Electoral incentives: the interaction between candidate selection and electoral rules*

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Abstract

One of the fundamental roles of democratic institutions is to provide incentives to politicians to perform. The literature has focused on the role of electoral rules as a central force to understand politicians' performance. In particular, plurality rule in single-member districts is believed to provide better incentives than proportional representation in multimember districts. In this paper, we argue that political parties also play an important role in providing incentives to politicians. We analyze the impact of the competition in the selection process of political parties as a source of incentives. Candidates exert costly effort to improve their appeal to voters. We contrast effort-based competitive selection to less competitive selection procedure. Unsurprisingly, we find that competitive selection systems are conducive to more effort. More interestingly, we show that the impact of competitive selection is larger under proportional representation than under plurality rule. When selection is not very competitive, plurality rule leads to more effort from politicians. When selection is very competitive, proportional representation leads to more effort. The intuition behind the result comes from the fact that under proportional representation, politicians compete not only to become a candidate but also for a better spot on the list parties offer to voters.

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1 Introduction

A central goal of political economy is to uncover how political institutions affect political and economic outcomes. The literature has made quite some progress along many dimensions. This being said, the focus of this literature is typically on voters and candidates. That is, the parties from which candidates originate are typically ignored. Alternatively, candidates and the political parties from which they originate are modelled as one and the same agent: parties, whenever they are modelled, are unitary actors (Downs, 1957). Yet, parties are not unitary actors but organisations. Once we allow for candidates and parties to be separate agents, a new set of research questions emerges, a prominent one being: how do political institutions affect the incentives and freedom of parties to influence the decisions of their candidates and representatives, and thus policy outcomes? Which political institutions ease the attempts by parties to control their candidates and representatives? Which institutions make this monitoring and control problem harder?

Our paper offers an answer to the above questions. The political institution we focus on is the electoral rule and the task we endow parties with is the selection of the set of candidates who will be allowed to stand for election. Our choice is dictated by the fact that there is still a clear disconnect between theoretical predictions about the effects of electoral rules on incentives of politicians and the available empirical evidence. The received theoretical wisdom, as summarised for example by Persson and Tabellini (2000) and Persson, Tabellini and Trebbi (2003), is that closed-list proportional representation in multi-member electoral districts (henceforth, PR) generates less powerful incentives and less accountability than plurality rule in single member districts (henceforth, FPTP). Yet, the evidence for this prediction is weak, as Persson et al. (2003) conclude themselves. To the best of our knowledge, this puzzle is still open.

We propose one solution to the above puzzle. We augment a standard moral hazard electoral game with an extra initial stage to the game, in which parties select the candidates who then run in the general election.[1] Once we let parties be active players in this moral hazard game, the effect of electoral rules on the decisions of politicians becomes more involved than the previous literature predicts.

In our basic model, individual politicians belong to one of two parties. On top of contributing to the party-specific tasks, requirements and objectives that may be more or less aligned with the

[1] As should be clear by now, our contribution focuses on incentive issues. Issues of selection are also central but left for future research.

views of the electorate at large, individual candidates can increase their chances of obtaining one of the available legislative seat by investing time and energy in the issues that are at the forefront on the current electoral race. We model such investments as a costly effort choice, for the sake of simplicity. This single effort decision of each politician influences both their probability of being selected and that of being elected. The rules of the game at the selection stage are governed by the candidate selection procedure (CSP). We let the CSP be either competitive and based on the costly effort choices of the different candidates, or not. To model the competitive CSP, we borrow Clark and Riis (1996)'s model of a contest for multiple prizes.[2]

In order to relate our theoretical findings to the empirical puzzle about the (lack of clearcut) relationship between electoral rules and effort by politicians, we compare the effort decisions of politicians across PR and FPTP under the two polar CSP's. Under FPTP, each party is required at the selection stage to file in a representative in every single member district. The effort decisions of the two party representatives then determine directly each representative's probability of being elected through a standard imperfectly discriminating contest as in Tullock (1980). Each candidate's probability of being elected is thus directly proportional to the ratio of their effort over the sum of the two candidates' efforts, net of some electoral randomness.

Under PR, each party needs to select the candidates it will put on its electoral list. The effort choices of all candidates on the party list determine their party's 'electoral output', which then determines each party's probability of winning a certain share of the available legislative seats. To replicate the individual moral hazard issues the previous literature has identified under PR, we let each party's electoral output be the unweighted mean of the effort choices of all individual politicians on a party's electoral list. As we need to be able to write down the exact probability that a party wins any number of seats, we extend the classical Tullock (1980) imperfectly discriminating contest success function to the case of multiple indivisible prizes, which in our specific case are the different legislative seats. Each seat is won by a party with a probability equal to the ratio of its own electoral output over the sum of the two parties' outputs. The number of prizes won by a party thus follows a binomial distribution. This generates the intuitive feature that it is easy for each party to win a few legislative seats, but it is hard for both parties to win nearly all seats.[3]

[2]Note that even under the non-competitive CSP politicians may still want to invest in effort, but only to the extend that such a decision improves their probability of election, as effort does not impact that of selection.

[3]Crutzen, Flamand and Sahuguet (2018) use the same binomial-Tullock technology to study incentive issues in a contest between teams for multiple indivisible prizes. One of their central results is the fact that, under a weak monotonicity condition, the use of an ordered list within each team – exactly like under closed-list PR – can be the

We start by comparing the above two electoral rules when selection is not competitive. We show that FPTP leads to stronger incentives than PR, in line with and for the exact same reasons found in the previous literature. To quote Persson et al. (2003, p. 961), in our model, just like in previous models, "politicians' incentives are [...] diluted by two effects. First, a free-rider problem arises among politicians on the same list. The reason is that under PR, the number of seats depends on the votes collected by the whole list, rather than the votes for each individual candidate. Second, [as] the list is closed and voters cannot choose their preferred candidate, an individual's chance of re-election depends on his rank on the list, not his individual performance". This implies that candidates on the top and bottom of the list have very little incentives to exert effort to get elected. The candidates on the top of the list are nearly certain of getting elected. The candidates at the bottom of the list know their chances are nearly zero. Finally, the effort chosen by the list members who face the highest incentives, namely the individuals in the middle of the list, are not sufficient to compensate the near-zero effort provision of the other list members.

We then compare average effort choices when the candidate selection procedure is competitive and effort-based. Under FPTP, we first solve for equilibrium efforts for a given number of candidates at the selection stage and then show that having two candidates competing in each single-member district maximises effort provision incentives. Next, we turn to the problem under PR. We solve for equilibrium efforts when party candidates compete for one of the positions on the list. The optimal number of candidates to have at the selection stage can vary with the number of legislative seats and thus the number of positions on each party's list. However, we need not pin it down as, fixing the number of intra-party candidates to be equal to the number of positions on the party list, we prove that average individual effort provision is larger under PR than under FPTP when the number of internal candidates is optimal under that rule, namely when two candidates compete in each electoral district. We thus show that accounting for the incentives generated at the party selection stage reverses the ranking of electoral rules in terms of average individual efforts chosen by party representatives.

The main reason for this result is that PR offers more incentive levers than FPTP. If under both electoral rules, competitive selection increases incentives, under PR competitive selection is associated with *two* incentive mechanisms. First, competition for being among the candidates who end up on the party list increases incentives to exert effort. Second, and this is the dimension that gives PR with the competitive CSP its incentive edge, competition for the best slots on the list

optimal intra-team prize allocation mechanism.

generates further incentives. Actually, the incentives generated by the competition for the best slots on each party list are sufficient to overturn the ranking of electoral rules when selection is not competitive, as our main result holds when the number of candidate in the selection stage is equal to the number of positions on the list under PR.

To go back to our initial question, the main message of our analysis is that, relative to FPTP, PR offers more room for parties to intervene in the electoral game between candidates and voters. Thus, provided parties wish to manipulate the incentives of their candidates, it should be more fruitful for parties to intervene under PR than under FPTP. And conditional on parties intervening, PR yields stronger individual incentives than FPTP. Our findings have obvious implications for empirical exercises on the incentives effects of electoral rules. The most important one is that any empirical exercise which does not include some party-level regressors that proxies for the behaviour of parties in the electoral game is plagued by an obvious omitted variable problem.

We close the analysis of our game by showing that this reversal of the ranking of electoral rules survives the extension of the basic model to account for many more realistic electoral scenarios, such as the introduction of ideological motives in the electorate, the extension of thee utility function of individual politicians to let them value their party winning a majority of seats (and thus control of government) and the increase of the number of parties beyond two under PR.

2 Related Literature

The paper contributes to several strands of the literature in political economy and the theory of contests. From a modelling perspective, we propose a novel mechanism to attribute seats to parties and offer one avenue to model the candidate selection procedure. From a substantive point of view, we contribute to the political economy literature on incentives and to the comparative politics literature on the differential effects of electoral rules and candidate selection procedures.

To the best of our knowledge, ours is the first moral hazard model to include a full-fledged intra-party candidate selection stage in the game. If there is an important literature on candidate selection procedures in political science, most contributions are either purely empirical or do not offer a formal model. Important works include Bille (2001), Hazan and Pennings (2001), Hazan and Rahat (2006, 2010), Katz and Mair (1994), Lundell (2004), Norris (1997, 2006), Rahat and Hazan (2001) and Shomer (2014, forthcoming). We return to the consequences of our findings for this literature in section 5.1.

We contribute to the political economy literature that deals with the incentive consequences of different party organisations. Caillaud and Tirole (2002) is arguably the first contribution to model *party organization* as a source of incentives for individual politicians.[4] Castanheira, Crutzen and Sahuguet (2010) extend the analysis of Caillaud and Tirole (2002) to allow for different political motives and general equilibrium effects. To the best of our knowledge, we are among the first to model the incentive effects of party lists under PR. We achieve this by borrowing Crutzen et al. (2017)'s extension of the classical Tullock (1980) contest model to the case of multiple indivisible prizes.

Our paper contributes to the comparative politics literature that tries to explain outcomes through differences in political institutions. If the literature is by now relatively large, no contribution examines the role of the candidate selection stage, even though, arguably, this is the fundamental stage to analyse how electoral rules and parties affect the choice set of voters, as the electorate can only choose from the set of candidates that parties are willing to offer them at the electoral stage. Important comparative politics works include Bawn and Thies (2003), Buisseret and Prato (2017), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002), Morelli (2004), Persson, Roland and Tabellini (199X?), Persson and Tabellini (1999, 2000 and 2003), Persson, Tabellini and Trebbi (2003) and Raffler (2016).[5] Buisseret and Prato (2017) are closest to us in terms of goals, as they analyse the consequences of varying the size of electoral districts for politicians' incentives to cater to the interests of voters, and PR is typically associated to the largest electoral districts whereas FPTP is typically run in single-member districts, a difference between electoral rules we assume in our model.

Turning to our specific modelling choices for the allocation of the legislative seats across parties and for the candidate selection procedure, our model contributes to the theory of team contests and to the theory of contests with multiple prizes. Starting with the latter, the selection stage within each party corresponds to a contest for multiple prizes (for a recent survey on contests with multiple prizes, see Sisak (2009)). To model this intra-party contest, we build on the allocation mechanism

[4] Previous models which focus on moral hazard problems between a party and its candidates without an explicit focus on party organisation include, amon others, Alesina and Spear (1988).

[5] There is also a theoretical and empirical literature on adverse selection problems in elections. Recent contributions focusing on selection issues under FPTP include Snyder and Ting (2011) and Galasso and Nannicini (2011). Contributions on selection effects under PR or multiple electoral rules include Besley et al. (2017), Dal Bo et al. (2017), Folke et al. (2016), Galasso and Nannicini (2016), Mattozzi and Merlo (2015) and Myerson (1993).

of Clark and Riis (1996).[6] The prizes in our intra-party contest correspond to the positions on the list that translate into (endogenous) probabilities of getting a seat as a function of the party's result in the general election. The higher on the list is an individual, the higher are their chances of getting elected, independently of their own effort choice.

Turning to our contribution to the literature on team contests, we introduce a novel way to model team contests for multiple homogeneous prizes. We assume that the number of prizes won by a party follows a binomial distribution with a success probability that corresponds to the Tullock contest function using the electoral outputs of the two parties. [7] Competing mechanisms to model the electoral contest between parties are the well known probabilistic model of Enelow and Hinich (1982) and Lindbeck and Weibull (1987) or the mechanism put forward by Nalebuff and Stiglitz (1983). The main advantage of our binomial-Tullock mechanism is its tractability and the fact that it generates the intuitive feature that it is easy for each party to win a few legislative seats, but it is hard for both parties to win nearly all seats.

3 The Model

We consider a society with a continuum of voters of mass K, K being an odd natural number. Under plurality rule, society is divided into K identical electoral districts, in which a unit mass of citizens vote. Each district elects one representative to the legislature. Under proportional representation, there is a single nationwide electoral district which elects all K legislators.

Candidates exert effort prior to the election to improve the quality of their platform, because a candidate's electoral chances increase in the electorate's perception of the quality of their platform (see Caillaud and Tirole (2002) or Castanheira et al. (2010) for contributions with similar strategies for candidates). The "quality" of a candidate's platform corresponds to the platform characteristics that the electorate values.

Candidates are office-motivated and choose their effort to maximize their expected utility. If elected, a candidate earns a payoff of V. If not elected, a candidate earns 0. We assume a quadratic effort cost function $C(e) = \frac{1}{2}e^2$. The objective function of a candidate is thus:

$$\Pr(\text{elected})\, V - \frac{1}{2}e^2 \tag{1}$$

[6] See also Clark and Riis (1998) and Fu and Lu (2009) for papers using this modelling.

[7] Crutzen et al. (2017) use the same binomial-Tullock contests success function between teams and prove that the use of a list as under closed-list PR can be optimal for incentive provision under a weak monotonicity constraint.

Candidates belong to one of two parties. Under plurality rule, each district-specific election is a contest between two party representatives, whose outcome is a function of the efforts chosen by the two candidates. Under proportional representation, parties offer voters a list of K candidates and voters choose which list to vote for. We thus consider the case of closed-list proportional representation. The quality of the list is determined by the individual effort decisions of the politicians who make up the list, as we explain in detail below.

Under plurality rule, each party selects one candidate per district. Under proportional representation, each party selects the K candidates and their place on the electoral list. To select their candidates, parties use one of two selection procedures. The first procedure is non-competitive: the party chooses the candidates who will represent it in the election via a system that is based on candidate characteristics that are non pliable and independent of the effort exerted by candidates, such as race, gender, seniority, etc.

Under the competitive procedure, the party organizes primary elections and let several candidates compete to become a candidate in the general election. The candidates' effort choices determine the outcome of the primaries. Under plurality rule, in each district, $N \geq 2$ candidates compete in a primary election for the right to run in the general election. Under proportional system, $N \geq K$ candidates compete nation-wide for the right to be on the party list and for their position on the list.

We now describe the contests corresponding to each scenario in more details.

3.1 Plurality Rule

Non-competitive selection

Voters in district d consider the efforts of the candidates selected by the two parties in that district, e_d^L and e_d^R. We assume that the probability that the representative of party L wins the seat in district d is given by a modified Tullock (1980) contest success function:

$$P_d^L\left(e_d^L, e_d^R\right) = \beta\left(\frac{e_d^L}{e_d^L + e_d^R}\right) + \frac{1-\beta}{2}. \tag{2}$$

Parameter β represents the importance of effort in the result of the election. A high β means that effort plays an important role in determining the result of the election. The case $\beta = 1$ corresponds to the standard Tullock contest function. The case $\beta = 0$ corresponds to a random election result. Parameter β thus determines the relative importance of inter-party and intra-party competition for electoral incentives.

Competitive selection

We also model the intra-party selection process as a Tullock contest between the n party candidates. The probability that candidate i of party L in district d is selected to represent their party in their district general election is given by :

$$Q_d^{iL} = \frac{e_d^{iL}}{e_d^{iL} + \sum_{k \neq i} e_d^{kL}}, \tag{3}$$

where e_d^{kL} denotes the effort of candidate k among the other $n-1$ candidates in i's party.

The degree of competition in the selection procedure varies with the number of candidates in the primary of each party. When $n = 1$, the system is non-competitive. When $n > 1$, the selection procedure is competitive.

Let e_d^R be effort by the representative of party R in district d. Candidate i in district d chooses effort e_d^{iL} to maximize:

$$Q_d^{iL} P_d^L \left(e_d^L, e_d^R\right) - \frac{1}{2}\left(e_d^{iL}\right)^2 \tag{4}$$

$$= \left(\frac{e_d^{iL}}{e_d^{iL} + \sum_{k \neq i} e_d^{kL}}\right)\left(\beta \frac{e_d^{iL}}{e_d^{iL} + e_d^R} + \frac{1-\beta}{2}\right) V - \frac{1}{2}\left(e_d^{iL}\right)^2 \tag{5}$$

3.2 Proportional Representation

Under PR, a party presents to voters an ordered list of K candidates. The party list determines the allocation of seats between a party's candidates. If a party wins m seats, the first m candidates on the list get elected while the $K - m$ remaining candidates don't get elected.

The electorate bases its voting decision on the expected quality of the parties' platforms. These are a function of the sum of the party candidates' individual efforts: $E^L = \sum_{m=1}^{K} e_m^L$ and $E^R = \sum_{m=1}^{K} e_m^R$, where e_m^P is effort by the candidate of party P, $P = L, R$, who is in mth position on the party list.

Non competitive selection

To model the way effort decisions of the party representatives map into the parties' seat shares of the legislature, we extend the usual Tullock contest function to the case of multiple prizes. We assume that any given seat is won according to the following Tullock contest success function:

$P_L = \beta\left(\frac{E^L}{E^L+E^R}\right) + \frac{1-\beta}{2}$. The total number of seats won by party L thus follows a binomial distribution with parameters K and P_L. The probability that party L wins k seats is given by:

$$P_L(k) = C_k^K P_L^k \left(1 - P_L\right)^{K-k} \tag{6}$$

Under the noncompetitive candidate selection procedure, each politician on the party electoral list chooses how much effort to exert with a view to maximizing their chances of being elected by voters. The candidate in mth position on the electoral list of party L, say, chooses effort e_m^L to maximize:

$$\sum_{k=m}^{K} P_L(k)V - \frac{1}{2}\left(e_m^L\right)^2 \tag{7}$$

Competitive selection

For the competitive selection procedure, $N \geq K$ candidates compete for the K positions on the party list. Given that within each party legislative seats are allocated to candidates in the order of the list, the first position on the list is more valuable than the second position and so on and so forth. We thus model the competitive selection procedure as a contest between N candidates for K prizes of different values.

To model each intra-party contest, we rely on the imperfectly discriminating contest model of Clark and Riis (1996). In particular, denoting e_i the effort of politician i, the probability that i ends up in position k or higher on the party list is given by:

$$Q_i(k) = p_1 + \sum_{j=2}^{k} p_j \left[\prod_{s=1}^{j-1}(1 - p_s)\right] \tag{8}$$

where p_j is the probability that i ends up in position $j = 1, ..., k$ on the list and is given by the standard Tullock ratio contest success function among the candidates who have not yet been attributed a slot on the list (thus, for the jth prize, there are $n - j + 1$ candidates competing with each other):

$$p_j = \frac{e_i}{e_i + \sum_{k \neq i} e_k}, \#k = n - j. \tag{9}$$

One can interpret these probabilities as the result of a sequential process. The contestants make one contribution that is valid for the entire contest. The winner of the first prize (the first spot the list) is decides using the Tullock contest function with the contributions of the N contestants. The winner and his contribution disappear and the winner of the second prize is then decided using the Tullock contest function with the contributions of the remaining $N-1$ contestants. We continue this process until all spots on the list have been awarded. The values of a spot on the party list are then endogenously determined by the probabilities that the party wins a given number of seats in the election. These probabilities are computed using the average effort of the candidates on the list to determine the party aggregate effort.

In the competitive system, an individual candidate chooses effort e_i^L to maximize:

$$\sum_{m=1}^{K} P_L(m)Q_i(m)V - \frac{1}{2}\left(e_i^L\right)^2, \tag{10}$$

where $Q_i(m) = p_1 + \sum_{j=2}^{m} p_j \left[\prod_{s=1}^{j-1}(1-p_s)\right]$ and $P_L(m) = C_m^K P_L^m \left(1-P_L\right)^{K-m}$.

3.3 Discussion of the model

A few features of the models are worth discussing. First, we model political competition as contests. While elections are more often modelled as games in which parties (or candidates) choose ideological positions, candidates do spend considerable efforts and resources to improve their electoral platforms. In section 4 we extend our model to allow for the presence of ideological preferences among voters and show that our key comparative politics results still hold.

The assumption that candidates choose their effort only once (even when they participate in two contests) is useful to make direct comparisons of the incentives under different systems. This assumption also has the benefit to simplify the analysis. Hirano and Snyder (2014) offer evidence that the candidates' choices in their intra-party primaries matters for the general election too. See Amegashie (2006) for a model of contests with two rounds in which candidates select their effort in both rounds. With two rounds, incentives to exert effort could be even stronger in the first round, and would be followed by rent extraction in the second. yet, it would be difficult to compare both level of efforts across electoral rules.

The assumption that candidates only care about their individual result is a strong assumption and we relax it in section 4. Candidates most likely also care about their party winning a majority of legislative seats, as this guarantees control of government. We show that when candidates also care about their party winning a majority of the seats, the main results still obtain.

Our modelling of the inter-party competition via a binomial-Tullock imperfectly discriminating contest success function is a promising novel way of describing any competitive environment between teams of players. For example, Crutzen, Flamand and Sahuguet (2017) and Crutzen, Konishi and Sahuguet (2017) use this technology to contribute to the analysis of moral hazard problems in teams. Our use of the technology developed by Clark and Riis (1996) is again driven by the fact that it allows for closed-form solutions but also and perhaps more importantly that it has been shown to possess all the desirable axiomatic properties of contest success functions; see Fu and Lu (20XX).

4 Equilibrium individual and aggregate efforts

In this section, we solve for equilibrium behaviour of candidates under each electoral rule and selection procedure. In the next section, we exploit our findings to derive our comparative politics predictions.

4.1 Non-competitive candidate selection procedure

4.1.1 Plurality Rule

Party L's candidate in district d, say, chooses e_d^L to maximize equation (2) above. The first-order condition associated to this problem is:[8]

$$\frac{e_d^R}{\left(e_d^L+e_d^R\right)^2}\beta V - e_d^L = 0.$$

At the symmetric Nash equilibrium, we have:

$$e_d^{L*} = e_d^{R*} = \frac{\sqrt{\beta V}}{2}.$$

Proposition 1 *Under plurality rule with no competitive selection, each candidate exerts effort* $e^* = \frac{\sqrt{\beta V}}{2}$.

4.1.2 Proportional Representation

Under proportional representation, when the candidate selection procedure is non-competitive, party representatives exert effort only because of the incentives generated by the inter-party contest.

[8]The second order conditions are satisfied.

Then, candidates exert effort according to their position on the list, as is clear from equation (7). Indeed, the first order condition to the problem of the candidate in position m on the list of party L is given by:

$$\begin{aligned} e_m^L &= \beta V \sum_{k=m}^{K} C_k^K P_L^{k-1} \left(1-P_L\right)^{K-k} k \tfrac{E^R}{\left(E^L+E^R\right)^2} \\ &\quad - \beta V \sum_{k=m}^{K} C_k^K \left(P_L\right)^k \left(1-P_L\right)^{K-k-1} \left(K-k\right) \tfrac{E^R}{\left(E^L+E^R\right)^2} \end{aligned}$$

In the symmetric equilibrium, effort choices of candidates in the same position on the list are equal across parties and thus $E^{L*} = E^{R*} = E^*$. We can simplify the above first order condition to find:

$$e_m^L = \frac{\beta V}{4E^*} \left(\frac{1}{2}\right)^{K-1} \sum_{k=m}^{K} \left(2k-K\right) C_k^K$$

Remark that $\sum_{k=m}^{K} \left(2k-K\right) C_k^K = mC_m^K$. The above first order condition thus boils down to:[9]

$$e_m^L = \frac{\beta V}{4E^*} m \left(\frac{1}{2}\right)^{K-1} C_m^K.$$

Summing these optimal effort decisions over all party list members and exploiting the fact that $\sum_{m=1}^{K} m \left(\frac{1}{2}\right)^{K-1} C_m^K = K$, we get:

$$E^* = \frac{\sqrt{\beta V K}}{2} \tag{11}$$

and thus:

$$e_{mL}^* = mC_m^K \left(\frac{1}{2}\right)^K \sqrt{\frac{\beta V}{K}} \tag{12}$$

Proposition 2 *Under closed-list proportional representation with no competitive selection, the effort exerted by the candidate in mth position is* $e_m^L = mC_m^K \left(\frac{1}{2}\right)^K \sqrt{\frac{\beta V}{K}}$. *Aggregate party effort is* $E^L = E^R = \frac{\sqrt{\beta V K}}{2}$.

As anticipated, candidates in different positions on the list exert different levels of effort. Further, given the properties of combinatorials, the distribution of individual efforts along each party

[9] The second-order conditions are satisfied in this equilibrium.

list is bell-shaped and symmetric about the median list member. Thus candidates at the top and bottom of the party list exert very little effort (this first because they are certain or nearly certain of getting elected independently of the effort they exert, the last because they are certain or nearly certain of not being elected) whereas those in the middle of the list are the candidates who exert most effort. The median list member is the candidate exerting the highest amount of effort. Indeed, this is the position where the marginal benefit of effort, driven by the binomial-Tullock mapping, is greatest.

4.2 Competitive candidate selection procedure

4.2.1 Plurality Rule

With a competitive selection procedure, candidates in each district now exert effort to be first selected by their party and then to win the general district election. The party lets n candidates run in the intra-party selection procedure. Then, Candidate i from party L in district d, chooses effort e_d^{iL} to maximize equation (5) above. The first order condition is :

$$e_d^{iL*} = V \left\{ \frac{\sum_{k \neq i} e_d^{kL} - e_d^{iL*}}{\left[\sum_{j=1}^{n} e_d^{jL}\right]^2} \left(\beta \frac{e_d^{iL*}}{e_d^{iL*} + e_d^R} + \frac{1-\beta}{2} \right) + \frac{e_d^{iL*}}{\sum_{j=1}^{n} e_d^{jL}} \frac{\beta e_d^R}{\left(e_d^{iL*} + e_d^R\right)^2} \right\}.$$

The first argument within the curly bracket corresponds to the marginal benefit of exerting effort on a candidate's chances of being selected, whereas the second term corresponds to the marginal benefit associated to the effect effort has on a candidate's general election prospects, given that they were selected by their party, which happens with probability $\frac{e_d^{iL*}}{\sum_{j=1}^{n} e_d^{jL}}$.

In the symmetric equilibrium in which all candidates exert the same effort, we get:

$$e_d^{iL*} = e_d^{R*} = e^* = \sqrt{V \left(\frac{n-1}{2n^2} + \frac{\beta}{4n} \right)}. \tag{13}$$

and we thus have:

Proposition 3 *Under plurality rule, when there are n candidates in the primary of each district, in the symmetric equilibrium, candidates exert effort equal to: $e^* = \frac{1}{2n}\sqrt{V\left((2+\beta)n-2\right)}$. Thus, if $\beta < 1$, $n = 2$ maximises individual effort provision. If $\beta = 1$, $n = 1$ and $n = 2$ both maximize individual effort provision.*

Proof. See appendix. ■

The intuition behind this finding is as follows. Intra-party competition leads to two effects: a dilution and a competition effect; and the optimal degree of competition trades off these two effects optimally. In the present model, the dilution effect counterbalances the competition effect quite quickly, implying that it is optimal for parties to restrict competition quite severely and never go beyond $n = 2$.

4.2.2 Proportional Representation

When the candidate selection procedure is competitive, there are two incentive devices that impact on a party's set of candidates: first, candidates want to be among the set of candidates who are selected by their party; second, conditional on being selected, each candidate would like to end up as high as possible on the party list, as this increases one's chances of getting elected to the legislature.

Maximizing equation (10), the first order condition to the problem of any candidate in either party yields, when evaluated in the symmetric equilibrium:

Proposition 4 *When N candidates compete in each party for one of the K spot on the list, the equilibrium effort is given by:*

$$e^* = \sqrt{V} * \sqrt{\frac{\beta}{4N} + \sum_{m=1}^{K} C_m^K \left(\frac{1}{2}\right)^K \left(1 - \frac{m}{N}\right) \left[\sum_{j=1}^{m} \frac{1}{N-j+1}\right]}$$

Further, for any given $N \geq K$, e^* is increasing in K.

Proof. See appendix. ■

The first term under the square root represents the marginal effect of effort on the party's electoral success given that every candidates has the same probability of getting a seat. The second term represents the marginal effect of effort on getting a better spot on the list.

Unfortunately, it turns out that pinning down algebraically the optimal number of candidates N^* for any given K is a very difficult task. Numerical simulations suggest that the optimal number of candidates is equal to $K + 1$ for values of K below 5 (we can actually prove this by direct computation) and equal to K itself for larger values of K. To circumvent this problem, in what follows, we impose $N = K$. If we could lift this restriction, our comparative politics results below would be, if anything, reinforced, given that these are derived under the constraint that the number of candidates within each party under proportional representation may not be the optimal one.

5 Comparative Politics

Using our findings in the four scenarios we considered in the previous section, we have:

Theorem 5 *Comparing effort provision across electoral rules:*
When the candidate selection rule is non-competitive, aggregate candidate effort is higher under plurality rule than under proportional representation.
When candidate selection is competitive, parties choose the optimal number of internal candidates in each district under plurality rule ($n = 1$ or 2) and there are $N = K$ candidates running for selection within each party under proportional representation, individual and aggregate effort is higher under proportional representation than under plurality rule.

Proof. See appendix. ■

This first part of the theorem is in line with previous results in the literature. See for example Persson and Tabellini (2000) and Persson et al. (2003) and the references therein. The intuition behind this result is straightforward. FPTP gives direct incentives to candidates to exert effort. PR dilutes incentives. Candidates work for the success of the party and only indirectly work for the chance to get a seat. There is thus a public good aspect to the party's success and this leads to underprovision of effort.

The main intuition behind the second part of the theorem is that under FPTP, the benefit of a competitive CSP is always compromised by the dilution effect, while under PR, the benefits of competitive selection can be obtained without increasing the number of candidates. The intra-party competition for the best spots on the party list turns out to be a very strong incentive device for all candidates.

Our findings also complement those of Myerson (1993) and Buisseret and Prato (2017) on the effects of district magnitude. Both papers conclude that increasing the size of electoral districts leads to outcomes that increase voters' utility, because larger districts are either associated to more *inter-party* competition which gives voters more freedom of choice (Myerson) or allow for a better balancing of the objectives of voters and parties (Buisseret and Prato). We add that there is also an *intra-party* dimension to these incentive problems, which reinforces the positive, inter-party incentives effects of larger districts.

5.1 Empirical implications

Our findings suggest that the characteristics of the selection procedure used to select candidates within a party are a major source of incentives. In particular, intra-party incentives for the best slots on the party list under PR are particularly strong. Thus, as long as parties wish to maximise their candidates' electoral efforts, the strategic use of closed-list is a very efficient devise, in stark contrast to the quite bleak description of these lists the literature has offered so far.

More generally, our findings suggest that any empirical comparative politics exercise is likely to suffer from a serious omitted variable problem unless variables that account for or at least proxy the way candidates are selected are not included in the empirical exercise. To the best of our knowledge, such an exercise has not been carried out so far. One major stumbling block is the lack of reliable data on intra-party candidate selection procedures. For example, Shomer (2014, forthcoming) represents probably the current state of the art when it comes to data on the CSP, but even there the data is quite patchy and very much spread out in time and across countries. Other, promising works are slowly emerging on some specific countries, like Besley et al. (2017) and Dal Bo et al. (forthcoming), but here the data focuses on municipalities in Sweden only. Thus, we are still lacking a good enough dataset to carry out precise and satisfactory cross-country analyses. This is a pressing issue for empirical work in comparative politics and political economy more generally.

6 Extensions

6.1 Candidates care about their party winning a majority of seats

Suppose that candidates also care about their party winning a majority of seats and control of the executive.

How will their effort decisions be affected by this change in their utility function? We should expect that the positive effect of the incentives on effort of PR with a competitive candidate selection procedure shrink as the value candidates attribute to their party winning the executive grows relative to their utility from getting elected themselves. Indeed, in the limit, if candidates care only about their party winning, their position on the party list is immaterial to them. We show that our results are robust to this change in the candidates objective function.

The candidate is pivotal when party L wins in exactly $\frac{K-1}{2}$ districts out of the $K-1$ districts in which candidate d is not running. In equilibrium, this happens with probability $C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$. Each candidate's preferences are thus made of two terms now: the first relates to their probability of winning a legislative seat; the second relates to the probability that their party wins a majority of the seats in the legislature. Augmenting our analysis to allow for this modified candidate preferences yields:

Proposition 6 *Theorem 6 still holds when candidates care about their party winning a majority of seats.*

Proof. See appendix B. ■

6.2 Number of parties

So far we assumed that there were two parties competing under PR. What happens if we relax this assumption? The short answer is: not much. Suppose that there are $T > 2$ parties under proportional representation. We can then show – see Appendix B for the formal proof – that if the numbers of parties is not too large, then the average effort under a proportional system with competitive selection leads to more effort than FPTP with 2 parties. Indeed, the obvious effect of the increase in the number of parties under proportional representation is to dilute effort incentives but, as long as the effective number of parties is not too high, theorem 6 still goes through. This suggests a novel trade off under proportional representation between the desire to offer the best possible representation of citizens preferences – which speaks for the presence of many parties – and the need to keep electoral incentives at work – which favours having a small number of parties. Lizzeri and Persico (2005) propose a related trade off – centred around the need to balance general, country-wide goals and local or particularistic ones – which also points to a cost of having an 'excessive' number of parties under proportional representation.

6.3 Adding Ideology

In the main body of the paper, we assumed that ideology plays no role. The most efficient way of augmenting our model to allow for ideological preferences is as follows. Under plurality rule, allow for a distribution of K pairs of biases $\left(\tilde{b}_d^L, \tilde{b}_d^R\right)$ that is symmetric and unimodal about 0, so that there is one unbiased district and $K-1$ biased but symmetrically distributed districts about this median district.[10] These biases are common knowledge to all players of the game. Under proportional representation, these local biases cancel each other out and optimal effort decisions

[10] In our model, there is no obvious reason to assume that biases are asymmetrically distributed.

under this electoral rule is unaffected. We can thus study the effect of ideology by analysing its relative impact under plurality rule only.

Given that the following biased inter-party binomial-Tullock contest success function

$$\frac{\tilde{b}_d^L e_d^L}{\tilde{b}_d^L e_d^L + \tilde{b}_d^R e_d^R}$$

can always be rewritten as

$$\frac{b_d^L e_d^L}{b_d^L e_d^L + e_d^R}$$

or

$$\frac{e_d^L}{e_d^L + b_d^R e_d^R}$$

with $b_d^P < 1$, $P = L, R$, depending on which of $\tilde{b}_d^L$ or $\tilde{b}_d^R$ is smallest, there is no loss of generality in focussing on only one random district among the biased ones, like the one in which L's probability of winning the seat is given by:

$$P_d^L\left(e_d^L, e_d^R\right) = \beta\left(\frac{b_d^L e_d^L}{b_d^L e_d^L + e_d^R}\right) + \frac{1-\beta}{2} \tag{14}$$

In that district, if the selection procedure is non-competitive, the first order conditions pointing down optimal effort are given by

$$\frac{e_d^R}{\left(b_d^L e_d^L + e_d^R\right)^2}\beta b_d^L V - e_d^L = 0$$

for the representative of party L and

$$\frac{e_d^L}{\left(b_d^L e_d^L + e_d^R\right)^2}\beta b_d^L V - e_d^R = 0,$$

for the representative of party R, which yield, naturally:

$$e_d^{L*} = e_d^{R*} = \frac{\sqrt{\beta b_d^L V}}{2} \tag{15}$$

Thus, if the selection procedure is non-competitive, whenever a district is biased in favour of either L or R representatives exert less effort than they would if the district was unbiased, as is well known in the literature on the theory of contests.

If the selection procedure is competitive, the first order conditions for any candidate in party L boils down to, in equilibrium:

$$e_d^{iL*} = V\left\{\frac{n-1}{n^2 e_d^{iL*}}\left(\beta\frac{b_d^L e_d^{iL*}}{b_d^L e_d^{iL*} + e_d^R} + \frac{1-\beta}{2}\right) + \frac{1}{n}\frac{\beta b_d^L e_d^R}{\left(b_d^L e_d^{iL*} + e_d^R\right)^2}\right\},$$

whereas that for any candidate in party R boils down to

$$e_d^{iR*} = V\left\{\frac{n-1}{n^2 e_d^{iR*}}\left(\beta\frac{e_d^{iR*}}{e_d^{iR*}+b_d^L e_d^L}+\frac{1-\beta}{2}\right)+\frac{1}{n}\frac{\beta b_d^L e_d^L}{\left(e_d^{iR*}+b_d^L e_d^L\right)^2}\right\}.$$

Remark now that $e_d^{iR*} = b_d^L e_d^{iL*}$ is a possible solution to the above system of equations. Thus, imposing this restriction and solving the corresponding equation, we get that

$$e_d^{iR*} = b_d^L e_d^{iL*} < e_d^{iL*} = \sqrt{V\left(\frac{n-1}{2n^2}+\frac{\beta}{4n}\right)} \tag{16}$$

implying that the candidates in the advantaged party exert less effort than those in the disadvantaged one if selection is competitive, so as to equalise across parties the probability of winning the general election.

More importantly for our purposes, it follows immediately that theorem 6 is actually reinforced by the introduction of ideology, as the effort disadvantage of proportional representation is reduced when selection is non-competitive and the advantage of proportional representation is magnified when selection is competitive.

7 Conclusion

Comparative politics is common way to relate economic outcomes to the nature of political institutions. The literature so far , starting with Persson Tabellini and coauthors has focused mostly on electoral systems and political regimes as the key explanatory variables. The literature has shown that the institutions have a large impact on the incentives of politicians and can explain cross-country variations on public spending, public good provision and the degree of corruption.

In this paper, we show that the way political parties are organized and select their candidates is another important variable to understand the link between institutions and incentives. We analyze a simple model of electoral competition in which candidates exert effort to increase the quality of their platforms and the probability to get elected. We show that the electoral system has a strong impact on the incentives of politicians to exert effort. In particular, we show that proportional systems tend to dilute the incentives of politicians as compared to a majoritarian system. However, when we include a competitive selection procedure in the model, we show that the results are turned. In a majoritarian system, intraparty competition only happens when the number of potential number of candidates is increases. Intraparty competition advantages are mitigated by the increased number of candidates. A proportional system uses a list to allocate the seats won by the party. In that system, incentives comes not only from the desire to be included on the list but also to improve one's position on the list. This means that competition can be created without multiplying the number of candidates.

This system of a primary election that determines the list of the party in a proportional system creates good incentives. We show that the selection procedure with the optimal degree of competition in a proportional system leads to higher effort than the primary system with the optimal degree of competition in a majoritarian system.

This result has two important consequences. First of all, we show that the organization of parties and the degree of competition in their selection procedures are an important variable to take into account in future empirical work. Good data about the organization of parties, the barriers to entry for new politicians are difficult to obtain. (See Shomer (2014, forthcoming) for an exception) However, we believe that this is an important direction for future research. Second, we show that party selection procedures are not only important by themselves, but also in conjunction with the electoral system. The empirical literature on the effect of electoral systems does not find as clear-cut effects as those predicted by the theoretical literature (See Persson Tabellini for instance). We show that when the effects of a competitive selection of candidates are different in different electoral systems. Incentives to exert effort in a proportional system can be improved dramatically when competition is introduced while the effects of competitive selection are smaller in a majoritarian system.

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Appendix A: Proofs

Some proofs are preliminary and unchecked.

Proof of proposition 4

The first order condition to the problem faced by any politician i (in party L, say) is:

$$\left[\sum_{m=1}^{K}\frac{\partial P_L(l)}{\partial e_i}Q_i(m)+\sum_{m=1}^{K}P_L(m)\frac{\partial Q_i(m)}{\partial e_i}\right]V=e_i^*$$

and, in the symmetric equilibrium, we have that:

As there are N candidates competing for one of the list slots, the equilibrium probability of being offered slot m on the list is $Q_i^*(m)=\frac{m}{N}$;

We also have

$$\frac{\partial P_L(m)}{\partial e_i}=\beta\frac{C_m^K}{4E^*}(2m-K)\left(\frac{1}{2}\right)^{K-1}=\beta\frac{C_m^K}{Ke_i^*}(2m-K)\left(\frac{1}{2}\right)^{K+1}$$

Finally:

$$\frac{\partial Q_i(m)}{\partial e_{iL}}=\frac{1}{e_i^*}\left(1-\frac{m}{N}\right)\sum_{j=1}^{m}\frac{1}{N-j+1}$$

and thus the FOC boils down to in the symmetric equilibrium:

$$e_i^*=\beta V\sum_{m=1}^{K}\frac{C_m^K}{Ke_i^*}(2m-K)\left(\frac{1}{2}\right)^{K+1}\frac{m}{N}+V\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^K\left[\frac{1}{e_i^*}\left(1-\frac{m}{N}\right)\sum_{j=1}^{m}\frac{1}{N-j+1}\right]$$

which implies therefore that equilibrium effort e_i^* is given by:

$$\sqrt{V}*\sqrt{\frac{\beta}{4N}+\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^K\left(1-\frac{m}{N}\right)\left[\sum_{j=1}^{m}\frac{1}{N-j+1}\right]}.$$

We now show that increasing the number of legislative seats from K seats to $K+1$ while keeping the number of candidates at $N>K$ leads to an increase in effort.

The sign of $e_{K+1}^*-e_K^*$ is the same as the sign of

$$V\left(\frac{1}{2}\right)^{K+1}\left(1-\frac{K+1}{N}\right)\left[\sum_{j=1}^{K+1}\frac{1}{(N-j+1)}\right]$$
$$+\frac{V}{2}\sum_{m=1}^{K}\left(\frac{1}{2}\right)^K C_m^K\frac{2m-K-1}{K+1-m}\left(1-\frac{m}{N}\right)\left[\sum_{j=1}^{m}\frac{1}{(N-j+1)}\right]$$

Let $u(m) = \frac{2m-K-1}{K+1-m}\left(1-\frac{m}{N}\right)\left[\sum_{j=1}^{m}\frac{1}{(N-j+1)}\right]$. A sufficient condition for the above to be positive is that $\sum_{m=1}^{K}\left(\frac{1}{2}\right)^{K} C_m^K u(m) > 0$.

$\left(\frac{1}{2}\right)^{L} C_m^K$ corresponds to the density of the binomial $B(K, 1/2)$ which is symmetric. With a symmetric density, the mean of the distribution μ is positive if $u(\mu+x) > -u(\mu-x)$. Here, we need to show (as K is odd) that, for all K,

$$\frac{4k}{L+1-2k}(1-\frac{L+1+2k}{2n})\sum_{j=1}^{(L+2k+1)/2}(1/(n+1+j))$$
$$+\frac{-4k}{L+1+2k}(1-\frac{L+1-2k}{2n})\sum_{j=1}^{(L-2k+1)/2}(1/(n+1+j)) \geq 0$$

$\frac{1}{L+1-2k}(1-\frac{L+1+2k}{2n})\sum_{j=1}^{(L+2k+1)/2}(1/(n+1+j)) \geq \frac{1}{L+1+2k}(1-\frac{L+1-2k}{2n})\sum_{j=1}^{(L-2k+1)/2}(1/(n+1+j))$

Note that
$$\sum_{j=1}^{(L+2k+1)/2}(1/(n+1+j)) > \sum_{j=1}^{(L-2k+1)/2}(1/(n+1+j))$$
so we need to show that
$$\frac{1}{L+1-2k}(1-\frac{L+1+2k}{2n}) \geq \frac{1}{L+1+2k}(1-\frac{L+1-2k}{2n}).$$

It is true for $k=1$.

$$\frac{1}{L-1}(1-\frac{L+3}{2n}) + \frac{-1}{L+3}(1-\frac{L-1}{2n}) = -\frac{4}{n}\frac{L-n+1}{L^2+2L-3} \geq 0.$$

For a given k,

We need to show that

$$\frac{1}{L+1-2k}(1-\frac{L+1+2k}{2n}) - \frac{1}{L+1+2k}(1-\frac{L+1-2k}{2n}) \geq 0$$

which is equivalent to $-4\frac{k}{n}\frac{L-n+1}{L^2+2L-4k^2+1} \geq 0$,

which is true.

Proof of Theorem 5

We will compare the effort in the majoritarian system when there are two candidates in each primary with the effort in the proportional system when the number of candidates is $N = K$.

We need to compare $\sqrt{V\frac{(1+\beta)}{8}}$ and $\sqrt{V\left(\frac{\beta}{4K}+\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^K\left(1-\frac{m}{K}\right)\left[\sum_{j=1}^{m}\frac{1}{K-j+1}\right]\right)}$.

The effort in the majoritarian system does not change with K.

The effort in the proportional system is the square root of the sum of two terms. The first term $\frac{\beta}{4K}$ decreases in K. We now show that second term, $\lambda(K)=\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^K\left(1-\frac{m}{K}\right)\left[\sum_{j=1}^{m}\frac{1}{K-j+1}\right]$, is increasing in K, that its limit when K goes to infinity is larger than $\frac{(1+\beta)}{8}$ and that it grows faster than the speed at which $\frac{\beta}{4K}$ shrinks for any K.

The following result about combinatorial sums of finite differences will prove useful. Identity 14 in Spivey (2007) shows that $\sum_{m=1}^{K}2^{-K}C_m^K\left(H_K-H_{K-m}\right)=\sum_{m=1}^{K}\frac{1}{m2^m}$ with $H_K=\sum_{j=1}^{K}\frac{1}{j}$ is the Kth harmonic number.

We thus have that $\lambda(K)$ is equal to:

$$\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^K C_m^{K-1}\left(H_K-H_{K-m}\right)=\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^K C_m^{K-1}\left(H_{K-1}-H_{K-1-m}+\frac{1}{K}-\frac{1}{K-m}\right).$$

Thus

$$\lambda(K)=\frac{1}{2}\sum_{m=1}^{K-1}\frac{1}{m2^m}+\frac{1}{2}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1}C_m^{K-1}\left(\frac{1}{K}-\frac{1}{K-m}\right).$$

The second term in $\lambda(K)$ simplifies to:

$$\frac{1}{K}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1}C_m^{K-1}\left(\frac{-m}{(K-m)}\right)=-\frac{1}{K^2}\sum_{m=1}^{K-1}\left(\frac{1}{2}\right)^{K-1}C_m^K m=-\frac{1}{K}\left(1-\left(\frac{1}{2}\right)^{K-1}\right)$$

Thus

$$\lambda(K)=\frac{1}{2}\left(\sum_{m=1}^{K-1}\left(\frac{1}{m2^m}\right)-\frac{1}{K}+\frac{1}{K}\left(\frac{1}{2}\right)^{K-1}\right).$$

This implies in turn that:

$$\begin{aligned}\lambda(K+1)-\lambda(K)&=\left(\frac{1}{(K)2^K}-\frac{1}{K+1}+\frac{1}{K+1}\left(\frac{1}{2}\right)^K+\frac{1}{K}-\frac{1}{K}\left(\frac{1}{2}\right)^{K-1}\right)\\&=\frac{1}{2^K K}\frac{2^K-1}{K+1}>0.\end{aligned}$$

Finally, $\forall K \geq 3$, simple algebra implies that $\frac{1}{2^K K} \frac{2^K - 1}{K+1} > \frac{\beta}{4K} - \frac{\beta}{4(K+1)}$. Thus the effort under proportional system is increasing in K and tends to $\sqrt{log(2)/2} = 0.58$. This is higher than effort in the majoritarian system, as $\sqrt{(1+\beta)/8} < 1/2, \forall \beta \in [0,1]$.

Appendix B: Proofs for the Extensions Section

Candidates care about winning a majority of seats

We first derive the effort under the noncompetitive selection rule

Plurality Rule

Let us focus on candidate of party L in district d. The outcome in district d is the only outcome candidate dL can influence. The candidate is pivotal when party L wins in exactly $\frac{K-1}{2}$ districts out of the $K-1$ districts in which candidate d is not running. in equilibrium, this happens with probability $C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1}$.

The candidate chooses effort e_{dL} to maximize:

$$\left(V + C^{K-1}_{\frac{K-1}{2}}\left(\tfrac{1}{2}\right)^{K-1} M\right)\left(\tfrac{e_{dL}}{e_{dL}+e_{dR}}\right) - \frac{e_{dL}^2}{2}$$

Taking the first order condition and evaluating at the symmetric equilibrium, we get

$$e^*_{dL} = \frac{1}{2}\sqrt{V + C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1} M}.$$

We thus get:

$$E^* = \frac{K}{2}\sqrt{V + C^{K-1}_{\frac{K-1}{2}}\left(\frac{1}{2}\right)^{K-1} M}. \tag{17}$$

Proportional Representation

Every candidate's effort matters for the party's probability of winning at least a majority of the legislative seats. Candidate in position m on party L list chooses e_m to maximize:

$$\begin{aligned} &V\sum_{l=m}^{K} C^K_l \left(\tfrac{E_L}{E_L+E_R}\right)^l \left(1-\left(\tfrac{E_L}{E_L+E_R}\right)\right)^{K-l} \\ &+ M\sum_{j=\frac{K+1}{2}}^{K} C^K_j \left(\tfrac{E_L}{E_L+E_R}\right)^j \left(1-\left(\tfrac{E_L}{E_L+E_R}\right)\right)^{K-j} - \frac{e_m^2}{2} \end{aligned}$$

The first order condition evaluated in the symmetric equilibrium yields:

$$
\begin{aligned}
e_m^* &= \frac{V}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{l=m}^{K}(2l-K)\,C_l^K + \frac{M}{4E^*}\left(\frac{1}{2}\right)^{K-1}\sum_{j=\frac{K+1}{2}}^{K}(2j-K)\,C_j^K \\
&= \frac{1}{4E^*}\left(mC_m^K\left(\frac{1}{2}\right)^{K-1}V + \left(\frac{1}{2}\right)^{K-1}\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K M\right)
\end{aligned}
$$

where the second line exploits the fact that $\sum_{l=j}^{K}(2l-K)\,C_l^K = jC_j^K$.

Then, summing the effort over all candidates on the list, we get $E^* = \sum_{m=1}^{K} e_m$ or:

$$
\begin{aligned}
E^* &= \frac{V}{4E^*}\sum_{m=1}^{K}\left(\frac{1}{2}\right)^{K-1} mC_m^K + \frac{M}{4E^*}\left[\left(\frac{K+1}{2}\right)\left(\frac{1}{2}\right)^{K-1} C_{\frac{K+1}{2}}^K M\right] \\
&= \frac{1}{4E^*}\left(VK + MK\left(\frac{K+1}{2}\right)C_{\frac{K+1}{2}}^K\left(\frac{1}{2}\right)^{K-1}\right)
\end{aligned}
$$

and thus using the fact that $\frac{K+1}{2}C_{\frac{K+1}{2}}^K = KC_{\frac{K-1}{2}}^{K-1}$

$$
E^* = \sqrt{VK + MK^2C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}}. \tag{18}
$$

Comparing (14) and (15) it is easy to verify that total effort is always higher under the majoritarian system if $V > 0$, via the incentives regarding the candidates' individual reward: $\frac{1}{2}\sqrt{VK^2 + MK^2C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}} > \frac{1}{2}\sqrt{VK + MK^2C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}}$.

Indeed, the part of the incentive problem linked to the party's winning at least a majority generates the same incentives across the two electoral rules. Thus, proposition 1 carries through to the case in which candidates care about their party winning the executive office too as long as candidates care at least marginally about themselves getting elected too, as otherwise the electoral rule is immaterial for incentives.

We now derive the effort under the competitive candidate selection procedure.

Plurality Rule

As we have seen before, the optimal number of candidates is 2.

Equilibrium effort is thus the same as in the non-competitive case:

$$
e^* = \frac{1}{2}\sqrt{V + C_{\frac{K-1}{2}}^{K-1}\left(\frac{1}{2}\right)^{K-1}M}
$$

Proportional Representation

A candidate from party L chooses effort $E_L$to maximise:

$$V\sum_{m=1}^{K}\left\{\left\{p_1+\sum_{j=2}^{m}p_j\left[\prod_{s=1}^{j-1}(1-p_s)\right]\right\}\left\{C_m^K\left(\tfrac{E_L}{E_L+E_R}\right)^m\left(1-\tfrac{E_L}{E_L+E_R}\right)^{K-m}\right\}\right\}$$
$$+M\sum_{j=\frac{K+1}{2}}^{K}C_j^K\left(\tfrac{E_L}{E_L+E_R}\right)^j\left(1-\tfrac{E_L}{E_L+E_R}\right)^{K-j}-\frac{e_L^2}{2}$$

where the second term is party L's probability of winning at least a majority of legislative seats which, from the perspective of candidate i when they choose their effort level, is independent on where on the list candidate i ends up.

The FOC is thus given by:

$$\begin{aligned}e^* &= V\sum_{m=1}^{K}C_m^K\frac{1}{Ke^*}(2m-K)\left(\frac{1}{2}\right)^{\mathcal{L}+1}\frac{m}{K}\\ &+V\sum_{m=1}^{K}C_m^K\left(\frac{1}{2}\right)^K\left[\frac{1}{e^*}\left(1-\frac{m}{n}\right)\sum_{j=1}^{m}\frac{1}{n-j+1}\right]\\ &+\frac{M}{4Ke^*}\left(\frac{1}{2}\right)^{K-1}\sum_{j=\frac{\mathcal{L}+1}{2}}^{K}(2j-K)\,C_j^K\end{aligned}$$

which yields

$$e^*=\sqrt{V\tfrac{1}{4K}+V\sum_{m=1}^{K}C_m^K\left(\tfrac{1}{2}\right)^K\left[\tfrac{1}{e^*}\left(1-\frac{m}{K}\right)\sum_{j=1}^{m}\tfrac{1}{K-j+1}\right]+\tfrac{M}{4}C_{\frac{K-1}{2}}^{K-1}\left(\tfrac{1}{2}\right)^{K-1}}\tag{19}$$

As was the case when selection was not competitive, the incentive effects of candidates caring about their party winning a majority of seats enter in a similar way across electoral rules. Theorem 6 thus still holds as long as individual politicians do not care too much about their party winning at least a majority of the legislative seats, relative to themselves being elected.

Effect of having $T > 2$ parties under proportional representation

In the proportional system, let there be T identical parties.

The probability that team i wins l prizes is given by:

$$P_i(l) = \frac{K!}{(K-l)!l!} \left(P_p\right)^l \left(1-P_p\right)^{K-l}, \tag{20}$$

with $P_p = \frac{1-\beta}{T} + \beta \frac{E_p}{\sum_{j=1}^{T} E_j}$.

The problem for the candidate in position m on the list of party p is to maximise with respect to their own effort e_m^p:

$$\sum_{k=m}^{K} P_p(k)V - \frac{1}{2}\left(e_m^p\right)^2. \tag{21}$$

The first order condition to the problem of the candidate in position m on the list of party p is given by:

$$\begin{aligned} e_m^p = {} & \beta V \sum_{k=m}^{K} C_k^K \left(P_p\right)^{k-1} \left(1-P_p\right)^{K-k} k \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2} \\ & - \beta V \sum_{k=m}^{K} C_k^K \left(P_p\right)^{k} \left(1-P_p\right)^{K-k-1} (K-k) \frac{\sum_{j=1}^{T} E_j - E_p}{\left(\sum_{j=1}^{T} E_j\right)^2} \end{aligned}$$

In the symmetric equilibrium, effort choices of candidates in the same position on the list are equal across parties and thus $E^{1*} = ... = E^{P*} = E^*$ and $P_p = 1/T$. We can simplify the above first order condition to find:

$$\begin{aligned} e_m^L &= \frac{(T-1)\,\beta V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[k \left(\frac{1}{T}\right)^{k-1} \left(\frac{T-1}{T}\right)^{K-k} - (K-k)\left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k-1} \right] \\ &= \frac{(T-1)\,\beta V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[\left(Tk - \frac{T(K-k)}{T-1}\right) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\ &= \frac{(T-1)\,\beta V}{T^2 E^*} \sum_{k=m}^{K} C_k^K \left[\frac{T}{T-1} (Tk-K) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \\ &= \frac{\beta V}{T E^*} \sum_{k=m}^{K} C_k^K \left[(Tk-K) \left(\frac{1}{T}\right)^{k} \left(\frac{T-1}{T}\right)^{K-k} \right] \end{aligned}$$

Exploiting the fact that $\forall T$, $\sum_{k=m}^{K} C_k^K \left[(Tk-K)\left(\frac{1}{T}\right)^k\left(\frac{T-1}{T}\right)^{K-k}\right] = \left(\frac{1}{T}\right)^K (T-1)^{K-m+1} mC_m^K$, this simplifies further to:

$$e_m^L = \frac{\beta V}{TE^*}\left(\frac{1}{T}\right)^K (T-1)^{K-m+1} mC_m^K.$$

Summing these optimal effort decisions over all party list members and exploiting the fact that $\sum_{m=1}^{K}\left(\frac{1}{T}\right)^K (T-1)^{K-m+1} mC_m^K = K\frac{T-1}{T}$, we get:

$$E^* = \frac{\beta V}{TE^*}K\frac{T-1}{T} \iff E^* = \frac{\sqrt{K(T-1)\beta V}}{T} \tag{22}$$

Comparing the effort under the majoritarian system and propotional system, we see that the result of the first part of theorem 6 is reinforced. When there are more than 2 teams, the average effort is larger in the majoritarian system and the difference is larger when there are more parties in the proportional system.

We now turn to the competitive selection. Nothing changes under plurality rule.

Under proportional representation, the first order condition to the problem faced by any politician i (in party p, say) is:

$$\left[\sum_{m=1}^{K}\frac{\partial P_p(l)}{\partial e_i}Q_i(m) + \sum_{m=1}^{K}P_p(m)\frac{\partial Q_i(m)}{\partial e_i}\right]V = e_i^*$$

and, in the symmetric equilibrium, we have that:

As there are N candidates competing for one of the K list slots, the equilibrium probability of being offered slot m on the list is $Q_i^*(m) = \frac{m}{N}$;

Also, as $P_p(m) = C_m^K\left(\frac{1-\beta}{T} + \beta\frac{E_i}{\sum_{j=1}^{T}E_j}\right)^m\left(1 - \frac{1-\beta}{T} - \beta\frac{E_i}{\sum_{j=1}^{T}E_j}\right)^{K-m}$, in equilibrium we have that, exploiting some of the algebra above:

$$\frac{\partial P_p(m)}{\partial e_i} = \frac{\beta V}{TKe^*}C_m^K\left[(Tm-K)\left(\frac{1}{T}\right)^m\left(\frac{T-1}{T}\right)^{K-m}\right]$$

Also:

$$\frac{\partial Q_i(m)}{\partial e_{iL}} = \frac{1}{e_i^*}\left(1-\frac{m}{K}\right)\sum_{j=1}^{m}\frac{1}{K-j+1}$$

Finally:

$$P_p^*(m) = C_m^K \left(\frac{E_i^*}{\sum_{j=1}^T E_j^*} \right)^m \left(1 - \frac{E_i^*}{\sum_{j=1}^T E_j^*} \right)^{K-m} = C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \tag{23}$$

and thus the FOC boils down to in the symmetric equilibrium:

$$\begin{aligned} e_i^* = & \sum_{m=1}^{K} \frac{\beta V}{T K e_i^*} C_m^K \left[(Tm - K) \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \right] \frac{m}{K} \\ & + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \left[\frac{1}{e_i^*} \left(1 - \frac{m}{K} \right) \sum_{j=1}^{m} \frac{1}{N - j + 1} \right] \end{aligned}$$

$$\begin{aligned} = & \frac{T-1}{T^2 e_i^*} \frac{\beta V}{K} \\ & + V \sum_{m=1}^{K} C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \left[\frac{1}{e_i^*} \left(1 - \frac{m}{K} \right) \sum_{j=1}^{m} \frac{1}{K - j + 1} \right] \end{aligned}$$

which implies therefore that equilibrium effort e_i^* is given by:

$$\sqrt{V} * \sqrt{\frac{\beta (T-1)}{T^2 K} + \sum_{m=1}^{K} C_m^K \left(\frac{1}{T} \right)^m \left(\frac{T-1}{T} \right)^{K-m} \left(1 - \frac{m}{K} \right) \left[\sum_{j=1}^{m} \frac{1}{K - j + 1} \right]}. \tag{24}$$

PROOF TO BE COMPLETED