# Silent Promotion of Agendas 

Campaign Contributions and Ideological Polarization*

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#### Abstract

Until recently, both Republican and Democratic administrations have been promoting free trade and market deregulation for decades without intensive policy debates. We set up a two-party electoral competition model in a two-dimensional policy space with campaign contributions by an interest group that promotes a certain agenda that many voters disagree. Assuming that voters are impressionable to campaign spending for/against candidates, we analyze incentive compatible contracts between the interest group and the candidates on agenda policy positions and campaign contributions. If the interest group pushes its agenda more than the candidates want by providing contributions, then the candidates can compete only over the other (ideological) dimension. As the agenda is pushed further by the interest group, ideological policy polarization and campaign contributions surge.(JEL Codes C72, D72, F02, F13)


Keywords: electoral competition, probabilistic voting, campaign contributions, interest groups, impressionable voters, polarization

[^0]
## 1 Introduction

The ideological distance between congressional Democrats and Republicans has risen substantially in the last few decades (McCarty et al. 2016). DW-Nominate scores by Poole and Rosenthal (1985, 1991) show that the voting gap between congressional Democrats and Republicans is now larger than any point in the history. ${ }^{1}$ This rise coincided with globalization, market deregulation, rising income inequality, and an increase in campaign spendings and contributions in electoral politics.

These trends interact with each other. It is natural to assume that globalization trend has been affecting market deregulation, and it is widely acknowledged that globalization and market deregulation have contributed to growing income inequality in the US as well as other countries. However, the mechanism by which globalization and market deregulation can cause policy polarization has not been discussed frequently (a notable exception is Autor et al., 2016). In this paper, we propose a simple and tractable model with multidimensional policy space to explain these interactions.

Historically, both Republican and Democratic administrations have been promoting free trade and (more recently) financial market deregulation for decades, and there have been few hot debates on the pros and cons between them. Free trade policies have been promoted by US administrations irrespective of party despite the fact that many citizens might not be keen on them. Many citizens feared NAFTA (North American Free Trade Agreement), since it would open up the US market for Mexican goods produced using cheap labor. However, Bill Clinton made tremendous efforts to get approval from Congress to ratify NAFTA, which had been signed by George H. W. Bush. ${ }^{2}$ TPP and TTIP have been pushed by Barack Obama. As a result, neither protectionism nor free trade have been salient issues in presidential debates until the 2016 election. ${ }^{3}$

Although Democrats have been traditionally the primary opponent of financial deregulation, partisan convergence on this issue occurred from the 1980s until the Lehmann shock. The major deregulation was the removal of the interstate branching prohibitions in banking industry, the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994, which was introduced by Democrats and signed into a law by Bill Clinton. Keller and Kelly (2015) find this convergence since 1980's empirically and show that campaign finance played a role in this partisan convergence.

In order to analyze the relationship between ideological polarization and convergence in free trade/market

[^1]deregulation issues, we will set up a two-candidate electoral competition model over two-dimensional issues: an ideological dimension and an "agenda" ${ }^{4}$ dimension in which both presidential candidates and voters have a bliss point in the policy space. Another key player is an Interest Group (IG): a group of large corporations that are interested in promoting the agenda only - they do not care about ideological dimension. Voters are assumed to be impressionable, and IG can provide campaign contributions to candidates who would effectively enhance their likeability by spending money on political advertisements. ${ }^{5}$ If both party candidates receive campaign contributions, the risk of electoral competition endangering the agenda is removed, and they will not debate the agenda (free trade promotion and market deregulation). We will explore the relationship between IG's promotion of the agenda, the rise of campaign contributions, and political polarization.

We introduce a tractable probabilistic voting model with uncertain valences, in which two party candidates have both office and policy motivations. Although it is well-known that majority voting rule is ill-behaved if the policy space is multi-dimensional, we assure the existence of a median voter in our model by adopting a variation of strong assumptions used in Davis, deGroot, and Hinch (1972). We first establish the existence of electoral equilibrium when there is a median voter (Proposition 1). Then, we assure the existence of the median voter in our model (Proposition 2). After establishing that candidates' incentive compatibility constraints are binding (Proposition 3), we show that in a symmetric candidate case, symmetric equilibrium is unique (Proposition 4), and that candidates' ideological positions polarize and campaign contributions rise as the IG promotes the agenda more than the candidates want (Proposition 5). The mechanism behind this result is simple: as IG promotes an agenda more than the candidates want, the candidates' payoffs from winning go down. To compensate these losses, candidates choose policies closer to their ideal positions, causing an ideological polarization. ${ }^{6}$ We conduct numerical analysis for asymmetric candidate cases. Our results suggest that, if two candidates are asymmetric in their ideal positions in the agenda dimension, their ideological polarization is also asymmetric as IG promotes the agenda more - that is, the candidate who is less eager to promote the agenda tends to receive more contributions and polarizes her ideological policy more. In contrast, the one who is more eager to promote the agenda polarizes her ideological policy less to compensate for the scarcity of contribution money relative to her opponent. The same pattern can be observed when voters are more and more leaning toward protectionism. The candidate

[^2]who has ex ante valence advantage (a charismatic candidate) also polarizes her policy in the ideological dimension, while the disadvantageous candidate moves to the center ideologically. Although ideologicallymotivated individual campaign contributions might be the main source of asymmetric polarization patterns observed in the US, as suggested by McCarty et al (2016), our results on corporate campaign contributions may also explain asymmetric polarization.

The rest of Section 1 reviews related literature. We introduce the model in Section 2. In Section 3, we analyze properties of equilibrium in the electoral competition and incentive compatibility constraints. In Section 4, we provide analytical results when two candidates are symmetric. We discuss the optimal IG contract under different circumstances via numerical analysis in Section 5. Section 6 concludes. All proofs are collected in Appendix A.

### 1.1 Related Literature

Our framework is built on an influential electoral competition model with interest groups by Grossman and Helpman (1996), but there are a number of differences. Following Baron (1994), Grossman and Helpman (1996) assume that there are informed and uninformed voters, and that uninformed voters' voting behaviors are affected by campaign contributions (impressionable voters). Although Grossman and Helpman (1996) allow general policy space with multiple lobbies, our model restricts the attention to a special policy space with two dimensions-(a) an agenda dimension in which an Interest Group wants to promote while voters (and candidates) do not want that much, and (b) a standard Hotelling-type ideological dimension. Grossman and Helpman (1996) assume that lobbies influence the parties' policy platforms through contribution functions, while we simply use take-it-or-leave-it offers instead. They analyze one lobby case extensively, and show that the lobby contributes more to a candidate who has a better chance to win, though it makes contributions to both candidates. ${ }^{7}$ We also focus on one IG case, and explore the shapes of incentive compatible constraints and the interaction of policies both analytically and numerically.

In the voting stage, we need to use a two-dimensional voter space. It is hard to assure the existence of simple majority voting equilibrium for multiple dimensional voter spaces, even with probabilistic voting (Wittman 1983, Lindbeck and Weibull 1987, Roemer 2001, and Krasa and Polborn 2012). ${ }^{8}$ Although we need to adopt a simplifying assumption ("symmetry" in voter distribution), we manage to establish a tractable

[^3]probabilistic voting model with both office- and policy-motivated candidates, applying the result in Davis, deGroot, and Hinch (1972). Note, however, that candidates choose different policies in our model, although policy-convergence occurs in Davis et al. (1972). Besides the dimensionality issue, Roemer (1997) proves the existence of pure strategy Nash equilibrium in a setup where the candidates do not have complete information about median voter's bliss point. ${ }^{9}$ In contrast, we assume that the uncertainty comes from an additive valence shock following Londregan and Romer (1993).

There is a large body of literature about campaign spending which can be roughly divided into two approaches. The first one assumes that the contribution "impresses" voters directly. In addition to Grossman and Helpman (1996), an incomplete list of this branch includes Meirowitz (2008), Ashworth and Bueno de Mesquita (2009), and Pastine and Pastine (2012). Our paper resembles to Chamon and Kaplan (2013) in a way that IG makes take-it-or-leave-it offers. ${ }^{10}$ Our main focus is to analyze interactions between campaign contributions and ideological policy polarization. The second approach considers informative campaign spending. For example, Austen-Smith (1987) considers contributions as advertising efforts that can reduce uncertainty when voters observe candidates' proposed policies. Prat (2002a and 2002b) models contributions as a signal of unobservable candidate valences. Coate (2004) considers campaign spending as an informative advertisement about policy positions.

## 2 The Model

We consider a two-party multi-dimensional political competition with campaign contributions. There are an Interest Group (IG), two party candidates $j \in\{L, R\}$, and voters.

We assume that the voter cares about ideological policy, free trade/market deregulation policy, and campaign money spent. We will use free trade or simply "agenda" interchangeably in this paper. Formally, suppose that $p \in \mathcal{P}=\mathbb{R}$ stands for an ideological policy, $a \in \mathcal{A}=\mathbb{R}$ for an agenda policy, and $C$ for the campaign money spent. Here we follow Grossman and Helpman (1996) in assuming that voters are impressionable. There is a continuum of atomless voters, who differ from each other with their bliss points. A voter with her bliss point $(\bar{p}, \bar{a}) \in \mathcal{P} \times \mathcal{A}$ has a quadratic payoff function:

$$
\begin{equation*}
v_{(\bar{p}, \bar{a})}(p, a, C)=-(p-\bar{p})^{2}-\theta(a-\bar{a})^{2}+C \tag{1}
\end{equation*}
$$

[^4]where $\theta>0$ describes the relative importance of the agenda dimension for voters. Note that $v$ is increasing in $C$ (voters are impressionable). The distribution of voters is described by the distribution of voters' bliss points. Voters' bliss points are distributed with density function $f: \mathcal{P} \times \mathcal{A} \rightarrow \mathbb{R}_{+}$on policy space $\mathcal{P} \times \mathcal{A}$.

There is an Interest Group (IG) that cares about agenda dimension $a \in \mathcal{A}$. To simplify the analysis, we assume that IG intends to achieve $\tilde{a}$ no matter who wins, and that IG tries to spend as little as possible to achieve $\tilde{a}$ through the election process using its contributions to two candidates. Therefore, we can simplify its offer as $\left(C_{L}, C_{R}\right)$ (or $\left(C_{L}(\tilde{a}), C_{R}(\tilde{a})\right)$ ). IG proposes $\left(C_{L}, C_{R}\right)$, and the political contribution $C_{j}$ is contingent on candidate $j$ 's commitment to adopting policy $\tilde{a}$ ( $C_{j}$ will be spent as campaign expenses in the election). We denote $C_{j}=0$ if IG decides not to make the offer to candidate $j$. Candidate $j$ needs to decide whether to take IG's offer $C_{j}$ or not. If candidate $j$ chooses not to take the offer, she can choose $p_{j}$ and $a_{j}$ freely, but needs to run her campaign without IG's contributions. In this case, we normalize the campaign spending to 0 . On the other hand, if she chooses to take the offer, she can only compete with the $p_{j}$ (since she has committed to $a_{j}=\tilde{a}$ ), but with $C_{j}$ as her covered campaign expenses. ${ }^{11}$

We assume that there are uncertainties in election outcomes by introducing a random valence term for candidate $j$, which is common to all voters. The valence vector $\epsilon=\left(\epsilon_{L}, \epsilon_{R}\right)$ is composed of two random variables such that voter $(\bar{p}, \bar{a})$ evaluates $L$ and $R$ by ${ }^{12}$

$$
\begin{aligned}
& v_{(\bar{p}, \bar{a})}\left(p_{L}, a_{L}, C_{L}\right)+\epsilon_{L}, \\
& v_{(\bar{p}, \bar{a})}\left(p_{R}, a_{R}, C_{R}\right)+\epsilon_{R}
\end{aligned}
$$

where $\epsilon=\left(\epsilon_{L}, \epsilon_{R}\right)$ is drawn from a joint distribution with a density function $g: R^{2} \rightarrow R_{+}$. The candidate who collects the majority of votes is the winner of the election.

If candidate $j$ wins the election, she gets the utility

$$
W_{j}\left(p_{j}, a_{j}\right)=Q+w_{j}\left(\left|p_{j}-\bar{p}_{j}\right|,\left|a_{j}-\bar{a}_{j}\right|\right)
$$

where $Q>0$ is a payoff from winning the office (office motivation), and $w_{j}$ is concave, decreasing in both arguments, and $w_{j}(0,0)=0$ (policy motivation: $\left(\bar{p}_{j}, \bar{a}_{j}\right)$ is the bliss point for candidate $j$ ). If the candidate loses, she gets 0 as her payoff. Candidates can choose their policies from compact and convex subset of

[^5]$\mathcal{P} \times \mathcal{A}$, namely, $\left[p_{\min }, p_{\max }\right] \times\left[a_{\min }, a_{\max }\right]$, where $p_{\min }<\bar{p}_{L}<\bar{p}_{R}<p_{\max }$ and $a_{\min }<\bar{a}_{j}<a_{\max }$ for $j=L, R$.
The sequence of moves is as follows:

Stage 1 : The IG proposes $\left(C_{j}\right)_{j \in\{L, R\}}$ to candidates for policy commitment $a_{j}=\tilde{a}$ for $j=L, R$.
Stage 2 : Candidates simultaneously decide whether to take the offer or not.

Stage 3 : If candidate $j$ accepts the offer in the Stage 2 , then she chooses $p_{j} \in \mathcal{P}$ under fixed $a_{j}=\tilde{a}$ and $C_{j}$. Otherwise, she chooses $\left(p_{j}, a_{j}\right) \in \mathcal{P} \times \mathcal{A}$ under 0 campaign spending. The two candidates choose their policies simultaneously.

Stage 4 : Nature plays and $\epsilon$ realizes.

Stage 5 : The voter votes sincerely according to his preference, and all payoffs realize.

We will assume that the IG minimizes $C_{L}+C_{R}$ to implement an arbitrary level of the agenda policy $\tilde{a}$ in this paper. ${ }^{13}$ The equilibrium concept adopted is the subgame perfect Nash equilibrium (SPNE). We solve the political game by a backward induction.

## 3 The Policy Competition Stage

### 3.1 Existence of Equilibrium

Here, we assume that there is a median voter and prove the existence of equilibrium in electoral competition. It is well-known that there may not be a median voter when the policy space is multi-dimensional. We will present a sufficient condition for the existence of a median voter in the next section.

During the voting stage, the median voter compares two candidates by ( $p_{j}, a_{j}, C_{j} ; p_{i}, a_{i}, C_{i}$ ) given the realized valence bias. That is, the voter votes for $j$ over $i$ if and only if

$$
v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{j}, a_{j}, C_{j}\right)-v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{i}, a_{i}, C_{i}\right) \geq \epsilon_{i}-\epsilon_{j}
$$

where $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ is the median voter's bliss point. Let

$$
\begin{aligned}
& S_{L}\left(p_{L}, a_{L}, C_{L} ; p_{R}, a_{R}, C_{R}\right) \equiv \\
& \qquad\left\{\epsilon \in \mathbb{R}^{2} \mid v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{L}, a_{L}, C_{L}\right)-v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{R}, a_{R}, C_{R}\right) \geq \epsilon_{R}-\epsilon_{L}\right\}
\end{aligned}
$$

[^6]

Figure 1: The winning probability determination for given $\left(p_{j}, a_{j}, C_{j}\right)_{j=L, R}$. The $45^{\circ}$ line stands for the set of events where $L$ and $R$ are tied.
which is the set of events where the pivotal voter votes for $L$. Therefore, given $\left(p_{j}, a_{j}, C_{j}\right)_{j=L, R}$, the winning probability for $j$ is

$$
\Pi_{L}\left(p_{L}, a_{L}, C_{L} ; p_{R}, a_{R}, C_{R}\right)=\int_{S_{L}\left(p_{L}, a_{L}, C_{L} ; p_{R}, a_{R}, C_{R}\right)} g(\epsilon) d \epsilon
$$

Figure 1 depicts the determination of winning probability for a given policy pair.
However, both the candidates and IG make their decisions before the uncertainty is resolved. Therefore, given the decision in Stage 2, both candidates choose policies to maximize their expected utility

$$
\begin{aligned}
& V_{L}=\Pi_{L}\left(p_{L}, a_{L}, C_{L} ; p_{R}, a_{R}, C_{R}\right) w_{L}\left(\left|p_{L}-\bar{p}_{L}\right|,\left|a_{L}-\bar{a}_{L}\right|\right) \\
& V_{R}=\Pi_{R}\left(p_{R}, a_{R}, C_{R} ; p_{L}, a_{L}, C_{L}\right) w_{R}\left(\left|p_{R}-\bar{p}_{R}\right|,\left|a_{R}-\bar{a}_{R}\right|\right)
\end{aligned}
$$

where $\Pi_{R}\left(p_{R}, a_{R}, C_{R} ; p_{L}, a_{L}, C_{L}\right)=1-\Pi_{L}\left(p_{L}, a_{L}, C_{L} ; p_{R}, a_{R}, C_{R}\right)$. The following proposition shows that, under our assumptions on utility functions and the density function $g$, a Nash equilibrium exists. Proposition 1 can be proved as a corollary of Theorem A in Appendix A.

Proposition 1. (Existence) Suppose that there is a median voter with her bliss point $\left(\bar{p}_{m}, \bar{a}_{m}\right)$, and that $v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}(p, a, C)$ and $w_{j}\left(\left|p_{j}-\bar{p}_{j}\right|,\left|a_{j}-\bar{a}_{j}\right|\right)$ are quadratic in $(p, a)$, and concave in $\left(p_{j}, a_{j}\right)$, respectively. Suppose further that the density function $g(\epsilon)$ is log-concave in $\epsilon \in \mathbb{R}^{2}$. Then $\Pi_{j}\left(p_{j}, a_{j}, C_{j} ; p_{i}, a_{i}, C_{i}\right) w_{j}\left(p_{j}, a_{j}\right)$ is log-concave in $\left(p_{j}, a_{j}\right)$, and there exists a Nash equilibrium in policy competition subgame.

This proposition covers logit model ( $\epsilon$ follows a type-I extreme value distribution). Before concluding
this section, we provide another convenient way to represent $\Pi_{L}$ and $\Pi_{R}$. For any $\tilde{\epsilon} \in \mathbb{R}$, define $\tilde{S}_{L}(\tilde{\epsilon}) \equiv$ $\left\{\epsilon \in \mathbb{R}^{2} \mid \tilde{\epsilon} \geq \epsilon_{R}-\epsilon_{L}\right\}$ and

$$
\tilde{G}(\tilde{\epsilon}) \equiv \int_{\epsilon \in \tilde{S}_{L}(\tilde{\epsilon})} g(\epsilon) d \epsilon
$$

Then, we have

$$
\Pi_{L}\left(p_{L}, a_{L}, C_{L} ; p_{R}, a_{R}, C_{R}\right)=\tilde{G}\left(v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{L}, a_{L}, C_{L}\right)-v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{R}, a_{R}, C_{R}\right)\right)
$$

Similarly,

$$
\Pi_{R}\left(p_{R}, a_{R}, C_{R} ; p_{L}, a_{L}, C_{L}\right)=1-\tilde{G}\left(v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{L}, a_{L}, C_{L}\right)-v_{\left(\bar{p}_{m}, \bar{a}_{m}\right)}\left(p_{R}, a_{R}, C_{R}\right)\right)
$$

From the proof of Proposition 1, we can show that $\tilde{G}(\tilde{\epsilon})$ is log-concave (Prekopa's theorem, 1973). We denote voters' density function by $\tilde{g}(\tilde{\epsilon})=\frac{d \tilde{G}}{d \tilde{\epsilon}}$.

### 3.2 Sufficient Conditions for Existence of the Median Voter in Multidimensional Policy Space

In the previous subsection, we obtained a general existence result by assuming that there is a median voter in multidimensional policy space. However, it is well-known that we need very strong conditions to assure the existence of the Condorcet winner (Plott 1967) and the existence of the median voter (Davis, deGroot, and Hinich 1972). Davis et al. (1972) showed that a necessary and sufficient condition is that voters' distribution is symmetric in policy space when voters have Euclidean preferences in a voting model without uncertainty. We will provide sufficient conditions for the existence of the median voter in our random valence (thus cardinal) model by applying their approach. ${ }^{14}$ Voters whose bliss point ( $\bar{p}, \bar{a}$ ) satisfies the following condition vote for candidate $L$.

$$
\begin{equation*}
-\left|p_{L}-\bar{p}\right|^{2}-\theta\left|a_{L}-\bar{a}\right|^{2}+C_{L}+\epsilon_{L} \geq-\left|p_{R}-\bar{p}\right|^{2}-\theta\left|a_{R}-\bar{a}\right|^{2}+C_{R}+\epsilon_{R} \tag{2}
\end{equation*}
$$

Based on the formula above, we can show that voter $(\bar{p}, \bar{a})$ votes for $L$ if

$$
\bar{a} \leq \frac{1}{2\left(a_{R}-a_{L}\right)}\left[-2\left(p_{R}-p_{L}\right) \bar{p}+\left(p_{R}^{2}-p_{L}^{2}\right)+\theta\left(a_{R}^{2}-a_{L}^{2}\right)+\left(C_{L}-C_{R}\right)+\epsilon_{R}-\epsilon_{L}\right]
$$

[^7]

Figure 2: The cut-off line of indifferent voters.
holds. Figure 2 shows the above line of indifferent voters in the policy space. Note that if the area below the cut-off line in Figure 2 has more voters than the above, candidate $L$ wins. This observation together with a symmetric distribution assumption in Davis et al. (1972), yields the following proposition.

Proposition 2. (Median Voter Result) Suppose that voters' preferences are represented by a quadratic utility function (2). Let $B_{\delta}(\bar{p}, \bar{a})$ be $\delta$-neighborhood of $(\bar{p}, \bar{a})$. Suppose that there exists $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ with (i) there is a $\delta>0$ such that $f(p, a)>0$ holds for any $(p, a) \in B_{\delta}\left(\bar{p}_{m}, \bar{a}_{m}\right)$, and (ii) for all $\left(e_{p}, e_{a}\right) \in$ $\mathbb{R}^{2}, f\left(\left(\bar{p}_{m}, \bar{a}_{m}\right)+\left(e_{p}, e_{a}\right)\right)=f\left(\left(\bar{p}_{m}, \bar{a}_{m}\right)-\left(e_{p}, e_{a}\right)\right)$. Then, $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ is the median voter whose preference determines voting outcome.

Although the "symmetric distribution" assumption (ii) in Proposition 2 is certainly very restrictive, it is often employed in the literature of voting problems for multi-dimensional policy spaces. ${ }^{15}$ In the rest of the paper (except for the numerical analysis section for the purpose of comparative static analysis), we will normalize the median voter's bliss point at $(0,0)$ without loss of generality. We will also assume that $\bar{a}_{m}=0<\bar{a}_{j}$ for $j=L, R$, and $\bar{p}_{L}<0\left(=\bar{p}_{m}\right)<\bar{p}_{R}$.

### 3.3 First-Order Characterization of Equilibrium in Policy Competition Game

Although we assume that voters' utility functions are quadratic, we write in a general form for conciseness; i.e.,

$$
\begin{equation*}
u_{m}(p, a, C)=v_{p}(|p|)+v_{a}(|a|)+C \tag{3}
\end{equation*}
$$

[^8]where $v_{p}(|p|)=-(|p|)^{2}$ and $v_{a}(|a|)=-\theta(|a|)^{2}$. Clearly, we have $v_{p}^{\prime}<0, v_{p}^{\prime \prime}<0, v_{a}^{\prime}<0$, and $v_{a}^{\prime \prime}<0$. We will assume additive separability for candidates' utility functions to avoid unnecessary interaction between ideology and the agenda; i.e.,
\[

$$
\begin{equation*}
W_{j}(p, a)=w\left(\left|p-\bar{p}_{j}\right|,\left|a-\bar{a}_{j}\right|\right)=Q+w_{p}\left(\left|p-\bar{p}_{j}\right|\right)+w_{a}\left(\left|a-\bar{a}_{j}\right|\right) \tag{4}
\end{equation*}
$$

\]

where $w_{p}\left(\left|p-\bar{p}_{j}\right|\right)$ and $w_{a}\left(\left|a-\bar{a}_{j}\right|\right)$ are concave functions with $w_{p}(0)=w_{a}(0)=0$, satisfying $w_{p}^{\prime}<0$, $w_{p}^{\prime \prime}<0, w_{a}^{\prime}<0$, and $w_{a}^{\prime \prime}<0$.

Each candidate $j$ 's maximization problem is

$$
\max _{p_{j}, a_{j}} \Pi_{j}\left(p_{j}, a_{j}, C_{j}, p_{i}, a_{i}, C\right)\left\{Q+w_{p}\left(\left|p-\bar{p}_{j}\right|\right)+w_{a}\left(\left|a-\bar{a}_{j}\right|\right)\right\}
$$

Naturally assuming $\left|\bar{p}_{j}\right|>\left|p_{j}\right|$ and $\bar{a}_{j}>a_{j}>0$ in an equilibrium when candidate $j$ can choose $p_{j}$ and $a_{j}$ freely, we have $\frac{\partial w_{p}\left(\left|p_{j}-\bar{p}_{j}\right|\right)}{\partial\left|p_{j}\right|}=-w_{p}^{\prime}\left(\left|p_{j}-\bar{p}_{j}\right|\right)>0$ and $\frac{\partial w_{a}\left(\left|a_{j}-\bar{a}_{j}\right|\right)}{\partial\left|a_{j}\right|}=-w_{a}^{\prime}\left(\left|a_{j}-\bar{a}_{j}\right|\right)>0$. The first order conditions for candidate $j \in\{L, R\}$ with $j \neq i \in\{L, R\}$ are

$$
\begin{align*}
& \frac{\partial \Pi_{j}\left(p_{j}, a_{j}, C_{j}, p_{i}, a_{i}, C_{i}\right)}{\partial\left|p_{j}\right|}\left\{Q+w_{p}\left(\left|p-\bar{p}_{j}\right|\right)+w_{a}\left(\left|a-\bar{a}_{j}\right|\right)\right\}-\Pi_{j}\left(p_{j}, a_{j}, C_{j}, p_{i}, a_{i}, C_{i}\right) w_{p}^{\prime}\left(\left|p_{j}-\bar{p}_{j}\right|\right)=0  \tag{5}\\
& \frac{\partial \Pi_{j}\left(p_{j}, a_{j}, C_{j}, p_{i}, a_{i}, C_{i}\right)}{\partial a_{j}}\left\{Q+w_{p}\left(\left|p-\bar{p}_{j}\right|\right)+w_{a}\left(\left|a-\bar{a}_{j}\right|\right)\right\}-\Pi_{j}\left(p_{j} a_{j}, C_{j}, p_{i}, a_{i}, C_{i}\right) w_{a}^{\prime}\left(\left|a_{j}-\bar{a}_{j}\right|\right)=0 \tag{6}
\end{align*}
$$

where the second equation is omitted when candidate $j$ commits to $a_{j}=\tilde{a}$. Thus, the Nash equilibrium $\left(p_{j}, a_{j}, p_{i}, a_{i}\right)$ of policy competition is characterized by the above equations (5) and (6) for $i, j \in\{L, R\}$ with $i \neq j$.

### 3.4 Incentive Compatible Contracts

For simplicity, we assume that IG intends to achieve $\tilde{a}$ no matter which candidate wins by offering $C_{L}$ and $C_{R}$ to candidates $L$ and $R$, respectively. In order to analyze the incentive compatibility of the contracts, let $\left(p_{L}^{*}, \tilde{a}, p_{R}^{*}, \tilde{a}\right),\left(p_{L}^{* *}, a_{L}^{* *}, p_{R}^{* *}, \tilde{a}\right)$, and $\left(p_{L}^{* * *}, \tilde{a}, p_{R}^{* * *}, a_{R}^{* * *}\right)$ be equilibria in the policy competition stage when (i) both candidates accept the offer, (ii) only candidate $L$ rejects the offer, and (iii) only candidate $R$ rejects
the offer, respectively. The incentive compatibility conditions are characterized by

$$
\begin{align*}
& \quad \Pi_{L}\left(p_{L}^{*}, \tilde{a}, C_{L}, p_{R}^{*}, \tilde{a}, C_{R}\right)\left\{Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right)\right\}  \tag{7}\\
& \quad \geq \Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)\left\{Q+w_{p}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)\right\} \\
& \Pi_{R}\left(p_{R}^{*}, \tilde{a}, C_{R}, p_{L}^{*}, \tilde{a}, C_{L}\right)\left\{Q+w_{p}\left(\left|p_{R}^{*}-\bar{p}_{R}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{R}\right|\right)\right\}  \tag{8}\\
& \geq \\
& \Pi_{R}\left(p_{R}^{* * *}, a_{R}^{* * *}, 0, p_{L}^{* * *}, \tilde{a}, C_{L}\right)\left\{Q+w_{p}\left(\left|p_{R}^{* * *}-\bar{p}_{R}\right|\right)+w_{a}\left(\left|a_{R}^{* * *}-\bar{a}_{R}\right|\right)\right\}
\end{align*}
$$

The IG's problem is:

$$
\begin{equation*}
\min C_{L}+C_{R} \quad \text { s.t. } \quad(7) \text { and (8) } \tag{9}
\end{equation*}
$$

Thus, in equilibrium, IG provides the minimal $\left(C_{L}, C_{R}\right)$ to implement $\tilde{a}$. In order to analyze the incentive constraint in relation to $\left(\tilde{a}, C_{L}, C_{R}\right)$, we need to know the properties of the Nash equilibrium of off-equilibrium subgames. We will start with an equilibrium analysis of a subgame in which $L$ rejects the contract.

### 3.4.1 Equilibrium when $L$ rejects the contract

A Nash equilibrium when $L$ rejects the contract is characterized by

$$
\begin{aligned}
& \frac{\partial \Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)}{\partial\left|p_{L}\right|}\left(Q+w_{p L}^{* *}+w_{a L}^{* *}\right)-\Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right) w_{p}^{\prime}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)=0 \\
& \frac{\partial \Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)}{\partial a_{L}}\left(Q+w_{p L}^{* *}+w_{a L}^{* *}\right)-\Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right) w_{a}^{\prime}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)=0 \\
& \frac{\partial \Pi_{R}\left(p_{R}^{* *}, \tilde{a}, C_{R}, p_{L}^{* *}, a_{L}^{* *}, 0\right)}{\partial p_{R}}\left(Q+w_{p R}^{* *}+w_{a R}^{* *}\right)-\Pi_{R}\left(p_{R}^{* *}, \tilde{a}, C_{R}, p_{L}^{* *}, a_{L}^{* *}, 0\right) w_{p}^{\prime}\left(\left|p_{R}^{* *}-\bar{p}_{R}\right|\right)=0
\end{aligned}
$$

where $w_{p j}^{* *}=w_{p}\left(\left|p_{j}^{* *}-\bar{p}_{j}\right|\right)$ and $w_{a j}^{* *}=w_{a}\left(\left|a_{j}^{* *}-\bar{a}_{j}\right|\right)$. Recalling that $\Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)=\tilde{G}\left(v_{L}^{* *}-v_{R}^{* *}\right)$, we have

$$
\begin{aligned}
& \frac{\frac{\partial \Pi_{L}}{\partial\left|p_{L}\right|}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)}{\Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)}=\frac{\tilde{g}\left(v_{L}^{* *}-v_{R}^{* *}\right)}{\tilde{G}\left(v_{L}^{* *}-v_{R}^{* *}\right)} v_{p}^{\prime}\left(\left|p_{L}^{* *}\right|\right) \\
& \frac{\frac{\partial \Pi_{L}}{\partial a_{L}}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)}{\Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, C_{R}\right)}=\frac{\tilde{g}\left(v_{L}^{* *}-v_{R}^{* *}\right)}{\tilde{G}\left(v_{L}^{* *}-v_{R}^{* *}\right)} v_{a}^{\prime}\left(\left|a_{L}^{* *}\right|\right) \\
& \frac{\frac{\partial \Pi_{R}}{\partial p_{R}}\left(p_{R}^{* *}, \tilde{a}, C_{R}, p_{L}^{* *}, a_{L}^{* *}, 0\right)}{\Pi_{R}\left(p_{R}^{* *}, \tilde{a}, C_{R}, p_{L}^{* *}, a_{L}^{* *}, 0\right)}=\frac{\tilde{g}\left(v_{R}^{* *}-v_{L}^{* *}\right)}{\tilde{G}\left(v_{R}^{* *}-v_{L}^{* *}\right)} v_{p}^{\prime}\left(\left|p_{R}^{* *}\right|\right),
\end{aligned}
$$

where $v_{L}^{* *}=v_{p}\left(\left|p_{L}^{* *}\right|\right)+v_{a}\left(\left|a_{L}^{* *}\right|\right)$ and $v_{R}^{* *}=v_{p}\left(\left|p_{R}^{* *}\right|\right)+v_{a}(|\tilde{a}|)+C_{R}$ are the median voter's utilities from the policies of candidates $L$ and $R$, respectively. Here we also assume that $p_{L}^{* *}<0<p_{R}^{* *}$ in equilibrium. Substituting them back into the first order conditions, we have

$$
\begin{aligned}
& \varphi\left(v_{L}^{* *}-v_{R}^{* *}\right) v_{p}^{\prime}\left(\left|p_{L}^{* *}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)\right\}-w_{p}^{\prime}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)=0 \\
& \varphi\left(v_{L}^{* *}-v_{R}^{* *}\right) v_{a}^{\prime}\left(\left|a_{L}^{* *}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)\right\}-w_{a}^{\prime}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)=0 \\
& \varphi\left(v_{R}^{* *}-v_{L}^{* *}\right) v_{p}^{\prime}\left(\left|p_{R}^{* *}\right|\right)\left\{Q+w_{p}\left(\left|p_{R}^{* *}-\bar{p}_{R}\right|\right)+w_{a}\left(\left|a_{R}^{* *}-\bar{a}_{R}\right|\right)\right\}-w_{p}^{\prime}\left(\left|p_{R}^{* *}-\bar{p}_{R}\right|\right)=0
\end{aligned}
$$

where $\varphi\left(v_{L}-v_{R}\right) \equiv \frac{\tilde{\tilde{g}}\left(v_{L}-v_{R}\right)}{\tilde{G}\left(v_{L}-v_{R}\right)}$. Letting $\Delta^{* *}=v_{L}^{* *}-v_{R}^{* *}$, we have the following system of equations

$$
\begin{array}{r}
\varphi\left(\Delta^{* *}\right) v_{p}^{\prime}\left(\left|p_{L}^{* *}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)\right\}-w_{p}^{\prime}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)=0 \\
\varphi\left(\Delta^{* *}\right) v_{a}^{\prime}\left(\left|a_{L}^{* *}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)\right\}-w_{a}^{\prime}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)=0  \tag{10}\\
\varphi\left(-\Delta^{* *}\right) v_{p}^{\prime}\left(\left|p_{R}^{* *}\right|\right)\left\{Q+w_{p}\left(\left|p_{R}^{* *}-\bar{p}_{R}\right|\right)+w_{a}\left(\left|a_{R}^{* *}-\bar{a}_{R}\right|\right)\right\}-w_{p}^{\prime}\left(\left|p_{R}^{* *}-\bar{p}_{R}\right|\right)=0 \\
v\left(\left|p_{L}^{* *}\right|,\left|a_{L}^{* *}\right|, 0\right)-v\left(\left|p_{R}^{* *}\right|,|\tilde{a}|, C_{R}\right)-\Delta^{* *}=0
\end{array}
$$

Recall that we are assuming $\left|\bar{p}_{j}\right|>\left|p_{j}\right|$ and $\bar{a}_{L}>a_{L}>0$ in an equilibrium naturally so that we have $\frac{\partial w_{p}\left(\left|p_{j}-\bar{p}_{j}\right|\right)}{\partial\left|p_{j}\right|}=-w_{p}^{\prime}\left(\left|p_{j}-\bar{p}_{j}\right|\right)>0$ and $\frac{\partial w_{a}\left(\left|a_{L}-\bar{a}_{L}\right|\right)}{\partial a_{L}}=-w_{a}^{\prime}\left(\left|a_{L}-\bar{a}_{L}\right|\right)>0$. In contrast, if we take the derivative with respect to $\tilde{a}$ for candidate $R$ 's f.o.c., we have $w_{a}^{\prime}\left(\left|\tilde{a}-\bar{a}_{R}\right|\right)=\frac{\partial w_{a}\left(\left|\tilde{a}-\bar{a}_{R}\right|\right)}{\partial \tilde{a}}<0$, since $\tilde{a}>\bar{a}_{R}$. Since $g$ is log-concave, $\tilde{G}$ is log-concave as well (Prékopa 1973), and we have $\varphi^{\prime}(\Delta)<0$. Totally differentiating the system, we obtain

$$
\begin{aligned}
\left(\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 \\
0 & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
v_{p L}^{\prime} & v_{R}^{\prime}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{a L}^{\prime} w_{L} \\
& \times\left(\begin{array}{c}
d\left|p_{L}\right| \\
d a_{L} \\
d p_{R} \\
d \Delta
\end{array}\right)=\left(\begin{array}{c}
0 \\
0 \\
-\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} \\
v_{\tilde{a} R}^{\prime}
\end{array}\right) d \tilde{a}+\left(\begin{array}{c}
0 \\
0 \\
0 \\
1
\end{array}\right) d C_{R}
\end{array}\right.
\end{aligned}
$$

where $v_{p j}^{\prime}=v_{p}^{\prime}\left(\left|p_{j}\right|\right)=-2\left|p_{j}\right|, v_{p j}^{\prime \prime}=v_{p}^{\prime \prime}\left(\left|p_{j}\right|\right)=-2, v_{a L}^{\prime}=v_{a}^{\prime}\left(\left|a_{L}\right|\right)=-2\left|a_{L}\right|, v_{a L}^{\prime \prime}=v_{a}^{\prime \prime}\left(\left|a_{L}\right|\right)=-2$, $v_{\tilde{a} R}^{\prime}=v_{a}^{\prime}(|\tilde{a}|)=-2|\tilde{a}|, w_{p j}^{\prime}=w_{p}^{\prime}\left(\left|p_{j}-\bar{p}_{j}\right|\right), w_{a L}^{\prime}=w_{a}^{\prime}\left(\left|a_{L}-\bar{a}_{L}\right|\right), w_{\tilde{a} R}^{\prime}=w_{a}^{\prime}\left(\left|\tilde{a}-\bar{a}_{R}\right|\right), \varphi_{L}=\varphi(\Delta)$, $\varphi_{R}=\varphi(-\Delta)$, and we drop all double-asterisk superscripts for conciseness. Denoting the LHS matrix by $D$, we can show that the determinant of $D$ has a positive sign.

Lemma 1. $|D|>0$.

With Lemma 1, we can conduct comparative static exercises.

Lemma 2. In the subgame where candidate $L$ rejects the offer, comparative static results on the Nash equilibrium of policy competition are:

1. $\frac{d\left|p_{L}\right|}{d C_{R}}<0, \frac{d a_{L}}{d C_{R}}<0, \frac{d p_{R}}{d C_{R}}>0$, and $\frac{d \Delta}{d C_{R}}<0$.
2. $\frac{d\left|p_{L}\right|}{d \tilde{a}}>0, \frac{d a_{L}}{d \tilde{a}}>0$, and $\frac{d \Delta}{d \tilde{a}}>0$, and $\frac{d p_{R}}{d \tilde{a}}>0$.
3. Candidate L's equilibrium payoff in this subgame is decreasing in $C_{R}$.

The case where candidate $R$ rejects the offer is symmetrically analyzed.

### 3.4.2 Equilibrium when both candidates accept the offer

Letting $\Delta^{*}=v_{L}^{*}-v_{R}^{*}=v_{p}\left(\left|p_{L}^{*}\right|\right)+v_{a}(|\tilde{a}|)+C_{L}-v_{p}\left(\left|p_{R}^{*}\right|\right)-v_{a}(|\tilde{a}|)-C_{R}$, the system of equation that characterizes the equilibrium in this case is written as

$$
\begin{align*}
& \varphi\left(\Delta^{*}\right) v_{p L}^{\prime}\left\{Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{*}-\bar{a}_{L}\right|\right)\right\}-w_{p L}^{\prime}=0 \\
& \varphi\left(-\Delta^{*}\right) v_{p R}^{\prime}\left\{Q+w_{p}\left(\left|p_{R}^{*}-\bar{p}_{R}\right|\right)+w_{a}\left(\left|a_{R}^{*}-\bar{a}_{R}\right|\right)\right\}-w_{p R}^{\prime}=0 \\
& v_{L}^{*}-v_{R}^{*}-\Delta^{*}=0 \tag{11}
\end{align*}
$$

By simplifying the notations in the same way as in the previous subsection and totally differentiating the system, we obtain

$$
\left(\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right)\left(\begin{array}{c}
d\left|p_{L}\right| \\
d p_{R} \\
d \Delta
\end{array}\right)
$$

$$
=\left(\begin{array}{c}
-\varphi_{L} w_{\tilde{a} L}^{\prime} v_{p L}^{\prime} \\
-\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} \\
0
\end{array}\right) d \tilde{a}+\left(\begin{array}{c}
0 \\
0 \\
-1
\end{array}\right) d C_{L}+\left(\begin{array}{c}
0 \\
0 \\
1
\end{array}\right) d C_{R}
$$

by noting $v_{\tilde{a} L}^{\prime}=v_{\hat{a} R}^{\prime}$ by additive separability. Denote the LHS matrix by $\hat{D}$. Then, we have the following:
Lemma 3. $|\hat{D}|<0$
We conduct comparative statics in this case, too.
Lemma 4. When both candidates accept IG's offer, comparative static results on policy competition equilibrium are: $\frac{d\left|p_{L}\right|}{d \bar{a}}>0, \frac{d p_{R}}{d \bar{a}}>0, \frac{d\left|p_{L}\right|}{d C_{L}}>0, \frac{d p_{R}}{d C_{L}}<0, \frac{d \Delta}{d C_{L}}>0, \frac{d\left|p_{L}\right|}{d C_{R}}<0, \frac{d p_{R}}{d C_{R}}>0, \frac{d \Delta}{d C_{R}}<0$. Moreover, $L$ 's equilibrium payoff in this subgame is decreasing in $C_{R}$ and increasing in $C_{L}$.

### 3.4.3 Incentive compatible contracts

Now, we will identify and impose a sufficient condition under which both incentive compatibility constraints must be binding when the IG minimizes the cost. Let us take a look at candidate $L$ 's incentive compatibility constraint (7). If the condition is binding, we have

$$
\Pi_{L}^{*} w_{L}^{*}=\Pi_{L}^{* *} w_{L}^{* *}
$$

where

$$
\begin{gathered}
\Pi_{L}^{*}=\tilde{G}\left(C_{L}-\left(\bar{p}_{m}-p_{L}^{*}\right)^{2}-C_{R}+\left(\bar{p}_{m}-p_{R}^{*}\right)^{2}\right), \\
\Pi_{L}^{* *}=\tilde{G}\left(-\left(\bar{p}_{m}-p_{L}^{* *}\right)^{2}-\left(\bar{a}_{m}-a_{L}^{* *}\right)^{2}-C_{R}+\left(\bar{p}_{m}-p_{R}^{*}\right)^{2}+\left(\bar{a}_{m}-\tilde{a}\right)^{2}\right), \\
w_{L}^{*}=Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right), \\
w_{L}^{* *}=Q+w_{p}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}^{* *}-\bar{a}_{L}\right|\right) .
\end{gathered}
$$

The key difference between $\Pi_{L}^{*}$ and $\Pi_{L}^{* *}$ is that in the former, candidate $L$ receives $C_{L}$ for committing to setting her agenda level at $\tilde{a}$, while in the latter, candidate $L$ can reduce her agenda level to $a_{L}^{* *}$ to attract median voter but without IG's contribution. Suppose that $C_{R}=0$. Then, there is a value $\bar{C}_{L}>0$ that achieves $\Pi_{L}^{*} w_{L}^{*}=\Pi_{L}^{* *} w_{L}^{* *}$ given a fixed value of $\tilde{a}$ (other policy variables are determined in equilibrium). Starting from $\left(C_{L}, C_{R}\right)=\left(\bar{C}_{L}, 0\right)$, increase $C_{R}$ by adjusting $C_{L}$, keeping the incentive compatibility


Figure 3: $A$ downward-sloping $I C_{L}-A s$ long as $C_{L}-C_{R}$ is fixed, $\Pi_{L}^{*} W_{L}^{*}$ remains unchanged (along $45^{\circ}$ lines). Similarly, as long as $C_{R}$ is fixed, $\Pi_{L}^{* *} W_{L}^{* *}$ is unchanged (along vertical lines). $I C_{L}$ will be determined by the intersection of indifference curves with the same payoff level.
constraint binding. Totally differentiating the above, we have

$$
\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{L}} d C_{L}+\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}} d C_{R}-\frac{d\left(\Pi_{L}^{* *} w_{L}^{* *}\right)}{d C_{R}} d C_{R}=0
$$

or

$$
\begin{equation*}
\left.\frac{d C_{L}}{d C_{R}}\right|_{I C_{L}}=\frac{-\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}}+\frac{d\left(\Pi_{L}^{* *} w_{L}^{* *}\right)}{d C_{R}}}{\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{L}}} \tag{12}
\end{equation*}
$$

By Lemmas 2 and 4, we can see that $\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}}<0$ and $\frac{d\left(\Pi_{L}^{* *} w_{L}^{* *}\right)}{d C_{R}}<0$. The slope of IC curves are completely determined by the relative size of these two terms $\left(-\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}}=\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{L}}\right.$ by definition). When $\tilde{a}$ is very far from $\bar{a}_{L}, w_{L}^{*}$ should be significantly less than $w_{L}^{* *}$. Figure 3 shows how the incentive compatibility constraint for $L$ looks like in such a case, and $L$ 's indifference curves when she accepted and rejected IG's offer. Thus, the impact on $\Pi_{L}^{*} w_{L}^{*}$ by an increase in $C_{R}$ tends to be dominated by the one on $\Pi_{L}^{* *} w_{L}^{* *}$, since a decrease in winning probability leaves more impact on the latter. In this case, $I C_{L}$ curve is downward sloping, and thus, the numerator of (12) is negative. In contrast, when $\tilde{a}$ is close to $\bar{a}_{L}$ while $\bar{a}_{m}$ is distant from $\bar{a}_{L}, w_{L}^{*}$ would be larger than $w_{L}^{* *}$. In this case, the numerator of (12) is positive. Figure 4 shows how the incentive compatibility constraints for $L$ and $R$ look like and illustrates IG's minimization problem. We will impose the following assumption.


Figure 4: Binding constraints for both candidates - If Regularity in IC Constraints is respected, two constraints can only cross each other once. Moreover, at point $A, I C_{L}$ is binding, but $I C_{R}$ is not. IG can reduce its cost by moving along $I C_{L}$. At point $B, I G$ 's cost is minimized.

Regularity in IC Constraints. Candidates' incentive compatibility constraints satisfy the following con-
ditions: $\left.\frac{d C_{L}}{d C_{R}}\right|_{I C_{L}}>-1$ and $\left.\frac{d C_{R}}{d C_{L}}\right|_{I C_{R}}>-1$
(or equivalently, $\left|\frac{d\left(\Pi_{L}^{* *} w_{L}^{* *}\right)}{d C_{R}}\right|<2\left|\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}}\right|$ and $\left|\frac{d\left(\Pi_{R}^{* *} w_{R}^{* *}\right)}{d C_{L}}\right|<2\left|\frac{d\left(\Pi_{R}^{*} w_{R}^{*}\right)}{d C_{L}}\right|$ ).
This condition is not imposed on economic primitives, since $\Pi_{j}^{*} w_{j}^{*}$ and $\Pi_{j}^{* *} w_{j}^{* *}$ are equilibrium payoffs in subgames. However, it appears to be quite a reasonable requirement. ${ }^{16}$ This condition requires that the impacts of an increase in the opponent's campaign contribution do not differ too much between accepting and rejecting IG's offer. We will impose this assumption for the rest of the paper.

Proposition 3. (Binding Incentive Compatibility Constraints) Suppose that ( $\tilde{a}, C_{L}, C_{R}$ ) is an incentive compatible contract at the minimum cost. Then, the incentive compatibility constraints (7) and (8) are binding.

## 4 Symmetric Equilibria

In order to get analytical results, we assume that two candidates are symmetric: that is, $\bar{p}_{m}=0, \bar{p}_{L}=$ $-\bar{p}_{R}=\bar{p}$, and $\bar{a}_{L}=\bar{a}_{R}=\bar{a}$, and $g(\epsilon)$ is a symmetric density function at $\epsilon=0$. In a symmetric equilibrium,

[^9]$\tilde{a}_{L}=\tilde{a}_{R}=\tilde{a}$ and $C_{L}=C_{R}=\tilde{C}$ hold. Therefore, in a symmetric equilibrium, $L$ 's equilibrium payoff when both candidates accept IG's offer becomes
\[

$$
\begin{equation*}
\Pi_{L}\left(p_{L}^{*}, \tilde{a}, \tilde{C}, p_{R}^{*}, \tilde{a}, \tilde{C}\right)\left\{Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right)\right\}=\frac{1}{2}\left\{Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right)\right\} \tag{13}
\end{equation*}
$$

\]

where $p_{L}^{*}=-p_{R}^{*}$. Now, increase $\tilde{a}$ by keeping the incentive compatibility constraints binding by adjusting $\tilde{C}$. By symmetry, $\tilde{C}$ will be adjusted equally, and the probability of winning does not change at $\frac{1}{2}$. Thus, candidate $L$ 's first order condition for $p_{L}$ is

$$
\begin{equation*}
\eta_{0} v_{p}^{\prime}\left(\left|p_{L}^{*}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right)\right\}-w_{p}^{\prime}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)=0 \tag{14}
\end{equation*}
$$

where $\eta_{0}=2 \tilde{g}(0)$ and $v_{p}^{\prime}\left(\left|p_{L}^{*}\right|\right)=-2\left|p_{L}^{*}\right|$.
With this first order condition, we observe the uniqueness of symmetric equilibrium.

Proposition 4. (Unique Symmetric Equilibrium) There is a unique symmetric equilibrium if candidates are symmetric.

When the IG pursues free trade policy more aggressively, what will happen to candidates' ideology positions and contribution money? The following is the main analytical result of this paper.

Proposition 5. (Polarization and Contributions Rise by an Increase in $\tilde{a}$ ) In symmetric equilibrium in which both candidates accept IG's offer, an increase in $\tilde{a}$ causes policy polarization and campaign contributions rise when $\tilde{a}>\bar{a}_{L}=\bar{a}_{R}>0$.

This result can be interpreted intuitively as follows. If IG pursues its agenda more aggressively, then the candidates' winning payoffs decrease since $\tilde{a}>\bar{a}_{j}$. This means that each candidate tries to increase her winning payoff by choosing a more extreme policy, even though such a move reduces her winning probability. Also, in order to keep the incentive compatibility constraints binding, $\tilde{C}$ has to be increased as $\tilde{a}$ increases to keep the payoff on the off-equilibrium path low enough to implement the offer.

Can we drop symmetry to get the same result? From Lemma 4, we know $\frac{d\left|p_{L}\right|}{d \tilde{a}}>0$ and $\frac{d p_{R}}{d \tilde{a}}>0$, so it might appear that it is possible to do so. However, if the candidates are asymmetric, to satisfy incentive compatibility constraints, $C_{L}$ and $C_{R}$ need to be adjusted in asymmetric manner in response to an increase in $\tilde{a}$. This in turn affects supported equilibrium allocation. The benefit of the symmetry assumption comes from the fact that a symmetric increase in contribution money per se has no direct effect on candidates'
policy choice, since candidates care about campaign contributions only when their winning probabilities are affected by them.

## 5 Numerical Analysis: A Logit Model

In the previous section, we analyzed how the agenda interacts with ideological policy and contribution money in the symmetric equilibrium. For the asymmetric cases as well as comparative static exercises with respect to other parameters, the signs of the determinants are undetermined. Therefore, in this section, we use a numerical example to explore how equilibrium strategies change in various cases.

Hereafter, we assume that $\epsilon_{j}$ is independently drawn from a Type-I Extreme Value Distribution. ${ }^{17}$ Moreover, we follow the quadratic utility function for both the voter and candidates: that is,

$$
v_{m}\left(p_{j}, a_{j}, C_{j}\right)=-\left(p_{j}-\bar{p}_{m}\right)^{2}-\theta_{a}\left(a_{j}-\bar{a}_{m}\right)^{2}+C_{j}
$$

and

$$
W_{j}=Q-\left(p_{j}-\bar{p}_{j}\right)^{2}-\left(a_{j}-\bar{a}_{j}\right)^{2}
$$

Note that we will allow $\bar{p}_{m} \neq 0$ and $\bar{a}_{m} \neq 0$ so that we can conduct comparative static exercises in the median voter's bliss point. We again assume that $\tilde{a}>\bar{a}_{L} \geq \bar{a}_{R}>\bar{a}_{m} \geq 0$ and $-1=\bar{p}_{L}<\bar{p}_{m}<\bar{p}_{R}=1$.

According to Train (2003), the distribution assumption implies that

$$
\Pi_{L}=\frac{\exp \left(v_{m L}\right)}{\exp \left(v_{m L}\right)+\exp \left(v_{m R}\right)}
$$

Here, we only list the f.o.c's for the equilibrium in which $L$ rejects the offer. Other cases are similar.

$$
\begin{aligned}
& \frac{\exp \left(v_{m R}\right)}{\exp \left(v_{m L}\right)+\exp \left(v_{m R}\right)}\left(\bar{p}_{m}-p_{L}\right)\left\{Q-\left(p_{L}-\bar{p}_{L}\right)^{2}-\left(a_{L}-\bar{a}_{L}\right)^{2}\right\}-\left(p_{L}-\bar{p}_{L}\right)=0 \\
& \frac{\exp \left(v_{m R}\right)}{\exp \left(v_{m L}\right)+\exp \left(v_{m R}\right)} \theta_{a}\left(a_{L}-\bar{a}_{m}\right)\left\{Q-\left(p_{L}-\bar{p}_{L}\right)^{2}-\left(a_{L}-\bar{a}_{L}\right)^{2}\right\}-\left(\bar{a}_{L}-a_{L}\right)=0 \\
& \frac{\exp \left(v_{m L}\right)}{\exp \left(v_{m L}\right)+\exp \left(v_{m R}\right)}\left(p_{R}-\bar{p}_{m}\right)\left\{Q-\left(p_{R}-\bar{p}_{R}\right)^{2}-\left(a_{R}-\bar{a}_{R}\right)^{2}\right\}-\left(\bar{p}_{R}-p_{R}\right)=0
\end{aligned}
$$

Here, we deal with the case in which $\bar{p}_{L} \leq p_{L} \leq \bar{p}_{m} \leq p_{R} \leq \bar{p}_{R}$ and $\bar{a}_{m} \leq a_{L} \leq \bar{a}_{L}$.

[^10]We use the following benchmark parameter values: $\bar{a}_{m}=\bar{p}_{m}=0, \theta_{a}=1$, and $Q=5$.

## Asymmetric Candidates

First, we list the symmetric equilibrium where $\bar{a}_{R}=\bar{a}_{L}=0.5,\left|\bar{p}_{L}\right|=\bar{p}_{R}=1$, and $\tilde{a}=0.8:\left|p_{L}\right|=p_{R}=$ 0.3108 , and $C_{L}=C_{R}=0.6021$.

## Asymmetric Agenda Bliss Points

We first consider $\bar{a}_{L}=0.5>\bar{a}_{R}=0.3$ and analyze the effects of an increase in $\tilde{a}$. From Lemma 4, we know that candidates polarize on the ideology dimension to get a higher winning payoff as a compensation for accepting a more aggressive agenda. ${ }^{18}$ In the asymmetric equilibrium, there is one more effect. As $\tilde{a}$ goes up, $R$ suffers more than $L$, and $R$ has a stronger incentive to deviate from taking IG's offer. Therefore, $C_{R}$ has to increase more than $C_{L}$ as $\tilde{a}$ goes up. According to Lemma 4, this change in difference of contributions tends to lower winning probability $\Pi_{L}$. To balance this effect, $L$ has an incentive to choose a more ideologically central position (that is, $\frac{d\left|p_{L}\right|}{d C_{R}}<0$ ). Which effect dominates depends on the parameter values. Intuitively, when $\tilde{a}$ is close to $\bar{a}_{L}$, the marginal loss from an increase in $\tilde{a}$ is nearly 0 for $L$, which means the incentive to take an extreme position (i.e., the first effect) is minimal. Therefore, the impact from increasing $C_{R}-C_{L}$ dominates, and $L$ moves to the center while $R$ moves to the right. On the other hand, when $\tilde{a}$ is much higher than $\bar{a}_{L}$, the loss from accepting an aggressive agenda dominates and polarization happens. ${ }^{19}$

We demonstrate this by increase $\tilde{a}$ from 0.5 to 1 . Note that $\bar{a}_{L}=0.5$ and, as a result, $L$ has minimal incentive to take an extreme ideological position initially. The computational results are listed in Table 1. As we expect from the above argument, $L$ initially moves to the center when $\tilde{a}$ is close to $\bar{a}_{L}$ but turns back to extreme as $\tilde{a}$ becomes larger and larger. Meanwhile, $R$ monotonically moves to his/her own extreme. Therefore, candidates show an asymmetric pattern of polarization in the sense that the more conservative candidate on the agenda becomes more extreme on the ideology dimension as IG becomes more aggressive in promoting the agenda. Moreover, in order to promote $\tilde{a}$ more aggressively, IG needs to contribute more to both candidates. It might be surprising that IG contributes more to the candidate who prefers a lower agenda, and this candidate wins more often in the equilibrium. This result is a consequence of the incentive

[^11]compatibility constraints: $R$ has a stronger incentive to reject IG. Therefore, IG contributes more to $R$. ${ }^{20}$

| $\tilde{a}$ | $p_{L}$ | $p_{R}$ | $C_{L}$ | $C_{R}$ | $\Pi_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | -0.3008 | 0.3148 | 0.1706 | 0.2400 | 0.4848 |
| 0.6 | -0.2997 | 0.3187 | 0.2855 | 0.3743 | 0.4808 |
| 0.7 | -0.2994 | 0.3238 | 0.4296 | 0.5389 | 0.4765 |
| 0.8 | -0.2999 | 0.3301 | 0.6026 | 0.7341 | 0.4719 |
| 0.9 | -0.3012 | 0.3377 | 0.8046 | 0.9604 | 0.4669 |
| 1.0 | -0.3033 | 0.3468 | 1.0355 | 1.2186 | 0.4614 |

Table 1: Asymmetric equilibrium where $\tilde{a} \in[0.5,1], \bar{a}_{L}=0.5>0.3=\bar{a}_{R}$.

Before moving on the next example, it is worth pointing out that the candidate who is more reluctant with agenda promotion is also the one proposing a more extreme ideology platform. This is a general trend in our logit example: when $\bar{a}_{R}$ decreases, IG contributes more to $R$ to compensate for the loss from committing to $\tilde{a}$. Therefore, $\Pi_{L}$ decreases and $\left|p_{L}\right|\left(p_{R}\right)$ decreases (increases).

## Asymmetric Ideology Bliss Point

Here, we consider the case where $\bar{p}_{R}=1.5>\left|\bar{p}_{L}\right|=1, \bar{a}_{L}=\bar{a}_{R}=0.5$ and $\tilde{a} \in[0.5,1]$. This exercise allows us to see the robustness of Proposition 5 under ideological asymmetry. The first-order effect of increasing $\tilde{a}$ creates polarization by Lemma 4, the same as before. Moreover, intuitively, an increase in $\tilde{a}$ raises the incentive for candidates to deviate from accepting the offer regardless of where the ideology bliss point is. However, it is not clear how the difference in contribution money, $C_{R}-C_{L}$, changes. Our numerical result is shown in Table 2.

[^12]| $\tilde{a}$ | $p_{L}$ | $p_{R}$ | $C_{L}$ | $C_{R}$ | $\Pi_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | -0.3201 | 0.4825 | 0.1628 | 0.1657 | 0.5318 |
| 0.6 | -0.3206 | 0.4832 | 0.2781 | 0.2807 | 0.5320 |
| 0.7 | -0.3221 | 0.4851 | 0.4241 | 0.4260 | 0.5324 |
| 0.8 | -0.3247 | 0.4883 | 0.6008 | 0.6014 | 0.5331 |
| 0.9 | -0.3284 | 0.4929 | 0.8084 | 0.8072 | 0.5340 |
| 1.0 | -0.3333 | 0.4989 | 1.0474 | 1.0435 | 0.5354 |

Table 2: Asymmetric equilibrium where $\tilde{a} \in[0.5,1], \bar{p}_{R}=1.5>1=\bar{p}_{L}$.

Our numerical exercise suggests that the candidate representing a stronger party line ( $R$ in our case) has a stronger incentive to deviate from accepting the offer when $\tilde{a}$ is relatively low. But, $C_{R}-C_{L}$ decreases as $\tilde{a}$ increases. By Lemma 4, this change increases $\Pi_{L}$, increases $\left|p_{L}\right|$, and decreases $p_{R}$ consequently. Another effect is the first-order effect of $\tilde{a}$ on polarization, $\frac{d\left|p_{L}\right|}{d \tilde{a}}>0$ and $\frac{d p_{R}}{d \tilde{a}}>0$. However, the first-order effect dominates so that $R$ still polarizes when $\tilde{a}$ increases. In fact, $R$ always proposes a relatively more extreme ideology policy, and moves to his own bliss point faster than $L$ does. ${ }^{21}$

We find that the result in Proposition 5 is still valid even for asymmetric candidates. Moreover, we observe asymmetric polarization when the candidates are asymmetric. Barber and McCarty (2015) report the ideological positions of Republican representatives have polarized much more than those of Democratic representatives. Our model suggest that we can explain such patterns of polarization if Republicans are more reluctant to promote free trade than Democrats, and if they have stronger ideological views.

## The Trend of Rising Protectionism

Recently, there has been a growing worldwide sentiment of anti-globalism and protectionism. Examples include 2016 US presidential election and the 2017 French presidential election. In our framework, this trend can be interpreted as decreasing $\bar{a}_{m}$. As $\bar{a}_{m}$ decreases, both candidates get more contributions from IG, which is rather intuitive. However, unlike the effect of increasing $\tilde{a}$, decreasing $\bar{a}_{m}$ has no effects on for the equilibrium in which both candidates accept IG's offer. In system (11), an increasing $\bar{a}_{m}$ changes neither $\Delta$ nor the first order conditions for both candidates. Therefore, in the symmetric case, only contributions increase and nothing else changes. However, in the asymmetric case, as $\bar{a}_{m}$ decreases, the payoff from

[^13]rejecting an offer is higher for the candidate whose ideal level of $a$ is lower (it is candidate $R$ in our setup). This is because the policy cost function is convex. When deviating from the agreement with IG, candidate $R$ finds it less costly to win an increasingly protectionist median voter. Thus, IG tends to contribute more to $R$ compared to $L$ in order to provide $R$ an enough incentive to accept an offer. According to Lemma 4 , the winning probability is biased toward $R$, which causes $p_{R}$ to move to extremes and $p_{L}$ to move to center. Moreover, the distance between $p_{L}^{*}$ and $p_{R}^{*}$ increases as well, which means ideology positions are more divergent as voters become more conservative on the agenda. We compute the equilibrium by setting $\bar{a}_{m} \in[-0.2,0.1], \bar{a}_{L}=0.5>\bar{a}_{R}=0.3$ and $\tilde{a}=0.8$. See the results in Table 3.

| $\bar{a}_{m}$ | $p_{L}$ | $p_{R}$ | $C_{L}$ | $C_{R}$ | $\Pi_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | -0.3010 | 0.3288 | 0.4809 | 0.5996 | 0.4747 |
| 0 | -0.2999 | 0.3301 | 0.6026 | 0.7341 | 0.4719 |
| -0.1 | -0.2989 | 0.3313 | 0.7379 | 0.8818 | 0.4691 |
| -0.2 | -0.2979 | 0.3325 | 0.8864 | 1.0426 | 0.4664 |

Table 3: Increasing trend of protectionism- $\bar{a}_{m}$ decreases from 0.3 to 0 .

Again, we observe an asymmetric polarization. The Republican's ideological position polarizes as $\bar{a}_{m}$ goes down, while the Democrat's position does not change much and even moves toward center slightly. Thus, if Republican candidates are more reluctant to promote free trade than Democrat's, then the asymmetric polarization can be explained by the increasing trend of protectionism. ${ }^{22}$

## Ex Ante Valence Advantage

In the benchmark case, we assume that the voter is unbiased toward the two candidates in the sense that, as long as the policy proposals and campaign contributions are symmetric, the winning probability is also the same. However, it is often the case that one candidate may have a "non-policy" advantage, such as incumbency or strong personal charisma. To incorporate this effect, we assume the voters evaluate $L$ and $R$

[^14]by
\[

$$
\begin{aligned}
& v\left(\left|p_{L}-\bar{p}_{m}\right|,\left|a_{L}-\bar{a}_{m}\right|, C_{L}\right)+\epsilon_{L}+\eta, \\
& v\left(\left|p_{R}-\bar{p}_{m}\right|,\left|a_{R}-\bar{a}_{m}\right|, C_{R}\right)+\epsilon_{R},
\end{aligned}
$$
\]

where $\eta$ stands for a nonrandom advantage that $L$ has at the beginning of the election (a disadvantage if $\eta$ is negative). It is relatively straightforward to show that, in the equilibrium where both candidates accept IG's offer, an increase in $L$ 's advantage causes $\left|p_{L}\right|$ to increase and $p_{R}$ to decrease. Moreover, this should increase $L$ 's winning probability and payoff. However, it is more difficult to decide how the change in $\eta$ affects $C_{L}$ and $C_{R}$, which are decided by the incentive constraints. Our numerical example shows that $C_{L}$ increases relatively to $C_{R}$ in most of the parameter space. Also, the winning probability is more biased toward $L$ as $\eta$ increases. The following table shows the results for $\bar{a}_{L}=\bar{a}_{R}=0.3$ and $\eta \in[0,0.5]$.

| $\eta$ | $p_{L}$ | $p_{R}$ | $C_{L}$ | $C_{R}$ | $\Pi_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | -0.3182 | 0.3182 | 0.7296 | 0.7296 | 0.5 |
| 0.1 | -0.3282 | 0.3089 | 0.7335 | 0.7261 | 0.5238 |
| 0.2 | -0.3388 | 0.3001 | 0.7378 | 0.7230 | 0.5474 |
| 0.3 | -0.3500 | 0.2920 | 0.7426 | 0.7202 | 0.5708 |
| 0.4 | -0.3619 | 0.2843 | 0.7479 | 0.7176 | 0.5939 |
| 0.5 | -0.3744 | 0.2773 | 0.7537 | 0.7154 | 0.6166 |

Table 4: Candidate $L$ has ex ante advantage $-\bar{a}_{L}=\bar{a}_{R}$ case.

Naturally, the same pattern of changes applies to the asymmetric $\bar{a}_{L} \neq \bar{a}_{R}$. In the following table, we list the results for $\bar{a}_{L}=0.5>\bar{a}_{R}=0.3$ and $\tilde{a}=0.8$. Also, in this case, we consider $\eta \in[-0.2,0.2]$. It is natural to interpret as $R$ having ex ante advantage, when $\eta$ is negative. Also, recall that $R$ has some advantages even at small positive $\eta$, since her/his preference is more in line with the voter. In this situation, candidate $R$ 's policy position is most polarized while $L$ 's position moves slightly towards the center. Notice that $p_{L}$ and $p_{R}$ move in the same direction as the symmetric case.

| $\eta$ | $p_{L}$ | $p_{R}$ | $C_{L}$ | $C_{R}$ | $\Pi_{L}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.2 | -0.2839 | 0.3514 | 0.6035 | 0.7431 | 0.4264 |
| -0.1 | -0.2917 | 0.3404 | 0.6031 | 0.7384 | 0.4491 |
| 0 | -0.2999 | 0.3301 | 0.6026 | 0.7341 | 0.4719 |
| 0.1 | -0.3087 | 0.3203 | 0.6022 | 0.7303 | 0.4948 |
| 0.2 | -0.3181 | 0.3112 | 0.6018 | 0.7268 | 0.5177 |

Table 5: Shifting ex ante advantage from one candidate to the other- $\bar{a}_{L}>\bar{a}_{R}$ case.

This result is in stark contrast with the one in Groseclose (2001), which shows that the advantageous candidate moves toward the center while the disadvantageous candidate moves away from the center when one candidate has a small advantage. Unlike our uncertain valence model, the source of uncertainty is from the median voter's position in Groseclose (2001). In his model, the median voter's position can be very sensitive to proposed policies when the utility function has high curvature and the ex ante advantage is small. Therefore, it is possible that the advantageous candidate proposes a more central policy under such a situation. This suggests that different ways to incorporate uncertainty have distinct comparative statics. ${ }^{23}$

## 6 Conclusion

Despite anti-free trade sentiment among the voters, protectionism and globalization have not been salient issues in US presidential elections until 2016. During this same period of time, we have also observed increasing ideological polarization in policies and the surge of campaign contributions from industries. This paper proposes a multi-dimensional policy competition model with an interest group that provides campaign contributions to two candidates and asking them to commit to a certain level in an agenda dimension (say, free trade). If they take contributions, they only compete in policies in an ideological dimension. Our probabilistic voting model with uncertain valence allows us to analytically investigate the structure of incentive constraints. We show that when candidates are symmetric, the candidates' ideological positions polarize as IG promotes their agenda more aggressively, providing more contributions for both candidates

[^15]to get them to stick to their commitment. The mechanism behind this is simple: if IG promotes the agenda despite the candidates' reluctance to agree, then policy-motivated candidates' winning payoffs go down, and they try to compensate by choosing ideological positions closer to their ideal ones, resulting in polarization. Except for analytical results in symmetric case, we also lay down the foundation of a computational method which shows that the above argument is not limited to the symmetric case. It is commonly recognized that polarization in the US politics is mainly a right-wing phenomenon. Using our model, we can show that such asymmetric polarization can be caused by the IG's promotion of their agenda, if Republican candidates are more reluctant to promote IG's agenda than Democratic candidates, and/or if Republican candidates is more charismatic than Democratic ones.

In this paper, we assume away the budget constraint of the Interest Group. One direct implication of this assumption is that both parties would accept IG's contract in equilibrium. Since the focus in this paper is the connection between ideology polarization and IG's promoting agenda via the incentive compatible contracts, this assumption seems to be appropriate in capturing the essentials in US politics before 2016. To explore the effect of budget constraint, it is needed to specify IG's payoff function. In the companion paper, Konishi and Pan (2017) endogenize the agenda policy by formulating IG's maximization problem subject to a budget constraint. We find that, when IG's budget is binding, IG tends contribute only to the candidate who is less eager in promoting the agenda. This is a result of IG's risk aversion, that is, it would prevent less eager candidate to win by proposing a low level agenda.

## Appendix A: Electoral Competition

In this part, we shall provide a general existence result of electoral equilibrium in a two-party setting by assuming that there is a single voter (or a median voter) in $K$-dimensional policy space. By a slight abuse of notation, we denote a policy as $p=\left(p^{1}, p^{2}, \ldots, p^{K}\right) \in \mathbb{R}^{K}$ instead of $(p, a)$ in this subsection. Here, we will set up a version of the Wittman model with valence (Wittman 1983). Following Wittman, we assume that candidate $j$ 's payoff function is

$$
V_{j}\left(p_{j}, p_{i}\right)=\Pi_{j}\left(p_{j}, p_{i}\right) w_{j}^{1}\left(p_{j}\right)+\left(1-\Pi_{j}\left(p_{j}, p_{i}\right)\right) w_{j}^{0}\left(p_{i}\right)
$$

where $w_{j}^{1}\left(p_{j}\right)$ and $w_{j}^{0}\left(p_{i}\right)$ are candidate $j$ 's payoffs when she wins or loses an election, respectively. By setting $w_{j}^{0}\left(p_{i}\right)=0$ for all $p_{i}$, the theorem below covers Proposition 1 as a special case. We also drop $C_{j} \mathrm{~s}$ from the voter's utility function since $C_{j}$ s are fixed here. During the voting stage, voters compare two candidates by $p_{j}$ and $p_{i}$ given the realized valence bias. That is, the median voter votes for $j \in\{L, R\}$ over $i \in\{L, R\}$ with $i \neq j$ if and only if

$$
v\left(\left|p_{j}-\bar{p}_{m}\right|\right)-v\left(\left|p_{i}-\bar{p}_{m}\right|\right) \geq \epsilon_{i}-\epsilon_{j}
$$

where $\epsilon_{j}$ denotes a random valence term for candidate $j$. Let

$$
\begin{aligned}
& S_{j}\left(p_{j}, p_{i}\right) \equiv \\
& \quad\left\{\epsilon \in \mathbb{R}^{2} \mid v\left(\left|p_{j}-\bar{p}_{m}\right|\right)-v\left(\left|p_{i}-\bar{p}_{m}\right|\right) \geq \epsilon_{i}-\epsilon_{j}\right\}
\end{aligned}
$$

which is the set of events where the pivotal voter votes for $j$. Note that $S_{j}\left(p_{j}, p_{i}\right)$ is a convex set in $\mathbb{R}^{2}$. Therefore, the winning probability for $j$ is

$$
\Pi_{j}\left(p_{j}, p_{i}\right)=\int_{S_{j}\left(p_{j}, p_{i}\right)} g(\epsilon) d \epsilon
$$

The following mathematical result is useful in proving the existence of equilibrium.

The Prékopa Theorem (Prékopa 1973). Let $\psi$ be a probability density function on $\mathbb{R}^{K}$ with convex support $C$. Take any measurable sets $A_{0}$ and $A_{1}$ in $\mathbb{R}^{K}$ with $A_{0} \cap C \neq \emptyset$ and $A_{1} \cap C \neq \emptyset$. For any $0 \leq \lambda \leq 1$,
define $A_{\lambda}=(1-\lambda) A_{0}+\lambda A_{1}$, the Minkowski average of the two sets. ${ }^{24}$ If $\psi(\alpha)$ is log concave, then

$$
\log \int_{A_{\lambda}} \psi(\alpha) d \alpha \geq(1-\lambda) \log \int_{A_{0}} \psi(\alpha) d \alpha+\lambda \log \int_{A_{1}} \psi(\alpha) d \alpha .
$$

We prove the following theorem by utilizing the Prékopa theorem:
Theorem A. (Existence) Let $P_{j} \subset \mathbb{R}^{K}$ be a compact and convex policy space. Suppose that there is a median voter, and that $v\left(\left|p_{j}-\bar{p}_{m}\right|\right)$ and $w_{j}^{1}\left(p_{j}\right)$ are continuous and concave in $p_{j}$, respectively, $w_{j}^{0}\left(p_{i}\right)$ is continuous in $p_{i}$, and the density function $g(\epsilon)$ is log-concave in $\epsilon \in \mathbb{R}^{2}$. Then, there exists a Nash equilibrium in the policy competition subgame.

Proof. Since $C_{L}$ and $C_{R}$ are fixed in this proposition, we will drop them from $u_{m}$ 's arguments. Since $v$ is concave, note that for all $p_{j}, p_{j}^{\prime}$, and all $\lambda \in[0,1]$,

$$
v\left(\left|\lambda p_{j}+(1-\lambda) p_{j}^{\prime}-\bar{p}_{m}\right|\right) \geq \lambda v\left(\left|p_{j}-\bar{p}_{m}\right|\right)+(1-\lambda) v\left(\left|p_{j}^{\prime}-\bar{p}_{m}\right|\right)
$$

By Prékopa's theorem (Prékopa 1973), we have

$$
\int_{\lambda S\left(p_{j}, p_{i}\right)+(1-\lambda) S\left(p_{j}^{\prime}, p_{i}\right)} g(\epsilon) d \epsilon \geq \lambda \int_{S\left(p_{j}, p_{i}\right)} g(\epsilon) d \epsilon+(1-\lambda) \int_{S\left(p_{j}^{\prime}, p_{i}\right)} g(\epsilon) d \epsilon
$$

Now, by definition of $S_{j}$ and concavity of $v$, we have

$$
S\left(\lambda p_{j}+(1-\lambda) p_{j}^{\prime}, p_{i}\right) \supseteq \lambda S\left(p_{j}, p_{i}\right)+(1-\lambda) S\left(p_{j}^{\prime}, p_{i}\right) .
$$

This implies

$$
\int_{S\left(\lambda p_{j}+(1-\lambda) p_{j}^{\prime}, p_{i}\right)} g(\epsilon) d \epsilon \geq \int_{\lambda S\left(p_{j}, p_{i}\right)+(1-\lambda) S\left(p_{j}^{\prime}, p_{i}\right)} g(\epsilon) d \epsilon
$$

and

$$
\int_{S\left(\lambda p_{j}+(1-\lambda) p_{j}^{\prime}, p_{i}\right)} g(\epsilon) d \epsilon \geq \lambda \int_{S\left(p_{j}, p_{i}\right)} g(\epsilon) d \epsilon+(1-\lambda) \int_{S\left(p_{j}^{\prime}, p_{i}\right)} g(\epsilon) d \epsilon .
$$

Therefore, we conclude that $\Pi_{j}\left(p_{j}, p_{i}\right)=\int_{S\left(p_{j}, p_{i}\right)} g(\epsilon) d \epsilon$ is log-concave in $p_{j}$ if $g$ is log-concave in $\epsilon$.

[^16]Let candidate $j$ 's best response $\beta_{j}: P_{i} \rightarrow P_{j}$ be such that

$$
\beta_{j}\left(p_{i}\right) \equiv \arg \max _{p_{j} \in P_{j}} V_{j}\left(p_{j}, p_{i}\right)
$$

This correspondence is nonempty-valued and upper hemicontinuous (continuity of $V_{j}$ ).
Using a trick by Roemer (1997), we can rewrite candidate $j$ 's payoff function in a convenient way:

$$
V_{j}\left(p_{j}, p_{i}\right)=\Pi_{j}\left(p_{j}, p_{i}\right)\left(w_{j}^{1}\left(p_{j}\right)-w_{j}^{0}\left(p_{i}\right)\right)+w_{j}^{0}\left(p_{i}\right)
$$

Thus, we have

$$
\log \left(V_{j}\left(p_{j}, p_{i}\right)-w_{j}^{0}\left(p_{i}\right)\right)=\log \Pi_{j}\left(p_{j}, p_{i}\right)+\log \left(w_{j}^{1}\left(p_{j}\right)-w_{j}^{0}\left(p_{i}\right)\right)
$$

and $V_{j}\left(p_{j}, p_{i}\right)-w_{j}^{0}\left(p_{i}\right)$ is shown to be log-concave in $p_{j}$ ( $p_{i}$ is fixed). Hence, $V_{j}\left(p_{j}, p_{i}\right)$ is quasi-concave in $p_{j}$. Thus, candidate $j$ 's best response correspondence $\beta_{j}: P_{i} \rightarrow P_{j}$ is convex-valued.

Since $P_{i} \times P_{j}$ is nonempty, compact, and convex, candidate $j$ 's best response correspondence $\beta_{j}$ is nonempty-valued, upper hemicontinuous, and convex-valued. By Kakutani's fixed point theorem, there exists a Nash equilibrium $p^{*}=\left(p_{L}^{*}, p_{R}^{*}\right)$

Remark 1. Proposition 1 is a special case of this theorem ( $w_{j}^{0}=0$, and $v_{j}^{m}$ is quadratic). If the policy space is one-dimensional, then there exists a median voter, and thus Theorem A guarantees the existence of electoral competition. Note that this theorem shows existence of equilibrium when uncertainty is generated only by valence terms. Roemer (1997) and Duggan and Martinelli (2017) use a model with uncertain median voter's position, which behaves differently, making the best response correspondence potentially discontinuous or nonconvex-valued. Note also that Duggan and Martinelli (2017) assumes log concavity of $G$. Here we assume a stronger condition: log concavity of $g$.

Proof of Proposition 2. The two candidates' policies are ( $p_{L}, a_{L}, C_{L}$ ) and ( $p_{R}, a_{R}, C_{R}$ ). Suppose that we have

$$
\bar{a}_{m} \leq \frac{1}{2\left(a_{R}-a_{L}\right)}\left[-2 \theta_{p}\left(p_{R}-p_{L}\right) \bar{p}_{m}+\left(p_{R}^{2}-p_{L}^{2}\right)+\theta\left(a_{R}^{2}-a_{L}^{2}\right)+\left(C_{L}-C_{R}\right)+\epsilon_{R}-\epsilon_{L}\right]
$$

Then, in $(p, a)$-space, $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ is below the voting cut-off line, the voter $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ votes for candidate $L$, who receives more votes than candidate $R$ under the assumptions (i) and (ii). If ( $\bar{p}_{m}, \bar{a}_{m}$ ) is above the voting
cut-off line, the voter $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ votes for candidate $R$, who receives more votes than candidate $L$. If $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ is right on the voting cut-off line, candidates $L$ and $R$ get exactly the same number of votes, and voter $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ is indifferent between $L$ and $R$. This proves that voter $\left(\bar{p}_{m}, \bar{a}_{m}\right)$ is the median voter. $\square$

Lemma 1. $|D|>0$.

Proof of Lemma 1. Direct calculations.

$$
\begin{aligned}
& |D|=-v_{p L}^{\prime}\left|\begin{array}{ccc}
-\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& +v_{a L}^{\prime}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& +v_{p R}^{\prime}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& -\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 \\
0 & 0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}
\end{array}\right| \\
& =\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} \\
& \times\left\{v_{p L}^{\prime}\left|\begin{array}{cc}
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L}
\end{array}\right|-v_{a L}^{\prime}\left|\begin{array}{cc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L}
\end{array}\right|\right\} \\
& +\left\{-\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)-w_{p R}^{\prime \prime}-\varphi_{R}^{\prime}\left(v_{p R}^{\prime}\right)^{2} w_{R}\right\} \\
& \times\left|\begin{array}{cc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime}
\end{array}\right| \\
& =\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} \\
& \times\left\{-\varphi_{L} \varphi_{L}^{\prime}\left(v_{p L}^{\prime}\right)^{2} v_{a L}^{\prime \prime} w_{L}^{2}-\varphi_{L}^{\prime}\left(v_{p L}^{\prime}\right)^{2} w_{L} w_{a L}^{\prime \prime}-\varphi_{L} \varphi_{L}^{\prime}\left(v_{a L}^{\prime}\right)^{2} v_{p L}^{\prime \prime} w_{L}^{2}-\varphi_{L}^{\prime}\left(v_{a L}^{\prime}\right)^{2} w_{L} w_{p L}^{\prime \prime}\right\} \\
& +\left\{-\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)-w_{p R}^{\prime \prime}-\varphi_{R}^{\prime}\left(v_{p R}^{\prime}\right)^{2} w_{R}\right\} \\
& \times\left\{\varphi_{L}^{2} w_{L}\left(v_{p L}^{\prime \prime} v_{a L}^{\prime \prime} w_{L}-v_{p L}^{\prime} w_{p L}^{\prime} v_{a L}^{\prime \prime}-v_{a L}^{\prime} w_{a L}^{\prime} v_{p L}^{\prime \prime}\right)+w_{p L}^{\prime \prime} w_{a L}^{\prime \prime}\right. \\
& \left.+\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right) w_{a L}^{\prime \prime}+\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right) w_{p L}^{\prime \prime}\right\} \\
& >0
\end{aligned}
$$

We have completed the proof.

Now, we are ready to conduct comparative static exercises.

Lemma 2. When candidate $L$ rejects the offer, the comparative static results on policy competition are:

1. $\frac{d\left|p_{L}\right|}{d \tilde{C}}<0, \frac{d a_{L}}{d \tilde{C}}<0, \frac{d p_{R}}{d \tilde{C}}>0$, and $\frac{d \Delta}{d \tilde{C}}<0$.
2. $\frac{d\left|p_{L}\right|}{d \tilde{a}}>0, \frac{d a_{L}}{d \tilde{a}}>0$, and $\frac{d \Delta}{d \tilde{a}}>0$, and $\frac{d p_{R}}{d \tilde{a}} \gtreqless 0$.
3. Candidate L's equilibrium payoff in this subgame is decreasing in $C_{R}$.

Proof of Lemma 2. Let's start with comparative statics in $\tilde{C}$.

$$
\begin{aligned}
& \frac{d\left|p_{L}\right|}{d C_{R}}=\frac{1}{|D|}\left|\begin{array}{cccc}
0 & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
1 & v_{a L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{-\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|D|}\left[\varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\left\{\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime}\right\}\right. \\
& \left.+\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L}\right] \\
& =\frac{-\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|D|}\left[\varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\left\{\varphi_{L} v_{a L}^{\prime \prime} w_{L}+w_{a L}^{\prime \prime}\right\}\right] \\
& <0 \\
& \begin{aligned}
\frac{d a_{L}}{d C_{R}} & =\frac{1}{|D|}\left|\begin{array}{cccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & 0 & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & 1 & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{v_{C R}^{\prime}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L}\left[-\varphi_{L} v_{p L}^{\prime \prime} w_{L}-w_{p L}^{\prime \prime}\right]}{|D|} \\
& <0
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d p_{R}}{d C_{R}} & =\frac{1}{|D|}\left|\begin{array}{cccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L} \frac{\partial v_{L}}{\partial a_{L}} \frac{\partial w_{L}}{\partial p_{p L}} & \varphi_{L}\left(\frac{\partial v_{L}}{\partial a_{L}} \frac{\partial w_{L}}{\partial a_{L}}+\frac{\partial^{2} v_{L}}{\partial a_{L}^{2}} w_{L}\right)+\frac{\partial^{2} w_{L}}{\partial a_{L}^{2}} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & 0 & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & v_{a L}^{\prime} & v_{C R}^{\prime} & -1
\end{array}\right| \\
& =\frac{\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}}{|D|}\left\{\varphi_{L}^{2} w_{L}\left(v_{p L}^{\prime \prime} v_{a L}^{\prime \prime} w_{L}-v_{p L}^{\prime} w_{p L}^{\prime} v_{a L}^{\prime \prime}+v_{a L}^{\prime} w_{a L}^{\prime} v_{p L}^{\prime \prime}\right)+w_{p L}^{\prime \prime} w_{a L}^{\prime \prime}\right. \\
& \left.+\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right) w_{a L}^{\prime \prime}+\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right) w_{p L}^{\prime \prime}\right\} \\
& >0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d \Delta}{d C_{R}} & =\frac{1}{|D|}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L} w_{a L}^{\prime} & 0 \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 \\
0 & 0 & 0 \\
v_{p L}^{\prime} & v_{a L}^{\prime} & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} \\
& 0 \\
& =\frac{1}{|D|}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} & -v_{p R}^{\prime}
\end{array}\right| \\
& \times\left\{\varphi_{L}^{2} w_{L}\left(v_{p L}^{\prime \prime} v_{a L}^{\prime \prime} w_{L}-v_{p L}^{\prime} w_{p L}^{\prime} v_{a L}^{\prime \prime}-v_{a L}^{\prime} w_{a L}^{\prime} v_{p L}^{\prime \prime}\right)+w_{p L}^{\prime \prime} w_{a L}^{\prime \prime}\right. \\
& \left.+\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right) w_{a L}^{\prime \prime}+\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right) w_{p L}^{\prime \prime}\right\} \\
& <0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d\left|p_{L}\right|}{d \tilde{a}}=\frac{1}{|D|}\left|\begin{array}{cccc}
0 & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
-\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & 0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{\tilde{a} R}^{\prime} & v_{a L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{-\varphi_{R} w_{a}^{\prime} v_{p R}^{\prime}}{|D|}\left|\begin{array}{ccc}
-\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
v^{\prime} a_{L} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& -\frac{v_{a}^{\prime} R}{|D|}\left|\begin{array}{ccc}
-\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& =\frac{-\varphi_{R} w_{a}^{\prime}\left(v_{p R}^{\prime}\right)^{2}+v_{\hat{a} R}^{\prime}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|D|} \\
& \times\left[-\varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\left\{\varphi_{L} v_{a L}^{\prime \prime} w_{L}+w_{a L}^{\prime \prime}\right\}\right]>0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d a_{L}}{d \tilde{a}}=\frac{1}{|D|}\left|\begin{array}{cccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & 0 & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & \varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & v_{\tilde{a} R}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{\varphi_{R} w_{a}^{\prime} v_{p R}^{\prime}}{|D|}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
v_{p L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& +\frac{v_{a R}^{\prime}}{|D|}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& =\frac{\left[\varphi_{R} w_{\tilde{a} R}^{\prime}\left(v_{p R}^{\prime}\right)^{2}-v_{\tilde{a} R}^{\prime}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}\right]}{|D|} \times\left[\varphi_{L}^{\prime} v_{a L}^{\prime} w_{L}\left\{\varphi_{L} v_{p_{L}}^{\prime \prime} w_{L}+w_{p L}^{\prime \prime}\right\}\right] \\
& >0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d p_{R}}{d \tilde{a}}=\left|\begin{array}{cccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & -\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & v_{a L}^{\prime} & v_{\tilde{a} R}^{\prime} & -1
\end{array}\right| \\
& =-v_{p L}^{\prime}\left|\begin{array}{ccc}
-\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & -\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& +v_{a L}^{\prime}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & -\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& -v_{\tilde{a} R}^{\prime}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}
\end{array}\right| \\
& -\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 \\
0 & 0 & -\varphi_{R} w_{a}^{\prime} v_{p R}^{\prime}
\end{array}\right| \\
& =\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} \\
& \times\left\{\varphi_{L} \varphi_{L}^{\prime}\left(v_{p L}^{\prime}\right)^{2} v_{a L}^{\prime \prime} w_{L}^{2}+\varphi_{L}^{\prime}\left(v_{p L}^{\prime}\right)^{2} w_{L} w_{a L}^{\prime \prime}+\varphi_{L} \varphi_{L}^{\prime}\left(v_{a L}^{\prime}\right)^{2} v_{p L}^{\prime \prime} w_{L}^{2}-\varphi_{L}^{\prime}\left(v_{a L}^{\prime}\right)^{2} w_{L} w_{p L}^{\prime \prime}\right\} \\
& +\left\{v_{p R}^{\prime}\left(\varphi_{R}^{\prime} v_{\tilde{a} R}^{\prime} w_{R}+\varphi_{R} w_{\tilde{a} R}^{\prime}\right)\right\} \\
& \times\left\{\varphi_{L}^{2} w_{L}\left(v_{p L}^{\prime \prime} v_{a L}^{\prime \prime} w_{L}-v_{p L}^{\prime} w_{p L}^{\prime} v_{a L}^{\prime \prime}-v_{a L}^{\prime} w_{a L}^{\prime} v_{p L}^{\prime \prime}\right)+w_{p L}^{\prime \prime} w_{a L}^{\prime \prime}\right. \\
& \left.+\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right) w_{a L}^{\prime \prime}+\varphi_{L}\left(-v_{p L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right) w_{p L}^{\prime \prime}\right\}
\end{aligned}
$$

$\frac{d \Delta}{d \tilde{a}}$

$$
\begin{aligned}
& =\frac{1}{|D|}\left|\begin{array}{cccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & 0 \\
-\varphi_{L} v_{a L}^{\prime} w_{p L}^{\prime} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & 0 \\
0 & 0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R} w_{a}^{\prime} v_{p R}^{\prime} \\
v_{p L}^{\prime} & v_{a L}^{\prime} & -v_{p R}^{\prime} & v_{\tilde{a} R}^{\prime}
\end{array}\right| \\
& =\frac{-\left(v_{p R}^{\prime}\right)^{2} \varphi_{R} w_{a}^{\prime} R+v_{a}^{\prime} R\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|D|} \\
& \times\left\{\varphi_{L}^{2} w_{L}\left(v_{p L}^{\prime \prime} v_{a L}^{\prime \prime} w_{L}-v_{p L}^{\prime} w_{p L}^{\prime} v_{a L}^{\prime \prime}-v_{a L}^{\prime} w_{a L}^{\prime} v_{p L}^{\prime \prime}\right)+w_{p L}^{\prime \prime} w_{a L}^{\prime \prime}\right. \\
& \left.+\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right) w_{a L}^{\prime \prime}+\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right) w_{p L}^{\prime \prime}\right\} \\
& =\frac{1}{|D|}\left[v_{\tilde{a} R}^{\prime}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}-\left(v_{p R}^{\prime}\right)^{2} \varphi_{R} w_{\tilde{a} R}^{\prime}\right] \\
& \times\left\{\varphi_{L}^{2} w_{L}\left(v_{p L}^{\prime \prime} v_{a L}^{\prime \prime} w_{L}-v_{p L}^{\prime} w_{p L}^{\prime} v_{a L}^{\prime \prime}-v_{a L}^{\prime} w_{a L}^{\prime} v_{p L}^{\prime \prime}\right)+w_{p L}^{\prime \prime} w_{a L}^{\prime \prime}\right. \\
& \left.+\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right) w_{a L}^{\prime \prime}+\varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right) w_{p L}^{\prime \prime}\right\} \\
& >0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\left|p_{L}\right|}{d \bar{a}_{m}} & =\frac{1}{|D|}\left|\begin{array}{cccc}
0 & -\varphi_{L} v_{p L}^{\prime} w_{a L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
\varphi_{L} v_{a L}^{\prime \prime} w_{L} & \varphi_{L}\left(-v_{a L}^{\prime} w_{a L}^{\prime}+v_{a L}^{\prime \prime} w_{L}\right)+w_{a L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{a L}^{\prime} w_{L} \\
0 & 0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{a L}^{\prime}-v_{a R}^{\prime} & v_{a L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{1}{|D|}\left(-\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)-w_{p R}^{\prime \prime}-\varphi_{R}^{\prime}\left(v_{p R}^{\prime}\right)^{2} w_{R}\right)\left(\varphi_{L}^{2} v_{p L}^{\prime} w_{a L}^{\prime} v_{a L}^{\prime \prime} w_{L}\right) \\
& +\frac{1}{|D|} v_{a L}^{\prime}\left(\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right)\left(\varphi_{L} \varphi_{L}^{\prime} v_{a L}^{\prime \prime} v_{p L}^{\prime} w_{L}^{2}\right) \\
& +\frac{1}{|D|}\left(v_{a L}^{\prime}-v_{a R}^{\prime}\right)\left(\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right)\left(-\varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\right)\left(\varphi_{L} v_{a L}^{\prime \prime} w_{L}+w_{a L}^{\prime \prime}\right) \\
& <0
\end{aligned}
$$

The last part of Lemma 2 can be proved by

$$
\frac{d \Pi_{L} W_{L}}{d C_{R}}=\tilde{g}(\Delta) \frac{d \Delta}{d C_{R}} w_{L}+\tilde{G}(\Delta)\left(-w_{p L}^{\prime} \frac{d\left|p_{L}\right|}{d C_{R}}-w_{a L}^{\prime} \frac{d a_{L}}{d C_{R}}\right)
$$

This is negative by the comparative statics results above.
Lemma 3. $|\hat{D}|<0$
Proof of Lemma 3. Direct calculations.

$$
\begin{aligned}
& \left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =-\left\{\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right\}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} \\
& -\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} v_{p L}^{\prime} \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
& -\left\{\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right\} v_{p R}^{\prime} \varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
& <0
\end{aligned}
$$

Lemma 4. $\frac{d\left|p_{L}\right|}{d \tilde{a}}>0, \frac{d\left|p_{L}\right|}{d C_{L}}>0, \frac{d p_{R}}{d \tilde{a}}>0, \frac{d p_{R}}{d C_{L}}<0, \frac{d \Delta}{d C_{L}}>0, \frac{d\left|p_{L}\right|}{d C_{R}}<0, \frac{d p_{R}}{d C_{R}}>0, \frac{d \Delta}{d C_{R}}<0$.

## Proof of Lemma 4.

$$
\begin{aligned}
\frac{d\left|p_{L}\right|}{d \tilde{a}} & =\frac{1}{|\hat{D}|}\left|\begin{array}{ccc}
-\varphi_{L} w_{\tilde{a} L}^{\prime} v_{p L}^{\prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
-\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
0 & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{1}{|\hat{D}|}\left\{\varphi_{L} w_{\tilde{a} L}^{\prime} v_{p L}^{\prime}\left[\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right]\right. \\
& \left.+v_{p R}^{\prime}\left[\varphi_{L} w_{\tilde{a} L}^{\prime} v_{p L}^{\prime} \varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}+\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\right]\right\}>0 \\
\frac{d p_{R}}{d \tilde{a}} & =\frac{1}{|\hat{D}|}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & -\varphi_{L} w_{\tilde{a} L}^{\prime} v_{p L}^{\prime} & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & -\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & -1
\end{array}\right| \\
& =\frac{1}{|\hat{D}|}\left\{\varphi_{R} w_{\tilde{a} R}^{\prime} v_{p R}^{\prime}\left[\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right]\right.
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\left|p_{L}\right|}{d C_{L}} & =\frac{1}{|\hat{D}|}\left|\begin{array}{ccc}
0 & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
-1 & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{\varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|\hat{D}|}>0 \\
\frac{d p_{R}}{d C_{L}} & =\frac{1}{|\hat{D}|\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & 0 & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & -1 & -1
\end{array}\right|} \begin{array}{l}
|\hat{D}|
\end{array}>\frac{-\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}\left\{\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right\}}{|\hat{D}|}<0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d \Delta}{d C_{L}} & =\frac{1}{|\hat{D}|}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & 0 \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & 0 \\
v_{p L}^{\prime} & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =-\frac{\left\{\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right\}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|\hat{D}|}>0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\left|p_{L}\right|}{d C_{R}} & =\frac{1}{|\hat{D}|}\left|\begin{array}{ccc}
0 & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
1 & -v_{p R}^{\prime} & -1
\end{array}\right| \\
& =\frac{-\varphi_{L}^{\prime} v_{p L}^{\prime} w_{L}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\}}{|\hat{D}|}<0
\end{aligned}
$$

$$
\begin{gathered}
\frac{d p_{R}}{d C_{R}}=\frac{1}{|\hat{D}|}\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & \varphi_{L}^{\prime} v_{p L}^{\prime} w_{L} \\
0 & 0 & -\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R} \\
v_{p L}^{\prime} & 1 & -1
\end{array}\right| \\
=\frac{\varphi_{R}^{\prime} v_{p R}^{\prime} w_{R}\left\{\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right\}}{|\hat{D}|}>0 \\
\frac{d \Delta}{d C_{R}}=\frac{1}{|\hat{D}|\left|\begin{array}{ccc}
\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime} & 0 & 0 \\
0 & \varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime} & 0 \\
v_{p L}^{\prime} & -v_{p R}^{\prime} & 1
\end{array}\right|} \begin{array}{c}
\left\{\varphi_{L}\left(-v_{p L}^{\prime} w_{p L}^{\prime}+v_{p L}^{\prime \prime} w_{L}\right)+w_{p L}^{\prime \prime}\right\}\left\{\varphi_{R}\left(-v_{p R}^{\prime} w_{p R}^{\prime}+v_{p R}^{\prime \prime} w_{R}\right)+w_{p R}^{\prime \prime}\right\} \\
|\hat{D}|
\end{array} 0
\end{gathered}
$$

Proof of Proposition 3. First, note that $\left.\frac{d C_{L}}{d C_{R}}\right|_{I C_{L}}<1$ holds from (12). This is because $\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}}<0$, $\frac{d\left(\Pi_{L}^{* *} w_{L}^{* *}\right)}{d C_{R}}<0$, and $-\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{R}}=\frac{d\left(\Pi_{L}^{*} w_{L}^{*}\right)}{d C_{L}}$. Thus, Regularity in IC Constraints assure that $\left.\left|\frac{d C_{L}}{d C_{R}}\right|_{I C_{L}} \right\rvert\,<1$ and $\left.\left|\frac{d C_{R}}{d C_{L}}\right|_{I C_{R}} \right\rvert\,<1$ hold. These imply that $I C_{L}$ and $I C_{R}$ intersect with each other at most once.

Let us suppose that at a contract neither (7) nor (8) is binding. In this case, by reducing $C_{L}$ and $C_{R}$ simultaneously without changing the winning probabilities, both candidates will still accept the new contracts. This contradicts the assumption that $\left(C_{L}, C_{R}\right)$ is cost minimizing.

Thus, assume that only $I C_{R}(8)$ is not binding. By Regularity in IC Constraints, we have $\left.\frac{d C_{L}}{d C_{R}}\right|_{I C_{L}}>-1$. Since (7) is binding, $C_{L}+C_{R}$ can be reduced by moving along $I C_{L}$ (see Figure 4). This proves that at the minimum, both IC constraints are binding.

Proof of Proposition 4. Since $\left|p_{L}-\bar{p}_{L}\right|=\left|\bar{p}_{L}\right|-\left|p_{L}\right|, \frac{\partial\left|p_{L}-\bar{p}_{L}\right|}{\partial\left|p_{L}\right|}=-1$ holds if $\left|\bar{p}_{L}\right|>\left|p_{L}\right|$. Let

$$
\phi\left(\left|p_{L}\right|\right) \equiv \eta_{0} v_{p}^{\prime}\left(\left|p_{L}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}-\bar{a}_{L}\right|\right)\right\}-w_{p}^{\prime}\left(\left|p_{L}-\bar{p}_{L}\right|\right) .
$$

Since $v_{p L}^{\prime}(0)=0, w_{p L}^{\prime}(0)=0$, and $\eta_{0}$ is a constant in symmetric equilibria, we have $\phi(0)>0$ and $\phi\left(\left|\bar{p}_{L}\right|\right)<0$.

Differentiating $\phi\left(\left|p_{L}\right|\right)$ with respect to $\left|p_{L}\right|$, we obtain

$$
\begin{aligned}
\phi^{\prime}\left(\left|p_{L}\right|\right) & \equiv \eta_{0} v_{p}^{\prime \prime}\left(\left|p_{L}\right|\right)\left\{Q+w_{p}\left(\left|p_{L}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|a_{L}-\bar{a}_{L}\right|\right)\right\}-\eta_{0} v_{p}^{\prime}\left(\left|p_{L}\right|\right) w_{p}^{\prime}\left(\left|p_{L}-\bar{p}_{L}\right|\right)+w_{p}^{\prime \prime}\left(\left|p_{L}-\bar{p}_{L}\right|\right) \\
& <0
\end{aligned}
$$

Thus, symmetric equilibrium is unique.

Proof of Proposition 5. We first assume that both candidates accept the offers, and analyze how an increase in $\tilde{a}$ affects their ideological policy positions, then we check how contribution money needs adjustment to provide the candidates incentives to accept the offers. For the first part, we can conclude that $\frac{d\left|p_{\tilde{L}}^{*}\right|}{d \tilde{a}}=\frac{d p_{R}^{*}}{d \tilde{a}}>0$ by Lemma 4 and the symmetry.

Second, we focus on contribution money. Under symmetry, the binding incentive compatibility constraint is written as

$$
\frac{1}{2}\left\{Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right)\right\}=\Pi_{L}\left(p_{L}^{* *}, a_{L}^{* *}, 0, p_{R}^{* *}, \tilde{a}, \tilde{C}\right) w_{L}\left(\left|p_{L}^{* *}-\bar{p}_{L}\right|,\left|a_{L}^{* *}-\bar{a}_{L}\right|\right)
$$

Since $\left|p_{L}^{*}\right|$ increases as $\tilde{a}$ is raised, $\left|v_{p}^{\prime}\left(\left|p_{L}^{*}\right|\right)\right|$ increases and $\left|w_{p}^{\prime}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|\right)\right|$ decreases which means that $Q+w_{p}\left(\left|p_{L}^{*}-\bar{p}_{L}\right|+w_{a}\left(\left|\tilde{a}-\bar{a}_{L}\right|\right)\right.$ must decrease as $\tilde{a}$ increases by observing (14). Therefore, the LHS of the above incentive compatibility constraint decreases as $\tilde{a}$ increases.

In contrast, without adjustment in $\tilde{C}$, the contents of the RHS is increased by an increase of $\tilde{a}$ :

$$
\frac{d R H S}{d \tilde{a}}=\tilde{g}(\Delta) \frac{d \Delta}{d \tilde{a}} w_{L}+\tilde{G}(\Delta)\left(-w_{p L}^{\prime} \frac{d\left|p_{L}\right|}{d \tilde{a}}-w_{a L}^{\prime} \frac{d a_{L}}{d \tilde{a}}\right)>0
$$

The inequality is determined by the comparative static results in Lemma 2.2. Thus, without an adjustment in $\tilde{C}$, the incentive compatibility constraint is violated. According to Lemma 2.1, an increase in $\tilde{C}$ affects the RHS by

$$
\frac{d R H S}{d \tilde{C}}=\tilde{g}(\Delta) \frac{d \Delta}{d \tilde{C}} w_{L}+\tilde{G}(\Delta)\left(-w_{p L}^{\prime} \frac{d\left|p_{L}\right|}{d \tilde{C}}-w_{a L}^{\prime} \frac{d a_{L}}{d \tilde{C}}\right)<0
$$

Hence, to keep the incentive constraint binding, an increase in $\tilde{a}$ must be accompanied by an increase in $\tilde{C} . \square$

## Appendix B: Incentives for Exporting Firms to Make Campaign

 ContributionsThe main beneficiaries of free trade are clearly exporting firms. If trade barriers by foreign countries are reduced, they can increase exports and profits tremendously. However, these countries have no reason to reduce their tariffs unilaterally for the US. They also want to protect their domestic firms. This was precisely the reason that the Reciprocal Trade Agreement Act (RTAA) was passed in 1934. In the early 1930s, high tariffs caused by the Smoot-Hawley Act contributed to the downward spiral of trade as other countries retaliated against the United States. Passing RTAA, Congress effectively gave up control over the US tariffs, authorizing President Franklin Roosevelt to enter into tariff agreements with foreign countries to reduce import duties in order to speed the recovery from the Depression. ${ }^{25}$ Irwin (2015) argues: "The RTAA explicitly linked foreign tariff reductions that were beneficial to exporters to lower tariff protection for producers competing against imports. This enabled exporters to organize and oppose high domestic tariffs because they want to secure lower foreign tariffs on their products." (Irwin, 2015, pp. 242) After World War II, the General Agreement on Tariffs and Trade (GATT) broadened the tariff negotiation talks to a multilateral system under the "reciprocity" and "nondiscrimination" principles, through the 'most-favorednation' (MFN) clause (Bagwell and Staiger, 1999). ${ }^{26}$ RTAA and GATT helped to bolster the lobbying position of exporters in the political process, and expanding trade through tariff reductions increased the size of strong industries and decreased the size of import competing industries (Irwin, 2015). As long as negotiation tables with other countries are set up and a good negotiation team is appointed, exporting firms can lobby for lowering the tariff rates. Thus, exporting firms have incentives to make campaign contributions to (possibly both) presidential candidates as to keep free trade/globalization issue nonsalient. ${ }^{27}$

Reciprocity is one of the key principles of international negotiations in tariff reductions in GATT and preferential trade agreements (Bagwell and Staiger 1999). For exporting firms to enjoy low foreign tariff rates,

[^17]the home country also needs to reduce its tariff rates. Otherwise, the negotiation will not be agreeable. In a recent paper, Kim (2017) finds that the variation in US applied tariff rates arises within industry, and explains how product differentiation leads to firm-level lobbying in tariff reduction. Using a quasi-linear product differentiation model by Melitz and Ottaviano (2008), reciprocity in two-country trade negotiation is analyzed (Bagwell and Staiger, 1999). Kim (2017) shows that productive exporting firms are more likely to lobby for reduced tariffs than less productive firms when products are more differentiated, and he provides empirical evidences for his predictions. He obtains this result by employing the protection-for-sale model in Grossman and Helpman (1994) as a proxy of the tariff negotiation process between two countries, assuming that the countries are symmetric.

Kim's paper shows that as long as countries are at the negotiation table for trade deals, productive exporting firms can lobby hard for lower tariffs for their products, gaining access to large foreign markets. ${ }^{28}$ However, the presence of international negotiation tables is not always assured, as with the tariff wars in early 1930s. Without a negotiation table, exporting firms have no way to lobby for lower tariff rates levied by foreign countries. GATT provided this service with the principles of reciprocity and most favored nations clause (MFN), and preferential trade agreements such as NAFTA, TPP, and TTIP provide additional negotiation tables. ${ }^{29}$ Thus, it is indeed in exporting firms' interests to have a president who is willing to commit to promoting free trade.

## Appendix C: The 2016 Presidential Race

Recently, we can observe an increasing trend of negative sentiments toward globalism in the US and other Western countries. Autor, Dorn, and Hanson (2013) report that the rise of competition with China and other developing countries explains $25 \%$ of the decline in US manufacturing employment between 1990 and 2007. ${ }^{30}$ In the 2016 US presidential campaign, anti-globalism/protectionism became one of the most salient issues, and industries' contributions to the two party nominees showed quite different patterns relative to prior presidential election years. In prior years, for almost all sectors/industries, the top two recipients of campaign contributions are most likely to be the Republican and Democratic party nominees, but in the 2016 presidential election race, Donald Trump received significantly lower contributions from industries that

[^18]have interests in trade agreements.
The Center of Responsive Politics provides detailed information on US politics (https://www.opensecrets.org/). We can get information on sector/industry-level contributions to each candidate who ran in presidential races (detailed decompositions are available from at least 2008 on). Each sector/industry provides contributions to a number of candidates including both parties' presidential nominees and other candidates who drop out as party primaries proceed. Sector/industries often have a party bias.

In usual presidential election years (Tables 2 and 3), for almost all sectors/industries, the two top recipients of contribution money are often Republican and Democratic party nominees, but other candidates in the two major parties also collected significant amounts of contribution money before they drop out.

In the 2016 presidential election race (Table 1), the two candidates who got most total campaign contributions (from industries, individuals, and other sources) are Hilary Clinton and Donald Trump (\$770 millions and $\$ 408$ millions, respectively). But sector/industry contributions to Clinton and Trump in 2016 display a different pattern relative to presidential campaigns in prior years. Clinton got the highest amount of contributions in most sectors/industries, but this is not a particularly interesting observation. The financial sector (commercial banks, hedge funds, insurance, and security investment) tends to contribute to many candidates from early stage, but in the end they contribute the highest amounts to the two candidates nominated by the two parties. However, in 2016, the financial sector gave significantly higher contributions to Clinton than to Trump. For example, Clinton's contributions from hedge funds were 100 times that of Trump, and Jeb Bush and Marco Rubio's contributions from hedge funds were also much higher than Trump's. In terms of the financial sector's campaign contributions to Republican candidates, Trump ranked 4th (commercial banks), 11th (hedge funds), 4th (insurance), and 10th (securities and investment). Even in the oil and gas industry, Trump got less money than Clinton and less than Jeb Bush who got ten times more than what Clinton did. The agricultural business sector is usually a Republican stronghold, but Trump got less than Clinton (4th in the Republican party). These observations are consistent with the idea that Donald Trump was a very unconventional Republican candidate. Industries usually contribute some money to most candidates in the initial stages of their campaigns. Thus, we can safely say that these industries did not contribute money to Trump after he was nominated, although data is only available for cumulative contributions.

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| 2016 | 1 | 2 | 3 | Clinton | Trump |
| :---: | :---: | :---: | :---: | :---: | :---: |
| commercial banks | Clinton 2.8 | Bush 1.1 | Rubio 0.4 | $2.8(1)$ | $0.37(5)$ |
| electronics/mfg equipment | Clinton 13 | Rubio 5.6 | Paul 2.4 | $13(1)$ | $0.6(6)$ |
| internet | Clinton 6.3 | Sanders 0.9 | Bush 0.22 | $6.3(1)$ | $0.06(9)$ |
| hedge funds \& private equity | Clinton 59 | Bush 17 | Rubio 16 | $59(1)$ | $0.3(12)$ |
| insurance | Bush 12 | Rubio 5.7 | Clinton 2.5 | $2.5(3)$ | $0.7(4)$ |
| oil gas | Bush 11 | Perry 1.6 | Kaisch 1.6 | $0.9(6)$ | $0.8(8)$ |
| pharma/health products | Clinton 12 | Bush 1.5 | Cruz 0.8 | $12(1)$ | $0.3(7)$ |
| securities \& investment | Clinton 87 | Bush 34 | Rubio 20 | $87(1)$ | $1.1(11)$ |
| telephone utilities | Clinton 0.7 | Sanders 0.2 | Cruz 0.1 | $0.7(1)$ | $0.1(4)$ |
| TV/movies/music | Clinton 24 | Rubio 2.3 | Sanders 1.5 | $24(1)$ | $0.4(5)$ |

Table A1. 2016 Selected Industry Contributions (https://www.opensecrets.org/)
The top three recepients of campaign contributions, and the two party nominees (unit: millions of dollars: numbers in parentheses are the rankings).

| 2012 | 1 | 2 | 3 | Obama | Romney |
| :---: | :---: | :---: | :---: | :---: | :---: |
| commercial banks | Romney 4.8 | Obama 1.7 | Perry 0.2 | $1.7(2)$ | $4.8(1)$ |
| computer/internet | Obama 5.9 | Romney 3.2 | Paul 0.6 | $5.9(1)$ | $3.2(2)$ |
| hedge funds \& private equity | Romney 7.7 | Obama 1.8 | Pawlenty 0.2 | $1.8(2)$ | $7.7(1)$ |
| insurance | Romney 4.7 | Obama 1.7 | Perry 0.5 | $1.7(2)$ | $4.7(1)$ |
| oil gas | Romney 5.9 | Perry 1.0 | Obama 0.8 | $0.8(3)$ | $5.9(1)$ |
| pharma/health products | Obama 2.0 | Romney 2.0 | Perry 0.9 | $2.0(1)$ | $2.0(2)$ |
| securities \& investment | Romney 23 | Obama 6.8 | Pawlenty 0.7 | $6.8(2)$ | $23(1)$ |
| telephone utilities | Obama 0.5 | Romney 0.5 | Paul 0.0 | $0.5(1)$ | $0.5(2)$ |
| TV/movies/music | Obama 6.5 | Romney 1.1 | Sanders 1.5 | $6.5(1)$ | $1.1(2)$ |

Table A2. 2012 Selected Industry Contributions (https://www.opensecrets.org/)
The top three recepients of campaign contributions, and the two party nominees (unit: millions of dollars: numbers in parentheses are the rankings).

| 2008 | 1 | 2 | 3 | Obama | McCain |
| :---: | :---: | :---: | :---: | :---: | :---: |
| commercial banks | Obama 3.4 | McCain 2.3 | Clinton 1.5 | $3.4(1)$ | $2.3(2)$ |
| computer/internet | Obama 9.7 | Clinton 2.3 | McCain 1.7 | $9.7(1)$ | $1.7(3)$ |
| hedge funds \& private equity | Obama 3.7 | McCain 2.1 | Clinton 1.8 | $3.7(1)$ | $2.1(2)$ |
| insurance | McCain 2.8 | Obama 2.6 | Clinton 1.2 | $2.6(2)$ | $2.8(1)$ |
| oil gas | McCain 2.7 | Obama 1.0 | Giuliani 0.7 | $1.0(2)$ | $2.7(1)$ |
| pharma/health products | Obama 2.4 | McCain 0.8 | Clinton 0.7 | $2.4(1)$ | $0.8(2)$ |
| securities \& investment | Obama 16.6 | McCain 9.3 | Clinton 7.3 | $16.6(1)$ | $9.3(2)$ |
| telephone utilities | Obama 0.6 | McCain 0.5 | Clinton 0.3 | $0.7(1)$ | $0.5(2)$ |
| TV/movies/music | Obama 9.9 | Clinton 3.5 | McCain 1.1 | $9.9(1)$ | $1.1(3)$ |

Table A3. 2008 Selected Industry Contributions (https://www.opensecrets.org/)
The top three recepients of campaign contributions, and the two party nominees (unit: millions of dollars: numbers in parentheses are the rankings).


[^0]:    *We thank Jim Anderson, Filipe Campante, Taiji Furusawa, Maria Gallago, Rossella Greco, David Hopkins, In Song Kim, Lisa Lynch, and Fabio Schiantarelli for helpful comments and useful conversations.
    ${ }^{\dagger}$ Hideo Konishi: Department of Economics, Boston College, US. Email: hideo.konishi@bc.edu
    ${ }^{\ddagger}$ Chen-Yu Pan: School of Economics and Management, Wuhan University, PRC. Email: panwhu@126.com

[^1]:    ${ }^{1}$ DW-Nominate score is to measure congressional legislators' position on liberal-conservative dimension according to their roll-call votes. It was introduced by Poole and Rosenthal $(1985,1991)$, and has been used to measure ideological positions of congressmen.
    ${ }^{2}$ In the 1992 Presidential election, neither Bush nor Clinton talked much about NAFTA, although a third party candidate, Ross Perot, denounced NAFTA strongly.

    3 "Both Parties Used to Back Free Trade. Now They Bash It." New York Times, July 29, 2016. Appendix C illustrates how the 2016 presidential election race was different from usual presidential election years.

[^2]:    ${ }^{4}$ Appendix B describes the incentives for exporting firms to make campaign contributions to presidential election candidates.
    ${ }^{5}$ Campaign contributions include individual contributions and PAC (Political Action Committees) contributions. Barber and McCarty (2015), and McCarty et al. (2016) report that the share of individual contributions continue to increase, suggesting that ideologically motivated individual contributions may be one of the causes for recent polarization (for an interesting behavioral model that deals with individual contributions, see Campante, 2011). However, PAC contributions from industries in presidential elections are also steadily increasing (see Tables A1, A2, and A3 in the Appendix). This paper focuses on this factor.
    ${ }^{6}$ Bafumi and Herrero (2010) report that candidates' policy positions are more extreme than their party's median position, and the distance between the positions of representatives and their constituents are expanding.

[^3]:    ${ }^{7}$ Grossman and Helpman (1996) analyze the multi-lobby case by applying the insights developed in a single lobby case.
    ${ }^{8}$ Krasa and Polborn (2014) provide an interesting electoral competition model in which income redistribution is discouraged as the Republican party's ideological position polarizes. Greco (2016) provides a simple model of the same nature, and provides empirical evidence for this.

[^4]:    ${ }^{9}$ In a similar setup, Bernhardt et al. (2009) provide a sufficient condition for the existence of symmetric equilibrium. See Duggan and Martinez (2017).
    ${ }^{10}$ They consider an offer with a threat to contribute to the other candidate if an offer is rejected. Although we do not allow for such a threat, a similar policy polarization result should apply even with this possibility.

[^5]:    ${ }^{11}$ If $C_{j}=0$, there is no contribution money to commit. Therefore, candidate $j$ can freely choose $p_{j}$ and $a_{j}$ in the election as if she rejects a 0 offer.
    ${ }^{12}$ Common valence shocks come from gaffes, scandals, and debate performances by the candidates.

[^6]:    ${ }^{13}$ In a companion paper, Konishi and Pan (2017), we analyze the optimal contract for a single IG (the export industry).

[^7]:    ${ }^{14}$ Since our model is a cardinal model, a remark on the quadratic transportation cost model in Caplin and Nalebuff (1991) is also relevant. The following discussion and Proposition 2 extends to the case for a $K$-dimensional policy space.

[^8]:    ${ }^{15}$ The above result holds for any finite $K$-dimensional policy space as long as voter cost function is quadratic in distance.

[^9]:    ${ }^{16}$ In the numerical examples in the next section, regularity condition is always satisfied with plenty of slack. In fact, IC-curves tend to be very inelastic to a change in the opponent's contribution.

[^10]:    ${ }^{17}$ Another commonly used distribution is a normal distribution. Our results do not change qualitatively under normality assumption.

[^11]:    ${ }^{18}$ In symmetric equilibrium, polarization causes no winning probability loss. Moreover, a symmetric increase in $\tilde{C}$ also has no effect on winning probability.
    ${ }^{19}$ Although this analysis (and those in this section) is very intuitive, we implicitly assume both incentive constraints are binding. However, in the parameter ranges used in this section, the slope of incentive constraints are all very close to 0 . This example is typical because two incentive constraints have very small negative slopes. Therefore, by Proposition 3, both incentive constraints are in fact binding.

[^12]:    ${ }^{20}$ Obviously, IG is likely to stop supporting candidate $R$ as it becomes prohibitively expensive to support candidate $R$. Here, we only consider the case where IG support both candidates.

[^13]:    ${ }^{21} \mathrm{~A}$ similar pattern of polarization is robust to other parameter settings. For example, when $\bar{p}_{R}=1.25$ or 1.75 , the result are qualitatively the same.

[^14]:    ${ }^{22}$ One might interpret rising protectionism as voter becoming more sensitive to the agenda, i.e., an increase in $\theta$. We obtain similar results in the case of increasing sensitivity, which is unsurprising.

[^15]:    ${ }^{23}$ Our result can also be seen as a theoretical base for a so-called marginality hypothesis, that is, electoral competition increases responsiveness on policy. (Fiorina, 1973). The empirical evidence of this hypothesis is mixed depending on how the valence advantage is defined. Recent supporting evidence includes Ansolabehere, Snyder, and Steward (2001) and Griffin (2006).

[^16]:    ${ }^{24}$ The Minkowski average $A_{\lambda}$ is defined as all points of the form $x_{\lambda}=(1-\lambda) x_{0}+\lambda x_{1}$, with $x_{0} \in A_{0}, x_{1} \in A_{1}$, and $0 \leq \lambda \leq 1$.

[^17]:    ${ }^{25}$ Anderson and Zanardi (2009) point out that this delegation of political power could also be explained by political pressure deflection-incumbent congressmen avoided revealing their preferences on trade policy for fear that opposing lobbies would confer viability on a challenger who will support their position.
    ${ }^{26}$ Bagwell and Staiger (1999) presents a general theory of GATT with reciprocity and MFN to evaluate whether or not regional trade agreements would be good for achieving efficient multinational outcomes. Bagwell, Bown, Staiger (2016) survey research on international trade agreements to date, concluding strong support to GATT (WTO).
    ${ }^{27}$ In the US, campaign contributions play an important role in determining the election results whether through bolstering their supporting candidates and/or running negative campaigns on the opposing candidates. Political Action Committees (PACs) raise money from individuals to elect or defeat candidates, but corporations and unions can sponsor a PAC inviting their members to contribute by covering administrative costs. Super PACs can raise money from corporate and unions directly without limit, but super PACs themselves decide how to run campaigns to support candidates (or oppose rivals). Therefore, campaign contributions are specific to supporting or opposing particular candidates, and they are not directly related to special interests' lobbying activities.

[^18]:    ${ }^{28}$ Hansen and Mitchell (2000) investigate the determinants of different corporate political activities, such as campaign contributions (through PACs) and lobbying expenses. Many firms with PACs have a lobbying presence in Washington.
    ${ }^{29}$ Although GATT Article 24 allows regional trade agreements as exceptions of the MFN principle, Bagwell and Staiger (1999) and Bagwell, Bown, and Staiger (2016) are more cautious about regional trade agreements.
    ${ }^{30}$ Bown (2016) argues that the other part of lost jobs were caused by automation, switching to cleaner energy, and the reduction of construction jobs by the Lehman shock.

