

Should the most efficient firm invest in its capacity? A value capture approach

Abstract

Recently, cooperative game theory and the stand-alone core have been introduced to value capture theory to establish lower and upper bounds on the profits of firms. Within these bounds, firms' profitability depends on many unobservable factors including individual bargaining abilities and market-specific practices. A counterintuitive paradox has emerged from previous studies: a firm with a cost advantage might actually be worse off when it decides to expand its capacity. We show that this paradox is extremely persistent and can resist to most extensions of the model, including the presence of additional buyers that were not served originally and economies of scale for the expanding firm. By expanding, the firm now has to attract more consumers, which considerably limits its bargaining power.

Keywords: value capture; expansion; bargaining power; core

1. Introduction

It is an empirical fact that firms in similar economic positions can have drastically different levels of profits, a fact that is counter to classic economic theories. A recent explanation provided using the value capture approach is that firms with market power often deal with firms (clients, suppliers, etc.) that also have market power. Thus, the terms of trade between them will be determined through bargaining. Results of these negotiations are difficult to predict, and will often depend on factors not easy to identify by outside observers, notably negotiation skills and industry norms. Instead of trying to predict the exact price on which they will agree, a recent approach (see for example Brandenburger and Stuart (1996, 2007), MacDonald and Ryall (2004), Adegbsan (2009), Obloj and Zemsky (2015), Montez et al. (2017), and the review of Gans and Ryall (2017)) consists in defining the range of potential prices. To obtain that range, the notion of the core (Gillies 1953), borrowed from cooperative game theory, is used. For all prices outside the range, at least one of the partners has incentives to move away from the partnership, as there are better deals attainable with other partners.

In this paper, we are interested in the paradox described by Gans and Ryall (2017) for a market with two consumers and two producers. These producers offer identical products but are limited by their capacity, initially such that they can only serve a single client. We suppose that one of the producers is more efficient than its rival, i.e. that it has a lower marginal cost of production. Gans and Ryall (2017) show that, under some conditions, expanding capacity is a bad idea for the more efficient firm, even if capacity itself is costless: both the lower and upper bounds of the profit range decrease with the expansion. This counter-intuitive phenomenon happens because the firm, by increasing its capacity, affects its negotiation power: pre-expansion it could put in competition the two consumers, which cannot be done anymore after the expansion, when the firm is trying to sell to both.

The notion that capacity impacts bargaining power is not new, and some real-life examples include airports limiting slots to increase bids by airline companies (Fan and Odoni 2002), retailers limiting shelf space to increase competition between manufacturers (Marx and Shaffer 2010) and insurance companies limiting the number of drugs covered by their plans to increase competition among pharmaceutical companies (Huskamp et al. 2005).

Our contributions consist in widening the scope and deepening our understanding of the paradox. We extend the model in many ways. We first add a third buyer, initially unserved, and distinguish between the expansion from 1 to 2 units and the expansion from 2 to 3 units. In the first case, expansion allows the rival firm to remain in the market but will move it to the consumers with lower willingness to pay, while in the second case it completely eliminates the rival firm. We also allow for more general cost functions, in particular considering the case in which the expanding firm has decreasing marginal costs, for which expansion should be more beneficial.

A more technical contribution of the paper is in terms of terminology, in precisely defining what we mean by “worse off,” when we compare ranges of profits. We define three ways in which we can examine the changes to the range of profits and conclude that it has worsened. We also provide a complete analysis of the determinants of these lower and upper bounds on profits, allowing a better understanding on how they are determined.

Our paradox result can serve as a warning for managers, as it highlights the importance of bargaining power and how it can affect the profitability of some decisions. Furthermore, it shows the impact of bilateral trades in which terms of trades are bargained: the most efficient firm might not have incentives to expand, which has important implications on the efficiency of such markets.

It is worth noting that before the expansion, the model is a special case of the (one-to-one) assignment problem (Shapley and Shubik 1971), while after the expansion it is a special case of the many-to-one assignment problem (Kelso and Crawford 1982). Both models are well-studied in the economics literature, both from the cooperative and non-cooperative points of view.

2 The model

We describe in this section the market in which firms evolve, as well as the concept of the core, used to establish lower and upper bounds to the profits of the firm.

2.1 The market

Two firms are competing on a market. The firms are identified as e (for expanding) and r

(for rival). Firms produce an identical good or service. We have three potential consumers, labeled 1, 2 and 3, which either consume zero or one unit of the good/service. The valuation of agent i for the good/service is u_i , with $u_1 \geq u_2 \geq u_3$.¹ Initially, both firms have only one unit of capacity, i.e. they can only serve one consumer each. We consider the expansion of firm e 's capacity from 1 to 2 and then from 2 to 3. We do not make any assumptions about the returns to scale of firm e , but we suppose that $c_{ei} \leq c_r$, where c_{ei} is the marginal cost of firm e for its i th unit and c_r is the marginal cost of the rival for its lone unit. Notice that before the expansion of the capacity, the market is an assignment market, as introduced by Shapley and Shubik (1971).

2.2 The core and the lower and upper bounds on profits

The various actors in the market will bargain over the shares of the surplus generated by their market transactions. However, these shares are constrained by stability conditions: if the combined share of a group of market participants is lower than the surplus they can jointly generate by trading only among themselves, then this group will secede from the grand coalition and trade among themselves. The set of allocations of the surplus that removes any incentives for groups to secede in such a manner is called the core (Gillies 1959). Our set of (potential) market participants is $N = \{e, r, 1, 2, 3\}$. For each $S \subseteq N$, let $V(S)$ be the (maximal) surplus generated by the group. For a vector $x \in R_+^N$ and a coalition $S \subseteq N$, let $x_S = \sum_{i \in S} x_i$. An allocation is $y \in R_+^N$ such that $y_N = V(N)$. A core allocation y is an allocation such that $y_S \geq V(S)$ for all $S \subseteq N$. The set of core allocations is denoted as $Core(V)$: Let π_e^{min} be the minimal core allocation of firm e and π_e^{max} be its maximal core allocation.

To distinguish between cases with various capacities for firm e , we denote as $\pi_e^{min}(k)$ and $\pi_e^{max}(k)$ the minimal and maximal core allocations when firm e has capacity k . In addition, V^k is the value game when capacity of firm e is k . Notice that since k only affects firm e we have that $V^k(S) = V^l(S)$ for all k, l if $e \notin S$. Also, since capacity affects the

¹ More precisely, agents have a quasi-linear utility function in money, with the willingness of agent i to pay for the first unit is u_i and zero for all subsequent units.

value only if we have enough buyers, we have that $V^k(S) = V^l(S)$ for all $k, l \geq |S \cap \{1,2,3\}|$. When there is no risk of confusion, we write $V(\{e, i\})$ instead of $V^1(\{e, i\})$, for $i = 1, 2, 3$.

Given that we are dealing with intervals, we need to define precisely the changes to an interval that clearly can be interpreted as negative for the firm. The first type of change to the interval that we consider is straightforward: all points in the interval after capacity expansion are worse than all points in the interval before the expansion. Without any doubts, in that case, the firm is in a worse position.

Definition 1. We say that increasing capacity is *unambiguously detrimental* for firm e if $\pi_e^{max}(k+1) \leq \pi_e^{min}(k)$

Unfortunately, this is too strong of a definition. We focus on two weaker versions. The first one is such that both the upper and lower bounds decline following the change in capacity.

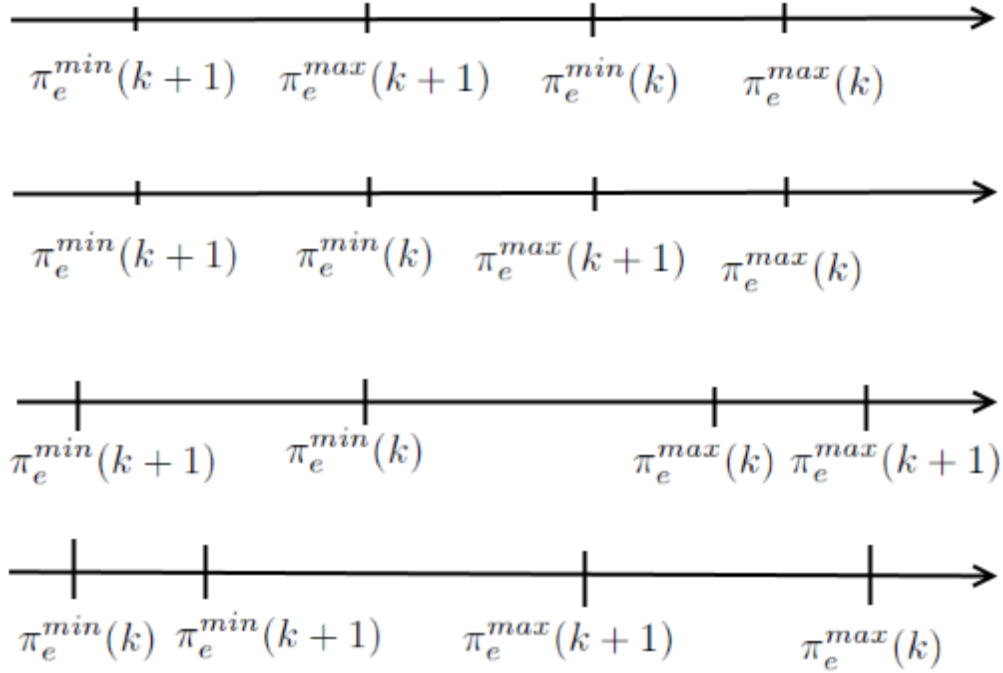
Definition 2. We say that increasing capacity results in a detrimental shift of profits of firm e if $\pi_e^{min}(k) \geq \pi_e^{min}(k+1)$ and $\pi_e^{max}(k) \geq \pi_e^{max}(k+1)$

In that case, the intervals overlap, meaning that it is still possible for the firm to do better than before the capacity expansion. But doing so requires a significant change in its ability to obtain profits that are closer to its maximum. We also consider another version that is slightly weaker and applies when bounds are moving in opposite directions. It applies when the change to the upper bound is less than the change to the lower bound. In that case, the middle point of the interval is decreasing.

Definition 3. We say that increasing capacity results in a detrimental spread of profits of firm e if $\pi_e^{max}(k+1) - \pi_e^{max}(k) \leq \pi_e^{min}(k) - \pi_e^{min}(k+1)$

The three cases are illustrated in Figure 1.

Figure 1: Illustration of Definitions 1, 2, and 3



Note: Unambiguously detrimental changes are on the top line, detrimental shifts are on the second line and detrimental spreads are in the third and fourth lines.

To further distinguish between the three cases, suppose that with capacity k , firm e is at the end able to secure profit level $\pi_e(k)$. We then define the relative bargaining ability of firm e with capacity k as $A_e^k = \frac{\pi_e(k) - \pi_e^{\min}(k)}{\pi_e^{\max}(k) - \pi_e^{\min}(k)}$.² A_e^k thus measures the ability to get profits closer to the maximum level. By construction, it is a number between 0 and 1, and the closer to 1 the larger is that ability.

Notice that there is no reason to expect A_e^k to change with k . Increasing the capacity will affect the bargaining power (which are the upper and lower bounds on profits), but the intrinsic negotiating abilities of the parties involved should remain. Using this notion of relative bargaining ability, we can restate the definitions shown above.

² We suppose $\pi_e^{\max}(k) > \pi_e^{\min}(k)$. Otherwise, bargaining ability is irrelevant.

- Lemma 4.** i) If increasing capacity from k to $k + 1$ is unambiguously detrimental, then the profit of firm e decreases for all values of A_e^k and A_e^{k+1} ;
- ii) If increasing capacity from k to $k + 1$ results in a detrimental shift, then the profit of firm e decreases if $A_e^k = A_e^{k+1} \equiv A$, for any values of A ;
- iii) If increasing capacity from k to $k + 1$ results in a detrimental spread, then the profit of firm e decreases if $A_e^k = A_e^{k+1} = \frac{1}{2}$.

We can see the clear ranking between the three versions. An unambiguously detrimental expansion will result in worse profits even if the firm goes from being terrible at bargaining to fantastic at it. An expansion resulting in a detrimental shift will negatively impact the firm's profits if its bargaining ability does not improve, regardless of what was the level initially. Suppose that the firm does not know its bargaining ability, and suppose a symmetric distribution on its value between 0 and 1. Then, in expectations, an expansion resulting in a detrimental spread reduces profits.

We can actually say more about detrimental spreads, but the analysis depends on if we are in a case in which bounds have become more extreme (as in the third line of Figure 1) or less extreme (as in the fourth line). If we suppose that bounds have become more extreme (which will be the more frequent case studied) and that the bargaining ability stays the same when expanding capacity, we obtain that the detrimental spread reduces profits if $A \leq \frac{\pi_e^{\min}(k) - \pi_e^{\min}(k+1)}{\pi_e^{\min}(k) - \pi_e^{\min}(k+1) + \pi_e^{\max}(k+1) - \pi_e^{\max}(k)}$. By definition, that fraction is larger than $\frac{1}{2}$. Thus, it requires a better than average relative negotiation ability to avoid a reduction of profits. If bounds become less extreme, the inequality is reversed and it requires a worse than average negotiation ability to avoid a reduction in profits. In both cases, again supposing a symmetric distribution of negotiation abilities, whenever we have a case with detrimental spreads, more than half of the population would be worse off. Stated otherwise, the average negotiator would be worse off.

3 Results

We suppose that $c_{e1}, c_{e2}, c_{e3} \leq c_r$ and u_3 . We show the values for minimal and maximal core allocations for firm e in Table 1. Details are provided in Appendix 1.

Table 1: Values for minimal and maximal core allocations for firm e

	π_e^{min}
$k = 1$	$V(\{r, 3\}) + V(\{e, 2\}) - V(\{r, 2\})$
$k = 2$	$V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - V^2(\{e, 1, 2\}) - 2V(\{r, 3\})$
$k = 3$	$\text{Max}(0, V^2(\{e, 1, 2\}) + V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\}))$
	π_e^{max}
$k = 1$	$V(\{r, 2\}) + V(\{e, 1\}) - V(\{r, 1\})$
$k = 2$	$V^2(\{e, 1, 2\}) - V(\{r, 2\}) - V(\{r, 1\}) + 2V(\{r, 3\})$
$k = 3$	$V^3(\{e, 1, 2, 3\}) - V(\{r, 1\}) - V(\{r, 2\}) - V(\{r, 3\})$

For $k = 1$, the lower bound is such that firm e extracts the extra value as if it had linear cost and an extra unit of capacity: agent 2 would buy from e instead of r , yielding $V(\{e, 2\}) - V(\{r, 2\})$, while agent 3 would (potentially) buy from firm r , yielding $V(\{r, 3\})$. The upper bound is the value it generates by being on the market. Without firm e , agent 1 would buy from r and agent 2 would not buy at all. Thus the gains are $V(\{e, 1\}) - V(\{r, 1\})$ (1 buying from e instead of r) and $V(\{r, 2\})$ (2 (potentially) buying from r instead of not at all). Notice that this upper bound simplifies to $V(\{r, 2\}) + c_r - c_{e1}$.

For $k = 2$, notice that the lower bound simplifies to $2u_3 - c_{e1} - c_{e2} - 2V(\{r, 3\})$. It can be interpreted as follows: firm e can threaten each of its buyers to replace it with agent 3, allowing it to extract the average gain in surplus compared to what agent 3 obtains with firm r : $u_3 - (c_{e1} + c_{e2})/2 - V(\{r, 3\})$. The upper bound is equal to the surplus its presence creates, compared to an hypothetical scenario in which firm r has linear costs and no capacity constraints, $V^3(\{e, 1, 2, 3\}) - V(\{r, 1\}) - V(\{r, 2\})$, to which we add $V(\{r, 3\})$ for each of its buyers, obtained by threatening them of being replaced by agent 3.

For $k = 3$, the lower bound simplifies to $\max\{0, 2c_{e3} - c_{e1} - c_{e2}\}$. Thus, the lower

bound is strictly positive only if we have strictly increasing marginal costs. Unlike with other capacities, firm e cannot threaten to replace a buyer by somebody else. With increasing returns to scale, the worse scenario is no profits for firm e . With decreasing returns to scale, it can offer every buyer $u_i - c_{e3}$, the surplus it creates by being the last convinced to buy. There remains profits of $2c_{e3} - c_{e1} - c_{e2}$ in that case. Once again, the upper bound is equal to the surplus its presence creates, compared to a hypothetical scenario in which firm r has linear costs and no capacity constraints.

We are now ready to examine when expansion has a negative effect on the profits of firm e .

3.1 Increasing capacity from 1 to 2

3.1.1 Detrimental shift

To obtain a detrimental shift, we need

$$V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) + V(\{r, 2\}) \leq V^2(\{e, 1, 2\}) + 3V(\{r, 3\}) + V(\{e, 2\})$$

and $V^2(\{e, 1, 2\}) + 2V(\{r, 3\}) \leq 2V(\{r, 2\}) + V(\{e, 1\})$.

If $c_r \geq u_2 \geq u_3$, there is never a detrimental shift.

If $u_2 \geq c_r \geq u_3$, the conditions simplifies to $u_3 \leq (c_{e1} + c_{e2})/2$ (the non-served agent must have a reserve price lower than the average of the marginal cost of the last units produced by each firm) and $c_r - c_{e2} \leq u_2 - c_r$ (the value created by agent 2 with the rival firm must be higher than the cost advantage for the second unit of the expanding firm).

If $u_2 \geq u_3 \geq c_r$, the conditions simplifies to $u_3 - c_{e2} \leq u_2 - u_3$ (the benefit that the expanding firm would obtain by taking agent 3 as its second client must not be larger than the gain from serving agent 2 instead) and $c_r - c_{e2} \leq u_3 - c_r$ (the value created by agent 3 with the rival firm must be higher than the cost advantage for the second unit of the expanding firm).

3.1.2 Detrimental spread

To obtain a detrimental spread, we need

$$V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) \leq V(\{r, 2\}) + V(\{r, 3\}) + V(\{e, 1\}) + V(\{e, 2\})$$

which simplifies to

$$2(u_3 - c_{e2}) \leq V(\{r, 2\}) + V(\{r, 3\}).$$

If $c_r \geq u_2 \geq u_3$, there is never a detrimental spread.

If $u_2 \geq c_r \geq u_3$, the conditions simplifies to $2(u_3 - c_{e2}) \leq u_2 - c_r$ (the surplus of agent 2 with firm r must be twice as large as the added surplus when agent 3 is the second served by firm e).

If $u_2 \geq u_3 \geq c_r$, the conditions simplifies to $2(c_r - c_{e2}) \leq u_2 - u_3$ (the difference in reserve prices between agents 2 and 3 must be twice as large as the differences in marginal cost of the last units produced by the two firms).

3.2 Increasing capacity from 2 to 3

3.2.1 Detrimental shift

To obtain a detrimental shift, we need

$$\begin{aligned} \text{Max}(\{0, V^2(\{e, 1, 2\}) + V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\}) \leq \\ V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - V^2(\{e, 1, 2\}) - 2V(\{r, 3\}) \end{aligned}$$

and $V^3(\{e, 1, 2, 3\}) \leq V^2(\{e, 1, 2\}) + 3V(\{r, 3\})$.

The first condition is always satisfied. If we have increasing returns to scale, the left - hand side is zero, while the right - hand side is non-negative. With decreasing returns to scale, it simplifies to

$$2V^2(\{e, 1, 2\}) + 2V(\{r, 3\}) \leq 2V^3(\{e, 1, 2, 3\})$$

which further simplifies to $2V(\{r, 3\}) \leq 2(u_r - c_{e3})$ which is always satisfied.

The second condition simplifies to $u_3 - c_{e3} \leq 3V(\{r, 3\})$: the surplus created by firm r

with agent 3 must be thrice as large as the extra surplus when agent 3 is the third agent served by firm e .

If $c_r \geq u_3$, there is never a detrimental shift.

If $u_3 \geq c_r$, we need $(c_r - c_{e3}) \leq 2(u_3 - c_r)$: the surplus created by firm r with agent 3 must be twice as large as the differences in marginal cost of the last units produced by the two firms.

3.2.2 Detrimental spread

To obtain a detrimental spread, we need

$$V^3(\{e, 1, 2, 3\}) - V^2(\{e, 1, 2\}) - 3V(\{r, 3\}) \leq \\ V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - V^2(\{e, 1, 2\}) - 2V(\{r, 3\}) - \max\{0, 2c_{e3} - c_{e1} - c_{e2}\}$$

which simplifies to

$$c_{e1} + c_{e2} + \max\{0, 2c_{e3} - c_{e1} - c_{e2}\} \leq V(\{r, 3\}) + u_3 + c_{e3}$$

If we have decreasing returns to scale, then $\max\{0, 2c_{e3} - c_{e1} - c_{e2}\} = 2c_{e3} - c_{e1} - c_{e2}$. We then have that $2c_{e3} \leq u_3 + c_{e3} \leq V(\{r, 3\}) + u_3 + c_{e3}$ and the inequality is satisfied. We thus always have a detrimental spread.

If we have increasing returns to scale, then $\max\{0, 2c_{e3} - c_{e1} - c_{e2}\} = 0$. We then have a detrimental spread if $c_{e1} + c_{e2} \leq u_3 + c_{e3} + V(\{r, 3\})$. We can rewrite the inequality as $c_{e1} + c_{e2} - 2c_{e3} \leq u_3 - c_{e3} + V(\{r, 3\})$. On the left – hand side is a measure of the returns to scale: we have the cost of producing the first two units minus twice the cost of producing the third one. On the right – hand side are the benefits the third buyer obtains with the rival and with the expanding firm if it is sold the third unit produced.

3.3 General results

We summarize the results of the previous subsections in the following Theorem.

Theorem 5. i) If the rival firm was not selling to anybody prior to the expansion, then we cannot have a detrimental shift. Otherwise, there always exist some parameter values for

which we have a detrimental shift.

ii) If the rival firm was not selling to anybody prior to the expansion, and there are unserved consumers after the expansion, then we cannot have a detrimental spread. Otherwise, there always exist some parameter values for which we have a detrimental spread.

iii) If the returns to scale are small enough, expanding to sell to all potential customers on the market always yields a detrimental spread. In particular, if marginal cost is not decreasing, then expanding to sell to all customers on the market always yields a detrimental spread.

4 Extending the paradox

In this section, we show that we might still have detrimental shifts when the firm expands even in cases in which it had the initial disadvantage, i.e. when $c_{e1} > c_r$ but c_{e2} and $c_{e3} < c_r$. It would be natural to think that since expanding allows the firm to move in front of the rival firm, it would be profitable to expand, but that is not always so. We do not provide an extensive study of that case, as the changes in the structure of the problem make it such that the core might become empty. Instead, we focus on an example.

Suppose that $c_{e1} = 4$, $c_{e2} = 2$, $c_{e3} = 1$ and $c_r = 3$, with $u_3 \geq c_{e1}$. The lower and upper bounds in this specific example are shown in Table 2. Details are provided in Appendix 2.

Table 2: Lower and upper bounds with $c_{e1} = 4$, $c_{e2} = 2$, $c_{e3} = 1$ and $c_r = 3$

	π_e^{min}	π_e^{max}
$k=1$	$u_3 - 4$	$u_2 - 4$
$k=2$	0	$2u_3 - 6$
$k=3$	0	2

Now suppose that $u_2 = 9$ and $u_3 = 5$. Then we have that $\pi_e^{min}(1) = 1$, $\pi_e^{max}(1) = 5$, $\pi_e^{min}(2) = 0$, $\pi_e^{max}(2) = 4$, $\pi_e^{min}(3) = 0$, $\pi_e^{max}(3) = 2$. Thus, both expansions from 1 to 2 units and from 2 to 3 units yield detrimental shifts.

5 Implications

Our results show that the paradox is remarkably persistent. We discuss in this section the implications of the paradox.

5.1 Managerial decisions

The idea that a firm with a cost advantage should expand to capitalize on this advantage seems like a no-brainer. But this is without taking into account the impacts on the bargaining powers of the agents involved. We show that unless the market was already monopolized (with the rival firm not selling anything prior to the expansion), then there are always parameter values that yield detrimental shifts in the lower and upper bounds for profits. The milder notion of detrimental spreads of the bounds is even more prominent, as there are always parameter values that yield them, unless the market is monopolized and there are unserved consumers after the expansion. Expanding to sell to all potential customers always results in a detrimental spread of the profits unless the returns to scale are particularly important. We also expand on the paradox by showing that it might still hold even if the firm has a cost disadvantage for the first unit, but advantages for further units.

Thus, a manager thinking about expanding should be careful before undergoing such an operation. Section 3 also provides some insights on when it might be a bad idea to expand.

When expanding from 1 unit to 2, what mostly matters are the value of the second buyer and the cost of producing that second unit. Both the probabilities for detrimental shifts and detrimental spreads increase with the value of the second buyer (as it gives him more bargaining power) and with the cost of producing the second unit.

For detrimental spreads, we also have clear impacts for the value put on by the third buyer (a high value gives more bargaining power to the expanding firm and reduces the probability of a detrimental spread) and the cost of the rival firm (which has a similar impact). For the detrimental shifts, the effects are ambiguous. If the third buyer can't buy from the rival firm but the second buyer can ($u_2 \geq c_r \geq u_3$) then an increase in the value put on by the third buyer increases the lower bound and thus decreases the probability of a detrimental spread. If the third buyer can buy from the rival firm ($u_3 \geq c_r$) then an increase in the value of buyer 3 had ambiguous effects, as it has conflicting effects on the lower and upper bounds. The effect

of the cost of the rival firm is similarly ambiguous when it cannot sell to firm 3 ($u_2 \geq c_r \geq u_3$) but it clearly reduces the probability of detrimental shifts if the rival can sell to all consumers.

When expanding from 2 to 3 units, high values for buyer 3 or for the cost of producing the third unit and low values for the cost of the rival all increase the probabilities of detrimental shifts and spreads. Detrimental spreads also depend on scale effects, with higher returns to scale decreasing the probability of a detrimental shift.

5.2 Efficiency of the market

Regardless of the effects on bargaining powers, the expansion of the capacity of the most efficient firm always generates extra surplus and improves the efficiency of the market. The paradox shows however that the firm with the cost advantage might not always have the incentives to do such a capacity expansion. In turn, this might generate significant losses in welfare, from the point of view of society. Counter-intuitively, policies that help dominant firms establish their bargaining power (setting industry standards on surplus sharing through government contracts, or setting minimum prices, for example) might help circumvent this result.

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Appendix 1: Derivation of lower and upper bounds for the standard case

We provide calculations for minimal and maximal core allocations for firm e with $c_{e1}, c_{e2}, c_{e3} \leq c_r$.

i) $k = 1$

Since agent 3 is not served, $y_3 = 0$ in any core allocations. Since $V^1(\{e, r, 2, 3\}) = V(\{e, 2\}) + V(\{r, 3\})$, we must have $y_{\{e, r, 2\}} = V(\{e, 2\}) + V(\{r, 3\})$ in all core allocations. Since $V^1(N) = V(\{e, 1\}) + V(\{r, 2\})$, we have that $y_{\{r, 2\}} = V(\{r, 2\})$ in any core allocations. Combining these last two results, we obtain that $y_e \geq V(\{e, 2\}) + V(\{r, 3\}) - V(\{r, 2\})$ in any core allocation. We can verify that the allocation $y = (V(\{e, 2\}) + V(\{r, 3\}) - V(\{r, 2\}), V(\{r, 3\}), V(\{e, 1\}) + V(\{r, 2\}) - V(\{e, 2\}) - V(\{r, 3\}), V(\{r, 2\}) - V(\{r, 3\}), 0)$ is in the core. Thus, $\pi_e^{\min}(1) = V(\{r, 3\}) + V(\{e, 2\}) - V(\{r, 2\})$.

Since $V^1(\{r, 1, 2, 3\}) = V(\{r, 1\})$, we have that $y_e \leq V(\{e, 1\}) + V(\{r, 2\}) - V(\{r, 1\})$ in any core allocation. Since the allocation $y = (V(\{e, 1\}) + V(\{r, 2\}) - V(\{r, 1\}), V(\{r, 2\}), V(\{r, 1\}) - V(\{r, 2\}), 0, 0)$ is in the core, $\pi_e^{\max}(1) = V(\{e, 1\}) + V(\{r, 2\}) - V(\{r, 1\})$.

ii) $k = 2$

Since $V^2(N) = V^2(\{e, 1, 2\}) + V(\{r, 3\})$, we have that $y_{\{r, 3\}} = V(\{r, 3\})$. Since $V^2(\{e, r, 1, 3\}) = V^2(\{e, 1, 3\})$, we must have that $y_{\{e, 1\}} \geq V^2(\{e, 1, 3\}) - V(\{r, 3\})$ in any core allocation. We also have that $y_{\{e, r, 2, 3\}} \geq V^2(\{e, 2, 3\})$ in any core allocation. Summing up these two constraints we obtain, in any core allocation, that $y_e + y_N \geq V^2(\{e, 1, 3\}) - V(\{r, 3\}) + V^2(\{e, 2, 3\})$ and $y_e \geq V^2(\{e, 1, 3\}) - 2V(\{r, 3\}) + V^2(\{e, 2, 3\}) - V^2(\{e, 1, 2\})$, as we use the fact that $y_N = V^2(\{e, 1, 2\}) + V(\{r, 3\})$. Since the allocation $y = (V^2(\{e, 1, 3\}) - 2V(\{r, 3\}) + V^2(\{e, 2, 3\}) - V^2(\{e, 1, 2\}), 0, V^2(\{e, 1, 2\}) + V(\{r, 3\}) - V^2(\{e, 2, 3\}), V^2(\{e, 1, 2\}) + V(\{r, 3\}) - V^2(\{e, 1, 3\}), V(\{r, 3\}))$ is in the core, $\pi_e^{\min}(2) = V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - V^2(\{e, 1, 2\}) - 2V(\{r, 3\})$.

Since $y_{\{r, 3\}} = V(\{r, 3\})$ in all core allocations and $V^2(\{r, 2, 3\}) = V(\{r, 2\})$, we have that $y_2 \geq V(\{r, 2\}) - V(\{r, 3\})$ in all core allocations. In addition, we have that $y_{\{r, 1, 3\}} \geq V^2(\{r, 1, 3\}) = V(\{r, 1\})$ for all core allocations. Combining these two results yields

$y_{\{r,1,2,3\}} \geq V(\{r, 1\}) + V(\{r, 2\}) - V(\{r, 3\})$ and thus

$y_e \leq V^2(\{e, 1, 2\}) + 2V(\{r, 3\}) - V(\{r, 1\}) - V(\{r, 2\})$ in all core allocations. Since the allocation $y = (V^2(\{e, 1, 2\}) + 2V(\{r, 3\}) - V(\{r, 1\}) - V(\{r, 2\}), V(\{r, 3\}), V(\{r, 1\}) - V(\{r, 3\}), V(\{r, 2\}) - V(\{r, 3\}), 0)$ is in the core, $\pi_e^{max}(2) = V^2(\{e, 1, 2\}) + 2V(\{r, 3\}) - V(\{r, 1\}) - V(\{r, 2\})$.

iii) $k = 3$

Since $V^3(N) = V^3(\{e, 1, 2, 3\})$, we have that $y_r = 0$ in any core allocation. This implies that $y_i \geq V(\{r, i\})$ in any core allocation, for $i = 1, 2$ or 3 . Combining, we obtain $y_{\{r,1,2,3\}} \geq V(\{r, 1\}) + V(\{r, 2\}) + V(\{r, 3\})$ and thus $y_e \leq V^3(\{e, 1, 2, 3\}) - V(\{r, 1\}) - V(\{r, 2\}) - V(\{r, 3\})$ in any core allocation. Since the allocation $y = (V^3(\{e, 1, 2, 3\}) - V(\{r, 1\}) - V(\{r, 2\}) - V(\{r, 3\}), 0, V(\{r, 1\}), V(\{r, 2\}), V(\{r, 3\}))$ is in the core, we obtain $\pi_e^{max}(3) = V^3(\{e, 1, 2, 3\}) - V(\{r, 1\}) - V(\{r, 2\}) - V(\{r, 3\})$.

To find the minimum core allocation, we need to distinguish two subcases

a) $c_{e1} \geq c_{e2} \geq c_{e3}$

Since the allocation $y = (0, 0, V^3(\{e, 1, 2, 3\}) - V^3(\{e, 2, 3\}), V^3(\{e, 2, 3\}) - V(e, 3), V(\{e, 3\}))$ is in the core, $\pi_e^{min}(3) = 0$.

b) $c_{e1} \leq c_{e2} \leq c_{e3}$

We have that $V^3(\{e, r, 1, 2\}) = V^3(\{e, 1, 2\})$ and $V^3(\{e, r, 1, 3\}) = V^3(\{e, 1, 3\})$ and thus $y_{\{e,r,1,2\}} \geq V^3(\{e, 1, 2\})$ and $y_{\{e,r,1,3\}} \geq V^3(\{e, 1, 3\})$ in any core allocation. Adding these constraints, we obtain $y_{\{e,r,1\}} + y_N \geq V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\})$ and $y_{\{e,r,1\}} \geq V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) - V^3(\{e, 1, 2, 3\})$ as we use the fact that $y_N = V^3(\{e, 1, 2, 3\})$. Notice that since $c_{e1} \leq c_{e2} \leq c_{e3}$, $V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) - V^3(\{e, 1, 2, 3\}) \geq V^3(\{e, r, 1\}) = V(\{e, 1\})$. We also have that $y_{\{e,2,3\}} \geq V^3(\{e, 2, 3\})$. Adding these two constraints, we

obtain $y_e + y_N \geq V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) + V^3(\{e, 2, 3\}) - V^3(\{e, 1, 2, 3\})$ and $y_e \geq V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) + V^3(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\})$ as we again use the fact $y_N = V^3(\{e, 1, 2, 3\})$. Notice that since $c_{e1} \leq c_{e2} \leq c_{e3}$, $V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) + V^3(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\}) \geq 0$.

Since the allocation

$y = (V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) + V^3(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\}), 0, V^3(\{e, 1, 2, 3\}) - V^3(\{e, 2, 3\}), V^3(\{e, 1, 2, 3\}) - V^3(\{e, 1, 3\}), V^3(\{e, 1, 2, 3\}) - V^3(\{e, 1, 2\}))$ is in the core, we have $\pi_e^{min}(3) = V^3(\{e, 1, 2\}) + V^3(\{e, 1, 3\}) + V^3(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\})$. Notice that we can also write $\pi_e^{min}(3) = V^2(\{e, 1, 2\}) + V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\})$. Combining the two cases, we have, in general, that $\pi_e^{min}(3) = \max\{0, V^2(\{e, 1, 2\}) + V^2(\{e, 1, 3\}) + V^2(\{e, 2, 3\}) - 2V^3(\{e, 1, 2, 3\})\} = \max\{0, 2c_{e3} - c_{e1} - c_{e2}\}$.

Appendix 2: The case for the initial disadvantage

i) $k = 1$

Agent 3 is not served, and thus $y_3 = 0$ in any core allocation. Since $V(\{e, 3\}) = u_3 - 4$, we must have $y_e \geq u_3 - 4$. Since the allocation $(u_3 - 4, u_3 - 3, u_1 - u_3, u_2 - u_3, 0)$ is in the core, $\pi_e^{min}(1) = u_3 - 4$.

We have that $V^1(N) = u_1 + u_2 - 7$ and $V^1(\{r, 1, 2, 3\}) = u_1 - 3$, we must have that $y_e \leq u_2 - 4$ in any core allocation. Since the allocation $(u_2 - 4, u_2 - 3, u_1 - u_2, 0, 0)$ is in the core, $\pi_e^{max}(1) = u_2 - 4$.

ii) $k = 2$

Since the allocation $(0, 0, u_1 - 3, u_2 - 3, u_3 - 3)$ is in the core, $\pi_e^{min}(2) = 0$. We have that $V^2(\{e, 1, 2\}) = u_1 + u_2 - 6$, $V^2(\{r, 1, 3\}) = u_1 - 3$ and $V^2(N) = u_1 + u_2 - u_3 - 9$. We obtain: $y_{\{e, 1, 2\}} + y_{\{r, 1, 3\}} = y_1 + y_N \geq 2u_1 + u_2 - 9$ and $y_1 \geq u_1 - u_3$.

Consider next that $V^2(\{r, 2, 3\}) = u_2 - 3$ and we obtain, by combining, that $y_{\{r, 1, 2, 3\}} \geq u_1 + u_2 - u_3 - 3$ in any core allocation, which implies that $y_e \leq 2u_3 - 6$. Since the allocation $(2u_3 - 6, u_3 - 3, u_1 - u_3, u_2 - u_3, 0)$ is in the core, $\pi_e^{max}(2) = 2u_3 - 6$.

iii) $k = 3$

Since the allocation $(0, 0, u_1 - 1, u_2 - 3, u_3 - 3)$ is in the core, $\pi_e^{min}(3) = 0$. Since $V^3(N) = V^3(\{e, 1, 2, 3\}) = u_1 + u_2 + u_3 - 7$, we obtain that $y_r = 0$ in any core allocation. In addition, for $i = 1, 2, 3$, we have that $V(\{r, i\}) = u_i - 3$ which implies that $y_i \geq u_i - 3$. Therefore, $y_{\{1, 2, 3\}} \geq u_1 + u_2 + u_3 - 9$ in any core allocation, which implies that $y_e \leq 2$ in any core allocation. Since the allocation $(2, 0, u_1 - 3, u_2 - 3, u_3 - 3)$ is in the core, we obtain that $\pi_e^{max}(3) = 2$.