Death and Capital

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Abstract

This paper examines the impact of adult mortality on the pattern of investment and economic development. In the presence of high mortality risk and imperfect annuities market, altruistic parents invest more in tangible assets (physical capital, land) that are readily transferable to future generations compared to intangible human capital. This differential effect of mortality can translate into divergent growth paths for economies, differing willingness to adopt modern skill-intensive technologies as well as a late transition from physical to human capital accumulation. Parental altruism can substitute for the absence of annuities reasonably well: investment in tangible assets is typically higher under missing annuities.

KEYWORDS: Life Expectancy, Mortality, Health, Physical Capital, Human Capital, Parental Altruism, Growth

JEL CLASSIFICATION: I10, O10, O40

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1 Introduction

This paper studies the impact of adult mortality on the choice between different income generating assets and its consequence for intergenerational transfer and economic development. We differentiate between physical assets and human capital as alternative sources of future income, one of the key distinction being the latter's "inalienability" (Hart and Moore, 1994). Physical assets such as land and capital are readily transferable across people in a way that human capital is not. This difference becomes especially significant when an investor faces lifetime uncertainty that can cut short his amortization period.

Transferability of physical assets implies that well-functioning annuity markets can deliver a risk-free return on it, but not on human capital. Lifetime uncertainty will therefore tilt portfolio allocation in favour of physical assets. In developing countries where mortality risks are high, the predominant form of asset accumulation will consequently be land and physical capital. Patterns of investment and production will shift towards human capital only when adult survival rates improve with the process of development.

This differential impact of mortality on human capital is not predicated on the availability of annuities, however. Markets in such instruments may be absent in developing countries and returns on both physical and human capital subject to lifetime uncertainties. This is where the transferability of physical assets becomes salient. Apart from its economic returns, when altruistic parents derive pleasure from bequests, the utility of an asset depends on its transferability to the future generation. The possibility that an investor may die prematurely but leave some of his physical assets for his survivors enhances the internal return on physical assets vis-a-vis human capital, even when the annuity market is completely missing. Indeed in the early stages of development when markets are yet to fully develop, strong parental altruism can substitute for a missing annuities market, creating incentives to invest in physical over human capital.

That adult mortality impacts the return on physical or human capital and thereby, overall investment and growth is well known in the literature (see, for example, Andersen and Bhattacharya, 2014, Blackburn and Cipriani, 1998, Bhattacharya and Qiao, 2007, Bembrilla, 2016, Chakraborty, 2004 and Zhang *et al.*, 2013 for various mechanisms). These studies either focus on the relationship between mortality and the effective rate of time preference or on a single growth-promoting asset, usually human capital. Razin (1976) is an early work to recognize that mortality risk distinguishes human capital investment from other types over the lifecycle. Razin's analysis, however, is restricted to partial equilibrium where rates of return are exogenous. In a dynamic general equilibrium framework, asset returns respond to factor accumulation and incentives change over time. By identifying more clearly the portfolio choice margin in a dynamic setting, we highlight its relative importance at various stages of development and ascertain its robustness to the availability of insurance. If longevity is positively associated with modern economic development, physical capital will be the prime engine of prosperity in the early stages of development, gradually displaced by human capital as living conditions improve.

Economic underdevelopment is often attributed to market imperfections. We show that parental altruism, if strong enough, can substitute for imperfect annuity markets even though complete diversification of mortality risks may not be possible. In this our work is an extension of Kotlikoff and Spivak (1981) who show that resource sharing between household members with independent mortality risk can substantially compensate for missing annuities. In the present context, that altruistic families over-accumulate physical assets implies that the cost of epidemic shocks would be relatively small in developing countries that face substantially high mortality risks. Quantitative evidence presented in Chakraborty and Perez-Sebastian (2016) point to the importance of this self-insurance mechanism.

The existing literature on annuities identifies the bequest motive as one reason why households do not opt for full annuitization of wealth under lifetime uncertainty. For example, Lockwood (2012) shows that agents with plausible bequest motives are better off by not annuitizing any wealth at available (not actuarially fair) rates (see also Davidoff *et al.*, 2005). Thus existence of bequest motives compensates for incomplete annuities market in terms of welfare. In comparison, here the existence of bequest motives is shown to compensate for incomplete annuities market in terms of other outcomes, specifically the investment rate, thereby connecting market incompleteness and lifetime uncertainty to economic growth.[1]

The structure of the paper is as follows. The next two sections present the overall framework and analyze the extent to which intergenerational altruism can compensate for missing annuity markets when individuals invest in a single tangible asset. Human capital is introduced in section 4. A general equilibrium version in section 5 incorporates pecuniary externalities and life insurance. Section 6 concludes.

[1] This paper also relates to recent works on adult survival and economic development in lifecycle models, e.g. Boucekkine *et al.* (2009), Lancia and Prarolo (2012), Prettner and Canning, (2014), Ricci and Zachariadis (2013) and Varvarigos and Zakaria (2013).

2 Structure of the Economy

In a discrete-time overlapping-generations economy individuals potentially live for two periods that we label "youth" and "middle-age". Individuals live in youth for sure but their survival into middle-age is dictated by a constant survival probability $p \in [0, 1]$. At the end of youth, each individual gives birth to a single offspring prior to realizing the mortality shock. Parents are purely altruistic in the sense of Becker (1981).

Individuals are endowed with a share of the family income in youth, which constitutes their first period income. They also inherit the tangible asset stock of the family (e.g. physical capital and/or land) upon the death of the parent. First period income is used for consumption, for investment in physical assets, and (in some cases) for acquiring human capital. The latter two activities determine future income. If an agent survives into middle-age he consumes a part of his second period income and transfers the remainder to his offspring as intended bequest. When he does not survive, his share of second period income either goes to the annuity issuer (in the case of perfect annuities) or to her offspring as unintended bequests (when annuities markets are absent). Parents derive utility from both types of bequests.

All agents in a generation are identical *ex ante*. The expected lifetime utility V_t of a young adult at t with income endowment y_t received either as intended or unintended bequest is

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}. \tag{1}$$

Here $\beta \in (0, 1)$ is the subjective discount rate, $\gamma > 0$ represents the intensity of parental altruism and utility from death has been normalized to zero. Even though altruism is pure in that parents care about their offsprings' lifetime welfare, they do not necessarily discount their offsprings' lifetime utility at the same rate as they discount their own future consumption. In fact it may be plausibly assumed that altruism is limited in the sense $\gamma \leq \beta$.

3 Altruism Substituting for Missing Annuities: A Single Tangible Asset

We first study how the presence of a bequest motive affects investment under mortality risk. Assume there is a single tangible asset, land. A land stock of T generates the income $f(T)$ where the production function satisfies $f(0) = 0$, $f' > 0$ and $f'' < 0$.

Let $1-\theta$ denote the share of output in any period that a parent intends to share with his offspring; $\theta \in (0,1)$ is exogenously given by social customs and convention. If the parent is alive in middle-age, he consumes $\theta f(T)$ leaving the rest for his offspring. If he does not survive, that $1-\theta$ share goes either to the annuity issuer (under perfect annuities) or to the offspring (when annuities are unavailable).

3.1 Optimization under Perfect Annuities

A young adult's decision at t is

$$\max\ u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t &= (1-\theta) f(T_t), \\ c_{2t+1} &= \theta f(T_{t+1})/p, \\ T_{t+1} &= (1-\delta) T_t + x_t, \end{aligned}$$

where x is land investment and $\delta \in [0,1]$ is the depreciation rate of land quality.[2] The middle-age constraint incorporates perfect annuities. Since the parent is committed to sharing $1-\theta$ fraction of the family income that period, she can pledge only $\theta f(T_{t+1})$ to the annuity issuer. Zero expected profits in the annuity market ensure actuarially fair annuities, that is, the annuity pays $\theta f(T_{t+1})/p$ in the event of survival while the annuity issuer keeps $\theta f(T_{t+1})$ in the event of death. The expected return on investment is independent of mortality risk.

As mentioned before, parental premature death has no effect on the offspring's budget constraints (given T_t). Rewrite the optimization problem as the dynamic programming problem (DPP)

$$V(T_t) = \max_{\{T_{t+1}\}} \{u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma V(T_{t+1})\}$$

subject to

$$\begin{aligned} c_{1t} &= (1-\theta) f(T_t) + (1-\delta) T_t - T_{t+1}, \\ c_{2t+1} &= \theta f(T_{t+1})/p. \end{aligned}$$

[2] Land investment blurs the distinction between replicable physical capital and non-replicable land. T should be interpreted as the quality-adjusted stock of a family's landholdings.

The necessary and sufficient first order condition for T_{t+1}

$$u'(c_{1t}) = \beta\theta u'(c_{2t+1})f'(T_{t+1}) + \gamma V'(T_{t+1})$$

combined with the envelope condition

$$V'(T_t) = [1-\delta + (1-\theta)f'(T_t)]u'(c_{1t})$$

leads to the Euler equation

$$u'(c_{1t}) = \beta\theta u'(c_{2t+1})f'(T_{t+1}) + \gamma[1-\delta+(1-\theta)f'(T_{t+1})]u'(c_{1t+1}) \tag{2}$$

that equates the marginal cost of land investment to the expected marginal benefit appropriately discounted. It is instructive to first establish some closed-form solutions for the investment rate under logarithmic and linear utility, full depreciation of land quality and constant output elasticity of land.

Example 1: Logarithmic Preferences

Suppose $u(c) = \ln c$, $f(T) = AT^{\alpha}$, $\delta = 1$. Then (2) becomes

$$\frac{1}{(1-\theta)AT_t^{\alpha} - T_{t+1}} = \frac{\alpha\beta p A T_{t+1}^{\alpha-1}}{AT_{t+1}^{\alpha}} + \frac{\alpha\gamma(1-\theta)AT_{t+1}^{\alpha-1}}{(1-\theta)AT_{t+1}^{\alpha} - T_{t+2}} \tag{3}$$

Using a guess-and-verify approach it can be obtained that

$$T_{t+1} = \frac{\alpha(\beta p+\gamma)}{1+\alpha\beta p}(1-\theta)AT_t^{\alpha}. \tag{4}$$

The investment rate $\alpha(\beta p+\gamma)/(1+\alpha\beta p)$ is increasing in the survival probability as long as $\alpha\gamma < 1$. Concavity of the policy function ensures that irrespective of initial land holding T_0, all dynasties eventually converge to the same T^*, the non-trivial fixed point of the mapping above, as long as they face the same p. If families differed in their survival rates as well, those facing longer lives would converge to a higher steady-state wealth. Even though actuarially fair annuity markets ensure that consumption smoothing does not depend on lifetime uncertainty (Barro and Friedman, 1977), an increase in p affects the saving rate since annuities now offer a lower return on saving for the same level of investment.

Example 2: Linear Preferences

Under linear preferences, $\delta = 1$ and $f(T) = AT^{\alpha}$, equation (2) leads to

$$T_{t+1} = [\alpha A \{\beta\theta + \gamma(1-\theta)\}]^{1/(1-\alpha)},$$

as long as

$$T_t \geq \left[\frac{[\alpha A \{\beta\theta + \gamma(1-\theta)\}]^{1/(1-\alpha)}}{A(1-\theta)} \right]^{1/\alpha} \equiv \hat{T}.$$

Otherwise the individual is at a constrained optimum where he invests his entire first period income

$$T_{t+1} = (1-\theta)AT_t^{\alpha}$$

and consumes only in middle-age. In neither case, though, does investment depend on p. All that the consumer values is expected returns which is independent of p.

3.2 Optimization under Missing Markets

When annuities are unavailable, the offspring's initial income depends on parental survival whose realization we denote by $z_t \in \{a, d\}$ corresponding to "alive" and "deceased" respectively. Denote the offspring's initial endowment as

$$y_t = y(T_t, z_t) = \begin{cases} (1-\delta)T_t + (1-\theta)f(T_t), & \text{if } z_t = a \\ (1-\delta)T_t + f(T_t), & \text{if } z_t = d \end{cases}$$

The DPP in this case is

$$V(T_t, z_t) = \max \{u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V(T_{t+1}, z_{t+1})\}$$

subject to

$$\begin{aligned} c_{1t} &= y(T_t, z_t) - T_{t+1} \\ c_{2t+1} &= \theta f(T_{t+1}) \\ z_{t+1} &\sim iid \end{aligned}$$

where expectations are taken with respect to z_{t+1}. The corresponding Euler equation is

$$u'(c_{1t}) = \beta p \theta u'(c_{2t+1}) f'(T_{t+1}) + \gamma E_t \left[u'(c_{1t+1}) y_1(T_{t+1}, z_{t+1}) \right] \tag{5}$$

where y_i denotes the partial derivative with respect to the i-th argument. Note how land yields a psychic return due to unintended bequests.

Example 1: Logarithmic Preferences

Under the same assumptions as before

$$y_1(T,z) = \begin{cases} (1-\theta)\alpha A T_t^{\alpha-1}, & \text{if } z_t = a \\ \alpha A T_t^{\alpha-1}, & \text{if } z_t = d \end{cases}$$

Optimal land investment at t will obviously depend on z_t so that $T_{t+1} = T(T_t, z_t)$. But since z takes discrete values and y depends on z only through a scaling constant, z affects $T(\cdot)$ through a scaling constant alone.

Given T_t, suppose we denote future assets as $T_{a,t+1}$ and $T_{d,t+1}$ for the two realizations of parental survival. Then it is easy to establish from (5) that

$$T_{a,t+1} = \alpha(1-\theta-\mu)\left[\beta p + \frac{\gamma p(1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu}\right] A T_t^{\alpha},$$

and

$$T_{d,t+1} = \alpha(1-\nu)\left[\beta p + \frac{\gamma p(1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu}\right] A T_t^{\alpha}.$$

where

$$\nu = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p} \tag{6}$$

and

$$\mu = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p}(1-\theta). \tag{7}$$

The investment propensity in both cases is identical and exactly as before, $\alpha(\beta p + \gamma)/(1 + \alpha\beta p)$.

Example 2: Linear Preference

Once again, equation (5) becomes

$$1 = \beta p \theta f'(T_{t+1}) + \gamma\{p(1-\theta) + (1-p)\} f'(T_{t+1})$$

which gives, assuming the same functional form for f,

$$T_{t+1} = [\alpha A\{p\beta\theta + \gamma\,(1-p\theta)\}]^{1/(1-\alpha)}$$

if

$$T_t \geq \left[\frac{[\alpha A\,\{p\beta\theta + \gamma(1-p\theta)\}]^{1/(1-\alpha)}}{A(1-\theta)}\right]^{1/\alpha} \equiv \bar{T},$$

and

$$T_{t+1} = (1-\theta)AT_t^{\alpha}$$

otherwise. Since marginal utility is independent of consumption level, bequest/income uncertainty has no effect on land investment. Note also that investment in the unconstrained case is lower under missing markets as long as $\beta > \gamma$. It is only when $\beta = \gamma$ (which also ensures that $\bar{T} = \hat{T}$) that the altruistic motive completely eliminates utility loss due to lifetime uncertainty.

Average investment under missing annuity markets is higher than under annuities when preferences are logarithmic even though the investment *rate* itself is invariant to the availability of markets. For linear utility, however, a missing annuity market does depress investment unless parents value an extra unit of their offspring's consumption exactly as they would their own. In the first case, the degree of parental "selfishness" has no bearing on the investment rate. In the second case, it does.

These parametric examples identify two effects at work: the role of consumption smoothing and the relative valuation parents place on their own consumption vis-a-vis their children's. To understand how investment responds more generally to the two effects, consider next CES preferences.

3.3 A More General Case

Suppose that $u(c) = c^{1-\sigma}/(1-\sigma)$ where we restrict $\sigma \in (0,1)$ for now.

3.3.1 The Net Marginal Benefit of Missing Annuities

Whether or not investment suffers from a lack of annuities ultimately relates to whether or not altruism can compensate for the utility loss that a parent suffers from the missing market.

Let T denote the parent's landholding. Consider the effect of a sudden disappearance of actuarially fair annuities. Assuming $\delta = 1$, the expected marginal utility loss an individual suffers from this, discounted appropriately, is

$$\begin{aligned}
\Gamma &\equiv \beta\theta\left[u'\left(\frac{\theta f(T)}{p}\right) - pu'\left(\theta f(T)\right)\right] f'(T) \\
&= \left[\beta\theta[\theta f(T)]^{-\sigma} p^{\sigma}(1-p^{1-\sigma})\right] f'(T).
\end{aligned}$$

The marginal benefit, on the other hand, comes from the offspring enjoying higher endowment under parental death (unintended bequest) which the parent takes into consideration. Weighted by the degree of parental altruism, and denoting by T' the offspring's landholidng under parental death, this benefit is

$$\begin{aligned}\Psi &\equiv \gamma \left[\begin{array}{c} p(1-\theta)\left\{u'\left((1-\theta)f(T)-T\right)\right\} f'(T) + (1-p)u'\left(f(T)-T'\right) f'(T) \\ -(1-\theta)\left\{u'\left((1-\theta)f(T)-T\right)\right\} \end{array}\right] f'(T) \\ &= \gamma(1-p)\left[\{f(T)-T'\}^{-\sigma} - (1-\theta)\{(1-\theta)f(T)-T\}^{-\sigma}\right] f'(T).\end{aligned}$$

Denote by $\tilde{\phi}$ the investment propensity out of first period income. Under missing annuities this income is different (higher) for an offspring whose parent dies prematurely. But suppose the individual maintains his savings propensity when annuity markets "disappear". By exclusively identifying the effect of missing annuities on the incentive to invest, we foreshadow how optimal investment would respond to missing annuities and hence, whether missing markets are costly for investment. Given our assumptions above, the net marginal benefit from missing markets (ignoring the common terms) is

$$\Delta(p) \equiv \Psi - \Gamma = \gamma(1-p)(1-\tilde{\phi})^{-\sigma}[1-(1-\theta)^{1-\sigma}] - \beta\theta^{1-\sigma}p^{\sigma}(1-p^{1-\sigma}).$$

For linear utility ($\sigma = 0$), this simplifies to $\Delta = -(\beta-\gamma)(1-p)\theta$. As long as $\beta > \gamma$, the consumption loss from missing markets cannot compensate for the lifetime utility gain the offspring enjoys. The individual would lower his investment in this case (section 3.2.2). When $\beta = \gamma$, that is when parents value an extra unit of their offspring's consumption exactly as they would their own, altruism fully compensates for missing markets and investment is unaffected.

For logarithmic utility ($\sigma = 1$), on the other hand, $\Delta = 0$ irrespective of p, β and γ. This happens because both the marginal benefit and loss are zero. It is, in fact, the familiar balancing of income and substitution effects in a different guise (the value function is logarithmic too). To see this, suppose the individual is consuming an endowment of future consumption goods ω that is priced at unity and yield the marginal utility $u'(\omega)$. Suppose he were instead given a higher endowment, say $a\omega$, at the price $a > 1$. The individual would prefer the new endowment if $u'(a\omega)/u(\omega) > a$. The endowment effect is measured by a pure income effect while the price change is purely the substitution effect. Since under log preferences $u'(a\omega)/u(\omega) = a$, these two effects cancel out.

For CES preference, the response of Δ to p is shown numerically in Figures 1 and 2. The net marginal benefit is positive for all values of p and decreasing in p (Figure 1). In these cases, missing annuities leaves the individual no worse off and usually strictly better off (at the same investment rate as under annuities). It follows then that the lack of annuities would actually encourage investment relative to actuarially fair annuity markets. The first two panels of Figure 1 show that this result does not depend sensitively on the value of σ: lower σ has the effect of raising the relative return of accidental bequests since the marginal utility of the offspring is less sensitive to windfall gains.

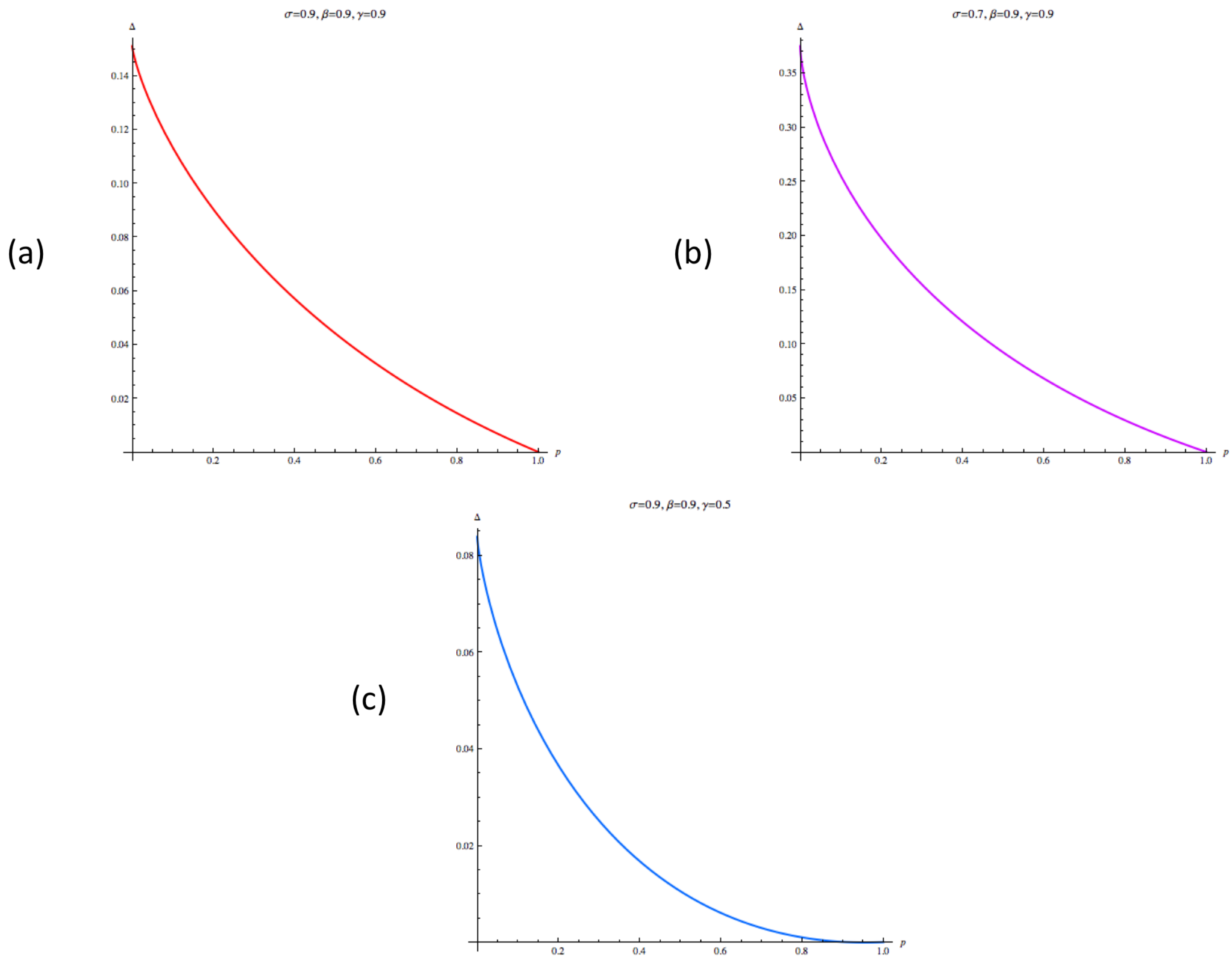

Figure 1: The Net Marginal Benefit of Investing in Tangible Assets without Annuities relative to with Annuities

As one expects from the linear case above, a higher subjective discount rate (β) relative to altruism intensity (γ) tends to reduce the net marginal benefit of missing markets. An increase in parental "selfishness" would then raise the cost of missing annuity markets. The third panel of Figure 1 uses the same set of numerical values as the other two except for $\gamma < \beta$.

The monotonicity of the net marginal benefit function does depend sensitively on θ. For instance, under $\sigma = 0.9$, when θ becomes smaller, from $3/4$ (first panel of Figure 2) to $1/2$ (second panel), p has a non monotonic effect on Δ. The offspring gets a relatively large share of output now, which decreases by a lot the marginal utility of the offspring's consumption under parental death. This reduces the attractiveness of the accidental bequest motive to the parent, unless p is relatively small as well in which case the individual's expected marginal utility from self-consumption is also small.

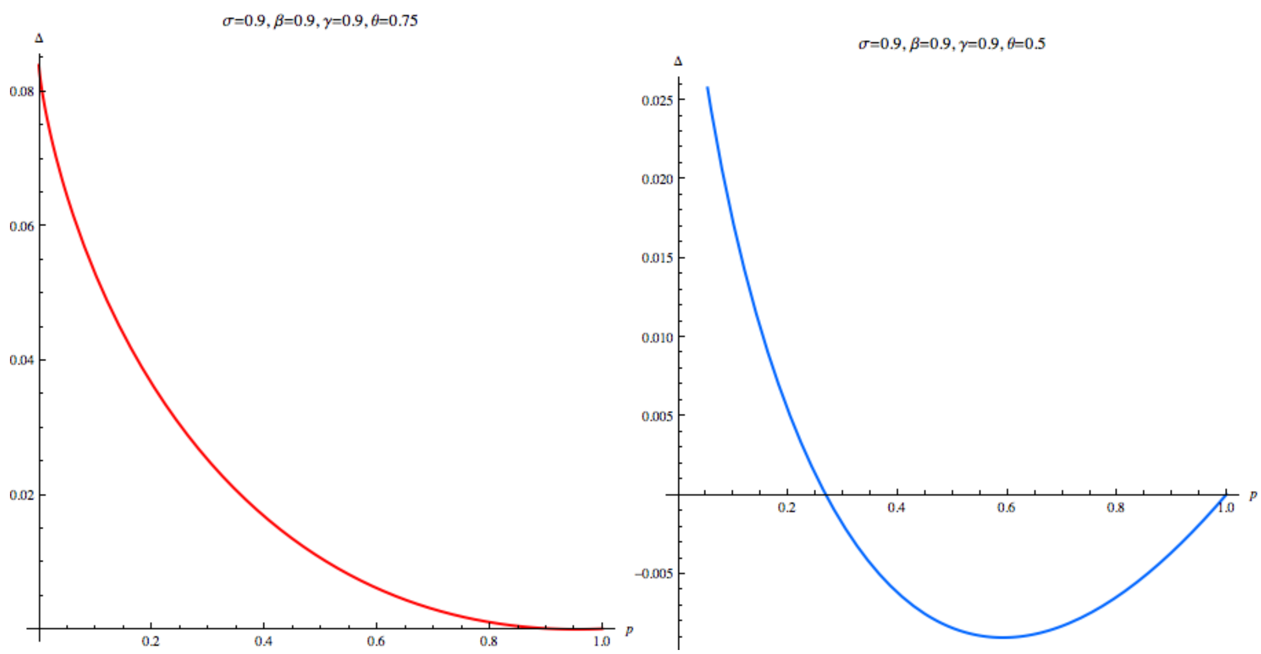

Figure 2: Non-monotonicity of Net Marginal Benefits

3.3.2 Optimal Investment with and without Annuity Markets

Consider now how the investment rate optimally responds to the presence or absence of annuities. First, we allow $\sigma > 1$[3] and assume $\alpha = 1$. We can make considerable progress under the latter assumption without having to solve the dynamic path of investment.

Qualitative results do not depend on this assumption. In fact, the log case from above (for $\alpha = 1$) will still be nested. But the linear case will not be because of corner solutions.

[3] Since this generates negative utility from being alive, we require a concomitantly large utility from death to counterbalance it. This is ignored as it has no bearing on optimal choices.

For linear utility and $f(T) = AT$, land investment is independent of p under annuity markets as long as $[\beta\theta + \gamma(1-\theta)]A \geq 1$. When annuities are missing, investment is positive and independent of p iff $p \geq [1-\gamma(1-p\theta)]/(\beta\theta A)$, zero otherwise. Investment is now a weakly increasing function of the survival probability.

Denote by ϕ the investment rate under annuity markets. Under actuarially fair annuities, consumption levels for a given T are

$$\begin{aligned} c_1 &= y - T', \\ c_2 &= \theta A T'/p, \end{aligned}$$

where $y = (1-\theta)AT$, T' denotes the land stock one period ahead and T'' two periods ahead. The Euler equation

$$[(1-\theta)AT - T']^{-\sigma} = \theta\beta A[\theta AT'/p]^{-\sigma} + \gamma(1-\theta)A[(1-\theta)AT' - T'']^{-\sigma}$$

under CES preference implicitly defines ϕ as a function of the survival probability p

$$\left(\frac{1-\phi}{\phi}\right)^{-\sigma} = A^{1-\sigma}\left[\beta p^{\sigma}\theta^{1-\sigma} + \gamma(1-\theta)^{1-\sigma}(1-\phi)^{-\sigma}\right].$$

When annuity markets are missing, consumption levels and investment choices depend on parental survival. But optimal investment *rates* under parental survival and death are identical for a linear production function. Let ψ denote the investment rates in this case and denote by T_a' and T_d' investments under parental survival and death respectively. The Euler equation, given an income endowment y, is now

$$\begin{aligned} [y - T_a']^{-\sigma} &= \theta\beta p A[\theta A T_a']^{-\sigma} + \gamma A[p(1-\theta)\{(1-\theta)AT_a' - T_a''\}^{-\sigma} \\ &\quad + (1-p)\{AT_a' - T_d''\}^{-\sigma}] \end{aligned}$$

where without loss of generality we have specified the problem for an adult whose parent survives in middle-age. Simplifying, the investment rate $\psi(p)$ solves[4]

$$\left(\frac{1-\psi}{\psi}\right)^{-\sigma} = A^{1-\sigma}\left[\beta p\theta^{1-\sigma} + \gamma p(1-\theta)^{1-\sigma}(1-\psi)^{-\sigma} + \gamma(1-p)(1-\psi)^{-\sigma}\right].$$

Figure 3 compares ϕ to ψ for various values of p. At $p = 0$, the two investment rates are

$$\begin{aligned} \phi &= [\gamma A^{1-\sigma}(1-\theta)^{1-\sigma}]^{1/\sigma}, \\ \psi &= [\gamma A^{1-\sigma}]^{1/\sigma}. \end{aligned}$$

[4]Substituting $\sigma = 1$ gives us the investment rates $\phi = \psi = (\gamma + \beta p)/(1 + \beta p)$, same as in (4), (11) and (12) under $\alpha = 1$.

Clearly $\psi(0) > \phi(0)$ whenever $\sigma < 1$, with the sign reversed for $\sigma > 1$.[5] At $p = 1$, the two rates are equal since parental bequests are same in both cases. As the first three panels of Figure 3 imply, investment is higher under missing annuities and this result is not qualitatively affected by γ or σ as long as the latter is below 1. The fourth panel of Figure 3 compares the investment rates for $\sigma = 1.1$. In this instance, investment under missing markets no longer uniformly dominates investment under annuities, the cost of missing markets being higher at higher values of the mortality risk (low p).

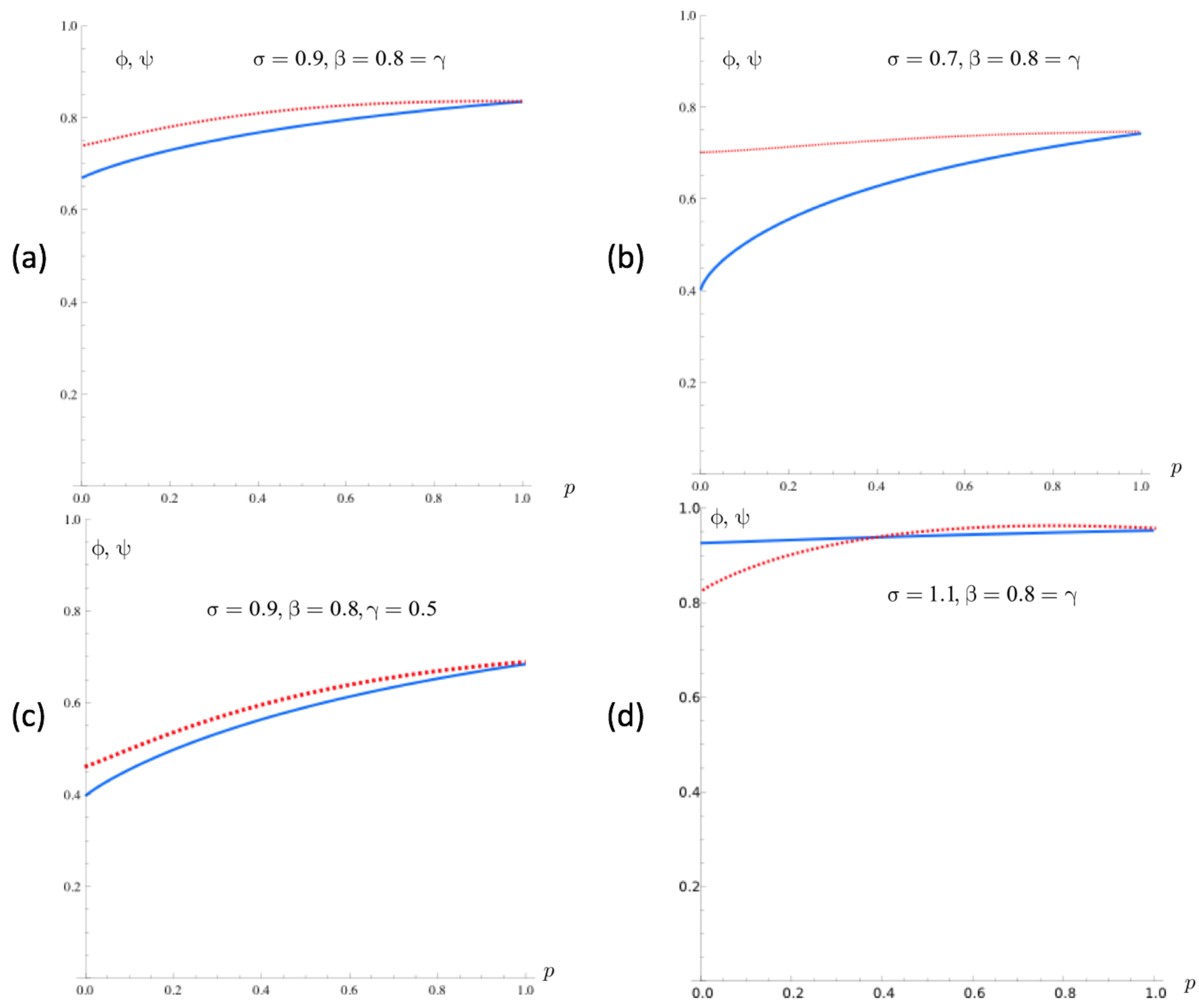

Figure 3: Tangible Investment with (solid blue) and without Annuities (dotted red)

[5]There is a discontinuity in ϕ at $p = 0$. For arbitrarily small p, annuity purchases are positive but zero at $p = 0$.

4 Mortality, Altruism and the Pattern of Investment

We turn to the effect of lifetime uncertainty on portfolio choice. Specifically suppose people now have access to a second investment vehicle, human capital. As before, suppose that the first asset is land. This specific interpretation is important now since we assume returns to the tangible asset or human capital do not depend on the other asset. This is more likely of traditional activities involving land – farming and small-scale business enterprise – than modern technologies involving physical capital.

The family shares its income from both land and human capital. Specifically a middle-aged parent shares $1 - \theta_1$ fraction of the family's land income and $1 - \theta_2$ fraction of his labor income with the child. If all that is being shared is family income, it is natural to assume $\theta_1 = \theta_2$. Human capital opportunities (urban areas), though, can be geographically removed from traditional farming (rural areas) requiring skilled workers to migrate elsewhere. If sharing of labor earnings is relatively more difficult, $\theta_2 < \theta_1$. It is also conceivable that land ownership in developing countries is not as well defined as human capital ownership (which is embodied in a person in any case). Consequently land is a family property with every member having some right over its produce: $\theta_1 > 0, \theta_2 = 0$. This, of course, implies the young can contribute to farm activities without seriously hampering their learning process.

All individuals are born with the same level of human capital (normalized to zero). Denote by e_t parental investment in human capital at time t and labor earnings in the second period of life as $h_{t+1} = g(e_t)$ where g is an increasing concave function satisfying $g(0) = 0$. Normalize the return to human capital to unity. We first establish results under linear utility which highlights the role of asset returns and their dependence on annuities and altruism. We show that the non-transferability of human capital across generations tilts investment in favor of tangible assets in the absence of annuities.

4.1 Optimization under Perfect Annuities

Given his income y_t, an adult in period t maximizes his expected lifetime utility

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t + e_t &= y_t, \\ c_{t+1} &= \theta_1 f(T_{t+1})/p + \theta_2 g(e_t), \\ T_{t+1} &= (1-\delta)T_t + x_t. \end{aligned}$$

Unlike before the offspring's first period income/inheritance is uncertain under annuities as long as $\theta_2 > 0$:

$$y_{t+1} = \begin{cases} (1-\theta_1)f(T_{t+1}) + (1-\theta_2)g(e_t), & \text{with prob. } p, \\ (1-\theta_1)f(T_{t+1}), & \text{with prob.} 1-p. \end{cases}$$

Note specifically the non-transferability of human capital – the offspring enjoys land income when the parent dies prematurely but not his labor earnings. Under $\delta = 1$, the first order and Envelope conditions lead to the pair of Euler equations:

$$u'(c_{1t}) = p\left[\theta_2 \beta u'(c_{2t+1}) + \gamma(1-\theta_2)u'(c^a_{1t+1})\right] g'(e_t)$$

for human capital investment and

$$u'(c_{1t}) = \left[\theta_1 \beta u'(c_{2t+1}) + \gamma(1-\theta_1)\left[p u'(c^a_{1t}) + (1-p)u'(c^d_{1t})\right]\right] f'(T_{t+1})$$

for land investment.

Example: Linear Preferences

Linear utility requires us to explicitly recognize non-negativity constraints on consumption levels. Reformulate the problem as

$$\begin{aligned} V(T_t, e_{t-1}, z_t) &= \max_{\{T_{t+1}, e_t\}} \{y_t - T_{t+1} - e_t + \beta p\left[\theta_1 f(T_{t+1})/p + \theta_2 g(e_t)\right] + \gamma E_t V(T_{t+1}, e_t, z_{t+1}\} \\ \text{subject to} &: \quad T_{t+1} + e_t \leq y_t. \end{aligned}$$

Denoting the Lagrange multiplier associated with the constraint by λ, optimality conditions are

$$-1 + p\left[\theta_2 \beta + \gamma(1-\theta_2)\right] g'(e_t) = \lambda$$

for e_t and

$$-1 + \left[\theta_1 \beta + \gamma(1-\theta_1)\right] f'(T_{t+1}) = \lambda$$

for T_{t+1}.

If $\lambda > 0$ (equivalently $c_{1t} = 0$), we have

$$-1 + p\left[\theta_2\beta + \gamma(1-\theta_2)\right] g'(e_t) = -1 + \left[\theta_1\beta + \gamma(1-\theta_1)\right] f'(T_{t+1})$$

or

$$\frac{g'(e_t)}{f'(T_{t+1})} = \frac{\theta_1\beta + \gamma(1-\theta_1)}{p\left[\theta_2\beta + \gamma(1-\theta_2)\right]}.$$

If we assume now that $f(T) = AT^\alpha$ and $g(e) = Be^\alpha$, then this equation gives us the optimal ratio of investment in human capital vis-a-vis land as

$$\rho \equiv \frac{e_t}{T_{t+1}} = \left[\frac{pB\left[\theta_2\beta + \gamma(1-\theta_2)\right]}{A\left[\theta_1\beta + \gamma(1-\theta_1)\right]}\right]^{1/(1-\alpha)}.$$

Higher longevity evidently tilts investment in favour human capital. Moreover, if the assets are treated symmetrically in terms of intergenerational income sharing ($\theta_1 = \theta_2$), the optimal ratio depends only on relative expected returns.

If, on the other hand, $c_{1t} > 0$ ($\lambda = 0$), the two Euler equations can be solved independently as

$$p\left[\theta_2\beta + \gamma(1-\theta_2)\right] g'(e_t) = 1,$$

and

$$\left[\theta_1\beta + (1-\theta_1)\gamma\right] f'(T_{t+1}) = 1.$$

Optimal solutions for land and human capital investment are now

$$e_t = \left[Bp\left[\theta_2\beta + \gamma(1-\theta_2)\right]\right]^{1/(1-\alpha)}$$

and

$$T_{t+1} = \left[A\left[\theta_1\beta + (1-\theta_1)\gamma\right]\right]^{1/(1-\alpha)}$$

which differ from the previous situation in that land investment is insensitive to mortality. The investment ratio ρ, however, is the same and an increase in p tilts investment in favor of human capital.

4.2 Optimization under Missing Markets

As before when annuities are not available, suppose the offspring enjoys the entire land income. Given his income y_t, an adult in period t maximizes his expected lifetime utility

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned}
c_{1t} + x_t + e_t &= y_t \\
c_{t+1} &= \theta_1 f(T_{t+1}) + \theta_2 g(e_t) \\
T_{t+1} &= (1-\delta) T_t + x_t
\end{aligned}$$

and recognizing that the child's first period income will be stochastic

$$y_{t+1} = \begin{cases} (1-\theta_1) f(T_{t+1}) + (1-\theta_2) g(e_t), & \text{w.p. } p \\ f(T_{t+1}), & \text{w.p.} 1-p \end{cases}$$

Optimality for investment requires

$$u'(c_{1t}) = p\left[\theta_2 \beta u'(c_{2t+1}) + \gamma(1-\theta_2) u'(c^a_{1t+1})\right] g'(e_{t+1})$$

for e_t and

$$u'(c_{1t}) = \left[\theta_1 \beta p u'(c_{2t+1}) + \gamma\left[p(1-\theta_1) u'(c^a_{1t}) + (1-p) u'(c^d_{1t})\right]\right] f'(T_{t+1})$$

for T_{t+1}.

Example: Linear Preferences

When $c_{1t} = 0$, the associated Euler equations lead to

$$-1 + p\left[\theta_2 \beta + \gamma(1-\theta_2)\right] g'(e_t) = -1 + \left[\theta_1 \beta + \gamma p(1-\theta_1) + (1-p)\right] f'(T_{t+1})$$

or

$$\frac{g'(e_t)}{f'(T_{t+1})} = \frac{\theta_1 \beta + \gamma(1-p\theta_1)}{p\left[\theta_2 \beta + \gamma(1-\theta_2)\right]}.$$

For the same technologies as above the optimal ratio of land vis-a-vis human capital investment is now

$$\rho = \left[\frac{B}{A} \frac{p\left[\theta_2 \beta + \gamma(1-\theta_2)\right]}{\left[\theta_1 \beta + \gamma(1-p\theta_1)\right]}\right]^{1/(1-\alpha)}$$

which is lower than under perfect annuities. This is specifically due to land investment being higher.

If $c_{1t} > 0$, the Euler equations can be solved independently and the solutions would be the same as in the previous section with the investment ratio given by

$$\rho = \left[\frac{B}{A} \frac{p\{\theta_2 \beta + \gamma(1-\theta_2)\}}{\gamma + \theta_1 p(\beta - \gamma)}\right]^{1/(1-\alpha)}.$$

4.3 A More General Case

To generalize this result we appeal to CES preferences, full depreciation of land and linear production technologies for the two assets, $f(T) = AT$ and $g(e) = Be$, with $B \geq A$. Without loss of generality we impose $\theta_1 \equiv \theta$ and $\theta_2 = 0$.[6]

We start with the annuity markets case. Let ϕ and η denote the investment propensities in land and human capital. The Euler equations for the two assets are

$$[y - T' - e']^{-\sigma} = \theta\beta A[\theta AT'/p + Be']^{-\sigma} + \gamma(1-\theta)A[(1-\theta)AT' - T'' - e'']^{-\sigma}$$

for an income endowment of y. The investment rates (ϕ, η) solve the pair of equations

$$\begin{aligned} (1-\phi-\eta)^{-\sigma} &= \theta\beta pB\left(\frac{\theta A\phi}{p} + B\eta\right)^{-\sigma} & (8)\\ \left(1 - \frac{A}{pB}\right) &= \gamma A^{1-\sigma}(1-\theta)^{1-\sigma}\phi^{-\sigma} & (9)\end{aligned}$$

The role of p in portfolio allocation can be examined by considering the relative investment rates η/ϕ in human-to-physical capital.

When annuities are missing let ψ and ν denote the investment propensities in land and human capital. The Euler equations for an individual whose parent has survived (and bequeaths y) are

$$\begin{aligned} [y - T'_a - e'_a]^{-\sigma} &= \theta\beta pA[\theta AT'_a + Be'_a]^{-\sigma} + \gamma p(1-\theta)A[(1-\theta)AT' - T''_a - e''_a]^{-\sigma} \\ &\quad + \gamma(1-p)A[AT' - T''_d - e''_d]^{-\sigma}. \end{aligned}$$

The investment rates now solve the pair of equations[7]

$$\begin{aligned} (1-\psi-\nu)^{-\sigma} &= \theta\beta pB(\theta A\psi + B\nu)^{-\sigma} & (10)\\ \left(1 - \frac{A}{B}\right) &= \gamma A^{1-\sigma}\psi^{-\sigma}[p(1-\theta)^{1-\sigma} + (1-p)]. & (11)\end{aligned}$$

Here our object of interest is the response of relative investments ν/ψ to changes in p.

Figures 4 and 5 illustrate the responsive of investments to p in the presence and absence of annuities. The blue solid lines correspond to η/ϕ and the red dashed lines to

[6] $\theta_2 > 0$ would only accentuate the effect of p on human capital investment since premature parental death would eliminate the ability to enjoy a share of parental labor income.

[7] Suppose $B = \omega A$ where $\omega > 1$. For very low values of p, the LHS of equation (9) can turn negative as returns to human capital are not high enough to compensate for mortality risk. To avoid that we restrict to $p \in [1/\omega, 1]$. $\omega > 1$ also ensures that $A < B$ for equation (11).

ν/ψ. In Figure 4(a), both relative investment rates are increasing in survival. While individuals may or may not diversify away mortality risks on physical assets via altruism, p still has a differentially higher impact on human capital investment. The relative investment rates rise faster in Figure 4(b) compared to Figure 4(a) where γ is higher. Since human capital investment is immune to the degree of parental altruism, a lower value of γ does not affect it as much as it dampens physical capital investment. Note also the curvature of the two relative investment rates. Under missing annuities, the switch from physical assets to human capital occurs at a faster rate. Indeed, as Figures 1–3 foreshadowed, land investment is higher for the parameter values used in Figure 4 so that human capital investment is lower relative to the annuities case.

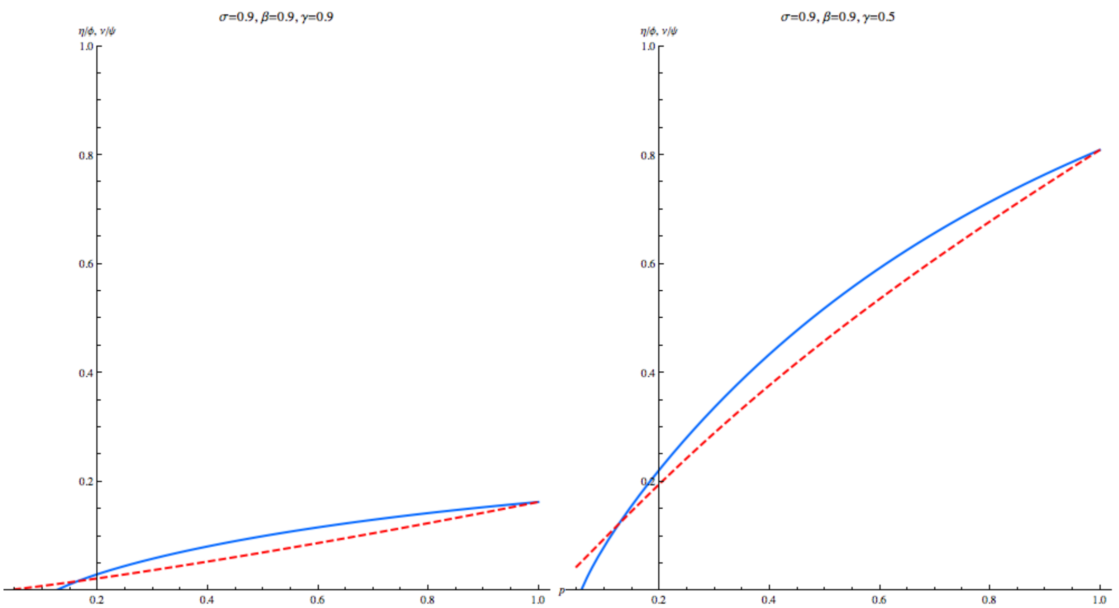

Figure 4: Relative Investment in Human-to-Physical Capital under Annuities (solid blue) and Missing Markets (dotted red)

Finally Figure 5 establishes that our results are not sensitive to the curvature of the utility function: investment in human capital rises faster for a smoother function (lower σ) under both cases because the parent does not have to "compensate" for strongly diminishing marginal utility of the offspring by investing more in land.

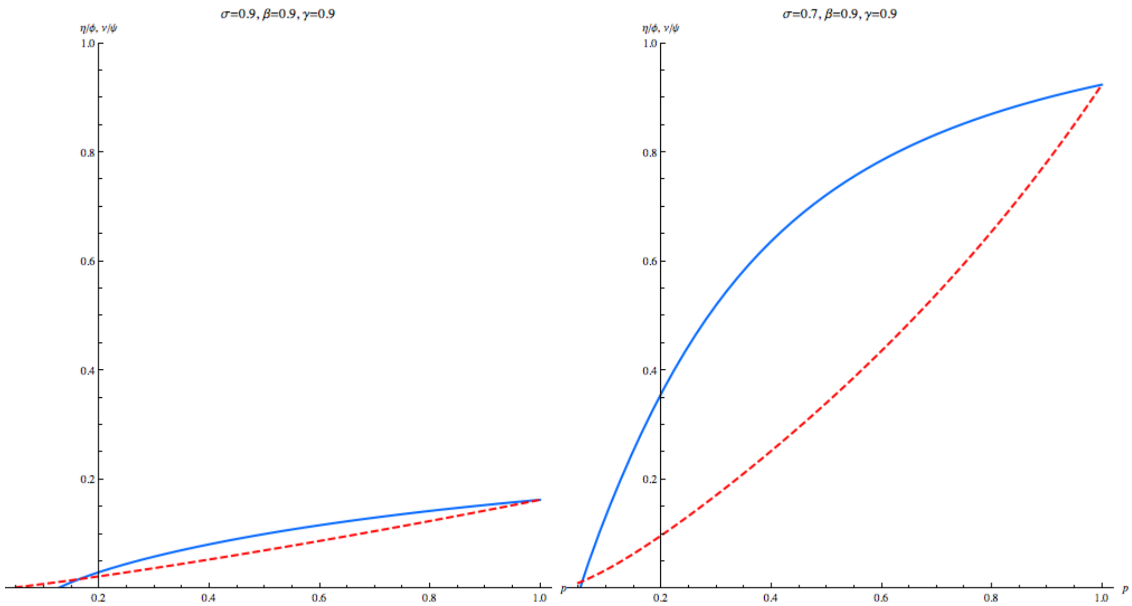

Figure 5: Response of Investment ratio to σ

5 Pecuniary Externalities

The interpretation of the tangible asset is more general than we have posited. Specifically, any form of physical asset that is transferable ought to face the same kind of incentive vis-a-vis inalienable human capital. But there is a key difference between modern forms of physical capital – equipment, machinery, business enterprise, workshop and cottage industry – and land. While it is easy to imagine traditional activities involving land do not involve skills or other forms of human capital, that assumption is harder to justify with physical capital. Physical and human capital may be complementary production inputs and, if so, the accumulation of physical assets can depend on the survival probability through pecuniary externalities that section 4 above rules out.

We establish, by means of a Cobb-Douglas technology that utilizes physical capital and skilled labor to produce a final consumption good, that our intuition from the previous sections generalize. An increase in p has a more pronounced effect on human capital investment and increases its aggregate supply. This raises the return on the complementary input, physical capital, encouraging its accumulation. The net effect is similar to section 4 except that it now tilts investment *and* production towards human capital.

A unique final good is produced from aggregate stocks of physical (K) and human capital (H) using

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha}$$

where $\alpha \in (0,1)$. In perfectly competitive factor and goods markets, wage per efficiency unit of labor and rental on capital (assume $\delta = 1$) are

$$\begin{aligned} w_t &= (1-\alpha)A(K_t/H_t)^{\alpha}, \\ r_t &= \alpha A(H_t/K_t)^{1-\alpha}. \end{aligned} \tag{12}$$

We assume a unit measure of people are born at each date, p fraction of whom survive into middle age. Since agents are *ex ante* identical in their preferences and survival and make the same optimizing choices, denote individual holdings of the two assets by k and h. The aggregate stocks are then $K_t = k_t$ and $H_t = ph_t$ where h is the human capital of each middle-aged person before experiencing their survival shock. Iindividuals do not possess any human capital endowment in youth and consume out of their shares $(1-\theta_1)$ and $(1-\theta_2)$ respectively of the parental capital and labour income. As with the CES case of the previous section, without loss of generality we impose $\theta_1 \equiv \theta$ and $\theta_2 = 0$.

In their youth individuals invest x_t in physical capital and e_t in their human capital that yields asset levels

$$\begin{aligned} k_{t+1} &= f(x_t) \\ h_{t+1} &= g(e_t) \end{aligned}$$

the following period. The production functions f and g are concave and satisfy $f(0) = 0 = g(0)$. Unlike the standard case the rate of transformation of investment into physical capital is not one, an assumption necessary here to allow for relative price effects on k since individuals are risk neutral (see below).

We focus exclusively on the case of missing annuities. The decision problem is to maximize expected lifetime utility

$$V_t \equiv u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} &= y_t - x_t - e_t, \\ c_{2t+1} &= \theta r_{t+1} k_{t+1} + w_{t+1} h_{t+1}, \end{aligned}$$

and stochastic bequests

$$y_{t+1} = \begin{cases} (1-\theta) r_{t+1} k_{t+1}, & \text{w.p. } p \\ r_{t+1} k_{t+1}, & \text{w.p. } 1-p \end{cases}$$

The middle-age budget constraint embodies the assumption that returns to physical capital are shared with the offspring and ownership of that asset is costlessly passed on to him if the parent dies prematurely.

The first order conditions for optimal investment x_t and e_t are

$$u'(c_{1t}) = [\theta\beta p u'(c_{2t+1}) + \gamma \left\{ p(1-\theta)u'(c^a_{1t+1}) + (1-p)u'(c^d_{1t+1}) \right\}] r_{t+1} f'(x_t) \tag{13}$$

and

$$u'(c_{1t}) = \beta p u'(c_{2t+1}) w_{t+1} g'(e_t) \tag{14}$$

respectively. Assume linear utility and

$$\begin{aligned} f(x) &= a x^{\chi}, \\ g(e) &= b e^{\chi}, \end{aligned} \tag{15}$$

where $\chi \in (0,1)$. Suppose also that all families start with a relatively high initial endowment of physical capital k_0 so that the individual is at an unconstrained optima and $c_1 > 0$. Equations (13) and (14) then lead to optimal investment decisions of

$$\begin{aligned} x_t &= [a\chi \{\gamma + p\theta(\beta-\gamma)\} r_{t+1}]^{1/(1-\chi)} \\ e_t &= [b\beta p \chi w_{t+1}]^{1/(1-\chi)} \end{aligned}$$

Individual stocks of the two assets are

$$\begin{aligned} k_{t+1} &= a^{1/(1-\chi)} [\chi\{\gamma + p\theta(\beta-\gamma)\}]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)} \\ h_{t+1} &= b^{1/(1-\chi)} (\chi\beta p)^{\chi/(1-\chi)} w_{t+1}^{\chi/(1-\chi)} \end{aligned}$$

and the ratio of aggregate capital stocks[8]

$$\frac{K_t}{H_t} = \frac{k_t}{p h_t} = \left(\frac{a}{b}\right)^{1/(1-\chi)} \left[\frac{\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi/(1-\chi)} \frac{1}{p} \left(\frac{r_t}{w_t}\right)^{\chi/(1-\chi)}. \tag{16}$$

From (12), on the other hand,

$$\frac{r_t}{w_t} = \frac{\alpha}{1-\alpha} \frac{H_t}{K_t} \tag{17}$$

Using (16) and (17), we can solve for the equilibrium factor ratio

$$\frac{K_t}{H_t} = \frac{a}{b} \left(\frac{\alpha}{1-\alpha}\right)^{\chi} \frac{1}{p^{1-\chi}} \left[\frac{\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi} \tag{18}$$

[8]The *ratio* of aggregate stocks would be the same if individuals were at a constrained optima.

which is a decreasing function of p for $\beta \geq \gamma$.

Investment in physical capital depends positively on its return, r. Since K and H are complementary inputs, an increase in the supply of human capital induced by p, would raise returns to physical capital and encourage its investment. Equilibrium supply of physical capital now depends positively on p. But as equation (18) shows, this second round effect is not enough to bias the equilibrium response away from human capital. There are two effects of p in this equation. The first term involving p on the right-hand side is the direct supply effect: changes in p shift aggregate labor supply for any level of h. The second term is the effect on individual portfolio choice. The ratio of physical-to-human capital evidently does not depend on p solely because of the supply effect.

It is easy to show that aggregate output

$$Y = \Gamma p^{\frac{1-\alpha}{1-\chi}} \left[\frac{1}{\beta} \left\{ \gamma + \theta p(\beta - \gamma) \right\} \right]^{\frac{\alpha\chi}{1-\chi}}$$

depends positively on longevity (as long as $\beta \geq \gamma$). Since both physical and human capital depreciate fully, the economy will jump straight to this steady-state output level assuming a high enough k_0. If capital did not fully depreciate, however, the transition path would also depend on p. Not only would low-p countries converge to a lower steady-state, their transition would be slower too. These high mortality economies would rely more intensively on physical capital, the switch from physical to human capitals as engines of development occurring later and remaining incomplete.

Two implications follow. First, from the expression above, aggregate output is a convex function of p as long as the return to human capital is not too low relative to the return to physical capital (e.g. Fig 6(a)). This means, a reduction in p at high levels of survival, lead to proportionately higher output loss than an equivalent reduction in p at low levels of survival. In effect, a low-p society self-insures against mortality shocks by allocating more towards transferable assets. Of course, output per worker, $y = Y/(1+p)$ may still be concave in p. For the same parameter values as in Fig 6(a), Fig 6(b) shows convexity of output per worker at low values of p, (slight) concavity at higher values. The cost of a mortality shock, such as HIV or ebola outbreaks that affect the adult population, will then tend to be contained because of this portfolio effect.[9]

A second implication is that a transition from physical to human capital based production can be facilitated by health and mortality improvements. The widespread mor-

[9] In other words, diminishing marginal product of a factor input does not necessarily imply a proportionately higher output loss due to an epidemic shock in developing countries.

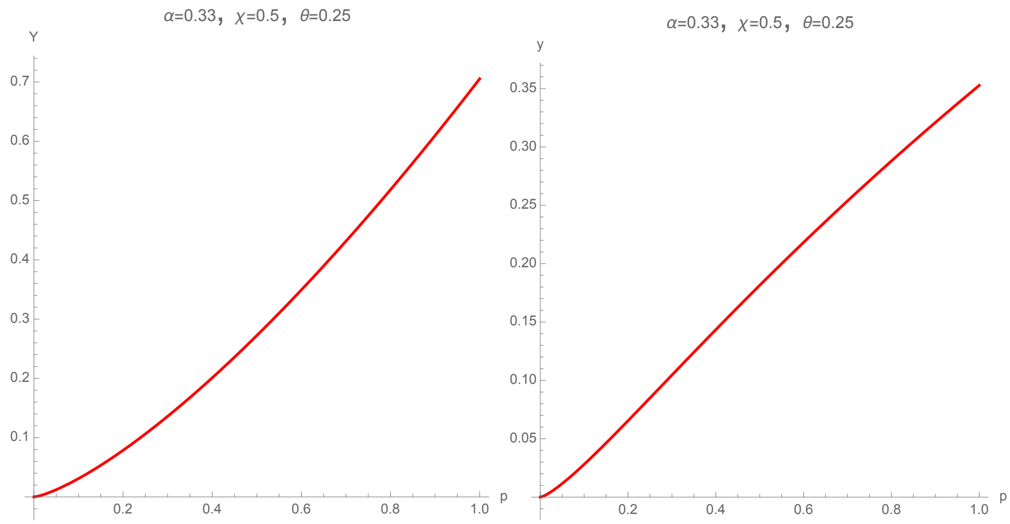

Figure 6: Effect of survival on output and output per worker

tality reductions (not limited to child and infant survival) in late nineteenth century Western Europe may have spurred accumulation and innovation towards newer generations of technologies that were biased towards human capital.[10] If newer technologies in the twentieth century have been skill oriented, as a body of work now argues, it has implications for developing countries. For instance an increase in the return to human capital B in a low-p country would see a more muted response in skill acquisition compared to a high-p country. High mortality, in other words, biases the response away from newer technologies. The lack of catch-up in parts of the developing world plagued by epidemics and health challenges may be as much to do with the low return from adopting modern technologies as with institutional constraints that prevent such adoption.[11]

[10]See Cutler *et al* (2006) on mortality reduction. On technological change, Abramovitch (1993) writes, as quoted in Galor and Moav (2004): "In the nineteenth century, technological progress was heavily biased in a physical capital-using direction. ... In the twentieth century, however, the physical capital-using bias weakened; it may have disappeared altogether. The bias shifted in an intangible (human and knowledge) capital-using direction and produced the substantial contribution of education and other intangible capital accumulation to this century's productivity growth..."

[11]Similar distributional implications are possible if households differed in their survival rates: low-p households would exhibit a preference towards tangible assets and benefit less from skill-biased technological change.

5.1 Availability of Life Insurance

We have so far ignored the possibility of buying a life insurance policy that guarantees a return (to the survivor) only in the event of premature parental death. The appeal of such a policy is that it allows an altruistic parent to circumvent the problem of non-transferability of human capital (Fischer, 1973). Qualitatively our results above turn out to be robust to the availability of life insurance.

Suppose an agent has the option of investing a part of his first period income in life insurance with the objective of transferring a part of his total earnings (from physical as well as human capital) to his child even in the event of premature death. Life insurance firms operate on a no-profit no-loss basis and invest the funds in the market. The returns from this are transferred to offsprings whose parents have died prematurely. Children whose parents are alive to make an end-of-the-period bequest get nothing. Since human capital is inalienable, the only investment vehicle available to life insurance companies is physical capital stock.

We focus exclusively on the existence of a life insurance market which allows parents to diversify bequest risks arising due to the possibility of premature death. Whether there exists a parallel annuities market (which allows for diversification of consumption risks) is irrelevant for this part of the analysis. Thus the results derived here can be compared directly to the results derived in the first part of this section which does not allow for life insurance.

The aggregate technology is as above and factor payments given by (12). The aggregate human capital stock is ph while the aggregate physical capital now consists of individual holdings of physical capital (denoted by k) and holdings of capital by the life insurance firms (κ per policy holder). Hence

$$
\begin{aligned}
K_t &= k_t + \kappa_t, \\
H_t &= ph_t.
\end{aligned}
$$

Suppose in their youth individuals invest x_t in physical capital, e_t in human capital and z_t in buying life insurance that yields asset levels in the following period,

$$
\begin{aligned}
k_{t+1} &= f(x_t) \\
\kappa_{t+1} &= f(z_t) \\
h_{t+1} &= g(e_t)
\end{aligned}
$$

where the production functions f and g are concave as specified in (15). The decision problem of a young adult at time t is to maximize expected lifetime utility

$$V_t \equiv u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} &= y_t - x_t - e_t - z_t, \\ c_{2t+1} &= \theta r_{t+1} k_{t+1} + w_{t+1} h_{t+1}, \end{aligned}$$

and

$$y_{t+1} = \begin{cases} (1-\theta) r_{t+1} k_{t+1}, & \text{w.p. } p \\ r_{t+1}\left(k_{t+1} + \dfrac{\kappa_{t+1}}{1-p}\right), & \text{w.p. } 1-p \end{cases}$$

Bequests include policy payouts by life insurance firms to children of prematurely deceased parents.

The first order conditions for optimal investment are

$$x_t : u'(c_{1t}) = [\theta\beta p u'(c_{2t+1}) + \gamma\left\{p(1-\theta)u'(c^a_{1t+1}) + (1-p)u'(c^d_{1t+1})\right\}] r_{t+1} f'(x_t) \quad (19)$$

$$e_t : u'(c_{1t}) = \beta p u'(c_{2t+1}) w_{t+1} g'(e_t) \quad (20)$$

and

$$z_t : u'(c_{1t}) = [\gamma u'(c^d_{1t+1})] r_{t+1} f'(z_t) \quad (21)$$

For comparability assume linear utility and that all dynasties start with a relatively high initial endowment of physical capital k_0 so that members are at their unconstrained optima ($c_1 > 0$). Equations (19) , (20) and (21) then lead to optimal investment decisions of

$$\begin{aligned} x_t &= [a\chi\{\gamma + p\theta(\beta-\gamma)\} r_{t+1}]^{1/(1-\chi)} \\ e_t &= [b\beta p\chi w_{t+1}]^{1/(1-\chi)} \\ z_t &= [a\chi\gamma r_{t+1}]^{1/(1-\chi)}. \end{aligned}$$

Consequently the individual stocks of the assets are

$$\begin{aligned} k_{t+1} &= a^{1/(1-\chi)} [\chi\{\gamma + p\theta(\beta-\gamma)\}]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)} \\ h_{t+1} &= b^{1/(1-\chi)} (\chi\beta p)^{\chi/(1-\chi)} w_{t+1}^{\chi/(1-\chi)} \\ \kappa_{t+1} &= a^{1/(1-\chi)} [\chi\gamma]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)} \end{aligned}$$

and the ratio of aggregate capital stocks are

$$\frac{K_t}{H_t} = \frac{k_t + \kappa_{t+1}}{ph_t} = \left(\frac{a}{b}\right)^{1/(1-\chi)} \left[\frac{2\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi/(1-\chi)} \frac{1}{p} \left(\frac{r_t}{w_t}\right)^{\chi/(1-\chi)}. \tag{22}$$

From (12), on the other hand,

$$\frac{r_t}{w_t} = \frac{\alpha}{1-\alpha}\frac{H_t}{K_t} \tag{23}$$

Using (16) and (17), we can solve for the equilibrium factor ratio

$$\frac{K_t}{H_t} = \frac{a}{b}\left(\frac{\alpha}{1-\alpha}\right)^{\chi} \frac{1}{p^{1-\chi}} \left[\frac{2\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi} \tag{24}$$

which once again is a decreasing function of p for $\beta \geq \gamma$.

The corresponding output per capita is given by

$$Y = \Gamma p^{\frac{1-\alpha}{1-\chi}} \left[\frac{1}{\beta}\left\{2\gamma + \theta p(\beta-\gamma)\right\}\right]^{\frac{\alpha\chi}{1-\chi}}$$

which also depends positively on longevity (as long as $\beta \geq \gamma$).

A direct comparison with the results derived without life insurance immediately tells us, for any p, the equilibrium K/H ratio is *higher* with life insurance than without. This apparently counter-intuitive result needs further explanation. The result is partially driven by the assumption of linear utility and that agents are assumed to have sufficient first period income which allows them to reach an unconstrained optima regarding portfolio choice. These two together imply that investment in each available asset is pushed to its maximum possible limit (where its marginal return equals unity). Availability of a third asset (life insurance) does not make any difference to the agent's investment in other assets.[12] Thus, while households' investment in physical and human capital remain unchanged, life insurance firms now invest in physical capital alone, which increases the aggregate K/H ratio under life insurance.

This result holds even if we relax the assumption of linear utility and/or tighten the budget constraint of the household so that investment in various assets is now limited by the availability of funds. In this case, the household will maintain a constant ratio of all the assets (such that their marginal returns are all equal), but availability of a third asset now does imply that the proportion of household income invested in physical capital goes down. However, at the aggregate level, K/H ratio still goes up with an increase in

[12]Observe from the respective first order conditions that optimal investments in x_t and e_t are exactly the same with and without life insurance.

p since investment in the third asset (life insurance) is again channeled towards physical capital formation (via the life insurance firms) which more than compensates for the fall in households' investment in physical capital. Consequently the overall balance tilts towards physical capital vis-a-vis human capital. We conclude that the availability of life insurance does not make a qualitative difference to the basic premise of this paper.

6 Conclusion

Two themes underlie our study of the effects of mortality on economic development. When people face uncertain lifetimes, they are more inclined to invest in tangible assets that can be passed on to their survivors. This has implications for long-run growth, convergence, and technology adoption. From an accounting point of view, moreover, high mortality societies would rely on physical capital accumulation more intensively relative to low mortality ones.

The second contribution of this paper has been to establish these results without appealing to the standard assumption of perfect annuities. Annuity markets are more likely to be underdeveloped in poorer societies. We have demonstrated that parental altruism can substitute for missing annuity markets reasonably well and in particular, for investment in tangible assets. In this paper we have assumed that the degree of parental altruism in exogenous. In ongoing work, Chakraborty and Das (2017) delve into the long-run implications of mortality when the degree of altruism is endogenously determined. If altruism requires parental time investment, developing a sufficiently high degree of altruism comes at the cost of time foregone in acquiring human capital. Thus altruism is likely to be high when parents are engaged in occupations which are less skill intensive, e.g., primary production. At the same time, high mortality itself makes investment in land more profitable than human capital following the logic of this paper. In the initial stages of development these two mechanisms work in tandem to generate a scenario where high mortality leads to concentration of production in the primary sector, which via endogenous altruism produces a high altruism coefficient that in turn sustains this scenario for a long period of time until some exogenous improvement in mortality breaks the vicious circle. In other words it is possible to have self-sustaining multiple equilibria where two economies with similar technologies and factor endowments exhibit different levels of development from mortality differences.

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