School Choice with Asymmetric Information: Priority Design and the Curse of Acceptance

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Abstract

An implicit assumption in most of the matching literature is that all participants know their preferences. If there is variance in the effort agents spend researching options, some will know their preferences, while others may not. When this is true, (ex-post) stable outcomes need not exist and informed agents gain at the expense of less informed agents, outcomes we attribute to a *curse of acceptance* for the less informed students. However, when all agents have a "secure school", we recover positive results: equilibrium strategies are simple, the outcome is ex-post stable, and less informed students are made better off. Our results have potential policy implications for the current debate in school choice over how priority design affects outcomes.

1 Introduction

"Please accept my resignation. I refuse to join any club that would have me as a member."
-*Groucho Marx*

In the last decade, school choice has rapidly expanded across the United States and around the world,[1] which has led to a vast and rapidly expanding economics literature, encompassing a wide array of theoretical, practical, and empirical papers. On the mechanism design side of this literature, the standard modeling approach emanates from the seminal paper of Abdulkadiroğlu and Sönmez (2003), and is the analogue of what the broader mechanism design/auctions literature refers to as *private values*: each student is perfectly informed

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[1] In a 2012 survey, almost 40% of parents in the United States reported having access to public school choice, a number which has certainly only increased since (National Center For Education Statistics, 2015).

about her own (usually strict) ordinal preference relation over all schools.[2] While this simplifies the analysis, it abstracts away from differentially informed students that likely exists in practice. For example, a key component of most school choice markets is a pre-assignment information acquisition stage, during which some parents may spend considerable time researching and visiting schools to learn about their quality. However, not all parents engage in such activities, perhaps due to a lack of time or other resources. In this setting, the standard framework will not apply. In fact, the other model of mechanism design and auctions, *common values*, is likely a better analogue: every agent has the same value for the object but agents have different information about what this value is. It is reasonable to assume that students preferences are correlated, although perhaps not perfectly so, because one dimension on which students rate schools is the quality of the school. In this paper, we investigate school choice where students preferences are correlated and some students have more information than others.[3]

Perhaps the most well-known result from the common value auction literature is the so-called winner's curse. We start by identifying a related "curse" in our environment, which we call the *curse of acceptance:* upon observing their assignments, less informed students infer that, whatever school they are assigned, it is a worse school on average. Intuitively, some students may not have as much information as others, but they are at least aware that there are other students who *do* have this information, which implies that, all else equal, the more informed students are more likely to list the better schools higher ex-ante. Thus, for a less informed student, even if she listed a school highly ex-ante and is admitted, after the fact, she will update her beliefs about the school's quality downward; in short, just like Groucho, less informed students would prefer not to enroll in any school that admits them.

The curse of acceptance leads to the failure of many standard properties that are familiar from the literature, such as strategy-proofness and stability of well-known assignment mechanisms.[4] First, the strategy-proofness of popular mechanisms such as deferred acceptance (DA) will no longer hold. Indeed, as we show, determining an optimal strategy can be very

[2]Mechanism design and auctions also usually consider independence in this setting which has not previously been seen as relevant in the matching literature, where it has simply been assumed that each agent is "born" with an innate ordinal preference relation, and the distribution from which they are drawn is not modeled. However, in order to understand and analyze correlations in preferences, as we will do in this paper, some type of distributional assumptions will obviously be necessary.

[3]The broader mechanism design literature usually assumes that all agents get different signals but from the same information structure. In school choice, it is more natural to think of students as getting signals from different information structures and so this is the assumption we will pursue.

[4]Strategy-proofness means that it is a dominant strategy for every student to report her true preferences to the mechanism. Stability means that no student prefers some school to her assignment and has higher priority than a student who is assigned to that school.

complex for the less informed students.[5] Second, stable matchings may no longer exist. In particular, less informed students may desire to rematch and have high enough priority to do so, based on the information they obtain by observing their match and the inherent information of knowing when a school would accept them; a property we call ex-post stability. Third, and perhaps most importantly, from a welfare perspective, the less informed students will be worse off relative to the more informed students. Both of these latter two facts can be directly attributed to the curse of acceptance: since the more informed students have more knowledge about which schools are better than others, they will be more likely to receive these schools in equilibrium. Therefore, less informed students know that any school that they receive is likely to be a worse school on average. This last point is particularly salient, as school choice is often justified on the basis of giving all students equal access to quality schools. To the extent that disadvantaged students are more likely to be less informed, the curse of acceptance will exacerbate this problem.[6]

While the first part of the paper contains mostly negative results, they rely on showing that desirable properties fail for some specific priority structure. In the second part of the paper, we show these negative results motivate looking at the problem in a new light, as one of priority design. We show that if priorities are designed in an appropriate way, positive results can be recovered. In particular, we introduce the idea of a "secure school", which is simply a school s such that a student j's priority ranking at s is below the capacity of s; in other words, it is a school that j can be certain she will receive (under any stable mechanism), so long as she ranks it first. The idea of a secure school is reminiscent of "neighborhood schools" or "walk-zone schools" that are often discussed in the literature. However, we emphasize that secure schools *need not* be neighborhood schools, and can be based on many other criteria besides geography. For example, a school district may divide the schools into certain quality levels, and allocate priorities based on this metric (as has been done recently in Boston; see below).

When the priority structure is designed so that each student has a secure school, many positive results are recovered. First, we show that ex-post stable matchings exist, and the deferred acceptance mechanism finds one. Second, while DA will still not be strategy-proof, the equilibrium strategies become very simple. Third, and perhaps most importantly, the existence of a secure school protects uninformed students from the curse of acceptance. Their

[5]DA will continue to be strategy-proof for students who know their preferences.

[6]Pathak and Sönmez (2008) argue that one of the main advantages of strategy-proof mechanisms is that they "level the playing field" between what they refer to as sophisticated students (who strategize optimally) and sincere students (who always report their true preferences, whether it is in their interest or not). Note, however, that they still follow the standard modeling approach and assume that all sincere students know their own preferences. When these assumptions do not hold, strategy-proof mechanisms may no longer "level the playing field". See also Pathak and Sönmez (2013).

secure school cannot be taken from them by the informed students, and so the informed students are no longer able to exploit the uninformed students by always taking the better schools for themselves and leaving the uninformed students with the lower quality schools.[7]

Recovering positive results differentiates our paper from Chakraborty et al. (2010), who are the first and only other we are aware of to take the "mechanism design" approach to studying matching under interdependent values. They consider a two-sided (college admissions) market and show that under various alternative notions of stability, a series of impossibility results are obtained. They are only able to recover positive results by imposing a strict assumption, which in our environment would translate to a common priority relation that all schools use to admit students. Under this assumption, they find that a serial dictatorship is stable in their sense.[8]

While there is a rich school choice literature comparing different mechanisms (Boston vs. DA vs. TTC, etc.), there is far less work devoted to the topic of how to design the priority structure that the mechanism will use, which is arguably just as important (if not moreso) than the ultimate algorithm that is applied to them.[9] Most formal models simply take priorities as given, but in reality, many school districts have the ability to design the priorities to achieve certain goals. For instance, Boston recently undertook a major redesign of the menu of schools from which students could choose. After intense debate in the city, Boston eventually settled on a plan proposed by Shi (2015) (see also Pathak and Shi, 2017). Previously, the district had been divided geographically, and a student's menu of possible choices she could list depended only on where she lived. Roughly, under the new plan, schools are divided into quality tiers, and a student's menu consists of the two closest Tier 1 schools, the four closest Tier 2 schools, etc. The key parallel to priority design is that any school that is not in a student's menu is just a school where they have no priority. Now, this redesign was done with many objectives in mind (see Shi (2015) and Pathak and Shi (2017) for further details) but giving each student some, but limited access, to Tier 1 schools

[7]We emphasize existence of secure schools does not preclude students from applying to other schools should they determine that others are a better match, and indeed in our model students who are informed will do so. The point is that it is unlikely that the mechanism designer will know which students will be more or less informed ex-ante, and the role of secure schools is to prevent less informed students from being exploited while still allowing choice for those who have the requisite information.

[8]Chade (2006) identifies a phenomenon he calls the "acceptance curse effect" in a dynamic marriage model where men and women randomly meet each period and must decide whether to accept their current partner and leave the market or wait and get a new draw in the next period. While intuitively similar to our curse of acceptance, the precise manifestation, as well his overall motivation and model, are quite different from ours, and thus the results are not formally comparable.

[9]In a survey of the school choice literature, Pathak (2016) argues that much of research comparing different mechanisms has turned out to be not as important for practical implementation as it was thought at the time, and that "the criteria used to allocate seats [the priorities] - taken as given by much of the literature - are just as important as how the mechanism processes applicants' claims".

is somewhat like how one would give students secure schools in the sense that they have a "good" school that they have high priority at, because it is not on many students' menus.

In order to provide a broader theoretical insight, we abstract away from particular institutional details that apply only to certain cities. However, our formal model captures relevant features that are present in many real-world settings, yet are absent from the standard framework, while still remaining tractable enough to produce clear results. Boston's priority redesign plan was undertaken for a variety of other reasons, but our paper shows that there may be an additional benefit: it mitigates the curse of acceptance for less informed students. This is something that could not be captured in standard models that take the priority structure as a given and assume private values and perfectly informed students. We hope that our model brings attention to an important issue that has heretofore been mostly overlooked, and will be an important step towards better-designed school choice markets in practice.

The rest of this paper proceeds as follows. Section 2 presents our model of school choice under asymmetric information. Section 3 discusses impossibility results and the curse of acceptance. Sections 4 and 5 show how priority design choices can be used to recover positive results. Section 6 concludes.

2 Preliminaries

2.1 Model

Let $J = \{j_1, \cdots, j_N\}$ be a set of students and $S = \{s_1, \cdots, s_M\}$ be a set of schools. Each school s has capacity q_s and a priority relation $\succ_s$, where $\succ_s$ is a strict, complete, and transitive binary relation over J. We write $q = (q_s)_{s\in S}$ and $\succ = (\succ_s)_{s\in S}$ to denote the capacity vector and priority profile for all schools, respectively. We assume that $\sum_{s\in S} q_s \geq N$.[10]

Student preferences depend on an underlying state ω. The state space Ω is finite, with an associated probability distribution $\Pr : \Omega \to [0,1]$ such that $\sum_{\omega\in\Omega} \Pr(\omega) = 1$. Every student has a (state-dependent) utility function, where $u_j^\omega(s)$ is student j's utility for school s in state ω. We assume that preferences are strict, i.e., given a state ω, $u_j^\omega(s) \neq u_j^\omega(s')$ for all $s \neq s'$. Students may have different levels of information about the underlying state (and therefore about their own preferences over schools). In particular, the agents are partitioned into a set of informed students, I, and a set of uninformed students, U, where $J = I \cup U$. All students $j \in I$ receive a perfectly informative signal of the state, while all students $j \in U$

[10] This is a natural assumption in school choice, where all students must legally be offered a seat in some school.

receive no signal (or receive a completely uninformative signal). Let $\mathcal{I}_j(\omega) \subseteq \Omega$ denote the states that student j thinks are possible after receiving their signal when the true state is ω. That is, for the informed students we have $\mathcal{I}_j(\omega) = \{\omega\}$ for all $\omega \in \Omega$, while for uninformed students we have $\mathcal{I}_j(\omega) = \Omega$ for all $\omega \in \Omega$.[11]

Where relevant, agents evaluate lotteries using von Neumann-Morgenstern preferences; that is, a student's expected utility from matching to school s given information that the true state lies in some subset of the state space $\mathcal{I}_j \subseteq \Omega$, is written

$$E(u_j^\omega(s)|\mathcal{I}_j) = \sum_{\omega \in \Omega} u_j^\omega(s)\Pr(\omega|\mathcal{I}_j),$$

where $\Pr(\omega|\mathcal{I}_j)$ is the posterior probability of the true state being ω conditional on information $\mathcal{I}_j$, and is obtained via Bayes' rule.

The state space Ω defined here is quite flexible, but in what follows we will concretely define state spaces where the students' utility for a school is determined by an intrinsic quality, which is common to all students, and, later in the paper, also an idiosyncratic component which is individual to each student. Informed students should be thought of as those whose parents have invested the time and energy to learn which schools are the high quality schools. Uninformed students also prefer schools with high intrinsic quality, though they are uncertain which ones have this quality at the time they will be asked to submit their preferences. We use the model presented because it is the most parsimonious formulation that exhibits our main theoretical insights.

2.2 Matchings and mechanisms

A **matching** is a function $\mu : J \cup S \to 2^{J \cup S}$ such that: (i) $\mu(j) \in S$ for all $j \in J$, (ii) $\mu(s) \subseteq J$ and $|\mu(s)| \leq q_s$ for all $s \in S$, and (iii) $\mu(j) = s$ if and only if $j \in \mu(s)$. Hereafter, we use the shorthands μ_j for $\mu(j)$ and μ_s for $\mu(s)$. In words, μ_j is the school assigned to student j, μ_s is the set of students assigned to school s, and the number of students assigned to school s cannot exceed its capacity q_s. Let $\mathcal{M}$ denote the set of all possible matchings.

In order to implement a "good" matching, market organizers (e.g., school districts) must elicit the private information of the agents. In most real-world settings, this is done by asking the agents to submit an ordinal preference relation over the set of schools, and then applying a particular algorithm or mechanism to these preferences to determine the final matching.

[11]Note that we model the informed students as observing the true state ω precisely, which includes information about the preferences of all other agents. This is for convenience only; the mechanisms we consider will be "strategy-proof" as long as an agent is perfectly informed about her own preferences, and so whether she knows the preferences of the other students or not will be irrelevant.

To model this formally, let $\mathcal{P}$ denote the set of all strict ordinal preference relations over S. Given a preference relation $P_j \in \mathcal{P}$, we write sP_js' to denote that s is strictly preferred to s', and we write sR_js' if either sP_js' or $s = s'$. Let $P = (P_j)_{j \in J}$ denote a profile of preference relations, one for each student. A **mechanism** is a function $\psi : \mathcal{P}^N \to \mathcal{M}$. We use the notation $\psi_j(P)$ to denote student j's assignment under mechanism ψ when the submitted reports are P; analogously, $\psi_s(P)$ denotes school s's assignment.

A mechanism ψ induces a game in which the action space for each player is $\mathcal{P}$. A *strategy* for student j in this game is a mapping $\sigma_j : \Omega \to \mathcal{P}$ that is measurable with respect to her information $\mathcal{I}_j(\omega)$.[12] A profile of strategies $\sigma = (\sigma_1, \ldots, \sigma_N)$ is a *(Bayesian) equilibrium* of the game induced by mechanism ψ if

$$E[u_j^\omega(\psi_j(\sigma(\omega)))|\mathcal{I}_j(\omega)] \geq E[u_j^\omega(\psi_j(\sigma_j'(\omega), \sigma_{-j}(\omega)))|\mathcal{I}_j(\omega)]$$

for all other strategies σ', all $j \in J$, and all $\omega \in \Omega$.

In standard matching models (where each student knows her own preferences and there is no correlation across students), a mechanism is said to be *strategy-proof* if, for each student, reporting her true preferences is a dominant strategy of the induced preference revelation game. Strategy-proofness is a desirable property because it makes the mechanism simple for students to play. Strategy-proof mechanisms will continue to be "strategy-proof" for the informed students in our model, in the following sense: if a mechanism ψ is strategy-proof in the standard sense,[13] then, in the game we consider, it will be weakly dominant for an informed student to use the *truthful* strategy $\sigma_j(\omega) = P_j(\omega)$, where $P_j(\omega)$ is her true ordinal preference ranking in state ω. For uninformed students, however, following the truthful strategy is not possible; indeed, the concept of a truthful strategy is not a meaningful one for uninformed students,[14] and therefore strategy-proofness is not a feasible property. We therefore must use Bayesian equilibrium to analyze the game, which, as we shall soon show, can be far more complicated for the uninformed students.

Deferred acceptance

While there are many possible mechanism choices, for the remainder of this paper we focus in particular on the student-proposing deferred acceptance (DA) mechanism. There are

[12] That is, for any two states ω and ω' where $\mathcal{I}_j(\omega) = \mathcal{I}_j(\omega')$, it must be that $\sigma_j(\omega) = \sigma_j(\omega')$. In particular, informed students can condition their reports on the state while uninformed student can not.

[13] Formally, we say ψ is *strategy-proof* if, for any j, there do not exist preferences P_j, P_j', P_{-j} such that $\psi_j(P_j', P_{-j})P_j\psi_j(P_j, P_{-j})$.

[14] More precisely, the truthful strategy $\sigma(\omega) = P_j(\omega)$ is not measurable with respect to the information of uninformed students.

several reasons for this. First, DA has a rich history in the literature, going back to the foundational work of Gale and Shapley (1962), and in their seminal paper, Abdulkadiroğlu and Sönmez (2003) propose DA as a promising solution for school choice markets. Second, in private values settings, DA satisfies many desirable properties; namely, it is (i) strategy-proof (ii) stable and (iii) Pareto dominates any other stable mechanism. Third, and most importantly, because of its appealing theoretical properties, DA is one of the most widely used mechanisms in practice, including in such large school districts as New York City, Boston, and New Orleans, and many others in the United States and around the world (see, for example, Abdulkadiroğlu et al. (2005a) and Abdulkadiroğlu et al. (2005b)). As most of the prior work and positive results for DA uses the standard "private values" model, it becomes particularly important to understand what happens to the appealing features of the DA mechanism when these assumptions are relaxed.

Below, we give a brief definition of the standard deferred acceptance algorithm as it applies to standard school choice problems (e.g., Abdulkadiroğlu and Sönmez, 2003). In the next section, we will present an example to highlight the major issues that arise in our model of asymmetrically informed students. The inputs to the algorithm consist of an ordinal preference relation P_j for each student j and a priority relation $\succ_s$ for each school s.

Step 1 Each student j applies to her most preferred school according to P_j. Each school s considers the set of students who have applied to it, and tentatively admits the q_s-highest priority students according to $\succ_s$ (if there are less than q_s applicants, it admits all students). All students not tentatively admitted to a school are rejected.

Step k**,**$k > 1$ Any student who was rejected in the previous step applies to her most preferred school that has not yet rejected her. Each school considers the pool of students tentatively held from the previous step plus any new applicants, and tentatively admits the q_s- highest priority students according to $\succ_s$. All students not tentatively admitted to a school are rejected.

The algorithm ends after the earliest step in which no students are rejected.

2.3 Full matchings and ex-post stability

Suppose the state is ω. In the classical matching literature, given a matching μ, a student and a school (j, s) are called a (classical) **blocking pair** if (i) $u_j^\omega(s) > u_j^\omega(\mu_j)$ and (ii) either $|\mu_s| < q_s$ or $j \succ_s j'$ for some $j' \in \mu_s$. A matching μ is then called **stable** if there are no blocking pairs.[15] This captures the notion that after the match is assigned, no two agents

[15] An additional component of stability is that every student prefers her assigned school to being unmatched, and no school wants to unilaterally drop one of its assigned students. We assume that students

can jointly deviate by dropping (one of) their current partners and matching themselves. In the school choice literature, stability is sometimes called *fairness* or *no justified envy,* and is given a normative interpretation in that, while a school district *can* enforce any assignment it wants, it *desires* to implement a fair outcome where no student will envy the assignment of a student over which she has higher priority at that school. Since our results can also be extended to two-sided matching models, and the terminology is more familiar, we stick with the word "stability".

The classical definition should still apply for the informed students. But the uninformed students do not know the state, so part (i) of the definition of a blocking pair needs to be modified to an expected utility. One might think this should be an unconditional expectation as the uninformed students know nothing about the state. However, blocking is a process that happens after the matching has been implemented and so uninformed students do actually have some information, namely, they know what school they were assigned to. Part (ii) also needs modification for the uninformed students, since having higher priority than some other student at a school is no longer a simple yes or no, but rather yes in some states and no in others. Both of these features can be folded into the conditional expected value for an uninformed student. In particular, the relevant expected utility for a student who is assigned to school s' and is considering blocking with school s should condition on states where the uninformed student is matched to the school s' and they have higher priority than some other student at school s (or s is not at full capacity).[16]

To capture this formally, we first introduce the concept of a full matching, which is a mapping from states to observed matchings. This is an important object in our setting, because strategies in the preference revelation game induce state-dependent (full) matchings. We then introduce a notion of ex-post stability, which is defined on full matchings, which extends the classical notion of stability to our environment.

Definition 1. A **full matching** is a function $\boldsymbol{\mu} : \Omega \to \mathcal{M}$ that assigns a matching to each state $\omega \in \Omega$.

We will use bold face type, $\boldsymbol{\mu}$, to denote full matchings, and write $\boldsymbol{\mu}^\omega$ for the matching in state ω, and $\boldsymbol{\mu}_j^\omega$ for the assignment of student j in matching $\boldsymbol{\mu}^\omega$.

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find all schools acceptable, and schools find all students acceptable, so this part of stability is not an issue. We assume this only for ease of exposition; none of our results are driven by this assumption.

[16]We take the traditional "game theory" approach here, meaning that the students play an equilibrium and so know the strategies and therefore final matchings that will occur in each state. Hence, the uninformed students "know" exactly in which states they will match to school s and exactly in which states they will have higher priority than some student at s'. Obviously, this is not exactly true in the real world, but is the best way to formalize the ideas and, as shall be seen, we think highlights the issues that information asymmetry presents.

There are two pieces of information that a student can use to update her information and form a potential blocking pair: first, the original school to which she is assigned, and second, whether the school she wishes to form a blocking pair with will accept her.[17] Given a full matching $\boldsymbol{\mu}$, define the following two sets (to avoid notational clutter, we suppress the dependence of these sets on $\boldsymbol{\mu}$):

$$\mathcal{A}_j(s) = \{\omega \in \Omega : \boldsymbol{\mu}_j^\omega = s\}$$
$$\mathcal{B}_j(s) = \{\omega \in \Omega : |\boldsymbol{\mu}_s^\omega| < q_s \text{ or } j \succ_s j' \text{ for some } j' \in \boldsymbol{\mu}_s^\omega\}$$

The set $\mathcal{A}_j(s)$ is the set of states in which student j is assigned to s, while the set $\mathcal{B}_j(s)$ is the set of states in which j has high enough priority to block with school s, either because she has higher priority over one of the current students of s, or s is not filled to capacity. Let $\mathcal{C}_j(s', s) = \mathcal{A}_j(s') \cap \mathcal{B}_j(s)$. In words, $\mathcal{C}_j(s', s)$ is the set of states in which student j is assigned to s', but could block with school s.

Definition 2. Given a full matching $\boldsymbol{\mu}$, student-school pair (j, s) are an **ex-post blocking pair** if there exists a state $\tilde{\omega}$ such that $\mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s) \neq \emptyset$ and

$$E[u_j^\omega(s) | \mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s) \cap \mathcal{I}_j(\tilde{\omega})] > E[u_j^\omega(\boldsymbol{\mu}_j^{\tilde{\omega}}) | \mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s) \cap \mathcal{I}_j(\tilde{\omega})].$$

Full matching $\boldsymbol{\mu}$ is **ex-post stable** if there are no ex-post blocking pairs.

To understand this definition, fix a state $\tilde{\omega}$ and notice that the right side of the inequality is student j's expected utility from her current assignment, $\boldsymbol{\mu}_j^{\tilde{\omega}}$, given her information that true state lies in the set $\mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s) \cap \mathcal{I}_j(\tilde{\omega})$. (The expected values are taken over all states $\omega \in \mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, c) \cap \mathcal{I}_j(\tilde{\omega})$; within each expected value, the school remains fixed, while the utility function varies with ω.) The left side of the inequality is j's expected utility from any other school s with which she may potentially want to form a blocking pair. If the latter is greater than the former, then (j, s) is an ex-post blocking pair, and the full matching $\boldsymbol{\mu}$ is not ex-post stable.

Informed students know the state exactly, $\mathcal{I}_j(\tilde{\omega}) = \{\tilde{\omega}\}$ for all $\tilde{\omega}$, and so for them $\mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s) \cap \mathcal{I}_j(\tilde{\omega}) = \tilde{\omega}$. Therefore, the inequality reduces to $u_j^{\tilde{\omega}}(s) > u_j^{\tilde{\omega}}(\boldsymbol{\mu}_j^{\tilde{\omega}})$. This, together with $\mathcal{C}_j(\boldsymbol{\mu}_j(\tilde{\omega}), s) \neq \emptyset$ (which implies that j has high enough priority to form a blocking pair with s), shows that the definition reduces to that of a classical blocking pair for informed

[17] It is possible to define ex-post stability under other informational assumptions, e.g., by allowing students to observe the entire final matching $\boldsymbol{\mu}^\omega$ (rather than just their own assignment). However, besides arguably being less realistic, under such a definition, almost nothing will be ex-post stable (see Chakraborty et al., 2010).

students. For uninformed students, $\mathcal{I}_j(\tilde{\omega}) = \Omega$ (they initially have no information), and so for them $\mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s) \cap \mathcal{I}_j(\tilde{\omega}) = \mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s)$. The information $\mathcal{C}_j(\boldsymbol{\mu}_j^{\tilde{\omega}}, s)$ is student j's updated information given both the observation of her current assignment *and* the knowledge of the set of states in which she has high enough priority at school s to form a blocking pair.

The following example is instructive for understanding the concepts of full matchings and ex-post stability.

Example 1. There are 3 students, $I = \{j_1\}$ and $U = \{j_2, j_3\}$, and 3 schools $S = \{A, B, C\}$ each with capacity one. The state space Ω is the set of all permutations of S (and therefore has size $|\Omega| = M! = 6$), and the probability distribution Pr over Ω is uniform. Each state $\omega \in \Omega$ is identified with an ordinal preference relation over the schools, and, given the state, each student has the same ordinal preferences. For shorthand, we will write $\omega = ABC$ to refer to the state where A is the most preferred school, B is the second most preferred school, and C is the least preferred school for all students and likewise for the other 5 states. For concreteness, we define utility as $u_j^\omega(s) = M - rank_\omega(s)$, where $rank_\omega(s)$ is the ranking of school s in permutation ω (although any utility function that preserves the common ordinal preferences would give the same results). We refer to this model as the *common ordinal preference model.*

The priority structure is as follows:

$\succ_A$	$\succ_B$	$\succ_C$
j_1	j_1	j_2
j_2	j_3	j_3
j_3	j_2	j_1

In this example, a full matching $\boldsymbol{\mu}$ is a function that gives a matching $\boldsymbol{\mu}^\omega$ for each of the 6 possible states $\omega \in \Omega$, and so, in this example, there are 36 possible full matchings (6 possible matchings per state $\times$ 6 states). The following table presents one possible (arbitrary) full matching $\boldsymbol{\mu}$:

	$\boldsymbol{\mu}_{j_1}^\omega$	$\boldsymbol{\mu}_{j_2}^\omega$	$\boldsymbol{\mu}_{j_3}^\omega$
$\omega = ABC$	A	C	B
$\omega = ACB$	A	C	B
$\omega = BAC$	B	C	A
$\omega = BCA$	B	C	A
$\omega = CAB$	A	C	B
$\omega = CBA$	B	C	A

We claim that this full matching is ex-post stable. To see why, start with the informed student j_1. As $\mathcal{I}_{j_1}(\omega) = \{\omega\}$, j_1 simply conditions potential blocks on the state to determine if they deliver a higher expected utility. In states ABC, ACB, BAC, and BCA, j_1 gets the best school, and so does not want to block. In state CAB and CBA, j_1 gets the second-best school, but has lower priority at the best school C than j_2, who is matched to it. So the stability constraints hold for j_1.

The stability constraints for the uninformed students are where we diverge from the standard model of stability. Consider j_2 first. It may be tempting to see that j_2 gets C in every state, so $\mathcal{A}_{j_2}(C) = \Omega$, and therefore conclude that j_2 learns nothing and so the stability constraints are satisfied. But one must be careful to also condition on whether potential blocks are possible or not. In particular, suppose j_2 is considering blocking at A. Then, $\mathcal{B}_{j_2}(A) = \{BAC, BCA, CBA\}$ as these are the three states where j_3 is matched to A so j_2 would have higher priority than the student matched to A. Combining these, we have $\mathcal{C}_{j_2}(C, A) = \mathcal{B}_{j_2}(A) = \{BAC, BCA, CBA\}$. Hence we compare $E[u^\omega_{j_2}(A)|\mathcal{C}_{j_2}(C, A)] = 1/3$ to $E[u^\omega_{j_2}(C)|\mathcal{C}_{j_2}(C, A)] = 1$ and see that j_2 would prefer to stay at C. Alternatively, j_2 does not have high enough priority to ever get school B, and so $\mathcal{B}_{j_2}(B) = \emptyset$ and $\mathcal{C}_{j_2}(C, B) = \emptyset$. Furthermore, as j_2 is never matched to A or B, we have that $\mathcal{A}_{j_2}(A) = \mathcal{A}_{j_2}(B) = \emptyset$ and so $\mathcal{C}_{j_2}(A, B) = \mathcal{C}_{j_2}(A, C) = \mathcal{C}_{j_2}(B, A) = \mathcal{C}_{j_2}(B, C) = \emptyset$ as well. We stress that it is true that the ex-post stability constraints are satisfied for j_2, but we must be careful as perhaps j_2, even though they learn nothing from observing their match, may find that they would only be accepted to a school when it is very good and therefore wish to block with that school.

Moving to j_3, consider first when j_3 is matched to B. Here, $\mathcal{A}_{j_3}(B) = \{ABC, ACB, CAB\}$. On the other hand, j_3 does not have high enough priority to block at A, as j_3 has the lowest priority there, or to block with C, as j_2 is matched to C in all 6 states. Hence $\mathcal{B}_{j_3}(A) = \mathcal{B}_{j_3}(C) = \emptyset$ and so $\mathcal{C}_{j_3}(B, A) = \mathcal{C}_{j_3}(B, C) = \emptyset$ as well. Similarly, when j_3 is matched to A, we have $\mathcal{A}_{j_3}(A) = \{BAC, BCA, CBA\}$ but $\mathcal{B}_{j_3}(B) = \mathcal{B}_{j_3}(C) = \emptyset$ and so $\mathcal{C}_{j_3}(A, B) = \mathcal{C}_{j_3}(A, C) = \emptyset$ too. Thus, the full matching $\boldsymbol{\mu}$ is ex-post stable.

It should be clear now that given some mechanism ψ, any strategy σ induces a full matching $\boldsymbol{\mu}(\sigma)$ defined by $\boldsymbol{\mu}^\omega(\sigma) = \psi(\sigma(\omega))$. This leads naturally to the next definition.

Definition 3. Mechanism ψ is **ex-post stable** if there exists an equilibrium σ such that the induced full matching $\boldsymbol{\mu}(\sigma)$ is ex-post stable.

3 The Curse of Acceptance

As stated above, in the standard model where each student is informed about their own preferences, DA is strategy-proof and stable.[18] In this section, we highlight some of the issues these properties present in our model with uninformed students, and how they can be attributed to a so-called curse of acceptance. We begin with stability. Example 2 shows that when there is even one student that is not perfectly informed about her preferences, DA is no longer (ex-post) stable.

Example 2. There are 4 students, $I = \{j_1, j_2, j_3\}$ and $U = \{j_4\}$, and 4 schools, $S = \{A, B, C, D\}$, each with capacity one. We continue to work in the common ordinal preference model of Example 1.[19] There are four schools now, so we write, for example, $\omega = ABCD$ to refer to the state where A is the most preferred school, B is the second most preferred school, etc., for all students, and define utility as $u_j^\omega(s) = 4 - rank_\omega(s)$.[20] The priority structure is as follows:

$\succ_A$	$\succ_B$	$\succ_C$	$\succ_D$
j_1	j_1	j_3	j_2
j_2	j_3	j_1	j_1
j_3	j_2	j_4	j_4
j_4	j_4	j_2	j_3

After receiving their signals, each student must submit a preference list, and the final assignment will be determined via deferred acceptance. First consider the informed students. For these students, playing the truthful strategy $\sigma_j(\omega) = \omega$ is weakly dominant. This means, for example, in state $\omega = DABC$, students j_1, j_2, and j_3 will all report $\sigma_j(\omega) = DABC$.

It has been shown (McVitie and Wilson, 1971; Dubins and Freedman, 1981) that an equivalent way to run DA is that at each step, to arbitrarily choose one unmatched student and have them apply to their most-preferred school where they have not yet been rejected. The order in which students are chosen to apply does not matter; in particular, we can run DA with only students j_1, j_2, and j_3 first and then, when they are all tentatively matched, have j_4 apply to her most-preferred school in her report and continue until DA ends.

Now, note that before j_4 makes her first application in this method of DA, j_1, j_2, and j_3 are matched to the best three schools according to the state. Student j_4 does not have

[18] While not Pareto efficient, another desirable property is that it Pareto dominates any other stable match.

[19] As with many similar impossibility results, this market can easily be embedded in other markets of arbitrary size and preference structure.

[20] Again, any utility values that preserve the common ordinal preference will give the same result.

very good priority at any school, and so will usually get the worst school regardless of the preference they have submitted. In particular, she only has a chance to match to a better (than the worst) school if, before she enters the market, either (i) j_2 is matched to C or (ii) j_3 is matched to D. Calculating DA on $\{j_1, j_2, j_3\}$ shows that this only happens in 2 of the 24 states: in state $\omega' = ABCD$, j_2 is matched to C and in state $\omega'' = BADC$, j_3 is matched to D. The final matching in these 2 states will depend on what preference relation j_4 reports. If j_4 reports a preference relation P_{j_4} with $CP_{j_4}D$ then j_4 will match to C in *both* ω' and ω'', whereas if j_4 reports a preference relation P'_{j_4} with $DP'_{j_4}C$ then j_4 will match to D in both ω' and ω''.[21] The key point to notice is that no matter what she submits, j_4 will receive the same assignment in ω' and ω''. In one of these states, this assignment will be the third-best school, and in the other, it will be the worst. Combining this with the previous discussion, we see that j_4 will get the worst school in 23 states and the third-best school in 1 state, regardless of what she reports.

We claim that for either report (P_{j_4} or P'_{j_4}), the resulting full matching will not be ex-post stable. Suppose j_4 reports $CP_{j_4}D$ and is matched to C. Now, she is matched to C in 7 states: the 6 states where C is worst and $\omega' = ABCD$. But j_3 is only matched to D in $\omega'' = BADC$, so if she proposes a block with D it will only be accepted in this state. Her utility from D in state ω'' is 1, while her utility from C is 0, and so she wishes to propose the block.[22] That is, the match is not ex-post stable. If j_4 were instead to report P'_{j_4} with $DP'_{j_4}C$, a symmetric argument shows that she will want to propose blocking with C when she matches to D.

In summary, we have shown the following.

Proposition 1. *The deferred acceptance mechanism is not ex-post stable.*[23]

In fact, there are even larger problems with deferred acceptance other than the fact that it is not ex-post stable. In the last example, the uninformed student gets the worst possible school in 23 of the 24 states. Now, part of reason for this is that she has quite low priority at

[21] The locations of A and B in $j_4's$ report are irrelevant as she will just be immediately rejected from either if she applies.

[22] In the notation of the previous section, $\mathcal{A}_{j_4}(C) = \{ABCD, ABDC, ADBC, BADC, BDAC, DABC, DBAC\}$ and $\mathcal{B}_{j_4}(D) = \{BADC\}$ and so $\mathcal{C}_{j_4}(C, D) = \{BADC\}$. Thus, $1 = E[u^{\omega}_{j_4}(D)|\mathcal{C}_{j_4}(\boldsymbol{\mu}^{\tilde{\omega}}_{j_4}, D) \cap \mathcal{I}_{j_4}(\tilde{\omega})] > E[u^{\omega}_{j_4}(\boldsymbol{\mu}^{\tilde{\omega}}_{j_4})|\mathcal{C}_{j_4}(\boldsymbol{\mu}^{\tilde{\omega}}_{j_4}, D) \cap \mathcal{I}_{j_4}(\tilde{\omega})] = 0$, where $\boldsymbol{\mu}$ is the full matching induced by DA. Complete calculations for this example are provided in the appendix.

[23] While DA is not ex-post stable, it is also possible to ask whether any ex-post stable mechanism exists. In the appendix, we show that there is in fact a unique ex-post stable mechanism we call the *state-learning mechanism.* Intuitively, this mechanism works by learning the state from the reports of the informed students, and using this information to assign the uninformed students. This mechanism not only ignores the stated preferences of some students, it also requires the mechanism designer to *know* which students are informed and which are uninformed. For these reasons, we do not believe it is a relevant mechanism in practice.

each school. But, as we will show more carefully in the next section, even when her priority is higher, she will always do worse than average.

These two features, no ex-post stability and generally poor outcomes for the uninformed students, can be attributed to a so-called *curse of acceptance*: uninformed students do not know which schools are the good schools, but upon seeing their assignment, they update their beliefs about the quality of the school they were assigned downwards. The reason is that they know that the more informed students know which schools are the good schools and will end up taking them for themselves. This leaves the less informed students with the low-quality schools, which not only makes them worse off from a welfare perspective, but also leads to ex-post instability.

This still is not the end of the story for how poorly we think DA performs in this environment, as we have yet to consider incentive issues. In the standard model, one of the most desirable features of DA is it's strategy-proofness. However, as argued previously, strategy-proofness is no longer a feasible property in our model, and it turns out that without it, determining the equilibrium can be quite complicated. In the previous example, this was not so bad, as in fact every strategy for j_4 yielded the same expected utility and therefore every strategy was an equilibrium. This is just an artifact of the example, however, and usually determining an optimal strategy will be far more complicated and depend very much on the details of the full priority structure. This can make playing the mechanism very difficult for some parents, which is unappealing to many school districts (see Pathak and Sönmez, 2008).

The next example clearly illustrates this point. It actually considers two different problems, which barely differ in their primitives but have vastly different equilibria. Furthermore, the example shows that a student's own priority is not sufficient for determining their equilibrium strategy (e.g., reporting a preference list that orders schools by one's priorities at them is generally *not* an equilibrium strategy for the uninformed students).

Example 3. The students, schools, and states are the same as in Example 2. We consider two more examples with slightly different priority structures. The first structure is as follows:

$\succ_A$	$\succ_B$	$\succ_C$	$\succ_D$
j_3	j_2	j_2	j_2
j_4	j_1	j_1	j_1
j_2	j_4	j_3	j_3
j_1	j_3	j_4	j_4

Following the same argument as in the previous example, after running DA on the three informed students, j_4 can only get A when it is not worst if j_2 or j_1 is matched to A and can only get B when it is not worst if j_3 is matched to B. There are no states where j_2 or

j_1 are matched to A. However, j_3 is matched to B in both $CDBA$ and $DCBA$. So if j_4 reports a preference P_{j_4} with $AP_{j_4}B$, then j_4 will get matched to the worst school in every state, but if j_4 reports a preference P_{j_4} with $BP_{j_4}A$, then j_4 will get matched to B, the third best school in states $CDBA$ and $DCBA$. That is, the equilibrium strategies for j_4 are any reports where $BP_{j_4}A$ (the rankings of C and D are irrelevant).

But now let's slightly alter the priority structure by switching the priorities of j_1 and j_3 at school D (note in particular that the priorities of student j_4 have not changed):

$\succ_A$	$\succ_B$	$\succ_C$	$\succ_D$
j_3	j_2	j_2	j_2
j_4	j_1	j_1	j_3
j_2	j_4	j_3	j_1
j_1	j_3	j_4	j_4

With this priority structure, j_1 is matched to A in states $CDAB$ and $BDAC$ and j_3 is matched to B in $DCBA$. So if j_4 reports a preference P_{j_4} with $AP_{j_4}B$, then j_4 will get matched to A, the third best school, in states $CDAB$ and $BDAC$. If j_4 reports a preference P_{j_4} with $BP_{j_4}A$, then j_4 will get matched to B, the third best school, in state $DCBA$. Therefore, the equilibrium strategies for j_4 are any reports where $AP_{j_4}B$. So, just switching the priorities of two students at a school (who both have higher priority than j_4, and to which j_4 is never matched unless it is the worst school) completely reverses the equilibrium strategies. In one case, j_4 should favor the school at which they have second-highest priority, but in the other case, they should favor the school at which they have third-highest priority. There is no way for j_4 to determine their equilibrium strategy without considering the full priority structure of all students, and we think this challenge is rather unappealing.

In summary, DA fails to be strategy-proof or ex-post stable. Furthermore, the uninformed students end up with poor outcomes. One way to proceed would be to look at different mechanisms, although we do not think this would be a fruitful approach, since the general logic of the curse of acceptance will hold independently of the mechanism used. Thus, the next section re-frames the problem from one of mechanism design to one of *priority* design, and argues that school districts can alleviate all of these issues by choosing an appropriate priority structure.

4 Secure Schools

The importance of the problems identified in the preceding section clearly depended on the underlying priority structure. In school choice settings, school districts generally have some control over the priority structure, and may be able to design priorities to achieve particular objectives. For example, Dur et al. (2017) and Pathak and Shi (2017) explore consequences of different design decisions relating to the design of walk zones in Boston.[24] These models still worked in the standard framework of perfectly informed students. In this section, we show that when this assumption is relaxed, the design of priorities can have important consequences for incentives, ex-post stability, and welfare of the uninformed students. In particular, we show that designing priorities such that all students have a "secure school" makes DA ex-post stable, the equilibrium strategies simple, and improves the welfare of the uninformed students by protecting them from the curse of acceptance.

We say that school s is a **secure school** for student j if $|\{j' \in J : j' \succsim_s j\}| \leq q_s$. In words, a secure school is one with enough seats for j and every student who has higher priority than j. The rest of this section shows how secure schools allow uninformed students to guard themselves against the curse of acceptance. They allow for a natural and intuitive equilibrium that is ex-post stable and guarantees the uninformed students get an average utility.

Consider again Example 2. Note that in this example, student j_4 does not have a secure school. What happens if we modify the priority structure to give her a secure school? The priority structure below does so, where we have raised j_4's priority at school D.

$\succ_A$	$\succ_B$	$\succ_C$	$\succ_D$
j_1	j_1	j_3	j_4
j_2	j_3	j_1	j_2
j_3	j_2	j_4	j_1
j_4	j_4	j_2	j_3

Let us reconsider the DA mechanism under these new priorities. Just as before, it remains a dominant strategy for the informed students to submit their true preferences. However, it turns out for the uninformed student j_4, her optimal strategy is now to rank D, her secure school, first. Formally, this will follow from Theorem 1 below which applies to more general markets of arbitrary size, but the intuition is easy to see in the small example: by ranking D first, j_4 receives D in every state. If she instead ranked C above D,[25] she will not get C

[24]One of the most important priority design issues to many school districts is how to design priorities to ensure certain diversity goals at schools, objectives that we do not consider here. Starting with the seminal paper of Abdulkadiroğlu and Sönmez (2003), there is a large literature that follows the standard approach of taking a priority structure that includes diversity constraints as given and looking for an optimal mechanism. Papers in this vein include Hafalir et al. (2013), Ehlers et al. (2014), Echenique and Yenmez (2015), and Fragiadakis and Troyan (2017). However, there is less work on how to design the priorities to achieve desired diversity goals, which we think is an interesting open question for future work.

[25]Note that j_4 still has the lowest priority at A and B, and so once again only the relative rankings of C and D are relevant.

when it is "good", but will continue to get it when it is "bad". Essentially, taking her secure school protects her from the acceptance curse and being taken advantage of by the informed students.

Theorem 1 formalizes this intuition to more general markets of arbitrary size. For the formal results in this section, we continue to work in the common ordinal preferences model introduced earlier, where $|\Omega| = M!$ and each state corresponds to an ordinal ranking of schools that is common to all students in that state. This can be thought of as an approximation to highly correlated ordinal preferences, which is a common feature in real-world school choice settings. Many papers have used this assumption when generalizing classical matching models to include more realistic assumptions on preferences and information (e.g., Abdulkadiroğlu et al. (2011) and Troyan (2012)). This assumption is necessary to get a clear result in the discrete model, as the proofs rely on combinatoric arguments that quickly become intractable when this assumption is relaxed. However, the intuition applies much more generally, and in the next section, we will use a continuum model to analyze the more general case of heterogeneous (but correlated) preferences.

For the priority structure, we assume that each uninformed student has at least one secure school, but otherwise, the priority structure is arbitrary. Define a strategy profile $\sigma_j^*(\omega)$ as follows: for all informed $j \in I$, $\sigma_j^*(\omega) = P_j(\omega)$, and for all uninformed $j \in U$, $\sigma_j^*(\omega) = \tilde{P}_j$ for all ω, where $\tilde{P}_j$ is any preference ranking that lists one of j's secure schools first. As discussed above, following σ_j^* is weakly dominant for the informed students; i.e., after observing their signal, they just report their true preferences. For the uninformed students, we have the following result.

Theorem 1. *Assume that every uninformed student has a secure school. Then, σ^* is an equilibrium strategy profile.*

The proof of this theorem in the appendix actually shows something slightly stronger, which is that for each j, j's outcome under profile $(\sigma_j^*, \sigma_{-j}^*)$ first-order stochastically dominates her outcome under $(\sigma_j', \sigma_{-j}^*)$ for any other strategy σ_j' that j could choose.[26] This implies that the strategy profile σ^* is an equilibrium of the deferred acceptance mechanism (and is also the reason why the cardinal utility values we assigned in the common ordinal preference model could be changed to any other values that preserve the order).

While Theorem 1 simply constructs one equilibrium, it is a natural equilibrium. When uninformed students do not have a secure school, determining equilibrium strategies for the

[26] A strategy induces a distribution over the number of states where j gets the best school, the second-best school, the third-best school etc. according to the common ordinal preference. This is the distribution we are referring to in regards to comments on first-order stochastic dominance.

uninformed students can be quite complicated, as seen in Example 3. Designing priorities such that every student has a secure school results in equilibrium strategies that are focal and very simple to compute: informed students follow the familiar truth-telling strategy, while uninformed students simply take their secure school. Doing so is optimal because it protects them from the curse of acceptance identified previously.

Furthermore, we think that secure schools may be helpful in this environment, because the full matching induced by the equilibrium strategies σ^* described above is ex-post stable.

Theorem 2. *Suppose every uninformed student has a secure school. Then, the full matching induced by equilibrium σ^* in the deferred acceptance mechanism is ex-post stable.*

What is it about secure schools that make the full matching ex-post stable? The reason is that the uninformed students do not learn anything from observing their own match, because they are always matched to their secure school. However, it's actually somewhat subtle to see why this implies ex-post stability, because they can still condition on their proposed block being implementable (recall the set $\mathcal{B}_j(s)$ above). Perhaps a student considers a rematching proposal to some school s that she knows will be accepted in some state where s is very good. Then, because the informed students propose to the schools in the order of the common values, when s is worse, the students who match to s have weakly lower priority than when s is better. That is, if s will block with j when it is a good school, it will continue to block with j whenever it is worse. So, the best that j can hope for is to get s when it's best, second-best, etc. down to the worst, which gives her the same expected utility as her secure school. In fact, unless she gets s when it is the best, she strictly prefers matching to her secure school.[27]

5 Heterogeneous Preferences

While preferences over schools are likely correlated in the real-world, they are almost certainly not perfectly correlated, which raises the question of how our results extend to this setting. The proof strategies used for the discrete economies in the previous section rely on combinatoric arguments that explode and quickly become intractable when preferences are correlated but heterogeneous. Still, the intuition for the curse of acceptance still holds when preferences are partially correlated. We can obtain formal results in this setting by moving to a continuum economy.

[27]She can only get s when it is best if s is also a secure school or at least one other uninformed student has two secure schools, one of which is s, and chooses one other than s. So, in most cases, j strictly will prefer to stay at her secure school.

Continuum economies have recently been receiving significant interest in the matching literature because they simplify analysis considerably while still providing useful insights. For example, Miralles (2008) uses a continuum model to compare the Boston mechanism and DA from an ex-ante welfare perspective. Abdulkadiroğlu et al. (2015) use a continuum model to understand properties of their choice-augmented DA algorithm that are too complicated to analyze in the standard discrete model. Azevedo and Leshno (2016) provide a general framework for analyzing stable matchings in economies with a finite number of agents on one side and a continuum on the other. Besides being much more tractable and providing useful insights, continuum economies are particularly well-suited as an approximation in school choice, where market sizes are often very large. For example, the Boston public school district serves over 60,000 students, while in New York City, this number is over one million students.[28]

We now introduce the formal description of the continuum model we will consider. The set of schools is finite, still denoted $S = \{s_1, \ldots, s_M\}$, but each school now has a unit mass of seats to fill. On the student side, there is a total mass N of students. The underlying state space Ω is again the set of all permutations of S, and each state ω is still identified with an ordinal ranking of the schools. However, we no longer assume that students have common preferences that are completely determined by the state. Rather, preferences are determined as follows. First, nature draws a state $\omega = (s^{(1)}, \ldots, s^{(M)})$ uniformly from Ω, and there is a probability distribution $\tilde{\Pr}$ such that $\tilde{\Pr}(s^{(1)}) > \tilde{\Pr}(s^{(2)}) > \cdots > \tilde{\Pr}(s^{(M)})$. Student j's ordinal preferences P_j are then determined via repeated draws from this distribution, removing repetitions, until all M schools have been chosen.[29] Intuitively, the state ω determines which schools are "most likely" to be popular, and then preferences for each student are drawn from this distribution. This procedure for drawing preferences is very similar to that used in the large-market analyses of DA in Immorlica and Mahdian (2005), Kojima and Pathak (2009) and Kojima et al. (2013). A student's cardinal preference for school s, given that she has ordinal preferences P_j, is $u_j(s, P_j) = v(rank(s, P_j))$, where $rank(s, P_j)$ is the rank of school s according to preferences P_j and v is any decreasing function.[30] As before, students are either informed or uninformed. Informed students receive a perfectly informative signal about their preferences, while uninformed students receive no signal. A mass $\nu(I)$ of students are informed and $\nu(U)$ are uninformed, where $\nu(I) + \nu(U) = N$. Whether a student is informed

[28]Chade et al. (2014), Bodoh-Creed and Hickman (2016), and Che and Koh (2016) use similar continuum models to study decentralized college admissions problems.

[29]More formally, each state ω induces a measure τ_ω on $\mathcal{P}$, the space of ordinal preferences, where $\tau_\omega(P)$ is the measure of students with ordinal preferences P.

[30]Higher ranks correspond to more preferred schools, i.e., the most preferred school according to P_j has $rank(s, P_j) = 1$.

or uninformed is independent of her preferences.

The last piece of the model to discuss is priorities. In reality, schools do not usually have strict priority relations, but instead divide students into a few large priority classes, with many ties that are broken using a random lottery. Most models of school choice in continuum economies go one step further and assume away priorities completely, for tractability (e.g., Miralles, 2008 and Abdulkadiroğlu et al., 2015). However, this will not work for our purposes, since we want to understand the role of secure schools. The most parsimonious way to capture the relevant issues is to assume that each school s has two priority levels, which we will refer to as "high priority" and "low priority". Each school has a mass 1 of students with high priority, and each student has high priority at exactly one school. To break ties, each student randomly draws a unique lottery number $\ell_j(s)$ for each school s.[31] If school s is student j's high priority school, then $\ell_j(s)$ is drawn uniformly from $[0, 1]$; for all other schools, $\ell_j(s)$ is drawn uniformly from $[1, 2]$. In other words, this procedure ensures that all high priority students at school s have better lottery numbers than low priority students at school s (where we use the convention that a lower lottery number is better, in the sense that schools admit students with lower lottery numbers first).

To summarize, a student j can be thought of as a tuple (ι_j, P_j, ℓ_j) where $\iota_j \in \{I, U\}$ is her information type, P_j is her preference type, and $\ell_j = (\ell_j(s_1), \ldots, \ell_j(s_M))$ is her vector of lottery numbers. Informed students know P_j, while uninformed students do not. Whether students observe their precise lottery numbers before they submit their preferences is irrelevant, and all of the results below will hold in both cases, so long as they do know their high priority school.

As before, each student will submit an ordinal preference relation, which will then be turned into a matching using the DA algorithm. Intuitively, the deferred acceptance mechanism works in the same way as in the discrete case: students apply starting at the top of their (submitted) ordinal preference list, and if the total mass of students who apply to a school is greater than the capacity of the school, the school admits the mass of students equal to its capacity that have the highest priority. Rejected students apply to their next most preferred school, etc. However, following these rejection chains, which may only converge in the limit, is difficult and cumbersome. A more useful way to understand deferred acceptance in continuum models is given by Abdulkadiroğlu et al. (2015) and Azevedo and Leshno (2016), who show that the output of DA can be characterized by a vector of *cutoffs* $(\bar{\ell}_1, \ldots, \bar{\ell}_M)$. Each student j is then assigned to her most preferred school for which her priority score is lower than the school's cutoff.

[31] In particular, the mechanism we consider will be DA with multiple tie-breaking (DA-MTB), where each student receives a different lottery number at each school, drawn independently across schools and students.

Intuitively, an individual student has no ability to affect the cutoffs, and so effectively act as "price-takers". This simplifies the strategic analysis, since, to analyze the equilibrium outcomes of DA in our model, we only need to understand the structure of these cutoffs. We begin with the following key proposition. Let σ^* be a strategy profile

Proposition 2. *Fix a state $\omega = (s^{(1)}, \ldots, s^{(M)})$, and suppose all informed students report their true preferences and uninformed students report their secure school first in their preference list. Then, $\bar{\ell}_{s^{(1)}} \leq \bar{\ell}_{s^{(2)}} \leq \cdots \leq \bar{\ell}_{s^{(M)}}$.*

This proposition gives a characterization of the equilibrium cutoffs under the proposed strategies. Intuitively, the cutoffs are smaller (and so the school is harder to get) for the schools that are more likely to be popular. Indeed, if any of these inequalities were reversed, then there is some school that is both more popular in aggregate and is easier to get into. The proof in the appendix reaches a contradiction by showing that this would result in some school being over capacity.

We can now use Proposition 2 to find an equilibrium of the preference submission game. For the informed students, truthful reporting continues to be a weakly dominant strategy. What is left to show is that for an uninformed student, listing her secure school first gives a higher expected utility than any other strategy. The proof of the following theorem is technical, but of all the many cases, the one that matters is intuitive. For every state ω where j's secure school is less popular than some other school s', there is a corresponding state ω' in which the relative popularities of her secure school and s' are swapped. If j submits her secure school at the top of her list, she will get it in both states. If she instead puts s' above her secure school, then it is possible for her lottery number to only be small enough to get it when s' is less popular, i.e., in state ω', where by Proposition 2, the cutoff to get into s' is larger than in state ω, thereby giving her the less popular school in both states.

Theorem 3. *The strategy profile in which all informed students is an equilibrium of the deferred acceptance mechanism.*

The remaining question that needs to be answered is ex-post stability. As in the discrete case, there is no learning by uninformed students, but we still need to address the issue of conditioning on a potential block being accepted. This turns out to be much simpler in this setting than in the discrete case, because by Proposition 2 we can immediately see that if a school will accept a student in some state ω, then the cutoffs are larger in every state where the school is less popular and so will accept the student in all those states as well.

Theorem 4. *The strategy σ^* induces an ex-post stable matching and therefore the deferred acceptance mechanism is ex-post stable.*

In summary, we have shown that the prior results continue to hold with correlated, heterogeneous preferences. In general, uninformed students may be exploited by informed students, falling prey to the curse acceptance and ending up at worse-performing schools on average. Further, DA will no longer be (ex-post) strategy proof. However, we have also provided school districts worried about these issues with a potential design response to mitigate their effects. Giving each student a secure school protects uninformed students from the curse of acceptance while still allowing students to make other choices if so desired. By doing so, equilibrium strategies are simpler, uninformed students are made better off, and (ex-post) stability of DA is recovered.

6 Conclusion

This paper has built on the canonical (private values) school choice model by allowing student preferences to be correlated across schools and assuming some students are more informed about their preferences than others. This will be the case, for example, if each school has an intrinsic quality that all parents care about, but some parents have more time and resources to research and learn about the qualities of various schools. We show that many of the standard results in the literature fail to hold: the popular deferred acceptance mechanism is no longer strategy-proof or stable, and uninformed students are made worse off. These results are not specific to deferred acceptance, and can be attributed to a general *curse of acceptance*: under any mechanism, informed students are more likely to list the good schools highly precisely because they know which ones they are, and so uninformed students expect that any school that accepts them will be of low quality. However, we also show that these effects can be mitigated by casting the problem as one of priority design. By designing priorities such that all students are given a secure school, positive results are recovered: DA is (ex-post) stable, the equilibrium strategies are simple, and the uninformed students are protected from the worst consequences of the acceptance curse.

Introducing more realistic assumptions about preferences motivates the need to think about the school choice problem from the priority design perspective, rather than just the mechanism selection perspective. While our model provides broad guidance, it remains highly stylized. The goal of our paper is to point out issues regarding preferences that have gone understudied in the school choice literature and highlight how they underscore the importance of priority design. The specifics of precise implementation in particular settings (e.g., how to distribute secure schools) is a crucial open question for future work.

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A An ex-post stable mechanism for Example 2

Example 2 shows that the DA mechanism is not ex-post stable. This leaves open the question of whether any other ex-post stable matchings and/or mechanisms exist. In this appendix, we show that such a full matching may exist, but any mechanism that could be used to implement it would be unreasonable from a practical perspective.

Consider again the example, and recall that the ex-post stability constraints for the informed students uniquely determined the final matching in all but 2 of the 24 possible. Further, in these two remaining states, there were two possible matchings that could be implemented, states $ABCD$ and $BADC$. Consider state $ABCD$ first, and note that ex-post stability requires that j_1 be assigned A and j_3 be assigned B. Then, suppose in this state that j_2 is assigned to C and j_4 to D. While j_4 matches to D in a number of states (namely, whenever D is worst and possibly in state $BADC$), there is just one state where j_4 has priority over any other student at the school to which they are matched, state $ABCD$, where j_4 has higher priority than j_2 at C, where j_2 is matched. Since C is better than D in state $ABCD$, j_4 will block with C when matched to D, and so any ex-post stable mechanism must assign j_2 to D and j_4 to C in state $ABCD$. A similar argument shows that an ex-post stable mechanism must match j_3 to C and j_4 to D in state $BADC$. That is, there is only one ex-post stable full matching. Denote this matching $\tilde{\boldsymbol{\mu}}$.

Since there is a unique full matching $\tilde{\boldsymbol{\mu}}$, any mechanism must select matching $\tilde{\boldsymbol{\mu}}^\omega$ in each state ω. We call this mechanism the *state-learning mechanism*. While this mechanism is ex-post stable, it is unappealing for two reasons: first, it is paternalistic, in the sense that it ignores the actual submitted preferences of student j_4 and sometimes gives her a school she claims she does not like at the expense of other students who claim to like it more. Second, in order to accomplish this, the mechanism designer must *know* who is informed and uninformed, and the exact nature of the correlation across student preferences, which is information a school district is unlikely to have.

B Proofs

B.1 Proof of Theorem 1

Recall that a state $\omega \in \Omega$ can be identified with an ordinal ranking of the schools. We will write a generic state as

$$\omega = (s^{(1)}, s^{(2)}, \ldots, s^{(M)}),$$

where $s^{(k)}$ denotes the k^{th} best school in state ω. Let $\boldsymbol{\mu}$ denote the full matching induce by the deferred acceptance mechanism.

Lemma 1. *Suppose all students $j' \neq j$ follow the strategy $\sigma^*_{j'}(\cdot)$, and let i choose any arbitrary strategy. Consider a state $\omega = (s^{(1)}, \ldots, s^{(k)}, \ldots, s^{(\ell)}, \ldots, s^{(M)})$ such that under this strategy profile, we have $\boldsymbol{\mu}^{\omega}_j = s^{(k)}$. Then, in any other state $\omega' = (s^{(1)}, \ldots, s^{(k)}, s^{(\ell)}, \ldots)$, we have $\boldsymbol{\mu}^{\omega'}_j \notin \{s^{(1)}, \ldots, s^{(k-1)}, s^{(\ell)}\}$.*

Proof. Given the proposed strategies, all of the uninformed students other than j apply to a secure school in the first round and are admitted. Thus, it is without loss of generality to consider the submarket that removes these students and their seats. This means that we can consider a market with only one uninformed student, and the rest of the students are informed. Consider step k of the DA mechanism, and note that at step k of the DA mechanism, all of the informed students who were not assigned in earlier steps apply to school $s^{(k)}$. Label this set of students I_k. Note that no informed student ever applies to $s^{(k)}$ at any step after step k. To see why, note that if this were true for some informed student i, then at the beginning of step k, i must have been tentatively held by a school $s^{(k')}$ for some $k' < k$. This means that i must have been rejected from $s^{(k')}$ at some round $\bar{k} \geq k > k'$. Consider the first time this happens in the algorithm, i.e., $\bar{k}$ is the earliest step at which an informed student is rejected from a school $s^{k'}$ such that $k' < \bar{k}$. Note that the only way this can happen is if student j applies to $s^{(k')}$ at step $\bar{k}$. But this then implies that j will never be rejected from $s^{(k')}$, which contradicts that j ends up at $s^{(k)}$.[32]

Now, consider state ω'. Note that steps 1 to $k-1$ of DA are equivalent as to the previous case, because the first $k-1$ schools are the same for the informed students, and we are fixing the strategy of the (unique) uninformed student. This means that at the beginning of step k, the same set of students are unmatched under either ω' or ω. In state ω', the informed students apply to school $s^{(k)}$ at a step $k' > k$, and so the set students who apply to $s^{(k)}$, I'_k, will only be a subset of those who applied in state ω, i.e., $I'_k \subseteq I_k$. Since j was matched to

[32] To see why, note that j can only be rejected from $s^{(k')}$ by an informed student i'' who is matched to a school than $s^{(k'')}$ for some $k'' < k'$. However, all of the unmatched informed students have already been rejected from all more preferred schools, and so i'' cannot be rejected from $s^{(k'')}$.

$s^{(k)}$ in state ω, she had one of the $q_{s^{(k)}}$ highest priorities at $s^{(k)}$ among the students in I_k, and so if she applies to $s^{(k)}$ in state ω', she will also be matched to $s^{(k)}$.

What we have shown is that if j applies to a school in state ω', then she also applies to that school in state ω. Since the first $k-1$ steps of DA are the same in both states, and j was rejected from schools $s^{(1)}, \ldots, s^{(k-1)}$ in state ω, she will also be rejected from them in state ω'. Last, consider school $s^{(\ell)}$. Note that in state ω', school $s^{(\ell)}$ has improved in the rankings of all of the informed students. Thus, if j had applied to $s^{(\ell)}$ in state ω (and was therefore rejected), she will continue to be rejected in state ω'. If j did not apply to $s^{(\ell)}$ in state ω, then it must be that her submitted preferences ranked $s^{(k)}$ over $s^{(\ell)}$. But, as we just argued, if j applies to $s^{(k)}$ in state ω', she will be matched there at the final assignment. In either case, $\boldsymbol{\mu}_j^{\omega'} \neq s^{(\ell)}$, which completes the proof. □

Fixing the strategies of the other agents at $\sigma^*_{j'}$, for any strategy σ_j for the uninformed student j, let

$$F(k|\sigma_j) = \Pr(j \text{ receives a school ranked } k^{th} \text{ or better}|\sigma_j)$$

denote the rank distribution of j's assignment when she uses strategy σ_j. We now prove Theorem 1, which is restated below using this notation. Note that $F(k|\sigma^*_j)$ first-order stochastically dominating $F(k|\sigma_j)$ immediately implies that σ^*_j is optimal for student j.

Theorem. *For any strategy σ_j, $F(k|\sigma^*_j)$ first-order stochastically dominates $F(k|\sigma_j)$.*

Proof. Suppose not, i.e., there exists some σ_j and some k such that $F(k|\sigma_j) > F(k|\sigma^*_j)$. Note that if j follows strategy σ^*_j, she gets the same school in every state, call it $\bar{s}$. There are $(M-1)!$ states where $\bar{s}$ is the best school, $(M-1)!$ states where it is the second-best school, etc., and so it is easy to see that $F(k|\sigma^*_j) = \frac{(M-1)!}{M!}k$. Consider the minimum k such that $F(k|\sigma_j) > F(k|\sigma^*_j)$. For this k, it must be that j gets the k^{th} or better ranked school in strictly more than $k(n-1)!$ states, and, because k is the smallest index for which this is true, j must get exactly the k^{th} ranked school in strictly more than $(n-1)!$ states.

Partition the state space into groups such that each group contains all of the states for which the best k schools are the same; in other words, two states $\omega = (s^{(1)}, \ldots, s^{(M)})$ and $\tilde{\omega} = (\tilde{s}^{(1)}, \ldots, \tilde{s}^{(M)})$ belong to the same group G if and only if $s^{(\ell)} = \tilde{s}^{(\ell)}$ for all $\ell = 1, \ldots, k$. There are $n!/(n-k)!$ groups, and each group contains $(n-k)!$ states. Note that each group G can be uniquely identified by listing its top k schools in order, $(s^{(1)}, \ldots, s^{(k)})$.

Suppose that in state $\omega = (s^{(1)}, \ldots, s^{(k)}, \ldots, s^{(M)})$, student j receives school $s^{(k)}$. Let $G_{\tilde{s}}$ be the group such that $\tilde{s}^{(1)} = s^{(1)}, \ldots, \tilde{s}^{(k-1)} = s^{(k-1)}$, but $\tilde{s}^{(k)} = \tilde{s} \neq s^{(k)}$; in other words, $G_{\tilde{s}}$ is the group that has the same first $(k-1)$ best schools as G, but replaces $s^{(k)}$ with $\tilde{s}$. There are $n-k$ possible choices for $\tilde{s}$, and hence, $n-k$ such groups $G_{\tilde{s}}$. Let $\tilde{S} = S \setminus \{s^{(1)}, \ldots, s^{(k)}\}$, and define $\tilde{G} = \cup_{\tilde{s} \in \tilde{S}} G_{\tilde{s}}$. Lemma 1 implies that for all $\omega' \in \tilde{G}$, student j ends up with worse than the k^{th} ranked school. Note that $|\tilde{G}| = (n-k) \times (n-k)!$.

By our hypothesis, j gets the k^{th} best school in strictly more than $(n-1)!$ states. Every group G contains $(n-k)!$ different states, which implies that i must get the k^{th} best school in at least $(n-1)!/(n-k)!$ different groups. But, by the previous paragraph, for each of these groups G, there is an associated $\tilde{G}$ such that j gets strictly worse than the k^{th} best school for all $\omega' \in \tilde{G}$. Since $|\tilde{G}| = (n-k) \times (n-k)!$ and there must be at least $(n-1)!/(n-k)!$ such $\tilde{G}$'s, that means that there are at least

$$(n-k) \times (n-k)! \times \frac{(n-1)!}{(n-k)!} = (n-k) \times (n-1)!$$

states where j gets worse than the k^{th} ranked school. Since there are $n!$ total states, this leaves at most

$$n! - (n-k) \times (n-1)! = k \times (n-1)!$$

states where j can get the k^{th} ranked or better school. However, this contradicts that j gets the k^{th} ranked or better school in strictly more than $k \times (n-1)!$ states. □

B.2 Proof of Theorem 2

Choose some uninformed student j. As with the proof of Theorem 1, it is without loss of generality to consider a submarket that has removed all of the other uninformed students $j' \in U \setminus \{j\}$ together with the seats they take at their secure schools in equilibrium. In the submarket we consider, we (with slight abuse of notation) let q_s be the remaining capacity at school s once these seats are removed. The set of students is $I \cup \{j\}$.

We start with the following lemma.

Lemma 2. *Assume that $j \succ_{s'} j'$ for some j' assigned to school s' in state $\omega = (s^{(1)}, \ldots, s^{(k-1)}, s', s^{(k+1)}, \ldots, s^{(M)})$. Then, for any state $\tilde{\omega}$ that permutes the ranking of schools k through n (i.e., in any $\tilde{\omega} = (\tilde{s}^{(1)}, \ldots, \tilde{s}^{(M)})$ such that $\tilde{s}^{(k')} = s^{(k')}$ for all $k' \leq k-1$), we have $j \succ_{s'} j''$ for some j'' assigned to school s' in state $\tilde{\omega}$.*

In other words, this lemma says that if student j's proposed rematching is accepted by school s' in some state ω, it will *also* be accepted in any state $\tilde{\omega}$ where s' is ranked lower; i.e., if s' accepts j when it is "good", it will also accept j when it is "bad".

Proof of lemma. First consider the DA algorithm in state ω. Since all uninformed students take their secure school, we only need to consider the informed students. Thus, at each step k' of the algorithm, there is a set of unmatched students $I_{k'}$, and all of these students apply to school $s^{(k')}$. The $q_{s^{(k')}}$-highest priority students are thus admitted, and, since no new students apply to $s^{(k')}$ at any later step of the algorithm, these students are the ones who will ultimately be assigned to school $s^{(k')}$. In state ω, $s^{(k)} = s'$, and so the $q_{s'}$-highest priority students from I_k are admitted to s'. By assumption, in this set there is some j' such that $j \succ_{s'} j'$.

Now, consider any $\tilde{\omega}$ that permutes the schools that were ranked k through M in state ω. By construction, in state $\tilde{\omega}$, school s' is ranked weakly worse than k^{th}: $s' = \tilde{s}^{(k')}$ for some $k' \geq k$. Since the ranking of the first $k-1$ schools are the same as in state ω, the set of students who apply to s' in state $\tilde{\omega}$ is a subset of those who applied in state ω. This means that the lowest-ranked student admitted to s' in state $\tilde{\omega}$, j'', is ranked (weakly) worse than j' according to $\succ_{s'}$, and so $j \succ_{s'} j' \succsim_{s'} j''$. □

Continuing with the main proof, let $\bar{s}$ be the secure school that student j lists first in equilibrium. Clearly, j she receives $\bar{s}$ in every state ω. After the assignment is implemented, consider j proposing to form a blocking pair with some other school s'. Suppose there is a state ω where j has higher priority than some j' that is assigned to s'. In particular, let the ranking of s' in state ω be k, and the ranking of $\bar{s}$ be ℓ:

$$\omega = (s^{(1)}, \ldots, s^{(k-1)}, s', s^{(k+1)}, \ldots, s^{(\ell-1)}, \bar{s}, s^{(\ell+1)}, \ldots, s^{(M)}).$$

By the lemma, j also has priority over some j'' assigned to s' in the state

$$\tilde{\omega} = (s^{(1)}, \ldots, s^{(k-1)}, \bar{s}, s^{(k+1)}, \ldots, s^{(\ell-1)}, s', s^{(\ell+1)}, \ldots, s^{(M)}).$$

So, for every state where s' accepts j as the k^{th} ranked school and $\bar{s}$ is the ℓ^{th} ranked school for $\ell > k$, there is a symmetric state where s' accepts j as the ℓ^{th} ranked school, while had j stuck with $\bar{s}$, she would have received the k^{th} ranked school. Since each of these states are equally likely, student j is not better off conditional on being admitted to s'. □

(Side note: It is tempting to look at this proof and conclude that every school gives the same expected payoff (conditional on a block being accepted), which is obviously not true. The reason is that the above logic does not apply in reverse: that is, if we start with the fact that j's block with s' is accepted in state $\tilde{\omega}$, we *cannot* conclude that j's block will be accepted in state ω. But j is worse off when making the switch in state $\tilde{\omega}$, and if she is not accepted in state ω to offset this "loss", she will be worse off overall.)

B.3 Proof of Proposition 2

Fix a state ω. All of the uninformed students receive their secure school in every state, which leaves an equal capacity of $\frac{\mu(I)}{M}$ at each school for the informed students. Note that each school will be entirely filled to capacity with informed students. For any ordinal preference ranking $P \in \mathcal{P}$, write $P(r)$ for the r^{th} ranked school according to P. Consider two ordinal preference rankings, P and P', such that $P(r) = P'(t) = s_A$ and $P(t) = P'(r) = s_B$ for some $r < t$, and $P(k) = P'(k)$ for all $k \neq r, t$. In words, P and P' are exactly the same, except that the ranking of schools s_A and s_B are swapped. Let $\lambda_\omega(P)$ and $\lambda_\omega(P')$ denote the measure of informed students who have preferences P and P', respectively. Without loss of generality, suppose that in state ω, we have $rank_\omega(A) < rank_\omega(B)$, which implies that $\lambda_\omega(P) > \lambda_\omega(P')$.

We want to show that $\bar{\ell}_{s_A} \leq \bar{\ell}_{s_B}$. Towards a contradiction, suppose that $\bar{\ell}_{s_A} > \bar{\ell}_{s_B}$. First, note that for any school s, the mass of students with preferences P and high priority at any s is $\frac{1}{N}\lambda_\omega(P)$. It will be helpful to divide these students into three distinct classes: (1) students who have high priority at school they strictly prefer to s_A; (2) students who have high priority at s_A; and (3) students who have high priority at a school ranked worse than s_A (where the rankings are according to P). The measure of students in class (1) is $\frac{r-1}{M}\lambda_\omega(P)$, the measure of students in class (2) is $\frac{1}{M}\lambda_\omega(P)$, and the measure of students in class (3) is $\frac{M-r}{M}\lambda_\omega(P)$.

Now, no student in class (1) is matched to s_A, since DA will never give them worse than their high priority school. Students in class (2) match to s_A if and only if their lottery numbers at the $r-1$ schools they prefer to s_A are higher than the cutoffs at (all of) these schools. Thus, the mass of students matched to s_A is $\prod_{x=1}^{r-1}(1-\bar{\ell}_{P(x)})$. Students in class (3) are similar, except that they also must have a high enough lottery number at s_A, and so the total mass of students in class (3) matched to s_A is $\prod_{x=1}^{r-1}(1-\bar{\ell}_{P(x)}) \times \bar{\ell}_{s_A}$. Combining all of this, the total measure of students with preference P matched to school s_A in state ω is

$$\frac{\lambda_\omega(P)}{M} \times \left[\prod_{x=1}^{r-1}(1-\bar{\ell}_{P(x)}) \times (1+(M-r)\bar{\ell}_{s_A})\right] \tag{1}$$

We can do an equivalent analysis for the number of students with preference P' who get matched to s_A (recalling that under P', school s_A is ranked t^{th}):

$$\frac{\lambda_\omega(P')}{M} \times \left[\prod_{x=1}^{t-1}(1-\bar{\ell}_{P'(x)}) \times (1+(M-t)\bar{\ell}_{s_A})\right] \tag{2}$$

Now, recall that $P(k) = P'(k)$ for all $k < t$, with the exception of $k = r$. In particular, we can re-write equation 2 as

$$\frac{\lambda_\omega(P')}{M} \times \left[\prod_{x=1}^{r-1}(1-\bar{\ell}_{P'(x)}) \times \prod_{x=r+1}^{t-1}(1-\bar{\ell}_{P'(x)}) \times (1-\bar{\ell}_{s_B}) \times (1+(M-t)\bar{\ell}_{s_A})\right]$$

Now, total mass of students assigned to s_A that are of preference type either P or P'

$$\delta(P, s_A)\frac{\lambda_\omega(P)}{M} + \delta(P', s_A)\frac{\lambda_\omega(P')}{M},$$

where, to simplify the notation, we define $\delta(P, s_A)$ and $\delta(P', s_A)$ as

$$\begin{aligned}
\delta(P, s_A) &= \prod_{x=1}^{r-1}(1-\bar{\ell}_{P(x)}) \times (1+(M-r)\bar{\ell}_{s_A}) \\
\delta(P', s_A) &= \prod_{x=1}^{r-1}(1-\bar{\ell}_{P'(x)}) \times \prod_{x=r+1}^{t-1}(1-\bar{\ell}_{P'(x)}) \times (1-\bar{\ell}_{s_B}) \times (1+(M-t)\bar{\ell}_{s_A})
\end{aligned}$$

We can do the same analysis for school s_B. By symmetry, the expressions are the same as above, except P is swapped with P' and s_A is swapped with s_B. to find the total measure of students matched to s_B is

$$\delta(P, s_B)\frac{\lambda_\omega(P)}{M} + \delta(P', s_B)\frac{\lambda_\omega(P')}{M},$$

where the δ's in this case are defined as

$$\begin{aligned}
\delta(P, s_B) &= \prod_{x=1}^{r-1}(1-\bar{\ell}_{P(x)}) \times \prod_{x=r+1}^{t-1}(1-\bar{\ell}_{P(x)}) \times (1-\bar{\ell}_{s_A}) \times (1+(M-t)\bar{\ell}_{s_B}) \\
\delta(P', s_B) &= \prod_{x=1}^{r-1}(1-\bar{\ell}_{P'(x)}) \times (1+(M-r)\bar{\ell}_{s_B})
\end{aligned}$$

We are now interested in comparing the δ's. In particular, recall our contradiction hypothesis that $\bar{\ell}_{s_A} > \bar{\ell}_{s_B}$. This immediately implies that $\delta(P, s_A) > \delta(P', s_B)$ and $\delta(P', s_A) > \delta(P, s_B)$. Adding these two equations and re-arranging gives

$$\delta(P, s_A) - \delta(P, s_B) > \delta(P', s_B) - \delta(P', s_A) \tag{3}$$

Note also that $\delta(P, s_A) - \delta(P, s_B) \geq 0$ (since $r < t$). Further, $\lambda_\omega(P) > \lambda_\omega(P')$, so we can multiply equation 3 and maintain the inequality to get

$$[\delta(P, s_A) - \delta(P, s_B)]\, \lambda_\omega(P) > [\delta(P', s_B) - \delta(P', s_A)]\, \lambda_\omega(P') \tag{4}$$

This re-arranges to

$$\delta(P, s_A)\lambda_\omega(P) + \delta(P', s_A)\lambda_\omega(P') > \delta(P', s_B)\lambda_\omega(P') + \delta(P, s_B)\lambda_\omega(P) \tag{5}$$

Note what this says: among those students whose ordinal preference types $P_j \in \{P, P'\}$, a greater measure are matched to s_A than to s_B. But now, we can partition the entire ordinal preference space $\mathcal{P} = \mathcal{P}_A \cup \mathcal{P}_B$, where $P \in \mathcal{P}_A$ if and only if $s_A P s_B$ (and $P \in \mathcal{P}_B$ if and only if $s_B P s_A$). Then, for every $P \in \mathcal{P}_A$, we can find a corresponding $P' \in \mathcal{P}_B$ that is the same as P, except s_A and s_B are swapped. We can then write an analogue of equation 5 for each pair (P, P'). Summing over all of these equations for every possible pair, we conclude that in state ω, the total mass of students matched to s_A is strictly greater than the total mass of students matched to s_B. However, this contradicts that the schools have the same measure of informed students in all states.

B.4 Proof of Theorem 3

Consider an uninformed student j. Recall that the cardinal utility for a school is some function $v(x)$ of its ordinal rank x, where v is strictly decreasing (so that $x = 1$ is the best possible rank). Thus, the expected utility for school s in state ω is

$$u_j^\omega(s) = \sum_{P \in \mathcal{P}} \Pr(j \text{ draws ordinal preferences } P|\omega) v(rank_P(s))$$

For a state $\omega = (s^{(1)}, \ldots, s^{(M)})$, it is clear that $u_j^\omega(s^{(1)}) > u_j^\omega(s^{(2)}) > \cdots > u_j^\omega(s^{(M)})$, and further, by symmetry, for any other state $\tilde{\omega} = (\tilde{s}^{(1)}, \ldots, \tilde{s}^{(M)})$, we have $u_j^{\tilde{\omega}}(\tilde{s}^{(m)}) = u_j^\omega(s^{(m)}) := \bar{v}_m$ for all $m = 1, \ldots, M$. In other words, there are M numbers $\bar{v}_1 > \cdots > \bar{v}_M$ such that, conditional on any state $\omega = (s^{(1)}, \ldots, s^{(M)})$, j's expected utility for school $s^{(1)}$ is $\bar{v}_1$, for $s^{(2)}$ is $\bar{v}_2$, etc., i.e., her expected utility for a school s depends only on the state-level ranking of school s (note that this may be different from j's own ordinal ranking of s, which he only learns ex-post).

Assume that all other players are playing their equilibrium strategy. Since j's strategy must be measurable with respect to her information, we can identify each of her possible strategies σ_j with an ordinal preference relation P_j, and her strategy space is the space of all ordinal preferences $\mathcal{P}$. Let $EU_j(P_j)$ be j's expected utility when she reports P_j (and everyone else plays their equilibrium strategy). More formally, using the above definitions,

$$EU_j(P_j) = \sum_{\omega \in \Omega} \sum_{s \in S} \Pr(\omega) \times \Pr(j \text{ receives } s | \sigma^*_{-j}, \omega, P_j) \times \bar{v}_{rank_\omega(s)}.$$

As in the previous proof, for any ordinal preference relation P_j, let (with slight abuse of notation) $P_j(r)$ be a function that returns the school that is ranked r^{th} according to P_j; where necessary, we also use the notation $P_j^{-1}(s)$ to denote the ranking of school s under preferences P_j. The proof relies on the following lemma.[33]

Lemma 3. *Let s_A and s_B be two schools such that $\ell_j(s_A) > \ell_j(s_B)$, and consider a preference report P_j such that $P_j(r) = s_A$ and $P_j(r+1) = s_B$. Let P'_j be the alternative report such that $P'_j(r) = s_B$, $P'_j(r+1) = s_A$, and, for all other $t \neq r, r+1$, $P'_j(t) = P'_j(t)$. Then, $EU_j(P_j) < EU_j(P'_j)$.*

Given this lemma, any arbitrary strategy P_j for student j that does not rank her secure school, $\bar{s}$, first, i.e., $P_j : s_1, s_2, \ldots, s_{r-2}, s_{r-1}, \bar{s}, s_{r+1}, \ldots$. By the lemma, $EU_j(P_j) \leq EU_j(P'_j)$, where $P'_j : s_1, s_2, \ldots, s_{r-2}, \bar{s}, s_{r-1}, s_{r+1}, \ldots$. Applying the lemma again, $EU_j(P'_j) \leq EU_j(P''_j)$, where $P''_j : s_1, s_2, \ldots, \bar{s}, s_{r-2}, s_{r-1}, s_{r+1}, \ldots$. Continuing in this manner, we eventually find a strategy $P^*_j : \bar{s}, s_1, s_2 \ldots$ such that $EU_j(P^*_j) \geq EU_j(P_j)$. All strategies that rank $\bar{s}$ first give the same expected utility, and so any such strategy is optimal for player j.

We now prove the lemma.

Proof of Lemma 3.

Proposition 2 shows that for any state $\omega = (s^{(1)}, \ldots, s^{(M)})$, the lottery cutoffs can be written $\bar{\ell}_{s^{(1)}} \leq \cdots \leq \bar{\ell}_{s^{(M)}}$. In addition, note that by symmetry, the cutoffs are independent of the state; that is, given any two states $\omega = (s^{(1)}, \ldots, s^{(M)})$ and $\tilde{\omega} = (\tilde{s}^{(1)}, \ldots, \tilde{s}^{(M)})$ and corresponding vectors of cutoffs, we have $\bar{\ell}_{s^{(m)}} = \bar{\ell}_{\tilde{s}^{(m)}}$ for all m. In other words, we can just write $\bar{\ell}_1 \leq \cdots \leq \bar{\ell}_M$ for the schools ranked first to last in any state.

Start by partitioning the state space into $\Omega = \Omega_A \cup \Omega_B$, where $\omega \in \Omega_A$ if and only if $rank_\omega(s_A) < rank_\omega(s_B)$ (and $\Omega_B = \Omega \setminus \Omega_A$). For each $\omega_A \in \Omega_A$, there is a corresponding ω_B that swaps the positions of s_A and s_B, and leaves all other schools the same. Consider one such pair (ω_A, ω_B). Let $\boldsymbol{\mu}_j^{P_j}(\omega_A)$ be j's match in state ω_A when she reports preferences P_j. Let $k_{s,\omega_A} = rank_{\omega_A}(s)$ be the rank of school s in state ω_A.

First, suppose that $P_j^{-1}(\boldsymbol{\mu}_j^{P_j}(\omega_A)) < r$. In other words, j is matched to a school she reported as preferred to s_A (and note that this implies that her match is also preferred to

[33] While this lemma is written for a fixed lottery draw $\ell_j(s_A)$ and $\ell_j(s_B)$, in the equilibrium we construct, students need not actually know their realized lottery numbers; all they need to know is that their (high) priority at their secure school is good enough that they will be admitted for sure if they list it first.

s_B). This implies that $\ell_j(\boldsymbol{\mu}_j^{P_j}(\omega_A)) < \bar{\ell}_{\boldsymbol{\mu}_j^{P_j}(\omega_A)}$ and $\ell_j(s') > \bar{\ell}_{k_{s'}}$ for all s' ranked strictly higher than $\boldsymbol{\mu}_j^{P_j}(\omega_A)$ in state ω_A. But, note that in moving to state ω_B, the rankings (and hence, cutoffs) of all schools that are P_j-weakly preferred to $\boldsymbol{\mu}_j^{P_j}(\omega_A)$ do not change (nor do they change in either of these states if j were to report P_j'). In summary, we can conclude that j's match is the same in all of these scenarios: $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j}(\omega_B) = \boldsymbol{\mu}_j^{P_j'}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_B) = s$.

Next, suppose that $P_j^{-1}(\boldsymbol{\mu}_j^{P_j}(\omega_A)) \geq r$. For ease of notation, define $rank_{\omega_A}(s_A) = rank_{\omega_B}(s_B) = k$ and $rank_{\omega_A}(s_B) = rank_{\omega_B}(s_A) = k'$, where $k > k'$, and $\bar{\ell}_k$ is the cutoff of the k^{th} ranked school in any state. There are several subcases, depending on the relative magnitudes of $\ell_j(s_A), \ell_j(s_B), \bar{\ell}_k$, and $\bar{\ell}_{k'}$. Recall that $\ell_j(s_B) < \ell_j(s_A)$ (by assumption) and $\bar{\ell}_k \leq \bar{\ell}_{k'}$ (by Proposition 2), which will eliminate many possibilities.

Subcase (i): $\ell_j(s_B) < \ell_j(s_A) \leq \bar{\ell}_k \leq \bar{\ell}_{k'}$. Note that j has a lottery number low enough to be admitted to both s_A and s_B in both states ω_A and ω_B. Thus, she will be admitted to whichever school she ranks higher in her preferences, regardless of the state. That is, $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j}(\omega_B) = s_A$ and $\boldsymbol{\mu}_j^{P_j'}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_B) = s_B$.[34]

Subcase (ii): $\ell_j(s_B) \leq \bar{\ell}_k < \ell_j(s_A) \leq \bar{\ell}_{k'}$. In this case, if j submits P_j', then she will be admitted to s_B in both states. However, if she submits P_j, then she will only be admitted to s_A in state ω_B, since her lottery number is not low enough in state ω_A. Thus, $\boldsymbol{\mu}_j^{P_j}(\omega_B) = s_A$ and $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_B) = s_B$.

Subcase (iii): $\ell_j(s_B) \leq \bar{\ell}_k \leq \bar{\ell}_{k'} < \ell_j(s_A)$: In this case, j's lottery number is not low enough to be admitted to s_A in either state, but is low enough for s_B in both states. That is, $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j}(\omega_B) = \boldsymbol{\mu}_j^{P_j'}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_B) = s_B$.

Subcase (iv): $\bar{\ell}_k < \ell_j(s_B) < \ell_j(s_A) < \bar{\ell}_{k'}$: In this case, j can never be admitted to the "better" school among s_A and s_B. That is, $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_A) = s_B$ and $\boldsymbol{\mu}_j^{P_j}(\omega_B) = \boldsymbol{\mu}_j^{P_j'}(\omega_B) = s_A$.

Subcase (v): $\bar{\ell}_k < \ell_j(s_B) \leq \bar{\ell}_{k'} < \ell_j(s_A)$: Note that here, j's lottery number is not low enough to be admitted to s_A in either state, but is low enough to be admitted to s_B in state ω_A, which happens under both P_j and P_j'. In state ω_B, j does not have a low enough lottery number for s_A or s_B, and so she gets some school s that is ranked (strictly) worse than $(r+1)^{th}$. Recall that $P_j(t) = P_j'(t)$ for all $t > r+1$, and so this will be the same school s under both reports in state ω_B. To summarize, in this case we have $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j'}(\omega_A) = s_B$ and $\boldsymbol{\mu}_j^{P_j}(\omega_B) = \boldsymbol{\mu}_j^{P_j'}(\omega_B) = s$ for some s such that $P_j^{-1}(s) = P_j'^{-1}(s) = t > r+1$.

Subcase (vi): $\bar{\ell}_k \leq \bar{\ell}_{k'} < \ell_j(s_B) < \ell_j(s_A)$: In this case, j does not have a low enough lottery number for either s_A or s_B in either state ω_A or ω_B. By similar reasoning to subcase

[34]Note that she can do no "better" (according to her reported preferences), since we are in the case $P_j^{-1}(\boldsymbol{\mu}_j^{P_j}(\omega_A)) \geq r$.

(v), we have $\boldsymbol{\mu}_j^{P_j}(\omega_A) = \boldsymbol{\mu}_j^{P'_j}(\omega_A) = \boldsymbol{\mu}_j^{P_j}(\omega_B) = \boldsymbol{\mu}_j^{P'_j}(\omega_B) = s$ for some s such that $P_j^{-1}(s) = P_j'^{-1}(s) = t > r + 1$.

Looking back through all of the previous cases, j's assignment is independent of her choice between reporting P_j and P'_j (for a fixed state) in all cases except subcases (i) and (ii). In subcase (i), if she reports P_j, she gets s_A in both states. Since both states are equally likely, her expected utility conditional on the true state being $\omega \in \{\omega_A, \omega_B\}$ is $\frac{1}{2}(\bar{v}_k + \bar{v}'_k)$. If she reports P'_j, she gets s_B in both states, and again her expected utility conditional on the true state being $\omega \in \{\omega_A, \omega_B\}$ is $\frac{1}{2}(\bar{v}_k + \bar{v}_{k'})$. Thus, in this subcase again, j is actually indifferent between P_j and P'_j. Last, consider subcase (ii). In this case, if she reports P_j, she receives the $(k')^{th}$-ranked school (the worse school of s_A and s_B) in both states ω_A and ω_B, for a expected utility conditional on $\omega \in \{\omega_A, \omega_B\}$ of $\bar{v}_{k'}$. If she reports P'_j, she receives s_B in both states, for a conditional expected utility of $\frac{1}{2}(\bar{v}_k + \bar{v}_{k'}) > \bar{v}_{k'}$.

Therefore, we have shown that we can partition the state space into $M!/2$ pairs (ω_A, ω_B) such that, for each pair, $EU_j(P_j|\omega \in \{\omega_A, \omega_B\}) < EU_j(P'_j|\omega \in \{\omega_A, \omega_B\})$. Since every pair of states is equally likely ex-ante, summing over all such pairs gives $EU_j(P_j) < EU_j(P'_j)$. $\square$

B.5 Proof of Theorem 4

It is obvious that informed students have no justified claims at a school they prefer. Thus, consider an uninformed student j. Let her secure school where she has high priority be $\bar{s}$ (and note that she is matched to $\bar{s}$ in the equilibrium). Consider j potentially proposing a block with some other school s'. We use many of the ideas and notation from the proof of Theorem 3. In particular, we again partition the states into two groups: those where $\bar{s}$ is more popular (in aggregate) and those where s' is more popular (in aggregate). Take two states ω and $\hat{\omega}$ that differ only in that the relative positions of $\bar{s}$ and s' are swapped. Let $rank_\omega(s') = rank_{\hat{\omega}}(\bar{s}) = k$ and $rank_\omega(\bar{s}) = rank_{\hat{\omega}}(s') = k'$, where $k' > k$. From the previous proof, recall that $\bar{\ell}_k < \bar{\ell}_{k'}$ and $\bar{v}_k > \bar{v}_{k'}$. There are three possibilities.

Case (i): $\ell_j(s') \leq \bar{\ell}_k$. In this case, i will be rematched with s' in both state ω and $\hat{\omega}$. In this case, i's payoff conditional on the true state being in $\{\omega, \hat{\omega}\}$ is $\frac{1}{2}(\bar{v}_k + \bar{v}_{k'})$, whether she stays with $\bar{s}$ or proposes a block with s'.

Case (ii): $\bar{\ell}_k < \ell_j(s') \leq \bar{\ell}_{k'}$. In this case, then i is rematched with s' in state $\hat{\omega}$, but not in state ω. Thus, conditional on the true state being in $\{\omega, \hat{\omega}\}$, i's payoff from proposing a block with s' is $\bar{v}_{k'}$, while her payoff from staying in at $\bar{s}$ is $\frac{1}{2}(\bar{v}_k + \bar{v}_{k'})$. As $\bar{v}_k > \bar{v}_{k'}$, she is worse off from proposing a block.

Case (iii): $\bar{\ell}_{k'} \leq \ell_j(s')$. In this case, i will not rematch to s' in either state ω or $\hat{\omega}$, and hence she indifferent between proposing a block or not.

Combining these three cases, we see that conditional on the true state lying in $\{\omega, \hat{\omega}\}$, i strictly prefers to stay at school $\bar{s}$ (she is indifferent in cases (i) and (iii), and is strictly better off in case (ii)). Since the choice of s' was arbitrary, as with the previous theorem, we can partition the state space into $M!/2$ pairs $(\omega, \hat{\omega})$ such that, for each pair, i prefers to stay at $\bar{s}$ than to propose a block with any other a s'. Since every pair of states is equally likely ex-ante, summing over all such pairs implies that i prefers staying at $\bar{s}$ over proposing a block with any other s'. $\square$