Selective Incentives and Intra-Group Heterogeneity in Collective Contests [1]

by

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Abstract

A group taking part in a contest has to confront the collective-action problem among its members, and devices of selective incentives are possible means of resolution. We argue that heterogeneous prize-valuations in a competing group normally prevent effective use of such selective incentives. To substantiate this claim, we adopt cost sharing as a means of incentivizing the individual group members. We confirm that homogeneous prize valuations within a group result in a cost-sharing rule inducing the first-best individual contributions. As long as the cost-sharing rule is dependent only on the members' contributions, however, such a first-best rule does not exist for a group with intra-group heterogeneity. Our main result clarifies how unequal prize valuations affect the cost-sharing rule and, in particular, the degree of cost sharing. If the relative rate of change of the marginal effort costs is decreasing, it is reduced by intra-group heterogeneity. If the rate is increasing, the cost is fully shared, but it cannot induce the first-best contributions for the group.

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1. Introduction

In a collective contest the contestants for a prize are groups. Applications of such contests include confrontations between labor unions and the employers, ethnic or religious conflicts, military conflict between countries or allies of countries, promotional competitions by firms with marketing activities, a championship by sports teams, competition among academic institutes on quality-based recognition or on financial support, and so on[4]. This paper examines how heterogeneity of individuals in a group affects the incentive device in such a contest.

Usually a group contains various individuals situated at different positions, politically, economically and sometimes ethnically or culturally. Such heterogeneity could prevent them from reaching a consensus on the value of the contested prize, and they naturally have different prize valuations. Their contributions to the contest are diversified then, which could be inefficient for the group as a whole[5]. Most of the existing literature on group contests with such intra-group heterogeneity, however, ignores the possibility that a competing group introduces an incentive device. When individuals win or lose the prize as a group, they are usually tempted to be free-riders while considering contribution to the teamwork to enhance the group winning probability. This tendency results in a collective-action problem, as argued by Olson (1965, 1982)[6]. He argues that the problem can be amended by "selective incentives," - incentives applied selectively to individuals depending on their actions[7]. In the literature on collective contests, such an incentive device has been incorporated into the model typically as a prize-sharing rule of a group, which prescribes how much of the won prize is distributed according to the contributions by the group members[8]. But the existing models with such incentive devices assume that the individuals in a group are homogeneous; they have the same valuation of the prize and the form of their effort cost function is also the same.

Competing groups in a real world face at least two sources of inefficiency; free riding and individual heterogeneity. The selective incentive devices chosen by them must cope with the both sources of inefficiency. To make the effect of intra-group heterogeneity

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[4] For more examples of contests in general, see Konrad (2009). On the basic theory of contests, see Hillman and Riley (1989) and Cornes and Hartley (2005).

[5] Intra-group heterogeneity of competing groups is not an old topic in the literature of collective contests. Baik (2008) examines a model of collective contests for group-specific public-good prize, assuming linear cost functions, i.e., constant marginal costs. Esteban and Ray (2011) study a model of ethnic conflict with two parameters of intra-group heterogeneity: ethnic radicalism and income. Epstein and Mealem (2009) and Ryvkin (2011) examine how the composition of individuals in competing groups affects the equilibrium effort levels. Nitzan and Ueda (2013) clarify how the form of effort cost functions determines the relation between intra-group heterogeneity and the equilibrium group effort.

[6] As similar concepts to free-riding, we could count shirking and social loafing (Kidwell and Bennett. 1993). The former is a term used in the context of the economics of organization, and the latter is mainly used in studies of social psychology. All three concepts concern individuals withholding effort in a group. Such overlapping of concepts in the different areas stresses the substantial role played by the free riding problem in determining the performance of a group.

[7] Olson's several conjectures on the collective action problems are neatly arranged and evaluated by Sandler (1992). For two recent surveys on the development of research on collective-action problems see Pecorino (2015) and Sandler (2015).

[8] The idea of a prize-sharing rule is introduced to the research on collective contests by Nitzan (1991). Baik (1994), Lee (1995), and Ueda (2002) develop models in which the prize sharing rule is endogenously determined by each of the competing groups. Nitzan and Ueda (2011) consider a collective contest in which the prize-sharing rule in each group is endogenously determined to maximize the utilitarian group welfare (i.e. the sum of the expected utility of the group members), and derive the result that every group can realize the collectively optimal contributions by its members in a consistent way with the group's objective.

explicit is therefore important to understand the properties of the appropriate selective incentive devices to be adopted in collective contests. For this purpose, we use a modified model of Nitzan and Ueda (2011) containing two new important features.

The first is, of course, the existence of intra-group heterogeneity. Unfortunately, however, it is not easy to analyze a model of prize-sharing with intra-group heterogeneity and, in particular, to characterize the equilibrium prize-sharing rules. It is especially so when the effort cost of an individual is non-linear, and such non-linearity is essential to get general insights on the collective-contest problem.[9] Therefore, a second new feature is introduced; instead of prize-sharing rules, our model assumes that the competing groups use *cost-sharing* rules as a means of selective incentives. It has been recently pointed out by Vázquez (2014) that commitment to a transfer rule of the costs among individual group members can work as a substitute for a prize-sharing rule. Actually, once we notice that cost-sharing makes the resources of the individuals in a group a common pool resource, it is not surprising that such a device enhances their activity levels. In general, transfer schemes within a group depending on the sacrifice to enhance the common interest can work as selective incentives. An individual who contributes more can shift more cost to the others relative to the cost imposed on him/her by the others, and as a result, get a net transfer. Those schemes can be considered as examples of cost-sharing[10].

With the device of cost-sharing, it is possible to derive similar results on the effectiveness of selective incentives as in Nitzan and Ueda (2011); if the individuals in a group are homogeneous, full sharing of the costs among the group members is the first-best cost-sharing rule maximizing the utilitarian group-welfare and the equilibrium rule chosen by each group coincides with this rule[11]. This case can therefore be used as a benchmark to examine the effect of intra-group heterogeneity. Furthermore, we will see that the model with cost-sharing has an easily tractable equilibrium, even when allowing intra-group heterogeneity and non-linear effort costs[12]. Assuming cost-sharing as the incentive devices

[9] Several impressive results derived from contest models with linear effort costs are not necessarily preserved under non-linear cost functions. For example, Esteban and Ray (2001) reveal that collective contests for a pure private-good prize with linear effort cost functions belong to a special case where the "group size paradox" is always obtained, i.e., a smaller group attains a higher win probability. They prove that the paradox is overturned if the elasticity of marginal effort costs is large. Another puzzling possibility is that an individual with a higher prize valuation can get a lower expected payoff in a group (it could be called a strong version of "the "exploitation of the great by the small"). This is a normal case in contests with linear effort costs, unless the largest value of the valuations is very prominent. The reason is, as shown by Baik (2008), that in this case only individuals with the largest valuation of the prize in a group put effort, and all the other group members become pure free riders (see also Lee (2012) for interesting related results). But Nitzan and Ueda (2013) point out that such exploitation is impossible if the elasticity of marginal effort costs is large. We will present another example of the peculiarity of the model with linear cost functions in Subsection 5.2. These examples imply that we should be careful regarding the robustness of the results obtained under linear costs.

[10] For example, more active members of a labor union confronting the employers are sometimes entitled to spend a higher amount of the fund collected as the union dues relative to less active members. In ethnic conflicts to establish some symbolic constructions or privileges, money and alternative resources are transferred to those directly and/or intensely engaged in the competition from their fellow associates. And competitions among universities or departments within the universities on financial support are usually determined on the basis of the research achievements of the faculty members (publications, citations, etc.), which to some extent certainly depend on their exerted efforts. In those academic institutes, teaching obligations are often inversely related to a faculty's achievements in research. These three practices can be conceived as actual transfer schemes (depending on the members' sacrifices) applied by groups competing on some kind of a prize.

[11] The formal definition of the first-best cost-sharing rule is given in subsection 3.2. It should be noted that higher selective incentives are not necessarily better for a group. When the members are rewarded for their effort, each member's effort has a negative externality for the others because their shares are cut. The result might be an excessive group effort. See Sen (1966).

[12] When the contested prize is a group-specific public good, the device of prize sharing cannot be applied, at

for competing groups, we have a handy and workable model for investigating the relationship between intra-group heterogeneity of prize valuations and the effectiveness of selective incentives.

We find that intra-group heterogeneity of the prize valuations requires the first-best cost-sharing rule to impose different degrees of cost sharing on the group members. That is, the group needs to set a discriminatory (personally varied) cost-sharing rule to induce such contributions. This finding points to a serious limitation intra-group heterogeneity imposes on a competing group. A device of selective incentives normally operates uniformly and impersonally, i.e., it does not discriminate individuals by their names. It specifies a reward that *hinges only on the individual's behavior*. This means that usually a group would not be able to implement the first-best incentive device. If a group has to set a uniform cost-sharing rule to individuals with heterogeneous valuations of the prize, how does intra-group heterogeneity affect the extent of cost-sharing or selective incentives? We identify a condition that *determines* to which direction unequal valuations of the prize within a group shift the degree of cost-sharing. If the relative rate of change of the marginal effort costs is decreasing, the degree of cost-sharing is reduced by intra-group heterogeneity. If the rate is increasing, the cost is fully shared, but it cannot induce the first-best contributions for the group. As argued above, the result is due to the inefficient realization of the voluntary contributions induces by heterogeneous valuations of the prize.

In the next section, our basic model is introduced. It treats the case where each competing group applies a uniform cost-sharing rule to its members. Section 3 deals with first-best cost-sharing rules. We start with the analysis of a collective group contest where the individuals in a group act cooperatively for enhancing their group's interest (we will call it a *contest by fully regulated groups*). Such a contest is a convenient tool to treat the cases in which the first-best selective incentives are available. Section 4 is the main part of the paper; it examines the relation between the form of equilibrium cost-sharing rules uniformly applied to all group members and intra-group heterogeneity in prize valuations. Section 5 focuses on the case of constant elasticity of marginal effort costs, a convenient special case to make our main story transparent. The conclusions are presented in Section 6. All the proofs appear in Section 7.

2. The Model

2.1 Players, strategies and payoffs

We consider a model of contest with m competing groups. The number of individuals participating in the contest as members of group i is N_i. They are assumed to be risk-neutral and individually and simultaneously deciding how much to contribute to enhance the winning of their group. The individual contributions are aggregated in every group, and the group probability of winning the prize is determined depending on those aggregated group efforts.

Assumption 1 (Contest success function): The win probability of group i is given by $\frac{A_i}{A}$, where A_i is the effort of group i, and $A = \sum_{j=1}^{m} A_j$ is the *total amount of effort* by all

least in a straightforward way. It also does not work well when the prize is a group-specific commons openly accessible to all members of the winning group, which is the case studied in Nitzan and Ueda (2009). Even in such cases, a group could commit to a *cost-sharing* rule that imposes partial sharing of the cost of the members' sacrificed efforts.

competing groups. The *group effort* of i, A_i, is given by $A_i = \sum_{k=1}^{N_i} a_{ik}$,[13] where $a_{ik} \geq 0$ denotes the effort made by member k of group i.

Preceding the contest, each group can set a cost-sharing rule applied to its members as the selective incentive device. For the sake of simple exposition, we assume the existence of a benevolent group leader in each group, who does not directly participate in the contest, but determines and implements the cost sharing rule to maximize the *utilitarian group welfare*, that is, the *sum* of the expected utility of the group members.[14] So the model has the two kinds of players; the *group leaders* who set the cost-sharing rules in the pre-stage of the contest, and the *individuals* belonging to the competing groups who determine their effort levels to win the prize in the contest stage.

The distinguishing feature of our model is the cost sharing among the heterogeneous individuals in a group. The heterogeneity has the form of different valuations of the prize. Let $v_{ik} > 0$ be the valuation of the prize by the kth individual belonging to the ith group[15]. Without loss of generality, we can set $v_{i1} \leq \cdots \leq v_{iN_i}$. We will use the notation $V_i = \sum_{k=1}^{N_i} v_{ik}$. The distribution of the members' stakes in the contest can be represented by the *stake vector* of group i, $\mathbf{v}_i = (v_{i1}, \ldots, v_{iN_i})$.

The individuals in the same group are assumed to have the same form of the effort cost function. The effort cost borne by individual k in group i is denoted by $c_i(a_{ik})$, if he puts the effort $a_{ik} \geq 0$.

Assumption 2 (Effort cost function): The effort cost function of individuals in group i, c_i, is a thrice differentiable function with $c_i(0) = 0$, $c_i'(a) > 0$ and $c_i''(a) > 0$ for all $a > 0$. Also, $\lim_{a \to 0} c_i'(a) = 0$.[16]

Let us now describe in more detail the device of selective incentives in our model. It takes the following form of uniform cost-sharing in each group. It is assumed that part of the cost of the members' contributions is shared within the group. Formally, group i specifies the value $0 \leq \delta_i \leq 1$, the ratio of the effort cost of *every* member compensated by making equal payback transfers that sum up to δ_i of the total exerted efforts. In other words, the cost of individual k belonging to group i has the form

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[13] See Kolmar and Rommeswinkel (2013) for contests with more general ways to aggregate efforts by individuals in a group.

[14] Such an assumption regarding the decision on the cost-sharing rule could be justified if keeping the position is the intrinsic objective of the leaders, and the nomination requires the consensus of the group members who share the utilitarian value judgement. Another possible assumption is that the rule is directly agreed by the individuals belonging to a group.

[15] Although the valuation of the prize of a specific individual is private information, the stake vectors are common knowledge in our model. We may interpret it as the valuation of a mixed private-public-good prize. Then v_{ik} is a function of two variables, the kth individual's share of the private good prize and the public good prize.

[16] The assumption $\lim_{a \to 0} c_i'(a) = 0$ excludes the possibility of non-contributors. As Nitzan and Ueda (2013) argue, such possibilities have important implications on the relation between the group's performance and intra-group heterogeneity in terms of the stakes. The main concern of this paper is, however, the relation between the equilibrium cost-sharing rule and intra-group heterogeneity. Hence, ignoring the possibility would be justified by the transparency of the analysis.

$$\left(1-\delta_i\right)\cdot c_i\left(a_{ik}\right)+\delta_i\frac{\sum_{p=1}^{N_i}c_i\left(a_{ip}\right)}{N_i}. \tag{1}$$

We may call a value of δ_i the *degree of cost-sharing*. The higher it is in a group, the larger the transfer to an individual as the return of the contribution.[17] When $\delta_i=0$, no part of the cost of individuals in a group is shared, and there exists no selective incentives. If $0<\delta_i<1$, the enforced cost-sharing is *partial*. When $\delta_i=1$, the cost is *fully shared*. We assume that it is the highest degree of cost-sharing[18]. Since each individual is risk-neutral, the utility of member *k* of group *i* is given by

$$EU_{ik}=\frac{A_i}{A}v_{ik}-\left\{\left(1-\delta_i\right)\cdot c_i\left(a_{ik}\right)+\delta_i\frac{\sum_{p=1}^{N_i}c_i\left(a_{ip}\right)}{N_i}\right\}. \tag{2}$$

The member chooses the contribution $a_{ik}\geq 0$ to maximize his own utility (2) in the contest, given the cost sharing rule in the group. The leader of the group *i* determines the value of $0\leq\delta_i\leq 1$ to maximize $\sum_{k=1}^{N_i}EU_{ik}$ prior to the contest, anticipating such the contributions by the group members.

2.2 Information structure and strategies

In this model, we assume that the sharing rule in a group is *unobservable* by those belonging to other groups. If an incentive scheme of a group is observable, it works as a strategic tool to threaten the competing groups, because the individuals belonging to them get the information how strong are the incentives given in that group. Reality of this secondary effect of the incentive scheme is, however, rather questionable; such a scheme results in redistribution within a group, and is applied only with respect to insiders. Those inside rules would be changeable by notification only to the group members, and the changes could be made secretly, that is, without informing other groups. Even if a group sharing rule is openly announced to outsiders, they would hardly believe that the announced rule is the final one the group has really committed itself to. Hence, it is uncertain whether the redistribution rules applied to the insiders can actually work as strategic tools for the opposing groups.[19] Even for cases where such observable commitment is possible, checking what happens if the sharing rules are unobservable is meaningful to isolate the pure strategic effects they have.

With unobservable cost-sharing rules, the timing and the information structure of the model is described as follows. Preceding to the contest, every group leader simultaneously determines the degree of cost-sharing in her own group, and announces it to the members.

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[17] Notice that the function (1) can be written as $c_i\left(a_{ik}\right)-\delta_i\frac{\sum_{p\neq k}\left(c_i\left(a_{ik}\right)-c_i\left(a_{ip}\right)\right)}{N_i}$, which implies that an individual who makes larger contributions relative to others in the same group gets a net transfer. The amounts get larger as the value of δ_i rises. This is how cost-sharing rules work as a device creating selective incentives.

[18] One may further consider the case of *over cost-sharing*, i.e. $\delta_i>1$. In such a case, the amount exceeding the real costs is redistributed in the group. We exclude this possibility for two reasons: first, we are not aware tof real examples of over cost-sharing. Second, it is ambiguous how far over cost-sharing can be advanced. The marginal cost of the individuals could be even less than or equal to zero by setting $\delta_i\geq\frac{N_i}{N_i-1}$, which makes the whole model intractable.

[19] This problem is firstly pointed out by Katz (1991). Following this line, Baik and Lee (2007) and Nitzan and Ueda (2011) consider collective contests with unobservable prize-sharing rules.

The strategy of the leader of group i is $0 \le \delta_i \le 1$. The individuals in each group enter to the contest *without knowing the other groups' cost-sharing rules*, and determine their contributions simultaneously and non-cooperatively. Since their information is incomplete on the sharing rules of the other groups, this contest stage is *not* a proper subgame separable from the preceding stage played by the leaders. At the beginning of the contest, an individual contributor of a group is only told the rule of his own group, and finds himself in the information set containing the nodes at which the other groups have chosen any combination of the cost-sharing rules. Member k of group i can make the strategy be contingent only on δ_i. We denote it by $a_{ik}(\delta_i) \ge 0$.

2.3 Reasonable beliefs

We apply the perfect Bayesian equilibrium concept to select the unique equilibrium of the game. To simplify the arguments on the reasonable beliefs of the players, let us assume that each player can use only pure strategies in equilibrium, and omit the possibility of randomization in the information sets. Since the decisions on the cost-sharing rules by the leaders are made simultaneously at the beginning of the game, their beliefs are trivial. Consider the beliefs of an individual who makes effort for the contest. They are at most conditional on the degree of cost-sharing in his own group. The belief of the kth individual belonging to group i is denoted by $\mu_{ik}(\delta_i)$, where δ_i is the degree of cost-sharing told by his group leader. It is a probability distribution defined over $[0,1]^{m-1}$, the space of possible configurations of the cost-sharing rules in the other groups $\delta_{-i} = (\delta_1, \cdots, \delta_{i-1}, \delta_{i+1}, \cdots, \delta_m)$.

Suppose that $\left(\delta_1^*, \cdots, \delta_m^*\right)$ is the equilibrium profile of cost-sharing rules chosen by the group leaders. Let us examine what property the beliefs of the individuals should have. At the information set lying on the equilibrium path, the belief of individual k in group i satisfies $\mu_{ik}\left(\delta_{-i}^* \,\middle|\, \delta_i^*\right) = 1$, by the requirement of consistency; being told the equilibrium sharing rule of his group, he calculates the expected utility on condition that the other groups have adopted the equilibrium sharing rules. We could also restrict the beliefs in the information sets outside of the equilibrium path, appealing to the "no-signaling-what-you-don't-know" condition, proposed by Fudenberg and Tirole (1991) as a reasonable restriction to the equilibrium beliefs; any player's deviation should not be treated as containing information about things the player does not know. In the context of our model, it means requiring that a deviation by the leader of a group does not change the beliefs of the group members on the sharing rules in the other groups. At an un-reached information set, the associated individual should keep the same belief as that held if the equilibrium group sharing rule is announced. The belief of the kth individual in group i must satisfy the condition $\mu_{ik}\left(\delta_{-i}^* \,|\, \delta_i\right) = 1$, for all $\delta_i \ne \delta_i^*$. Then we can characterize the equilibrium contributions of individuals in a competing group by the following lemmas.

Lemma 1. *The equilibrium contribution by individual k of group i who is aware of the cost-sharing rule δ_i (i.e. at the information set indexed by δ_i) is described by a strictly increasing differentiable function $a_{ik}^*(\delta_i)$ defined by the following equations*:

$$\frac{\sum_{j \ne i} A_j^*(\delta_j^*)}{\left\{\sum_{j \ne i} A_j^*(\delta_j^*) + A_i^*(\delta_i)\right\}^2} v_{ik} - \left\{(1 - \delta_i) + \frac{\delta_i}{N_i}\right\} \cdot c_i'\left(a_{ik}^*(\delta_i)\right) = 0, \quad k = 1, \cdots, N_i, \tag{3}$$

where $A_i^*(\delta_i) = \sum_{k=1}^{N_i} a_{ik}^*(\delta_i)$, *given the other groups' equilibrium cost-sharing rules*

$\delta_j^*, \quad j \neq i$.

2.4 Distorted group cost

Lemma 1 shows that there exists a one-to-one relation between the degree of cost-sharing and the equilibrium contributions by individuals in a group, given the efforts of the other groups, $A_j^*(\delta_j^*), \quad j \neq i$. Then the leader of a competing group can control the level of group effort via the choice of the degree of cost-sharing.[20] This observation suggests that our model of collective contests can be reduced to a simpler model of a contest played by group leaders. To go along this line of analysis, the following lemma is useful;

Lemma 2. (a) *For each level of efforts by the other groups* $\sum_{j \neq i} A_j$ *, group i can attain the aggregate group effort* A_i *if and only if it belongs to the closed interval* $\left[A_i^L\left(\sum_{j\neq i} A_j\right), \quad A_i^H\left(\sum_{j\neq i} A_j\right)\right]$, *where* $A_i^L\left(\sum_{j\neq i} A_j\right)$ *and* $A_i^H\left(\sum_{j\neq i} A_j\right)$ *are uniquely given by the equations*

$$\sum_{k=1}^{N_i} (c_i')^{-1}\left[\frac{\sum_{j\neq i} A_j}{\left(\sum_{j\neq i} A_j + A_i^L\left(\sum_{j\neq i} A_j\right)\right)^2} v_{ik}\right] = A_i^L\left(\sum_{j\neq i} A_j\right), \textit{ and}$$

$$\sum_{k=1}^{N_i} (c_i')^{-1}\left[N_i \cdot \frac{\sum_{j\neq i} A_j}{\left(\sum_{j\neq i} A_j + A_i^H\left(\sum_{j\neq i} A_j\right)\right)^2} v_{ik}\right] = A_i^H\left(\sum_{j\neq i} A_j\right).$$

(b) *For each level of group effort* $A_i > 0$ *mentioned in (a), the allotment of the contribution in the group is given by the functions* $a_{ik}(A_i;\mathbf{v}_i)$ *'s defined by the following equations*:

$$\sum_{k=1}^{N_i} a_{ik}(A_i;\mathbf{v}_i) = A_i \quad \text{and} \quad \frac{1}{v_{ik}} \cdot c_i'\left(a_{ik}(A_i;\mathbf{v}_i)\right) = \frac{1}{v_{i1}} \cdot c_i'\left(a_{i1}(A_i;\mathbf{v}_i)\right) \quad \text{for all} \quad k = 1,\cdots,N_i. \tag{4}$$

Lemma 2 (a) specifies the range of group effort that the leader can attain by controlling the degree of cost-sharing, and 2 (b) specifies how each level of group effort is bore within a group.[21] It is noticeable that the allotment of group effort is determined independently of the efforts by the other groups. We can then define the function specifying how much cost is sacrificed in the group if the leader wants to induce a given level of group effort;

$$E_i(A_i;\mathbf{v}_i) = \sum_{k=1}^{N_i} c_i\left(a_{ik}(A_i;\mathbf{v}_i)\right).$$

It does *not necessarily* coincide with the minimum sum of the group members' effort cost to induce a given aggregate effort A_i, because they voluntarily and individually choose their contributions. So we refer to $E_i(A_i)$ as the *distorted group cost function* of *i*. The next lemma on this function is useful.

[20] Notice that the individuals in a group retain the same belief on the degrees of cost-sharing (and the group efforts) in the other groups, when being told that different cost-sharing rules are applied in their own group.

[21] The representations of the contribution by an individual in the two lemmas can be related by the equation: $a_{ik}\left(A_i^*(\delta_i);\mathbf{v}_i\right) = a_{ik}^*(\delta_i)$.

Lemma 3. *The equation*

$$\frac{\partial}{\partial A_i} E_i(A_i;\mathbf{v}_i) = \sum_{k=1}^{N_i} c_i'(a_{ik}(A_i;\mathbf{v}_i)) \cdot \frac{\dfrac{v_{ik}}{c_i''(a_{ik}(A_i;\mathbf{v}_i))}}{\sum_{p=1}^{N_i} \dfrac{v_{ip}}{c_i''(a_{ip}(A_i;\mathbf{v}_i))}} \tag{5}$$

holds for $A_i > 0$. *Furthermore,* $\lim_{A_i \to 0} \frac{\partial}{\partial A_i} E_i(A_i;\mathbf{v}_i) = 0$

With the distorted group cost function, the utilitarian group welfare of group *i*, which is the payoff of the group leader, can be represented as a function of group efforts A_j, $j = 1,\cdots,m$, as follows;

$$\frac{A_i}{\sum_{j=1}^{m} A_j} V_i - E_i(A_i;\mathbf{v}_i). \tag{6}$$

As long as keeping one reservation in mind, therefore, we can reduce the original model into a model of contest by the group leaders, in which the strategies are group efforts and the payoff functions are given by (6). And the important reservation is that the strategy set of each player is variable with the strategies chosen by the other players. A group leader cannot choose the level of group effort independently of the competing group leaders.

Formally, the equilibrium of this reduced contest is a configuration of group efforts A_j^*, $j = 1,\cdots,m$ such that A_i^* is a solution of the maximization problem:

$$\max_{A_i \geq 0} \frac{A_i}{\sum_{j\neq i} A_j^* + A_i} V_i - E_i(A_i;\mathbf{v}_i) \text{ subject to } A_i^L\left(\sum_{j\neq i} A_j^*\right) \leq A_i \leq A_i^H\left(\sum_{j\neq i} A_j^*\right), \tag{7}$$

for all $i = 1,\cdots,m$. The range of possible group effort in (7) corresponds to that of the possible degree of cost-sharing, $0 \leq \delta_i \leq 1$. Hence we can find a profile of the sharing rules $\delta_1^*,\cdots,\delta_m^*$ described by the equations $\sum_{k=1}^{N_i} a_{ik}^*(\delta_i^*) = A_i^*$, $i = 1,\cdots,m$, using the functions defined in Lemma 1. It is a profile of cost-sharing rules in the pure-strategy perfect Bayesian equilibrium of the original model. Each A_i^* is the group effort in equilibrium. Conversely, if A_j^*, $j = 1,\cdots,m$ are group efforts in equilibrium of the original model of collective contests, then they constitute equilibrium of the reduced contest by group leaders. If equilibrium of this reduced model of contest uniquely exists, therefore, the same is true for pure-strategy perfect Bayesian equilibrium of our collective contest model with cost-sharing.

To complete the proof on existence and uniqueness of equilibrium with this approach, we introduce a regularity condition on the distorted group cost function.

Regularity Condition: The distorted group cost $E_i(A_i;\mathbf{v}_i)$ is convex in A_i.

Since this function is a reduced form containing the best-responses of the individuals, one may feel uneasy to directly assume its property. But we can find several cases where the convexity actually holds. It applies to the case of *intra-group homogeneity*, i.e. all individuals in the same group have the same valuation of the prize; $v_{ik} = v_{i1}$ for all $k = 1,\cdots,N_i$. When a group exhibits intra-group heterogeneity, one sufficient condition for convexity is that the relative rate of change of the marginal effort cost $\frac{c_i''(a)}{c_i'(a)}$ is decreasing and the inverse $\frac{c_i'(a)}{c_i''(a)}$ is convex. The convenient class of cost functions with *constant elasticity of marginal effort* *costs*, in which $c_i(a)=\dfrac{K_i}{1+\alpha_i}a^{1+\alpha_i}$ for all $i=1,\cdots,m$, where $\alpha_i>0$ and $K_i>0$, satisfies these conditions. So the regularity condition can be applied in an important frequently assumed case. Furthermore, it is compatible with the increasing relative rate of change of the marginal effort costs, and the main results of this paper include such cases.

The reduced contest by the group leaders can be seen as a generalization of the model by Cornes and Hartley (2005) to a case that the strategy sets of the players are not independent. Extending their "share function" approach, we can prove the following result.

Proposition 1. *If the above regularity condition holds, then there exists a unique pure-strategy perfect Bayesian equilibrium in our model of group contest with cost-sharing.*

3. The Possibility of First-Best Cost-Sharing

3.1. Contests by fully regulated groups

If individuals in a group perfectly obey the group leader, she could directly assign them the contributions that maximize the objective function of the group. In equilibrium of such a *contest by fully regulated groups*, each group leader sets the efforts of her group members such that

$$\sum\nolimits_{k=1}^{N_i}\left(\frac{\sum\nolimits_{j=1}^{N_i}a_{ij}}{\sum\nolimits_{j\neq i}A_j+\sum\nolimits_{j=1}^{N_i}a_{ij}}v_{ik}-c_i(a_{ik})\right)=\frac{\sum\nolimits_{j=1}^{N_i}a_{ij}}{\sum\nolimits_{j\neq i}A_j+\sum\nolimits_{j=1}^{N_i}a_{ij}}V_i-\sum\nolimits_{k=1}^{N_i}c_i(a_{ik})$$

is maximized with respect to $a_{ik}\geq 0,\quad k=1,\cdots,N_i$, given the other groups' efforts. It is easy to see that the solution must minimize the aggregate group cost $\sum\nolimits_{k=1}^{N_i}c_i(a_{ik})$ given the equilibrium aggregate group effort. So we can apply the popular two-stage approach to a maximization problem: define the *group cost function*

$$E_i^*(A_i)=\min\left\{\sum\nolimits_{k=1}^{N_i}c_i(a_{ik})\,\middle|\,\sum\nolimits_{k=1}^{N_i}a_{ik}=A_i,\ a_{ik}\geq 0,\quad k=1,\cdots,N_i\right\}.$$

The first-order condition for the solution of this minimization problem is

$$c_i'(a_{ik})=c_i'(a_{i1})\ \text{ for all }\ k=1,\cdots,N\,. \tag{8}$$

That is, the equalization of the individuals' marginal effort costs. Since the marginal effort cost function is strictly increasing, it requires equal contributions by all individuals in the group.

Hence it holds that $E_i^*(A_i)=N_i\cdot c_i\left(\dfrac{A_i}{N_i}\right)$ and the equilibrium choice by the group leader of a fully regulated group is simplified to choosing the group effort $A_i\geq 0$ to maximize

$$\frac{A_i}{\sum\nolimits_{j\neq i}A_j+A_i}V_i-N_i\cdot c_i\left(\frac{A_i}{N_i}\right),$$

given the other groups' efforts. The first-order (necessary and sufficient) condition for the solution of this problem is

$$\frac{\sum\nolimits_{j\neq i}A_j}{\left(\sum\nolimits_{j\neq i}A_j+A_i\right)^2}V_i-c_i'\left(\frac{A_i}{N_i}\right)=0\,. \tag{9}$$

Viewing each group leader just as an individual competing on the prize, we can treat the contest by fully regulated groups as a contest by individuals with convex technologies, which is considered by Szidarovszky and Okuguchi (1997) and Cornes and Hartley (2005). Hence the existence of unique equilibrium on group efforts A_i's is guaranteed. Each individual in a group contributes $1/N_i$ of the equilibrium group effort.

3.2. Efficiency of cost-sharing rules

Formally, the first-best cost-sharing rule induces the same equilibrium contributions of individuals as those in a contest by fully regulated groups. That is, a cost-sharing rule that induces the contributions satisfying the conditions (8) and (9) in equilibrium is first-best. If there exists such a cost-sharing rule for each group, every group leader will choose it thus ensuring that the equilibrium group efforts coincide with those of the unique equilibrium of the contest by fully regulated groups. Such a situation is obtained in a group consisting of homogeneous individuals.

When the valuation of the prize is common among the individuals belonging to group i, we can write $v_{ik} = \frac{V_i}{N_i}$ for all $k = 1, \cdots, N$. The condition (4) implies that the marginal costs of the individuals are equalized, and they contribute equally $1/N_i$ of the group effort A_i. Now, let us set $\delta_i = 1$ and observe that the condition (3) under such full cost-sharing always coincides with the condition (9), because $a_{ik} = \frac{A_i}{N_i}$ for all $k = 1, \cdots, N$. Hence we have the following result (formally, it will be proved as a special case of Proposition 4):

Proposition 2. *When all individuals in a group are identical in their prize valuations, the cost is fully shared in equilibrium. This cost-sharing rule is first-best.*[22]

When a group consists of individuals with heterogeneous prize valuations, any uniform cost-sharing rule cannot be first-best. This result is directly obtained from the comparison between (4) and (8). The former requires that the ratio of the marginal effort costs of individual group members is equal to that of their prize valuations, as long as they contribute voluntarily. Hence the marginal costs are never equalized, and the condition (8) is not satisfied. Basically, this is how intra-group heterogeneity of the prize valuations prevents the cost-sharing rules from resolving the collective action problem.[23]

The problem can also be viewed and analyzed as that of discriminating cost-sharing

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[22] In the model of collective contests with prize-sharing considered by Nitzan and Ueda (2011), comparison between two homogeneous groups of the same size reveals that the group attaining the *lower* winning probability is the one dividing a *larger* part of the prize according to the relative effort rule (See Nitzan and Ueda (2011) for the details). Interestingly, this seemingly strange pattern is equivalent to full cost-sharing in contests by homogeneous groups with cost-sharing. Once we notice that the pattern is caused by the incentives depending not on the costs but on the prize, it becomes more understandable; strong incentives are needed if the win probability is low because the reward for the contribution is less probable.

[23] Actually it is an intrinsic problem for a group with intra-group heterogeneity in prize valuations, because the marginal costs are not equal already in the case where $\delta_i = 0$, the case without cost-sharing. Misallocation of burdens in a group is usually regarded as a fairness problem in the collective action literature, i.e. the "exploitation of the great by the small" emphasized by Olson (1965) and Olson and Zeckhauser (1966). But it also causes an efficiency problem as just has been shown. We will see that it is a critical factor in determining the degree of cost-sharing.

rules. Suppose that it is possible for group *i* to differentiate the size of the shared part of the cost among its members as follows. For individual *k*, the effort cost is compensated at the individual-specific rate of $\delta_{ik} \geq 0$, $k = 1, \cdots, N_i$. The amount $\sum_{p=1}^{N_i} \delta_{ip} c_i(a_{ip})$ needed for the compensation is collected equally from all the individual group members. Then the cost of individual *k* has the form

$$(1 - \delta_{ik}) \cdot c_i(a_{ik}) + \frac{\sum_{p=1}^{N_i} \delta_{ip} c_i(a_{ip})}{N_i}.$$

It could be called the *sophisticated cost-sharing* rule. With this rule, we obtain the next result.

Proposition 3. *By adequate choice of the sophisticated cost-sharing rule, group i can induce the first-best contribution by its members, in the sense that their contributions maximize the utilitarian group welfare, given the levels of aggregate group efforts of the other competing groups. This first-best rule is characterized by* the *degrees of cost-sharing*

$$\delta_{ik} = \frac{1 - \frac{v_{ik}}{V_i}}{1 - \frac{1}{N_i}}, \quad k = 1, \cdots, N_i. \tag{10}$$

When all individuals share the same valuation of the prize, $\delta_{ik} = 1$ holds for all $k = 1, \cdots, N_i$. Otherwise, the degree of cost-sharing must be different from person to person to induce the first-best contributions. Equation (10) implies that, under sophisticated cost-sharing, an individual with a lower prize valuation must receive a larger compensation. But such a requirement seems too stringent for a device of selective incentives because usually it has to treat individuals uniformly and is not allowed to apply discrimination based on their names. A reward to an individual is provided only according to his/her behavior. Why is it difficult to let incentive schemes depend on specific individual characteristics? At least two plausible reasons can be mentioned. First, differences in behavior among individuals are easily observable, but this is not necessarily the case for individual characteristics such as prize valuations. Second, different treatment of individuals depending on their names, status, or other observable characteristics often involves political problems, even though such variable treatment is systematically related to different valuations. Hence we assume that the available cost-sharing for a group is uniform and therefore the existence of intra-group heterogeneity precludes the application of effective selective incentives. The next section examines how this problem affects the choice of cost-sharing rules by a competing group.

4. Cost-sharing Rules under Intra-group Heterogeneity

4.1 Basic observations

To characterize the equilibrium rules of cost-sharing chosen by competing groups, it is convenient to introduce a new variable, $\gamma_i = 1 - \delta_i + \frac{\delta_i}{N_i}$. Using the first order condition (3) to determine each group member's contribution, notice that it can be interpreted as the *discount factor of marginal effort costs* reflecting cost-sharing. Since this factor is strictly decreasing in δ_i, we can argue on the equilibrium cost-sharing scheme of group *i* applying γ_i instead of δ_i. It takes the value 1 when none of the costs are shared ($\delta_i = 0$), and the value $\frac{1}{N_i}$ when the whole costs are shared ($\delta_i = 1$)[24]. By using the properties of the distorted group cost function $E_i(A_i)$ and the first-order conditions for the maximization problem (7), the basic result on the equilibrium cost-sharing rules is derived;

Proposition 4. *In equilibrium, the cost-sharing scheme chosen by group i satisfies the inequalities*

$$\gamma_i \leq (\geq) \sum_{k=1}^{N_i} \frac{v_{ik}}{V_i} \cdot \frac{v_{ik} \big/ c_i''(a_{ik}^*)}{\sum_{p=1}^{N_i} \left(v_{ip} \big/ c_i''(a_{ip}^*) \right)} = \frac{\frac{\partial E_i}{\partial A_i}\left(A_i^*; \mathbf{v}_i\right)}{\sum_{p=1}^{N_i} c_i'(a_{ip}^*)} \tag{11}$$

if $\gamma_i > \frac{1}{N_i}$ ($\gamma_i < 1$), *where* a_{ik}^* *is the equilibrium contribution by individual k belonging to group i, and* A_i^* *is the aggregate equilibrium effort of the group.*[25]

Notice that the middle term of (11) is strictly less than one. Also notice that $\gamma_i < 1$ must hold if γ_i is larger than this term. Thus $\gamma_i < 1$ always holds and a direct corollary of Proposition 4 is:

Corollary 1. *Under the pure strategy Bayesian perfect equilibrium of the model, at least some degree of cost-sharing is implemented in every group.*

Hence a competing group always adopts cost-sharing when such a device gives selective incentives to its members. Also, Proposition 4 permits us to directly derive the cost-sharing rule in case of intra-group homogeneity. Condition (4) requires that contestants with the same stake will choose the same level of effort. This fact combined with Proposition 4 proves that $\gamma_i = \frac{1}{N_i}$, i.e. the costs of the individual group members are fully shared.

4.2 Distortions of the degree of cost-sharing

Let us see what happens to the equilibrium cost-sharing rules under intra-group prize-valuation heterogeneity. To provide an answer, let $\bar{a}_i$ be the effort level determined by the equation $\bar{a}_i = (c_i')^{-1}(V_i)$. Since an individual member of group *i* does not expect to get more than his stake, in equilibrium every group member contributes less than $\bar{a}_i$. The following important result holds:

Proposition 5. *Suppose that the valuations of the prize by individuals in group i are not even.*

(a) *If the relative rate of change of the marginal effort costs,* $\frac{c_i''(a)}{c_i'(a)}$, *is strictly*

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[24] The equation $\delta_i = \frac{1-\gamma_i}{1-\frac{1}{N_i}}$ may be useful to see the following results.

[25] It may be noteworthy that the degree of cost-sharing of a group is not affected by the cost conditions of the competing groups, if the cost-sharing is partial. The relative strength of a group compared to the rivals does not matter to determine this selective incentive device. It is a distinctive property comparing to prize-sharing (see Proposition 1 of Nitzan and Ueda (2011)).

decreasing in a on the interval $(0, \bar{a}_i)$*, then in equilibrium* $0 < \delta_i < 1$ *(i.e.,* $\frac{1}{N_i} < \gamma_i < 1$ *).*

(b) *If* $\frac{c_i''(a)}{c_i'(a)}$ *is increasing in a on the interval* $(0, \bar{a}_i)$*, then in equilibrium* $\delta_i = 1$ *(i.e.,* $\gamma_i = \frac{1}{N_i}$ *).*

The proposition reveals that intra-group heterogeneity works differently on the form of cost-sharing depending on a property of the relative rate of change of the marginal costs. If it is strictly decreasing, partial sharing of costs is chosen by the group. If it is increasing, the costs are still fully shared in the group.

The former case seems more probable because of the following reasons. First, investigating conditions (4) from which the distorted group cost function is derived, we can see that $\frac{\partial}{\partial A_i} a_{ik}(A_i;\mathbf{v}_i) = \frac{\frac{c_i'(a_{ik}(A_i;\mathbf{v}_i))}{c_i''(a_{ik}(A_i;\mathbf{v}_i))}}{\sum_{p=1}^{N_i} \frac{c_i'(a_{ip}(A_i;\mathbf{v}_i))}{c_i''(a_{ip}(A_i;\mathbf{v}_i))}}$. Also notice that the larger the valuation of the prize of an individual, the larger his contribution. If $\frac{c_i''(a)}{c_i'(a)}$ is increasing, an individual with a lower valuation of the prize would increase his contribution more than others as the aggregate group effort rises. This seems rather unusual. Second, a strictly decreasing $\frac{c_i''(a)}{c_i'(a)}$ permits the elasticity of marginal effort costs $\frac{c_i''(a)}{c_i'(a)} \cdot a$ to be constant, or to show up-and-down changes. This possibility seems more natural than the situation where the elasticity steadily rises, which is necessary if $\frac{c_i''(a)}{c_i'(a)}$ is increasing. We could therefore normally expect that intra-group heterogeneity in the valuations of the prize results in partial cost-sharing. Also notice that, even if the relative rate of change of the marginal costs is increasing, the full cost-sharing in this case cannot be the first-best. The contributions by the members are still not equalized.

Why does heterogeneity of stakes within a group affect the choice of the degree of selective incentives in the form of cost-sharing? Distributional concerns for the net benefit of group members would play a minor role in the decisions of a group leader because her objective is assumed to be the utilitarian group welfare. As we have already argued in Section 3, cost-sharing rules uniformly applied to heterogeneous individuals fail to induce efficient contributions minimizing the sum of effort costs. Equal contributions by every individual in the group should be made, but intra-group heterogeneity of valuations of the prize does differentiate the voluntary contributions. The existence of such inefficiency and the direction of change associated with a rise in a group effort determine how intra-group heterogeneity affects the degree of cost-sharing.

If $\frac{c_i''(a)}{c_i'(a)}$ is strictly decreasing, the inefficiency gets worse as the aggregate effort grows because an individual with a higher valuation of the prize would more rapidly enhance his contribution.[26] This reallocation of contributions piles up the extra burden of enhancing group effort, causing the leader to hesitate when considering the application of a high degree of cost-sharing. In contrast, the inefficiency would be alleviated if $\frac{c_i''(a)}{c_i'(a)}$ is increasing, because the lower the prize valuation, the larger the expansion of effort by the individual. The reallocation effect is now desirable to equalize the voluntary contributions, and the leader would like to promote cost-sharing. The result is the highest possible degree of cost sharing.

This intuition is supported by confirming that the reallocation of contributions actually affects the level of the marginal group costs. Notice that $\frac{1}{N_i}\sum_{k=1}^{N_i} c_i'(a_{ik}(A_i;\mathbf{v}_i))$ is the increment in the group's costs provided that the individual group members equally expand their contributions. Remembering that $\delta_i < 1$ is equivalent to $\gamma_i > \frac{1}{N_i}$, we can combine the equality of the middle and the right terms in (11) with Proposition 5 to get the following observations: $\frac{1}{N_i} < \gamma_i = \frac{\frac{\partial E_i}{\partial A_i}\left(A_i^*;\mathbf{v}_i\right)}{\sum_{p=1}^{N_i} c_i'\left(a_{ip}^*\right)}$ holds if $\frac{c_i''(a)}{c_i'(a)}$ is strictly decreasing in *a*, and $\frac{1}{N_i} = \gamma_i \geq \frac{\frac{\partial E_i}{\partial A_i}\left(A_i^*;\mathbf{v}_i\right)}{\sum_{p=1}^{N_i} c_i'\left(a_{ip}^*\right)}$ holds if $\frac{c_i''(a)}{c_i'(a)}$ is increasing in *a*. These observations are summarized in the next corollary.

Corollary 2. *Suppose that the valuations of the prize by individuals in group i are not homogeneous. If* $\frac{c_i''(a)}{c_i'(a)}$ *is strictly decreasing in a,* $\frac{\partial}{\partial A_i}E_i(A_i;\mathbf{v}_i) > \frac{1}{N_i}\sum_{k=1}^{N_i} c_i'(a_{ik}(A_i;\mathbf{v}_i))$. *If* $\frac{c_i''(a)}{c_i'(a)}$ *is increasing in a,* $\frac{\partial}{\partial A_i}E_i(A_i;\mathbf{v}_i) \leq \frac{1}{N_i}\sum_{k=1}^{N_i} c_i'(a_{ik}(A_i;\mathbf{v}_i))$.

4.3 Intra-group heterogeneity and win probability

Our analysis proves that, when competing groups use cost-sharing rules as a device of selective incentives, intra-group heterogeneity prevents effective use of them. Unless discriminating and individualistic rules are available, a cost-sharing scheme induces inefficient responses of the heterogeneous group members and the leader cannot provide them with the first-best selective incentives. This problem would put such a group in an inferior position to a homogeneous group. Under some conditions, the inferiority takes the plain form of lower win probabilities.

To study this issue, set V_i, the sum of the valuations of the individuals in group *i*, at a constant value. Let us consider a contest in which all individuals of group *i* have the same

[26] As we have already noted, $\frac{\partial}{\partial A_i}a_{ik}(A_i;\mathbf{v}_i) = \frac{\frac{c_i'(a_{ik}(A_i;\mathbf{v}_i))}{c_i''(a_{ik}(A_i;\mathbf{v}_i))}}{\sum_{p=1}^{N_i}\frac{c_i'(a_{ip}(A_i;\mathbf{v}_i))}{c_i''(a_{ip}(A_i;\mathbf{v}_i))}}$. If the proportional rate of marginal effort costs is strictly decreasing, this derivative is larger for an individual with a larger stake. If it is increasing, the reverse relation holds.

valuation of the prize, i.e. every member has the valuation $\frac{V_i}{N_i}$ of the prize. Starting from the equilibrium of such a contest, we examine the effect of changing the stake vector of group *i* by making it unequal (but keeping V_i, or the average prize valuation in the group, unchanged). Does the equilibrium win probability of group *i* decline relative to the initial equilibrium?

Even though inefficient contributions by individuals raise the group costs, the answer to this question is not clear. Intra-group heterogeneity in valuations of the prize causes an inefficient assignment of group efforts, but it does not necessarily lower the equilibrium level of the group's effort. As Nitzan and Ueda (2013) argue, the intra-group heterogeneity itself (not the effect through inadequate selective incentives) can enhance the group effort. Furthermore, strategic interaction among the competing groups can eventually enhance the group efforts. We can identify, however, one of the cases in which intra-group heterogeneity has a negative effect on the group's prospect of winning.

Proposition 6. *Let the average prize valuation in group i be constant.*

(a) *Consider the case where the relative rate of change of the marginal effort costs, $\frac{c_i''(a)}{c_i'(a)}$, is strictly decreasing in a. If the marginal effort costs of group i, $c_i'(a)$, is convex, the equilibrium win probability of group i is reduced when its members have heterogeneous prize valuations rather than the same prize valuation.*

(b) *Consider the case where $\frac{c_i''(a)}{c_i'(a)}$ is increasing in a. If the marginal effort costs of group i is strictly convex, the equilibrium win probability of group i is reduced when its members have heterogeneous prize valuations rather than the same prize valuation.*

It is widely argued in the literature of collective contests that, if the function of marginal effort costs is *strictly* convex, then a competing group attains the maximum win probability only when its members share the same stakes.[27] But Proposition 6 is not a simple repetition of the existing results because we are considering collective contests *with selective incentives*. The prospect of a competing group is restricted by intra-group heterogeneity via selective incentives, particularly in the case where the relative rate of change of the marginal effort costs is strictly decreasing. Proposition 6 illustrates how the merit of selective incentives in collective contests is influenced by intra-group heterogeneity.

4.4 Intra-group heterogeneity with respect to cost conditions

In subsection 4.2, we have argued that inefficient contributions caused by intra-group heterogeneity is an essential factor for inducing a group to deviate from full cost-sharing. To confirm this point, let us resort to the following argument. Keeping all other components of the basic model, individual valuations of the prize and the cost functions are now changed as follows; the cost function of individual *k* who belongs to group *i* becomes $\frac{1}{v_{ik}} \cdot c_i(a_{ik})$, and the valuation of the prize is just 1. Then, his equilibrium choice of contribution satisfies the modified first-order condition

[27] See Section 5 of Nitzan and Ueda (2013).

$$\frac{\sum_{j\neq i} A_j^*(\delta_j^*)}{\left\{\sum_{j\neq i} A_j^*(\delta_j^*)+A_i^*(\delta_i)\right\}^2}-\left\{(1-\delta_i)+\frac{\delta_i}{N_i}\right\}\cdot\frac{1}{v_{ik}}\cdot c_i'\left(a_{ik}^*(\delta_i)\right)=0\,, \tag{12}$$

which replaces the first-order condition (3) in the original model. It is obvious that both conditions (12) and (3) result in the same reaction to any given degree of cost-sharing, under any combination of the values v_{ik}'s. If intra-group heterogeneity can be represented as a distribution of those v_{ik}'s, the relation between intra-group heterogeneity and the induced contributions by cost-sharing is the same as in the original setting. However, individual contributions no longer cause inefficiency here, even though the v_{ik}'s are different within the group; heterogeneous individuals in a group contribute differently because of their different technologies, i.e., cost functions, but their marginal costs in equilibrium are now always equal. This modified model can be referred to as the *associated contest with intra-group heterogeneous cost functions*. In such an associated contest, how do the group leaders choose the degree of cost-sharing?

The summed group cost in the associated contest is defined by

$$\hat{E}_i(A_i;\mathbf{v}_i)=\sum_{k=1}^{N_i}\frac{1}{v_{ik}}\cdot c_i\left(a_{ik}(A_i;\mathbf{v}_i)\right),$$

which equalizes the marginal effort costs of the individuals belonging to the group.
Since this is the function of the minimized group costs, we can see that the first-best choice of aggregate group effort for the leader maximizes

$$\frac{A_i}{A}N_i-\hat{E}_i(A_i;\mathbf{v}_i).$$

The following result is therefore obtained:

Proposition 7. *Assume that the group cost function* $\hat{E}_i(A_i;\mathbf{v}_i)$ *is convex*[28]*. Then, the individuals in a group fully share their costs in equilibrium of the associated contest with intra-group heterogeneous cost functions.*

That is, in the modified setting the cost is fully shared. It should also be clear that it is the first-best cost sharing rule. How much contributions can be induced from heterogeneous individuals is, therefore, not critical for a group leader in avoiding full cost-sharing. The created inefficiency makes the difference.

5. Cases of Constant Elasticity of Marginal Effort Costs

If the class of effort cost functions is confined to the form of constant elasticity of marginal costs, i.e. $c_i(a)=K_i\cdot\frac{a^{1+\alpha_i}}{1+\alpha_i}$ ($\alpha_i>0$), the model becomes surprisingly tractable. In this special case, the equilibrium discount factor of marginal effort costs, γ_i, is explicitly expressed with the individual valuations of the prize. Intra-group heterogeneity can then be directly related to the degree of cost-sharing. Notice that, in this case, our Regularity Condition actually holds. In addition, $\frac{c_i''(a)}{c_i'(a)}=\frac{\alpha_i}{a}$ is decreasing, which means that partial cost-sharing is expected. The marginal effort cost function $c_i'(a)=K_i a^{\alpha_i}$ is convex if $\alpha_i\geq 1$, and strictly convex if $\alpha_i>1$.

28 The convexity holds in the cases of intra-group homogeneity and of constant elasticity of marginal effort costs.

5.1 Elasticity of marginal effort costs and the degree of cost-sharing

Some calculations using the equations in (4) show that under constant elasticity of marginal costs, $a_{ik}\left(A_i;\mathbf{v}_i\right)=\frac{v_{ik}^{\frac{1}{\alpha_i}}}{\sum_{p=1}^{N_i}v_{ip}^{\frac{1}{\alpha_i}}}A_i$.[29] Also, the next lemma can be derived as a special case of Proposition 4. The equilibrium cost-sharing is partial in this case, as Proposition 5 predicts.

Lemma 4. *Assume that the effort cost function of every member of group i has the form of constant elasticity of marginal costs,* $c_i(a)=K_i\cdot\frac{a^{1+\alpha_i}}{1+\alpha_i}$ ($\alpha_i>0\,,K_i>0$). *Then, the equilibrium cost-sharing rule has the following explicit form;*

$$\gamma_i=\frac{1}{\sum_{p=1}^{N_i}v_{ip}}\sum_{k=1}^{N_i}\frac{v_{ik}^{\frac{1}{\alpha_i}+1}}{\sum_{p=1}^{N_i}v_{ip}^{\frac{1}{\alpha_i}}}\,. \tag{13}$$

The main attraction of equality (13) is that it can be related to a well-known mean concept. For an n-dimensional vector $\mathbf{v}=(v_1,\ldots,v_n)$, the *Lehmer mean* with index q is defined by $L_n(\mathbf{v},q)=\sum_{k=1}^{n}\frac{v_k^{\,q}}{\sum_{p=1}^{n}v_p^{\,q-1}}$ (Lehmer (1971)). Then, the equilibrium cost-sharing scheme (13) can be represented as follows;

$$\gamma_i=\frac{L_{N_i}\left(\mathbf{v}_i\,,\frac{1}{\alpha_i}+1\right)}{\sum_{p=1}^{N_i}v_{ip}}\,. \tag{13$'$}$$

From the well-known properties of the Lehmer mean, we directly get the following results on the relation between the cost-sharing schemes and the elasticity of marginal costs.

Proposition 8. *Assume that the effort cost function of an individual in group i has the form* $c_i(a)=K_i\cdot\frac{a^{1+\alpha_i}}{1+\alpha_i}$ ($\alpha_i>0$). *If the valuations of the prize are not the same among individuals under given* **v**$_i$, *the degree of equilibrium cost-sharing is strictly increasing in* α_i. *Furthermore, we have the two limit cases on the equilibrium discount factor of marginal effort costs,* $\lim_{\alpha_i\to\infty}\gamma_i=\frac{1}{N_i}$ *and* $\lim_{\alpha_i\to 0}\gamma_i=\frac{\max\{v_{i1},\ldots,v_{iN_i}\}}{V_i}$.[30]

Hence, given a distribution of stakes in a group, the higher the elasticity of marginal costs, the nearer to full sharing the implemented cost-sharing. The value of $\lim_{\alpha_i\to\infty}\gamma_i$ shows that such propensity eventually results in the full sharing of effort cost. This result is

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[29] Notice that since $c_i'(a)=K_i\cdot a^{\alpha_i}$, the condition (4) implies that $\frac{a_{ik}}{a_{i1}}=\left(\frac{v_{ik}}{v_{i1}}\right)^{\frac{1}{\alpha_i}}$ and $\sum_{k=1}^{N_i}a_{ik}=A_i$.

[30] Since the right-hand-side of this equality is larger than or equal to the arithmetic mean of the stakes of the individuals, $\lim_{\alpha_i\to 0}\gamma_i$ is in fact larger than or equal to $1/N_i$.

understandable in light of the argument regarding the reallocation of contributions presented in Subsection 4.2. When the elasticity of marginal effort costs is constant, we have $\frac{\partial a_{ik}}{\partial A_i}(A_i;\mathbf{v}_i)=\frac{v_{ik}^{\frac{1}{\alpha_i}}}{\sum_{p=1}^{N_i} v_{ip}^{\frac{1}{\alpha_i}}}$. If the elasticity is high, the increments of contributions caused by enhancing selective incentives are almost the same for every group member. In such a case, reallocation of contributions caused by expansion of group effort is negligible, and the caused deviation from full cost sharing is small.

5.2 The peculiarity of linear cost functions

Our basic model assumes convex effort cost functions, and does not include the case of linear effort cost function or constant marginal effort costs. But it is a limit case of constant elasticity of marginal effort costs, because $K_i \cdot \frac{a^{1+\alpha_i}}{1+\alpha_i}$ has the form $K_i \cdot a$ when $\alpha_i = 0$. Actually, the value $\lim_{\alpha_i \to 0} \gamma_i = \frac{\max\{v_{i1},\ldots,v_{iN_i}\}}{V_i}$ in Proposition 8 coincides with the discount factor of the equilibrium cost-sharing rule in the case of constant marginal costs. In that case, only the contestants with the largest stake in the group are active, as established in Baik (2008). Then the aggregate effort of group *i*, A_i, is determined by the equation

$$\frac{A-A_i}{A^2}\cdot \max\{v_{i1},\ldots,v_{iN_i}\} = \gamma_i \cdot K_i .$$

We can also show that the group effort maximizing the utilitarian group welfare is determined by the equation

$$\frac{A-A_i}{A^2}\cdot V_i = K_i .$$

Combining the above two equations, we can see that the first-best cost-sharing rule is induced if the value of the discount factor of marginal effort costs is set at $\lim_{\alpha_i \to 0} \gamma_i$. The group leader will of course choose it.

A remarkable point in this argument is that the equilibrium cost-sharing rule induces the effort maximizing the group welfare, *even with* heterogeneous valuations of the prize. By virtue of constant marginal costs, the allocation of contributions among heterogeneous individuals does not matter in a group. With linear costs, therefore, the requirement that the sharing rule should be dependent only on contributions causes no problem, and the sophisticated cost-sharing discussed in subsection 3.2 is redundant.[31] This is another example of the peculiarity associated with linear costs in the theory of collective contests.

5.3 Inequality of stakes within a group and cost-sharing

In the case of constant elasticity of marginal costs, we can find a much more regular relation between a distribution of the stakes and the degree of cost-sharing than in the general model. This is especially clear for the case of a quadratic cost function, i.e. $\alpha_i = 1$, where γ_i coincides with the Herfindahl-Hirschman Index (HHI);

[31] This observation is true even in the case of observable cost-sharing rules. Hence we can generally say that, *as far as linear effort cost functions are assumed, the simple cost-sharing rules used in our basic model are sufficient* to analyze the problem of intra-group heterogeneity of prize valuations. More complicated rules are unnecessary.

$$\frac{L_{N_i}(\mathbf{v}_i, 2)}{\sum_{p=1}^{N_i} v_{ip}} = \sum_{k=1}^{N_i}\left(\frac{v_{ik}}{\sum_{p=1}^{N_i} v_{ip}}\right)^2 .$$

Then, the degree of cost-sharing δ_i directly declines with the inequality of stakes in a group measured by the HHI. This observation suggests that, in the cases of constant elasticity of marginal costs, cost-sharing should be loosened up as the distribution of stakes gets more unequal.

To examine this conjecture, we can use the concept of *Lorenz-dominance* to determine whether a stake vector is more unequal (or "less nearly equal") than another vector. Take two stake vectors $\mathbf{v} = (v_1, \cdots, v_n)$ and $\mathbf{v}' = (v'_1, \cdots, v'_n)$. The latter distribution of group members' prize valuations is called more unequal than the former in the sense of Lorenz-dominance, if $\sum_{k=1}^{h} v_k \geq \sum_{k=1}^{h} v'_k$ for all $h \leq n$, with strict inequality for at least one *h*, and $\sum_{k=1}^{n} v_k = \sum_{k=1}^{n} v'_k$. Straightforward calculations are needed to confirm the following results regarding the relationship between the Lehmer mean and this concept of inequality:[32]

Lemma 5. *Let* $\mathbf{v}'_i = (v'_{i1}, \cdots, v'_{in})$ *be a vector more unequal than* $\mathbf{v}_i = (v_{i1}, \cdots, v_{iN_i})$ *in the sense of Lorenz-dominance. Then, for* $2 \geq q > 0$, $L_n(\mathbf{v}'_i, q) > L_n(\mathbf{v}_i, q)$.

Hence we can argue as follows; if $\alpha_i \geq 1$, or the *marginal* cost function is convex, then $2 \geq \frac{1}{\alpha_i} + 1$ and $L_{N_i}\left(\mathbf{v}_i, \frac{1}{\alpha_i} + 1\right)$ is eligible for the above lemma: the more unequal stake vector in the sense of Lorenz dominance the group has, the larger the value of the Lehmer mean; that is, the degree of cost-sharing the group applies is reduced. When the marginal cost function is strictly concave or $0 < \alpha_i < 1$, we do not obtain a clear-cut result as in the case where $\alpha_i \geq 1$. Since $L_{N_i}\left(\mathbf{v}_i, \frac{1}{\alpha_i} + 1\right)$ is decreasing with α_i, however, the value of the discount factor γ_i is never lower than the HHI if $0 < \alpha_i < 1$. The HHI goes up and approaches 1 as the stake vector becomes more unequal[33]. Then the value of γ_i *asymptotically* rises as the stake vector gets worse in the sense of Lorenz dominance, even in the case of $0 < \alpha_i < 1$. Thus our results support the conjectures made at the beginning of this subsection. We summarize them as our last proposition.

Proposition 9. *Assume that the effort cost function of an individual in group i has the form* $c_i(a) = K_i \cdot \frac{a^{1+\alpha_i}}{1+\alpha_i}$ ($\alpha_i > 0$). *When* $\alpha_i \geq 1$, *a more unequal stake vector in the sense of Lorenz dominance results in a lower degree of cost-sharing. Even if* $0 < \alpha_i < 1$, *the degree of cost-sharing asymptotically declines as the distribution of stakes becomes more unequal.*

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[32] The statement "$\mathbf{v}'$ is more unequal than $\mathbf{v}$ in the sense of Lorenz-dominance" is equivalent to the statement "$\mathbf{v}$ can be obtained from $\mathbf{v}'$ by a finite sequence of transformations (called the Dalton transfers)" of the form $v_{il}(t+1) = v_{il}(t) + \varepsilon(t) \leq v_{ih}(t)$, $v_{ih}(t+1) = v_{ih}(t) - \varepsilon(t) \geq v_{il}(t)$ with $\varepsilon(t) > 0$ for some h and l ($h > l$), and $v_{ik}(t+1) = v_{ik}(t)$ for all $k \neq h, l$, where at least in one of the transformations the inequality must be strict. Lemma 5 is derived applying this property.

[33] Since Lorenz-dominance is a partial ordering relation, we would need to adequately specify a sequence of stake vectors.

6. Conclusion

Each group in a contest has to confront and manage its own collective-action problem, and devices of selective incentives are possible means of resolution. The widely observed persistence of collective action problems suggests that those devices are not perfect in resolving the problem. In this paper, we have argued that one of the reasons is intra-group heterogeneity. When the disagreement on the valuation of the prize is not negligible among the individual group members, the group faces the problem: to what extent it should apply the device of selective incentives. Such a group intrinsically has a problem of inefficient voluntary contributions, and selective incentives can aggravate it.

We have examined how such a problem affects the adopted selective incentive devices that take the form of cost-sharing rules. It has been found that the relative rate of change of marginal effort costs determines whether unequal valuations of the prize in a group reduce the degree of cost-sharing. We have argued that usually this rate would be strictly decreasing, and the equilibrium degree of cost-sharing in a heterogeneous group is lowered such that its winning probability is reduced considerably. Even if the relative rate is increasing, groups with intra-group heterogeneity at best choose the same cost-sharing rule as homogeneous groups. Our results suggest therefore that heterogeneous prize-valuations in a competing group prevent effective use of selective incentives.

Our conclusion is amended if a group can depend on the selective incentive rule responding not only to the individual actions but also to their characteristics. Actual rules of selective incentives may apply some degree of discrimination toward different categories of individuals. How discriminating such rules are and how widely they are adopted are interesting questions that deserve attention. Another topic for further study is the generalization of the group welfare function. If group leaders embrace non-linear group welfare functions implying distributional concerns, how does it affect the equilibrium selective incentives? Are such concerns good or bad in enhancing the win probability of a competing group? Comparing cost-sharing with prize-sharing under intra-group heterogeneity is also interesting. Prize sharing is a much more popular type of device in the literature of group contests to mitigate collective action problems. It seems natural to ask how the outcomes of the two types of sharing rules differ. In particular, it is interesting to identify the conditions that justify the group's selection of cost-sharing rather than prize-sharing. We may also need to identify the conditions justifying the simultaneous (optimal) use of both sharing modes. A model of collective contests with prize-sharing rules affording intra-group heterogeneity seems to be a prerequisite for such research.

7. Proofs

Proof of Lemma 1

The restrictions on the beliefs of the individuals by the "no-signaling-what-you-don't-know" condition discussed in subsection 2. 3, the contribution by individual k of group i at the information set indexed by δ_i is described as the solution of

$$\max_{a\geq 0} \frac{\sum_{p\neq k} a_{ip}^*(\delta_i)+a}{\sum_{j\neq i} A_j^*(\delta_j^*)+\sum_{p\neq k} a_{ip}^*(\delta_i)+a} v_{ik} - \left\{ (1-\delta_i)\cdot c_i(a)+\delta_i\cdot \frac{\sum_{p\neq k} c_i\left(a_{ip}^*(\delta_i)\right)+c_i(a)}{N_i} \right\}.$$

Since $a_{ik}^*(\delta_i)>0$ always holds due to the assumption $\lim_{a\to 0} c_i'(a)=0$, equation (3) is the first-order necessary and sufficient condition for $a_{ik}^*(\delta_i)$, the solution of this maximization problem.

Q.E.D.

Proof of Lemma 2

(a) The contribution of each individual in a group i, which is determined by equation (3), is increasing in δ_i. The highest value of aggregate group effort is attained when $\delta_i = 1$. Denoting this value by A_i^H, the contribution by individual k is derived from the equation $\frac{\sum_{j\neq i} A_j}{\left(\sum_{j\neq i} A_j + A_i^H\right)^2} v_{ik} - \frac{1}{N_i} c_i'(a_{ik}) = 0$, or $(c_i')^{-1}\left[N_i \cdot \frac{\sum_{j\neq i} A_j}{\left(\sum_{j\neq i} A_j + A_i^H\right)^2} v_{ik}\right] = a_{ik}$. The sum of the contributions by all members is equal to A_i^H. The definition of $A_i^H\left(\sum_{j\neq i} A_j\right)$ is thus derived. Similar arguments hold for $A_i^L\left(\sum_{j\neq i} A_j\right)$, by setting $\delta_i = 0$. Since the contribution by each individual is continuous in δ_i, so is the aggregate group effort. Hence, the group leader can attain any value of group effort in the interval $\left[A_i^L\left(\sum_{j\neq i} A_j\right),\ A_i^H\left(\sum_{j\neq i} A_j\right)\right]$, by the intermediate value theorem.

(b) The first equation in (4) is straightforward. The second can be derived by combining the condition (3) of different individuals in the same group.

Q.E.D.

Proof of Lemma 3

By the conditions in (4), we have

$$\sum_{k=1}^{N_i} \frac{\partial a_{ik}(A_i;\mathbf{v}_i)}{\partial A_i} = 1 \quad \text{and} \quad \frac{\partial a_{ik}(A_i;\mathbf{v}_i)}{\partial A_i} = \frac{v_{ik}}{v_{i1}} \cdot \frac{c_i''(a_{i1}(A_i;\mathbf{v}_i))}{c_i''(a_{ik}(A_i;\mathbf{v}_i))} \cdot \frac{\partial a_{i1}(A_i;\mathbf{v}_i)}{\partial A_i}.$$

Hence

$$\frac{\partial a_{ik}(A_i;\mathbf{v}_i)}{\partial A_i} = \frac{\dfrac{v_{ik}}{c_i''(a_{ik}(A_i;\mathbf{v}_i))}}{\sum_{p=1}^{N_i} \dfrac{v_{ip}}{c_i''(a_{ip}(A_i;\mathbf{v}_i))}} \quad \text{for all } k = 1,\cdots,N_i. \tag{14}$$

and we get equation (5) by definition of $E_i(A_i;\mathbf{v}_i)$.

Hence, $0 \le \frac{\partial}{\partial A_i} E_i(A_i;\mathbf{v}_i) \le \sum_{k=1}^{N_i} c_i'(a_{ik}(A_i;\mathbf{v}_i))$ for all $A_i > 0$. By Assumption 2, these inequalities imply that $\lim_{A_i \to 0} \frac{\partial}{\partial A_i} E_i(A_i;\mathbf{v}_i) = 0$.

Q.E.D.

Proof of Proposition 1

We want to show that there uniquely exists a configuration of group efforts A_j^*, $j = 1,\cdots,m$ such that A_i^* is a solution of the maximization problem (7) for all $i = 1,\cdots,m$. Differentiating the objective function in (7), define $A_i^{UC}\left(\sum_{j\neq i} A_j\right)$ by

$$\frac{\sum_{j\neq i} A_j}{\left(\sum_{j\neq i} A_j + A_i^{UC}\left(\sum_{j\neq i} A_j\right)\right)^2} V_i - \frac{\partial}{\partial A_i} E_i\left(A_i^{UC}\left(\sum_{j\neq i} A_j\right);\mathbf{v}_i\right) = 0. \tag{15}$$

Since we assume the regularity condition, the objective function is strictly concave. Then, the following equality holds for A_i^*;

$$A_i^* = \max\left\{ \min\left\{ A_i^{UC}\left(\sum\nolimits_{j\neq i} A_j^*\right), A_i^H\left(\sum\nolimits_{j\neq i} A_j^*\right) \right\}, A_i^L\left(\sum\nolimits_{j\neq i} A_j^*\right) \right\}. \tag{16}$$

We will show that this representation can be replaced by that with the share function, in which the variable is the total amount of effort $A = \sum_{j=1}^m A_j$.

To begin with, define the functions $\pi_i^L(A;\mathbf{v}_i)$ and $\pi_i^H(A;\mathbf{v}_i)$ by the equations

$$\begin{cases} \sum_{k=1}^{N_i} (c_i')^{-1}\left[\dfrac{1-\pi_i^L(A;\mathbf{v}_i)}{A} \cdot v_{ik} \right] = A \cdot \pi_i^L(A;\mathbf{v}_i) \\ \sum_{k=1}^{N_i} (c_i')^{-1}\left[N_i \cdot \dfrac{1-\pi_i^H(A;\mathbf{v}_i)}{A} \cdot v_{ik} \right] = A \cdot \pi_i^H(A;\mathbf{v}_i) \end{cases} \tag{17}$$

for $A > 0$. Clearly, $\pi_i^L(A;\mathbf{v}_i) < \pi_i^H(A;\mathbf{v}_i)$. Consider a configuration of group efforts in which the sum of the effort put by all of the competing groups is A and also group i is constrained to solve problem (7) by the minimal feasible group effort, $A_i^L\left(\sum_{j\neq i} A_j\right)$. Then we have the equality $A_i^L\left(\sum_{j\neq i} A_j\right) = A \cdot \pi_i^L(A;\mathbf{v}_i)$. If the total effort is A and group i is constrained by the maximal feasible group effort, then group i's effort is given by $A_i^H\left(\sum_{j\neq i} A_j\right) = A \cdot \pi_i^H(A;\mathbf{v}_i)$. Also define the function $\pi_i^{UC}(A;\mathbf{v}_i)$ by the equation

$$\left(1-\pi_i^{UC}(A;\mathbf{v}_i)\right)\cdot V_i - A \cdot \frac{\partial}{\partial A_i} E_i\left(A \cdot \pi_i^{UC}(A;\mathbf{v}_i);\mathbf{v}_i\right) = 0 \tag{18}$$

for $A > 0$. This function is well defined because $\lim_{A_i \to 0} \dfrac{\partial}{\partial A_i} E_i(A_i;\mathbf{v}_i) = 0$ holds by Lemma 3. If the total effort is A and group i chooses an interior solution of (7), group i's effort is given by $A_i^{UC}\left(\sum_{j\neq i} A_j\right) = A \cdot \pi_i^{UC}(A;\mathbf{v}_i)$. Note that $\pi_i^L(A;\mathbf{v}_i)$, $\pi_i^H(A;\mathbf{v}_i)$ and $\pi_i^{UC}(A;\mathbf{v}_i)$ are all continuous and strictly decreasing in $A > 0$. They also converge to 1 as A is reduced to 0, and converge to 0 as A rises to infinity.

Define the share function of group i by

$$\pi_i(A;\mathbf{v}_i) = \max\left\{ \min\left\{ \pi_i^{UC}(A;\mathbf{v}_i), \pi_i^H(A;\mathbf{v}_i) \right\}, \pi_i^L(A;\mathbf{v}_i) \right\}$$

for $A > 0$. Then, $\pi_i(A;\mathbf{v}_i)$ is continuous and strictly decreasing in $A > 0$. In addition, $\lim_{A\to 0} \pi_i(A;\mathbf{v}_i) = 1$ and $\lim_{A\to\infty} \pi_i(A;\mathbf{v}_i) = 0$. We can see that $\sum_{j=1}^m \pi_j(A;\mathbf{v}_j)$ is continuous, strictly decreasing in $A > 0$, $\lim_{A\to 0} \sum_{j=1}^m \pi_j(A;\mathbf{v}_i) = m > 1$ and $\lim_{A\to\infty} \sum_{j=1}^m \pi_j(A;\mathbf{v}_i) = 0$. It should be clear that there exists a unique $A^* > 0$ such that $\sum_{j=1}^m \pi_j(A^*;\mathbf{v}_j) = 1$ for given $\mathbf{v}_1,\cdots,\mathbf{v}_m$ and $\sum_{j=1}^m \pi_j(A^*;\mathbf{v}_j)\cdot A^* = A^*$. Denote $\pi_i(A^*;\mathbf{v}_i)\cdot A^* = \hat{A}_i$.

If $\hat{A}_i = \pi_i^{UC}(A^*;\mathbf{v}_i)\cdot A^*$, then $\pi_i^L(A^*;\mathbf{v}_i) \le \pi_i^{UC}(A^*;\mathbf{v}_i) \le \pi_i^H(A^*;\mathbf{v}_i)$. By comparing (15) and (18), we can see that $\hat{A}_i = A_i^{UC}\left(A^* - \hat{A}_i\right)$ holds by strict concavity of the payoff function. Suppose that $\hat{A}_i < A_i^L\left(A^* - A_i^*\right)$. Then

$$\frac{A^* - \hat{A}_i}{\left(A^* - \hat{A}_i + A_i^L\left(A^* - A_i^*\right)\right)^2} < \frac{A^* - \hat{A}_i}{\left(A^*\right)^2} \le \frac{A^* - A^* \cdot \pi_i^L\left(A^*;\mathbf{v}_i\right)}{\left(A^*\right)^2}.$$

But these inequalities imply

$$\hat{A}_i \ge A^* \cdot \pi_i^L\left(A^*;\mathbf{v}_i\right) = \sum_{k=1}^{N_i} (c_i')^{-1}\left[\frac{1-\pi_i^L\left(A^*;\mathbf{v}_i\right)}{A^*} \cdot v_{ik} \right]$$

$$> \sum_{k=1}^{N_i} (c_i')^{-1} \left[\frac{A^* - \hat{A}_i}{\left(A^* - \hat{A}_i + A_i^L\left(A^* - \hat{A}_i\right)\right)^2} \cdot v_{ik} \right] = A_i^L\left(A^* - \hat{A}_i\right).$$

This contradiction proves that $\hat{A}_i \geq A_i^L\left(A^* - \hat{A}_i\right)$. In a similar way, we can prove that $\hat{A}_i \leq A_i^H\left(A^* - \hat{A}_i\right)$. Hence, $\hat{A}_i = \max\left\{\min\left\{A_i^{UC}\left(A^* - \hat{A}_i\right), A_i^H\left(A^* - \hat{A}_i\right)\right\}, A_i^L\left(A^* - \hat{A}_i\right)\right\}$.

Consider the case where $\pi_i\left(A^*; \mathbf{v}_i\right) = \pi_i^H\left(A^*; \mathbf{v}_i\right)$. Comparing Lemma 2(a) and (17), we can see that $\hat{A}_i = A_i^H\left(A^* - \hat{A}_i\right) > A_i^L\left(A^* - \hat{A}_i\right)$ holds. Since $\hat{A}_i \leq \pi_i^{UC}\left(A^*; \mathbf{v}_i\right) \cdot A^*$,

$$\frac{A^* - \hat{A}_i}{A^*} V_i - A^* \cdot \frac{\partial}{\partial A_i} E_i\left(\hat{A}_i; \mathbf{v}_i\right) \geq \left(1 - \pi_i^{UC}\left(A^*; \mathbf{v}_i\right)\right) \cdot V_i - A^* \cdot \frac{\partial}{\partial A_i} E_i\left(A^* \cdot \pi_i^{UC}\left(A^*; \mathbf{v}_i\right); \mathbf{v}_i\right) = 0$$

and concavity of the objective function implies that $\hat{A}_i \leq A_i^{UC}\left(A^* - \hat{A}_i\right)$. Similarly, we can prove that $\pi_i\left(A^*; \mathbf{v}_i\right) = \pi_i^L\left(A^*; \mathbf{v}_i\right)$ implies $\hat{A}_i = A_i^L\left(A^* - \hat{A}_i\right) < A_i^H\left(A^* - \hat{A}_i\right)$ and $\hat{A}_i \geq A_i^{UC}\left(A^* - \hat{A}_i\right)$. Thus in any case, $\hat{A}_i = \max\left\{\min\left\{A_i^{UC}\left(A^* - \hat{A}_i\right), A_i^H\left(A^* - \hat{A}_i\right)\right\}, A_i^L\left(A^* - \hat{A}_i\right)\right\}$. The configuration of group effort $\hat{A}_j$, $j = 1, \cdots, m$ is such that $\hat{A}_i$ is a solution of the maximization problem (7) for all $i = 1, \cdots, m$. The existence of equilibrium is therefore established.

Now let us prove the uniqueness of equilibrium. It is clear that the total effort by all of the competing groups must be positive in equilibrium. Let $A_1^*, \cdots, A_m^*$ be a configuration of the group efforts such that A_i^* is a solution of the maximization problem (7) for all $i = 1, \cdots, m$. Let $A^* = \sum_{j=1}^m A_j^* > 0$.

If $A_i^* = A_i^{UC}\left(\sum_{j \neq i} A_j^*\right)$, we can see $A_i^* = A^* \cdot \pi_i^{UC}\left(A^*; \mathbf{v}_i\right)$ by comparing (15) and (18). Suppose that $\pi_i^{UC}\left(A^*; \mathbf{v}_i\right) > \pi_i^H\left(A^*; \mathbf{v}_i\right)$. Since $A_i^L\left(A^* - A_i^*\right) \leq A_i^* \leq A_i^H\left(A^* - A_i^*\right)$,

$$\sum_{k=1}^{N_i} (c_i')^{-1} \left[N_i \cdot \frac{A^* - A_i^*}{\left(A^* - A_i^* + A_i^H\left(A^* - \hat{A}_i\right)\right)^2} \cdot v_{ik} \right] = A_i^H\left(A^* - A_i^*\right) \geq A^* \cdot \pi_i^{UC}\left(A^*; \mathbf{v}_i\right)$$

$$> A^* \cdot \pi_i^H\left(A^*; \mathbf{v}_i\right) = \sum_{k=1}^{N_i} (c_i')^{-1} \left[N_i \cdot \frac{1 - \pi_i^H\left(A^*; \mathbf{v}_i\right)}{A^*} \cdot v_{ik} \right].$$

But $\pi_i^{UC}\left(A^*; \mathbf{v}_i\right) > \pi_i^H\left(A^*; \mathbf{v}_i\right)$ also implies that

$$\frac{A^* - A_i^*}{\left(A^* - A_i^* + A_i^H\left(A^* - \hat{A}_i\right)\right)^2} \leq \frac{A^* - A_i^*}{\left(A^*\right)^2} < \frac{1 - \pi_i^H\left(A^*; \mathbf{v}_i\right)}{A^*}.$$

This contradiction proves that $\pi_i^{UC}\left(A^*; \mathbf{v}_i\right) \leq \pi_i^H\left(A^*; \mathbf{v}_i\right)$. We can also prove $\pi_i^{UC}\left(A^*; \mathbf{v}_i\right) \geq \pi_i^L\left(A^*; \mathbf{v}_i\right)$, in a similar way. Hence $A_i^* = A^* \cdot \pi_i\left(A^*; \mathbf{v}_i\right)$.

If $A_i^* = A_i^H\left(\sum_{j \neq i} A_j^*\right)$, we can derive $A_i^* = A^* \cdot \pi_i^H\left(A^*; \mathbf{v}_i\right)$ from (17). We can use the inequality $A_i^H\left(A^* - A_i^*\right) \leq A_i^{UC}\left(A^* - A_i^*\right)$ to derive

$$\frac{1 - \pi_i^H\left(A^*; \mathbf{v}_i\right)}{A^*} V_i - \frac{\partial}{\partial A_i} E_i\left(A^* \cdot \pi_i^H\left(A^*; \mathbf{v}_i\right); \mathbf{v}_i\right) \geq \frac{A^* - A_i^{UC}\left(A^* - A_i^*\right)}{\left(A^* - A_i^* + A_i^{UC}\left(A^* - A_i^*\right)\right)^2} V_i - \frac{\partial}{\partial A_i} E_i\left(A_i^{UC}\left(A^* - A_i^*\right); \mathbf{v}_i\right)$$

$$= 0 = \frac{1 - \pi_i^{UC}\left(A^*; \mathbf{v}_i\right)}{A^*} V_i - \frac{\partial}{\partial A_i} E_i\left(A^* \cdot \pi_i^{UC}\left(A^*; \mathbf{v}_i\right); \mathbf{v}_i\right),$$

and $\pi_i^{UC}\left(A^*; \mathbf{v}_i\right) \geq \pi_i^H\left(A^*; \mathbf{v}_i\right)$ is derived by strict concavity of the objective function. Similarly, we can show that $A_i^* = A_i^L\left(\sum_{j \neq i} A_j^*\right)$ implies $\pi_i^L\left(A^*; \mathbf{v}_i\right) \geq \pi_i^{UC}\left(A^*; \mathbf{v}_i\right)$.

We have proved that $A_j^* = A^* \cdot \pi_j\left(A^*; \mathbf{v}_i\right)$ must hold for all $j = 1, \cdots, m$. Since the total effort is positive in equilibrium, the equation $\sum_{j=1}^{m} \pi_j\left(A^*;\mathbf{v}_j\right)=1$ is derived. We have already proved that such A^* is unique, and so is $A_j^* = A^* \cdot \pi_j\left(A^*;\mathbf{v}_j\right)$ for all $j=1,\cdots,m$.

Q.E.D.

Proof of Proposition 3

Given the efforts by the other competing groups, the best configuration of the contributions by the individuals in group *i*, $a_{i1}^B,\cdots,a_{iN_i}^B$, maximizes the group objective function $\dfrac{\sum_{k=1}^{N_i} a_{ik}}{\sum_{j\neq i} A_j + \sum_{k=1}^{N_i} a_{ik}} V_i - \sum_{k=1}^{N_i} c_i\left(a_{ik}\right)$. Then $\sum_{k=1}^{N_i} c_i\left(a_{ik}^B\right)$ must be minimized to realize the best group effort $A_i^B = \sum_{k=1}^{N_i} a_{ik}^B$. It is necessary therefore that every individual must bear the same marginal effort costs, i.e. $c_i'\left(a_{ik}^B\right)=c_i'\left(a_{i1}^B\right)$ holds for all $k=1,\cdots,N_i$. The implication is that $a_{ik}^B = \dfrac{A_i^B}{N_i}$ holds for all $k=1,\cdots,N_i$. Suppose that a_{ik}^B can be induced by a sophisticated cost-sharing rule. The first-order condition

$$\frac{\sum_{j\neq i} A_j}{A^2}\cdot v_{ik} - \left(1-\delta_{ik}+\frac{\delta_{ik}}{N_i}\right)\cdot c_i'\left(a_{ik}^B\right)=0,$$

must hold for each individual *k*. The condition $c_i'\left(a_{ik}^B\right)=c_i'\left(a_{i1}^B\right)$ is realized by setting

$$\frac{1-\delta_{ik}+\dfrac{\delta_{ik}}{N_i}}{1-\delta_{i1}+\dfrac{\delta_{i1}}{N_i}} = \frac{v_{ik}}{v_{i1}} \quad \text{for all } k=1,\cdots,N_i. \tag{19}$$

Since the best group effort $A_i^B = \sum_{k=1}^{N_i} a_{ik}^B$ maximizes $\dfrac{A_i^B}{\sum_{j\neq i} A_j + A_i^B} V_i - N_i \cdot c_i\left(\dfrac{A_i^B}{N_i}\right)$, the following first-order condition also holds:

$$\frac{\sum_{j\neq i} A_j}{A^2}\cdot V_i - c_i'\left(\frac{A_i^B}{N_i}\right)=0.$$

Comparing this equation with the first-order condition for individual *k*, we get that $1-\delta_{ik}+\dfrac{\delta_{ik}}{N_i}=\dfrac{v_{ik}}{V_i}$ or $\delta_{ik}=\dfrac{1-\dfrac{v_{ik}}{V_i}}{1-\dfrac{1}{N_i}}$ for all $k=1,\cdots,N_i$. Since these equations are compatible with equation (19), we obtain the desired result.

Q.E.D.

Proof of Proposition 4

We can use Lemma 3 to write the first-order conditions for problem (7) as follows:

$$\frac{\sum_{j\neq i} A_j}{\left(\sum_{j=1}^{m} A_j\right)^2} \sum_{k=1}^{N_i} v_{ik} - \sum_{k=1}^{N_i} c_i'\left(a_{ik}\left(A_i;\mathbf{v}_i\right)\right)\cdot \frac{\dfrac{v_{ik}}{c_i''\left(a_{ik}\left(A_i;\mathbf{v}_i\right)\right)}}{\sum_{p=1}^{N_i} \dfrac{v_{ip}}{c_i''\left(a_{ip}\left(A_i;\mathbf{v}_i\right)\right)}} \leq (\geq)\, 0,$$

if $\delta_i < 1$ ($\delta_i > 0$).

Since the contribution of each individual belonging to the group satisfies the equation

$$\frac{\sum_{j\neq i} A_j}{\left(\sum_{j=1}^{m} A_j\right)^2}\cdot\frac{v_{ik}}{\gamma_i} = c_i'\left(a_{ik}\left(A_i;\mathbf{v}_i\right)\right) \text{ for all } k=1,\cdots,N_i,$$

we can derive the inequality in (11) by inserting these equations into the above inequality. Let us turn to the equation in (11). Notice that the equilibrium contributions by the individuals in group i satisfy the conditions in (4). In particular, $v_{ik}=v_{i1}\cdot\frac{c_i'\left(a_{ik}^*\right)}{c_i'\left(a_{i1}^*\right)}$ holds for all $k=1,\cdots,N_i$.

Hence we obtain the equation:

$$\sum_{k=1}^{N_i}\frac{v_{ik}}{\sum_{p=1}^{N_i} v_{ip}}\cdot\frac{v_{ik}/c_i''\left(a_{ik}^*\right)}{\sum_{p=1}^{N_i}\left(v_{ip}/c_i''\left(a_{ip}^*\right)\right)} = \sum_{k=1}^{N_i}\frac{\frac{v_{i1}}{c_i'\left(a_{i1}^*\right)}c_i'\left(a_{ik}^*\right)}{\frac{v_{i1}}{c_i'\left(a_{i1}^*\right)}\sum_{p=1}^{N_i} c_i'\left(a_{ip}^*\right)}\cdot\frac{v_{ik}/c_i''\left(a_{ik}^*\right)}{\sum_{p=1}^{N_i}\left(v_{ip}/c_i''\left(a_{ip}^*\right)\right)}$$

$$=\frac{\sum_{k=1}^{N_i} c_i'\left(a_{ik}^*\right)\cdot\frac{v_{ik}/c_i''\left(a_{ik}^*\right)}{\sum_{p=1}^{N_i}\left(v_{ip}/c_i''\left(a_{ip}^*\right)\right)}}{\sum_{p=1}^{N_i} c_i'\left(a_{ip}^*\right)}.$$

By using (5), the equation appearing in the condition (11) is directly obtained.

Q.E.D.

Proof of Proposition 5

Let $x_k=\frac{v_{ik}}{\sum_{p=1}^{N_i} v_{ip}}$ and $y_k=\frac{c_i'\left(a_{ik}^*\right)/c_i''\left(a_{ik}^*\right)}{\sum_{p=1}^{N_i}\left(c_i'\left(a_{ip}^*\right)/c_i''\left(a_{ip}^*\right)\right)}$ for $k=1,\cdots,N_i$.

Then, $\sum_{k=1}^{N_i} x_k=\sum_{k=1}^{N_i} y_k=1$ and $x_1\leq\cdots\leq x_{N_i}$. Also $y_k=\frac{v_{ik}/c_i''\left(a_{ik}^*\right)}{\sum_{p=1}^{N_i}\left(v_{ip}/c_i''\left(a_{ip}^*\right)\right)}$ holds by the condition (4).

(a) Since $a_{i1}^*\leq\cdots\leq a_{iN_i}^*$, we have $y_1\leq\cdots\leq y_{N_i}$. Furthermore, intra-group heterogeneity of the valuations of the prize implies that there exists some k such that $x_k<x_{k+1}$ and $y_k<y_{k+1}$ hold. Hence we can apply Chebyshev's sum inequality to derive $\sum_{k=1}^{N_i} x_k\cdot y_k>\frac{1}{N_i}\cdot\left(\sum_{k=1}^{N_i} x_k\right)\cdot\left(\sum_{k=1}^{N_i} y_k\right)=\frac{1}{N_i}$. If $\gamma_i=\frac{1}{N_i}$, Proposition 4 requires that $\gamma_i\geq\sum_{k=1}^{N_i} x_k\cdot y_k$, contradicting the above strict inequality.

(b) In this case, $y_1\geq\cdots\geq y_{N_i}$ holds. Again by Chebyshev's sum inequality,

$$\sum_{k=1}^{N_i} x_k\cdot y_k\leq\frac{1}{N_i}\cdot\left(\sum_{k=1}^{N_i} x_k\right)\cdot\left(\sum_{k=1}^{N_i} y_k\right)=\frac{1}{N_i}.$$

If $\gamma_i>\frac{1}{N_i}$, Proposition 4 requires $\gamma_i=\sum_{k=1}^{N_i} x_k\cdot y_k$, contradicting this inequality.

Q.E.D.

Proof of Proposition 6

Denote by $\mathbf{1}_{N_i}$ the N_i vector with all elements being equal to 1. When all individuals of group

i have the same valuation of the prize, the stake vector can be represented by $\frac{V_i}{N_i}\mathbf{1}_{N_i}$. For any degree of cost-sharing, they will choose the same effort level. For any level of group effort A_i, therefore, the equation $a_{ik}\left(A_i;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)=\frac{A_i}{N_i}$ holds. We can show that $E_i\left(A_i;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)=N_i\cdot c_i\left(\frac{A_i}{N_i}\right)$, and $\frac{\partial E_i}{\partial A_i}\left(A_i;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)=c_i'\left(\frac{A_i}{N_i}\right)$.

Consider a stake vector $\hat{\mathbf{v}}_i\neq\frac{V_i}{N_i}\mathbf{1}_{N_i}$. As the average of the prize valuations is constant, $\sum_{k=1}^{N_i}\hat{v}_{ik}=V_i$ holds.

(a) When $\frac{c_i''(a)}{c_i'(a)}$ is strictly decreasing, Corollary 2 and the convexity of $c_i'(a)$ imply

$$\frac{\partial E_i}{\partial A_i}(A_i;\hat{\mathbf{v}}_i)>\frac{1}{N_i}\sum_{k=1}^{N_i}c_i'(a_{ik}(A_i;\hat{\mathbf{v}}_i))\geq c_i'\left(\sum_{k=1}^{N_i}\frac{a_{ik}(A_i;\hat{\mathbf{v}}_i)}{N_i}\right)$$
$$=c_i'\left(\frac{A_i}{N_i}\right)=\frac{\partial E_i}{\partial A_i}\left(A_i;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right).$$

Hence the marginal distorted group costs with the stake vector $\hat{\mathbf{v}}_i$ is always larger than that with $\frac{V_i}{N_i}\mathbf{1}_{N_i}$. On the other hand, the equilibrium degree of cost-sharing of group i with the stake vector $\hat{\mathbf{v}}_i$ is always strictly less than 1 by Proposition 5(a). Hence, we can set $\pi_i(A;\hat{\mathbf{v}}_i)=\pi_i^{UC}(A;\hat{\mathbf{v}}_i)$ without loss of generality. The assumption $\sum_{k=1}^{N_i}\hat{v}_{ik}=V_i$ and the equation (18) imply that

$$\pi_i(A;\hat{\mathbf{v}}_i)=\pi_i^{UC}(A;\hat{\mathbf{v}}_i)<\pi_i^{UC}\left(A;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)\leq\pi_i\left(A;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)\text{ for all } A>0.$$

Let A^* be the equilibrium total effort when the stake vector of group i is $\frac{V_i}{N_i}\mathbf{1}_{N_i}$, and let A^{**} be the equilibrium effort when the stake vector of group i is $\hat{\mathbf{v}}_i$. Remembering the necessary condition of the equilibrium total effort with the share functions, we have

$$\sum_{j\neq i}\pi_j(A^{**};\mathbf{v}_j)+\pi_i(A^{**};\hat{\mathbf{v}}_i)=1=\sum_{j\neq i}\pi_j(A^*;\mathbf{v}_j)+\pi_i\left(A^*;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)$$
$$>\sum_{j\neq i}\pi_j(A^*;\mathbf{v}_j)+\pi_i(A^*;\hat{\mathbf{v}}_i).$$

and $A^{**}<A^*$ because the share functions are decreasing in A. Then, $\sum_{j\neq i}\pi_j(A^{**};\mathbf{v}_j)>\sum_{j\neq i}\pi_j(A^*;\mathbf{v}_j)$ and we get that $\pi_i(A^{**};\hat{\mathbf{v}}_i)<\pi_i\left(A^*;\frac{V_i}{N_i}\mathbf{1}_{N_i}\right)$.

(b) When $\frac{c_i''(a)}{c_i'(a)}$ is increasing, a change in the stake vector from $\frac{V_i}{N_i}\mathbf{1}_{N_i}$ to $\hat{\mathbf{v}}_i$ in group i does not lead to a change in the cost-sharing rule. The costs of individuals belonging to the group are still fully shared. Then, the choices by individuals in the group obey the same first-order condition as in the case where they have the effort cost function $\frac{1}{N_i}\cdot c_i(a)$ under no device of selective incentives. When $c_i(a)$ is strictly convex, so is $\frac{1}{N_i}\cdot c_i(a)$. Hence we can apply a result by Nitzan and Ueda (2013) on the relation of intra-group heterogeneity and strictly convex marginal effort costs (their Proposition 4) to derive the desired result.

Q.E.D.

Proof of Proposition 7

Since the contributions induced under a cost-sharing rule are the same as in the original model, we have (14) as the relation between each individual's contribution and the aggregate effort intended by the group leader. Because of the convexity of $\hat{E}_i(A_i;\mathbf{v}_i)$,

$$0=\frac{\sum_{j\neq i}A_j}{A^2}N_i-\hat{E}_i'(A_i;\mathbf{v}_i)=\frac{\sum_{j\neq i}A_j}{A^2}N_i-\sum_{k=1}^{N_i}\frac{c_i'(a_{ik}(A_i;\mathbf{v}_i))}{v_{ik}}\cdot\frac{\dfrac{v_{ik}}{c_i''(a_{ik}(A_i;\mathbf{v}_i))}}{\sum_{p=1}^{N_i}\dfrac{v_{ip}}{c_i''(a_{ip}(A_i;\mathbf{v}_i))}}$$

is a first-order necessary and sufficient condition for the first-best group effort.

The contribution of each individual belonging to the group satisfies the equation

$$\frac{\sum_{j\neq i}A_j}{A^2}=\gamma_i\cdot\frac{1}{v_{ik}}\cdot c_i'(a_{ik}(A_i;\mathbf{v}_i))\quad\text{for all}\quad k=1,\cdots,N_i.$$

Hence, the leader induces the first-best group effort by setting $\gamma_i=\frac{1}{N_i}$.

Q.E.D.

Proof of Proposition 8

The Lehmer mean has the following properties. (i) $L_n(\mathbf{v},q)$ is strictly increasing in q if at least two components of $\mathbf{v}$ are unequal; (ii) $L_n(\mathbf{v},1)=\frac{\sum_{k=1}^n v_k}{n}$; and (iii) $\lim_{q\to\infty}L_n(\mathbf{v},q)=\max\{v_1,\ldots,v_n\}$. Applying these properties to the result of Lemma 4, i.e. equation (13′), the proof of the claims in Proposition 8 are directly derived.

Q.E.D.

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