Abstract

We develop a model that explores the link between services and fragmentation. In our model components used in the manufacturing process are made with services and labor. Low foreign wages create an incentive to outsource component production (fragmentation). However, the importance of services in the manufacturing process can play a role in limiting fragmentation. Although advanced economies have higher costs of supplying simple services, because of the size of their home market they have a greater variety of services. As a result, their price of “aggregate services” tends to be lower than those of less developed economies. When services are not tradable advanced economies will tend to produce components that are service-intensive, and outsource the manufacturing of components that are less service-intensive (labor intensive) to developing countries. Complete free trade in services, however, eliminates the home market effect of the developed country, and in the absence of the Dornbush-Fischer-Samuelson type of comparative advantage will lead to complete outsourcing of components to the low wage economy. However, trade in services in the presence of transportation costs that are biased against the specialized services supplied by the home country may lead to reduced fragmentation of manufacturing.
1. Introduction

There has been a marked increase in the fragmentation of production in recent years. A given final good (for example, a personal computer, a car) may include components that are produced at many different parts of the world. The emergence of contract manufacturers of chips, such as Taiwan’s UMC (United Microelectronics Corporation) and TSMC (Taiwan Semiconductor Manufacturing Company), and Singapore’s CSM (Chartered Semiconductor Manufacturing) is a spectacular illustration of such fragmentation. It was Morris Chang, founder and chairman of TSMC, who pointed out to the world that the semiconductor industry was in fact not one but two industries, and that it would be best to separate them out. “One industry is about designing chips, which requires lots of talent but little capital. The other is about making chips, which requires more capital than talent” (The Economist, May 19, 2001, p 62). Another well known example of fragmentation concerns the famous camera, Leica, with lenses produced in Germany, and other parts produced in Canada, Spain, and the Far East.

To explain the tendency for increased fragmentation, economists have cited factors such as low labor costs in developing countries that

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1In 1987, Mr Chang founded TSMC, the world’s first “fab”, or fabrication plan. Many “fabless” designers (among which are university-based research centers) have bought capacity from TSMC. Integrated design manufacturers (IDMS) such as Intel still maintain that chip design and manufacture should go hand in hand.
have opened up and welcome foreign investment, and reduced transport costs. These factors alone do not explain why computer chips are made in Taiwan and Singapore, and car parts are made in Mexico, but these are not made in, say, Nepal, Vietnam, or Bangladesh. To answer this question, one must take into account the fact that components are not just “manufactured” by manufacturing labor. They have to be marketed, their transportation have to be speedy, and insured, deliveries must be timely, communications with head office or customers must be efficient, contracts must be reliable, etc. All these factors point to the importance of a service industry that supports and facilitates manufacturing. It follows that it is not necessarily optimal to produce components in a low wage economy that does not have an efficient service sector.

In this paper, we develop a general equilibrium model with a manufacturing sector, and a service sector. We show how the growth of the service sector enables the economy to manufacture and export components. The range of components that a country can export is shown to depend on the price of services. We also show how international trade in services may affect the trade in components, and the degree of fragmentation.

Our story may be summarized as follows. There are two countries, and advanced country, and a developing country. There are two final goods, say bananas and cars. Bananas are produced by labor, and no fragmentation is possible in banana production. Cars are produced by assembling components, such as spark plugs, seat belts, tires, gear box, transmission, engine, and so on. To produce these components and connect them with other “production blocks”, an economy needs both manufacturing labor, and services. A manufacturer of a given component hires manufacturing labor and buys and combines services from “service providers”, such as an accounting firm, a marketing firm, a security firm, a bank, a delivery firm, a cleaning contractor, and so on. These service firms have a constant variable cost, and a fixed cost. Thus their average cost falls as their volume of service output rises. Decreasing average cost is not compatible with price-
taking behavior. Therefore we assume that the service industry is
characterized by monopolistic competition: each service firm faces a
downward sloping demand curve, and sets its price, taking as given
the prices of imperfect substitutes set by other service firms.

The number of different services that an economy can offer depends
on its stage of development and the size of the economy. For exam-
ple, a matured economy such as the US has more specialized services
than less matured ones, such as Indonesia. A large economy, such
as Germany, can offer more services than a small economy, such as
New Zealand, even if the two economies have the same level of human
capital per person.

We argue that the greater is the range of services, the more effi-
cient is the production of components. On the other hand, a country
that has a greater range of services may have a more expensive ser-
vice price, perhaps due to higher labor cost. For example, a computer
programmer in the US earns more than his counterpart working in
India. The trade-off between range of services and costs of individ-
ual services determine what types of components will be produced in
which country. We expect that a component that has a high ratio of
service to manufacturing labor will be produced in the country that
has a greater range of services.

Our model is basically Ricardian in structure; labor is the only
primary factor of production. Intermediate inputs (components) are
produced by labor and services which are themselves produced by
labor alone. We postulate a continuum of intermediate inputs, an idea
that we borrow from Dornbush, Fischer and Samuelson (1977). We
use the model to examine the extent of outsourcing from a developed
economy to a less developed economy.

The assumptions that drive the results of our model are exogenous
differences in (i) the number of specialized services that exist in the
two economies (ii) the pattern of comparative advantage in produc-
tion of components, and (iii) the productivity of labor in the numeraire
good. In writing our paper, we have benefited from the insightful dis-
cussions on fragmentation, contained in Jones and Kierzkowski (1990,
Jones and Kierzkowski (1990) proposed a distinction between integrated and fragmented technologies. Under the integrated technology, the manufacturing of a good takes place within a single production block, and the role of services is rather limited. Fragmented technology requires that service links connect individual production blocks. “These links can be best thought of as consisting of bundles of activities—coordination, transportation, telecommunication, administration, insurance, financial services, and so on.” (Jones and Kierzkowski (2001a), p 368). However, Jones and Kierzkowski (2001a) did not model the service sector. They focussed attention on the case in which one integrated production activity gets segmented into two components as a result of an exogenous reduction in the cost of international service links. Jones and Kierzkowski (2001b) devoted a short section to the role of services in fragmentation. They mentioned two stylized facts about the costs of service links: “First, purely domestic service links are less costly than those required to connect production blocks located in more than one country...Second, production of services displays strong increasing returns to scale.” The second stylized fact seems to call for a departure from a model based on perfect competition. Our paper is attempt to move in that direction.

Various aspects of fragmentation have been studied by a number of authors. Arndt (1997) showed that intra-product specialization can be a source of gains from trade. Harris (1993, 1995) focussed on the role of telecommunications. Hanson (1996) mentioned fragmentation in the context of trade between Mexico and the United States. Feenstra (1998) linked the integration of trade with the disintegration of production. Harris (2001) considers increasing returns to component production combined with coordination costs associated with fragmented production. Deardorff (2001) shows that liberalizing trade in services can have larger than anticipated benefits. He argues that liberalizing service trade facilitates fragmentation which in turn, stimulates commodity trade. Raff (2001) discussed the role of direct foreign investment in services.
This paper is organized as follows. In section 2, we develop a simple model of a closed economy with a service sector. In section 3, we characterize the autarkic equilibrium. Section 4 considers two economies with different numbers of specialized service firms, and shows how the pattern of trade in components is determined, given that there is no trade in services. Section 5 allows trade in services. Section 6 discusses some generalizations of the basic model.

2. A Closed Economy with Services and Components as Intermediate Inputs

In this section, we present our basic model of a closed economy with two final consumption goods, where labor is the only primary factor of production, and where services are essential intermediate inputs in the production process of components of a final consumption good.

2.1. Production structure

The labor endowment is denoted by $L$. The economy produces two final consumption goods, $X$ and $Y$. To fix ideas, call the consumption good $X$ cars, and the consumption good $Y$ food. Food is produced using labor alone, according to the technology

$$Y = \frac{L_Y}{a_Y}$$

where $L_Y$ is the quantity of labor employed in sector $Y$ and $a_Y > 0$ is the labor requirement per unit of food output.

Throughout the paper, good $Y$ is the numeraire good, i.e. $P_Y = 1$. The wage rate in terms of good $Y$ is $w = 1/a_Y$.

The production function of good $X$ is assumed to be of the Leontief type. To produce good $X$, exactly $k$ distinct components are needed. By choice of units, we can suppose without loss of generality that one unit of good $X$ needs one unit of each type of component. The production function of cars is

$$X = \min [Q_1, Q_2, ..., Q_k]$$
where $Q_i$ is the quantity of component $i$. (Here, for simplicity, we assume there are no other inputs such as direct labor or services.)

Let $\pi_i$ denote the price of component $i$ (in terms of the numeraire good). Under the assumption that producers of good $X$ are perfectly competitive firms, the price of good $X$ (in terms of the numeraire good) is

$$P_X = \sum_{i=1}^{k} \pi_i$$

(3)

Turning now to the production of components, we assume that to produce one unit of component $i$, one needs one unit of labor and $e_i$ units of “aggregate service”. The concept of aggregate service needs some explanation. We suppose that the economy has $n$ types of specialized services. By combining these specialized services, the aggregate service is produced. The technology of combining services is assumed to be of the CES type. Thus, denoting by capital letter $S$ the quantity of aggregate service, and by small letter $s_j$ the quantity of specialized service $j$, we postulate the aggregate service production function

$$S = \left[ \sum_{j}^{n} s_j^\alpha \right]^{1/\alpha}$$

(4)

where $0 < \alpha < 1$. For example, if $\alpha = 1/2$ and $n = 2$, then using 4 units of specialized service $s_1$ and zero unit of specialized service $s_2$ will result in 4 units of aggregate service, while using 2 units of each specialized service will result in 8 units of aggregate service. This formulation of aggregate services reflects the notion that there are gains from specialization. This is an old idea which goes back to Adam Smith’s pin factory example.

Because our focus in this paper is on short-run analysis we take $n$ as exogenous. One could argue that $n$ depends on the stage of development of the economy since a highly developed economy is likely to have more specialized services than a less developed economy. However, we leave more detailed analysis of this for future work.
Component manufacturers, who are perfectly competitive, hire labor and buy specialized services which they transform into aggregate service. Let $p_j$ denote the price of specialized service $j$ (where $j = 1, 2, \ldots, n$) which component manufacturers take as given. The unit cost of aggregate service is then

$$P_S = \left[ \sum_{i=1}^{n} p_i^{\alpha/(\alpha-1)} \right]^{(\alpha-1)/\alpha}$$  \hspace{1cm} (5)

Under the assumption that all specialized services are produced under identical technology (the symmetry assumption), we have $p_j = p$ for all $j$, and thus

$$P_S = pn^{(\alpha-1)/\alpha}$$  \hspace{1cm} (6)

It follows that the cost of a unit of component $i$ is

$$\pi_i = w + e_i P_S = w + e_i pn^{(\alpha-1)/\alpha}$$  \hspace{1cm} (7)

It remains to describe the technology of the production of specialized services. We assume that there are $n$ service firms, each specializing in one type of service. These firms use labor as the only input. If service firm $j$ produces $s_j$ units of specialized service $j$, it must employ $cs_j + f$ units of labor. This means each firm must incur a fixed cost $F = wF$ and the marginal cost is $wc$. Thus their average cost curve is negatively sloped. These firms therefore can survive only if they have some market power, which is what we assume. Each specialized service firm then equates marginal revenue with marginal cost.

2.2. Equilibrium prices under autarky

Now, the conditional demand function for specialized service $j$ is obtained by applying Shephard’s Lemma to equation (5)

$$s_j = S \left[ \frac{\partial P_S}{\partial p_j} \right] = ASP_j^{1/(\alpha-1)}$$  \hspace{1cm} (8)
where $S$ is the quantity of aggregate service demanded in the economy, and

$$A \equiv \left[ \sum_{j=1}^{n} p_j \frac{\alpha}{(\alpha-1)} \right]^{-1/\alpha} \quad (9)$$

Following the standard approach, we assume that the specialized service firm $j$ takes both $A$ and $S$ as given. Then its perceived elasticity of demand is, from (8):

$$\varepsilon = \frac{1}{1 - \alpha} > 1 \quad (10)$$

The price $p_j$ is set by equating marginal revenue with marginal cost

$$MR_j = p_j \left[ 1 - \frac{1}{\varepsilon} \right] = MC_j = wc \quad (11)$$

This yields

$$p_j = \frac{cw}{\alpha} \equiv p \quad (12)$$

which is the same for all $j$.

Note that since $p_j = p$ for all $j$, if $S$ units of aggregate service are demanded, then the demand for each specialized service is $s$ where

$$S = [ns^{\alpha}]^{1/\alpha} = n^{1/\alpha} s \quad (13)$$

The profit of each specialized service firm is

$$r = (p - wc) s -wf = \left( \frac{1}{\alpha} - 1 \right) cws -wf \quad (14)$$

Clearly, $r$ is non-negative if and only if the equilibrium service output satisfies

$$s \geq \frac{\alpha f}{(1 - \alpha)c} \quad (15)$$

We will show in a later subsection (see equation (32) below) that this condition is satisfied under certain parameter restrictions.
Thus, as one would expect from a Ricardian-type model, the equilibrium prices (in terms of the numeraire good $Y$) in this economy are independent of demand:

$$ p_j = p = cw/\alpha, \quad w = \frac{1}{a_Y} \quad (16) $$

$$ P_S = \frac{cn^{1-(1/\alpha)}}{\alpha a_Y} \quad (17) $$

$$ \pi_i = w + e_i p n^{(\alpha-1)/\alpha} = w + e_i \frac{cw}{\alpha} \left( n^{(\alpha-1)/\alpha} \right) \quad (18) $$

$$ P_X = \sum_{i=1}^{k} \pi_i = kw + \frac{cw}{\alpha} \left( n^{(\alpha-1)/\alpha} \right) E = \frac{1}{a_Y} \left[ k + \frac{cE}{\alpha} \left( n \right)^{1-(1/\alpha)} \right] \quad (19) $$

where

$$ E \equiv \sum_{i=1}^{k} e_i. \quad (20) $$

Also, as expected, the greater the number of specialized services, the lower is the price of good $X$.

Finally, we must look at the allocation of labor. If $S$ units of aggregate service are to be produced, each specialized service firm must supply $s = Sn^{-1/\alpha}$ units of its service, and hence must employ $f + cSn^{-1/\alpha}$ units of labor. Total employment of labor in the service sector is thus

$$ L_S = nf + cSn^{1-(1/\alpha)} \quad (21) $$

Suppose $X$ cars are to be produced. Then the demand for aggregate service by the component sector is $S = EX$, and equation (21) becomes

$$ L_S = nf + cEXn^{1-(1/\alpha)} \quad (22) $$

The total amount of labor hired by the component sector is

$$ L_C = kX \quad (23) $$
Full employment of labor means

\[ L_Y + L_C + L_S = L \]  \hspace{1cm} (24)

From equation (22), (23), and (24) we obtain

\[ L_Y = L - n_f - [cEn^{1-(1/\alpha)} + k] X \]  \hspace{1cm} (25)

The economy’s final-goods production possibilities frontier is, from (1) and (25),

\[ Y = \frac{L - n_f}{a_Y} - \left[ \frac{cEn^{1-(1/\alpha)} + k}{a_Y} \right] X \]  \hspace{1cm} (26)

where we assume that \( n_f < L \). The slope of this frontier is

\[ \frac{dY}{dX} = -\left[ \frac{cEn^{1-(1/\alpha)} + k}{a_Y} \right] \]  \hspace{1cm} (27)

Comparing the slope of the production possibilities frontier (PPF) with the equilibrium relative price \( P_X \) (price of cars in terms of food) in (19) we find that the relative price exceeds the absolute value of the slope of the PPF. This implies that the autarkic equilibrium is not Pareto efficient. The intuition is that there is market power in the service sector which results in the undersupply of each service. (If there were a benevolent social planner, he would ask specialized service firms to equate marginal cost to price, and pay them a subsidy to cover the fixed cost.)

2.3. Equilibrium outputs under autarky

We have been able to determine equilibrium prices without reference to the demand side because the economy is Ricardian in nature (in spite of the market power of the specialized service firms). To determine equilibrium output, we must look at the demand side.

Assume all consumers are identical and have equal shares in the profits of specialized service firms. (Recall that no other firms make
Profits.) We denote by $M$ the national income (in terms of the numeraire good $Y$.) It consists of wage income ($wL = L/a_Y$) and profits. Using (14):

$$M = \frac{1}{a_Y} \left[ L + n \left( \frac{1}{\alpha} - 1 \right) cs - nf \right]$$

(28)

where $s$ must be determined (see below).

The representative consumer takes $M$ and all prices as given. With a homothetic utility function $U(X,Y)$, his demand function for $X$ takes the form

$$X^d = \phi(P_X)M$$

(29)

In equilibrium,

$$s = n^{-1/\alpha}S = n^{-1/\alpha}XE = n^{-1/\alpha}X^dE = n^{-1/\alpha}\phi(P_X)ME$$

(30)

Substituting (30) into (28), we can solve for equilibrium income

$$M = \frac{(L - fn)}{a_Y - (\frac{1-\alpha}{\alpha})cn^{(\alpha-1)/\alpha}\phi(P_X)E}$$

(31)

where $P_X$ is given by equation (19). Thus the equilibrium output of good $X$ is

$$X = \frac{(L - fn)\phi(P_X)}{a_Y - (\frac{1-\alpha}{\alpha})cn^{(\alpha-1)/\alpha}\phi(P_X)E}$$

The equilibrium output of good $Y$ is obtained from (26) with $X = \overline{X}$:

$$\overline{Y} = \frac{L - nf}{a_Y} - \left[ cEn^{1-(1/\alpha)} + k \right] \overline{X}$$

The equilibrium output of each specialized service firm is

$$\overline{s} = n^{-1/\alpha}\overline{XE}$$

Thus profits are non-negative if and only if

$$\frac{(L - fn)\phi(P_X)n^{-1/\alpha}E}{a_Y - (\frac{1-\alpha}{\alpha})cn^{(\alpha-1)/\alpha}\phi(P_X)E} > \frac{\alpha f}{(1-\alpha)c}$$

(32)
2.4. A continuum of components

It will be convenient, in dealing with trade between countries, to modify the basic model by assuming that there is a continuum of components instead of a finite number of components. Let \( \tau \) be the index for components. Without loss of generality, let the continuum be the interval \([0, 1]\). Let the price of component \( \tau \) be \( \pi(\tau) \). Then, the price of \( X \) is

\[
P_X = \int_0^1 \pi(\tau) d\tau
\]

where

\[
\pi(\tau) = w + e(\tau) P_S
\]

Without loss of generality, similarly to the discrete case, we take it that \( e'(\tau) > 0 \). Assume for simplicity that

\[
\theta(\tau) = b\tau, \text{ where } b > 0.
\]

Then we can draw the graph of \( \pi(\tau) \) as a function of \( \tau \) in Figure 1 below.

\[
\pi(\tau) = w + \tau b P_S
\]

In \((\pi, \tau)\) space the graph of equation (33) is a straight line with slope \( bP_S \).

In autarky equilibrium

\[
\pi(\tau) = w + \tau bn^{(\alpha-1)/\alpha} \left( \frac{cw}{\alpha} \right)
\]

3. Free Trade in Goods and Components

We now turn to consideration of free trade equilibrium. In this section, we assume that goods and components are freely traded while services are not internationally traded. The motivation for this assumption is that since services are very labor intensive the effective transportation cost of many services is quite high. Rather than model
\begin{align*}
\pi(\tau) &= w + \tau b P_s
\end{align*}

Figure 1: Price of Components
the transportation cost directly we begin with the simplest case of no trade in services. Later on, we allow trade in services to see how the results change.

3.1. Assumptions

Consider a two country world. The home country is the US, the foreign country is Mexico. All consumers have identical preferences. The endowments of labor are \( L \) and \( L^* \). There are \( n \) specialized service firms in the US, and \( n^* \) specialized service firms in Mexico, where we assumed \( n^* < n \). It is important to emphasize that our assumption that \( n^* < n \) reflects the stylized fact that more developed economies have a greater number of specialized services. We take the level of development of each economy as given.

Mexico has the component price curve

\[
\pi^*(\tau) = w^* + \tau b^* \left( \frac{c^* w^*}{\alpha} \right) \frac{1}{n^*(1-\alpha)/\alpha}
\]

The slope of this curve is

\[
b^* P^*_S \equiv b^* \left( \frac{c^* w^*}{\alpha} \right) \frac{1}{n^*(1-\alpha)/\alpha}
\]

We assume that the US has absolute advantage in producing the \( Y \) good, \( a^*_Y > a_Y \). Then under free trade, recalling that good \( Y \) is the numeraire good, we have

\[
1 = P_Y = \alpha_Y w = \alpha_Y^* w^* = P_Y^*
\]

This implies that at free trade equilibrium Mexico’s wage is lower than the US wage:

\[
w^* < w.
\]

If the slope \( b^* P^*_S > b P_S \), then the two curves \( \pi^*(\tau) \) and \( \pi(\tau) \) will intersect at a point \( \tau_I \).
For points to the left of $\tau_I$, Mexico is the cheaper producer of components while to the right of $\tau_I$ the US is the low cost producer of components. Thus when there is trade in components, Mexico will export those components with index $\tau < \tau_I$ and the US will export those components with $\tau_I < \tau < 1$. The intuition is that Mexico exports those components that are relatively labor intensive, and the US exports the relatively service intensive components.

It is important to note that the two assumptions $n > n^*$ and $w > w^*$ alone do not ensure that the component-price curve for US has a flatter slope than that of Mexico. The interplay between the cross country differences in wage, number of service firms, and $b$ will be explored in more detail below.

3.2. Free Trade Equilibrium

Our task in this sub-section is to analyze the determinants of trade in components, given that services are not traded.
Given our assumptions, it is clear that trade is driven by differences in three types of characteristics:

(i) An exogenous difference in labor productivity in the production of the numeraire good,

(ii) An exogenous difference in the fixed number of specialized services. This is the basis for the comparative advantage, through the variety effect, of lower cost for the aggregate service.

(iii) A Dornbush-Fischer-Samuelson-type pattern of comparative advantage across the ordering of components.

The key variable that we need to solve for is $\tau_I$. Since $\tau_I$ is the intersection of the two component-price curves, it must be the case that for component $\tau_I$ the production costs in both countries are equal:

$$w^* + b^* P^*_S \tau_I = w + b P_S \tau_I$$

(35)

This condition implies

$$w^* + \tau_I b^* (n^*)^{(\alpha - 1)/\alpha} \left( \frac{cw^*}{\alpha} \right) = w + \tau_I b n^{(\alpha - 1)/\alpha} \left( \frac{cw}{\alpha} \right)$$

(36)

Equation (36) determines $\tau_I$ as a function of $n, n^*, w$ and $w^*$. Then

$$\tau_I = \tau_I (n, n^*, w, w^*) = \frac{w - w^*}{\Delta}$$

(37)

where

$$\Delta \equiv (n^*)^{(\alpha - 1)/\alpha} \left( \frac{b^* c^* w^*}{\alpha} \right) - n^{(\alpha - 1)/\alpha} \left( \frac{bcw}{\alpha} \right) = b^* P^*_S - b P_S$$

It is possible that two degenerate cases can occur. If $\tau_I = 0$ then all components are made in the US. If $\tau_I \geq 1$ all components are produced in Mexico. Since we want to restrict attention to the more interesting case in which both countries produce components, we assume that
$\Delta > 0$ which along with our earlier assumption that $w > w^*$ implies that $\tau_I > 0$. We will need further restriction to ensure that $\tau_I < 1$.  

Note that if $b^* = b$ and $c^* = c$, the assumption $\Delta > 0$ would be consistent with $w^* < w$ only if the number of specialized services in Mexico, $n^*$, is less than the number of specialized services in the US. More generally, $\Delta > 0$ if and only if

$$\frac{n}{n^*} > h \text{ where } h \equiv \left[ \frac{bcw}{b^*c^*w^*} \right]^{\frac{\alpha}{\mu}}$$

This condition says that, for a diversified equilibrium where components are produced in both countries, the ratio of specialized services of the home country to that of the foreign country must be great enough to compensate for the adjusted wage ratio.

Assume that condition (38) holds, and that

$$1 > \frac{w - w^*}{\Delta}$$

Then the following results follow:

(i) The foreign country exports components in the range $[0, \tau_I]$ (i.e., components that are less service-intensive) and the home country exports components in the range $[\tau_I, 1]$ (that are more service-intensive).

(ii) An increase in $n^*$ or a decrease in $c^*$ will result in a greater range of components exported by the foreign country.

(iii) An increase in $w^*$ will result in a decrease (increase) in the range of components exported by the foreign country.

To prove these results, we must rule out corner solutions that would mean that only one of the countries produces components. To do that we need to show that $0 < \tau_I < 1$. Given our earlier result that $w^* < w$, then $\tau_I > 0 \iff \Delta > 0 \iff$

$$\left( \frac{n}{n^*} \right)^{\frac{1-\alpha}{\alpha}} > \frac{bcw}{b^*c^*w^*}$$

$$\iff n > \left( \frac{bcw}{b^*c^*w^*} \right)^{\frac{\alpha}{1-\alpha}} n^*$$
Next, $\tau_I < 1 \iff w - w^* < \Delta$. To show that result (i) holds note that at autarky, the price of components in the range $[0, \tau_I]$ are cheaper in the foreign country and will be exported by them. Components in the range $[\tau_I, 1]$ are cheaper in the home country and exported by the home country. Result (ii) follows by directly calculating the appropriate derivatives using equation (37). Result (iii) follows directly from the fact that $\tau_I$ is increasing in $w$ and $n^*$ and decreasing in $w^*$ and $n$.

The results (i) to (iii) above establish that comparative advantage in producing components depends on the interplay of two forces. A greater variety of services (more $n$) leads to cheaper aggregate services and more efficient production of components. Lower wages also means more component production. That means that an increase in the size of the service sector in the foreign country or a lower relative wage in the foreign country will lead to more “fragmentation”, or “outsourcing” by the home country’s car industry.

We next compute the price of $X$. Since the price of good $X$ must equal the cost of production, in a free trade equilibrium, we must have

$$P_X^* = P_X = \hat{P}_X = \left[ \int_0^{\tau_I} (w^* + b^* \tau P_S^*)d\tau \right] + \left[ \int_{\tau_I}^1 (w + b\tau P_S)d\tau \right] \tag{40}$$

More precisely

$$P_X^* = P_X = \hat{P}_X = \left[ \int_0^{\tau_I} (w^* + b^* \tau P_S^*(n^*, w^*)) d\tau \right] + \left[ \int_{\tau_I}^1 (w + b\tau P_S(n, w)) d\tau \right]$$

Thus, as shown in Appendix 2,

$$\hat{P}_X = \frac{1}{2} bn^{1 - \frac{1}{n}} \left( \frac{cw}{\alpha} \right) + \left[ w - \frac{1}{2\Delta} (w - w^*)^2 \right] > 0 \tag{41}$$
where the expression inside the square brackets is positive because of (39).

Clearly, higher wages in either country translate into higher car prices while expanding the service sector in either country results in lower car prices. The equilibrium world output of cars can then be determined. See the Appendix.

4. Free Trade in Goods, Components and Services

Now suppose that services are internationally tradable. For simplicity we assume that there are no transport costs associated with individual services\(^2\). Later, we discuss relaxing the assumption of zero transport costs. This tradability of services means that in equilibrium the prices of all services will be the same across countries and the same set of services will be available in both countries.

\(^2\)We maintain the assumption throughout that aggregate services are not traded.
We must be careful about the specification of overlap of services. Recall that the foreign country supplies $n^*$ specialized services, and the home country supplies $n$ specialized services. Assume $n > n^*$. Does the world as a whole have $n + n^*$ distinct specialized services, or only $n$ distinct specialized services? If the second case applies, we would have a set of pair of of firms (one in the home country, one in the foreign country) that supply identical services. If each pair is Bertrand rivals, typically only one firm will survive. If they are Cournot rivals, both firms may co-exist.

Let us assume that in the case of overlap, each pair is Bertrand rivals. Under autarky, non-negative profit requires that

\[ p_i \geq \frac{fw}{s_i} + cw \]  
\[ p_i^* \geq \frac{f^*w^*}{s_i^*} + c^*w^* \]  

With free trade in services equation (42) becomes

\[ p_i \geq \frac{fw}{s_i + s_i^*} + cw \]  
\[ p_i^* \geq \frac{f^*w^*}{s_i^* + s_i} + c^*w^* \]

So, the home market effect disappears with free trade in services and, for any given specialized service for which a pair of firms exist, the country with the lowest variable cost will provide that service for the world market. In the special case in which $f = f^*$ and $c = c^*$ then all services for which both countries are able to produce are only produced in the foreign country.

With trade in specialized services, the production function of aggregate services in the home country is

\[ S = \left[ \sum_i (s_i + s_i^*)^\alpha \right]^{1/\alpha} \]
and thus the price is

\[
P_S = \left[ \sum_{i=1}^{n^f} \min(p_i, p_i^*)^{\alpha/(\alpha-1)} \right]^{(\alpha-1)/\alpha}
\]

where \(n^f\) is the number of services that exist with free trade in services.

Hence, when there is free trade in services and there are no transport costs for individual services then in both countries the price and number of services available will be the same. This means that the price of aggregate services will be the same. It follows that, in the special case where \(b = b^*\) (i.e., in the absence of the Dornbush-Fischer-Samuelson type of comparative advantage profile), the costs of producing all components are lower in the foreign country since the price of the aggregate service is the same as in the home country and the wage rate is lower. The home country will in that case import all components—all car components are produced in Mexico. This is the case of total outsourcing from the point of view of the home country. This conclusion also applies when \(b > b^*\). In the case where \(b < b^*\) the common value \(P_S\) does not preclude the possibility that the two component-price curves still intersect at some \(\tau_i^+\) that is between zero and 1 (where clearly \(\tau_i^+\) is bigger than the \(\tau_f\) that is depicted in Figure 2).

5. Extensions

In this section we discuss several extensions. The first is to assume that trade in services involves transport cost (of the iceberg type). Further suppose that the transport cost of specialized services from the foreign country to the home country is low (say zero), while the transport cost of specialized services from home to the foreign country is prohibitively expensive. For example, typesetting or telephone answering services can be supplied by developing countries and exported without transport costs, while the exportation of forensic auditing services from developed countries may involve very high transport costs.
The effect of service trade on the prices of aggregate service are: $P_S$ becomes smaller, while $P^*_S$ is unchanged. Under these circumstances, the range of components produced in the foreign country will contract. Thus trade in services can lead to less fragmentation when there are transportation costs that are biased against services produced by the home country. Hence, we conjecture that we can establish the following: Trade in services in the presence of transportation costs that are biased against the specialized services supplied by the home country may lead to reduced outsourcing of manufacturing.

In order to motivate the idea that it might be more difficult to transport those services produced in the home country, one could introduce the idea that some services are more complex than others. It is convenient to have a continuum of services, indexed by $\rho$ in $[0, n]$. The greater is $\rho$, the higher is the variable cost $c(\rho)$. If more complex services are more difficult to transport and the home country specializes in the more complex services then trade in services could reduce fragmentation.

6. Concluding Remarks

We have developed a model that explores the link between services and fragmentation. In our model components used in the manufacturing process are made with services and labor. Low foreign wages create an incentive to outsource component production (fragmentation). However, the importance of services in the manufacturing process can play a role in limiting outsourcing of components to a low wage economy. Although advanced economies have higher costs of supplying simple services, because of the size of their home market they have a greater variety of services. As a result, their price of “aggregate services” tends to be lower than those of less developed economies. When services are not tradable advanced economies will tend to produce components that are service-intensive, and outsource the manufacturing of components that are less service-intensive (labor intensive) to developing countries. Complete free trade in services, however, elimi-
nates the home market effect of the developed country, and will lead to complete outsourcing of components. However, trade in services in the presence of transportation costs that are biased against the specialized services supplied by the home country may lead to reduced fragmentation of manufacturing.

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A APPENDICES

Appendix 1: An explicit expression for $\hat{P}_X$.
Denote the equilibrium value by $\hat{P}_X$. Then

$$\hat{P}_X = w^* \tau_I(n, n^*, w, w^*) + w \left[ 1 - \tau_I(n, n^*, w, w^*) \right] +$$

$$\left( \frac{1}{2} \right) b^* P^*_s(n^*, w^*) \left[ \tau_I(n, n^*, w, w^*) \right]^2$$

$$+ \left( \frac{1}{2} \right) b P_s(n, w) \left\{ 1 - \left[ \tau_I(n, n^*, w, w^*) \right]^2 \right\}$$

(44)

$$\hat{P}_X = w + \left( \frac{1}{2} \right) b P_s(n, w) + (w^* - w) \tau_I + \frac{1}{2} (\tau_I)^2 (b^* P^*_s - b P_s)$$

$$= w + \left( \frac{1}{2} \right) b P_s(n, w) - \frac{1}{2 \Delta} (w - w^*)^2$$

$$= \frac{1}{2} \frac{b n^{1-\frac{1}{\alpha}} (cw)}{\alpha} + \left[ w - \frac{1}{2 \Delta} (w - w^*)^2 \right] > 0$$

Appendix 2: Equilibrium world output of cars and trade pattern under free trade in final goods and in components
Let us compute the equilibrium output of cars. First, we must calculate world’s income, denoted by $Z$. In terms of good $Y$, world income (the sum of labor income and profits) is:

$$
Z \equiv wL + w^*L^* + n \left[ \left( \frac{1 - \alpha}{\alpha} \right) cm - f \right] w + n^* \left[ \left( \frac{1 - \alpha}{\alpha} \right) cm^* - f^* \right] w^*
$$

(45)

where

$$
w = \frac{1}{a_Y}
$$

$$
w^* = \frac{1}{a_Y^*}
$$

World demands for good 1 and good 2 are

$$
X^{\omega} = \phi(P_X)Z
$$

(46)

$$
Y^{\omega} = [1 - P_X\phi(P_X)]Z
$$

(47)

Suppose that in equilibrium, the foreign country exports components with index $\tau$ in the range $[0, \tau_I]$ and the home country exports components with index $\tau$ in the range $[\tau_I, k]$. Then the total quantity of aggregate services supplied by the foreign country is

$$
S^* = \left[ \int_0^{\tau_I} b^* \tau d\tau \right] X^{\omega} = \frac{1}{2} \tau_I^2 X^{\omega}
$$

(48)

Similarly, for the home country,

$$
S = \left[ \int_{\tau_I}^1 b \tau d\tau \right] X^{\omega} = \frac{1}{2} \left[ 1 - \tau_I^2 \right] X^{\omega}
$$

(49)

We also have

$$
s = n^{-1/\alpha} S
$$

$$
s^* = (n^*)^{-1/\alpha} S^*
$$

From (45),

$$
Z = wL + w^*L^* - wn f - w^*n^* f + \xi
$$
where $\xi$ is profit of the service sector (before subtracting fixed cost):

$$\xi \equiv \left( \frac{1 - \alpha}{\alpha} \right) [cwns + c^* w^* n^* s^*]$$

Using

$$s = n^{-1/\alpha} S = n^{-1/\alpha} b[1 - \tau^2_I] X^\omega$$  \hspace{1cm} (50)

$$s^* = (n^*)^{-1/\alpha} S^* = (n^*)^{-1/\alpha} b^* \tau^2_I X^\omega$$  \hspace{1cm} (51)

we get

$$\xi = q X^\omega$$

where

$$q \equiv (1 - \alpha) \left[ \frac{(w - w^*)^2}{\Delta} \right] + \left( \frac{1 - \alpha}{\alpha} \right) bcwn^{(1 - 1/\alpha)}$$

Thus (46) gives

$$X^\omega = \phi(\hat{P}_X) [wL + w^* L^* - wn f - w^* n^* f + q X^\omega]$$  \hspace{1cm} (52)

Therefore world equilibrium output of cars is

$$\hat{X}^\omega = \frac{\phi(\hat{P}_X) [wL + w^* L^* - wn f - w^* n^* f]}{1 - q \phi(\hat{P}_X)} > 0$$

Note that since $1 - P_X \phi(P_X) > 0$ and since $P_X > q$, we have $1 - q \phi(\hat{P}_X) > 0$. $(q X^\omega < P_X X^\omega$ because the value of output of $X$ must exceed the value of profits to the service industry, by the accounting identity that revenue equals the sum of payments to inputs.)

Now let us turn to the pattern of trade. Recall that cars can be assembled costlessly by assumption, using components that are freely traded. Thus there is no need for a country to import or export cars: component trade suffices. As argued in the text, under the assumption that $\tau_f$ is greater than zero and smaller than 1, the US will export components that are relatively service-intensive, i.e., components with $\tau > \tau_I$, and import components that are labor-intensive ($\tau < \tau_I$).
Let $\gamma$ denote US’s share of world income

$$\gamma = \frac{M}{M + M^*}$$

Then the consumption of cars in the US is $\gamma \hat{X}^\omega$ and that in Mexico is $(1 - \gamma)\hat{X}^\omega$. The value of US’s import of components is

$$I = \gamma \hat{X}^\omega \int_0^{\tau_I} \pi^*(\tau) d\tau$$

and the value of US’s export of components (i.e. Mexico’s import of components) is

$$I^* = (1 - \gamma)\hat{X}^\omega \int_{\tau_I}^1 \pi(\tau) d\tau$$

Thus, US net imports of component is $I - I^*$ which may be positive or negative. If it is positive, then to achieve overall trade balance, the US will export food. Otherwise, it will import food.

Note that

$$M = \frac{1}{a_Y} [L - nf] + \left(\frac{1 - \alpha}{\alpha a_Y}\right) [cn] = \frac{1}{a_Y} [L - nf] + \left(\frac{1 - \alpha}{\alpha a_Y}\right) cn^{1-(1/\alpha)} b [1 - \tau_I^2] \hat{X}^\omega$$

and

$$M^* = \frac{1}{a_Y^*} [L^* - n^* f^*] + \left(\frac{1 - \alpha}{\alpha a_Y^*}\right) c^* (n^*)^{1-(1/\alpha)} b^* \tau_I^2 \hat{X}^\omega$$

References


