Identifying Skilled Analysts

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Abstract

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Abstract

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1. Introduction

The literature has widely examined the historical performance of portfolio managers and financial analysts.\(^1\) U.S. investors have entrusted trillions of dollars to the care of these individuals, so clearly a lot is at stake. Private investors are concerned whether fund managers generate excess returns above the fees charged for managing their portfolios. Institutional investors hope that analysts provide value beyond the cost of acquiring their services. Meanwhile, brokers and investment banks strive to determine whether the analysts they employ generate additional underwriting fees and increase net order flow.

A number of recent papers have empirically examined the abilities of financial analysts and market forecasters. These studies include O’Brien (1990), Butler and Lang (1991), Stickel (1992), Desai and Jain (1995), Graham (1996), Graham and Harvey (1996), and Womack (1996). The conclusions from this research generally agree with those from the mutual fund literature.\(^2,3\) As a group, financial analysts do not seem to possess superior long-term forecasting ability, at least not enough that investors utilizing

\(^{1}\) This body of research has its roots in the early work of Cowles (1933), Treynor (1965), Sharpe (1966), Jensen (1968), and Bjerring, Lakonishok, and Vermaelen (1983). The field has been extensively examined over the past decade. Part of this popularity is likely due to the rapid growth in the mutual fund industry during the same period.

\(^{2}\) Evidence from this research generally maintains that investment managers, especially those engaging in active portfolio management, do not outperform passively managed benchmarks on average. See Jensen (1968) and Malkiel (1995) among others. The poor performance of mutual fund managers documented by these studies corresponds with the growth in popularity of passively-managed index funds in recent years.

\(^{3}\) At a basic level, the mutual fund manager’s job closely resembles that of the financial analyst; both try to forecast which securities will make the best investments. Thus, a fund manager may only be as good as the analysts that he or she employs. However, portfolio managers must also consider the timing and dollar amount of security transactions, bringing an additional dimension of skill into the assessment of their ability as compared to individual analysts.
their services would expect to earn excess returns beyond the cost of obtaining the information.

Despite the results cited in the literature, the popular press reports escalating salaries and bonuses for the industry’s top analysts.\(^4\) This trend is not unlike the “superstars” phenomenon reported by Rosen (1981) where a relatively small number of individuals earn enormous amounts of money and dominate the activities in which they engage. In the case of financial analysts, small differences in forecasting ability may be associated with large differences in compensation. Therefore, the financial institutions employing these investment professionals have an even greater need to identify those individuals with superior stock-picking and forecasting skills.

However, the predominant focus within the empirical investment management literature has been at the macro level. While the average money manager or analyst may lack superior stock picking ability, we cannot completely reject the existence of some skilled analysts or fund managers in the profession. In fact, a few studies even document that certain individuals appear to possess some short-term forecasting skill.\(^5\) The question is whether skilled individuals can be distinguished from the rest of the population. As Desai and Jain (1995) note, “Even if such ‘superstar’ money managers exist, it is difficult to identify them, as data for a large number of years are needed for a reliable statistical analysis (p. 1257).”

\(^4\) See “The 1997 All-America Research Team,” *Institutional Investor*, Oct. 1997, p. 79-91. According to the article, compensation has begun to “escalate through the roof.” The top analysts in fields such as technology and health care can “easily command in excess of $1.5 million and a contract for as long as three years.” However, annual salaries for the hottest analysts can reach upwards of $3 million.

\(^5\) See Graham (1996) and Graham and Harvey (1997).
In contrast to the literature, the theme of this paper is at the micro level, beginning under the assumption that the population contains some analysts with superior forecasting skill. We examine how investors and financial institutions determine the ability or reputation of individual analysts. Through simulations of a multi-period herding model, we measure the number of observations necessary to accurately and reliably conclude whether a given analyst has superior talent. The model also suggests rules for increasing the probability of selecting a skilled analyst.

Several studies formulate models of the investment decision facing financial analysts and investment managers. Generally, this body of research centers on the economic incentives and conditions leading investment professionals to herd on the actions of their peers. The basis for our analysis is the reputational herding model originally developed by Scharfstein and Stein (1990), that was extended by Graham (1999). Scharfstein and Stein demonstrate equilibrium conditions where an investment manager ignores his private information and mimics the investment decision of another manager. Managers in their two-agent model are evaluated relative to one another and the private signals of skilled analysts are correlated. Therefore, it is more costly to an analyst’s reputation to incorrectly announce a signal different from the other analyst than

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6 Trueman (1994) models the forecast behavior of financial analysts in both simultaneous and sequential frameworks. Analysts in this two-agent model select one of four possible earnings announcements after observing a private signal. The analysis reveals that under certain circumstances analysts have a tendency to release earnings forecasts that are too close to prior earnings expectations even when the analyst’s private information justifies a more extreme forecast. In addition, analysts have a tendency to release forecasts similar to those previously announced by other analysts. In another study of herd behavior, Banerjee (1992) analyzes a sequential decision model where each agent views the actions of other agents before making their own decisions. The economy contains N agents, but not all agents receive a private signal. Since no one knows whether the previous mover was informed, the model is characterized by herd behavior. Other managerial herding or reputational learning models include Brandenburger and Polak (1996), Hershleifer (1993), and Prendergast and Stole (1996).
to incorrectly announce the same signal. Therefore, the labor market judges analysts who act as part of a group more favorably than those who behave in a contrarian fashion.

The Scharfstein and Stein (1990) and Graham (1999) model was originally formulated to examine analyst forecast decisions for a single period. However, the model can be easily applied to a multi-period setting. Using this framework, we develop one and two-analyst models and then simulate each over several hundred periods. The results provide insights into the process of identifying skilled analysts from the population.

The results confirm that identifying the most talented individuals can be highly challenging. With certain parameter assumptions, as many as 400-500 observations may be needed to distinguish a skilled analyst from an unskilled analyst with reasonable certainty. The difficulty of selecting superior analysts increases when agents are known to engage in herd behavior. However, institutional and private investors can increase the likelihood of identifying a skilled analyst simply by selecting the first analyst to announce a recommendation. This result develops because more talented analysts desire to reveal their ability by taking on the leadership role. Meanwhile, less skilled individuals prefer to hide their ability by herding on the recommendations of the leader. Unskilled analysts will not deviate from this strategy since their true type is likely to be revealed serving as the leader.

These results suggest a couple of practical investment policy implications. First, investment banks or financial institutions that are looking to hire an analyst may improve their probability of choosing a talented individual by focusing on the leaders. Second, institutional or private investors can concentrate on the recommendations of the last analyst to announce. When herd behavior is likely, the follower should at least be as
accurate as the leader. The follower also has the opportunity to observe a private signal in addition to the leader’s recommendation. Therefore, if the follower announces a forecast different from the leader, then we may conclude that his or her private information must have been strong enough to justify such a move.

The rest of the paper is organized as follows. Section 2 introduces the one-analyst model within a multi-period framework. Section 3 extends the model to the two-analyst case. Simulation results of the one and two-analyst models under varying parameters are described in sections 4 and 5, respectively. Section 6 examines the sensitivity of the models and simulation results to the parameter assumptions. Finally, the paper concludes with section 7 where we discuss the practical implications of these results and possible extensions for future research.

2. The One-Analyst Model

The one-analyst model is a subset of the two-agent investment model originally developed by Scharfstein and Stein (1990) and later modified by Graham (1999). Both studies examine the agents’ decision-making process within the context of a single period. This paper simulates these models within a multi-period framework. Graham’s notation is followed throughout the paper.

A single risk-neutral agent is randomly chosen from the population at time $t = 0$ to examine an investment. The agent is the only analyst publicly monitoring the security.

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7 Neither Graham (1999) nor Scharfstein and Stein (1990) implicitly develop a single-agent investment model. However, the one-analyst model is merely the two-analyst model where the second agent does not participate. Graham (1999) extends Scharfstein and Stein (1990) by allowing for the possibility of imperfect correlation among the private signals of skilled analysts and for variability in the initial state probabilities.
At the end of each period, the investment returns either a high state \((X_{H,t})\) with prior probability \(\alpha\) or a low state \((X_{L,t})\) with probability \((1-\alpha)\).\(^8\) Analysts announce investment recommendations each period based on their interpretation of a private signal and assessment of their individual ability. The analysts interpret a high signal \((s_{H,t})\) as an indicator that the high state is more likely to occur and vice versa for a low signal \((s_{L,t})\).

The population is known to initially contain two types of agents: skilled and unskilled.\(^9\) Let \(\theta_0\) denote the common knowledge, prior probability of choosing a skilled analyst, where \(\theta_0 \in (0,1)\). An analyst’s type is unknown to everyone, including the analyst. The two types of agents differ by the accuracy of their private signals. Skilled analysts receive informative signals, while unskilled analysts receive random signals. This information structure is symmetric whereby the following holds:

\[
\begin{align*}
\Pr(s_{H,t}\vert X_{H,t},\text{skilled}) &= \Pr(s_{L,t}\vert X_{L,t},\text{skilled}) = p, \text{ where } p \in (\frac{1}{2},1] \\
\Pr(s_{H,t}\vert X_{H,t},\text{unskilled}) &= \Pr(s_{L,t}\vert X_{L,t},\text{unskilled}) = \frac{1}{2}
\end{align*}
\]

Therefore, skilled analysts are expected to receive the correct signal with probability \(p > \frac{1}{2}\), while unskilled analysts randomly receive the correct signal only half of the time.

The structure of the analyst’s information set is modeled in Figure 1 for one period. Although analysts do not know their type, they use their knowledge of this structure to update their beliefs each period. Given the state, the probability of receiving the correct and incorrect signal for any period can be computed as follows:

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\(^8\) The high state can be interpreted as a period where the security’s return outperforms the market, while the low state occurs when the security underperforms the market. Based on this interpretation, a reasonable assumption with efficient markets is that the high and low state probabilities are equal \((\alpha = \frac{1}{2})\).

\(^9\) Scharfstein and Stein (1990) originally labeled agents as “smart” and “dumb.” Graham (1999) continued with this notation. While these labels are not intended as a commentary on the analyst community, we abandon this classification in favor of the labels “skilled” and “unskilled.” Even “dumb” analysts in our model must be smart enough to evaluate their alternatives each period via Bayesian updating.
\[ \Pr(s_{H,t}|X_{H,t}) = \Pr(s_{L,t}|X_{L,t}) = \theta_{t-1}p + (1-\theta_{t-1})^{1/2} \]
\[ \Pr(s_{L,t}|X_{H,t}) = \Pr(s_{H,t}|X_{L,t}) = \theta_{t-1}(1-p) + (1-\theta_{t-1})^{1/2} \]

These formulas weight the accuracy of skilled and unskilled analysts by their beginning-of-the-period reputation \( (\theta_{t-1}) \).

Since investors and employers do not know the analyst’s type, they form opinions of her ability each period based on the investment outcomes from previous periods and the accuracy of her recommendations. The analyst’s wages are therefore assumed to be a linear function of her reputation, and the analyst’s reputation represents the updated probability that the analyst is skilled.\(^{10}\) The final reputation from one period becomes the prior reputation of the next period. A risk-neutral analyst always prefers higher wages, so one possible strategy is for the analyst to choose the investment recommendation each period maximizing the probability that investors or employers will believe she possesses forecasting skill. Section A.1 of the Appendix demonstrates an equilibrium where state-by-state reputational optimization is the analyst’s preferred strategy.

Investors and employers use Bayes’ rule to update the analyst’s end-of-the-period reputation at time \( t \), denoted by \( \theta_t \). After observing the agent’s announcement and the investment outcome each period, they revise their initial probability that the analyst is skilled. When the parameter values \( \alpha, p, \) and \( \theta_t \) are such that the analyst is known to truthfully announce her private signal, the agent’s reputation will update according to equation (1) and (2):

\(^{10}\) In practice, analysts employed by brokerages or investment banks may also be compensated based on the amount of business they generate for the firm. Therefore, we implicitly assume that more skillful analysts generate more business through increased order flow from investors or higher newsletter subscriptions. The advice of a skilled analyst will have a greater value than the advice of an unskilled analyst to both the investor and the analyst’s employer.
\[ \hat{\theta}_t(\hat{s}_{H,t}, X_{H,t}) = \frac{\theta_{t-1} p}{\theta_{t-1} (1-p) + (1-\theta_{t-1}) \frac{1}{2}} \]  

\[ \hat{\theta}_t(\hat{s}_{L,t}, X_{L,t}) = \frac{\theta_{t-1} (1-p)}{\theta_{t-1} (1-p) + (1-\theta_{t-1}) \frac{1}{2}} \]  

Here, the analyst’s publicly-announced recommendation for period \( t \) is denoted by \( \hat{s}_{H,t} \) and \( \hat{s}_{L,t} \) for the high and low states, respectively. When the parameter values are such that the analyst ignores the private signal in equilibrium, her posterior reputation remains unchanged from her initial reputation.\(^{11}\)

Investors reward analysts for making accurate recommendations. From the standpoint of the investor, it does not matter if an analyst recommends the high state or the low state as long as her announcement is correct. Because of the binary nature of the recommendations and the stability of the parameters, the order of correct and incorrect announcements has no influence on the analyst’s long-run reputation.\(^{12}\) Therefore, an analyst who makes two correct recommendations followed by two incorrect recommendations will have the same reputation as an analyst who is correct, incorrect, correct, and then incorrect. When the agent is known to tell the truth, equation (3)

\(^{11}\) For instance, if the analyst is known to announce the high state even after receiving a low signal, then her reputation will not change from the prior period. In essence, the analyst herds on her prior state beliefs and does not utilize any skill in making her recommendation.

\(^{12}\) A number of studies from the investment management literature have examined persistence in mutual fund performance, including Grinblatt and Titman (1992), Hendricks, Patel, and Zeckhauser (1993), Brown and Goetzmann (1995), Malkiel (1995), Elton, Gruber, and Blake (1996), and Carhart (1997). In addition, studies such as Ippolito (1992) and Sirri and Tufano (1998) have documented that investment dollars flow into those funds with strong recent track records. Therefore, to the extent that mutual fund managers can exhibit a “hot hand” or persistent performance, they are likely to be rewarded with an increase in the amount of assets under management. In this sense, a manager’s most recent performance record may play a greater role in investors’ evaluation of his abilities than his career “won-loss” record. It does not seem unreasonable that this same pattern might play out for financial analysts as well. In fact, Graham and Harvey (1997) document that certain individuals seem to encounter a short-term hot streak forecasting future market movements. If the investment community favors analysts with a hot hand, then the order of correct and incorrect recommendations may have an impact on their long-run reputation and earnings.
demonstrates how the single analyst’s reputation can be calculated for future periods given the total number of correct recommendations.

\[ \theta_N(\theta_0, M, N) = \frac{\theta_0 p^M (1 - p)^{N-M}}{\theta_0 p^M (1 - p)^{N-M} + (1 - \theta_0)^\frac{1}{2} N} \]  

In this equation (3), \( N \) is the total number of trials, and \( M \) is the number of correct investment decisions. Therefore, an analyst’s reputation as of a point in time will be the product of her historical rate of accuracy.

The Scharfstein and Stein (1990) and Graham (1999) models examine only pure-strategy Bayesian Nash equilibria. Therefore, the analyst forms rational conjectures each period about the possible state given her private signal and knowledge of the parameter values. For instance, if the analyst receives a high signal, her revised probabilities that the high and low states will occur are determined as follows:

\[
\Pr\left(X_{H,t} | \hat{s}_{H,t}\right) = \frac{\left[\theta_{t-1} p + (1 - \theta_{t-1}) \frac{1}{2}\right] \alpha}{\left[\theta_{t-1} p + (1 - \theta_{t-1}) \frac{1}{2}\right] \alpha + \left[\theta_{t-1} (1 - p) + (1 - \theta_{t-1}) \frac{1}{2}\right] (1 - \alpha)}
\]

\[
\Pr\left(X_{L,t} | \hat{s}_{H,t}\right) = \frac{\left[\theta_{t-1} (1 - p) + (1 - \theta_{t-1}) \frac{1}{2}\right] (1 - \alpha)}{\left[\theta_{t-1} p + (1 - \theta_{t-1}) \frac{1}{2}\right] \alpha + \left[\theta_{t-1} (1 - p) + (1 - \theta_{t-1}) \frac{1}{2}\right] (1 - \alpha)}
\]

The analyst uses these beliefs to select the announcement that maximizes her expected reputation at the end of each period. If the analyst receives a high signal in period \( t \), then she announces truthfully when the following equation holds:

\[
\hat{\theta}_t(\hat{s}_{H,t}, X_{H,t}) \Pr(X_{H,t} | \hat{s}_{H,t}) + \hat{\theta}_t(\hat{s}_{H,t}, X_{L,t}) \Pr(X_{L,t} | \hat{s}_{H,t}) > \hat{\theta}_t(\hat{s}_{L,t}, X_{H,t}) \Pr(X_{H,t} | \hat{s}_{H,t}) + \hat{\theta}_t(\hat{s}_{L,t}, X_{L,t}) \Pr(X_{L,t} | \hat{s}_{H,t})
\]

The left-hand side of equation (4) represents the analyst’s expected reputation when telling the truth, while the right-hand side is the expected reputation from deviating.
The analyst only deviates when it is the more attractive equilibrium strategy. Based on the analysis presented by Graham (1999), the single analyst’s incentive to honestly announce her private information i) increases in ability ($p$), ii) increases in initial reputation ($\theta_{t-1}$), and iii) increases (decreases) in the strength of prior information ($\alpha$) when it is consistent (inconsistent) with her private information. In section A.2 of the Appendix, we solve for those parameter values where the analyst will tell the truth for a given period.

3. The Two-Analyst Model

The multi-period, single-analyst model was outlined in Section 2. This model is now extended to examine how the agent’s behavior changes with the addition of a second analyst. Once again, the basis for the model is Scharfstein and Stein (1990) and Graham (1999) which are applied to a multi-period setting. The notation and definitions remain the same, except superscripts are now added to the decision variables in order to distinguish between analyst A and analyst B.

Two risk-neutral agents are initially chosen from a large population to provide investment recommendations for the same security. The asset returns either the high state ($X_{H,t}$) with probability $\alpha$ or the low state ($X_{L,t}$) with probability $(1-\alpha)$. In this model, analysts move sequentially. At time $t=0$, one of the agents, analyst A, is chosen as the leader, while the remaining agent, analyst B, becomes the follower. The analysts do not switch their announcement order throughout the model; however, we extend the model to allow for this possibility in section 5.2. Since both analysts are chosen from the same
population, each has the same initial reputation denoted by \( \theta_0 \), where \( \theta_0 \in (0,1) \). The type of each analyst is unknown to everyone.

Figure 2 diagrams the informational structure of the two-analyst model. Like the one-analyst model, each agent receives a private signal. However, the private signals of skilled analysts are positively correlated. We denote the level of signal correlation by \( \rho \), where \( \rho \in (0,1) \). Meanwhile, the random signals of unskilled analysts are assigned independently. According to Graham (1999), the signal correlation of skilled analysts is critical to his reputational herding results:

“If smart analysts’ private information is positively correlated, they have a tendency to choose the same investment projects; that is, smart analysts often act as part of a group. In contrast, dumb analysts following their private information would appear to act independently. Analysts therefore deduce that by acting as part of a group they can ‘look smart,’ which provides an incentive to discard private information and ‘herd’ to be part of a group.”

The probability that two skilled analysts receive the correct signal is \((1-\rho)p^2 + \rho p\). The correlation parameter weights the case where skilled analysts receive independent signals \((p^2)\) with the case where their signals are perfectly correlated \((p)\). Likewise, the possibility that two skilled analysts both receive the wrong signal is \((1-p)(1-p)^2 + \rho(1-p)\), and the likelihood that they receive different signals is \(2p(1-p)(1-p)\). All parameter values \((\alpha, \rho, p, \text{ and } \theta_0)\) are common knowledge at the start of the game.

\[13\] When the announcement order remains fixed, allowing the analysts to have different initial reputations does not dramatically alter the decision variables. However, section 5.2 allows for the possibility that the analysts may switch the announcement order based on their prior period reputations. Under this scenario, the analyst with the highest reputation assumes the role of leader while the analyst with the lowest reputation becomes the follower.

\[14\] Scharfstein and Stein (1990) assume that skilled analysts always receive the same signal \((\rho = 1)\). The introduction of the correlation parameter \((\rho)\) is one of the extensions Graham (1999) adds to the Scharfstein and Stein model.
The leader, analyst A, begins each period by announcing her investment recommendation after considering the possible actions which B may choose. If analyst B herds in equilibrium, analyst A’s announcement decision is exactly the same as that of the one-analyst model. In this case, A is evaluated independently from analyst B, using equations (1) or (2), because the follower is known to mimic A’s recommendation. However, if the parameter values indicate that analyst B will announce truthfully in equilibrium, then the two analysts are evaluated relative to one another based on the updating rules described in section A.3 of the Appendix.

Analyst B holds an informational advantage over analyst A because he observes the leader’s announcement prior to making his own investment recommendation. The follower conditions on the leader’s announcement and his own private signal to determine the revised state probabilities. When analyst B receives a signal different from that released by A, the informational values cancel, and the following relationships hold:

$$\Pr(X_{H,t} \mid \hat{S}_{L,t}^A, s_{H,t}^B) = \Pr(X_{H,t} \mid \hat{S}_{H,t}^A, s_{L,t}^B) = \alpha$$

$$\Pr(X_{L,t} \mid \hat{S}_{L,t}^A, s_{H,t}^B) = \Pr(X_{L,t} \mid \hat{S}_{H,t}^A, s_{L,t}^B) = 1 - \alpha$$

Section A.3 of the Appendix presents additional updating rules for the case where the follower receives the same private signal as announced by the leader.

Analyst B makes an investment decision after updating his state probabilities and assessing whether the leader honestly revealed her signal. Assume that the follower receives the low signal and analyst A truthfully reveals the high signal. Then, there is an equilibrium in which analyst B ignores his private signal and herds on the leader when the following condition holds:
\[
\hat{\theta}_t^B (\hat{s}_{H,t}^A, \hat{s}_{L,t}^B, X_{L,t}) \Pr(X_{L,t} | \hat{s}_{H,t}^A, \hat{s}_{L,t}^B) + \hat{\theta}_t^B (\hat{s}_{H,t}^A, \hat{s}_{L,t}^B, X_{H,t}) \Pr(X_{H,t} | \hat{s}_{H,t}^A, \hat{s}_{L,t}^B) < \theta_{t-1}^B \quad (5)
\]

The left side of equation (5) is analyst B’s expected reputation when he truthfully announces his signal, while the right side is his expected reputation when he deviates. Therefore, the follower’s reputation changes only when he announces opposite the leader’s signal. When analyst B receives the same signal that analyst A reports, investors cannot distinguish between this case and that where analyst B herds on the leader.\(^{15}\) Therefore, the follower is only evaluated when he deviates from the leader, but analyst B will diverge from herding on A’s forecast only when it is a more attractive equilibrium strategy.

According to Graham (1999), when the leader’s equilibrium strategy is to announce truthfully, the follower’s incentive to honestly reveal his private information 1) increases in ability \((p)\), 2) decreases in informative signal correlation \((\rho)\), 3) decreases in initial reputation \((\theta_{t-1})\), and 4) increases (decreases) in the strength of prior information \((\alpha)\) when it is consistent (inconsistent) with his private information. In section A.4 of the Appendix, we solve for those parameter values where the follower tells the truth in equilibrium for a given period.

4. Simulations of the One-Analyst Model

A better understanding of the model’s parameter dependence is gained by simulating the one-analyst model in a multi-period framework. To discover how quickly and how accurately the analyst’s type can be identified, Monte Carlo simulations were

\(^{15}\) Graham (1999) demonstrates that when both analysts have the same private information, they always make the same investment recommendation.
conducted using *Crystal Ball* under a variety of parameter values. These results offer a valuable insight into how the abilities of investment professionals are evaluated.

To study the single analyst’s reputation throughout her career, the number of periods is set equal to 500. Assuming the average analyst makes approximately 20-25 investment recommendations per year, this implies that the analyst has at least a twenty-year career. The single-agent model is simulated for both skilled and unskilled analysts where the initial probability of being skilled ($\theta_0$) was 0.10, 0.30, and 0.50. The probability that skilled analysts receive the correct signal ($p$) was tested for accuracy rates of 0.55, 0.60, 0.65, and 0.70. Signals and states were randomly and independently drawn each period from a uniform distribution. Each simulation represents the results of 10,000 independent trials.

An assumption of market efficiency underlies the analysis, as the probability of both the high and low state was set equal to $\frac{1}{2}$ for all simulations. The states can be interpreted as whether a particular security outperforms (high state) or underperforms (low state) a market benchmark over the recommendation period. The analysts’ recommendations can therefore be viewed as predictions of whether the asset will return either positive or negative excess returns. Based on this definition, the 50 percent probability of each state seems most realistic. It can also be easily demonstrated, by solving for the truth-telling parameter ranges provided in section A.2 of the Appendix, that the analyst always tells the truth when $\alpha = \frac{1}{2}$.

Figures 3, 4, and 5 display the average reputation from Monte Carlo simulations of the multi-period, single-analyst model when skilled analysts initially comprise 50, 30, and 10 percent of the population ($\theta_0 = 0.50, 0.30, \text{and } 0.10$), respectively. The charts plot...
the reputation of skilled and unskilled analysts when skilled analysts have expected accuracy rates of 55, 60, and 65 percent. With equal state probabilities, the analyst tells the truth in equilibrium.

These figures allow us to visualize the evolution of the single agent’s reputation under different parameter assumptions. To interpret these graphs, one must compare the reputation trend of skilled and unskilled analysts assuming the same skilled-analyst accuracy rate \( p \). The ability to identify an individual agent’s type increases as the accuracy rate increases. In other words, an agent can be classified as skilled or unskilled more quickly with higher values of \( p \). The average reputation also depends on the number of skilled agents in the underlying population. When \( \theta_0 \) is low, additional time is needed to identify talented analysts because a given agent is more likely to be unskilled. Therefore, a skilled analyst requires a greater number of observations to prove her ability.

Unfortunately, identifying skilled analysts is not as easy as Figures 3-5 may indicate. The figures display only the average reputation of the agents. However, individual reputations vary considerably over time for both skilled and unskilled agents. This volatility may lead us to the wrong conclusion about an analyst’s ability. Beckers (1997) demonstrates this fact by simulating the investment decisions of fund managers. Using a variety of portfolio formation techniques, the study concludes that almost any manager can achieve above average performance after only five years. In fact, a number of skilled managers may be forced out of the industry by bad luck while some unskilled managers prosper. Therefore, we want to understand how reputational variability influences our capacity to determine an analyst’s skill at different periods in time.
Table 1 and Table 2 display descriptive statistics from simulations of the multi-period, single-analyst model for both a skilled and unskilled analyst. For these simulations, skilled analysts comprised 30 percent of the population ($\theta_0 = 0.30$) and received the correct signal 60 percent ($p = 0.60$) of the time. The probability of the high and low state was set equal to $\frac{1}{2}$; therefore, the agent was known to tell the truth in equilibrium. These statistics expose the potential reputation variability across the trials. The fourth and fifth columns of these tables list the percentage of trials where the reputation was below 5 percent and above 95 percent for a given time period. The confidence intervals represent a range where 95 percent of the observed reputations land for the given period.

The variability of the skilled analyst’s reputation in Table 1 reaches its peak around period 200. With a large standard deviation and a wide confidence interval, we encounter considerable noise in the reputation of the skilled analyst. In fact, between periods 90 and 175 we find that over five percent of the trials reported the skilled analyst’s reputation to be below 0.05. Investors would incorrectly interpret that the analysts from these outlying observations are at least 95 percent likely to be unskilled. Therefore, a reasonable conclusion is that these analysts might be fired if their reputation drops this low. By comparison, we find much less variability in the reputation of the unskilled analyst in Table 2. This simulation generated a smaller standard deviation, a narrower confidence interval, and no trials where the analyst’s ability was misclassified.

Although not directly reported in this paper, we discover that skilled analysts have greater reputational variability than unskilled analysts for values of $\theta_0$ less than 0.50. However, this result is not surprising. When skilled analysts comprise a minority of the
initial population, agents believe that a given analyst is more likely to be unskilled. Figures 3-5 also demonstrate that skilled analysts require more observations to reveal their type for lower values of $\theta_0$. As a result, a skilled analyst is more likely to be incorrectly identified than is an unskilled analyst.

A primary goal of this paper is to determine the number of observations needed to identify an analyst’s type with reasonable certainty. We have already determined that the answer is likely to depend heavily on the underlying parameter assumptions. Table 3 reveals the minimum number of periods for a variety of scenarios before the 95 percent confidence intervals of a skilled and unskilled analyst diverge. Assuming that 10 percent of the initial population is skilled and skilled analysts are accurate 55 percent of the time, almost 500 observations are required to determine a given agent’s type. However, when skilled analysts have an accuracy rate of 70 percent, we find the skill levels can be distinguished after only about 80 observations.

Table 3 demonstrates that determining an analyst’s ability with a reasonable level of assurance may take a large number of observations. For an analyst releasing 20 recommendations per year, this process may take anywhere from several years to an entire career to determine the individual’s ability depending on the underlying parameter assumptions. Obviously, the frequency of each analyst’s recommendations will determine the amount of time really needed. Some analysts deliver quarterly forecasts for each stock that they follow, while newsletter editors might issue only 12 recommendations per year.

Ideally, we would like to know the true underlying parameter values of the model. The vast amount of empirical research investigating financial analysts and money
managers may provide some insight. To identify the accuracy rate \( (p) \) of skilled agents, we want to examine studies where individual analysts are tracked throughout a portion of their career, rather than pooled together. However, most empirical studies of individual analysts, such as O’Brien (1990), measure accuracy by the individual’s mean forecast error of earnings per share estimates. This definition does not coincide with the assumption in our model that analysts forecast the performance of the security over the upcoming period. Given that the average analyst does not seem to possess superior long-term forecasting ability, we estimate that the skilled-analyst accuracy rate \( (p) \) is perhaps somewhere in the range of 50 to 60 percent.

We also desire to discover the proportion of skilled analysts \( (\theta_0) \) in the underlying population. Our initial estimates are that this number is also rather low, possibly less than 30 percent. Survivorship of talented analysts may elevate this figure somewhat, but only if the abilities of individual analysts can be properly identified. Evidence from the mutual fund industry suggests that fewer than 35 percent of all general equity fund managers where able to outperform the S&P 500 Index over the past 25 years.\(^ \text{16} \) As previously noted, fund managers do operate in a slightly different environment than financial analysts, but their underlying duties are closely related. In fact, the long-term success of both groups is based on their fundamental ability to identify the best investments.

\(^ {16} \) See "Be Not The First... Nor Yet The Last," a speech presented by John C. Bogle, Chairman of the Board of The Vanguard Group of Investment Companies at the 1996 AIMR Annual Conference in Atlanta, Georgia on May 8, 1996. Text of this address was originally downloaded from http://www.vanguard.com.
5. Simulations of the Two-Analyst Model

5.1 The Announcement Order Remains Fixed

The one-analyst simulations of section 4 provided valuable insight into the model’s parameter dependence. The results also measured the number of observations necessary to determine an analyst’s type. Unfortunately, most analysts are not the sole forecasters of the securities they follow. In fact, some blue-chip companies may be monitored by as many as 20 or more analysts. Individual recommendations in this setting are unlikely to be made completely independent of the forecasts of other analysts. Therefore, we now turn to simulating the multi-period two-agent model.

The number of periods is again set equal to 500 for each of the 10,000 independent trials. The two-analyst simulations include models with zero, one, and two skilled analysts. The model is also examined when the initial proportion of skilled analysts \((\theta_0)\) is set to 0.10, 0.30, and 0.50. The probability that skilled analysts receive the correct signal is tested for values of \(p\) equal to 0.55, 0.60, 0.65, and 0.70. The signals and states are randomly and independently drawn each period from a uniform distribution.

The previously noted market efficiency assumption continues to underlie our analysis, as the probability of both the high and low state was set to \(\frac{1}{2}\) for all simulations. Section A.4 of the Appendix solves for those parameter values where the follower ignores his private information and herds on the leader’s announcement. When \(\alpha = \frac{1}{2}\), the follower always herds in equilibrium. Therefore, the leader knows that she will be evaluated independently of analyst B. Her announcement decision becomes equal to the
one-analyst case, and she announces her signal truthfully in equilibrium as described in section A.2 of the Appendix.

Figure 6 and Figure 7 display the average reputation of the leader and the follower from Monte Carlo simulations of the two-agent model where skilled analysts comprise 30 percent of the population ($\theta_0 = 0.30$). The leader in Figure 6 is skilled, while the leader in Figure 7 is unskilled. These charts confirm our expectations that when the follower herds in equilibrium, his reputation remains constant. Agents only learn about the follower’s ability when he announces a signal opposite the leader.

When analysts herd in equilibrium, investors and financial institutions have an even more difficult time determining whether an individual is skillful. Herd behavior among analysts and money managers has been documented in several studies, including Grinblatt, Titman, and Wermers (1995), Welch (1996), Wermers (1999), and Graham (1999). Therefore, our two-analyst simulations may provide evidence of how difficult evaluating these individuals can be in a multi-analyst setting.

When the second analyst herds on the recommendation of the leader, investors do not receive additional information beyond what a single analyst would revealed. Analyst B’s private information is not disclosed, and only the leader’s signal is informative. When an unskilled analyst announces first, investors will receive accurate forecasts from the two analysts only half of the time on average. Therefore, society would benefit from skilled analysts announcing their recommendations before unskilled analysts, but unfortunately, all agents are unaware of their individual type.
5.2 The Analyst with the Highest Reputation Moves First

One drawback of the sequential model up to this point is that the role of leader and follower remained fixed throughout each trial. The consequence of this assumption is that agents can never infer any knowledge about the skill of the second analyst when herd behavior is likely. In addition, this assumption may distort the equilibrium strategies of the analysts in practice. Therefore, to make the simulations results more realistic, this section allows the analysts to switch their announcement order. This change requires us to devise a rule by which the analysts determine who assumes the role of leader and follower each period.

A simple rule is to let the analyst with the highest reputation at the start of each period move first. In the event that the analysts have identical prior period reputations, they will continue announcing in the same order as the previous period. This strategy allows skilled analysts to migrate towards the leadership role even if the individual begins the simulation as the second mover. As previously demonstrated, investors benefit from more accurate forecasts when a skilled analyst moves first as opposed to an unskilled analyst, especially when the follower has a tendency to herd. An unskilled analyst favors moving second to hide his lack of ability from the public, while a talented analyst prefers to reveal her superior skill by leading. However, analysts know only their reputation and not their individual type, but given two analysts, the one with the highest reputation is also more likely to have the most talent.

Figure 8 displays the average reputation from simulations of the multi-period, mixed-order, two-agent model with one skilled and one unskilled analyst. The individual with the highest reputation at the start of each period moves first. Smart analysts
comprise 30 percent of the population and receive signals with an 80 percent rate of correlation. The probability of the high state remains at 50 percent for this analysis. Analysts may switch the order several times, especially during the early periods of the simulation, so the public receives some information about the ability of both analysts even though the follower herds in equilibrium. On average, skilled analysts announcing truthfully will provide more accurate forecasts than unskilled analysts, and skilled analysts migrate toward the leadership role over time.

The average reputation from simulations of the mixed-order, two-analyst model is presented in Figure 9 for the cases where both analysts have the same type. Initial parameter values are the same as in Figure 8, and the analyst with the highest reputation each period continues to moves first. Investors learn some information about both agents over time; however, two unskilled analysts are identified more quickly than two skilled analysts. When both analysts are skilled, one analyst eventually assumes the leadership role on a permanent basis, and the public stops learning about the other analyst. However, two unskilled analysts will continually switch the order of announcement, since neither is likely to emerge as the “smarter” analyst.

Figures 8 and 9 reveal the benefits from the mixed-order announcement strategy. The likelihood of identifying a skilled analyst increases when the analyst with the highest reputation moves first. With a fixed-order of announcement, the probability of selecting a skilled analyst to report first equals the initial proportion of skilled analysts in the population ($\theta_0$). However, skilled analysts emerge as leaders in the limit of the mixed-
order model, where the probability of having a skilled leader increases to $\theta(2-\theta)$.\textsuperscript{17} Simulation results confirmed this finding when the analyst selection was randomized.

Perhaps more important than identifying the smartest analysts, investors benefit from more accurate forecasts with the mixed-order model. The initial probability of an accurate forecast from the leader is only $p\theta + \frac{1}{2}(1-\theta)$ when the announcement order remains fixed. However, the initial expected rate of accuracy increases to $p\theta(2-\theta) + \frac{1}{2}(1-\theta)(2-\theta)$ when the most skilled analyst moves first.

6. Parameter Sensitivity and Model Limitations

In his Proposition 2, Graham (1999) notes that the leader’s (and single analyst’s) incentive to truthfully announce her private information increases in ability ($p$), skilled analyst signal correlation ($\rho$), initial reputation ($\theta$), and the strength of the state probability information ($\alpha$) when it is consistent with her private information. Likewise, Graham notes that when the leader announces truthfully, the follower’s incentive to announce truthfully (avoid herding) decreases in skilled analyst signal correlation and initial reputation, and increases in ability and strength of the state probability information when it is consistent with his private information. The possible combinations of these parameter values are infinite, so an appropriate strategy for examining parameter sensitivity might be to first determine a realistic range for each variable. Unfortunately, little empirical evidence is available to provide guidance.

\textsuperscript{17} The probability of selecting two skilled analysts is $\theta^2$, while the probability that just one of the two chosen analysts is skilled equals $2\theta(1-\theta)$. Therefore, the model will contain at least one skilled analyst with probability $\theta + 2\theta(1-\theta) = \theta(2-\theta)$. 


The accuracy rate and signal correlation of smart analysts appears related. If skilled analysts report highly accurate forecasts, than skilled analysts must also receive the correct signal with a relatively high degree of correlation. Two skilled analysts that rarely agree will not seem as skilled relative to one another. Scharfstein and Stein (1990) assumed a correlation of one; however, a more reasonable estimate is probably between 50 and 90 percent, \( \rho \in (0.50, 0.90) \).

Empirical evidence has demonstrated that the average analyst does not possess superior forecasting ability. This result implies that the population contains few skilled analysts and/or skilled analysts have low forecast accuracy rates. The likely scenario is a little of both. Analysts with high rates of accuracy should be easier to identify; however, these individuals might also leave the profession and invest on their own. An accuracy rate close to 55 percent \( (p=0.55) \) is likely to be considered very good for most analysts. Evidence from the mutual fund literature suggest that less than 30 percent of fund managers can outperform the S&P 500 on a regular basis. If this figure applies to analysts as well, we might find about 10 to 30 percent of the analyst population is skilled, \( \theta_0 \in (0.10, 0.30) \).

The probability of the high state \( (\alpha) \) is set equal to \( \frac{1}{2} \) for the simulations. As noted in sections A.2 and A.4 of the appendix, this assumption implies that the single analyst and leader always announce truthfully, while the follower always herds in equilibrium. This conjecture obviously has a dramatic impact on the behavior of the analysts. If \( p=0.60 \) and \( \theta_0=0.30 \), the single analyst will not deviate from telling the truth unless \( \alpha<0.47 \) when the high signal was received or \( \alpha>0.53 \) when the low signal was received. Likewise, with these same parameter values and \( \rho=0.80 \), the follower will herd
regardless of $\alpha$ or the leaders signal. Simulations confirmed these results. The dynamic process of this model is the reputation of the individual analyst. As an analyst’s reputation changes over time, so too does his incentive to deviate from a truth-telling equilibrium.

One limitation of the model is that analysts lack the ability to learn their true identity. Reputation provides an understanding of an agent’s ability, but his or her skill can never be determined with certainty. This shortcoming may distort the incentives of analysts with very high or very low reputations. For example, assume the reputation of an analyst falls to 0.05, implying only a five percent chance that the individual is skilled and a strong likelihood of being fired. The model does not allow the analyst to conclude that he is unskilled. Instead, the model suggests that the analyst will herd on the leader in equilibrium to avoid revealing his type. However, the analyst may actually benefit from the leadership role because it allows a “last ditch” opportunity to boost his reputation. Likewise, a high reputation analyst, who believes she is skilled, may actually prefer herding to protect her reputation and future wages.\(^{18}\)

Another drawback of this model is that it does not consider the time value of money. Analyst wages are a linear function of reputation, and the model does not include the discounted value of future reputation in the decision process. A large discount rate may induce agents to follow more short-term decision strategies.

Finally, the announcement process is modeled for only two analysts, but often, several analysts follow a particular company. This model can be extended to the three-

\(^{18}\) Hong, Kubik, and Solomon (1998) identify evidence inconsistent with this proposal. The authors discover that older analysts, who presumably have higher reputational capital, are less likely to herd and more likely to release earnings forecasts before younger analysts.
analyst case, but the dynamics become very complicated. The number of possible outcomes makes proofs unmanageable, yet such a model would provide valuable insight into the herding decision in a larger group. For instance, one strategy may be for the second analyst to deviate from the leader to draw the third analyst into herding on his forecast. Likewise, the third analyst, faced with a strong signal that contradicts the observed announcements of the first two analysts, may abandon this information to remain part of the herd. The herding incentive is likely to be stronger under some possible scenarios of the three-analyst case and weaker under others.

7. Summary and Conclusions

The simulation results demonstrate the difficulty of evaluating analysts based solely on the accuracy of their recommendations. A large number of observations are needed to identify an analyst’s skill with reasonable certainty. However, in practice, forecast accuracy is not the only way analysts add value. Financial institutions also judge analysts based on their client service record and their volume of business generated.

Every October, *Institutional Investor* publishes its All-America Research Team using the results of a peer-based survey where financial institutions rate the top analysts across six different areas. These categories include stock picking, written reports, earnings estimates, industry knowledge, accessibility and responsiveness, and useful and timely calls. Since the standings reflect the investment community’s perception of an analyst’s skill, members of the All-America team should earn higher salaries than non-members earn. The rankings also appear to identify those analysts with the best forecasting ability, since Brown and Chen (1991) and Stickel (1992) discover that All-
America team members generate more accurate earnings forecasts than non-members. Therefore, while customer service is an important aspect of the analyst’s job, forecast accuracy must ultimately come first and foremost. After all, without accurate recommendations, an analyst is unlikely to have many customers to service.

The evaluation process is complicated even further by the fact that senior analysts are often supported by several junior-level analysts. Within this context, the performance of a senior analyst is not unlike that of a fund manager; the individual is likely to be only as good as the analysts working beneath her. The appraisal of a senior analyst effectively becomes a judgment of the entire research team.

Results from the two-analyst simulations suggest some interesting and testable applications. First, when private or institutional investors choose among recommendations from more than one analyst, they should more heavily weight the signal released by the last analyst. Followers observe the forecast of the leader and all previous analysts, so these signals will be incorporated into their own announcements. Therefore, when herd behavior is likely, the recommendation of the last analyst should be at least as accurate as the leader’s forecast. In practice, the most recent forecast also incorporates the most recent news and information about the firm.

The second implication of the results derives from evidence presented in section 5.2. When a skilled analyst assumes the leadership role, she has a greater opportunity to demonstrate her true talent to the market. Likewise, an unskilled analyst has an incentive to hide his ability from the market by following. Evaluating the ability of an individual analyst can be extremely difficult, but conditioning upon the announcement order may
increase the probability of identifying a skilled individual. Thus, financial institutions should focus their hiring efforts on analysts who are leaders in their respective sectors.

This hypothesis is testable with the proper data. The All-America team represents those individuals that the analyst community believes are the most skilled in their field. If the proposed conjecture was true, then these individuals should release their forecasts before other analysts in the sector. The research question is whether All-Americans are more likely to serve as the leader both prior to and after being named to the team. The other side of this proposal is to examine if other analysts herd on the recommendations of the All-America team members.

Another research extension involves the persistence of an analyst’s reputation across time. The model presented in this paper derives the perceived ability of an analyst from his or her past announcement history. Because of the binomial nature of the recommendations, an analyst’s reputation can be computed from the total number of correct and incorrect forecasts. However, the question arises whether the assessment of an individual’s skill is in fact myopic. The analyst community may actually focus more on an individual’s recent track to judge ability than on his or her career performance. This possibility warrants further examination.

Finally, the literature has not identified the model’s true underlying parameter values, mainly the accuracy rate of skilled analysts and the proportion of skilled analysts in the population. Most analyst performance studies focus on the mean forecast error of earnings estimates rather than the basic probability of selecting the best investments. In addition, most analyst studies generally focus on the aggregate performance of the group, rather than on specific individuals. To solve for the accuracy parameter in the model, one
must condition the evaluation on analysts believed to be skilled, such as the All-America team. While analysts have a wide range of skill levels, the more interesting task would be to identify those individuals with enough skill to distinguish themselves from the rest of the herd.
Appendix A.1

The Single Analyst’s Equilibrium Strategy in a Multi-Period Model

Financial analysts and investment managers have finite careers. Therefore, let the career of the single analyst in the model last for T periods. The optimal equilibrium strategy is for the analyst to maximize her expected reputation each period. The uniqueness of this equilibrium is shown through backward induction.

Assume the analyst receives a high signal in period T. The analyst updates her state probability beliefs and determines whether to truthfully reveal her private information. She tells the truth when the expected reputation from doing so exceeds that from deviating:

\[ E_T[\hat{\theta}_T (\text{truth} | s_{H,T}, \theta_{T-1}, p, \alpha)] > E_T[\hat{\theta}_T (\text{deviate} | s_{H,T}, \theta_{T-1}, p, \alpha)] \]

Let the parameter values be such that the optimal strategy for the analyst is to tell the truth in each period. Therefore, the following inequality must hold in equilibrium:

\[ E_T[\hat{\theta}_T(\hat{s}_{i,T} | s_{i,T}, \theta_{T-1}, p, \alpha)] > E_T[\hat{\theta}_T(\hat{s}_{j,T} | s_{i,T}, \theta_{T-1}, p, \alpha)] \]

In this equation, \( s_{i,T} \) is the analyst’s private signal, while \( s_{j,T} \) is a signal opposite the analyst’s private information. When \( s_{i,T} \) is the high signal, \( s_{j,T} \) will be the low signal. The analyst knows that deviating in period T will not maximize expected reputation, so the analyst truthfully reveals her signal.

Now examine the analyst’s decision at time T-1. We want to demonstrate that the sub-optimal, deviation strategy does not enhance current or future reputation. Since truth telling is the optimal strategy each period, the following relationship holds.

\[ E_{T-1}[\hat{\theta}_{T-1}(\hat{s}_{i,T-1} | s_{i,T-1}, \theta_{T-2}, p, \alpha)] > E_{T-1}[\hat{\theta}_{T-1}(\hat{s}_{j,T-1} | s_{i,T-1}, \theta_{T-2}, p, \alpha)] \]

Deviation does not increase the expected reputation of period T-1. Could this strategy enhance the analyst’s expected reputation in period T?

Let \( \gamma_{T-1} = E_{T-1}[\hat{\theta}_{T-1}(\hat{s}_{i,T-1} | s_{i,T-1}, \theta_{T-2}, p, \alpha) \) represent the period T-1 expected reputation when the analyst tells the truth, and \( \lambda_{T-1} = E_{T-1}[\hat{\theta}_{T-1}(\hat{s}_{j,T-1} | s_{i,T-1}, \theta_{T-2}, p, \alpha) \) be the expected reputation when the analyst deviates. We know that \( \gamma_{T-1} > \lambda_{T-1} \). Since the analyst truthfully reveals her private signal in period T, deviation in period T-1 will only be beneficial if it provides a higher ending reputation in period T.

\[ E_T[\hat{\theta}_T(\hat{s}_{i,T} | s_{i,T}, \lambda_{T-1}, p, \alpha)] > E_T[\hat{\theta}_T(\hat{s}_{j,T} | s_{i,T}, \gamma_{T-1}, p, \alpha)] \]
\[
\hat{\theta}_T (\hat{s}_{H,T}, X_{H,T}) \Pr(X_{H,T} | s_{H,T}) + \hat{\theta}_T (\hat{s}_{L,T}, X_{L,T}) \Pr(X_{L,T} | s_{H,T}) > \\
\Rightarrow \hat{\theta}_T (\hat{s}_{L,T}, X_{H,T}) \Pr(X_{H,T} | s_{H,T}) + \hat{\theta}_T (\hat{s}_{L,T}, X_{L,T}) \Pr(X_{L,T} | s_{H,T})
\]

For the deviation strategy to enhance the analyst’s reputation in a future period \(t+1\), it must be the case that the following inequalities hold for both correct and incorrect signals.

\[
\hat{\theta}_{t+1} (\text{correct} | \lambda_i) > \hat{\theta}_{t+1} (\text{correct} | \gamma_i)
\]

\[
\Rightarrow \lambda_{t+1} \frac{p}{\lambda_{t+1} + (1 - \lambda_{t+1})} > \gamma_{t+1} \frac{p}{\gamma_{t+1} + (1 - \gamma_{t+1})}
\]

\[
\Rightarrow \lambda_{t+1} \gamma_{t+1} p^2 + \lambda_{t+1} p (1 - \gamma_{t+1}) \frac{1}{2} > \lambda_{t+1} \gamma_{t+1} p^2 + \gamma_{t+1} p (1 - \lambda_{t+1}) \frac{1}{2}
\]

\[
\Rightarrow \lambda_{t+1} (1 - \gamma_{t+1}) > \gamma_{t+1} (1 - \lambda_{t+1})
\]

\[
\Rightarrow \lambda_{t+1} > \gamma_{t+1} \text{ which violates the definition that } \lambda_{t+1} < \gamma_{t+1}
\]

\[
\hat{\theta}_{t+1} (\text{incorrect} | \lambda_i) > \hat{\theta}_{t+1} (\text{incorrect} | \gamma_i)
\]

\[
\Rightarrow \lambda_{t+1} (1 - p) \frac{1}{\lambda_{t+1} (1 - p) + (1 - \lambda_{t+1})} > \gamma_{t+1} (1 - p) \frac{1}{\gamma_{t+1} (1 - p) + (1 - \gamma_{t+1})}
\]

\[
\Rightarrow \lambda_{t+1} (1 - p) (1 - \gamma_{t+1}) \frac{1}{2} > \gamma_{t+1} (1 - p) (1 - \lambda_{t+1}) \frac{1}{2}
\]

\[
\Rightarrow \lambda_{t+1} > \gamma_{t+1} \text{ which violates the definition that } \lambda_{t+1} < \gamma_{t+1}
\]

This result shows that deviating from the reputation maximizing strategy in one period cannot increase the analyst’s reputation in the next period. The analyst’s reputation for all future periods is built upon the reputation from previous periods. Therefore, deviating from the reputation maximizing strategy in any given period will be a sub-optimal equilibrium strategy. Analysts need only focus on selecting the announcement recommendation that maximizes the expected reputation for each period independent of all future periods.


Appendix A.2

Parameters Values Where Single Analyst Truthfully Reveals Signal

Assume that the analyst receives the high signal for period $t$. We know that the analyst will truthfully reveal this signal as long as her expected reputation from doing so exceeds that from deviating and equation (4) holds:

\[
\hat{\theta}_t (\hat{s}_{H,t}, X_{H,t}) \Pr(X_{H,t} | s_{H,t}) + \hat{\theta}_t (\hat{s}_{H,t}, X_{L,t}) \Pr(X_{L,t} | s_{H,t}) > \hat{\theta}_t (\hat{s}_{L,t}, X_{H,t}) \Pr(X_{H,t} | s_{H,t}) + \hat{\theta}_t (\hat{s}_{L,t}, X_{L,t}) \Pr(X_{L,t} | s_{H,t})
\]

\[
\Rightarrow [\hat{\theta}_t (\hat{s}_{H,t}, X_{H,t}) - \hat{\theta}_t (\hat{s}_{L,t}, X_{H,t})] \Pr(X_{H,t} | s_{H,t}) > [\hat{\theta}_t (\hat{s}_{L,t}, X_{L,t}) - \hat{\theta}_t (\hat{s}_{H,t}, X_{L,t})] \Pr(X_{L,t} | s_{H,t})
\]

We know that $[\hat{\theta}_t (\hat{s}_{H,t}, X_{H,t}) - \hat{\theta}_t (\hat{s}_{L,t}, X_{H,t})] = [\hat{\theta}_t (\hat{s}_{L,t}, X_{L,t}) - \hat{\theta}_t (\hat{s}_{H,t}, X_{L,t})]$ and is greater than 0, because the reputation from being correct exceeds the reputation from being wrong. Therefore, to determine when the analyst will announce truthfully, we only need find the parameter values for which $\Pr(X_{H,t} | s_{H,t}) > \Pr(X_{L,t} | s_{H,t})$.

\[
\Pr(X_{H,t} | s_{H,t}) > \Pr(X_{L,t} | s_{H,t})
\]

\[
\Rightarrow \left[ \theta_{t-1} p + (1 - \theta_{t-1}) \frac{1}{2} \right] \alpha > \left[ \theta_{t-1} (1 - p) + (1 - \theta_{t-1}) \frac{1}{2} \right] (1 - \alpha)
\]

\[
\Rightarrow \left[ \theta_{t-1} p + (1 - \theta_{t-1}) \frac{1}{2} \right] \alpha > \left[ \theta_{t-1} (1 - p) + (1 - \theta_{t-1}) \frac{1}{2} \right] (1 - \alpha)
\]

\[
\Rightarrow \alpha > \theta_{t-1} (1 - p) + (1 - \theta_{t-1}) \frac{1}{2}
\]

By definition, we know that $\alpha \in (0,1)$, $\theta_{t-1} \in (0,1)$, and $p \in (\frac{1}{2},1]$. Therefore, the analyst will tell the truth when the following relationships hold.

\[
\alpha > \theta_{t-1} (\frac{1}{2} - p) + \frac{1}{2}
\]

\[
\theta_{t-1} > \frac{\alpha - \frac{1}{2}}{\frac{1}{2} - p}
\]

\[
p > \frac{\alpha (\theta_{t-1} + 1) - \alpha}{\theta_{t-1}}
\]

If the analyst receives the low signal for period $t$, she will truthfully reveal this signal as long as her expected reputation from doing so exceeds that from deviating and equation \(4'\) holds:
\[
\hat{\theta}_t(\hat{s}_{L,t}, X_{L,t}) \Pr(X_{L,t} \mid s_{L,t}) + \hat{\theta}_t(\hat{s}_{H,t}, X_{H,t}) \Pr(X_{H,t} \mid s_{L,t}) > \\
\hat{\theta}_t(\hat{s}_{H,t}, X_{L,t}) \Pr(X_{L,t} \mid s_{L,t}) + \hat{\theta}_t(\hat{s}_{H,t}, X_{H,t}) \Pr(X_{H,t} \mid s_{L,t})
\]  

(4')

Solving equation (4') for the parameter values where the analyst always announces truthfully involves algebraic steps very similar to those for equation (4). Doing so reveals that when given a low signal, the analyst will tell the truth as long as the following parameter values apply.

\[
\alpha < \theta_{t-1}(p - \frac{1}{2}) + \frac{1}{2}
\]

\[
\theta_{t-1} > \frac{\alpha - \frac{1}{2}}{p - \frac{1}{2}}
\]

\[
p > \frac{\alpha - \frac{1}{2}}{\theta_{t-1}} + \frac{1}{2}
\]
Appendix A.3

Two-Analyst Updating Equations for Reputation and State Probabilities

When both analyst A and B are known to truthfully reveal their private information in equilibrium, their end of the period reputation is determined relative to one another based on the following equations. We present these updating formulas from the standpoint of analyst B’s reputation.

\[
\hat{\theta}_t^B \left( \hat{s}_{L,t}^A, \hat{s}_{H,t}^B, X_{H,t} \right) = \hat{\theta}_t^B \left( \hat{s}_{L,t}^A, \hat{s}_{H,t}^B, X_{L,t} \right) = \frac{\frac{1}{2} p \theta_{t-1} (1 - \theta_{t-1}) + p (1 - p) (1 - \rho) \theta_{t-1}^2}{\frac{1}{2} \theta_{t-1} (1 - \theta_{t-1}) + \frac{1}{4} (1 - \theta_{t-1})^2 + p (1 - p) (1 - \rho) \theta_{t-1}^2}
\]

\[
\hat{\theta}_t^B \left( \hat{s}_{H,t}^A, \hat{s}_{L,t}^B, X_{L,t} \right) = \hat{\theta}_t^B \left( \hat{s}_{L,t}^A, \hat{s}_{H,t}^B, X_{L,t} \right) = \frac{\frac{1}{2} (1 - p) \theta_{t-1} (1 - \theta_{t-1}) + p (1 - p) (1 - \rho) \theta_{t-1}^2}{\frac{1}{2} \theta_{t-1} (1 - \theta_{t-1}) + \frac{1}{4} (1 - \theta_{t-1})^2 + p (1 - p) (1 - \rho) \theta_{t-1}^2}
\]

\[
\hat{\theta}_t^B \left( \hat{s}_{H,t}^A, \hat{s}_{H,t}^B, X_{H,t} \right) = \hat{\theta}_t^B \left( \hat{s}_{L,t}^A, \hat{s}_{L,t}^B, X_{L,t} \right) = \frac{\frac{1}{2} p \theta_{t-1} (1 - \theta_{t-1}) + \theta_{t-1}^2 \left[ (1 - \rho) p^2 + \rho p \right]}{p \theta_{t-1} (1 - \theta_{t-1}) + \frac{1}{4} (1 - \theta_{t-1})^2 + \theta_{t-1}^2 \left[ (1 - \rho) p^2 + \rho (1 - p) \right]}
\]

\[
\hat{\theta}_t^B \left( \hat{s}_{L,t}^A, \hat{s}_{L,t}^B, X_{L,t} \right) = \hat{\theta}_t^B \left( \hat{s}_{H,t}^A, \hat{s}_{H,t}^B, X_{L,t} \right) = \frac{\frac{1}{2} (1 - p) \theta_{t-1} (1 - \theta_{t-1}) + \theta_{t-1}^2 \left[ (1 - \rho) p^2 + \rho (1 - p) \right]}{(1 - p) \theta_{t-1} (1 - \theta_{t-1}) + \frac{1}{4} (1 - \theta_{t-1})^2 + \theta_{t-1}^2 \left[ (1 - \rho) p^2 + \rho (1 - p) \right]}
\]

Analyst B updates his state probability beliefs each period conditional on A’s investment recommendation. These probabilities for the case where the two agents receive opposite information were presented in the text. The conditional state probabilities when the analysts receive the same signal are provided below. To shorten the length of these equations, we use simplifying algebraic definitions.

\[
\Omega = (1 - p) \theta_{t-1} (1 - \theta_{t-1}) + \frac{1}{4} (1 - \theta_{t-1})^2 + \theta_{t-1}^2 \left[ (1 - \rho) p^2 + \rho (1 - p) \right]
\]

\[
\Psi = p \theta_{t-1} (1 - \theta_{t-1}) + \frac{1}{4} (1 - \theta_{t-1})^2 + \theta_{t-1}^2 \left[ (1 - \rho) p^2 + \rho p \right]
\]

\[
\Pr(X_{H,t} | \hat{s}_{H,t}^A, \hat{s}_{H,t}^B) = \frac{\Psi \alpha}{\Psi \alpha + \Omega (1 - \alpha)}
\]

\[
\Pr(X_{L,t} | \hat{s}_{H,t}^A, \hat{s}_{H,t}^B) = \frac{\Omega (1 - \alpha)}{\Psi \alpha + \Omega (1 - \alpha)}
\]

\[
\Pr(X_{L,t} | \hat{s}_{L,t}^A, \hat{s}_{L,t}^B) = \frac{\Psi (1 - \alpha)}{\Psi (1 - \alpha) + \Omega \alpha}
\]
\[ \Pr(X_{H,H} \mid \hat{s}_{L,L}^A, s_{L,L}^B) = \frac{\Omega \alpha}{\Psi(1 - \alpha) + \Omega \alpha} \]

The following relationships are true:

\[ \Pr(X_{H,H} \mid \hat{s}_{H,H}^A, s_{H,H}^B) > \Pr(X_{H,H} \mid \hat{s}_{L,L}^A, s_{H,L}^B) \]

\[ \Pr(X_{L,L} \mid \hat{s}_{L,L}^A, s_{H,L}^B) > \Pr(X_{L,L} \mid \hat{s}_{H,H}^A, s_{L,L}^B) . \]
Appendix A.4

Parameter Values Where Follower Herds on Leader’s Recommendation

Assume that the leader truthfully announces the high signal and the follower receives the low signal. Analyst B will truthfully reveal this signal only when his expected reputation from doing so exceeds his expected reputation from herding on the leader’s announcement. Therefore, B will tell the truth only when equation (5) does not hold.

\[
\hat{\theta}_t^B (\hat{s}_{H,t}^A, \hat{s}_{L,t}^B, X_{L,t}) \Pr(X_{L,t} | \hat{s}_{H,t}^A, \hat{s}_{L,t}^B) + \hat{\theta}_t^B (\hat{s}_{H,t}^A, \hat{s}_{L,t}^B, X_{H,t}) \Pr(X_{H,t} | \hat{s}_{H,t}^A, \hat{s}_{L,t}^B) < \theta_{t-1} \tag{5}
\]

\[
\Rightarrow \left[ \frac{1}{2} p \theta_{t-1}^B (1 - \theta_{t-1}^B) + \frac{1}{4} (1 - \theta_{t-1}^B)^2 + p(1-p)(1-\rho)(\theta_{t-1}^B)^2 \right] (1 - \alpha) + \frac{1}{2} (1-p) \theta_{t-1}^B (1 - \theta_{t-1}^B) + \frac{1}{4} (1 - \theta_{t-1}^B)^2 + p(1-p)(1-\rho)(\theta_{t-1}^B)^2 \alpha < \theta_{t-1}^B
\]

\[
\Rightarrow \frac{p(1-p)(1-p)(\theta_{t-1}^B)^2 + \frac{1}{2} p \theta_{t-1}^B (1 - \theta_{t-1}^B) (1 - \alpha) + \frac{1}{2} (1-p) \theta_{t-1}^B (1 - \theta_{t-1}^B) \alpha}{\frac{1}{2} \theta_{t-1}^B (1 - \theta_{t-1}^B) + \frac{1}{4} (1 - \theta_{t-1}^B)^2 + p(1-p)(1-\rho)(\theta_{t-1}^B)^2} < \theta_{t-1}^B
\]

\[
\Rightarrow \alpha > \frac{1}{2} \theta_{t-1}^B (1 - \theta_{t-1}^B) + \frac{1}{4} (1 - \theta_{t-1}^B)^2 + p(1-p)(1-\rho)(\theta_{t-1}^B - 1) - \frac{1}{2} p(1-\theta_{t-1}^B) (\frac{1}{2} - p)(1-\theta_{t-1}^B)
\]

\[
\Rightarrow \alpha > \frac{1}{2} \theta_{t-1}^B + \frac{1}{4} (1 - \theta_{t-1}^B) - p(1-p)(1-\rho)(\theta_{t-1}^B - \frac{1}{2} p)
\]

Likewise, when the leader truthfully announces the low signal and the follower receives the high signal, analyst B will ignore his private information and herd on the leader when equation (5’) holds:

\[
\hat{\theta}_t^B (\hat{s}_{L,t}^A, \hat{s}_{H,t}^B, X_{H,t}) \Pr(X_{H,t} | \hat{s}_{L,t}^A, \hat{s}_{H,t}^B) + \hat{\theta}_t^B (\hat{s}_{L,t}^A, \hat{s}_{H,t}^B, X_{L,t}) \Pr(X_{L,t} | \hat{s}_{L,t}^A, \hat{s}_{H,t}^B) < \theta_{t-1} \tag{5’}
\]

\[
\Rightarrow \alpha < \frac{1}{2} \theta_{t-1}^B + \frac{1}{4} (1 - \theta_{t-1}^B) - p(1-p)(1-\rho)(\theta_{t-1}^B - \frac{1}{2} p)
\]

\[
\Rightarrow \alpha < \frac{1}{2} (p - \frac{1}{2})
\]
References


Brandenburger, Adam and Ben Polak, 1996, When managers cover their posteriors: Making the decisions the market wants to see, RAND Journal of Economics 27, 523-541.


Table 1  Reputation Variability of a Skilled Analyst

$\theta_0 = 0.30$, $p = 0.60$, $\alpha = 0.50$

<table>
<thead>
<tr>
<th>Period No.</th>
<th>Average Reputation</th>
<th>Standard Deviation</th>
<th>Percent &lt; 0.05</th>
<th>Percent &gt; 0.95</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.354</td>
<td>0.136</td>
<td>-</td>
<td>-</td>
<td>[0.13, 0.64]</td>
</tr>
<tr>
<td>20</td>
<td>0.403</td>
<td>0.191</td>
<td>-</td>
<td>-</td>
<td>[0.11, 0.83]</td>
</tr>
<tr>
<td>30</td>
<td>0.447</td>
<td>0.222</td>
<td>0.60</td>
<td>-</td>
<td>[0.09, 0.86]</td>
</tr>
<tr>
<td>40</td>
<td>0.488</td>
<td>0.239</td>
<td>0.63</td>
<td>0.60</td>
<td>[0.08, 0.92]</td>
</tr>
<tr>
<td>50</td>
<td>0.530</td>
<td>0.251</td>
<td>3.02</td>
<td>2.98</td>
<td>[0.04, 0.95]</td>
</tr>
<tr>
<td>60</td>
<td>0.567</td>
<td>0.266</td>
<td>2.46</td>
<td>2.98</td>
<td>[0.05, 0.96]</td>
</tr>
<tr>
<td>70</td>
<td>0.601</td>
<td>0.282</td>
<td>3.05</td>
<td>8.34</td>
<td>[0.04, 0.97]</td>
</tr>
<tr>
<td>80</td>
<td>0.634</td>
<td>0.289</td>
<td>4.21</td>
<td>10.13</td>
<td>[0.04, 0.97]</td>
</tr>
<tr>
<td>90</td>
<td>0.659</td>
<td>0.294</td>
<td>5.44</td>
<td>16.72</td>
<td>[0.03, 0.98]</td>
</tr>
<tr>
<td>100</td>
<td>0.681</td>
<td>0.296</td>
<td>4.84</td>
<td>18.58</td>
<td>[0.04, 1.00]</td>
</tr>
<tr>
<td>125</td>
<td>0.725</td>
<td>0.314</td>
<td>5.44</td>
<td>29.42</td>
<td>[0.03, 1.00]</td>
</tr>
<tr>
<td>150</td>
<td>0.756</td>
<td>0.327</td>
<td>4.17</td>
<td>49.12</td>
<td>[0.04, 1.00]</td>
</tr>
<tr>
<td>175</td>
<td>0.773</td>
<td>0.340</td>
<td>5.44</td>
<td>56.87</td>
<td>[0.05, 1.00]</td>
</tr>
<tr>
<td>200</td>
<td>0.784</td>
<td>0.341</td>
<td>0.60</td>
<td>62.83</td>
<td>[0.05, 1.00]</td>
</tr>
<tr>
<td>225</td>
<td>0.820</td>
<td>0.313</td>
<td>1.19</td>
<td>71.21</td>
<td>[0.06, 1.00]</td>
</tr>
<tr>
<td>250</td>
<td>0.858</td>
<td>0.274</td>
<td>-</td>
<td>74.86</td>
<td>[0.13, 1.00]</td>
</tr>
<tr>
<td>300</td>
<td>0.922</td>
<td>0.202</td>
<td>-</td>
<td>84.43</td>
<td>[0.22, 1.00]</td>
</tr>
<tr>
<td>350</td>
<td>0.973</td>
<td>0.091</td>
<td>-</td>
<td>91.58</td>
<td>[0.63, 1.00]</td>
</tr>
<tr>
<td>400</td>
<td>0.998</td>
<td>0.004</td>
<td>-</td>
<td>100.00</td>
<td>[0.99, 1.00]</td>
</tr>
<tr>
<td>450</td>
<td>1.000</td>
<td>0.001</td>
<td>-</td>
<td>100.00</td>
<td>[1.00, 1.00]</td>
</tr>
<tr>
<td>500</td>
<td>1.000</td>
<td>0.000</td>
<td>-</td>
<td>100.00</td>
<td>[1.00, 1.00]</td>
</tr>
</tbody>
</table>

The table displays descriptive statistics for the reputation of a skilled analyst from simulations of the multi-period, single-analyst model. Skilled analysts comprised 30 percent of the initial population and received the correct signal with probability $p$ equal to 60 percent. The probability of the high and low state was set equal to $\frac{1}{2}$; therefore, the agent was known to tell the truth in equilibrium. The statistics were compiled from the results of 10,000 independent trials. The average reputation denotes the probability that the analyst is skilled given her performance history. The fourth and fifth columns provide the percentage of trials where the reputation is below 5 percent and above 95 percent for the given time period. Finally, the confidence interval represents a range where 95 percent of the observed reputations fall for the given period.
Table 2  Reputation Variability of an Unskilled Analyst
\(\theta_0 = 0.30, \ p = 0.60, \ \alpha = 0.50\)

<table>
<thead>
<tr>
<th>Period No.</th>
<th>Average Reputation</th>
<th>Standard Deviation</th>
<th>Percent &lt; 0.05</th>
<th>Percent &gt; 0.95</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.277</td>
<td>0.109</td>
<td>0.60</td>
<td>-</td>
<td>[0.13, 0.54]</td>
</tr>
<tr>
<td>20</td>
<td>0.254</td>
<td>0.133</td>
<td>-</td>
<td>-</td>
<td>[0.05, 0.59]</td>
</tr>
<tr>
<td>30</td>
<td>0.233</td>
<td>0.153</td>
<td>4.80</td>
<td>-</td>
<td>[0.04, 0.64]</td>
</tr>
<tr>
<td>40</td>
<td>0.215</td>
<td>0.169</td>
<td>6.00</td>
<td>-</td>
<td>[0.04, 0.68]</td>
</tr>
<tr>
<td>50</td>
<td>0.200</td>
<td>0.187</td>
<td>18.61</td>
<td>-</td>
<td>[0.02, 0.73]</td>
</tr>
<tr>
<td>60</td>
<td>0.184</td>
<td>0.193</td>
<td>19.21</td>
<td>-</td>
<td>[0.02, 0.76]</td>
</tr>
<tr>
<td>70</td>
<td>0.172</td>
<td>0.200</td>
<td>29.40</td>
<td>-</td>
<td>[0.01, 0.80]</td>
</tr>
<tr>
<td>80</td>
<td>0.161</td>
<td>0.206</td>
<td>33.59</td>
<td>-</td>
<td>[0.01, 0.83]</td>
</tr>
<tr>
<td>90</td>
<td>0.152</td>
<td>0.212</td>
<td>48.52</td>
<td>-</td>
<td>[0.01, 0.80]</td>
</tr>
<tr>
<td>100</td>
<td>0.144</td>
<td>0.213</td>
<td>49.09</td>
<td>-</td>
<td>[0.01, 0.83]</td>
</tr>
<tr>
<td>125</td>
<td>0.130</td>
<td>0.212</td>
<td>60.48</td>
<td>-</td>
<td>[0.00, 0.84]</td>
</tr>
<tr>
<td>150</td>
<td>0.117</td>
<td>0.196</td>
<td>61.68</td>
<td>-</td>
<td>[0.00, 0.72]</td>
</tr>
<tr>
<td>175</td>
<td>0.102</td>
<td>0.171</td>
<td>66.47</td>
<td>-</td>
<td>[0.00, 0.56]</td>
</tr>
<tr>
<td>200</td>
<td>0.091</td>
<td>0.158</td>
<td>67.08</td>
<td>-</td>
<td>[0.00, 0.48]</td>
</tr>
<tr>
<td>225</td>
<td>0.086</td>
<td>0.163</td>
<td>70.68</td>
<td>-</td>
<td>[0.00, 0.61]</td>
</tr>
<tr>
<td>250</td>
<td>0.080</td>
<td>0.174</td>
<td>73.66</td>
<td>-</td>
<td>[0.00, 0.63]</td>
</tr>
<tr>
<td>300</td>
<td>0.057</td>
<td>0.140</td>
<td>81.43</td>
<td>-</td>
<td>[0.00, 0.58]</td>
</tr>
<tr>
<td>350</td>
<td>0.032</td>
<td>0.081</td>
<td>84.42</td>
<td>-</td>
<td>[0.00, 0.25]</td>
</tr>
<tr>
<td>400</td>
<td>0.016</td>
<td>0.039</td>
<td>88.03</td>
<td>-</td>
<td>[0.00, 0.15]</td>
</tr>
<tr>
<td>450</td>
<td>0.008</td>
<td>0.020</td>
<td>94.01</td>
<td>-</td>
<td>[0.00, 0.09]</td>
</tr>
<tr>
<td>500</td>
<td>0.005</td>
<td>0.018</td>
<td>96.40</td>
<td>-</td>
<td>[0.00, 0.05]</td>
</tr>
</tbody>
</table>

The table displays descriptive statistics for the reputation of an unskilled analyst from simulations of the multi-period, single-analyst model. Skilled analysts comprised 30 percent of the initial population and received the correct signal with probability \(p\) equal to 60 percent. The probability of the high and low state was set equal to \(\frac{1}{2}\); therefore, the agent was known to tell the truth in equilibrium. The statistics were compiled from the results of 10,000 independent trials. The average reputation denotes the probability that the analyst is skilled given her performance history. The fourth and fifth columns provide the percentage of trials where the reputation is below 5 percent and above 95 percent for the given time period. Finally, the confidence interval represents a range where 95 percent of the observed reputations fall for the given period.
Table 3  Confidence Interval Divergence of Skilled and Unskilled Analysts

<table>
<thead>
<tr>
<th></th>
<th>Skilled Accuracy</th>
<th>Period No.</th>
<th>Skilled Analyst</th>
<th></th>
<th>Unskilled Analyst</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Percent Mean</td>
<td>&gt; 0.95</td>
<td>95% C.I.</td>
<td>Percent Mean</td>
</tr>
<tr>
<td><strong>Panel A: θ₀ = 0.50</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p = 0.55</td>
<td>400</td>
<td>0.873</td>
<td>25.72</td>
<td>[0.65,0.98]</td>
<td>0.190</td>
<td>4.22</td>
</tr>
<tr>
<td>p = 0.60</td>
<td>350</td>
<td>0.987</td>
<td>93.39</td>
<td>[0.80,1.00]</td>
<td>0.061</td>
<td>78.43</td>
</tr>
<tr>
<td>p = 0.65</td>
<td>80</td>
<td>0.941</td>
<td>69.43</td>
<td>[0.64,1.00]</td>
<td>0.143</td>
<td>59.82</td>
</tr>
<tr>
<td>p = 0.70</td>
<td>60</td>
<td>0.938</td>
<td>77.27</td>
<td>[0.67,1.00]</td>
<td>0.104</td>
<td>71.88</td>
</tr>
<tr>
<td><strong>Panel B: θ₀ = 0.30</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p = 0.55</td>
<td>400</td>
<td>0.693</td>
<td>27.53</td>
<td>[0.44,0.96]</td>
<td>0.102</td>
<td>35.89</td>
</tr>
<tr>
<td>p = 0.60</td>
<td>350</td>
<td>0.973</td>
<td>91.58</td>
<td>[0.63,1.00]</td>
<td>0.032</td>
<td>84.42</td>
</tr>
<tr>
<td>p = 0.65</td>
<td>175</td>
<td>0.937</td>
<td>79.05</td>
<td>[0.47,1.00]</td>
<td>0.015</td>
<td>90.42</td>
</tr>
<tr>
<td>p = 0.70</td>
<td>70</td>
<td>0.920</td>
<td>74.26</td>
<td>[0.46,1.00]</td>
<td>0.047</td>
<td>80.91</td>
</tr>
<tr>
<td><strong>Panel C: θ₀ = 0.10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p = 0.55</td>
<td>500</td>
<td>0.639</td>
<td>0.01</td>
<td>[0.34,0.89]</td>
<td>0.060</td>
<td>65.31</td>
</tr>
<tr>
<td>p = 0.60</td>
<td>300</td>
<td>0.887</td>
<td>73.01</td>
<td>[0.07,1.00]</td>
<td>0.002</td>
<td>99.98</td>
</tr>
<tr>
<td>p = 0.65</td>
<td>125</td>
<td>0.916</td>
<td>58.08</td>
<td>[0.57,1.00]</td>
<td>0.022</td>
<td>90.40</td>
</tr>
<tr>
<td>p = 0.70</td>
<td>80</td>
<td>0.874</td>
<td>67.63</td>
<td>[0.18,1.00]</td>
<td>0.017</td>
<td>94.04</td>
</tr>
</tbody>
</table>

The table displays the approximate time period when the reputation confidence intervals of skilled and unskilled analysts diverge. Statistics are based on simulations of a multi-period, single-analyst model. Skilled analysts were assumed to comprise 50, 30, and 10 percent of the population in Panels A, B, and C, respectively. The probability of the high and low state was set equal to ½; therefore, all agents are known to tell the truth in equilibrium. The statistics are compiled from the results of 10,000 independent trials. The mean reputation denotes the probability that an analyst is skilled given his or her performance history. The fourth and seventh columns provide the percentage of trials where the reputation is above 95 percent for skilled analysts and below 5 percent for unskilled analysts for the given time period. Finally, the confidence interval represents a range where 95 percent of the observed reputations fall for the given period.
The extensive form of the information structure facing a single analyst is presented for one period. Initially, the analyst is randomly chosen from a large population, containing both skilled and unskilled analysts, to provide investment recommendations for a single security. \( \theta_0 \) denotes the common knowledge prior probability of selecting a skilled analyst. The analyst and the public are both unaware of the analyst’s type. After observing a private signal, the analyst publicly predicts whether the high state or low state will occur. Skilled analysts receive informative signals that are accurate \( p \) percent of the time, while the signals of unskilled analysts are randomly assigned. The actual state is realized after announcement. Analysts are paid based on their reputation, which is measured by the probability of being skilled given the accuracy of their predictions.
The extensive form information structure of the two-analyst model is presented for one period. The analysts are initially chosen from a large population of skilled and unskilled analysts to provide investment recommendations for a common security. $\theta_0$ denotes the common knowledge prior probability of selecting a skilled analyst, who receives informative signals with an accuracy rate of $p$ percent. The signals of unskilled analysts are randomly provided. The analysts and the public are unaware of their type. After observing a private signal, the analysts predict whether the high or low state will occur. However, analyst B moves second and incorporates analyst A’s signal into his decision. The actual state is realized after announcement.
Figure 3 Average Reputation from Simulations of the One-Analyst Model
θ₀ = Initial Pr(Skilled) = 0.50, α = 0.50

The chart displays the average reputation from simulations of the multi-period, single-analyst model, when skilled analysts comprised 50 percent of the initial population. Simulations were conducted for both skilled and unskilled analysts when skilled analysts receive the correct signal with probability \( p \) equal to 55, 60, and 65 percent. The probability of the high and low state was set equal to ½; therefore, agents are known to tell the truth in equilibrium. While analysts are unaware of their individual type, they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.
The chart displays the average reputation from simulations of the multi-period, single-analyst model, when skilled analysts comprised 30 percent of the initial population. Simulations were conducted for both skilled and unskilled analysts when skilled analysts receive the correct signal with probability $p$ equal to 55, 60, and 65 percent. The probability of the high and low state was set equal to $\frac{1}{2}$; therefore, agents are known to tell the truth in equilibrium. While analysts are unaware of their individual type, they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.
Figure 5  Average Reputation from Simulations of the One-Analyst Model

$\theta_0 = \text{Initial Pr(Skilled)} = 0.10, \alpha = 0.50$

The chart displays the average reputation from simulations of the multi-period, single-analyst model, when skilled comprised 10 percent of the initial population. Simulations were conducted for both skilled and unskilled analysts where skilled analysts receive the correct signal with probability $p$ equal to 55, 60, and 65 percent. The probability of the high and low state was set equal to $\frac{1}{2}$; therefore, agents are known to tell the truth in equilibrium. While analysts are unaware of their individual type, they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.
Figure 6  Average Reputation from Simulations of the Two-Analyst Model With a Skilled Leader

\[ \theta_0 = \text{Initial Pr(Skilled)} = 0.30, \ \alpha = 0.50, \ \rho = 0.80 \]

The chart displays the average reputation from simulations of the multi-period, two-analyst model with a skilled leader. Skilled analysts comprised 30 percent of the population and received signals with an 80 percent rate of correlation. Simulations were conducted for cases where the follower was skilled and unskilled. Skilled analysts received the correct signal with probability \( p \) equal to 55, 60, or 65 percent. The probability of the high and low state was set equal to \( \frac{1}{2} \). Therefore, the follower is known to herd on the leader’s announcement in equilibrium, and his reputation does not change. The follower’s reputation only changes when he announces a signal opposite the leader. Knowing this, the leader’s equilibrium strategy is to always tell the truth. When the follower is known to herd, the leader is evaluated independently as in the one-analyst model. While analysts are unaware of their individual type, they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.
This chart displays the average reputation from simulations of the multi-period, two-analyst model with an unskilled leader. Skilled analysts comprised 30 percent of the initial population and received signals with an 80 percent rate of correlation. Simulations were conducted for cases where the follower was skilled and unskilled. Skilled analysts received the correct signal with probability $p$ equal to 55, 60, or 65 percent. The probability of the high and low state was set equal to $\frac{1}{2}$. Therefore, the follower is known to herd on the leader’s announcement in equilibrium, and his reputation does not change. The follower’s reputation only changes when he announces a signal opposite the leader. Knowing this, the leader’s equilibrium strategy is to always tell the truth. When the follower is known to herd, the leader is evaluated independently as in the one-analyst model. While analysts are unaware of their individual type, they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.
The chart displays the average reputation from simulations of the multi-period, mixed-order, two-analyst model when analyst A is unskilled and analyst B is skilled. The mixed-order model assumes that the analyst with the highest reputation moves first. Skilled analysts comprised 30 percent of the initial population and received signals with an 80 percent rate of correlation. Skilled analysts received the correct signal with probability \( p \) equal to 55, 60, or 65 percent. The probability of the high and low state was set equal to \( \frac{1}{2} \). Therefore, the follower is known to herd on the leader’s announcement in equilibrium, and his reputation does not change. The follower’s reputation only changes when he announces a signal opposite the leader. Knowing this, the leader’s equilibrium strategy is to always tell the truth. When the follower is known to herd, the leader is evaluated independently as in the one-analyst model. Analysts are unaware of their individual type, but they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.
The chart displays the average reputation from simulations of the multi-period, mixed-order, two-analyst model when analyst A and B have the same type. The mixed-order model assumes that the analyst with the highest prior reputation moves first. Skilled analysts comprised 30 percent of the initial population and received signals with an 80 percent rate of correlation. Skilled analysts received the correct signal with an accuracy rate $p$ equal to 60 percent. The probability of the high and low state was set equal to $\frac{1}{2}$. Therefore, the follower is known to herd on the leader’s announcement in equilibrium, and his reputation does not change. The follower’s reputation only changes when he announces a signal opposite the leader. Knowing this, the leader’s equilibrium strategy is to always tell the truth. When the follower is known to herd, the leader is evaluated independently as in the one-analyst model. Analysts are unaware of their individual type, but they have knowledge of all parameter values. The average reputation was compiled from the simulation results of 10,000 independent trials.