Why polls can be wrong but still informative*

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Abstract

I introduce a polling stage to Feddersen and Pesendorfer's (1996) two-candidate election model in which some voters are uncertain about the state of the world. While Feddersen and Pesendorfer find that less informed, indifferent voters strictly prefer abstention, which they refer to as the swing voter's curse, I show that there exists an equilibrium in which everyone truthfully reveals his/her preference in the poll and participates in voting. Moreover, I find that even in the truth-telling equilibrium, the candidate who wins the poll may be defeated in the election. However, in a large election polls are still welfare improving.

1 Introduction

> *The most important function of polls, however, is not in telling us who is going to win, but in revealing what is on the voters' minds (Barone 2015).*

Polls are often to blame for wrong predictions. Donald Trump's victory in the 2016 US presidential election is a good example, as nearly all the pollsters predicted that Hillary Clinton would win. A day after Election Day, in *The New York Daily News*, Silverstein (2016) wrote, "The results are in: Predictive polls are a major loser of the 2016 race," and *USA Today*'s Bomey (2016) added, "Pollsters flubbed the 2016 presidential election in seismic fashion." Another example of a recent miss of the polls is the Brexit referendum in June 2016. Leading up to voting day, the vast majority

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of Brexit polls predicted that Remain would win. However, the Leave side won the referendum by 52 percent to 48 percent and the polling industries had to take the blame.

What went so wrong? The media and pollsters largely suggest three possible reasons. First, people may have lied about their true preferences (e.g., Bomey 2016; Whiteley 2016); they might have felt embarrassed to tell the pollsters their true opinion, especially with such controversial issues. Second, the response rate often fell below 10 percent which is too low to be representative of the population because those who are willing to answer polls might be quite different from the silent majority (e.g., Riddell 2016; Silverstein 2016; Whiteley 2016). Third, the weight schemes used to determine likely voters were inaccurate (e.g., Bomey 2016; Riddell 2016; Whiteley 2016). For example, in an interview with *USA Today*, Arie Kapteyn, the director of the University of Southern California's Dornsife Center for Economic and Social Research, suggested that many pollsters may have eliminated too many Trump supporters by assuming that those who did not vote in 2012 would not vote in 2016 either.

In this paper, I present a simple two-candidate election model and show that polls can be wrong (i.e., the candidate who led in the poll can be defeated in the election) even if no one lies (thus, no shy supporters), no one refuses to talk (thus, no sampling error or selection bias), and the probability of each citizen voting is known to everyone (thus, no weight problem). Specifically, I introduce a polling stage to Feddersen and Pesendorfer's (1996) model. Their model assumes that voters are either partisans or independents. The partisan voters prefer a particular candidate regardless of the state of the world. In contrast, the independent voters' preference depends on the state of the world and these voters can learn the state of the world only with some probability. I add a polling state to their model so that voters have an opportunity to answer the poll question about who they support and to learn about the poll results before voting.

I characterize the most informative equilibrium in which every player truthfully answers the poll question. Several important implications flow from the equilibrium. First, in the truth-telling equilibrium, the uninformed independent voters participate in voting. This is an important result because, without a polling stage, the uninformed voters may abstain to allow the informed voters to pick the winner, which Fedderson and Pesendorfer (1996) refer to as the "swing voter's curse." The swing voter's curse indicates that the uninformed voters are sometimes strictly better off abstaining even though voting is not costly. Once a polling stage is introduced, however, the curse may disappear, and the uninformed voters have strong incentives to participate in voting. Second, even if no one lies, the poll outcome can still be different from the election outcome even in large elections. As the uninformed independent voters decide whom to vote for after they learn about the poll result, a big lead in a poll may not be large enough to win the election. Third, although polls may not forecast the election outcome correctly, and even the loser in the polls may win the election, large elections successfully aggregate information in the sense that they can identify the correct candidate who would win if everyone were informed about the state of the world. The uninformed voters almost surely learn about the state of the world and make a correct decision for themselves.

Furthermore, I examine whether an election aggregates information more successfully with informative polls than without them. A naive answer to this question is that it should do so because agents have more information when deciding for whom to vote. I demonstrate this intuition is not always correct: When the size of the electorate is not large enough, polls may not be helpful because the uninformed voters may sway the election depending on few answers from the poll, which might be truthful but insufficient to reveal the state of the world. When the size of the electorate increases, the uninformed voters learn the state more often from the poll. But Fedderson and Pesendorfer (1996) show that a large election can successfully aggregate information without polls as well. Thus, I need to compare the probabilities of an election aggregating information with and without informative polls, which converge to 1 in both cases. By taking the large deviation approach, I show that the former converges more rapidly to 1, which implies that in a large election, informative polls help information aggregation.

Together, the results suggest that pre-election polls can be welfare improving, though they may sometimes seem dead wrong when interpreted naively. Polls may encourage the uninformed independent voters to participate in voting, and help them decide whom to vote for so that the election outcome can fully reflect what voters have in their mind.

In what follows, I first briefly discuss related literature. Then, I lay out the model, discuss the results and conclude. Finally, the appendix contains all the proofs.

2 Related Literature

This paper is related to several strands of literature, the first of which is the literature on elections and information aggregation. When do large elections identify the correct candidate who would win under no uncertainty? The classic Condorcet jury model suggests that they do so almost surely even if each voter is partially informed about the state of the world. Furthermore, Feddersen and Pesendorfer (1996, 1999) find that elections still fully aggregate information even if a substantial number of voters do not acquire any signals about the state of the world and remain uninformed (see also McMurray(2013) for an extension of the model). Experimental studies find support for this result (Battaglini et al. 2008, 2010).On the other hand, Kim and Fey (2007) and Bhattacharya (2013) show that this result may not be the case if voters' preferences are heterogeneous. In a similar vein, Feddersen and Pesendorfer (1997) and Mandler (2012) show that elections fail to aggregate information if voters are uncertain about the distribution from which their preferences or private signals are drawn. The model in my paper matches Feddersen and Pesendorfer (1996)'s except that there is now a pre-election polling stage. And I show that polls can help large elections aggregate information even more successfully.

Second, this paper is related to the broad literature on communication in committees. A large proportion of the literature pertains to the variants of the classic Condorcet jury model with various forms of a pre-voting communication stage. Studies' main focus tends to be on the relative performance of various voting rules in small committees with and without communication (e.g., Austen-Smith and Feddersen 2006; Coughlan 2000; Gerardi and Yariv 2007; Goeree and Yariv 2011; Jackson and Tan 2013, for a review of the literature, see Austen-Smith and Feddersen 2009) whereas my paper considers how a pre-election opinion poll changes voters' incentives to vote and whether such changes are desirable in large elections. Importantly, studies find that for the committee members to fully share private information in the communication stage, either they share identical preferences, or they are uncertain about one another's preferences (Coughlan 2000; Austen-Smith and Feddersen 2006). However, when everyone is uncertain about one another's preferences, it is not always possible to share information in the communication stage or to reach the correct collective decision. Moreover, a voting rule that extracts more information in the communication stage does not necessarily lead to a more efficient voting outcome (Jackson and Tan 2013). Thordal-Le Quement and Yokeeswaran (2015) find that it is more efficient to restrict communication to subgroups that share identical preferences rather than to have public plenary communication. The environment for communication is more hostile in my model than in these jury models because I have partisan voters who always prefer a particular candidate no matter what the state of the world is. These partisan voters may have a strong incentive to misrepresent their preferences if it helps their favorite candidate get elected. I show that even with these conflicting preferences and incentives, the players can manage to transmit information via public polling and to vote more efficiently with the acquired information.

Third, this paper is related to research on public information and costly voting. Taylor and Yildirim (2010) show that when public information about the distribution of voters' preferences is available, the candidate the majority prefer is less likely to win the election than when no such information is available. Similarly, Goeree and Grosser (2007) find that when a public signal on voters' preferences is available, the candidate the minority group prefers has a higher chance of winning. Both studies assume that voting is costly, and therefore the decision to vote depends on how pivotal each voter will be. Thus, in both articles, public information on other voters' preferences mobilizes the minority group to participate and results in a less desirable outcome with higher chances. By contrast, I assume that the information about other voters' preferences is not exogenously given, but rather endogenously generated through pre-election polling, and that voting is costless.

Finally, the paper is related to research on public opinion polls. Many studies mainly examine the strategic behavior of respondents in polls to influence policy makers' decision or the voting behavior of others. For example, Meirowitz (2005) shows that respondents may misrepresent their preferences if candidates take their policy positions after observing polling data. Morgan and Stocken (2008) also find similar results and argue that strategic behavior should be taken into account when interpreting polling statistics. Burke and Taylor (2008) show that in the absence of independent voters, truth-telling is not an equilibrium if the electorate is large or voting costs are not negligible. In these studies, polls may be wrong because respondents lie. By contrast, I analyze an equilibrium in which no one lies, but polls may still be wrong.[1]

3 The Model

In this section, I present a model of elections in which uninformed independent voters may learn about the state of the world by observing an opinion poll result. As I base the model on Feddersen and Pesendorfer (1996), I closely follow their notations whenever possible.

There are two states, $Z = \{0, 1\}$; two candidates $X = \{0, 1\}$; and $N + 1$ agents. The agents consist of partisan voters, independent voters, and non-voters. The partisan voters prefer either candidate 0 or candidate 1 regardless of the state, while the independent voters prefer candidate 0 in state 0 and candidate 1 in state 1. Among the

[1] When there are more than two candidates, this can happen because people may vote strategically. For example, see Andonie and Kuzmics (2012).

independent voters, some learn the state of the world but the others do not. In case one is ignorant of the state, he/she only knows the prior α, that is, the probability of nature choosing state 0. Finally, the non-voters do not vote. Formally, the set of types of agents is given by $T = \{\phi, 0, 1, u, i\}$ where type-ϕ agents are non-voters, type-0 and type-1 agents are partisan voters who prefer 0 and 1 respectively, type-u agents are uninformed independents, and type-i agents are informed independents who know the state. For $(x, z) \in X \times Z$, the utility of an independent voter (i.e., type-u or type-i agent) is

$$U(x, z) = \begin{cases} -1 & \text{if} \quad x \neq z, \\ 0 & \text{if} \quad x = z. \end{cases}$$

The game starts with nature choosing a state, $z \in Z$. State 0 is chosen with probability α and state 1 with probability $1-\alpha$. Nature then takes $N+1$ times of independent random draws to determine the type of each agent. The total number of voters and the number of voters of each type are not known to the players. In each draw, an individual is chosen as a voter with probability $1-p_\phi$. Among the voters, an individual is of type-1 with probability $p_1/(1-p_\phi)$, and type-0 with probability $p_0/(1-p_\phi)$. Finally, a voter is an independent with probability $p_s/(1-p_\phi)$, and each of the independent voters learns the state of the world with probability q. Thus, a voter is an informed independent (i.e., type-i) with probability $qp_s/(1-p_\phi)$ and an uninformed independent (i.e., type-u) with probability $(1-q)p_s/(1-p_\phi)$. The probabilities α, $p = (p_s, p_0, p_1, p_\phi)$, and q are common knowledge, whereas the type of each voter is private information.

Before an election is held, a public opinion poll asks whom the potential voters intend to vote for between candidates 0 and 1. I assume that every agent participates in the poll. Each agent may answer that he/she supports 0 or 1, or has not yet made up his/her mind: $A = \{0, 1, (ND)\}$ (see footnote 2 for an alternative set of responses). The vector of poll responses is denoted $a = (a_1, ..., a_{N+1}) \in A^{N+1}$. The poll result is summarized as $m(a) = (m_0, m_1, m_{\text{ND}}) \in M \equiv \left\{m \in \{0, 1, ..., N+1\}^3 : \sum_r m_r = N+1\right\}$, where m_r denotes the number of respondents whose answer was $r \in A$, and then publicly released. Finally, an election is held. In the election, each voter chooses an action $v \in \{0, 1, \phi\}$, where 0 or 1 denotes his/her vote for candidate 0 or 1 respectively, and ϕ indicates his/her abstention. The candidate who gains more votes wins the election. If there is a tie, each candidate wins with equal probability.

The solution concept is perfect Bayesian equilibrium. I restrict attention to symmetric equilibria in which agents of the same type choose the same voting and poll response strategies. I also assume that no agent plays a weakly dominated strategy in answering the poll question or in voting. Thus, every agent behaves as if he/she is pivotal. A poll response strategy is a function $r : T \times Z \to A$, which assigns an answer to each of the types in each state, where only the informed voters' poll strategy can depend on the state $z \in Z$. Note that only the uninformed independent agents (UIAs) are affected by the poll result when deciding what to do in the voting stage, because type-ϕ voters will not participate in voting and the partisan voters will support their preferred candidate in both states. Therefore, I can simply concentrate on the UIAs' voting strategy given their beliefs about z based on the summary statistic $m(a)$. A type-u's mixed voting strategy is a measurable function $\tau : M \to [0,1]^3$, which maps every poll result into probabilities to choose an action $v \in \{0, 1, \phi\}$, where $\tau_x(m)$, for $x \in X$, denotes the probability of voting for x and $\tau_\phi(m)$, the probability of abstaining.

4 Analysis

As is typical in private information games, this game has multiple equilibria. Among these multiple equilibria, I consider an uninformative and the most informative ones, depending on how much information can be delivered through the opinion poll. Specifically, I consider two poll response strategies which I subsequently show can be supported in equilibrium.

Definition 4.1. (Uninformative poll) Poll response strategy 1:

$$r_1(t|z) = \text{ND for all } t \in T \text{ and all } z \in Z.$$

Definition 4.2. (Informative poll) Poll response strategy 2:

$$r_2(t|z) = \begin{cases} z & \text{if} \quad t = i \\ t & \text{if} \quad t \in \{0,1\} \\ \text{ND} & \text{if} \quad t \in \{\phi, u\} \end{cases} \quad \text{for all } z \in Z.$$

Note that if players use the uninformative poll response strategy, $r_1(\cdot)$, the game is essentially identical to Feddersen and Pesendorfer's (1996). In poll response strategy 1, agents' choices of answers do not depend on their types. Thus, the poll result delivers no information to the UIAs, and as a result, they will behave as in Feddersen and Pesendorfer's model.

By contrast, given the set of responses A, poll response strategy 2 forms the most informative equilibrium.[2] Agents sincerely answer which candidate they support if they surely prefer one candidate to the other. If they do not have such preferences or are not sure about who the right candidate is for them, they indicate they have not decided yet. This is a natural and plausible poll response strategy that one would expect to see in reality.

The UIAs update their beliefs about the state based on the poll result. Specifically, if players use poll response strategy $r_1(\cdot)$, the belief that the state is 0 remains the same as the prior belief, that is, $\mu(z=0|m,r_1)=\alpha$ for all $m=(m_0,m_1,m_{\text{ND}})$. On the other hand, if players use poll response strategy $r_2(\cdot)$, an UIA indexed by j can update his/her belief on $z=0$ from the poll result $m_{-j}=m(a_{-j})=(m_0,m_1,m_{\text{ND}}-1)$, $a_{-j}=(a_1,...,a_{j-1},a_{j+1},...,a_{N+1})$ using Bayes' rule,

$$\begin{aligned}\mu_j(z=0|m_{-j},r_2) &= \frac{\alpha\Pr(m_{-j}|z=0,r_2)}{\alpha\Pr(m_{-j}|z=0,r_2)+(1-\alpha)\Pr(m_{-j}|z=1,r_2)} \\ &= \frac{\alpha(p_0+p_sq)^{m_0}p_1^{m_1}}{\alpha(p_0+p_sq)^{m_0}p_1^{m_1}+(1-\alpha)p_0^{m_0}(p_1+p_sq)^{m_1}}.\end{aligned} \tag{4.1}$$

As respondents answer truthfully, the UIAs can infer whose side the informed independents are on between the two candidates. It is easy to see that as m_0 increases, the UIAs believe that candidate 0 is more likely to match the state. Likewise, as m_1 increases, the UIAs believe that candidate 1 is more likely to be the right one. Since belief μ is the same for any UIA, I omit the index j hereinafter.

To decide UIAs' voting strategy given μ, I need to consider the probabilities of an UIA being pivotal because only in those cases will his/her decision make a difference in his/her expected utility. There are three possible cases: a tie, candidate 0 losing by exactly 1 vote, and candidate 1 losing by exactly 1 vote. In what follows, I specify

[2] A fully revealing equilibrium in which each type can choose a distinctive response to the poll question does not exist even if I enlarge the set of responses A so that $\#A \geq \#T$. This is because the partisan voters have an incentive to choose the same response as the informed independents. Since the UIAs will decide whom to vote for according to the informed independents' responses in the end, the partisan voters will mimic the informed independents to raise their candidate's chance.

As a small variation of poll response strategy 2, one can imagine an alternative poll response strategy with an enlarged A in which the non-voters respond differently from the UIAs or other types of voters. However, this strategy delivers essentially the same information as poll response strategy 2. That is, the additional information from this new poll response strategy does not affect the UIAs' voting behavior. Moreover, empirical studies show that people do not want to reveal their intention to abstain and that many respondents (25-50%) lie about their voting participation when asked (Silver et al., 1986; Harbaugh 1996; Belli et al., 1999; DellaVigna et al., 2016). Thus, I use poll response strategy 2 with set A rather than this alternative situation.

the probabilities of each of these events for an agent, given state z, N other agents, strategy profile τ, poll result m, and poll response strategy r. I denote the probability of a tie by $\pi_e(z, \tau \mid m_{-j}, r)$ and the probability of candidate x receiving one less vote than candidate y by $\pi_x(z, \tau \mid m_{-j}, r)$. In addition, for a given profile τ and poll response strategy r, I define $\sigma_{v,z,a}(\tau|r)$ as the probability that a random selection among whose poll response was $a \in A$, results in a choice of voting behavior $v \in \{0, 1, \phi\}$ in state z. In the following two lemmas, I compute these probabilities under the two poll response strategies.

Lemma 4.1. *(Feddersen and Pesendorfer 1996, p. 412-413) If players use poll response strategy $r_1(\cdot)$, then*

$$\pi_e(z, \tau \mid m_{-j}, r_1) = \sum_{j=0}^{\lfloor N/2 \rfloor} \frac{N!}{j!j!(N-2j)!} \times \sigma_{\phi,z,ND}(\tau|r_1)^{N-2j} \left(\sigma_{0,z,ND}(\tau|r_1)\, \sigma_{1,z,ND}(\tau|r_1)\right)^j$$

and

$$\pi_x(z, \tau \mid m_{-j}, r_1) = \sum_{j=0}^{\lfloor (N-1)/2 \rfloor} \frac{N!}{(j+1)!j!(N-2j-1)!} \times \sigma_{\phi,z,ND}(\tau|r_1)^{N-2j-1} \sigma_{y,z,ND}(\tau|r_1) \left(\sigma_{x,z,ND}(\tau|r_1)\, \sigma_{y,z,ND}(\tau|r_1)\right)^j,$$

where $x, y \in \{0, 1\}$, $x \neq y$, and

$$\sigma_{v,z,ND}(\tau|r_1) = \begin{cases} p_\phi + p_s(1-q)\tau_\phi & \text{if } v = \phi \\ p_v + p_s(1-q)\tau_v + p_s q & \text{if } z = v,\ v \in \{0,1\} \\ p_v + p_s(1-q)\tau_v & \text{if } z \neq v,\ v \in \{0,1\} \end{cases}.$$

Lemma 4.1 shows that the probabilities that an UIA is pivotal vary depending on the state under the uninformative poll response strategy r_1. This is because the probability of a random voter voting for either candidate depends on the state. By contrast, the next lemma shows that this is not the case if players use the informative poll response strategy, $r_{2.}$

Lemma 4.2. *When players use poll response strategy $r_2(\cdot)$, for any m and τ,*

$$\pi_e(0, \tau \mid m_{-j}, r_2) = \pi_e(1, \tau \mid m_{-j}, r_2) \text{ and}$$

$$\pi_x(0, \tau \mid m_{-j}, r_2) = \pi_x(1, \tau \mid m_{-j}, r_2) \text{ for all } x \in \{0, 1\}.$$

Lemma A.1 in the appendix specifies the probabilities $\pi_e\left(z, \tau \mid m_{-j}, r_2\right)$ and $\pi_x\left(z, \tau \mid m_{-j}, r_2\right)$ for $z \in \{0, 1\}$. Lemma 4.2 establishes that the expected number of votes for each candidate does not depend on state z when the poll is informative. Since the partisan and informed independent voters will vote as they did in the poll, the poll result m reveals how many votes each candidate will receive from these types of voters. Thus, the uncertainty remains only in the UIAs' decisions, and the UIAs cannot condition their decision on the state because they do not know in what state they are.

Lemmas 4.1 and 4.2 imply that an opinion poll helps the UIAs learn more about not only for whom they should vote but also how much their votes matter. Lemma 4.1 shows that under r_1, the pivotality of UIAs' vote in state 0 is in general different from that in state 1 because the informed voters vote differently depending on the state. In contrast, Lemma 4.2 shows that their vote matters equally in either state under r_2. The poll result reveals how many decided voters there are for each candidate, and the pivotality of an UIA's vote is also determined accordingly.

Now, I am ready to prove that the *swing voter's curse — that abstention is strictly better when indifferent between the two candidates* — disappears when the poll is informative. Let $Eu\left(v, \tau | m_{-j}, r\right)$ be the expected payoff to an UIA of taking action v when the other UIAs use a strategy profile τ given poll results m and a poll response strategy r. Proposition 4.1 shows that when agents answer the poll question truthfully (i.e., use poll response strategy r_2), announcing the poll result may encourage people to vote.

Proposition 4.1. *Suppose $p_\phi > 0$, $q > 0$, and $N \geq 2$. For any symmetric strategy profile τ in which no agent plays a strictly dominated strategy,*
1) (Feddersen and Pesendorfer 1996, p. 413) if $Eu\left(1, \tau | m_{-j}, r_1\right) = Eu\left(0, \tau | m_{-j}, r_1\right)$, then

$$Eu\left(1, \tau | m_{-j}, r_1\right) < Eu\left(\phi, \tau | m_{-j}, r_1\right).$$

In contrast,
2) $Eu\left(\phi, \tau | m_{-j}, r_2\right) \leq \max\left\{Eu\left(1, \tau | m_{-j}, r_2\right), Eu\left(0, \tau | m_{-j}, r_2\right)\right\}$.

Before discussing the intuition on why informative polls make the swing voter's curse disappear, I turn to the uninformative poll case first. Proposition 4.1 Part 1) is from Feddersen and Pesendorfer (1996) and shows that with uninformative polls (or no polls), the UIAs sometimes have a strong incentive to abstain even though voting is not costly. Why not vote for one of the candidates if they are indifferent between them when doing so is not costly? To see this, consider a case in which $\mu\left(z = 0 | m_{-j}, r_1\right) = \alpha > \frac{1}{2}$.

Since

$$\begin{aligned}
&Eu\left(1,\tau|m_{-j},r_1\right)-Eu\left(0,\tau|m_{-j},r_1\right)\\
=&\left(1-\mu\left(0|m_{-j},r_1\right)\right)\left(\pi_e\left(1,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_1\left(1,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_0\left(1,\tau\ |m_{-j},r_1\right)\right)\\
&-\mu\left(0|m_{-j},r_1\right)\left(\pi_e\left(0,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_1\left(0,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_0\left(0,\tau\ |m_{-j},r_1\right)\right),
\end{aligned}$$

the condition $Eu\left(1,\tau|m_{-j},r_1\right)=Eu\left(0,\tau|m_{-j},r_1\right)$ implies that

$$\begin{aligned}
&\pi_e\left(1,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_1\left(1,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_0\left(1,\tau\ |m_{-j},r_1\right)\\
>&\pi_e\left(0,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_1\left(0,\tau\ |m_{-j},r_1\right)+\frac{1}{2}\pi_0\left(0,\tau\ |m_{-j},r_1\right).
\end{aligned}$$

Roughly speaking, this means that the probability of being pivotal is higher when $z=1$ than when $z=0$. For example, consider $p_0>p_1$ with $p_sq\approx p_0-p_1$. As an UIA believes that candidate 0 is more likely to match the state (i.e., $\mu\left(z=0|m_{-j},r_1\right)>\frac{1}{2}$), it might seem that he/she should vote for 0 and not abstain. However, this is not the case, because an UIA's voting for 0 has a different effect on the election outcome depending on the state. Specifically, when $z=0$, candidate 0 is highly likely to win because all the type-0 and the informed voters vote for 0, and thus an UIA's vote might not affect the election outcome. If $z=1$, however, candidate 1's chance is relatively lower, and an UIA is now more likely to be pivotal because $p_0\approx p_1+p_sq$, which means that the number of the informed voters might not be large enough to fill the gap between the partisan supports. Thus, if an UIA votes for candidate 0, he/she marginally increases the chance of the right candidate winning in state 0 but, at the same time, greatly increases the chance of the wrong person winning in state 1. As he/she loses more than he/she gains on average by voting for 0, he/she would rather abstain.

If agents truthfully answer the poll, however, abstention can never be the single best option. As the poll result clears up the uncertainty about the probabilities of an UIA's vote being pivotal, voting for someone who is more likely to match the state is always (weakly) better than abstaining. For example, compare the expected utilities from voting for 0 and abstaining. Because

$$\begin{aligned}
&Eu\left(0,\tau|m_{-j},r_2\right)-Eu\left(\phi,\tau|m_{-j},r_2\right)\\
=&\frac{1}{2}\left[\begin{array}{c}-\left(1-\mu\left(0|m_{-j},r_2\right)\right)\left[\pi_e\left(1,\tau\ |m_{-j},r_2\right)+\pi_0\left(1,\tau\ |m_{-j},r_2\right)\right]\\+\mu\left(0|m_{-j},r_2\right)\left[\pi_e\left(0,\tau\ |m_{-j},r_2\right)+\pi_0\left(0,\tau\ |m_{-j},r_2\right)\right]\end{array}\right]\\
=&-\frac{1}{2}\left(1-2\mu\left(0|m_{-j},r_2\right)\right)\left[\pi_e\left(1,\tau\ |m_{-j},r_2\right)+\pi_0\left(1,\tau\ |m_{-j},r_2\right)\right]
\end{aligned}$$

by Lemma 4.2, it is easy to see that voting for 0 is strictly better than abstaining if $\mu(0|m_{-j}, r_2) > \frac{1}{2}$. It can be similarly shown that voting for 1 is strictly better than abstaining if $\mu(1|m_{-j}, r_2) > \frac{1}{2}$.

The following proposition shows that there is an equilibrium in which every agent answers the poll truthfully and the UIAs vote according to their beliefs.

Proposition 4.2. *Suppose $p_\phi > 0$, $q > 0$, and $N \geq 2$. There is a perfect Bayesian equilibrium in which players use poll response strategy 2, r_2, and all UIAs vote for candidate 0 if $\mu(z = 0|m_{-j}, r_2) \geq \frac{1}{2}$ and candidate 1 if $\mu(z = 0|m_{-j}, r_2) < \frac{1}{2}$.*

In the equilibrium specified in Proposition 4.2, all players but the UIAs vote as they did in the poll. The UIAs cannot indicate in the poll whom they are going to vote for because they are not sure about the state. After learning the poll outcome, however, they do vote for the candidate they believe is more likely to be the right one for them.

Because the UIAs decide whom to vote for after the poll result is disseminated, I get the following corollary stating that polls might be wrong in case there are enough UIAs and the leading candidate does not do well *enough* in the poll.

Corollary 4.1. *In the equilibrium specified in Proposition 4.2,*
1) there exists a threshold, d^*, such that if $d \equiv m_x - m_y < d^*$ for $x, y \in \{0, 1\}$ with $x \neq y$, then $\mu(z = x|m_{-j}, r_2) < \frac{1}{2}$.
2) there exists a parameter value such that $d^* > 0$.
3) if $0 < d < d^*$ and $m_{ND} > d$, then candidate y wins the election with probability w^*, where

$$w^* = \left(\frac{1}{p_\phi + p_s(1-q)}\right)^{m_{ND}} \sum_{j=1}^{m_{ND}-d} \frac{m_{ND}!}{(d+j)!\,(m_{ND}-d-j)!} (p_\phi)^{m_{ND}-d-j} (p_s(1-q))^{d+j}.$$

Corollary 4.1 shows that the candidate who leads in the poll might not win the election, even though everyone answers the poll question truthfully. Here, the threshold d^* shows how much more support a candidate needs to receive in the poll relative to the other candidate for the UIAs to believe that he/she is more likely to match the state. In particular, part 2) implies that the threshold to gain the UIAs' support might be strictly positive, and thus performing relatively better in the poll is not enough to win the election under some parameter values. If the leading candidate does not pass this threshold in the poll, all the UIAs will vote for the other candidate in election, which makes the poll result wrong.

Specifically, even if $m_x > m_y$ (i.e., $d > 0$), the belief $\mu(z = x|m_{-j}, r_2)$ might be smaller than $\frac{1}{2}$ when p_x is far higher than p_y. For example, Figure 4.1 shows the

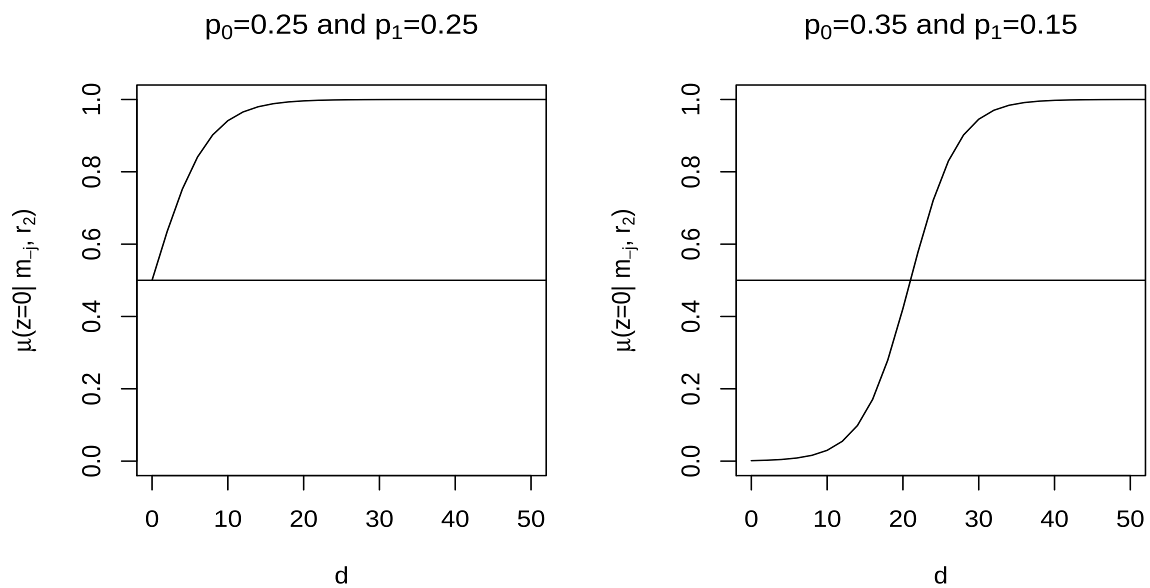

Figure 4.1: μ and d when $N+1=100$, $m_{\mathrm{ND}}=40$, $\alpha=0.5$, $p_s=0.4$, and $q=0.2$

relationship between $d=m_0-m_1$ and the posterior belief $\mu\left(z=0|m_{-j},r_2\right)$ when $N=99$, $m_{\mathrm{ND}}=40$, $\alpha=0.5$, $p_s=0.4$, and $q=0.2$. In the panel on the left, because $p_0=p_1$and $\alpha=\frac{1}{2}$, the posterior belief μ is larger than $\frac{1}{2}$ as long as $m_0{>}m_1$. Thus, in this case, the candidate who wins the poll will also win the election. This is not the case when p_0 is much larger than p_1, however. The panel on the right shows the case when $p_0=0.35>p_1=0.15$. In this case, $\mu\left(z=0|m_{-j},r_2\right)$ is smaller than $\frac{1}{2}$ if $d\leq 20$. Since $p_0>p_1$, the difference in m_0 and m_1 needs to be large enough—larger than 20—to infer that the informed independent agents support candidate 0. If candidate 0 does not lead by more than 20 points in the poll, all UIAs vote for candidate 1 according to their posterior belief. The probability that an agent whose answer was ND in the poll votes for candidate 1 is $\frac{p_s(1-q)}{p_\phi+p_s(1-q)}\approx 0.76$, and thus the probability of candidate 1 getting 20 or more votes from the forty agents is almost 1 ($w^*\approx 1$), which means candidate 1 is highly likely to win the election if candidate 0 leads the poll by less than 20 points.

Two important implications flow from the examples in Figure 4.1. First, polls can be wrong even if N is very large. Under the parameter settings on the right-hand side, no matter how large N is, candidate 0 leading candidate 1 by 20 percentage points in the poll is not enough to guarantee a victory on Election Day. The UIAs do not randomly choose for whom to vote. Rather, they learn from the poll outcome and choose the candidate who they think matches the state. Second, depending on parameter values, a seemingly close election might end up with a landslide, whereas a seemingly lopsided one might end up with an unexpected winner. When $p_0 = p_1$ with $p_s q = 0.08$, the gap between the two candidates will be relatively small in the pre-election poll. However, whoever (barely) wins the poll will gain far greater support on election day because all the UIAs will vote for this candidate. By contrast, when p_0 is much larger than p_1, the poll results are likely to be lopsided, favorable to candidate 0. However, if the lead is not big enough[3], candidate 1 is highly likely to be the winner on Election Day with a relatively small margin of victory. The victory of candidate 1 will be even more surprising because everyone knows that candidate 1 does not have enough loyal supporters relative to candidate 0.

In contrast with existing models of polls (e.g., Meirowitz 2005; Morgan and Stocken 2008), my model shows that polls might be wrong not because of strategic misrepresentation but because of learning. Here, no one intends to misrepresent his or her preference in the poll. Rather, refusing to choose between 0 and 1 in a poll is the best the UIAs can do because otherwise the poll is less informative. If they answer either 0 or 1 in the poll, it is more difficult to learn about the state. Because they decide whom to vote for *after* the poll result is known, the poll outcome will always differ from the election outcome. Sometimes, the difference might be large enough to surprise everyone if the learning process is not taken into account.

The next result establishes that in a large election, information is successfully aggregated with informative polls, in the sense that the winner of the election is the same as the winner if all agents were fully informed about the state of the world. Consider $z = 0$ without loss of generality, and let n_t denote the number of agents whose type is $t \in T$. If $|(n_0 + n_i) - n_1| > n_u$, the election fully aggregates information no matter what the UIAs do. If $|(n_0 + n_i) - n_1| \leq n_u$, then the election fully aggregates information only if the UIAs vote for the candidate who matches the state. Since the UIAs vote according to the belief μ in the informative equilibrium, it suffices to show that as the size of the electorate grows large, belief μ on the true state converges to 1 almost surely. Proposition 4.3 states this formally.

Proposition 4.3. *Suppose $q > 0$. For each $\overline{z} = 0, 1$, $\mu(z = \overline{z}|m_{-j}, r_2)$ converges to 1 almost surely conditional on $z = \overline{z}$. That is,* $\Pr\left(\lim_{N\to\infty} \mu(z = \overline{z}|m_{-j}, r_2) = 1 | z = \overline{z}\right) =$

[3]Specifically, to have $\mu(z = 0|m_{-j}, r_2) > \frac{1}{2}$, the following should hold:

$$m_0 > \frac{1}{\log\left(\left(p_0 + p_s q\right)/p_0\right)} \left(m_1 \log\left(\frac{p_1 + p_s q}{p_1}\right) + \log \frac{1-\alpha}{\alpha} \right).$$

1.

Proposition 4.3 implies that in a large election, the UIAs can learn the state almost surely, and thus, will vote correctly. Feddersen and Pesendorfer (1996, p. 415) show that information is fully aggregated with no polls (or uninformative polls) because the UIAs optimally abstain and thereby compensate the gap in partisan supports so that the informed independents get to choose the winner. My result verifies that informative polls encourage the UIAs to participate in voting and still achieve the efficient outcome.

Then, does an election with informative polls aggregate information *more* successfully than without them? A naive answer is that it does so because agents have more information to consider when making their voting decisions. However, depending on the realization of z and n_t's, the informative equilibrium performs better or worse than the uninformative equilibrium. For example, assume the same parameter values as in the right-hand panel of Figure 4.1 ($\alpha = 0.5$, $p_s = 0.4$, $p_0 = 0.35$, $p_1 = 0.15$, and $q = 0.2$) and consider the case in which the random draws of each agent's type and the state result in $n_0 = 30$, $n_1 = 20$, $n_i = 10$, $n_u = 30$, $n_\phi = 10$, and $z = 0$. Then, in the informative equilibrium, I have $m_0 - m_1 = (n_0 + n_i) - n_1 = 20$. Thus, as discussed previously, all the UIAs vote for candidate 1 because $\mu\left(z = 0 \middle| (m_0, m_1, m_{\mathrm{ND}}) = (40, 20, 39), r_2\right) < \frac{1}{2}$. As a result, candidate 1 wins the election for sure. On the other hand, in the uninformative equilibrium, candidate 0 has a chance to win the election, because an UIA votes for 0 with a strictly positive probability.[4] Since candidate 0 is the winner if the state of the world is publicly known (i.e., $n_0 + n_i + n_u > n_1$), the equilibrium with uninformative polls performs better in this case. However, if just one agent switches from type-1 to type-0, so that $n_0 = 31$ and $n_1 = 19$, all else being equal, candidate 0 wins for sure in the informative equilibrium because every UIA votes for candidate 0 by the fact $\mu\left(z = 0 \middle| (m_0, m_1, m_{\mathrm{ND}}) = (41, 19, 39), r_2\right) > \frac{1}{2}$. In the uninformative equilibrium, candidate 0 may lose because an UIA votes for 1 with a strictly positive probability.[5] Thus, the informative equilibrium works better in this case.

These examples imply that one equilibrium does not dominate the other for every (ex post) realization of z and n_t's. Thus I need to compute and compare the ex ante probabilities that an election aggregates information for the two equilibria. Let W denote the event of $n = (n_0, n_1, n_i, n_u, n_\phi)$ in which the winner of the election is different from the winner of the election if all agents were fully informed (i.e., the election fails to aggregate information). The event W happens if the number of agents who prefer

[4] When $N + 1 = 100$, $p_0 = 0.31$, $p_1 = 0.15$, $p_s = 0.4$, $q = 0.2$, and $\alpha = 0.5$, the probability of an UIA voting for 0 in the uninformative equilibrium is give by $\tau_0 \approx 0.61$.

[5] $\tau_1 = 1 - \tau_0 \approx 0.39$.

candidate x exceeds the number of agents who prefer candidate y, but candidate y wins the election where $x, y \in \{0,1\}$, $x \neq y$. In the uninformative equilibrium, this happens with probability 1 if $z = x$, $n_y < n_x + n_i + n_u$, and $n_y + n_{uy} > n_x + n_i + n_{ux}$, and with probability $\frac{1}{2}$ if $z = x$, $n_y < n_x + n_i + n_u$, and $n_y + n_{uy} = n_x + n_i + n_{ux}$, where n_{uy} and n_{ux} denote the number of the UIAs who vote for candidate x and y, respectively. Therefore,

$$\begin{aligned}&\Pr\left(W|r_1,\ z=x,\ N\right)\\ =&\sum_{n\in\Gamma_1}\frac{(N+1)!}{n_1!n_0!n_i!n_u!n_\phi!}\left(\prod_{t\in T}p_t^{n_t}\right)\times\Xi,\end{aligned}$$

where

$$\Gamma_1=\begin{cases}\left\{n\in\mathbb{R}_+^5|\sum_{t\in T}n_t=N+1,\ n_x+n_i-n_u<n_y<n_x+n_i+n_u\right\} & \text{if}\quad \tau_x=0\\ \left\{n\in\mathbb{R}_+^5|\sum_{t\in T}n_t=N+1,\ n_x+n_i<n_y<n_x+n_i+n_u\right\} & \text{if}\quad \tau_y=0\end{cases},$$

and

$$\Xi=\begin{cases}\frac{1}{2}\frac{n_u!}{h!(n_u-h)!}\tau_y^h\tau_\phi^{(n_u\,-\,h)}+\left(\sum_{n_{uy}\,=\,h+1}^{n_u}\frac{n_u!}{n_{uy}!(n_u\,-\,n_{uy})!}\tau_y^{n_{uy}}\tau_\phi^{(n_u\,-\,n_{uy})}\right) & \text{if}\quad \tau_x=0\text{ and }h\geq 0\\ \sum_{n_{uy}\,=\,0}^{n_u}\frac{n_u!}{n_{uy}!(n_u\,-\,n_{uy})!}\tau_y^{n_{uy}}\tau_\phi^{(n_u\,-\,n_{uy})} & \text{if}\quad \tau_x=0\text{ and }h<0\\ \left(\sum_{n_{ux}\,=\,0}^{h'\,-\,1}\frac{n_u!}{n_{uy}!(n_u\,-\,n_{uy})!}\tau_x^{n_{ux}}\tau_\phi^{n_{u\phi}}\right)+\frac{1}{2}\frac{n_u!}{h'!(n_u-h')!}\tau_x^{h'}\tau_\phi^{n_u-h'} & \text{if}\quad \tau_y=0\end{cases}$$

with

$$h=n_x+n_i-n_y\text{ and }h'=n_y-n_x-n_i.$$

Note that it is enough to consider two cases, $\tau_x = 0$ and $\tau_y = 0$, because the UIAs never mix between 0 and 1 by Proposition 4.1. The term Ξ is the probability of candidate y winning given n_t's, z, and τ.

Similarly, in the informative equilibrium, the event W happens with probability 1 if $z = x$, $n_y < n_x + n_i + n_u$, $n_y + n_u > n_x + n_i$, and $\mu\left(z = x|m_{-j}, r_2\right) < \frac{1}{2}$, and with probability $\frac{1}{2}$ if $z = x$, $n_y < n_x + n_i + n_u$, $n_y + n_u = n_x + n_i$, and $\mu\left(z = x|m_{-j}, r_2\right) < \frac{1}{2}$. Thus,

$$\begin{aligned}&\Pr\left(W|r_2,\ z=x,\ N\right)\\ =&\sum_{n\in\Gamma_2}\left(\frac{(N+1)!}{n_1!n_0!n_i!n_u!n_\phi!}\prod_{t\in T}p_t^{n_t}\right)+\frac{1}{2}\sum_{n\in\Gamma_2'}\left(\frac{(N+1)!}{n_1!n_0!n_i!n_u!n_\phi!}\prod_{t\in T}p_t^{n_t}\right),\end{aligned}$$

where

$$\Gamma_2=\{n\in\mathbb{R}_+^5|\sum_{t\in T}n_t=N+1,\ n_x+n_i-n_u<n_y<n_x+n_i+n_u,\text{ and }\mu\left(z=x|m_{-j},r_2\right)<\frac{1}{2}\},$$

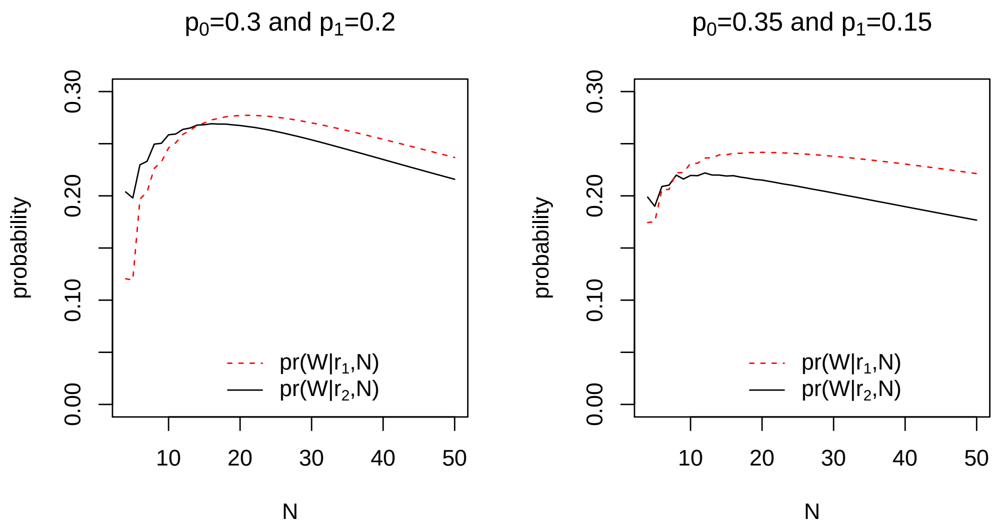

Figure 4.2: Probabilities $\Pr(W|r_2,\ \ N)$ and $\Pr(W|r_1,\ N)$ when $\alpha = 0.5$, $p_\phi = 0.1$, $p_s = 0.4$ and $q = 0.2$

$$\Gamma_2' = \{n \in \mathbb{R}_+^5 | \sum_{t \in T} n_t = N + 1,\ n_y + n_u = n_x + n_i, \text{ and } \mu\,(z = x|m_{-j}, r_2) < \frac{1}{2}\},$$

and

$$m_{-j} = (m_x, m_y, m_{\text{ND}}) = (n_x, n_y, N - (n_x + n_y))\,.$$

Instead of comparing $\Pr(W|r_2,\ z = x,\ N)$ and $\Pr(W|r_1,\ z = x,\ N)$ directly for finite N, I consider large elections. Since the election fully aggregates information in a large election under both poll response strategies, the probability of W converges to zero regardless of the poll response strategy being used. By taking the large deviation approach, I compare the converging speed of these probabilities to show that $\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\,(W|r_2,\ z = x,\ N) < \lim_{N\to\infty} \frac{1}{N+1} \log \Pr\,(W|r_1,\ z = x,\ N)$ for all x, which in turn implies that $\Pr(W|r_2,\ z = x,\ N) < \Pr(W|r_1,\ z = x,\ N)$ for sufficiently large N. Proposition 4.4 proves it formally.

Proposition 4.4. *Suppose $p_0 > 0$, $p_1 > 0$, $p_\phi > 0$, and $q > 0$. If $p_1 \neq p_0$ and $p_s\,(1 - q) > |p_1 - p_0|$, then, for sufficiently large N, $\Pr\,(W|r_2, z = x, N) < \Pr\,(W|r_1, z = x, N)$ for each $x \in \{0, 1\}$.*

Proposition 4.4 shows that polls can be welfare improving in a large election if the expected fractions of partisans, p_0 and p_1, are not exactly the same and the expected fraction of UIAs is larger than the difference between the fractions of partisans.[6] Figure 4.2 shows numerical examples of $\Pr(W|r,N)$ in which $\Pr(W|r_2, z = x, N) < \Pr(W|r_1, z = x, N)$ does not hold for small N but it does eventually. Parameters are set to $\alpha = 0.5$, $p_\phi = 0.1$, $p_s = 0.4$, and $q = 0.2$. Note that even when everyone tells the truth, polls are not always helpful when the size of electorate is not large enough. For example, for $N = 5$, the expected number of the informed independent agents is far less than 1. Since most of the times there are too few informed independent agents, if any, to signal the state, and all the UIAs are voting anyway in the informative equilibrium, the wrong candidate might be elected quite often. On the contrary, in the uninformative equilibrium, the UIAs just try to fill the partisan gap and optimally abstain.[7] This means that the UIAs might make a wrong decision, but not all of them do so. Thus, the chance of electing the wrong candidate is relatively lower. However, as N increases, $\Pr(W|r_2,\ N)$ converges to zero more rapidly than $\Pr(W|r_1,\ N)$ does, as illustrated in the figure.

5 Discussion and Conclusion

My analysis implies that in large elections, such as presidential or congressional elections, it is better to take public pre-election opinion polls regardless of whether these polls can clearly predict the winner. I show that there is an equilibrium in which everyone truthfully answers the poll question and that in such an equilibrium, the swing voters' curse disappears and everyone votes as if they had no uncertainty about the state of the world.

What if there are multiple rounds of polling, as is typical in real elections? Theoretically, all kinds of things can happen in this case. For example, in equilibrium, it is possible that players use the informative poll response strategy in the first few rounds and thereafter answer the poll as they would vote on election day based on their posterior belief. It is also possible that everyone keeps playing the informative poll response strategy throughout the campaign periods and then votes on election day according to the posterior belief. In either case, the equilibrium being played with multiple pollings is essentially the same as described in the game with one-round polling regarding information transmission, but the gap between the poll results and the election outcome

[6]When $p_0 = p_1$, the converging speeds coincide. The other assumption $p_s(1-q) > |p_1 - p_0|$ means that the expected fraction of UIAs is large enough to overturn the election outcome.

[7]For $N = 5$, $p_0 = 0.3$ and $p_1 = 0.2$, an UIA abstains with probability $\tau_\phi = 1$ and for $p_0 = 0.35$ and $p_1 = 0.15$ with probability $\tau_\phi \approx 0.71$.

will vary depending on which strategies are played.

It would be useful for future research to test the theoretical results using experiments. In particular, because there are multiple equilibria in the game, it should be empirically examined how agents answer the poll question and how they interpret the poll results. Existing experimental studies show considerable support for the swing voter's curse (Battaglini et al. 2008, 2010). Will the curse disappear when agents answer a public opinion poll? Will the uninformed voters be willing to participate in voting and do so in the right way? Answers to these questions would lead to further understanding of democratic institutions.

A Appendix

Lemma A.1 proves Lemma 4.2. I omit proof of Lemma A.1 as it is straightforward.

Lemma A.1. *For $x, y \in X$ with $x \neq y$, assume $m_x \geq m_y$. Let $\delta = m_x - m_y$ and $m'_{ND} = m_{ND} - 1$. When players use poll response strategy $r_2(\cdot)$, for all $z \in \{0, 1\}$,*
1) if $\delta = m'_{ND} = 0$, then $\pi_e(z, \tau\ |m_{-j}, r_2) = 1$ and $\pi_x(z, \tau\ |m_{-j}, r_2) = \pi_y(z, \tau\ |m_{-j}, r_2) = 0$,
2) if $\delta = 1$ and $m'_{ND} = 0$, then $\pi_e(z, \tau\ |m_{-j}, r_2) = \pi_x(z, \tau\ |m_{-j}, r_2) = 0$, and $\pi_y(z, \tau\ |m_{-j}, r_2) = 1$,
3) if $\delta - m'_{ND} = 1$ and $m'_{ND} > 0$, then $\pi_e(z, \tau\ |m_{-j}, r_2) = \pi_x(z, \tau\ |m_{-j}, r_2) = 0$, and

$$\pi_y(z, \tau\ |m_{-j}, r_2) = \frac{N!}{m_0! m_1! m'_{ND}!} \left(\sigma_{y,z,ND}(\tau|r_2)\right)^{m'_{ND}},$$

4) if $\delta - m'_{ND} > 1$, then $\pi_e(z, \tau\ |m_{-j}, r_2) = \pi_x(z, \tau\ |m_{-j}, r_2) = \pi_y(z, \tau\ |m_{-j}, r_2) = 0$,
5) if $\delta = m'_{ND} > 0$, then

$$\begin{aligned}
&\pi_e(z, \tau\ |m_{-j}, r_2) = \frac{N!}{m_0! m_1! m'_{ND}!} \left(\sigma_{y,z,ND}(\tau|r_2)\right)^{m'_{ND}}, \\
&\pi_x(z, \tau\ |m_{-j}, r_2) = 0, \text{ and} \\
&\pi_y(z, \tau\ |m_{-j}, r_2) = \frac{N!}{m_0! m_1! m'_{ND}!} m'_{ND} \sigma_{\phi,z,ND}(\tau|r_2) \left(\sigma_{y,z,ND}(\tau|r_2)\right)^{m'_{ND}-1},
\end{aligned}$$

5) if $0 \leq \delta < m'_{ND}$*, then*

$$\begin{aligned}
&\pi_e\left(z, \tau\ |m_{-j}, r_2\right) \\
&= \frac{N!}{m_0! m_1! m'_{ND}!} \sum_{j=0}^{\left\lfloor \left(m'_{ND}-\delta\right)/2 \right\rfloor} \frac{m'_{ND}!}{j!\left(\delta+j\right)!\left(m'_{ND}-\left(\delta+2j\right)\right)!} \\
&\times \sigma_{\phi,z,ND}\left(\tau|r_2\right)^{m'_{ND}-\left(\delta+2j\right)} \sigma_{x,z,ND}\left(\tau|r_2\right)^{\delta+j} \sigma_{y,z,ND}\left(\tau|r_2\right)^{j},
\end{aligned}$$

$$\begin{aligned}
&\pi_x\left(z, \tau\ |m_{-j}, r_2\right) \\
&= \frac{N!}{m_0! m_1! m'_{ND}!} \sum_{j=0}^{\left\lfloor \left(m'_{ND}-\left(\delta+1\right)\right)/2 \right\rfloor} \frac{m'_{ND}!}{j!\left(\delta+j+1\right)!\left(m'_{ND}-\left(\delta+2j+1\right)\right)!} \\
&\times \sigma_{\phi,z,ND}\left(\tau|r_2\right)^{m'_{ND}-\left(\delta+2j+1\right)} \sigma_{x,z,ND}\left(\tau|r_2\right)^{j} \sigma_{y,z,ND}\left(\tau|r_2\right)^{\delta+j+1},
\end{aligned}$$

and

$$\begin{aligned}
&\pi_y\left(z, \tau\ |m_{-j}, r_2\right) \\
&= \frac{N!}{m_0! m_1! m'_{ND}!} \sum_{j=0}^{\left\lfloor \left(m'_{ND}-\left(\delta-1\right)\right)/2 \right\rfloor} \frac{m'_{ND}!}{j!\left(\delta+j-1\right)!\left(m'_{ND}-\left(\delta+2j-1\right)\right)!} \\
&\times \sigma_{\phi,z,ND}\left(\tau|r_2\right)^{m'_{ND}-\left(\delta+2j-1\right)} \sigma_{x,z,ND}\left(\tau|r_2\right)^{j} \sigma_{y,z,ND}\left(\tau|r_2\right)^{\delta+j-1},
\end{aligned}$$

where

$$\sigma_{v,z,ND}\left(\tau|r_2\right) = \begin{cases} \frac{p_\phi + p_s(1-q)\tau_\phi}{p_\phi + p_s(1-q)} & \text{if} \quad v = \phi \\ \frac{p_s(1-q)\tau_v}{p_\phi + p_s(1-q)} & \text{if} \quad v \in \{0,1\} \end{cases}.$$

[**Proof of Proposition 4.1**] The proof of part 1) can be found in Fey and Kim (2002). To prove part 2), I need to make a pair-wise comparison between voting actions as follows. Since $\pi_e\left(1, \tau\ |m_{-j}, r_2\right) = \pi_e\left(0, \tau\ |m_{-j}, r_2\right)$, $\pi_1\left(1, \tau\ |m_{-j}, r_2\right) = \pi_1\left(0, \tau\ |m_{-j}, r_2\right)$, and $\pi_0\left(1, \tau\ |m_{-j}, r_2\right) = \pi_0\left(1, \tau\ |m_{-j}, r_2\right)$ by Lemma 4.2, I have

$$\begin{aligned}
&Eu\left(1, \tau|m_{-j}, r_2\right) - Eu\left(\phi, \tau|m_{-j}, r_2\right) \qquad \text{(A.1)} \\
&= \frac{1}{2}\left[\begin{array}{c} \left(1-\mu\left(0|m_{-j}, r_2\right)\right)\left[\pi_e\left(1, \tau\ |m_{-j}, r_2\right) + \pi_1\left(1, \tau\ |m_{-j}, r_2\right)\right] \\ -\mu\left(0|m_{-j}, r_2\right)\left[\pi_e\left(0, \tau\ |m_{-j}, r_2\right) + \pi_1\left(0, \tau\ |m_{-j}, r_2\right)\right] \end{array}\right] \\
&= \frac{1}{2}\left(1-2\mu\left(0|m_{-j}, r_2\right)\right)\left[\pi_e\left(1, \tau\ |m_{-j}, r_2\right) + \pi_1\left(1, \tau\ |m_{-j}, r_2\right)\right],
\end{aligned}$$

and

$$Eu\left(0,\tau|m_{-j},r_2\right)-Eu\left(\phi,\tau|m_{-j},r_2\right) \tag{A.2}$$
$$=\frac{1}{2}\left[\begin{array}{c}-\left(1-\mu\left(0|m_{-j},r_2\right)\right)\left[\pi_e\left(1,\tau\ |m_{-j},r_2\right)+\pi_0\left(1,\tau\ |m_{-j},r_2\right)\right]\\+\mu\left(0|m_{-j},r_2\right)\left[\pi_e\left(0,\tau\ |m_{-j},r_2\right)+\pi_0\left(0,\tau\ |m_{-j},r_2\right)\right]\end{array}\right]$$
$$=-\frac{1}{2}\left(1-2\mu\left(0|m_{-j},r_2\right)\right)\left[\pi_e\left(1,\tau\ |m_{-j},r_2\right)+\pi_0\left(1,\tau\ |m_{-j},r_2\right)\right].$$

Thus, if $\mu\left(0|m_{-j},r_2\right)\geq\frac{1}{2}$, I have $Eu\left(0,\tau|m_{-j},r_2\right)\geq Eu\left(\phi,\tau|m_{-j},r_2\right)$. If $\mu\left(0|m_{-j},r_2\right)<\frac{1}{2}$, then $Eu\left(1,\tau|m_{-j},r_2\right)\geq Eu\left(\phi,\tau|m_{-j},r_2\right)$, as needed. □

[**Proof of Proposition 4.2**] I first show that no player has an incentive to deviate from the specified voting strategy given the beliefs. Then, I show that given the voting strategy, no one has an incentive to deviate from the specified poll response strategy.

Voting strategy:

By Proposition 4.1, it suffices to compare $Eu\left(1,\tau|m_{-j},r_2\right)$ and $Eu\left(0,\tau|m_{-j},r_2\right)$. Since

$$\begin{aligned}&Eu\left(1,\tau|m_{-j},r_2\right)-Eu\left(0,\tau|m_{-j},r_2\right)\\&=\left(1-\mu\left(0|m_{-j},r_2\right)\right)\left(\pi_e\left(1,\tau\ |m_{-j},r_2\right)+\frac{1}{2}\pi_1\left(1,\tau\ |m_{-j},r_2\right)+\frac{1}{2}\pi_0\left(1,\tau\ |m_{-j},r_2\right)\right)\\&-\mu\left(0|m_{-j},r_2\right)\left(\pi_e\left(0,\tau\ |m_{-j},r_2\right)+\frac{1}{2}\pi_1\left(0,\tau\ |m_{-j},r_2\right)+\frac{1}{2}\pi_0\left(0,\tau\ |m_{-j},r_2\right)\right)\\&=\left(1-2\mu\left(0|m_{-j},r_2\right)\right)\left(2\pi_t\left(1,\tau\ |m_{-j},r_2\right)+\pi_1\left(1,\tau\ |m_{-j},r_2\right)+\pi_0\left(1,\tau\ |m_{-j},r_2\right)\right)\end{aligned}$$

by Lemma 4.2, I have $Eu\left(0,\tau|m_{-j},r_2\right)\geq Eu\left(1,\tau|m_{-j},r_2\right)$ if $\mu\left(0|m_{-j},r_2\right)\geq\frac{1}{2}$ and $Eu\left(1,\tau|m_{-j},r_2\right)>Eu\left(0,\tau|m_{-j},r_2\right)$ if $\mu\left(0|m_{-j},r_2\right)<\frac{1}{2}$.

Poll response strategy:

Note that the probability of candidate 0 winning weakly increases with $\mu\left(z=0|m_{-j},r_2\right)$. Since $\frac{\partial\mu}{\partial m_1}\leq 0$ and $\frac{\partial\mu}{\partial m_0}\geq 0$, a type-0 or type-1 voter does not have an incentive to deviate from r_2. For the same reason, an informed independent agent truthfully answers the poll question to signal the state. Now, to check an UIA's incentive, consider her/his deviation to 0 while the other players play r_2. The only case in which an UIA's deviation to 0 makes a difference is when $\mu\left(z=0|(m_0,m_1,m_{\mathrm{ND}}-1),r_2\right)<\frac{1}{2}$ and $\mu\left(z=0|\left(m_0+1,m_1,m_{\mathrm{ND}}-2\right),r_2\right)\geq\frac{1}{2}$. In this case, if an UIA deviates to 0, the other UIAs switch their votes to candidate 0, which gives an equal or worse expected utility to the UIA since $z=1$ is more likely. Deviation to 1 can be checked similarly. □

[**Proof of Proposition 4.3**] Suppose $z = 0$. The case in which the state is 1 can be proved similarly.

Note that

$$\begin{aligned}\mu\left(z=0|m_{-j},r_2\right) &= \frac{\alpha\left(p_0+p_sq\right)^{m_0}p_1^{m_1}}{\alpha\left(p_0+p_sq\right)^{m_0}p_1^{m_1}+\left(1-\alpha\right)p_0^{m_0}\left(p_1+p_sq\right)^{m_1}} \\ &= \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)\left[\left(\frac{p_0}{p_0+p_sq}\right)^{m_0/(N+1)}\left(\frac{p_1+p_sq}{p_1}\right)^{m_1/(N+1)}\right]^{N+1}}.\end{aligned}$$

By the Strong Law of Large Numbers, $m_0/\left(N+1\right)$ converges to p_0+p_sq a.s., and $m_1/\left(N+1\right)$ converges to p_1 a.s. as $N\to\infty$. Thus, it suffices to show that

$$\left(\frac{p_0}{p_0+p_sq}\right)^{p_0+p_sq}\left(\frac{p_1+p_sq}{p_1}\right)^{p_1}<1.$$

Note that

$$\begin{aligned}&\log\left[\left(\frac{p_0}{p_0+p_sq}\right)^{p_0+p_sq}\left(\frac{p_1+p_sq}{p_1}\right)^{p_1}\right] \\ &= \left(p_0+p_sq\right)\log\left(1-\frac{p_sq}{p_0+p_sq}\right)+p_1\log\left(1+\frac{p_sq}{p_1}\right) \\ &< \left(p_0+p_sq\right)\left(-\frac{p_sq}{p_0+p_sq}\right)+p_1\left(\frac{p_sq}{p_1}\right)=0.\end{aligned}$$

The inequality follows from the fact that $\log\left(1+a\right)<a$ for all $a\in\left(-1,0\right)\cup\left(0,\infty\right)$, which can be verified by observing that $\log\left(1+a\right)-a$ is strictly concave and maximized at $a=0$.

□

[**Proof of Proposition 4.4**]

Without loss of generality, assume $p_1>p_0$. Before proceeding with the proof, define new notations. Let $\left\{\tau^N\right\}_{N=0}^{\infty}$be a sequence of equilibrium voting strategy for type-u in which agents use uninformative poll response strategy. Let $p_i\equiv p_sq$, $p_u\equiv p_s\left(1-q\right)$, $p_{u0}^N\equiv p_s\left(1-q\right)\tau_0^N$, $p_{u1}^N\equiv p_s\left(1-q\right)\tau_1^N$ and $p_{u\phi}^N\equiv p_s\left(1-q\right)\tau_\phi^N$. Note that since I assume $p_u>p_1-p_0$, $p_{u0}^N\to p_{u0}\equiv p_1-p_0$ and $p_{u\phi}^N\to p_{u\phi}\equiv p_u-\left(p_1-p_0\right)$ (Feddersen and Pesendorfer 1996, Proposition 3). Further, let n_{u0} denote the number of agents whose type is u and chooses to vote for 0, n_{u1} the number of agents whose type is u and chooses to vote for 1, and $n_{u\phi}$ the number of agents whose type is u and chooses to abstain. Clearly, $n_u=n_{u0}+n_{u1}+n_{u\phi}$. I will abuse the notation and denote $n=\left(n_0,n_1,n_i,n_u,n_\phi\right)$ or $\left(n_0,n_1,n_i,n_{u0},n_{u\phi},n_\phi\right)$, depending on the context.

For a positive integer K, let $\Delta^K = \left\{f \in \mathbb{R}_+^K : \sum_k f_k = 1\right\}$. I may omit N when it is clear from the context.

I need the following lemmas. I will use the large deviation principle (for example, see DasGupta (2008, Theorem 23.2)) in the proofs below.

Lemma A.2. *Suppose $p_1 > p_0 > 0$, $p_u > p_1 - p_0$, $p_\phi > 0$ and $q > 0$. Then,*

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W | r_1,\ z = 0\right) = -\min_{g\in G_0} \sum_{k=0,1,i,u0,u\phi,\phi} g_k \log\left(g_k/p_k\right),$$

where $G_0 = \{g = (g_1, g_0, g_i, g_{u0}, g_{u\phi}, g_\phi) \in \Delta^6 : g_1 \leq g_0 + g_i + g_{u0} + g_{u\phi},\ g_1 \geq g_0 + g_i + g_{u0}\}$.

Proof: I ignore the event of ties and let W' be the event in which $n_1 \leq n_0 + n_i + n_u$ (more agents prefer candidate 0) and $n_1 + n_{u1} \geq n_0 + n_i + n_{u0}$ (more agents vote for candidate 1). Since the probability of a tie (the event that $n_1 + n_{u1} = n_0 + n_i + n_{u0}$) vanishes in the limit, I compute the probability of W' instead. Note that since $p_1 > p_0$, $n_{u0} \geq n_{u1} = 0$ by Proposition 4.1. Then,

$$\begin{aligned}
&\Pr\left(W' | r_1, z = 0\right) \\
=&\Pr\left(n_1 \leq n_0 + n_i + n_u \text{ and } n_1 \geq n_0 + n_i + n_{u0} | r_1, z = 0\right) \\
=&\Pr\left(n_1 \leq n_0 + n_i + n_{u0} + n_{u\phi} \text{ and } n_1 \geq n_0 + n_i + n_{u0} | r_1, z = 0\right) \\
=&\Pr\left(\frac{n}{N+1} \in G_0 | r_1, z = 0\right),
\end{aligned}$$

Here, G_0 is defined in the statement of the lemma.

By the large deviation principle, I have

$$\begin{aligned}
&\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(\frac{n}{N+1} \in G_0 | r_1, z = 0\right) \\
=& -\inf_{g\in G_0} \sup_{d\in\mathbb{R}^6} \left(g^T d - \phi(d)\right),
\end{aligned}$$

where T is the transformation operator and $\phi(d) = \lim_{N\to\infty} \frac{1}{N+1} \log E\left[\exp\left(d^T n\right)\right]$ is the limit of the cumulant generating function of n. Since the voter types are independent and identically distributed, I can consider the type distribution of the first agent only and hence $\phi(d) = \log\left(\sum_{k=0,1,i,u0,u\phi,\phi} p_k \exp(d_k)\right)$.

Simple algebra shows $\sup_d \left(g'd - \phi(d)\right) = \sum_k g_k \log\left(g_k/p_k\right)$. This implies

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W' | r_1,\ z = 0\right) = -\inf_{g\in G_0} \sum_k g_k \log\left(g_k/p_k\right).$$

Since the objective function is continuous and G_0 is compact, the minimum is achieved and the lemma is proved. $\square$

Lemma A.3. *Suppose $p_1 > p_0 > 0$, $p_u > p_1 - p_0$, $p_\phi > 0$ and $q > 0$. Then,*
(i) $\lim_{N\to\infty} \frac{1}{N+1} \log \Pr(W|r_1,\ z=0) = -\min_{g\in G_0'} \sum_{k=0,1,i,u0,u\phi,\phi} g_k \log(g_k/p_k)$*, where* $G_0' = \{g = (g_1, g_0, g_i, g_{u0}, g_{u\phi}, g_\phi) \in \Delta^6 : g_1 \geq g_0 + g_i + g_{u0}\}$ *and* $g_1 \geq g_0 + g_i + g_{u0}$ *is binding at the minimizer, and*
(ii) $\lim_{N\to\infty} \frac{1}{N+1} \log \Pr(W|r_1,\ z=0) = \lim_{N\to\infty} \frac{1}{N+1} \log \Pr(W|r_1, z=1)$.

Proof: I solve the constrained optimization problem in Lemma A.2 to compute

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr(W|r_1,\ z=0).$$

The Kuhn-Tucker conditions give the following solution at which only one inequality constraint $g_1 \geq g_0 + g_i + g_{u0}$ is binding:

$$\begin{aligned}
g_0 &= \frac{p_0}{p_0+p_i+p_{u0}} \frac{\sqrt{p_1(p_0+p_i+p_{u0})}}{2\left(\sqrt{p_1(p_0+p_i+p_{u0})}\right)+(p_{u\phi}+p_\phi)},\\
g_1 &= \frac{\sqrt{p_1(p_0+p_i+p_{u0})}}{2\left(\sqrt{p_1(p_0+p_i+p_{u0})}\right)+(p_{u\phi}+p_\phi)},\\
g_i &= \frac{p_i}{p_0+p_i+p_{u0}} \frac{\sqrt{p_1(p_0+p_i+p_{u0})}}{2\left(\sqrt{p_1(p_0+p_i+p_{u0})}\right)+(p_{u\phi}+p_\phi)},\\
g_{u0} &= \frac{p_{u0}}{p_0+p_i+p_{u0}} \frac{\sqrt{p_1(p_0+p_i+p_{u0})}}{2\left(\sqrt{p_1(p_0+p_i+p_{u0})}\right)+(p_{u\phi}+p_\phi)},\\
g_{u\phi} &= \frac{p_{u\phi}}{2\left(\sqrt{p_1(p_0+p_i+p_{u0})}\right)+(p_{u\phi}+p_\phi)}, \text{ and}\\
g_\phi &= \frac{p_\phi}{2\left(\sqrt{p_1(p_0+p_i+p_{u0})}\right)+(p_{u\phi}+p_\phi)}.
\end{aligned}$$

Plugging these into $-\sum_k g_k \log(g_k/p_k)$ gives

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr(W|r_1,\ z=0) = \log\left(2\sqrt{p_1(p_1+p_i)} + (p_u - (p_1-p_0) + p_\phi)\right). \tag{A.3}$$

Because the inequality $g_1 \leq g_0 + g_i + g_{u0} + g_{u\phi}$ is not binding, the minimization problem under G_0' has the same solution and thus 1) holds.

Turn to $z = 1$. Recall that the probability of a tie vanishes in the limit. Ignoring the probability of a tie, I consider the event denoted by W' again in which $n_0 \leq n_1 + n_i + n_{u0} + n_{u\phi}$ (more agents prefer candidate 1) and $n_0 + n_{u0} \geq n_1 + n_i$ (more agents vote for candidate 0). Applying the large deviation principle again (see the proof of Lemma A.2 for detail), I obtain

$$\lim_{N\to\infty}\frac{1}{N+1}\log\Pr\left(W'|r_1,z=1\right)=-\inf_{g\in G_1}\sum_k g_k\log\left(g_k/p_k\right)$$

where $G_1=\{g=(g_0,g_1,g_i,g_{u0},g_{u\phi},g_\phi)\in\Delta^6 : g_0\leq g_1+g_i+g_{u0}+g_{u\phi}, g_0+g_{u0}\geq g_1+g_i\}$. Then, the Kuhn-Tucker conditions give the following solution at which only one inequality constraint $g_0+g_{u0}\geq g_1+g_i$ is binding:

$$\begin{aligned}
g_0 &= \frac{p_0}{(p_0+p_{u0})}\left(\frac{\sqrt{(p_0+p_{u0})(p_1+p_i)}}{2\sqrt{(p_0+p_{u0})(p_1+p_i)}+(p_{u\phi}+p_\phi)}\right),\\
g_1 &= \frac{p_1}{(p_1+p_i)}\left(\frac{\sqrt{(p_0+p_{u0})(p_1+p_i)}}{2\sqrt{(p_0+p_{u0})(p_1+p_i)}+(p_{u\phi}+p_\phi)}\right),\\
g_i &= \frac{p_i}{(p_1+p_i)}\left(\frac{\sqrt{(p_0+p_{u0})(p_1+p_i)}}{2\sqrt{(p_0+p_{u0})(p_1+p_i)}+(p_{u\phi}+p_\phi)}\right),\\
g_{u0} &= \frac{p_{u0}}{(p_0+p_{u0})}\left(\frac{\sqrt{(p_0+p_{u0})(p_1+p_i)}}{2\sqrt{(p_0+p_{u0})(p_1+p_i)}+(p_{u\phi}+p_\phi)}\right),\\
g_{u\phi} &= \left(\frac{p_{u\phi}}{2\sqrt{(p_0+p_{u0})(p_1+p_i)}+(p_{u\phi}+p_\phi)}\right), \text{ and}\\
g_\phi &= \frac{p_\phi}{2\sqrt{(p_0+p_{u0})(p_1+p_i)}+(p_{u\phi}+p_\phi)}.
\end{aligned}$$

The statement of the lemma is verified by plugging these into the objective function and comparing with (A.3). □

Lemma A.4. *Suppose $p_1>p_0>0$, $p_u>p_1-p_0$, $p_\phi>0$ and $q>0$. Then,*

$$\lim_{N\to\infty}\frac{1}{N+1}\log\Pr\left(W|r_2,z=0\right)=-\sum_{k=0,1,i,u,\phi}f_k\log\left(f_k/p_k\right),$$

where

$$
\begin{aligned}
f_0 &= \frac{p_0}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right), \\
f_1 &= \frac{\eta}{1+C}, \\
f_i &= \frac{p_i}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right), \\
f_u &= \frac{p_u}{p_u+p_\phi}\left(\frac{C}{1+C}\right), \text{ and} \\
f_\phi &= \frac{p_\phi}{p_u+p_\phi}\left(\frac{C}{1+C}\right),
\end{aligned}
$$

with $\eta = \frac{\log((p_0+p_i)/p_0)}{\log((p_0+p_i)/p_0)+\log((p_1+p_i)/p_1)}$ *and* $C = (p_u+p_\phi)\left(\frac{\eta}{p_1}\right)^\eta \left(\frac{1-\eta}{p_0+p_i}\right)^{(1-\eta)}$.

Proof: As in Lemma A.2, the probability of a tie vanishes in the limit. Thus, instead of W, I can consider the event denoted by W' in which $n_1 < n_0+n_i+n_u$ (more agents prefer candidate 0), $n_1+n_u > n_0+n_i$ (more agents vote for candidate 1 if the UIAs vote for 1) and $\mu(z=0|m_{-j},r_2) < \frac{1}{2}$ (the UIAs vote for candidate 1). Observe

$$
\begin{aligned}
&\Pr(W'|r_2, z=0) \\
=&\Pr\left(n_1 < n_0+n_i+n_u, n_1+n_u > n_0+n_i, \mu(z=0|m_{-j},r_2) < \frac{1}{2}|r_2, z=0\right) \\
=&\Pr(n_0+n_i-n_u < n_1 < n_0+n_i+n_u, \\
&\qquad (n_0+n_i)\log\frac{p_0}{p_0+p_i} - n_1\log\frac{p_1}{p_1+p_i} > \log\frac{\alpha}{1-\alpha}|r_2, z=0) \\
=&\Pr\left(\frac{n}{N+1} \in F_0\left(\frac{1}{N+1}\log\frac{\alpha}{1-\alpha}\right)|r_1, z=0\right),
\end{aligned}
$$

where

$$
\begin{aligned}
F_0(b) = \{f = (f_0, f_1, f_i, f_u, f_\phi) \in \Delta^5 : f_0+f_i-f_u \le f_1 \le f_0+f_i+f_u \text{ and} \\
(f_0+f_i)\log\frac{p_0}{p_0+p_i} - f_1\log\frac{p_1}{p_1+p_i} \ge b\}.
\end{aligned}
$$

Claim: $\lim_{N\to\infty}\frac{1}{N+1}\log\Pr(W'|r_2, z=0) = -\inf_{f\in F_0(0)}\sum_k f_k\log(f_k/p_k)$.

To show this, note that $\frac{1}{N+1}\log\frac{\alpha}{1-\alpha}$ converges to 0. Thus, for any $\varepsilon > 0$, there is $\overline{N} > 0$ such that $N > \overline{N}$ implies

$$
-\varepsilon < \frac{1}{N+1}\log\frac{\alpha}{1-\alpha} < \varepsilon
$$

and also

$$\Pr\left(\frac{n}{N+1} \in F_0(\varepsilon)\,|r_1, z=0\right) < \Pr\left(W'|r_2, z=0\right) < \Pr\left(\frac{n}{N+1} \in F_0(-\varepsilon)\,|r_1, z=0\right).$$

Therefore, for any $\varepsilon > 0$,

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(\frac{n}{N+1} \in F_0(\varepsilon)\,|r_1, z=0\right) \le \lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W'|r_2, z=0\right) \tag{A.4}$$

$$\le \lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(\frac{n}{N+1} \in F_0(-\varepsilon)\,|r_1, z=0\right). \tag{A.5}$$

Now, apply the large deviation principle to obtain

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(\frac{n}{N+1} \in F_0(\varepsilon')\,|r_1, z=0\right) = -\inf_{f\in F_0(\varepsilon')} \sum_k f_k \log\left(f_k/p_k\right)$$

for each $\varepsilon' \in \mathbb{R}$. (See the proof of Lemma A.2 for detail.) Because $F_0(\varepsilon')$ is compact-valued and continuous in ε', the Maximum Theorem implies that

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(\frac{n}{N+1} \in F_0(\varepsilon')\,|r_1, z=0\right)$$

is continuous at $\varepsilon' = 0$. Therefore, by taking $\varepsilon \downarrow 0$ in (A.4) and (A.5) I have shown the claim.

Compute

$$-\inf_{f\in F_0(0)} \sum_k f_k \log\left(f_k/p_k\right).$$

The Kuhn-Tucker conditions give the solution in the lemma at which only one inequality constraint $(f_0+f_i)\log\left(\frac{p_0}{p_0+p_i}\right) - f_1 \log\left(\frac{p_1}{p_1+p_i}\right) \ge 0$ is binding. □

Lemma A.5. *Suppose $p_1 > p_0 > 0$, $p_u > p_1 - p_0$, $p_\phi > 0$ and $q > 0$. Then,*

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W|r_2, z=0\right) = \lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W|r_2, z=1\right).$$

Proof: In Lemma A.4, $\lim_{N\to\infty} \frac{1}{N} \log \Pr\left(W|r_2, z=0\right)$ is computed. The converging speed for $z=1$ is computed similarly as follows.

Assume $z=1$. Ignoring a tie, I consider the event denoted by W' in which $n_0 < n_1 + n_i + n_u$ (more agents prefer candidate 1), $n_0 + n_u > n_1 + n_i$ (more agents vote for candidate 0 if the UIAs vote for 0) and $\mu(z=1|m_{-j},r_2)<\frac{1}{2}$ (the UIAs vote for candidate 0). By the large deviation principle, it suffices to solve

$$-\inf_{f\in F_1(0)}\sum_k f_k\log(f_k/p_k)$$

where

$$F_1(b)=\{f=(f_0,f_1,f_i,f_u,f_\phi)\in\Delta^5: f_1+f_i-f_u\le f_0\le f_1+f_i+f_u,$$
$$f_0\log\left(\frac{p_0+p_i}{p_0}\right)-(f_1+f_i)\log\left(\frac{p_1+p_i}{p_1}\right)\ge b\}.$$

The Kuhn-Tucker conditions give the following solution at which only one inequality constraint $f_0\log\left(\frac{p_0+p_i}{p_0}\right)-(f_1+f_i)\log\left(\frac{p_1+p_i}{p_1}\right)\ge 0$ is binding:

$$f_0=\frac{1-\eta}{1+C}, \tag{A.6}$$
$$f_1=\frac{p_1}{p_1+p_i}\left(\frac{\eta}{1+C}\right), \tag{A.7}$$
$$f_i=\frac{p_i}{p_1+p_i}\left(\frac{\eta}{1+C}\right), \tag{A.8}$$
$$f_u=\frac{p_u}{p_u+p_\phi}\left(\frac{C}{1+C}\right),\ \text{and} \tag{A.9}$$
$$f_\phi=\frac{p_\phi}{p_u+p_\phi}\left(\frac{C}{1+C}\right). \tag{A.10}$$

Note that η and C are defined in Lemma A.4.

Note that, for each $z=0,1$, the converging speed is expressed as

$$-\sum_k f_k\log(f_k/p_k)$$

where the minimizer f is given in Lemma A.4 for $z=0$ and (A.6)-(A.10) for $z=1$.

I will plug the two solutions into the objective function and verify the two are the same. Because (f_u,f_ϕ) coincides between $z=0$ and $z=1$, I need to show

$$\begin{aligned}
&\frac{p_0}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right)\log\left(\frac{p_0}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right)/p_0\right)+\frac{\eta}{1+C}\log\left(\frac{\eta}{1+C}/p_1\right)\\
&+\frac{p_i}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right)\log\left(\frac{p_i}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right)/p_i\right)\\
=&\frac{1-\eta}{1+C}\log\left(\frac{1-\eta}{1+C}/p_0\right)+\frac{p_1}{p_1+p_i}\left(\frac{\eta}{1+C}\right)\log\left(\frac{p_1}{p_1+p_i}\left(\frac{\eta}{1+C}\right)/p_1\right)\\
&+\frac{p_i}{p_1+p_i}\left(\frac{\eta}{1+C}\right)\log\left(\frac{p_i}{p_1+p_i}\left(\frac{\eta}{1+C}\right)/p_i\right)
\end{aligned}$$

The left-hand side is

$$(LHS) = \left(\frac{1-\eta}{1+C}\right)\log\left(\frac{1}{p_0+p_i}\left(\frac{1-\eta}{1+C}\right)\right) + \frac{\eta}{1+C}\log\left(\frac{\eta}{1+C}/p_1\right),$$

and the right-hand side is

$$(RHS) = \frac{1-\eta}{1+C}\log\left(\frac{1-\eta}{1+C}/p_0\right) + \left(\frac{\eta}{1+C}\right)\log\left(\frac{1}{p_1+p_i}\left(\frac{\eta}{1+C}\right)\right).$$

Thus,

$$\begin{aligned}(LHS)-(RHS) &= \frac{1}{1+C}\left((1-\eta)\log\left(\frac{p_0}{p_0+p_i}\right) - \eta\log\left(\frac{p_1}{p_1+p_i}\right)\right)\\ &= \frac{1}{1+C}\left[\frac{\log\left(\frac{p_1}{p_1+p_i}\right)}{\log\left(\frac{p_0}{p_0+p_i}\right)+\log\left(\frac{p_1}{p_1+p_i}\right)}\log\left(\frac{p_0}{p_0+p_i}\right) - \right.\\ &\left.\frac{\log\left(\frac{p_0}{p_0+p_i}\right)}{\log\left(\frac{p_0}{p_0+p_i}\right)+\log\left(\frac{p_1}{p_1+p_i}\right)}\log\left(\frac{p_1}{p_1+p_i}\right)\right] = 0.\end{aligned}$$

□

Lemma A.6. *Suppose $p_1 > p_0 > 0$, $p_u > p_1 - p_0$, $p_\phi > 0$ and $q > 0$. Then,*

$$\lim_{N\to\infty}\frac{1}{N+1}\log\Pr(W|r_2, z=0) = -\sum_{k=0,1,i,u0,u\phi,\phi} g'_k \log(g'_k/p_k),$$

where $g' = (g'_0, g'_1, g'_i, g'_{u0}, g'_{u\phi}, g'_\phi)$, $g'_k = f_k$ for $k = 0,1,i,\phi$, $g'_{u0} = \frac{p_{u0}}{p_u} f_u$, $g'_{u\phi} = \frac{p_{u\phi}}{p_u} f_u$ and $(f_0, f_1, f_i, f_u, f_\phi)$ is defined in Lemma A.4.

Proof: The lemma holds because of Lemma A.4 and the fact

$$f_u\log(f_u/p_u) = \frac{p_{u0}}{p_u} f_u \log\left(\frac{p_{u0}}{p_u} f_u/p_{u0}\right) + \frac{p_{u\phi}}{p_u} f_u\log\left(\frac{p_{u\phi}}{p_u} f_u/p_{u\phi}\right).$$

□

Define $a_0 = \log(1+r_0)$ and $a_1 = \log(1+r_1)$ for positive r_0 and r_1. In addition, let

$$\Phi = a_0 \log\frac{(a_0-a_1)\,r_0\,(1+r_1)}{(r_0-r_1)\,a_0} + a_1\log\frac{(a_0-a_1)\,r_1}{(r_0-r_1)\,a_1}.$$

Lemmas A.7 through A.9 are needed for Lemma A.10 that shows $\Phi > 0$ for $r_0 > r_1 > 0$. Then, (A.11), the key step to prove Proposition 4.4, relies on Lemma A.10.

Lemma A.7. *If* $r_0 > r_1 > 0$,

$$\frac{1}{(1+r_0)(1+r_1)(a_0-a_1)^2} - \frac{1}{(r_0-r_1)^2} > 0.$$

Proof: Let $\Psi \equiv a_0 - a_1 - \frac{r_0-r_1}{\sqrt{1+r_0}\sqrt{1+r_1}}$ and it suffices to show $\Psi < 0$. If $r_0 = r_1$, $\Psi = 0$. I need to show that $\frac{d}{dr_0}\Psi < 0$. Compute

$$\frac{d}{dr_0}\Psi = \frac{1}{1+r_0}\left(1 - \frac{1}{2}\frac{1}{\sqrt{1+r_0}\sqrt{1+r_1}}(2+r_0+r_1)\right) < 0.$$

The inequality follows because

$$2 + r_0 + r_1 - 2\sqrt{(1+r_1)(1+r_0)} = \left(\sqrt{1+r_0} - \sqrt{1+r_1}\right)^2 > 0.$$

□

Lemma A.8. *If* $r_0 > r_1 > 0$, $\frac{d}{dr_0}\Phi > 0$.

Proof: Compute

$$\frac{d}{dr_0}\Phi = \frac{1}{1+r_0}\log\frac{(a_0-a_1)r_0(1+r_1)}{(r_0-r_1)a_0} + \frac{a_0+a_1}{a_0-a_1}\frac{1}{1+r_0} - \frac{a_0+a_1}{r_0-r_1} + \frac{a_0}{r_0} - \frac{1}{1+r_0}.$$

Then,

$$\lim_{r_1\downarrow 0}\frac{d}{dr_0}\Phi = 0.$$

Therefore, I only need to show $\frac{d^2}{dr_1 dr_0}\Phi > 0$. Observe

$$\frac{d^2}{dr_1 dr_0}\Phi = (a_0+a_1)\left(\frac{1}{(1+r_0)(1+r_1)(a_0-a_1)^2} - \frac{1}{(r_0-r_1)^2}\right) > 0$$

by Lemma A.7. □

Lemma A.9. *For* $r_1 > 0$, $2\log\frac{r_1}{a_1} - a_1 > 0$.

Proof: By L'Hôpital's rule,

$$\lim_{r_1\downarrow 0}\frac{r_1}{a_1} = \lim_{r_1\downarrow 0}\frac{1}{1/(1+r_1)} = 1.$$

Thus, $\lim_{r_1\downarrow 0} 2\log\frac{r_1}{a_1} - a_1 = 0$. Moreover,

$$\begin{aligned}\frac{d}{dr_1}\left(2\log\frac{r_1}{a_1} - a_1\right) &= 2\frac{1}{r_1} - 2\frac{1}{a_1}\frac{1}{1+r_1} - \frac{1}{1+r_1} = \frac{2+r_1}{r_1(1+r_1)} - 2\frac{1}{a_1}\frac{1}{1+r_1} \\ &= \frac{2+r_1}{r_1(1+r_1)a_1}\left(a_1 - \frac{2r_1}{2+r_1}\right) > 0,\end{aligned}$$

where the last inequality is implied by the inequality $\log(1+r) > \frac{2r}{2+r}$ for $r > 0$. □

Lemma A.10. *If $r_0 > r_1 > 0$, $\Phi > 0$.*

Proof: First, L'Hôpital's rule implies

$$\lim_{r_0 \downarrow r_1} \frac{a_0 - a_1}{r_0 - r_1} = \lim_{r_0 \downarrow r_1} \frac{1/(1+r_0)}{1} = \frac{1}{1+r_1},$$

and thus

$$\begin{aligned}\lim_{r_0 \downarrow r_1} \Phi &= a_1 \log \frac{r_1}{a_1} + a_1 \log \frac{r_1}{(1+r_1)\, a_1} \\ &= a_1 \log \frac{r_1}{a_1} + a_1 \log \frac{r_1}{a_1} - a_1 \log (1+r_1) \\ &= 2a_1 \log \frac{r_1}{a_1} - a_1^2 = a_1 \left(2 \log \frac{r_1}{a_1} - a_1 \right) > 0\end{aligned}$$

by Lemma A.9. In addition, I have shown $\frac{d}{dr_0}\Phi > 0$ in Lemma A.8. Therefore, $\Phi > 0$. $\square$

Lemma A.11. *Suppose $p_1 > p_0 > 0$ and $p_i > 0$. For g' and G_0' defined Lemmas A.3 and A.6, g' lies in the interior of G_0'.*

Proof: Set $r_0 = \frac{p_i}{p_0}$ and $r_1 = \frac{p_i}{p_1}$. Then, $r_0 > r_1 > 0$ and Lemma A.10 implies

$$a_0 \log \frac{(a_0 - a_1)\, r_0 (1 + r_1)}{(r_0 - r_1)\, a_0} + a_1 \log \frac{(a_0 - a_1)\, r_1}{(r_0 - r_1)\, a_1} > 0.$$

Recalling that $\eta = \frac{a_0}{a_0 + a_1}$, I can rewrite this inequality as

$$(a_0 - a_1)\, p_1^{\eta} (p_0 + p_i)^{1-\eta} > (p_1 - p_0)\, a_0^{\eta} a_1^{1-\eta}.$$

Then,

$$\begin{aligned}-g_1' + g_0' + g_i' + g_{u0}' &= -\frac{\eta}{1+C} + \frac{1-\eta}{1+C} + \frac{(p_1 - p_0)}{(p_u + p_\phi)} \frac{C}{1+C} \\ &= \frac{1}{1+C} \left(1 - 2\eta + \frac{(p_1 - p_0)}{(p_u + p_\phi)} C \right) \\ &= \frac{1}{1+C} \left(1 - 2\eta + (p_1 - p_0) \left(\frac{\eta}{p_1} \right)^{\eta} \left(\frac{1-\eta}{p_0 + p_i} \right)^{(1-\eta)} \right) \\ &= \frac{1}{1+C} \left(\frac{a_1 - a_0}{a_0 + a_1} + (p_1 - p_0) \left(\frac{\eta}{p_1} \right)^{\eta} \left(\frac{1-\eta}{p_0 + p_i} \right)^{(1-\eta)} \right) \\ &= \frac{1}{1+C} \left(\frac{-(a_0 - a_1)\, p_1^{\eta} (p_0 + p_i)^{1-\eta} + (p_1 - p_0)\, a_0^{\eta} a_1^{1-\eta}}{p_1^{\eta} (p_0 + p_i)^{1-\eta} (a_0 + a_1)} \right) < 0,\end{aligned}$$

which implies g' lies in the interior of G'_0. $\square$

Now I prove Proposition 4.4. I need to prove that

$$\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W|r_2, z=x\right) < \lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W|r_1, z=x\right) \tag{A.11}$$

for each $x = 0, 1$. Lemmas A.3(ii) and A.5 imply that it is enough to prove (A.11) for $z = 0$ only. Note that

$$\begin{aligned}
&\lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W|r_1, z=0\right) \\
&= -\min_{g\in G'_0} \sum_k g_k \log\left(g_k/p_k\right) \\
&> -\sum_{k=0,1,i,u0,u\phi,\phi} g'_k \log\left(g'_k/p_k\right) \\
&= \lim_{N\to\infty} \frac{1}{N+1} \log \Pr\left(W|r_2, z=0\right)
\end{aligned} \tag{A.12}$$

where Lemmas A.3(i) and A.6 define G'_0 and g', respectively, and prove the two equalities. Lemma A.11 shows g' does not satisfy $g_1 = g_0 + g_i + g_{u0}$ (the constraint in G'_0). Thus, g' cannot be a minimizer of (A.12) by Lemma A.3(i). This implies the strict inequality. $\square$

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