

# Product and Labor Market Entry Costs, Underemployment and International Trade\*

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## Abstract

We develop a small, open economy, two-sector model with heterogeneous agents and endogenous participation in a labor matching market. There are two types of agents: workers and entrepreneurs. Both populations are heterogeneous. Workers are distinguished by their potential ability as skilled workers and entrepreneurs by their potential ability to manage a firm. To capture the notion of decentralized labor markets we assume random matching. Those agents on the long side of the market who are not matched find employment in the unskilled sector as do those agents who decided not to attempt to enter the matching market. The output of matched pairs is a function of the two partners' abilities. We find that disparities in labor institutions become a source of comparative advantage. The exact patterns will depend not only on the costs of entering the skilled sector but also on the mechanism used for dividing the surplus. We analyze the implications of asymmetric market entry costs for the patterns of international trade and underemployment. We find that if labor market inefficiencies are sufficiently strong trade liberalization can lead to welfare losses. We also examine the robustness of our results when we allow for complementarities in the production function and for alternative matching mechanisms.

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# 1. Introduction

Establishing a competitive advantage in high-skilled sectors at the national level requires that a number of conditions must be met. The Ricardian theory of international trade emphasizes the need for technological know-how while from the Heckscher-Ohlin-Vanek model we learn that a sufficient endowment of skilled labor is necessary. While endowments and technologies are necessary pre-conditions they are by no means sufficient. Neoclassical trade theory is silent about the product and labor market institutions which play an important role in bringing the factors of production together. In particular, both the entry of workers into skilled labor markets and the establishment of new enterprises are costly.

When these costs are sufficiently high they discourage market participation. For example, Brixiova, Li and Yousef (2009) and Fan, Overland and Spagat (1999) suggest that the reluctance of workers to enter skilled labor markets can explain shortages of skilled labor in emerging economies and the consequent slow development of their private sector. In contrast, relatively low skill acquisition costs and minimal labor market frictions can potentially explain the phenomenon of overeducation and mismatch observed by researchers in many European countries, United States and Canada.<sup>1</sup> For example, Maynard, Joseph and Maynard (2006) summarizing research results by sociologists, psychologists and operation researchers report that in the United States and the United Kingdom at least one in five workers experience underemployment. Further evidence comes from studies looking at the labor market impact of the economic crisis that began in 2007 which find that underemployment levels have substantially risen since the onset of the crisis reaching 25% in many parts of the world (McKee-Ryan and Harvey, 2011; Bell and Blanchflower, 2011).

Looking at the other side of the labor market, Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) provide evidence that market entry costs incurred by start-up firms are significant and vary widely across countries. They find that "The official cost of following required procedures for a simple firm ranges from under 0.5 percent of per capita GDP in the United States to over 4.6 times per capita GDP in the Dominican Republic, with the worldwide average of 47 percent of annual per capita income."

In addition to market entry costs, we also need to consider frictions arising during the matching process of skilled workers to firms. The decision of young people to acquire skills is going to depend, in addition to any direct costs, on their expectations about the probability of getting a job in the skilled sector and, given that they do find a job, on the quality of the match. Similarly, the decision of potential entrepreneurs to establish new firms will depend on their expectations about the future availability of skilled labor and the latter's level of skills. Furthermore, both parties decisions will depend on the allocation of the surplus generated by the match.

These issues are well understood by labor economists.<sup>2</sup> In this paper, we analyze some of the implications for international trade. We develop a two-sector model with three

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<sup>1</sup>See McGuinness (2006) for a review of this literature.

<sup>2</sup>For example, the need for coordination between skill acquisition and job creation in order to avoid situations where the economy is locked in a low-skill/bad-job trap is emphasized by both Snower (1996) and Redding (1996).

factors of production; namely, unskilled labor, skilled labor and entrepreneurial ability.<sup>3</sup> One sector produces a low-tech good with a constant returns to scale technology that requires only unskilled labor. The second sector is a high-tech sector. To establish a production unit in that sector a skilled worker needs to be matched with an entrepreneur.

There are two types of agents: workers and entrepreneurs. Both populations are heterogeneous. Workers are distinguished by their potential ability as skilled workers and entrepreneurs by their potential ability to manage a firm. Initially, each type must decide whether to enter the matching market. Workers who decide to enter incur a fixed cost related to the acquisition of skills. Entrepreneurs who opt to enter incur a cost for establishing a new firm. To capture the notion of decentralized labor markets we assume random matching. Those agents on the long side of the market who are not matched find employment in the unskilled sector as do those agents who decided not to attempt to enter the matching market. The output of matched pairs is a function of the two partners' abilities.

Not surprisingly, we find that disparities in labor institutions become a source of comparative advantage. The exact patterns will depend not only on the costs of entering the skilled sector but also on the mechanism used for dividing the surplus. This suggests that in addition to traditional sources of comparative advantage, i.e. endowments and technologies, we also need to take into account those costs related to the acquisition of skills and those costs related to the creation of firms and the institutional structure of labor markets (unions, minimum wages, etc.). Thus, our work is related to a group of papers suggesting that differences in labor market rigidities across nations can be a major driving force of comparative advantage (Krugman, 1995; Davis, 1998a; Davis, 1998d; Kreickemeier and Nelson, 2006). Research in this area has paid particular attention to rigidities that have a direct impact on wage formation. In contrast, our main interest is on cross-country differences in (a) the costs of establishing new firms, and (b) the costs of entering skilled labor markets. Finally, our work is also related to some recent theoretical work that explores the implications of trade liberalization for inequality and labor market outcomes.<sup>4</sup>

Traditionally, matching models also include a search process thus generating frictional unemployment (see, for example, Davidson, Martin and Matusz, 1999; Davidson, Matusz and Shevchenko, 2008; Felbermayr, Prat and Schmerer, 2008; Felbermayr, Larch and Lechthaler, 2013). Our main concern is to examine issues related to long-term mismatch associated with underemployment. Our model generates either underemployment of skills or firm capacity that is not utilized depending of which side of the market is long. In particular, we show that unless the sharing rule satisfies a condition that it is equivalent to the one derived in Hosios (1990) for search models using the matching function, there

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<sup>3</sup>A simplified version of the model with one-sided uncertainty has been used by Bougheas and Riezman (2007) to examine the relationship between the distribution of endowments and the patterns of trade and by Davidson and Matusz (2006) and Davidson, Matusz and Nelson (2006) to examine redistribution policy issues.

<sup>4</sup>In Helpman, Itskhoki and Redding (2009) although both populations of firms and entrepreneurs are heterogeneous it is only the participation of the second group that is derived endogenously. Egger and Kreickemeier (2008) analyze a model with one heterogeneous population and generalized endogenous participation where agents in addition to their level of skills also decide in which sector to be employed. In our model, both workers and entrepreneurs can choose whether or not to enter the matching market.

will always be some type of imbalance. Clearly, as Mortensen and Wright (2002) argue, there is no good reasons to believe that when the sharing rule is taken as a primitive it will satisfy the Hosios condition. We demonstrate that the effect of trade liberalization on underemployment will depend on the pattern of trade. More specifically, we find that trade increases underemployment when the country has a comparative advantage in the high-tech sector. The level of underemployment will also depend on the sharing rule that divides the surplus between workers and entrepreneurs. Here, we find that the likelihood that the small-open economy has a comparative advantage in the high-tech sector is decreasing with the level of underemployment in autarky.

Most of our analytical results are derived from a benchmark version of our model that includes a linear production technology and a one-to-one matching mechanism. In Section 2 we develop the model and examine the autarky case and then in Section 3 we open the small-economy to international trade. In Section 4 we analyze two extensions of the benchmark version of our model. First, we allow for complementarities in the production function and we use this extended version to explore the welfare implications of trade liberalization. We show that trade can potentially be welfare reducing. We also identify conditions under which the patterns of international trade are not optimal. Second, we also examine alternative matching mechanisms and show that our results are fairly robust. We offer some final comments in Section 5.

## 2. The Closed-Economy Benchmark Model

The economy is populated by two types of agents and produces two goods. The two types of agents, workers and entrepreneurs, are each of unit mass. The first good, the numeraire, is a high-tech product and its production requires the joint efforts of an entrepreneur and a worker. The second good is a primary commodity and all types of agents can produce one unit should they decide to seek employment in that sector. Let  $P$  be its price in numeraire units. All agents are risk neutral, form expectations rationally and have identical Cobb-Douglas preferences allocating equal shares of their income on each good which implies that real income is equal to nominal income divided by  $\sqrt{P}$ .<sup>5</sup>

The populations of both workers and entrepreneurs are heterogeneous. Workers are differentiated by their ability  $\alpha$  to work in the high-tech sector and entrepreneurs by their ability  $z$  to manage in the high-tech sector. Both  $\alpha$  and  $z$  are private information and are randomly drawn from uniform distributions with support  $[0, 1]$ . Both workers and entrepreneurs have to incur a fixed utility (real income) cost  $0 < \gamma < 1$  and  $0 < c < 1$ , respectively, to enter the high-tech sector.<sup>6</sup> Entrepreneurs and workers that have

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<sup>5</sup>Let  $X$  denote the level of consumption of the high-tech product,  $Y$  the level of consumption of the primary commodity and  $I$  the level of nominal income. By maximizing  $\sqrt{XY}$  subject to  $I = PX + Y$ , we obtain the solutions  $X = \frac{I}{4P}$  and  $Y = \frac{I}{4}$ , which after substituting them back in the utility function and multiplying by 2 (because (a) the marginal utility of income is equal to 1, and (b) the measure of agents is equal to 2) we obtain the solution in the text.

<sup>6</sup>In an earlier version of the paper, we had the costs denominated in numeraire units (units of the high-tech good). As a result the relative price of the two goods depended on the size of these costs. By eliminating this bias we have simplified many derivations and we were able to derive some additional results.

incurred the fixed entry costs are randomly matched. If the two masses are not equal then unmatched agents enter the primary sector. Matched pairs produce  $\alpha + z$  units of the high-tech product.

To complete the description of the model we need to specify how matched pairs divide their joint output. The division of surplus normally depends on the outside options of the two parties and their relative bargaining power. Given that we have assumed away any recontracting the outside options of the two sides are the same and equal to  $P$  the income they will receive in their alternative employment option. Denote by  $\beta$  the share of output allocated to entrepreneurs. In this section we shall assume that all pairs divide the surplus equally, i.e.  $\beta = \frac{1}{2}$ . As we will see below, assuming equal division is analytically convenient and allows for analytical derivations. We will explore numerically the consequences of relaxing this restriction.<sup>7</sup>

Given that an agent's expected payoff is increasing in her own ability there exist two cut-off ability levels  $\alpha^*$  and  $z^*$  such that all workers with ability levels less than  $\alpha^*$  and all entrepreneurs with ability levels less than  $z^*$  do not incur the high-tech sector entry costs and find employment in the primary sector. Thus, a mass of workers of  $1 - \alpha^*$  and a mass of entrepreneurs of  $(1 - z^*)$  will enter the matching market. The decisions to enter the high-tech sector, and the cut-off levels, will depend on each agent's belief about their likelihood of being matched. Thus, there are three cases to consider that correspond to three potential rational expectations equilibria, namely **matching market clearing**  $(1 - \alpha^*) = (1 - z^*)$ , **surplus of entrepreneurs**  $(1 - \alpha^*) < (1 - z^*)$ , and **surplus of workers**  $(1 - \alpha^*) > (1 - z^*)$ . The one that prevails will depend on the values of the various model parameters. In particular, it will depend on the difference between the two entry costs and the level of bargaining power. When  $\beta = \frac{1}{2}$  in the benchmark model, the equilibrium type depends only on the relative size of the two entry costs. Thus, without any loss of generality we assume that  $c < \gamma$  in which case in equilibrium, as we verify below, there will be a mass of entrepreneurs who incur the fixed cost of entry but are not matched.

By definition an entrepreneur with ability  $z^*$  is indifferent between investing and market search and directly entering the primary sector. Given that the income of this threshold agent is equal to  $z^*$  if matched and equal to  $P$  if unmatched, the equilibrium condition for the cut-off level is given by

$$\frac{1}{2} \left( \frac{1 - \alpha^*}{1 - z^*} \right) \left( z^* + \frac{1 + \alpha^*}{2} \right) + \left( 1 - \frac{1 - \alpha^*}{1 - z^*} \right) P - c = P \quad (1)$$

where  $\frac{1 - \alpha^*}{1 - z^*}$  is the probability the entrepreneur is matched with a worker and  $z^* + \frac{1 + \alpha^*}{2}$  is equal to the expected output of a matched pair where the entrepreneur has ability equal to  $z^*$  keeping in mind that only those workers with ability higher than  $\alpha^*$  are attempting to enter the high-tech sector. The first term is multiplied by  $\frac{1}{2}$  which is equal to the share of output received by each member of a matched pair. Similarly,  $\alpha^*$  is determined by

$$\frac{1}{2} \left( \alpha^* + \frac{1 + z^*}{2} \right) - \gamma = P \quad (2)$$

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<sup>7</sup>Acemoglu (1996) also employs Nash bargaining in a random matching environment similar to the one in this paper.

To close the model we need the equilibrium condition for one of the two goods markets. Without loss of generality we focus on the market for the primary commodity

$$2\alpha^* = \frac{2\alpha^*P + (1 - \alpha^*)\frac{2+\alpha^*+z^*}{2}}{2P} \quad (3)$$

The left-hand side is equal to the gross supply of the primary commodity. All workers that enter the matching market are matched and thus there are  $\alpha^*$  unmatched workers which means there are  $\alpha^*$  unmatched entrepreneurs. Therefore, in total there is a mass of  $2\alpha^*$  agents that are employed in the primary sector and each produces one unit. The right-hand side is equal to the gross demand. The specification of preferences imply that an agent with income  $y$  demands an amount  $\frac{y}{2P}$  of the primary commodity. Furthermore, risk-neutrality implies that the marginal utility of income is constant and thus, for the derivation of the gross market demand it suffices to derive aggregate income and divide it by  $2P$ . Agents employed in the primary sector produce one unit and earn income  $P$  and the first term of the numerator on the right-hand side shows their gross income. The second term is equal to the total income of matched pairs.<sup>8</sup>

In the next Proposition we verify that the solution of the above system, that solves for the three endogenous variables  $\alpha^*$ ,  $z^*$  and  $P$ , is indeed a rational expectations equilibrium.

**Proposition 1** *If  $\gamma > c$  then  $z^* < \alpha^*$ .*

**Proof** (1) and (2) imply that  $\frac{1}{2}(z^* + \frac{1+\alpha^*}{2}) - \frac{1-z^*}{1-\alpha^*}c = \frac{1}{2}(\frac{1+z^*}{2} + \alpha^*) - \gamma$ . The equality can be written as  $\frac{1}{4}(1 - \alpha^*)(\alpha^* - z^*) = \gamma - \frac{1-z^*}{1-\alpha^*}c$ . For  $\gamma = c$  the last expression can be written as  $\frac{1}{4}(1 - \alpha^*)^2(\alpha^* - z^*) = \gamma(z^* - \alpha^*)$ . Given that  $\gamma > 0$  it follows that  $\alpha^* = z^*$ . Next consider the case  $\gamma > c$  and let  $\gamma \equiv c + \delta$ . Now we can write the equality as  $\frac{1}{4}(1 - \alpha^*)^2(\alpha^* - z^*) = c(z^* - \alpha^*) + \delta$ . Given that  $\delta > 0$  and given that when  $\gamma = c$ ,  $\frac{1}{4}(1 - \alpha^*)^2(\alpha^* - z^*) = c(z^* - \alpha^*)$  we have  $\alpha^* - z^* > 0$  which completes the proof.

**Remark 1** *There is another equilibrium where nobody participates in the matching market given that it is the best response for each type of agent not to participate if she believes that no agent of the other type will participate. However, this equilibrium is unstable given that any small deviation from any of the two types increases significantly the participation payoff of the other type. The interior equilibrium described by conditions (1), (2) and (3) is a unique strict strategy equilibrium. This is because in our model agents are heterogeneous.*

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<sup>8</sup>For the derivation of the last term, given that the output of a matched pair is equal to the sum of the abilities of its members, it suffices to add individual abilities. Thus, we have that aggregate income of matched pairs equals

$$\int_{\alpha^*}^1 \alpha d\alpha + \frac{1 - \alpha^*}{1 - z^*} \int_{z^*}^1 z dz$$

Notice that second term follows from random matching and  $z^* < \alpha^*$ .

## 2.1. Entry Costs and the Autarky Price

Comparative advantage is completely determined by comparing the autarky price with the foreign price and in the benchmark model the autarky price depends only on the two entry costs. With that in mind, in this section, we examine how changes in these costs affect the autarky price. The following lemma will be useful for subsequent proofs.

**Lemma 1**  $\alpha^* > \frac{1}{2}$ .

**Proof** See the Appendix.

The following Proposition describes some important comparative static results.

**Proposition 2** *Suppose that  $\gamma > c$ . Then, we have (a)  $\frac{d\alpha^*}{d\gamma} > 0$ , (b)  $\frac{dz^*}{dc} > 0$ , (c)  $\frac{d\alpha^*}{dc} < 0$  and (d)  $\frac{dz^*}{d\gamma} \geq 0$ .*

**Proof** See the Appendix

It is not surprising that an increase in  $\gamma$  discourages workers from participating in the matching market and thus the overall effect is to increase  $\alpha^*$ . Similarly, an increase in  $c$  discourages the entry of entrepreneurs in the matching market, i.e.  $z^*$  increases. Now, consider the effect of an increase in any of the two entry costs on the entry decisions in the other side of the market. Payoffs depend on the abilities of both agents so any increase in the threshold level of either workers' or entrepreneurs' entry costs increases the expected payoff of the other type of agent and thus their incentive to participate. Thus, an increase in  $c$  has a positive effect on workers' payoffs thus providing a positive incentive for workers to participate, ( $\alpha^*$  falls.). However, the effect of an increase in  $\gamma$  on the entrepreneurs' entry decision is ambiguous due to a second effect. Given that there are unmatched entrepreneurs, an increase in  $\gamma$  discourages the entry of workers which means that the likelihood of potential entrepreneurs being matched *declines* which discourages their entry. It is clear that this effect is larger the wider the gap between the two entry costs.

Next, we examine how entry costs affect autarky prices.

**Proposition 3** *Suppose that  $\gamma > c$ . Then, (a)  $\frac{dP}{dc} > 0$  and (b)  $\frac{dP}{d\gamma} < 0$ .*

**Proof** See the Appendix

The effect of a change in  $c$  on the autarky price is positive. This is because the decline in the participation rate by entrepreneurs increases the worker's expected payoff thus further increasing their participation rate. Thus, since there is a surplus of entrepreneurs, the supply of the high-tech product increases and this results in an increase in the autarky price. An increase in  $\gamma$  discourages the participation of workers in the matching market and as a consequence both the production of the high-tech product and the autarky price decline.

### 3. International Trade

We now consider international trade. Let  $P^T$  denote the international price. It is clear that if  $P^T > P$  the economy will export the primary commodity and if  $P^T < P$  the economy will export the high-tech product. The following Proposition follows directly from Proposition 3.

**Proposition 4** *Suppose that  $\gamma > c$ . Then, other things equal, economies with higher labor entry costs will export the primary commodity and economies with higher entrepreneur entry costs will export the high-tech product.*

**Remark 2** *In the statement of the Proposition the qualifier ‘other things equal’ is there to remind us that the pattern of international trade will depend not only on cross country differences in the gap between the two costs but also on the levels. The prediction will be reversed if we set entrepreneur entry costs higher than labor entry costs.*

Using the results stated in Proposition 4 we are able to make the following generalizations. Consider two countries  $A$  and  $B$ . If country  $B$ 's higher entry cost is higher than country  $A$ 's higher entry cost and country  $B$ 's lower entry cost is lower than country  $A$ 's lower entry cost then country  $B$ 's autarky price will be lower than country  $A$ 's autarky price. The last statement is due to the symmetry of the model which implies that the autarky price remains the same if we switch the entry costs of the two markets. In contrast, we cannot make any general statements about other rankings of entry costs.

#### 3.1. Underemployment and Trade

We know from the autarky case that when entry costs are asymmetric in equilibrium there are some agents who entered the matching market but were not matched. The total real income loss of unmatched agents due to entry costs  $(\alpha^* - z^*)c$  provides a measure of inefficiency. As the following proposition demonstrates the effect of international trade on inefficiency depends on the pattern of trade.

**Proposition 5** *As the economy moves from autarky to free trade the measure of inefficiency declines when the economy exports the primary commodity and increases when the economy exports the high-tech product.*

**Proof** <sup>9</sup>Setting  $P = P^T$ , rearranging and totally differentiating equations (1) and (2) we get the new system of equations

$$\frac{1}{2}d\alpha + \frac{1}{4}dz = dP^T$$

$$\left(\frac{1}{4} - \frac{1-z}{(1-\alpha)^2}c\right)d\alpha + \left(\frac{1}{2} + \frac{c}{1-\alpha}\right)dz = dP^T$$

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<sup>9</sup>The \* have been suppressed.



The determinant of the new system is equal to

$$\Delta = \frac{3}{16} + \frac{1}{4} \frac{1-z}{(1-\alpha)^2} c + \frac{1}{4} \frac{c}{1-\alpha} > 0$$

Then,

$$\frac{d\alpha}{dP^T} = \frac{\left(\frac{1}{4} + \frac{c}{1-\alpha}\right)}{\Delta} > 0$$

$$\frac{dz}{dP^T} = \frac{\left(\frac{1}{4} + \frac{1-z}{(1-\alpha)^2} c\right)}{\Delta} > 0$$

Lastly,

$$\frac{d\alpha}{dP^T} - \frac{dz}{dP^T} = \frac{\frac{c}{1-\alpha} \left(\frac{z-\alpha}{1-\alpha}\right)}{\Delta} < 0$$

Suppose that  $P < P^T$ . In this case the world price is higher than the autarky price so that the economy exports the primary product. The increase in the price will reduce inefficiency.

The intuition for this result is that if the economy exports the primary product when liberalized then trade will draw resources into that sector and out of the high-tech sector. The high-tech sector is where the matching inefficiencies occur and hence efficiency increases as trade increases.<sup>10</sup>

### 3.2. Division of Surplus and Trade

To this point we have assumed that workers and entrepreneurs share firm output equally. However, it is clear that any change in the division rule will affect all entry decisions and the autarky price. When the two parties share output equally but worker entry costs are higher than those of entrepreneurs it is not surprising that in equilibrium there is a surplus of entrepreneurs. Below we demonstrate that there always exists a sharing rule such that the two equilibrium cut-off levels are equal, i.e.  $\alpha^* = z^* = x$ . Denote by  $\beta^*$  the value of  $\beta$  that sets  $\alpha^* = z^* = x$ . Substituting these expressions in equilibrium conditions (1) and (2) we get

$$\beta^* \left( \frac{1+3x}{2} \right) - c = P$$

$$(1-\beta^*) \left( \frac{1+3x}{2} \right) - \gamma = P$$

Eliminating the autarky price from the first two conditions and rearranging we obtain

$$\beta^* = \frac{1}{2} - \frac{\gamma - c}{1+3x}$$

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<sup>10</sup>It is clear that symmetry implies that the result does not depend on which side of the market is short. What matters for our conclusions is the presence of underemployment and not on the type of underemployment.

The solution is very intuitive.<sup>11</sup> When the two entry costs are equal we also need to set the shares allocated to each side equal so that the entry masses of workers and entrepreneurs are also equal. If entrepreneur entry costs are higher then we need to increase the share of output allocated to entrepreneurs. The exact amount will depend on the gap between the two costs and their level.

Two countries that differ in their sharing rules but are otherwise identical will have different autarky prices and thus both can benefit from trade. Hence, we would like to know how a change in the sharing rule, affects a small open economy's pattern of trade. More specifically, suppose that we increase the share of output allocated to entrepreneurs, i.e.  $\beta$  increases. As Table 1 indicates, the effect on the autarky price will depend on the relationship between  $\beta$  and  $\beta^*$ .

**Table 1:** Sharing Rule and the Autarky Price

**Table 1a:**  $c = 0.3, \gamma = 0.4$

$\beta$	$P$	$\alpha$	$z$
0.3	0.187	0.608	0.821
0.4	0.313	0.610	0.727
0.467 ( $\beta^2$ )	0.405	0.674	0.674
0.5	0.355	0.700	0.620
0.6	0.225	0.788	0.547

**Table 1b:**  $c = 0.3, \gamma = 0.8$

$\beta$	$P$	$\alpha$	$z$
0.2	0.073	0.875	0.929
0.3	0.209	0.772	0.811
0.348 ( $\beta^2$ )	0.273	0.764	0.764
0.4	0.204	0.813	0.721
0.5	0.094	0.907	0.763

**Table 1c:**  $c = 0.7, \gamma = 0.8$

$\beta$	$P$	$\alpha$	$z$
0.35	$\sim 0$	$\sim 1$	$\sim 1$
0.4	0.061	0.921	0.941
0.472 ( $\beta^2$ )	0.148	0.863	0.863
0.5	0.115	0.891	0.876
0.6	$\sim 0$	$\sim 1$	$\sim 1$

The numerical results suggest that when  $\beta < \beta^*$ , an increase in the share of output allocated to entrepreneurs results in a higher autarky price and when  $\beta > \beta^*$  the autarky

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<sup>11</sup>Our solution for  $\beta^*$  corresponds to the Hosios (1990) condition derived from search models using the matching function.

price falls as  $\beta$  increases. Therefore, the autarky price reaches a maximum when  $\beta = \beta^*$ . Below we prove that this is indeed the case for values of  $\gamma$  and  $c$  sufficiently close.<sup>12</sup> Letting  $P(\beta; \gamma, c)$  be the autarky price as a function of the entrepreneur's share of the surplus, we obtain the following result.

**Proposition 6** *For  $\gamma$  and  $c$  sufficiently close  $P(\beta; \gamma, c)$  attains a maximum at  $\beta = \beta^*$ .*

**Proof** See the Appendix.

An important implication of the above result is that as the masses of the two types of entrants get closer the likelihood that the country has a comparative advantage in the high-tech sector goes up. This is intuitive given that when the two masses of entrants are equal underemployment and hence, inefficiency in the high-tech sector is minimized.

It is also interesting to note that with a variable sharing rule entrepreneurs are not necessarily on the long-side of the market as a relatively high proportion of output allocated to them can compensate for higher entry costs. Table 1 presents comparative static results for three distinct cases. In Tables 1a and 1c the two entry costs are relatively close but in the former both are low while in the latter both are high. In Table 1b the gap between the two entry costs is relatively large. The results suggest that there is a monotonic effect of a change in the sharing rule on the cut-off corresponding to the short-side of the market. Keep in mind that for  $\beta < \beta^*$  entrepreneurs are on the short-side of the market while for  $\beta > \beta^*$  workers are on the short-side of the market. In contrast, the effect of a change in the sharing rule on the long-side is ambiguous as we have an additional effect first mentioned in Proposition 2. Given that the change affects the short-side it effects the value of a match but also affects the likelihood that an agent on the long-side of the market is matched.

## 4. Beyond the Benchmark Model

### 4.1. Skill Complementarity

In this section, we extend the benchmark model by allowing for a more general production function. More specifically, we consider the case where the skills of workers and entrepreneurs are complementary. Now, matched pairs produce  $(\alpha + z)^2$  units of the high-tech product. Without any loss of generality, we are going to restrict our attention to the case where  $\gamma > c$ . To keep the analysis tractable we are also setting  $\beta = \frac{1}{2}$ . Given these restrictions, once more in equilibrium we must have  $z^* < a^*$ .

In this case all workers that invest in skills will be matched but only a proportion  $\frac{1-\alpha^*}{1-z^*}$  of entrepreneurs will find employment in the high-tech sector. The equilibrium condition

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<sup>12</sup>Demonstrating the result for values of  $\gamma$  and  $c$  sufficiently apart has proven to be a very daunting task. However, calibrations of the model (Table 1 provides just a few examples), where we have allowed the two entry costs and the sharing rule to take values in the interval  $[0, 1]$ , show that the result stated in Proposition 6 is not only valid when we allow the two values to differ considerably but also that the maximum is a global maximum.

for  $z^*$  is given by

$$\frac{1}{2} \left( \frac{1 - \alpha^*}{1 - z^*} \right) \frac{\int_{\alpha^*}^1 (\alpha + z^*)^2 d\alpha}{1 - \alpha^*} + \left( 1 - \frac{1 - \alpha^*}{1 - z^*} \right) P - c = P \quad (4)$$

where  $\frac{1}{2} \frac{\int_{\alpha^*}^1 (\alpha + z^*)^2 d\alpha}{1 - \alpha^*}$  is equal to the expected payoff of a matched entrepreneur with ability equal to the equilibrium cut-off level. The corresponding condition for  $\alpha^*$  is given by

$$\frac{1}{2} \frac{\int_{z^*}^1 (\alpha^* + z)^2 d\alpha}{1 - z^*} - \gamma = P \quad (5)$$

Now, we turn our attention to the goods market equilibrium concentrating again on the market for the primary commodity. As before, the gross supply is equal to  $2\alpha^*$ . Next, we derive the gross demand of the primary commodity. As before, the specification of preferences imply that an agent with income  $y$  demands an amount  $\frac{y}{2P}$  of the primary commodity. Agents employed in the primary sector produce one unit and earn income  $P$ . What remains is to derive the demand for the primary commodity by those agents who are matched.

The combined income of a matched pair comprising of an entrepreneur with ability  $z$  and a worker with ability  $\alpha$  is equal to  $(\alpha + z)^2$ . In order to find the expected income of a matched pair we need to derive the distribution of  $\alpha + z$  which is the sum of two independent, non-identically distributed uniform random variables.<sup>13</sup> More specifically,  $\alpha$  is uniformly distributed on  $[\alpha^*, 1]$  and  $z$  is uniformly distributed on  $[z^*, 1]$ .

**Lemma 2** *The distribution density function of  $\alpha + z$  for  $\alpha^* > z^*$  is given by*

$$\begin{aligned} \frac{\alpha + z - \alpha^* - z^*}{(1 - \alpha^*)(1 - z^*)} & \quad \text{for} \quad \alpha^* + z^* < \alpha + z \leq 1 + z^* \\ \frac{1}{(1 - z^*)} & \quad \text{for} \quad 1 + z^* < \alpha + z \leq 1 + \alpha^* \\ \frac{2 - \alpha - z}{(1 - \alpha^*)(1 - z^*)} & \quad \text{for} \quad 1 + \alpha^* < \alpha + z \leq 2 \end{aligned} \quad (6)$$

**Proof** Lusk and Wright (1982) provide the derivation when the two random variables are non-identically but independently uniformly distributed on intervals with a lower bound equal to 0. For our more general case we apply the following transformation. Let  $Z = z - z^*$  and  $A = \alpha - \alpha^*$ . Then  $Z$  is uniformly distributed on  $[0, 1 - z^*]$  and  $A$  is uniformly distributed on  $[0, (1 - \alpha^*)]$ . Also  $\alpha + z = A + Z + \alpha^* + z^*$ . So it is sufficient to find the distribution of  $A + Z$ .

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<sup>13</sup>This of course requires that this distribution is the same as the realized distribution resulting from random matching. Alós-Ferrer (2002) has shown that this is indeed the case.

Using the above density functions we can calculate the expected output of a matched pair ( $E\{(\alpha+z)^2 \mid \alpha^* \leq \alpha \leq 1, z^* \leq z \leq 1\}$ ). It follows that the primary market equilibrium condition is given by

$$2a^* = \frac{2\alpha^*P + (1 - \alpha^*)(E\{(\alpha+z)^2 \mid \alpha^* \leq \alpha \leq 1, z^* \leq z \leq 1\})}{2P} \quad (7)$$

The first term, on the right-hand side, is equal to the income of all workers employed in the primary sector. The second term is equal to the aggregate income of matched pairs.

As in the benchmark case, the system of equations (4), (5) and (7) solves for the three endogenous variables  $\alpha^*$ ,  $z^*$  and  $P$ . This new system is too complex to solve analytically but numerical calibrations of the model show that the results of Propositions 2 - 5 derived for the benchmark case are also valid when complementarities are present.<sup>14</sup> Notice that the qualitative results on the pattern of trade do not depend on the exact form of the production function. This is because here we are concentrating on cross-country differences in market entry costs. As Bougheas and Riezman (2007), Costinot and Fogel (2009), Grossman and Maggi (2000), Ohnsorge and Trefler (2007) and Sly (2010) have shown, this is not the case anymore when countries also differ in the distribution of endowments.

#### 4.1.1. Matching Efficiency with Skill Complementarities

When the technology is linear what matters for efficiency is who gets matched however, it does not matter with whom they are matched. The reason is that as long as we know who is matched on each side of the matching market we can find aggregate production in that sector by adding their respective ability levels. However, this is not the case when complementarities are present. Our function is a particular case of a super-modular function. As Grossman and Maggi (2000) have demonstrated efficiency requires that we match workers and entrepreneurs with identical abilities. Thus, we are going to use this more general framework to make some observations on the gains from trade and the pattern of trade. More specifically, using an example, we are going to demonstrate that (a) trade can lead to welfare losses, and (b) the patterns of trade may be sub-optimal. What drives these results is that the competitive equilibrium under autarky is inefficient. When our model allows for skill complementarities there are two sources of inefficiency. The first one is underemployment and this source of inefficiency, as we have already demonstrated, is present even in the absence of skill complementarities. The second one is due to mismatch that arises only when the technology is not linear. A social planner by restricting entry can eliminate the first source. However, given that types are private information a social planner cannot eliminate the second source of inefficiency. Nevertheless, this type of inefficiency must be significantly lower in markets with lower search costs. To keep things simple, we have assumed that search costs are sufficiently high so that agents do not have an incentive to separate. As these costs decline matching efficiency should improve. With that in mind, in the example below we compare the competitive equilibrium with a social optimum where both inefficiencies have been eliminated.<sup>15</sup>

<sup>14</sup>The numerical results are provided in a separate Appendix that is available from the authors.

<sup>15</sup>Our result that trade can potentially lead to welfare losses relies on comparisons between the competitive equilibrium under autarky and the corresponding equilibrium under trade and thus is not affected

**Example 1: Efficiency of Competitive Equilibrium** Consider the example:  $c = 0.5$  and  $\gamma = 0.6$ . We measure aggregate welfare by aggregating individual utilities yielding  $W = 2\sqrt{XY}$ , where  $X$  denotes the level of consumption of the high-tech product and  $Y$  the level of consumption of the primary commodity.<sup>16</sup> Aggregate welfare derived in autarky equilibrium,  $W_A^C$ , is given by<sup>17</sup>

$$W_A^C = \frac{2\alpha^*P + (1 - \alpha^*)(E\{(\alpha + z)^2 \mid \alpha^* \leq \alpha \leq 1, z^* \leq z \leq 1\} - c - \gamma)}{2\sqrt{P}} \quad (8)$$

Substituting the above values of entry costs in (4), (5) and (7) we find that  $\alpha^* = 0.64$ ,  $z^* = 0.59$  and  $P = 0.43$ . Finally, substituting these values in the welfare function we find that  $W_A^C = 0.84$ .

Next, we compare the above solution with the social optimum level of aggregate welfare under autarky,  $W_A^S$ . We begin by setting the mass of workers participating in the matching market equal to the corresponding mass of entrepreneurs. Let  $x^*$  denote the proportion of agents who decide not to enter the matching market and let  $X_A^S$  and  $Y_A^S$  denote the representative agent's consumption levels of the high-tech product and the primary commodity correspondingly. These consumption levels are equal to the aggregate quantities produced in the economy divided by 2 (given that the measure of agents is equal to 2) and given by

$$X_A^S = \left( \int_{x^*}^1 (2x)^2 dx - (c + \gamma)(1 - x^*) \right) / 2 \quad (9)$$

and

$$Y_A^S = x^* \quad (10)$$

Given that efficient matching requires that agents of equal ability are matched, the first term in the brackets in (9) captures the level of aggregate production of the high-tech product. The second term is equal to the aggregate cost of entry in the matching market. Equation (10) follows from the fact that each agent employed in the primary sector produces one unit. After we substitute (9) and (10) in the welfare function we maximize the latter by choosing the proportion of agents who will find employment in the primary sector to obtain  $x^* = 0.69$ . Substituting the solution in (9) and (10) and then those solutions in the welfare function we get  $X_A^S = 0.28$ ,  $Y_A^S = 0.69$  and  $W_A^S = 0.8746 > 0.84 = W_A^C$ .

The above results show that in autarky the market equilibrium is inefficient which is not surprising given that the social planner eliminates underemployment (every agent who incurs the entry cost finds employment in the high-tech sector) and matches agents efficiently. Furthermore, given that the high-tech sector operates more efficiently, optimal participation in that sector is below the corresponding market equilibrium level.<sup>18</sup>

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by this choice.

<sup>16</sup>Keep in mind that the size of the population has measure 2.

<sup>17</sup>See footnote 5.

<sup>18</sup>In fact, this is a third source of inefficiency due to a participation externality which is common in many matching and search models.

**Example 2: International Trade Under Free Competition May Reduce Welfare**

Next, we consider the corresponding welfare levels under international trade when  $P^T = 0.38 < P = 0.43$ . Given that the international price is below the autarky price the small open economy has a comparative advantage in the high-tech product. By substituting the international price in (4) and (5) and solving the system we find the equilibrium cut-off participation rates for the open economy are equal to  $\alpha^* = 0.61$  and  $z^* = 0.56$ . Substituting these values and the international price in the right hand side of (8) we find that  $W_T^C = 0.81 < 0.84 = W_A^C$ ; thus, in this particular case, welfare under international trade is lower than welfare under autarky. The intuition for this result is that when the economy opens to trade it expands the sector in which the inefficiencies arise and in this particular case, the costs due to these inefficiencies exceed the gains from trading at a price that differs from the autarky one.

**Example 3: International Trade is Socially Efficient**

We need to be very careful about interpreting the last result. To see why, let us see what a national social planner would have done when facing the same exogenous international price. The social planner, in addition to allocating agents to sectors, decides which goods and what quantities will be traded with the rest of the world. Let  $\tau_X \geq 0$  and  $\tau_Y \geq 0$  denote the units traded of each good, where positive numbers indicate imports and negative exports. These quantities must satisfy the trade balance condition

$$P^T \tau_Y = -\tau_X$$

The representative agent's consumption levels of the two goods are given by

$$X_T^S = X_A^S + \tau_X$$

and

$$Y_T^S = Y_A^S + \tau_Y$$

Substituting the above three conditions in the welfare function and choosing the participation rate to maximize welfare we obtain  $\tau_Y = 0.024$ ,  $x^* = 0.68$ ,  $X_T^S = 0.27$ ,  $Y_T^S = 0.70$  and  $W_T^S = 0.875 > 0.8746 = W_A^S$ .<sup>19</sup> This demonstrates that if the inefficiencies arising in the matching market are eliminated, trade always improves welfare.

Thus, if matching inefficiencies exist our results suggest that imposing trade restrictions might be welfare improving. However, the results also suggest that a better policy might be to improve labor and product market institutions thus facilitating more efficient matches. Once this is done, free trade is the preferred policy. So, it is not international trade that lowers welfare, rather it is labor market inefficiencies that cause welfare to fall in moving from autarky to free trade.

**Example 4: The Patterns of Trade Under Free Competition May be Sub-optimal**

In the above example the social planner chooses to export the high-tech product

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<sup>19</sup>Due to the choice of functional forms and parameter values the differences are small, however, they are robust in the sense that the qualitative results are obtained for a wide set of parameter values.

and thus the equilibrium patterns of trade are optimal. But in the absence of a social planner this is not always the case. Consider the following question: what must be the international price so that the social planner would choose not to trade; i.e.  $\tau_X = \tau_Y = 0$ ? It is clear that this would be the price that would induce the social planner to choose the same ability cut-off level as the one chosen in the case for autarky, i.e.  $x^* = 0.69$ .<sup>20</sup> We denote this price by  $P^S$ . This price solves

$$2x^* = \frac{2P^S x^* + \int_{x^*}^1 (2x)^2 dx - (c + \gamma)(1 - x^*)}{2P^S}$$

This is similar to (7) but now we have substituted the corresponding demand for and supply of the primary commodity given that production is determined by the social planner's allocation. Substituting the values for  $c$ ,  $\gamma$  and  $x^*$  we obtain  $P^S = 0.402$ . The implication for trade patterns is that if  $P^T > P^S$  then the social planner would choose to export the primary commodity and if  $P^T < P^S$  the social planner would choose to export the high-tech product. If the world price,  $P^T$  lies between the autarky price under a competitive equilibrium ( $P = 0.42$ ) and the social planner's autarky price ( $P^S = 0.402$ ) then the equilibrium pattern of trade will not be optimal. The intuition is that market inefficiencies affect the autarky price. If the world price lies between the two autarky prices then the patterns of trade are not optimal.

## 4.2. Alternative Matching Mechanisms

In this section, we examine the robustness of our comparative static results to alternative matching mechanisms. Up to this point we have assumed that exactly one entrepreneur (long-side of the market) is matched with one worker leaving the rest of the entrepreneurs to seek employment in the primary sector. Given our supposition that there is no possibility of recontracting (infinite search costs) we have assumed matched agents share the surplus equally.

Before we consider any alternative mechanisms we will show that our benchmark set-up is equivalent to one in which all unmatched entrepreneurs in the benchmark case are now matched with one single worker while each one of the rest of the entrepreneurs are matched again with one worker. In this new arrangement, the worker who is matched with multiple entrepreneurs is in a strong bargaining position. Given that the production technology requires a single entrepreneur, bargaining will push the payoff of all entrepreneurs matched with the single worker down to the outside option which in this case is equal to the price of the primary commodity. Now all these entrepreneurs are indifferent between staying in the high-tech sector and entering the primary market. Thus, in this new set up, with the exception of one pair, all other workers and entrepreneurs receive the same payoffs as those in the original set-up. The exception is because now there is one additional entrepreneur

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<sup>20</sup>This is an application of the second welfare theorem. Suppose that the agents in the economy are allocated to sectors by the social planner (this step follows from the fact that the equilibrium allocation is inefficient) and then exchange goods in competitive markets. The equilibrium price would be the one that decentralizes the the social planner's optimal allocation under autarky.



who receives the low payoff and a worker who receives a payoff that is equal to the total surplus generated by the pair minus the price of the primary commodity. Given that we have assumed that both populations are very large the two versions only differ in a set of measure zero.

Now consider the other extreme.<sup>21</sup> Suppose that all workers (short-side of the market) are again matched but now some of them are matched with one entrepreneur and some of them are matched with two entrepreneurs.<sup>22</sup> Thus, in this alternative arrangement underemployment is more evenly distributed in the economy. To keep this simple, we will ignore complementarities and focus on the linear technology case. Once more, under the supposition that  $c < \gamma$ , the mass of entrepreneurs who enter the matching market,  $1 - z^*$ , will be higher than the corresponding mass of workers,  $1 - \alpha^*$ . The proportion of workers matched with two entrepreneurs is equal to  $\frac{\alpha^* - z^*}{1 - \alpha^*}$  and the proportion of entrepreneurs matched with workers who are also matched with another entrepreneur is equal to  $2\frac{\alpha^* - z^*}{1 - z^*}$ . The equilibrium condition that determines  $z^*$  is given by

$$2\frac{\alpha^* - z^*}{1 - z^*}P + \left(1 - 2\frac{\alpha^* - z^*}{1 - z^*}\right)\frac{1}{2}\left(z^* + \frac{1 + \alpha^*}{2}\right) - c = P \quad (11)$$

where the left-hand side is equal to the marginal entrepreneur's expected payoff from entering the market. The equilibrium condition for  $\alpha^*$  is given by

$$\frac{\alpha^* - z^*}{1 - \alpha^*}\left(\alpha^* + \frac{1 + z^*}{2} - P\right) + \left(1 - \frac{\alpha^* - z^*}{1 - \alpha^*}\right)\frac{1}{2}\left(\alpha^* + \frac{1 + z^*}{2}\right) - \gamma = P \quad (12)$$

where if the marginal worker is matched with more than one entrepreneur they receive a payoff equal to the total surplus minus the price of the primary commodity (the entrepreneur's outside option) and if matched with a single entrepreneur they receive half the surplus. Once more, we need the market equilibrium condition (3) to close the model.

Numerical calibration shows that with one exception the comparative static results under this alternative mechanism are the same as those derived from the benchmark case.<sup>23</sup> The only exception relates to the effect of a change in the entry cost of entrepreneurs on  $\alpha^*$  that determines the mass of workers who enter the matching market. In the benchmark case we found that an increase in the entry cost of entrepreneurs has a negative effect on  $\alpha^*$  (more workers enter) because the expected ability of entrepreneurs is now higher. This result could be reversed with the alternative matching mechanism because there is an additional effect. Namely, as the mass of entrepreneurs entering the matching market declines the likelihood that a worker will be matched with more than one entrepreneur, and thus receiving the higher payoff, also declines. This effect discourages worker entry, so the net effect is ambiguous.

The original benchmark model is a special case of the matching arrangement considered in this section. Our paper is motivated by the existence of persistent surpluses of either

<sup>21</sup>We are indebted to Carl Davidson for suggesting this alternative mechanism.

<sup>22</sup>Of course, if the measure of entrepreneurs who enter the matching market is more than twice the measure of corresponding workers then all workers will be matched with multiple entrepreneurs.

<sup>23</sup>The numerical results are provided in a separate Appendix available from the Authors.

skilled workers or vacancies in certain labor markets, especially in emerging economies and their potential implications for comparative advantage. Our more general version suggests that it is not only underemployment that matters but also its distribution throughout the economy. To see this, think of each agent on the short-side of the market as occupying a distinct location. Agents on the long-side of the market know the locations and choose one at random.<sup>24</sup> There are many potential matching arrangements. Those that we have considered in this section correspond to two extreme cases.

## 5. Conclusions

Both workers and potential entrepreneurs who want to enter sectors that use advanced technologies must incur entry costs. For workers these costs might capture time and money spent on skill acquisition while for entrepreneurs these costs might be related to the establishment of new technologies or more directly to costly procedures related to the start-up of new enterprises. The decision to incur these costs will depend on expectations about future benefits from participating in these markets. In turn, these benefits will depend on the likelihood of finding a match and thus employment in these markets and on the productivity of that match. Competitive markets can ensure that *ex ante* all entry decisions are optimal but *ex post* it is very likely that some agents will fail to match and thus their new skills or know-how will be underemployed. Having argued that such imbalances are common we have built a simple two-sector model with heterogeneous agents in order to explore their implications for international trade.

Our first task has been to explore the impact of a change in market entry costs on competitiveness and the patterns of international trade. We have found that the results will depend on three factors. First, on the side of the market that faces the change in entry costs, second, on the distribution of underemployment in the economy, and third, on the sharing rule for dividing the surplus generated by a match. More specifically, we have found that an increase in the entry costs of the agents on the short-side of the market will decrease the international competitiveness of that sector. However, the effect of an increase in the entry costs of the long-side of the market would depend on the distribution of underemployment in the economy. Furthermore, we have shown that the lower the level of underemployment, where the latter directly depends on the sharing rule, the higher the likelihood that the sector's competitiveness is strong.

Calibrations have shown that our results also hold when we introduce complementarities in the production function. However, now in addition to inefficiencies arising because of social sub-optimal entry decisions we also have matching inefficiencies. Given that the autarkic equilibrium is not Pareto optimal it is not surprising that when the economy has a comparative advantage in the sector affected by those inefficiencies, international trade can be welfare reducing. In fact, we have also demonstrated that even the patterns of trade can be inefficient. We have also argued that the best policy response is to initiate measures that improve the functioning of the labor market rather than imposing restrictions on the cross-border movement of goods.

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<sup>24</sup>What matters is that they do not know how many others are trying to find a match in the same location.

## Appendix<sup>25</sup>

### Proof of Lemma 1

The system of equations (1), (2) and (3) can be rewritten as

$$\frac{1}{2} \left( \frac{1+z}{2} + \alpha \right) - \gamma = P \quad (A1)$$

$$\frac{1}{2} \left( z + \frac{1+\alpha}{2} \right) - \frac{1-z}{1-\alpha} c = P \quad (A2)$$

$$P = \frac{(1-\alpha)(2+\alpha+z)}{4\alpha} \quad (A3)$$

Differentiating (A3) with respect to  $\alpha$  we get

$$\frac{\partial P}{\partial \alpha} = \frac{-2 - \alpha^2 - z}{4\alpha^2} < 0$$

Then the difference  $\frac{1}{2} \left( \frac{1+z}{2} + \alpha \right) - P = \frac{1}{2} \left( \frac{1+z}{2} + \alpha \right) - \frac{(1-\alpha)(2+\alpha+z)}{4\alpha}$  is increasing in  $\alpha$ . When the expression is evaluated at  $\alpha = \frac{1}{2}$  we find that it is equal to  $-\frac{1}{8} < 0$ . Then, the Lemma follows from (A1) and  $\gamma > 0$ .

### Proof of Proposition 2

By substituting (A3) into (A1) and (A2) we can reduce the above system into two equations in the two unknowns  $\alpha$  and  $z$ . Totally differentiating the new system we get

$$\left( \frac{1}{2} - \frac{\partial P}{\partial \alpha} \right) d\alpha + \left( \frac{1}{4} - \frac{\partial P}{\partial z} \right) dz = d\gamma \quad (A4)$$

$$\begin{aligned} & \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2} c - \frac{\partial P}{\partial \alpha} \right) d\alpha + \left( \frac{1}{2} + \frac{1}{1-\alpha} c - \frac{\partial P}{\partial z} \right) dz \\ &= \left( \frac{1-z}{1-\alpha} \right) dc \end{aligned} \quad (A5)$$

where

$$\frac{\partial P}{\partial z} = \frac{1-\alpha}{4\alpha} > 0$$

Next, we proceed to show that the determinant  $\Delta$  is positive.

$$\begin{aligned} \Delta &= \left( \frac{1}{2} - \frac{\partial P}{\partial \alpha} \right) \left( \frac{1}{2} + \frac{1}{1-\alpha} c - \frac{\partial P}{\partial z} \right) - \left( \frac{1}{4} - \frac{\partial P}{\partial z} \right) \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2} c - \frac{\partial P}{\partial \alpha} \right) \\ &= \left[ \frac{3}{16} - \frac{1}{4} \frac{\partial P}{\partial z} - \frac{1}{4} \frac{\partial P}{\partial \alpha} \right] + \left[ \frac{1}{2} \frac{1}{1-\alpha} c - \frac{1}{1-\alpha} c \frac{\partial P}{\partial \alpha} + \frac{1}{4} \frac{1-z}{(1-\alpha)^2} c - \frac{1-z}{(1-\alpha)^2} c \frac{\partial P}{\partial z} \right] \end{aligned}$$

Lemma 1 implies that  $\frac{\partial P}{\partial z} < \frac{1}{4}$  and that  $-\frac{\partial P}{\partial \alpha} > \frac{3}{4}$ . The two inequalities imply that the expression in the first bracket is positive. The first inequality also implies that  $\frac{1}{4} \frac{1-z}{(1-\alpha)^2} c -$

<sup>25</sup>In all proofs we have suppressed the \*.

$\frac{1-z}{(1-\alpha)^2}c \frac{\partial P}{\partial z} > 0$  which, in turn, implies that the expression in the second bracket is also positive.

Thus,  $\Delta > 0$ .

(a)  $\Delta > 0$  implies that

$$\text{sign} \left\{ \frac{d\alpha^*}{dc} \right\} = \text{sign} \left\{ - \left( \frac{1-z}{1-\alpha} \right) \left( \frac{1}{4} - \frac{\partial P}{\partial z} \right) \right\} = \text{sign} \left\{ - \frac{1}{4} \left( \frac{1-z}{1-\alpha} \right) \frac{2\alpha-1}{\alpha} \right\}$$

where given that  $\alpha > \frac{1}{2}$  is negative.

(b)  $\Delta > 0$  implies that

$$\text{sign} \left\{ \frac{d\alpha^*}{d\gamma} \right\} = \text{sign} \left\{ \frac{1}{2} + \frac{1}{1-\alpha}c - \frac{\partial P}{\partial z} \right\} > 0$$

Given that  $\frac{\partial P}{\partial z} < \frac{1}{4}$  the expression is positive.

(c)  $\Delta > 0$  implies that

$$\begin{aligned} \text{sign} \left\{ \frac{dz^*}{d\gamma} \right\} &= \text{sign} \left\{ - \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2}c - \frac{\partial P}{\partial \alpha} \right) \right\} = \text{sign} \left\{ - \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2}c + \frac{2+\alpha^2+z}{4\alpha^2} \right) \right\} \\ &= \text{sign} \left\{ -4\alpha^2(1-z)c + (1-\alpha)^2(2+2\alpha^2+z) \right\} \geq 0 \end{aligned}$$

For sufficiently high values of  $\gamma$  (high  $\alpha$ ) the expression will be negative. For low values of  $\gamma$  the sign will depend on  $c$ , and given that an increase in  $c$  implies an increase in  $z$  (see below), for relative extreme values of  $c$  the expression will be positive.

(d)  $\Delta > 0$  implies that

$$\text{sign} \left\{ \frac{dz^*}{dc} \right\} = \text{sign} \left\{ \left( \frac{1}{2} - \frac{\partial P}{\partial \alpha} \right) \left( \frac{1-z}{1-\alpha} \right) \right\} > 0$$

where the inequality follows from  $\frac{\partial P}{\partial \alpha} < 0$ .

### Proof of Proposition 3

(a) Totally differentiating (A1) we get

$$\frac{dP}{dc} = \frac{1}{4} \frac{dz}{dc} + \frac{1}{2} \frac{d\alpha}{dc}$$

Given that  $\Delta > 0$  the sign of the above expression is the same as the sign of

$$\begin{aligned} &\frac{1}{4} \left( \frac{1}{2} - \frac{\partial P}{\partial \alpha} \right) \left( \frac{1-z}{1-\alpha} \right) - \frac{1}{2} \left( \frac{1-z}{1-\alpha} \right) \left( \frac{1}{4} - \frac{\partial P}{\partial z} \right) \\ &= \frac{1}{4} \left( \frac{1-z}{1-\alpha} \right) \left( \left( \frac{1}{2} + \frac{2+\alpha^2+z}{4\alpha^2} \right) - \frac{1}{2} \left( \frac{2\alpha-1}{\alpha} \right) \right) \\ &= \frac{1}{4\alpha} \left( \frac{1-z}{1-\alpha} \right) (2 - \alpha^2 + 2\alpha + z) > 0 \end{aligned}$$

(b) Totally differentiating (A1) we get

$$\frac{dP}{d\gamma} = \frac{1}{4} \frac{dz}{d\gamma} + \frac{1}{2} \frac{d\alpha}{d\gamma} - 1$$

The first two terms are equal to

$$\left\{ -\frac{1}{4} \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2} c - \frac{\partial P}{\partial \alpha} \right) + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{1-\alpha} c - \frac{\partial P}{\partial z} \right) \right\} / \Delta$$

To complete the proof we need to show that the numerator is less than  $\Delta$

$$\begin{aligned} & -\frac{1}{4} \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2} c - \frac{\partial P}{\partial \alpha} \right) + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{1-\alpha} c - \frac{\partial P}{\partial z} \right) - \\ & \left[ \frac{3}{16} - \frac{1}{4} \frac{\partial P}{\partial z} - \frac{1}{4} \frac{\partial P}{\partial \alpha} \right] - \left[ \frac{1}{2} \frac{1}{1-\alpha} c - \frac{1}{1-\alpha} c \frac{\partial P}{\partial \alpha} + \frac{1}{4} \frac{1-z}{(1-\alpha)^2} c - \frac{1-z}{(1-\alpha)^2} c \frac{\partial P}{\partial z} \right] \\ = & \left( \frac{1}{2} + \frac{1}{1-\alpha} c \right) \frac{\partial P}{\partial \alpha} - \left( \frac{1}{4} - \frac{1-z}{(1-\alpha)^2} c \right) \frac{\partial P}{\partial z} \\ = & -\frac{1}{16\alpha^2(1-\alpha)} \left( (2(1-\alpha) + 4c)(2 + \alpha^2 + z) + \alpha(1-\alpha)^2 - 4\alpha(1-z)c \right) \\ = & -\frac{1}{16\alpha^2(1-\alpha)} \left( 4(1-\alpha) + 2(1-\alpha)\alpha^2 + 2(1-\alpha)z + 8c + 4\alpha^2c + 4zc + \alpha(1-\alpha)^2 - 4\alpha(1-z)c \right) \end{aligned}$$

The proof follows from  $4\alpha(1-z)c < 4c < 8c$ .

### Proof of Proposition 6

The function  $P(\beta; \gamma, c)$  is continuous but not differentiable at  $\beta = \beta^*$ .

For  $\beta > \beta^*$  the equilibrium of the model is determined by

$$(1-\beta) \left( \frac{1+z}{2} + \alpha \right) - \gamma = P \quad (\text{A6})$$

$$\beta \left( z + \frac{1+\alpha}{2} \right) - \frac{1-z}{1-\alpha} c = P \quad (\text{A7})$$

$$P = \frac{(1-\alpha)(2+\alpha+z)}{4\alpha} \quad (\text{A8})$$

Given that the definition of  $\beta^*$  implies that  $\alpha^* > z^*$  we have obtained the above system from the system (A1), (A2) and (A3) after setting the entrepreneur's share of surplus equal to  $\beta$ . Let  $P^+(\beta)$  denote the price function for  $\beta > \beta^*$ . We will show that  $\frac{dP^+(\beta)}{d\beta} \Big|_{(\beta=\beta^*)} = \left( \frac{\partial P}{\partial \alpha} \frac{d\alpha}{d\beta} + \frac{\partial P}{\partial z} \frac{dz}{d\beta} \right) \Big|_{(\beta=\beta^*)} < 0$ . Notice that given that the market clearing condition (A8) does not directly depend on  $\beta$  the expressions for  $\frac{\partial P}{\partial \alpha}$  and  $\frac{\partial P}{\partial z}$  are the same as those derived in the proof of Proposition 2. After substituting (A8) in (A6) and (A7) and totally differentiating we get

$$\left( (1-\beta) - \frac{\partial P}{\partial \alpha} \right) d\alpha + \left( \frac{1-\beta}{2} - \frac{\partial P}{\partial z} \right) dz = \left( \frac{1+z}{2} + \alpha \right) d\beta \quad (\text{A9})$$

$$\left( \frac{\beta}{2} - \frac{1-z}{(1-\alpha)^2} c - \frac{\partial P}{\partial \alpha} \right) d\alpha + \left( \beta + \frac{c}{1-\alpha} - \frac{\partial P}{\partial z} \right) dz = - \left( z + \frac{1+\alpha}{2} \right) d\beta \quad (\text{A10})$$

Following the same steps as those used in the proof of Proposition 2 we can show that the determinant is positive for  $\beta$  close to  $\frac{1}{2}$ , and we have already shown that as  $\gamma$  and  $c$  come closer together  $\beta^*$  approaches  $\frac{1}{2}$ . Then,

$$\begin{aligned} & \text{sign} \left\{ \frac{\partial P}{\partial \alpha} \frac{d\alpha}{d\beta} + \frac{\partial P}{\partial z} \frac{dz}{d\beta} \right\}_{\beta=\beta^*} \\ = & \text{sign} \left\{ \begin{aligned} & \frac{\partial P}{\partial \alpha} \left( \left( \frac{1+z}{2} + \alpha \right) \left( \beta + \frac{c}{1-\alpha} - \frac{\partial P}{\partial z} \right) + \left( z + \frac{1+\alpha}{2} \right) \left( \frac{1-\beta}{2} - \frac{\partial P}{\partial z} \right) \right) \\ & - \frac{\partial P}{\partial z} \left( \left( (1-\beta) - \frac{\partial P}{\partial \alpha} \right) \left( z + \frac{1+\alpha}{2} \right) + \left( \frac{\beta}{2} - \frac{1-z}{(1-\alpha)^2} c - \frac{\partial P}{\partial \alpha} \right) \left( \frac{1+z}{2} + \alpha \right) \right) \end{aligned} \right\}_{\beta=\beta^*} \end{aligned}$$

Setting  $\alpha = z = x$  and simplifying we get

$$\frac{\partial P}{\partial \alpha} \left( \frac{1}{2} + \frac{\beta}{2} + \frac{c}{1-x} \right) - \frac{\partial P}{\partial z} \left( 1 - \frac{\beta}{2} - \frac{c}{1-x} \right) < 0$$

The inequality follows from  $\frac{\partial P}{\partial z} < \frac{1}{4}$  and  $-\frac{\partial P}{\partial \alpha} > \frac{3}{4}$ .

To complete the proof we need to show that when  $\beta < \beta^*$ ,  $\frac{dP^-(\beta)}{d\beta}_{(\beta=\beta^*)} = \left( \frac{\partial P}{\partial \alpha} \frac{d\alpha}{d\beta} + \frac{\partial P}{\partial z} \frac{dz}{d\beta} \right)_{(\beta=\beta^*)} > 0$ , where  $P^-(\beta)$  denotes the corresponding price function. In this case, the equilibrium is given by

$$(1-\beta) \left( \frac{1+z}{2} + \alpha \right) - \frac{1-\alpha}{1-z} \gamma = P \quad (\text{A11})$$

$$\beta \left( z + \frac{1+\alpha}{2} \right) - c = P \quad (\text{A12})$$

$$P = \frac{(1-z)(2+\alpha+z)}{4z} \quad (\text{A13})$$

Notice that we can obtain (A13) by substituting  $\gamma$  for  $c$ ,  $c$  for  $\gamma$ ,  $\alpha$  for  $z$  and  $z$  for  $\alpha$  in (A6). Thus the partial derivatives  $\frac{\partial P}{\partial \alpha}$  and  $\frac{\partial P}{\partial z}$  are given by

$$\frac{\partial P}{\partial \alpha} = \frac{1-z}{4z} > 0$$

and

$$\frac{\partial P}{\partial z} = \frac{-2-z^2-\alpha}{4z^2} < 0$$

After substituting (A13) in (A11) and (A12) and totally differentiating we get

$$\left( (1-\beta) + \frac{\gamma}{1-z} - \frac{\partial P}{\partial \alpha} \right) d\alpha + \left( \frac{1-\beta}{2} - \frac{1-\alpha}{(1-z)^2} \gamma - \frac{\partial P}{\partial z} \right) dz = \left( \frac{1+z}{2} + \alpha \right) d\beta \quad (\text{A9})$$

$$\left( \frac{\beta}{2} - \frac{\partial P}{\partial \alpha} \right) d\alpha + \left( \beta - \frac{\partial P}{\partial z} \right) dz = - \left( z + \frac{1+\alpha}{2} \right) d\beta \quad (\text{A10})$$

Because of symmetry the determinant of this system is identical to the one for the previous case for  $\beta = \frac{1}{2}$ . (Let the old matrix be  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ . Then the new matrix is  $\begin{bmatrix} D & C \\ B & A \end{bmatrix}$ ).

The determinant has not been affected by these changes, however, when the signs were different across a diagonal before now the positive sign has become negative and the other way around). Thus, at least for values of  $\beta$  close to  $\frac{1}{2}$ , the new determinant will also be positive. Then the sign of  $\frac{dP^-(\beta)}{d\beta}$  ( $\beta=\beta^*$ ) is the same as

$$\begin{aligned} & \text{sign} \left\{ \frac{\partial P}{\partial \alpha} \frac{d\alpha}{d\beta} + \frac{\partial P}{\partial z} \frac{dz}{d\beta} \right\}_{\beta=\beta^*} \\ = & \text{sign} \left\{ \begin{array}{l} \frac{\partial P}{\partial \alpha} \left( \left( \frac{1+z}{2} + \alpha \right) \left( \beta - \frac{\partial P}{\partial z} \right) + \left( z + \frac{1+\alpha}{2} \right) \left( \frac{1-\beta}{2} - \frac{1-\alpha}{(1-z)^2} \gamma - \frac{\partial P}{\partial z} \right) \right) \\ - \frac{\partial P}{\partial z} \left( \left( (1-\beta) + \frac{\gamma}{1-z} - \frac{\partial P}{\partial \alpha} \right) \left( z + \frac{1+\alpha}{2} \right) + \left( \frac{\beta}{2} - \frac{\partial P}{\partial \alpha} \right) \left( \frac{1+z}{2} + \alpha \right) \right) \end{array} \right\}_{\beta=\beta^*} \end{aligned}$$

Setting  $\alpha = z = x$  and simplifying we get

$$\frac{\partial P}{\partial \alpha} \left( \frac{1}{2} + \frac{\beta}{2} - \frac{\gamma}{1-x} \right) - \frac{\partial P}{\partial z} \left( 1 - \frac{\beta}{2} + \frac{\gamma}{1-x} \right) > 0$$

The above inequality follows from using the same logic as the one used for the proof of Lemma 1 to show that  $z > \frac{1}{2}$  and then using the last inequality to show that  $\frac{\partial P}{\partial \alpha} < \frac{1}{4}$  and  $-\frac{\partial P}{\partial z} > \frac{3}{4}$ .

This completes the proof of the proposition.

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