Optimal education policies and comparative advantage

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July, 2011

Abstract

We consider the optimal education policies of a small economy whose government has a limited budget. Initially, the economy is closed and the government chooses its education policy to maximize welfare under autarky. When the economy trades with the rest of the world the government chooses a new education policy that maximizes welfare under trade. Is it ever optimal for the government to choose its new policy so that it reverses the economy’s comparative advantage? We find that if the budget stays fixed when it is optimal to ‘move up the skills chain’ it is not feasible. In such a case a foreign loan is welfare improving. A move in the opposite direction can be optimal and when it is optimal it is also feasible.

JEL: F11, O15

Key Words: Patterns of Trade, Education Policy, Welfare

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1 Introduction

During the second half of the past century many developing countries, that have traditionally been inward looking, opened their economies and began to trade with the rest of the world. Initially these economies specialized in low-skill goods and most of them still do. But some countries (e.g. countries in the East Asian region) have managed to transform their economies by shifting resources to high-skill sectors and thus reversing their patterns of trade. Their exports are now dominated by goods whose production requires the use of highly-skilled labor. These economies that have successfully achieved this transformation had to devote resources to education in order to equip workers with the new skills that were necessary for employment in high tech sector. Today, many governments aim to achieve similar results. Good examples are India and China where there is evidence that they actively pursue policies to help producers ‘move up the value chain’.¹ ²

Intuitively, these policies cannot be globally optimal. In equilibrium, some countries must have comparative advantage in the low-skill sector. For developing countries with limited government budgets that constrain their choices, understanding where their comparative advantage lies is important. The following two figures show the export patterns and education attainment levels of four countries that have followed four distinct development paths over the last three decades.

Figure 1 depicts the percentage of high-tech exports in total manufacturing exports over the period 1980-2000 for Argentina, Korea, Poland and United States.³ Not surprisingly, we find that over the whole period the exports of United States are dominated by high-tech products. In contrast, Argentina’s exports over the period are dominated by relatively low-tech products. The other two countries are examples of economies that have seen a change in their patterns of trade. For Korea we observe a steady increase

¹This is clear from the World Economic Forum’s reports on the China Business Summit 2003 and on the India Economic Summit 2004 and from daily business magazines and newspapers in these two countries.

²‘Moving up the chain’ has a dual meaning. In some cases it is taken to mean ascending a quality ladder where the products are still the same however their quality is increasing. In our context it implies a move along the production possibilities frontier substituting high-skill intensive goods for low-skill intensive ones.

³The data on exports were obtained from the World Trade and Production Database. For the separation of sectors into high-tech and low-tech ones we used the OECD classification of sectors according to their level of skills employed. We have experimented with different threshold levels but with no consequence for our comparisons.

In the Appendix we show similar results for a much larger sample of countries.
in the proportion of high-tech exports as a percentage of total exports that highlights the transformation of the economy during its high-growth period. For Poland we observe the reverse pattern. Before the collapse of the Soviet Union and consequently COMECON (the East-European Common market) Poland’s exports were dominated by relatively high-tech products with the main destination being the former Soviet Union. These sectors proved to be non-competitive and after the collapse of communism Poland’s patterns of trade were reversed.

Figure 1: Patterns of Trade

Figure 2 shows the educational attainment levels for the same four countries over the same period. More specifically, it shows the proportion of the population aged over 25 with post-secondary education. We observe that the countries with the highest post-secondary education attainment levels

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4 The data comes from Barro and Lee (2001).
are those with exports dominated by high-tech products. In particular, notice that while in the beginning of the 1980s Korea's attainment level was close to that of Argentina's and Poland's by the end of the 1990s it had reached considerably higher levels.

![Figure 2: Percent of Population with Post-Secondary Education](image)

In this paper we determine the conditions under which it is optimal for governments to encourage shifts in production that will eventually lead to a reversal in their pattern of trade. A number of recent papers, (Ishikawa, 1996; Grossman and Maggi, 2000; Grossman, 2004; Bougheas and Riezman, 2007) examine the relationship between an exogenous distribution of human capital and the pattern of trade. In this paper, we step back to consider how this distribution of human capital arises.

In our model the government plays a crucial role. It influences production patterns through its education policy that determines the distribution of skills in the economy. Traditionally, in trade models decisions on human capital accumulation are taken by agents according to their level of ability.\(^5\)

\(^5\)The relationship between human capital accumulation and trade was first considered
In our model all agents are identical hence, the distribution of skills in the economy is entirely determined by the government’s education policy. We do this to focus attention on the role of government policy in shaping the distribution of human capital and by implication trade patterns. We feel that this model is particularly relevant for developing nations with limited government budgets and whose citizens have relatively little to invest in individual human capital accumulation.\textsuperscript{6}

Our small open economy consists of two sectors, namely, a low-skill sector that produces a primary commodity and a high-tech sector that employs high-skill workers. The productivity of each worker depends on both his sector of employment and his level of education. Both product and labor markets are competitive. Initially, we consider the closed economy case and derive the optimal education policy that maximizes aggregate welfare under autarky. Next, we allow the economy to trade and we compare trade patterns when the skill distribution is the same as autarky and when it optimally adjusts education policy to free trade conditions.\textsuperscript{7} \textsuperscript{8}

We find that depending on the terms of trade, a move up the skills chain can be optimal. However, when this is the case, a ‘move up the chain’ is also infeasible exactly because the limited budget will not allow any change in education policy. We then consider the possibility of a foreign loan to finance an increased government budget. We find that when ‘moving up the chain’ is optimal the budget constrained country can benefit from a foreign loan as the welfare gains resulted from relaxing the government budget constraint exceed the welfare losses due to the lump-sum taxation imposed in order


\textsuperscript{6}Our model implies that the distribution of wealth in the economy depends entirely on government policy and not on personal characteristics. It is only to keep things simple that we have not introduced any heterogeneity among agents by specifying a distribution of ability. Had we done so government policies would still determine the distribution of educational attainment but in that case efficiency would require that the level of educational attainment for each agent depend on his level of ability.

\textsuperscript{7}In a recent paper, Egger, Egger, Falkinger and Grossmann (2005) follow a similar procedure to consider how individual educational choice is affected by the integration of capital markets.

\textsuperscript{8}Our focus is on long-term trends and thus we have ignored any short-term adjustment costs. For some potential pitfalls of our approach, see Davidson and Matusz (2004, 2006).
to repay the loan.\footnote{The theoretical literature on dynamic comparative advantage suggests that if the initial gains of trade are so high that they relax the factors constraining growth (in our case the limited budget constraint) then a reversal of exports from low-skill goods to high-skill goods might be optimal. This suggests that economies that move up the chain must be economies that grow fast. In this paper we focus on distributional aspects of government policy and, for analytical tractability, we abstract from dynamic considerations. For theoretical work on dynamic comparative advantage see Bond, Trask and Wang (2003), Matsuyama (1992), Redding (1999) and Ventura (1997).} We also find that reversals in the opposite direction (moving "down the chain") can also be optimal and that such reversals are not budget constrained.

2 The Model

Consider a two-sector small open economy inhabited by a continuum of agents of unit measure. Sector $X$ produces a high-tech product while sector $Y$ produces a primary commodity (numeraire). In both sectors labor is the only input in production, however, the productivity of each worker depends on his level of education and his sector of employment. To keep things simple we assume that there are three levels of education, namely low, medium, and high. Workers with a low level of education (type $l$) can only find employment in sector $Y$ where they produce 1 unit while workers with a medium level of education (type $m$) can produce $v (>2)$ units in either sector.\footnote{Setting the productivity the same in the two sectors keeps the number of parameters low and, below, it will become clear that it is without any loss of generality.} The high level of education (type $h$) is useful only to workers employed in the high-tech sector where each produces $V (>v)$ units. Workers with a high level of education employed in the primary sector produce $v$ units of output.

The distribution of educational attainment in the economy is completely determined and financed by the government. Agents are initially identical but education separates them into three skill groups that correspond to the three levels of education. We assume that the low level of education is provided to all agents and that the fixed education budget of the government is sufficiently high to cover its cost. The remaining budget is equal to $b$ units of the numeraire. We assume that the size of the education budget is exogenously determined. We normalize to unity the cost of providing an agent with the medium level of education and denote by $c$ the cost of providing an agent with the high level of education. We impose the following restrictions on the parameters of the model:
\textbf{Condition 1} \quad \frac{v}{c} > v > 2

\textbf{Condition 2} \quad b < c

The first condition implies, as will demonstrate below, that it will be socially optimal to employ all workers with the medium level of education in the primary sector.\textsuperscript{11} The second condition implies that the government cannot provide all agents with the high level of education, however, it does not necessarily imply that the government is financially constrained. As long as both goods are consumed in equilibrium then it is inefficient to provide agents employed in the \(Y\) sector with the high level of education. A sufficient condition for a financially constrained government is that \(b = 1\) as either some agents employed in the \(X\) sector will be type \(m\) or some agents employed in the \(Y\) sector will be type \(l\). We do not force the government to allocate the whole budget to education. Rather, we allow the government to redistribute any unspent money from the education budget in a lump-sum fashion to all workers.

Let \(\theta_i\) (\(i = l, m, h\)) denote the proportion of type \(i\) agents. The government’s choice of \(\theta_i\)’s must satisfy the following two constraints:

\[ \theta_l + \theta_m + \theta_h = 1 \quad (1) \]

and

\[ b \geq \theta_m + c\theta_h \quad (2) \]

where the second constraint states that government spending on education cannot exceed the budget.

All agents have identical Cobb-Douglas preferences specified as:

\[ U_i = (X_iY_i)^{\frac{1}{2}} \quad i = l, m, h \quad (3) \]

where \(X_i\) and \(Y_i\) denote a type \(i\)’s worker consumption of the high-tech product and primary commodity, respectively.

\textbf{2.1 The Endogenous Production Possibilities Frontier}

The government’s choice of education policy determines the economy’s production possibilities which is shown graphically in Figure 3. The reason

\textsuperscript{11}Notice that if \(v < 2\) it is never optimal to employ agents with the medium level of education in the primary sector. The reason is that we do not force the government to allocate the whole budget on education. Since an agent with a low level of education can still produce 1 unit of the primary commodity it only makes sense to spend an additional unit to bring her education up to the medium level only if \(v > 2\).
that the frontier is vertical at the point where it intersects the horizontal line is that type $l$ workers can only produce the primary commodity. The maximum amount of $X$ that can be produced is attained when all type $m$ and type $h$ workers are employed in that sector. Given the education distribution in order to produce more than $\theta_l$ units of $Y$ (remember that a type $l$ worker can produce 1 unit of $Y$), efficiency requires that the workers first to change employment are type $m$ workers and, thus the middle section of the frontier has a slope equal to 1. As the production of $Y$ is further increased the slope takes the value $\frac{v}{V}$ because type $h$ workers can each produce either $V$ units of $X$ or $v$ units of $Y$. For what follows, it is important to notice that any changes in the allocation of the education budget will alter the frontier.

\[
\text{Slope} = \frac{v}{V} \\
\text{Slope} = 1
\]

Figure 3: Endogenous Production Possibilities Frontier

3 Autarky

We derive the equilibrium under autarky in two stages. Under the assumption that all markets are competitive, we begin by deriving the equilibrium
price and the corresponding production and consumption allocations for an arbitrary education policy. Then, we derive the education policy that maximizes aggregate welfare. The following preliminary result significantly simplifies the equilibrium analysis.

**Proposition 1** If the government is financially constrained, efficiency requires that type \( l \) and type \( m \) workers are employed in the \( Y \) sector and type \( h \) workers are employed in the \( X \) sector.

**Proof.** Suppose not. Then one of the following must be true:

a) Aggregate production of the \( X \) sector is less than \( V \theta_h \). But this implies that some type \( h \) workers are employed in the \( Y \) sector. Further, a binding government constraint means that \( \theta_l > 0 \). Then the government could have enhanced welfare by reducing \( \theta_h \) and increasing \( \theta_m \) as this change in policy would result in a higher output of the primary commodity without any reduction in the production of the high-tech product. We have a contradiction.

b) Aggregate production of the \( X \) sector is more than \( V \theta_h \). But this implies that some type \( m \) workers are employed in the \( X \) sector. Consider a small increase in the proportion of type \( h \) workers. Then the budget constraint implies that the proportion of type \( m \) workers has to be reduced with \( \frac{d\theta_m}{d\theta_h} = -c \). Now suppose that after this change you keep the production in sector \( X \) constant. Given that all type \( h \) workers are employed in sector \( X \) then the proportion of type \( m \) workers employed in this sector will be reduced and \( \frac{d\theta_m}{d\theta_h} = -\frac{V}{v} \). But since \( \frac{V}{v} > c \) the reduction in the proportion of type \( m \) workers in sector \( X \) is higher than the reduction in the overall proportion of type \( m \) workers in the economy and therefore after the above change the government can increase production in sector \( Y \) without decreasing production in sector \( X \). We have a contradiction. 

The above result implies that, given the government’s education policy, allocation efficiency requires that production in sector \( X \) must be equal to \( V \theta_h \) while production in sector \( Y \) must be equal to \( \theta_l + v \theta_m \). However, we also need to make sure that it is indeed in the interest of all agents with the medium level of education to find employment in the primary sector. Given that these agents produce exactly the same number of physical units in each sector they will choose the primary sector for their employment if and only if they expect that \( p^A < 1 \), where \( p^A \) denotes the autarky price (keeping in mind that the primary commodity is the numeraire.) We are going to demonstrate that there exists a rational expectations equilibrium where both agents’ expectations and the equilibrium price satisfy the above inequality.
Let \( I_i^A \) denote the income of a type \( i \) worker. Maximization of (3) subject to the budget constraint yields the demand functions:

\[
X_i = \frac{I_i^A}{2p^A}, \quad Y_i = \frac{I_i^A}{2}
\]

where proposition 1 implies that \( I_i^A = 1, I_m^A = v, \) and \( I_h^A = p^AV \). Equilibrium under autarky requires that the following market clearing conditions for sectors \( X \) and \( Y \) respectively, are satisfied:

\[
V\theta_h = \frac{1}{2} \left[ \frac{1}{p^A} (\theta_l + v\theta_m) + V\theta_h \right]
\]

and

\[
\theta_l + v\theta_m = \frac{1}{2} \left[ \theta_l + v\theta_m + p^AV\theta_h \right]
\]

where in both conditions the left-hand side equals the supply of that good and the right-hand side equals the corresponding demand. Solving either of the above market clearing conditions for the equilibrium autarky price we get:

\[
p^A = \frac{\theta_l + v\theta_m}{V\theta_h} \tag{5}
\]

It also follows from proposition 1 that \( 1 > p^A > v/V \).

### 3.1 Optimal education policy

The optimal education policy corresponds to the solution of the following program:\textsuperscript{12}

\[
\max_{\theta_l} \frac{1}{2} \left[ (\theta_l + v\theta_m) \left( \frac{1}{p^A} \right)^{\frac{1}{2}} + V\theta_h \left( p^A \right)^{\frac{1}{2}} \right]
\]

\[
= \frac{1}{2} (p^A)^{-\frac{1}{2}} \left[ \theta_l + v\theta_m + V\theta_h p^A \right] \tag{6}
\]

subject to (5),

\[
\theta_h = 1 - \theta_l - \theta_m \tag{7}
\]

\textsuperscript{12} Of course, this is the correct program under the assumption that it is optimal to spend the whole budget on education. Assuming that this is the case amounts to restricting the budget to be less than \( \frac{1}{2}(1 + c) \). To see this consider the case set \( \theta_l = 0 \) and maximize the same objective function subject to the constraint \( \theta_m + \theta_h = 1 \). The optimal solution is \( \theta_m = \theta_h = \frac{1}{2} \). Thus, if the government’s budget was above \( \frac{1}{2}(1 + c) \) then it would be optimal to allocate part of the excess budget to increase \( \theta_h \) and the rest to redistribute.
and

\[ \theta_m = \frac{c(1 - \theta_l) - b}{c - 1} \]  \quad (8)

where the last two constraints follow from (1) and (2).

The optimal proportion of type \( l \) workers under autarky is:

\[ \theta_l^A = \frac{1 - b - c + bc - bv + 2cv - bcv}{2(1 - c + cv)} \]  \quad (9)

By substituting the above solution in (7) and (8) we find the optimal solutions for \( \theta_h \) and \( \theta_m \), respectively, and then by substituting these solutions in (5) we can solve for the optimal price under autarky:\(^{13}\)

\[ p^{A*} = \frac{1 + c(v - 1)}{V} \]  \quad (10)

Notice that the autarky price does not depend on the size of the budget. This is because we have focused our attention to the case of an interior solution for the education policy; i.e. when \( \theta_l > 0 \), \( \theta_m > 0 \) and \( \theta_h > 0 \). In this case, because preferences are homothetic, the size of the budget does not affect the ratio of the production levels of the two goods and hence the equilibrium price. For intermediate values of budget size, as the budget changes, the proportions of the three types of agents adjusts so that the autarky price stays constant.

By substituting (9) in (8) and differentiating with respect to \( b \) we find that \( \theta_m \) is increasing as the budget increases. When the budget is sufficiently low we have \( \theta_m^A = 0 \). In that case

\[ \theta_l^A = \frac{c - b}{c}, \quad \theta_m^A = 0, \quad \text{and} \quad \theta_h^A = \frac{b}{c} \]

Using (5) we find that the equilibrium autarky price for this case, is given by

\[ p^{A1} = \frac{c - b}{bV} > \frac{1 + c(v - 1)}{V} = p^{A*} \]

Notice that \( v \) does not appear in the above solution because there are not any type \( m \) workers. Also notice that the relative price decreases as the budget increases. This is because the budget restrains output in the high-tech sector

\(^{13}\)Notice that the government maximizes the expected utility of each agent. Thus the autarky price does not make agents indifferent to their level of education as it would in a model with private accumulation of human capital. Put differently, in equilibrium there will be welfare inequality even if all agents are identical.
X. As the budget size increases the proportion of type $h$ workers increases while the proportion of type $l$ workers decreases. Equating $p^{A1}$ with $p^{A*}$ we find a threshold level for the budget, given by

$$b_1 = \frac{c}{2 + c(v - 1)}$$

such that when $b < b_1$, $\theta^A_m = 0$.

There is another threshold level for the budget, $b_2$, such that when the budget is higher than this threshold $\theta^A_l = 0$. In that case (5) implies that the corresponding autarky price is given by:

$$p^{A2} = \frac{v(c - b)}{V(b - 1)} < p^{A*}$$

Equating $p^{A2}$ with $p^{A*}$ we find that

$$b_2 = \frac{1 + c(v - 1) + cv}{1 + c(v - 1) + v}$$

To complete the solution we need to verify that the solution is a rational expectations equilibrium, i.e. $p^{A*} < 1$. It suffices to show that $V > 1 + c(v - 1)$. Rearranging we get $\frac{V}{c} - v > \frac{1}{2} - 1$. The demonstration is completed by noticing that the first inequality of Condition 1 implies that the expression on the left is positive while $c > 1$ implies that the expression on the right is negative.

Figure 4 shows the autarky price as a function of the budget. Notice that if $b \geq \frac{1}{2}(1 + c)$ the budget constraint under autarky is not binding.

4 Trade

Suppose that the small economy trades with the rest of the world at the world price $p^*$ and that the government does not adjust its education policy. Then it is clear that if $p^A > p^*$ the economy will export the primary commodity and if $p^A < p^*$ it will export the high-tech product. However, the government can further enhance welfare by adjusting its education policy after the change in the trade regime.

By substituting the world price for the autarky price in (6) we obtain the government’s problem under trade.

$$\max_{\theta} \frac{1}{2}(p^*)^{-\frac{1}{2}}(\theta_i + v\theta_m + V\theta_h p^*)$$
which using (7) and (8) can be written as:

$$\max_{\theta_l} \frac{1}{2} (p^*)^{-\frac{1}{2}} \left( \theta_l + \frac{c(1 - \theta_l) - b}{c - 1} + \left( 1 - \theta_l - \frac{c(1 - \theta_l) - b}{c - 1} \right) V p^* \right)$$

Differentiating with respect to $\theta_l$ we get

$$\frac{1}{2} (p^*)^{-\frac{1}{2}} \left( 1 - \frac{c}{c - 1} - V p^* + V p^* \frac{c}{c - 1} \right) \quad (11)$$

Notice that the above expression is independent of $\theta_l$ which implies that we obtain corner solutions. The intuition is that under free trade it is optimal for the economy to specialize as long as it is allowed by the budget constraint. When the budget is sufficiently high so that the corresponding constraint is not binding we also allow the government to redistribute any budgetary surplus.
Furthermore, the optimal education policy under free trade depends on the sign of (11). In particular, the expression is equal to 0 when the world price \( p^* \) is equal to \( p^A \). The following proposition outlines the optimal production patterns under trade.

**Proposition 2** *(Optimal Production Patterns)* (a) If \( p^* > p^A \) it is optimal that the economy produces as much as possible of the high-tech product, \( X \) (the budget will not allow complete specialization). (b) If \( p^* < p^A \) it is optimal that the economy specializes in the production of the primary commodity, \( Y \).

**Proof.** For this proof keep in mind that \( p^A \) is equal to the optimal autarky price, that is the autarky price given that the government has chosen its educational policy optimally. (a) In this case (11) is greater than 0 which implies that \( \theta_l \) must be set as high as possible. This is because, given the budget constraint, the only way that the economy can increase the production of \( X \) is by increasing \( \theta_h \) that can only be accomplished by increasing \( \theta_l \) while decreasing \( \theta_m \). At the optimum we have \( \theta_l = \frac{c-b}{c} \), \( \theta_m = 0 \), and \( \theta_h = \frac{b}{c} \). (b) In this case (11) is less than 0 and optimality requires to set \( \theta_l \) as low as possible so that \( \theta_m \) is at the maximum possible level. If \( b < 1 \), \( \theta_m = b \) and if \( b > 1 \), \( \theta_m = 1 \) (budget surplus).

Notice that the above optimal production decisions do not depend on the price under autarky. This is in contrast to traditional trade models where the optimal production decisions and hence the patterns of trade depend on the difference between the autarky price and the world price.

The reason is that in traditional models the production possibilities frontier is fixed. In our model, when the government changes the education mix it also changes the production possibilities frontier. We will see that this is crucial for understanding patterns of trade reversals. To see how this actually works consider Figure 4 and the case in which the budget is strictly less than \( b_1 = \frac{c}{2(c-1)} \). If \( p^A > p^* \) and the budget is unchanged the country would export the primary commodity \( Y \). If the government can increase the budget to \( b_1 \) then it would pay the country to export the high-tech product \( X \). So, if a country can increase its budget it would want to ‘move up the chain.’ Alternatively consider what happens if the budget is larger than \( b_2 = \frac{1+c(v-1)+cv}{1+c(v-1)+v} \). If \( p^A < p^* \) then with an unchanged budget the country would export the high-tech product \( X \). However, it will be welfare improving to reduce the budget to \( b_2 \) and export the primary commodity \( Y \), that is, in this situation it pays a country to ‘move down the chain.’ We spell out the details in the next two propositions.
The following proposition defines the patterns of trade before and after the change in education policy for all possible autarky prices assuming a fixed budget. Let $X^-$ or $Y^-$ denote the good that was exported before the change in education policy and $X^+$ or $Y^+$ denote the good that is exported after the change.

**Proposition 3** Optimal trade patterns generated by the education policies that are optimal (a) under autarky and (b) under free trade are as follows:

**Case 1:** $b < b_1$
1a: If $p^* > p^A$ then $X^-$ and $X^+$
1b: If $p^A > p^* > p^{A*}$ then $Y^-$ and $Y^+$
1c: If $p^A > p^{A*} > p^*$ then $Y^-$ and $Y^+$

**Case 2:** $b_1 < b < b_2$
2a: If $p^* > p^A = p^{A*}$ then $X^-$ and $X^+$
2b: $p^A = p^{A*} > p^*$ then $Y^-$ and $Y^+$

**Case 3:** $b_2 < b$
3a: If $p^* > p^{A*} > p^A$ then $X^-$ and $X^+$
3b: If $p^{A*} > p^* > p^A$ then $X^-$ and $Y^+$
3c: If $p^{A*} > p^A > p^*$ then $Y^-$ and $Y^+$

**Proof.** Consider the patterns of trade before the change in education policy. Then it is clear that when $p^* > p^A$ it was optimal for the economy to export the high-tech product $X$ while when $p^* < p^A$ it was optimal to export the primary commodity $Y$. Next, consider the patterns of trade after the change in education policy. With the exception of case 1b, they depend on the patterns of specialization derived in proposition 2. In case 1b, the education policy is determined by proposition 1 and welfare is maximized when the economy specializes in the high-tech product $X$. However, the binding budget constraint does not allow the government to further increase production in that sector and thus it keeps exporting the primary commodity $Y$.

The proposition identifies four type of economies. Sub-cases (1a), (2a) and (3a) identify economies for which it is always optimal to export the high-tech product while sub-cases (1c), (2b) and (3c) identify economies for which it is always optimal to export the primary commodity. Sub-case (1b) (discussed above) identifies economies that would like to move up the chain but cannot do so because of the limited size of their budget. In sub-case (3b) economies want to move down the chain and it is feasible for them to do so. Next, we take a closer look at the latter two cases.
Moving up the chain: Notice that case 1b is the only instance where it would be optimal for the government to adjust its education policy in order to reverse the pattern of trade so that the economy ‘moves up the chain’. What prevents the government from pursuing such a policy is the binding budget constraint. In this case, borrowing from a foreign country to finance the increase in education expenditures is beneficial. As the following proposition suggests, the welfare gains resulting from a change in the patterns of trade will be higher than the welfare loss incurred from a lump-sum tax imposed to finance the foreign loan.

**Proposition 4** Suppose that \( p^A > p^* > p^{A*} \). Then borrowing to finance increased education expenditures is welfare improving and the country will "move up the chain".

**Proof.** We know that in this case it is optimal for the economy to maximize the production of the high-tech product; thus \( \theta^A_l = \frac{c-b}{c} \), \( \theta^A_m = 0 \), and \( \theta^A_h = \frac{b}{c} \). Using (6) we find that welfare is equal to

\[
\frac{1}{2}(p^*)^{-\frac{1}{2}} \left[ \frac{c-b}{c} + V \frac{b}{c} p^* \right]
\]

Then the new welfare level after an increase in the budget by \( \Delta b \) that is financed by a lump-sum tax, is equal to

\[
\frac{1}{2}(p^*)^{-\frac{1}{2}} \left[ \frac{c-b-\Delta b}{c} \left( 1 - \frac{\Delta b}{c} \right) + \frac{b+\Delta b}{c} \left( V p^* - \Delta b \right) \right]
\]

where the increase in the budget allows for a greater proportion of agents receiving the high level of education. Subtracting the former expression from the latter we get

\[
\frac{1}{2}(p^*)^{-\frac{1}{2}} \left[ \frac{-\Delta b}{c} - \frac{c-b-\Delta b}{c} \Delta b + \frac{\Delta b}{c} V p^* - \frac{b+\Delta b}{c} \Delta b \right]
\]

\[
= \frac{1}{2}(p^*)^{-\frac{1}{2}} \left[ V p^* - 1 - c \right]
\]

Given that \( p^* > p^{A*} \) the expression above is larger than

\[
\frac{1}{2}(p^*)^{-\frac{1}{2}} \left[ V \frac{1+c(v-1)}{V} - 1 - c \right]
\]

which is positive given that \( v > 2 \). \( \blacksquare \)
Moving down the chain: Proposition 3 identifies one instance, that is case 3b, where a reversal in the patterns of trade is optimal and feasible without any outside intervention. The government can increase welfare by encouraging producers to specialize in the production of the primary commodity. This is because the world price of the high-tech good is relatively low and thus welfare is higher when the economy specializes in the production of the primary commodity. In contrast, when the economy exports the high-tech product the gains from trade are low because of the relatively small differential between the autarky price and the world price. Given that the economy completely specializes in the production of the primary commodity the government redistributes the surplus \( b - 1 \) using lump-sum payments.

5 Conclusion

Education policies affect an economy’s skill distribution, its competitiveness with the rest of the world and, as a consequence, its patterns of trade. It follows then, that a country’s optimal education policy would depend on whether it is a closed economy or one that is open to trade.

In the beginning of this paper we asked the following question. Is it ever optimal for a government of a small developing economy that moves from autarky to trade, and with an initial skill distribution that was optimal under autarky, to change its education policy so that its patterns of trade are reversed? We show that as long as the budget remains fixed, when it is optimal to change from an economy that exports low-skill goods to one that exports high-skill goods, such a change will not be feasible without any outside financial assistance.

Our results have also some interesting policy implications for the provision of aid. Any economy with a binding budget can benefit from aid. However, the size of the benefits can differ significantly. For those countries for which it is optimal to move up the chain, relaxing the budget constraint would allow them to switch their patterns of trade and in that case the welfare gains can be very high. In fact they can be sufficient to cover the cost of financing the original loan. Our model also suggests that under certain circumstances reversals in the opposite direction (moving "down the chain") might also be optimal.
References


