposed to use paths to incorporate foresight into his solution concepts. But Xue (1998) still used a framework in which players form arbitrary expectations based on optimism or pessimism to evaluate different sets of paths. Questions remain about these extreme expectations players hold to evaluate sets of paths, for details see Bhattacharya and Ziad (2012), Herings, Mauleon and Vannetelbosch (2004) and Ray and Vohra (2014).

Dutta and Vohra (2015) propose to deal with these issues by embodying the farsighted stable set with consistent expectations and introducing one-step deviations (also see Jordan (2006) for an earlier work concerning common expectations and farsighted stability and Bloch and van den Nouweland (2017), where players might hold heterogeneous expectations).

Given the problems of the static approach some authors found the way out in introducing an explicitly dynamic solution concept. The main solution concept in the dynamic approach is the EPCF (Konishi and Ray (2003) and Ray and Vohra (2014)), which models coalition formation as an explicitly dynamic process and the payoffs are discounted with a common discount factor.[15]

Other authors have tried to remedy the problems with the static approach by taking a cooperative domain such as the abstract game as the primitive and proposing a solution concept by defining a noncooperative game from the abstract game. Examples include Herings et al. (2004) and Granot and Hanany (2016). Herings et al. (2004) take the abstract game as the primitive and define a multistage game associated with the abstract game, then they define on appropriate notion of rationalizability on this multi stage game. In Granot and Hanany (2016), the evolution of play resulting from deviations is modeled as an extensive form game.

Recently, the literature have started to extend the solution concepts in two directions: 1) By incorporating history dependence and 2) By allowing for different degrees of foresight. Vartiainen (2011) has incorporated history dependent expectations into EPCF and showed that this greatly enhances the existence property of the solution concept. More recently, Dutta and Vartiainen (2017) and Ray and Vohra (2017) incorporate history dependent expectations to the REFS and SREFS. On the second development, Herings et al. (2014) introduced the concept of level-K farsightedness to incorporate differing levels of foresight. More recently, Herings et al. (2017) study matching when the players might

[14]For more on farsighted stable set see Diamantoudi and Xue (2003), Mauleon, Vannetelbosch and Vertoge (2011) and Ray and Vohra (2015). For more on the largest consistent set see Beal, Durieu and Solal (2008), Bhattacharya (2005), Herings, Mauleon and Vannetelbosch (2009), Mauleon and Vannetelbosch (2004), Page, Wooders and Kamat (2005) and Xue (1997).

[15]Also see Dutta, Ghosal and Ray (2004) and Vartiainen (2011).

have heterogeneous levels of foresight.[16]

6 Appendix

Proof of Lemma 1. The proof follows the same lines as the proof of Proposition 1 in Chwe (1994). Define the function $f : 2^\Phi \to 2^\Phi$, where $f(X) = \{\phi \in \Phi |$ for all $\phi' \in \Phi$ such that $\phi' >_D \phi, \exists \phi^* \in X$ with $\phi^* >_D \phi'\}$. Observe that a set Y is credible if and only if $f(Y) = Y$. Furthermore f is isotonic, i.e. $X \subseteq Y \implies f(X) \subseteq f(Y)$.

Let $\Delta = \{X \subseteq \Phi | f(X) \supseteq X\}$, which is nonempty since it contains the empty-set. Let $Y = \cup_{X \in \Delta} X$. Then $f(Y) \supseteq f(X)$ for all $X \in \Delta$. Hence, $f(Y) \supseteq \cup_{X\in\Delta} f(X) \supseteq \cup_{X\in\Delta} X = Y$. But, then $f(f(Y)) \supseteq f(Y)$, implying $f(Y) \in \Delta$, thus $f(Y) \subseteq Y$, and hence $f(Y) = Y$. This means that Y is a credible set. Furthermore since every other credible set is in Δ, it also contains all other credible sets.

Proof of Proposition 1. Assume S has a profitable (and credible) deviation from ϕ to ϕ', let z^* be such that $\phi(z^*) \neq \phi'(z^*)$. $\sigma'(z^*)$ is not a cycle and there are finitely many $z \in \sigma'(z^*)$ such that $\phi(z) \neq \phi'(z)$. Then, there exists $z' \in \sigma'(z^*)$ such that $\phi(z') \neq \phi'(z')$, but $\phi(z) = \phi'(z)$ for all $z \in \sigma'(z')$ such that $z \neq z'$. Think about the deviation in which S only changes $\phi(z')$ to $\phi'(z')$, let the resulting coalitional behavior be ϕ'', this is a one step deviation. Since the initial deviation is profitable we have $\sigma''(z') \succ_S \sigma(z')$, hence this one step deviation is also profitable. Finally, credibility also follows as $\Gamma(z', \phi') = \Gamma(z', \phi'')$.

Proof of Proposition 2.
1st Step: Showing Existence for a Basic Game:

Let Γ be a finite basic game. By Lemma 1 we already know that an LCRS exists, hence all we need to show is the non-emptiness of the LCRS. Let Y be the LCRS, I will show that Y satisfies the external stability property, which implies non-emptiness.

Towards a contradiction, assume that Y does not satisfy external stability. Then, there exists $\phi_1 \notin Y$ such that there does not exist $\phi \in Y$ with $\phi >_D \phi_1$. But then, by the definition of the LCRS, there exists $\phi_2 \notin Y$ such that $\phi_2 >_D \phi_1$, furthermore there does not exist $\phi \in Y$ that dominates ϕ_2.

But then the argument can be repeatedly applied to get a set of coalitional behaviors $\{\phi_1, ..., \phi_k\} \subseteq \{\Phi \setminus Y\}$ such that $\phi_i >_D \phi_{i-1}$ for all $i \geq 2$ and $\phi_1 >_D \phi_K$ (this last inequality follows by the finiteness of the set of coalitional behaviors).

Now, consider the set $X = \{\phi_1, ..., \phi_k\}$. Observe that $f(X) = X$ with the function f defined in the proof of Lemma 1, implying that X is a credible set and hence $X \subseteq Y$. A contradiction.

This shows that the LCRS satisfies the external stability property, non-emptiness follows.

2nd Step: Generalizing to any Finite Game:

Let Γ be any finite extended coalitional game. For any $z \in Z$, let $l(\Gamma(z))$ denote the length of the subgame at z, where length is the number of proper subgames $\Gamma(z)$ includes.[17] Since the game is finite, there exist $z \in Z$ for which $l(\Gamma(z)) = 0$.

For any z for which $l(\Gamma(z)) = 0$, by the 1st Step, there exists a CECB for $\Gamma(z)$, pick one CECB for each such z and call their union ϕ_0.

[16] Also see Kirchsteiger et al. (2016), who provide evidence of behavior in favor of limited foresight.

[17] Note that this definition is consistent with the usual definition of length of a subgame in acyclic games.

Suppose ϕ_i is defined for each $i < n$. For any z for which $l(\Gamma(z)) = n$, consider the reduced game at z given ϕ_{n-1}, since this is a basic game, there exists a CECB for this game, call it ϕ' and let $\phi_n = \phi_{n-1} \cup \phi'$.

Continue this process until all the nodes are exhausted. The process will define an action for each node of the game and hence a coalitional behavior for the whole game.

Suppose ϕ is the resulting coalitional behavior. I will show that ϕ is a CECB. Towards a contradiction assume that there exists a profitable and credible deviation to ϕ'. Let $Z^0 = \{z \in Z | \phi(z) \neq \phi'(z)\}$ and let $Z^1 = \{z \in Z_0 | l(\Gamma(z)) \leq l(\Gamma(z')) \forall z' \in Z_0\}$. For any node $z \in Z^1$, the reduced game at z will correspond to the game used in the construction and hence by the construction there cannot exist a profitable and credible deviation from z in this reduced game. A contradiction.

Proof of Proposition 3. By backward induction, it is sufficient to prove the result for a basic game. Let Γ be a basic game with root z and suppose that V is an e-stable set. It is sufficient to prove that V is a credible set. Let f be the function defined in the proof of Lemma 1, remember that a set of coalitional behaviors Y is credible if and only if $f(Y) = Y$. Hence, all we need to show is that $f(V) = V$.

Take any $a \in V$. Then for any action b with $b >_D a$, we have that $b \notin V$ and hence by external stability there exists $c \in V$ with $c >_D b$. This proves that $V \subseteq f(V)$. For the other direction, suppose that $a \in f(V)$, but $a \notin V$. Since, $a \notin V$, by external stability there exists $b >_D a$ with $b \in V$. By internal stability, there does not exist $c \in V$ with $c >_D b$. But this contradicts $a \in f(V)$.

Proof of Proposition 4. We just need to show that in a strongly acyclic basic game LCRS is an E-stable set.

Let Γ be a strongly acyclic basic game and let A^* be the LCRS. By the proof of proposition 2 we already know that A^* satisfies the external stability property. Hence, all we need to show is that A^* satisfies internal stability.

Towards a contradiction, assume that it does not. Then there exists $a_1, a_2 \in A^*$ such that $a_2 >_D a_1$. By the definition of the LCRS, since $a_1 \in A^*$, a_2 should also be dominated by some action $a_3 \in A^*$.

But then the argument can be repeatedly applied to get a set of coalitional behaviors $\{a_1, ..., a_k\} \subseteq A^*$ such that $a_i >_D a_{i-1}$ for all $i \geq 2$ and $a_1 >_D a_K$ (this last inequality follows by the finiteness of the set of coalitional behaviors). A contradiction to strong acyclicity.

Proof of Proposition 5. The result directly follows from Section 2.1, because the algorithm given in Section 2.1 is equivalent to Alg_c.

The following definition and the lemma will be useful in proving Proposition 6 and 7.

Definition 19. *In an acyclic basic game with root z, we say that an action $a \in A_z$ s-dominates an action $(z, b, S) \in A_z$ if and only if there exists $i \in S$ with $a \succ_i (z, b, S)$.*

An action $a \in A_z$ weakly s-dominates an action $(z, b, S) \in A_z$ if and only if there exists $i \in S$ with $a \succeq_i (z, b, S)$.

Lemma 2. *For an acyclic basic game, if a set of actions V satisfies internal stability with the s-dominance relation and external stability with the weak s-dominance relation, then any $a \in V$ can be supported as the stationary subgame perfect equilibrium of the bargaining game.*

Proof. Consider the following strategies: $i \in N$ offers the most preferred action in V. $i \in N$ accepts (z, x, S) if $(z, x, S) \in V$. If $(z, x, S) \notin V$ then the last player accepts if (z, x, S) is better than any stable action, rejects otherwise. Second to last player accepts if the last player accepts and if (z, x, S) is better than any stable outcome, rejects otherwise. And so on. Note that by weak external stability any action that is not in V will be rejected.

The strategy is stationary and it leads to an outcome in V, furthermore any action in V can be supported as the outcome with these strategies with an appropriate order of players. Hence, all we need to show is that the strategy is indeed subgame perfect.

First consider the acceptance-rejection stage. Suppose i accepts (z, x, S). First assume that $(z, x, S) \in V$, note that by rejecting, i can only induce actions in V, however by internal stability for any $(z, y, T) \in V$, $(z, x, S) \succeq_i (z, y, T)$. Now, assume that $(z, x, S) \notin V$, but then i only accepts the action if it is better than every stable action and everybody else responding after him accepts the action (by rejecting i can only induce actions in V)

Now suppose i rejects some action (z, x, S), but then this can only be because the implemented action is better than (z, x, S) or someone after i will reject and the eventual implemented action is (weakly) worse than the implemented when i rejects. Finally, it is easy to see that at the proposal stage everyone is taking the optimal action. $\square$

Proof of Proposition 6. By backward induction, it is sufficient to prove the result for a basic game. Let Γ be a strongly acyclic basic game with root z. And Let ϕ be an ECB for such a game.

Let $A_0 = \phi$. Let A_1 be the set of actions that are weakly s-dominated by ϕ. Let $A_2 = \phi'$ be an action that is undominated in $A \setminus (A_0 \cup A_1)$. Note that ϕ is not s-dominated by ϕ', as otherwise there would exist a profitable deviation from ϕ to ϕ'.[18] Let A_3 be the set of actions in $A \setminus (A_0 \cup A_1 \cup A_2)$ that are weakly s-dominated by ϕ'. Let $A_4 = \phi''$ be an action that is undominated in $A \setminus (A_0 \cup A_1 \cup A_2 \cup A_3)$. Continue until the set of all coalitional behaviors is exhausted (by strong acyclicity and finiteness, the set will eventually be exhausted).

Let $V = \cup_{k\, even} A_k$. The result follows by Lemma 2, since V satisfies internal stability with the s-dominance relation and external stability with the weak s-dominance relation

Proof of Proposition 7. All solution concepts can be found through backward induction, hence it is sufficient to show the result for a basic game. Let Γ be a basic game with root z. Let S^* be the potent coalition and ϕ^* be the corresponding coalitional behavior.

Let $\Phi^* = \{\phi \in \Phi | \phi \sim_S \phi^*\}$. It is clear that Φ^* is the set of ECBs of the game. Furthermore since from any other coalitional behavior there exists a profitable deviation to Φ^*, it is also the set of CECBs. Finally, any $\phi \in \Phi^*$ satisfies external stability with the weak s-dominance and trivially satisfies internal stability with s-dominance.[19] Hence by Lemma 2, it can be supported as a stationary subgame perfect equilibrium.

Now, all we need to show is that no other coalitional behavior can be supported as a stationary subgame perfect equilibrium. Towards a contradiction assume that in a basic game, $(z, x, T) \notin \Phi^*$ can be supported as a stationary subgame perfect equilibrium. Note

[18] Let us say $\phi' = (z, x, T)$ and $\phi = (z, y, S)$. We already know that ϕ' is not weakly s-dominated by ϕ, which implies that $(z, x, T) \succ_T (z, y, S)$. But then, if ϕ is s-dominated by ϕ', clearly there exists a deviation from ϕ to ϕ'

[19] The statement does not claim that Φ^* satisfies internal stability, indeed Φ^* need not satisfy internal stability. The statement claims that a singleton from the set Φ^* trivially satisfies internal stability.

that $S^* \cap T \neq \emptyset$, but no one in $S^* \cap T$ rejects (z, x, T) and offers ϕ^*, although they prefer ϕ^*. This could only be because there exists some $i \in S^*$, who rejects ϕ^* in anticipation of some ϕ'. But then $\phi' \succeq_i \phi^*$, implying $\phi' \sim_S \phi^*$. But then, it is better for someone in $S^* \cap T$ to reject (z, x, T) and offer ϕ', a contradiction.

Proof of Proposition 8. Finite extensive form games are strongly acyclic and both of the backward induction algorithms in Section 3 reduces down to the well-known way to find subgame perfect equilibrium through backward induction in finite extensive form games.

To show that ECB is always a subgame perfect equilibrium, suppose ϕ is a coalitional behavior that is not a subgame perfect equilibrium. Then there exists an individual $i \in N$ that can deviate to a coalitional behavior ϕ' such that $\sigma'(z^*) \succ_i \sigma(z^*)$ for some $z^* \in Z$. Let $Z_1 = \{z \in \sigma'(z^*) | \phi(z) \neq \phi'(z)\}$ and let $Z_2 = \{z \in Z_1 | \sigma(z) \succeq_i \sigma'(z)\}$. If $Z_2 = \emptyset$ then consider the deviation from ϕ by i that only includes the actions in Z_1, which is a deviation that increases the payoff of i at every node at which an action changes. If $Z_2 \neq \emptyset$ then let z' be the node in Z_2 that is closest to z^*. Consider the deviation by i from ϕ, that only involves the actions at the nodes in Z_1 that are in between z^* and z' (including z^*, not including z'). The resulting deviation increases the payoff of i at every node at which an action changes, hence it is profitable. That is, ϕ is not an ECB.

Proof of Proposition 9. Take Γ that satisfies *no indifference* and *monotonicity of actions.*

First suppose that V is an SREFS and ϕ is the coalitional behavior that supports it. I will show that ϕ is a static ECB. First note that static ECB satisfies the one step deviation property on this domain. This is because any profitable deviation necessarily leads to finite paths. Hence, it suffices to check one step deviations.

Suppose there is a profitable deviation at some unstable x in which some i blocks the move. But then, $v_i(x) > v_i(\mathcal{T}(\sigma(x)))$, which is a contradiction to ES. Suppose there is a profitable deviation at some unstable x where i blocks an action leading to z and T takes an action leading to y. By *monotonicity of actions* there is also a profitable deviation in which i blocks the action leading to z and $T \cup \{i\}$ takes an action leading to y. Furthermore since the initial deviation is profitable we have $v_j(\mathcal{T}(\sigma(y))) > v_j(\mathcal{T}(\sigma(x)))$ for all $j \in \{T \cup i\}$, which is a contradiction to M. Finally suppose there is a profitable deviation at some stable outcome x by coalition S to an outcome z. But then $v_j(\mathcal{T}(\sigma(z))) > v_j(\mathcal{T}(\sigma(x)))$ for all $j \in S$, a contradiction to IS. But we have exhausted all possible one step deviations, hence ϕ is a static ECB.

Now suppose that ϕ is a static ECB. We will show that $S(\phi)$ is an SREFS supported by ϕ. Note that at any state x with $\phi(x) = (x, x, \emptyset)$ we have that there does not exist y and S such that $(x, y, S) \in A_x$ and $v_i(\mathcal{T}(\sigma(y))) > v_i(\mathcal{T}(\sigma(x)))$ for all $i \in S$, as otherwise S has a profitable deviation. Hence IS is satisfied. Now take any state x for which $\phi(x) = (x, y, S)$ for some y and S. First note that $v_j(\mathcal{T}(\sigma(x))) > v_j(x)$ for all $j \in S$, since otherwise by NI there exists $i \in S$ for which $v_i(x) > v_i(\mathcal{T}(\sigma(x)))$ in which case i has a profitable deviation at x. Hence, ES is satisfied. Finally, if M is violated then trivially there exists a profitable deviation and ϕ is not an ECB, so M is also satisfied.

Proof of Proposition 10. Figure 5 shows that an EPCF may not be a dynamic ECB, so here I will only show that each dynamic ECB is an EPCF. Let ϕ be an ECB.

Take any $x \in Z$ with $\phi(x) = (x, y, S)$ where $x \neq y$, first observe that $\sigma(x) \succ_S (x, x, \emptyset)$, as otherwise by *no indifference* there exists $i \in S$ for which $(x, x, \emptyset) \succ_i \sigma(x)$, who would have a profitable deviation in which she blocks the taken action. This also implies that $\sigma(y) \succ_S \sigma(x)$. Towards a contradiction assume that there exists z with $(x, z, S) \in A_x$ and $\sigma(z) \succ_S \sigma(y)$. But then there exists a profitable deviation by S to z, a contradiction. So, ϕ satisfies the first condition.

Now assume that x is such that there exists y, S with $(x, y, S) \in A_x$ and $\sigma(y) \succ_S \sigma(x)$. Towards a contradiction assume $\phi(x) = (x, x, \emptyset)$. But then $\sigma(y) \succ_S (x, x, \emptyset)$, but this implies that $\{(x, y, S), \sigma(y)\} \succ_S (x, x, \emptyset)$. But then there is a profitable deviation by S to y, a contradiction. So, ϕ also satisfies the second condition and it is an EPCF.

Proof of Proposition 11. The construction used in the proof is similar to the ones found in Diamantoudi and Xue (2003) and Konishi and Ray (2003).

1st Step: If $(x^*, P^*) \in C(N, V)$ then there exists an ECB ϕ such that $\mathcal{T}(\sigma((x, P))) = (x^*, P^*)$ for all $(x, P) \in Z$.

The proof is by construction, I will construct an ECB with the desired property.

Take any $(x^*, P^*) = (x^*, \{S_1^*, S_2^*, .., S_K^*\}) \in C(N, V)$. For any coalition S, let $\overline{S}$ denote the partition of S composed of singletons. Let $\phi(x^*, P^*) = ((x^*, P^*), (x^*, P^*), \emptyset)$.

Let $(x_t, P_t) = (x_t, \{S_1^*, ..., S_t^*, \overline{\cup_{j=t+1,..,K} S_j^*}\})$, where $x_t(S_j^*) = x^*(S_j^*)$ for all $j = 1, .., t$ and $x_t(\{i\}) = \max v(\{i\})$ for any $i \in \cup_{j=t+1,..,K} S_j^*$. For any $t = 0, 1, .., K-1$, let $\phi(x_t, P_t) = ((x_t, P_t), (x_{t+1}, P_{t+1}), S_{t+1}^*)$.

For any $(x, P) \neq (x_t, P_t)$ for some $t = 0, 1, .., K$, let i be the player with the smallest index for which $x_i^* \geq x_i$ and $i \in S \in P$, where $|S| \geq 2$. Note that since $(x^*, P^*) \in C(N, V)$, such a player exists. Let $\phi((x, P)) = ((x, P), (x', P'), \{i\})$, where $x'(\{i\}) = \max v(\{i\})$ and $\{i\} \in P'$.

This completes the specification of ϕ, note that ϕ is a coalitional behavior as it assigns a unique action for each $z \in Z$. Furthermore $\mathcal{T}(\sigma(z)) = (x^*, P^*)$ for all $z \in Z$. Now we need to show that ϕ is an ECB. As any deviation leading to an infinite path would lead to the lowest utility, we can restrict attention to one step deviations.

As any one step deviation at (x^*, P^*) leads to a cycle there exists no profitable one step deviation at (x^*, P^*). Assume that there is a profitable one step deviation at some $(x, P) \neq (x^*, P^*)$. But if the deviating coalition is taking another action at (x, P) then the deviation is not profitable since it will again end up at (x^*, P^*). Then the deviating coalition is inducing 'no action' at (x, P), but since the coalition moving at (x, P) weakly prefers (x^*, P^*) to (x, P), this cannot be a profitable deviation. Contradiction. Hence, ϕ is an ECB.

2nd Step: Suppose ϕ is an ECB such that $\mathcal{T}(\sigma((x, P))) = (x^*, P^*)$ for all $(x, P) \in Z$, then $(x^*, P^*) \in C(N, V)$.

Towards a contradiction suppose ϕ is an ECB under foresight such that $\mathcal{T}(\sigma(x, P)) = (x^*, P^*)$ for every $(x, P) \in Z$ but (x^*, P^*) is not in the core. Then there exists (x, P) such that $S \in P$ and $x_S > x^*(S)$. But ϕ induces a finite path from (x, P) to (x^*, P^*) and at some point someone in S is active on this path. Let (x', P') be the first node on the path for which some $j \in S$ is active, let j deviate by refusing to take the action. The resulting deviation is profitable as j is getting $x'_j = x_j$ instead of x_j^*. A contradiction.

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