Trade and the Distribution of Human Capital*

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Abstract

We develop a two-country, two-sector model of trade where the only difference between the two countries is their *distribution* of human capital endowments. We show that even if the two countries have identical aggregate human capital endowments the pattern of trade depends on the properties of the two human capital distributions. We also show that the two distributions of endowments also completely determine the effects of trade on income inequality. Then, we prove that there are long-term gains from trade if the marginal utility of income is constant or as long as losers from trade are compensated by winners. Finally, we look at a simple majority voting model. It turns out depending on the distribution of human capital, autarky and free trade with and without compensation may be the outcome of majority voting.

Key Words: Patterns of Trade, Income Distribution, Welfare, Political Economy
JEL: F1

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1 Introduction

The impact of trade on income inequality has been a topic widely discussed in both academic and policy forums. What has triggered interest in this topic is a growing concern among industrialized nations about their ability to sustain high standards of wellbeing in the face of competition from low wage countries. These issues have been addressed theoretically by models in which trade occurs because of differences in technologies and endowments.[1] However, it has also been noted that a large volume of international trade takes place between rich countries and they have similar technologies and endowments.[2]

In order to address these issues in a way that accounts for these facts we develop a two-country, two-sector model of trade where the only difference between the two countries is in their *distribution* of human capital endowments.[3] Their technological capabilities and the preferences of their consumers are identical.[4] In each country there is a primary sector where output is produced using labor and a high-tech sector that uses human capital as its input. We will demonstrate that even if the two countries have identical aggregate human capital endowments they will trade with the patterns of trade depending on the properties of the two human capital distributions.[5]

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[1] Both the theoretical and empirical literatures are extensive and have recently been reviewed by Feenstra and Hanson (2001).

[2] See Brander (1981), Davis (1995), Grossman and Maggi (2000) and Krugman (1979) for theoretical attemts to account for this observation.

[3] We consider differences in both means (aggregate endowments) and varances.

[4] Yeaple (2005) has a model in which worker heterogeneity, technology differences and trade costs jointly determine firm heterogeneity.

[5] Bond (1986) considered a trade model where firm heterogeity arises because of variations in entrepreneurial ability. He analyzed the relationship between factor intensities, factor returns and and patterns of trade, however, he kept the distribution of ability fixed throughout the paper.

To our knowledge, Ishikawa (1996) was the first to explore the relationship between the distribution of human capital and the patterns of trade. However, he has restricted his attention to countries that differ in aggregate endowments while we are also interested in differences in the variance of the two distributions.

Grossman and Maggi (2000) using production technologies where workers' talents can be complementary in some sectors and substitutable in others have also found that the distribution of human capital can potentially matter for a country's patterns of trade. This is in contrast with our paper where as long as the distributions differ the two countries can benefit from trade.

In addition, both of the above papers focus on trade patterns while we are also interested on trade's consequences for inequality and welfare.

Lastly, distributions also matter in Grossman (2004) but in his model firms are not

We will also show that together, the two distributions of endowments also completely determine the effects of trade on income inequality. More specifically, we will find that inequality always rises in the country that exports the high-tech product and declines in the country that exports the primary commodity.

Next, we explore the welfare implications of our model. We compare total welfare under autarky with the corresponding welfare under free trade and find that, unless the marginal utility of income is constant, there exist free-trade equilibria that are welfare reducing. If we allow income redistribution, then there are always long-term gains from trade for each member of society as long as losers are compensated.[6]

Finally, we ask what outcome would emerge in a simple majority voting framework. We find that in the absence of redistribution, autarky or free trade could be the equilibrium choice of a majority of the population. There is also an equilibrium in which free trade is chosen but overall welfare declines. In that case, free trade is preferred by the majority but the losses of the losers outweigh the gains of the winners.

If, in addition to voting on free trade, we also allow voters to vote on whether there should be income redistribution that ensures no member of society loses from trade, then autarky can never be an equilibrium. However, there still is an equilibrium in which free trade is chosen, redistribution fails to be approved and overall welfare declines. We begin by developing the model.

2 The Model

There are two countries: A and B. Each country is populated by a continuum of agents of measure 1. Each agent (i for country A and j for country B) is endowed with one unit of labor and some level of human capital, $h_i\,(h_j)$, randomly drawn from the interval $[1, h_{MAX}]$. Let f_A and f_B denote the density functions and F_A and F_B the corresponding human capital distribution functions of countries A and B respectively.

There are two goods X and Y. Good Y is a primary commodity and each unit produced requires one unit of labor. In contrast, good X is a high-tech

perfectly informed about workers' productivity and their output is not verifiable by their employees.

[6]Here, we completely ignore any short-term adjustment costs as the economy moves from one regime to another. See Davidson and Matusz (2002, 2004) for interesting work in this area.

product and each unit produced requires one unit of human capital. The amount of good X produced by an agent corresponds to their level of human capital. So, an agent with human capital h_i produces h_i units of good X.

All agents derive utility from the consumption of both goods and they have identical homothetic preferences.

2.1 Autarky

In this section, we derive the equilibrium under autarky. Without any loss of generality we concentrate on country A. We first derive the production possibilities frontier. The maximum amount of good Y that can be produced is equal to 1. Each agent uses her single labor unit endowment to produce one unit of the primary good. The slope of the PPF at the point where it intersects the x axis is equal to $-(1/h_{MAX})$. This is because efficiency requires specialization according to comparative advantage, and the agent with the most comparative advantage in producing X is agent h_{MAX}. However, as production of the high-tech product increases the PPF gets steeper because the new producers have lower human capital endowments. The maximum amount of good X that the economy can produce, $\hat{h}_A$ is attained when all the agents produce good X, hence, is equal to the average endowment of human capital. That is,

$$\hat{h}_A = \int_1^{h_{MAX}} h_i f_A(h) dh$$

The marginal rate of transformation is equal to $\frac{dY}{dX} = 1/h'$ where h' is equal to the human capital endowment of the agent with the highest endowment among those producing good Y.

2.1.1 Equilibrium

Define as p_A the relative price (i.e. the price of good Y measured in units of good X), $q_A(X)$ the quantity produced of good X and $q_A(Y)$ the corresponding quantity of good Y. Then,

Proposition 1 *Equilibrium under Autarky:*

The equilibrium price satisfies $1 < p_A < h_{MAX}$ *and there exists a critical level of human capital endowment,* h_A^**, such that* $p_A = h_A^*$*, all agents with* $h_i < h_A^*$ *produce good* Y*, all agents with* $h_i > h_A^*$ *produce good* X*,* $q_A(Y) = \int_1^{h_A^*} f_A(h)dh$*, and* $q_A(X) = \int_{h_A^*}^{h_{MAX}} h f_A(h)dh$*.*

Proof. The proposition follows from straightforward arbitrage arguments. ■

Notice that agents with human capital endowments equal to h_A^* are indifferent between producing X or Y.

2.1.2 Income Distribution

Next we derive the economy's income distribution. In order to measure incomes we need a numeraire. It is clear that any income evaluation is affected by the choice of numeraire. However, as long as we are interested in changes in inequality this choice is inconsequential. With this in mind we use good X as the numeraire. Then, for each type of equilibrium we can derive the corresponding income distribution of the economy. Let z_i denote the income of agent i. Then,

Proposition 2 *Income Distribution under Autarky:*

Under Autarky, $z_i = h_A^*$ for all i such that $h_i \leqslant h_A^*$, and $z_i = h_i > h_A^*$ for all i such that $h_i > h_A^*$. The proportion of agents with income exactly equal to h_A^* is given by $F_A(h_A^*)$ and the proportion of agents with income higher than h_A^* ($h_A^* < h_i < h_{MAX}$) is given by $1 - F_A(h_A^*)$.

The intuition behind the above result is the following. Each agent's income is equal to the value of her marginal product. The marginal product of all agents employed in sector Y is equal to 1 and $p_A = h_A^*$. The marginal product of those agents employed in sector X is equal to their endowment of human capital h_i and the price of the high-tech product is equal to 1 (numeraire).

We next illustrate what autarky equilibrium looks like for a particular utility function.

Example 1 *Suppose that preferences are described by the utility function: $U(X,Y) = U(X,Y) = AX^{\gamma}Y^{\delta}$. For a given price p_A, those agents with $h_i \geqslant p_A$ (producers of X) maximize the above utility subject to the budget constraint: $h_i - X - p_A Y = 0$ that yields the following demand functions:*

$$X = h_i \frac{\gamma}{\gamma+\delta}, \quad Y = \frac{h_i}{p_A}\frac{\delta}{\gamma+\delta}$$

while those agents with $h_i \leqslant p_A$ (producers of Y) maximize the same utility subject to the budget constraint: $p_A - X - p_A Y = 0$ that yields the demand functions:

$$X = p_A \frac{\gamma}{\gamma+\delta}, \quad Y = \frac{\delta}{\gamma+\delta}$$

The equilibrium price h_A^* is such that the supply of Y (demand for X) is equal to the demand for Y (supply of X); in other words it satisfies the following equality:

$$\int_1^{h_A^*} \frac{\gamma}{\gamma+\delta} f_A(h)dh = \int_{h_A^*}^{h_{MAX}} \frac{h_i}{h_A^*} \frac{\delta}{\gamma+\delta} f_A(h)dh$$

Notice that each producer of Y produces 1 unit, consumes $\frac{\delta}{\gamma+\delta}$ units and supplies $\frac{\gamma}{\gamma+\delta}$ units of Y. Simplifying the above expression we get:

$$p_A = h_A^* = \frac{\delta}{\gamma} \frac{\int_{h_A^*}^{h_{MAX}} h_i f_A(h)dh}{F_A(h_A^*)} = \frac{\delta}{\gamma} \frac{q_A(X)}{q_A(Y)} \tag{1}$$

2.2 Free Trade

We next turn to consideration of opening up to international trade. In our two country model the only way that the two countries differ is in their *distributions* of human capital endowments. In general, this implies that $p_A \neq p_B$ ($h_A^* \neq h_B^*$) which means that autarky prices differ in the two countries. Different relative autarky prices imply that there are opportunities for trade. It is clear that the world price p_T will be between the two autarky prices and that the country with the higher autarky price will export good X and import good Y.

2.2.1 Patterns of Trade

Suppose that the two human capital distributions have the same mean which implies that the two countries have the same aggregate endowments. If the two human capital *distributions* are different then the autarky prices will be different. This implies that aggregate endowments may not be accurate predictors of the patterns of trade. This leads to two questions.

The first is under what conditions will a country that has a higher aggregate endowment in human capital export the human capital intensive good? The second question is what properties of human capital distributions provide reliable guides to predict trade patterns? We answer the first question with the following proposition.

Proposition 3 *Suppose that preferences are Cobb-Douglas the sizes of the two countries are equal and let $F_B(h)$ dominate $F_A(h)$ in the sense of first-order stochastic dominance. Then country B, that is the human capital abundant country, will export the human capital intensive good.*

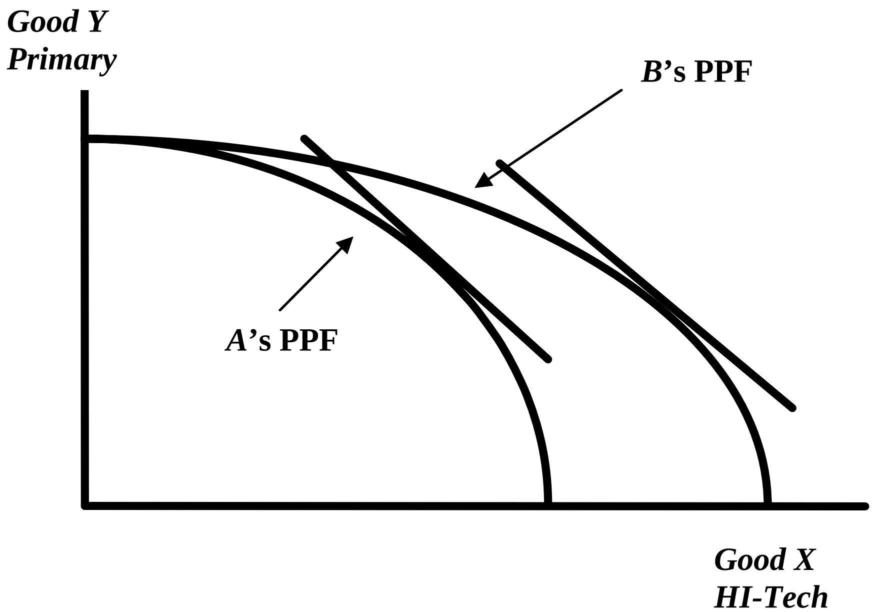

Figure 1: Comparative Advantage under First-Order Stochastic Dominance

Proof. We need to show that $h_A^* < h_B^*$. First-order stochastic dominance implies that $F_A(h) \geqslant F_B(h)$ and $\int_{h'}^{h_{MAX}} h_i f_A(h)dh \leqslant \int_{h'}^{h_{MAX}} h_i f_B(h)dh$ for every h'. Then the inequality follows directly from the autarky price equilibrium condition (1). ■

This proposition identifies the patterns of trade for the case in which one human capital distribution dominates the other in the sense of first-order stochastic dominance. In this case the variances of the two distributions do not matter. Hence, the pattern of trade depends only on aggregate endowments as in the Heckscher-Ohlin-Samuelson model. Figure 2 illustrates the above result.

Given that the populations of the two countries are equal both production possibilities frontiers intersect the y axis at the same point. However, if the two countries produce only the high-tech good then country B, that is the country with the higher endowment, will produce more. What the proposition demonstrates is that when the two production possibilities frontiers have the same slope then $\frac{q_A(Y)}{q_A(X)} > \frac{q_B(Y)}{q_B(X)}$. Thus, given homothetic preferences, country B exports the high-tech good.

The above result contrasts with the no-trade result that Grossman and Maggi (2000) derive under first-order stochastic dominance. In their model, both sectors exhibit constant returns to scale in talent (our human capital) and thus the slopes of the two PPFs at points where any ray through the origin crosses them are equal. Then homotheticity implies that under autarky the two countries produce exactly the same ratio of quantities of the two goods and thus there is no comparative advantage and hence, no trade. In contrast, in our model the returns to human capital vary across sectors and thus, there are gains from trading.

To provide an answer to the second question we consider the case of two distributions of human capital that have the same mean but different variance (mean-preserving spreads) with the additional restriction that their cumulative distribution functions cross only once. Let the variance of country A's distribution be higher than that of country B's. In the terminology of Grossman and Maggi (2000) the country A's distribution is more *diverse* than country B's. Figure 3 shows the two production possibilities frontiers.

Notice that the two PPFs share the same intercepts. This is because (a) the populations of the two countries are equal that implies that the maximum amount of good Y that they can produce is the same, and (b) aggregate endowments are the same which implies that the maximum amount of good X that they can produce is also the same. Notice that country B's PPF lies inside country A's PPF. This follows from the fact that if $F_A(h)$ is more diverse than $F_B(h)$ then $\int_{h'}^{h_{MAX}} hf_A(h)dh > \int_{h'}^{h_{MAX}} hf_B(h)dh$. That is, if the two countries produce both goods and also produce the same quantity of Y then country A will produce a higher quantity of X. The following proposition describes the patters of trade.

Proposition 4 *Suppose that preferences are Cobb-Douglas and that country A's distribution is more diverse than country B's. Then if the demand for the primary good is relatively strong country A will export the high-tech product and if the demand for the high-tech product is relatively strong country A will export the primary commodity.*

Proof. (Use Figure 2) First note that in the vicinity of the y intercept country A's PPF is flatter than B's while it is steeper in the vicinity of the x intercept. Then, continuity implies that there exists a unique ray through the origin ($\frac{q_A(Y^*)}{q_A(X^*)} = \frac{q_B(Y^*)}{q_B(X^*)}$) such that at the points where it crosses the two PPFs their slopes are equal ($MRT_A = MRT_B = MRT^*$). If we are to

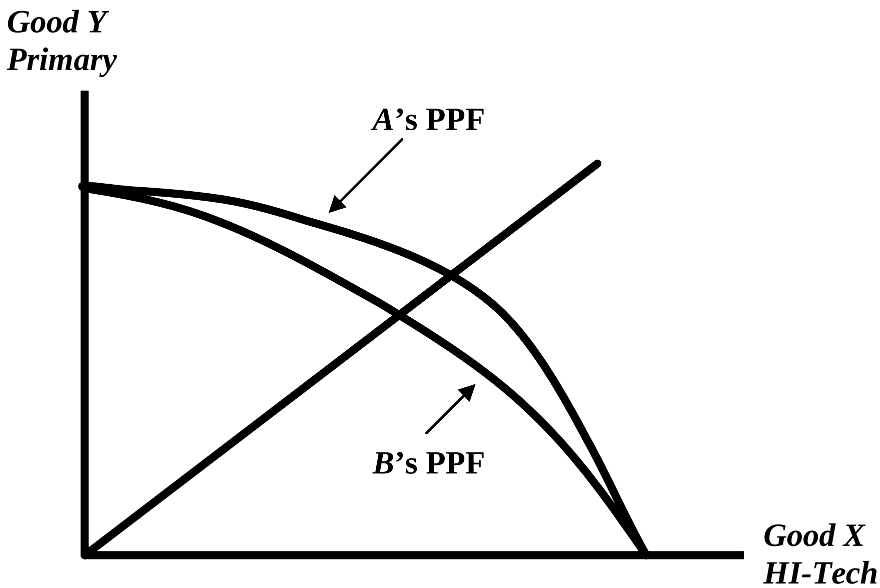

Figure 2: Comparative Advantage under Mean-Preserving Spreads

the left of that ray (relatively strong demand for the primary commodity), i.e.$MRS < MRT^*$, then $\frac{q_A(Y)}{q_A(X)} < \frac{q_B(Y)}{q_B(X)}$ meaning that Country A exports X, the high-tech good. If we are to the right of that ray (relatively strong demand for the high-tech product), i.e.$MRS > MRT^*$, then $\frac{q_A(Y)}{q_A(X)} > \frac{q_B(Y)}{q_B(X)}$ and country B exports the high-tech good. ■

One implication of the above discussion is that when countries only differ in the distributions of their endowments, these distributions alone are not sufficient to determine the pattern of trade. We also need to know the exact specification of preferences even when they are the same for all agents. Once more, this result contrasts with the corresponding result in Grossman and Maggi (2000). In their model the more diverse country always exports the good that is produced using a process characterized by input substitutability while the less diverse country exports the good that is produced using a process characterized by input complementarity. Therefore, the two distributions completely determine the pattern of trade.

2.2.2 Trade and Inequality

We compare the income distributions of each country under autarky and after trade. In this section, we are only interested in changes in inequality and thus the choice of numeraire does not matter. We begin with the following lemma.

Lemma 1 *Trade increases inequality in the country that exports the high-tech product and reduces inequality in the country that exports the primary commodity.*

Proof. Without any loss of generality, assume that $h_B^* < h_G^* < h_A^*$. therefore at the global equilibrium country A exports the high-tech product while country B exports the primary commodity. Using good X as the numeraire, we observe that, after trade, country A's income distribution is as follows: $z_i = h_G^*$ for all i such that $h_i \leqslant h_G^*$, and $z_i = h_i > h_G^*$ for all i such that $h_i > h_G^*$. Comparing this distribution to the corresponding one obtained under autarky we find that all agents with $h_i \leqslant h_G^*$ (proportion equal to $F_A(h_G^*)$) have experienced a decrease in income equal to $h_A^* - h_G^*$, those agents with $h_G^* < h_i < h_A^*$ (proportion equal to $F_A(h_A^*) - F_A(h_G^*)$) have experienced a decrease in income equal to $h_A^* - h_i$, while the income of the rest of the agents (proportion equal to $1 - F_A(h_A^*)$) has remained the same. Therefore, agents with low endowments of human capital have experienced the greatest relative loss in income, the loss of agents with moderate endowments group has been more moderate, while the incomes of those agents with high endowments has remained unchanged. Similarly, comparing country B's after trade income distribution to the corresponding one obtained under autarky we find that the income of agents with low endowments (proportion equal to $F_B(h_B^*)$) has increased by $h_G^* - h_B^*$, the income of those agents with moderate endowments (proportion equal to $F_B(h_G^*) - F_B(h_B^*)$) has increased by $h_G^* - h_i$, while the incomes of those agents with high endowments (proportion equal to $1 - F_B(h_G^*)$) has remained unchanged.[7] ■

We can now prove the main result of this section.

Proposition 5 *Suppose that preferences are Cobb-Douglas.*

Part 1: Suppose that country A's distribution dominates country B's distribution in the sense of first-order stochastic dominance. Then trade will increase inequality in country A and decrease inequality in country B.

Part 2: Suppose that countries A and B have the same aggregate endowments but country A's distribution is more diverse than country B's. Then trade will increase inequality in country A if there is relatively strong demand for the primary good while inequality in country B will increase if there is relatively strong demand for the high-tech good.

Proof. The proof of part 1 follows from Proposition 3 and Lemma 1 and that of part 2 follows from Proposition 4 and Lemma 1. ■

The above results suggest that trade has the opposite effect on the income inequality of the two trading partners. In contrast, Feenstra and Hanson (1996) and Zhu and Trefler (2005) find that inequality increases in both countries. The difference is that they are interested in trade between developed and developing nations where trade is driven because of a technological gap while we are interested in trade between countries with similar technologies.

Propositions 3, 4, and 5 together suggest that whether the gap between two countries' inequality measures increases or decreases after they trade depends on the patterns of trade which in turn, depend on the two endowment distributions and preferences.

[7] In general, we need to be cautious with inequality comparisons because one needs to take into account not only relative income changes but also absolute ones. For example, an increase in the gap between rich and poor does not necessarily imply an increase in inequality if it is also accompanied by an increase in per capita income that is uniformly distributed. Nevertheless, such concerns are clearly irrelevant for our model. When inequality increases, depending on the numeraire used either the rich get richer and the poor stay the same or the poor get poorer and the rich stay the same.

3 Welfare

In this section we demonstrate that uncompensated trade does not necessarily enhance social welfare. We know that not all agents gain from trade. But here we are going to prove a stronger result; namely that if the losers are not compensated then trade might reduce social welfare. We measure welfare by using a standard additive social welfare function:

$$W(h,p) = \int_1^{h_{MAX}} U(X(h,p),Y(h,p))f(h)dh \tag{2}$$

Let p^a denote the equilibrium relative price under autarky. If trade is welfare improving then the following must be true:

$$p^a = \arg\min \left\{ \int_1^{h_{MAX}} U(X(h,p),Y(h,p))f(h)dh \right\}$$

That is, if trade is welfare improving then the social welfare must be minimized when agents trade at autarky prices. We can prove the following result:

Proposition 6 *Suppose that the preferences of agents are described by the utility function $U(X,Y) = AX^{\gamma}Y^{\delta}$. Then, unless $\gamma + \delta = 1$, there exists a set of prices such that if the country trades at those prices its welfare will decrease.*

Proof. See the Appendix. ■

To understand the intuition for this result consider the postulated 'weighted utilitarian' social welfare function. One can think of this welfare function as representing the expected utility of an agent whose endowment is randomly drawn from a distribution that is the same as the distribution of aggregate endowments. Suppose we change γ and δ but we keep the ratio $\frac{\gamma}{\delta}$ constant. We know that such a change will only affect the marginal utility of income leaving equilibrium prices and quantities unaltered. However, expected utility valuations are affected by changes in the marginal utility of income.

We next identify the relationship between the marginal utility of income and the social welfare minimizing prices to better understand the circumstances under which uncompensated trade can reduce social welfare. The next proposition completely characterizes the prices for which social welfare falls.[8]

[8] It will become clear that the result must hold for any atomless distribution with a convex domain. However, our method of proof cannot be applied for general specifications of distribution functions.

Proposition 7 *Suppose that the preferences of agents are described by the utility function* $U(X,Y) = AX^{\gamma}Y^{\delta}$ *and that endowments are uniformly distributed on the interval* $[1,2]$*. If* $\gamma+\delta < 1$ *then there exists an interval* $(\underline{p}, p^a)$ *such that if the country trades at a price in that interval its social welfare will be lower relative to autarky. Similarly, if* $\gamma + \delta > 1$ *then there exists an interval* $(p^a, \overline{p})$ *such that if the country trades at a price in that interval its social welfare will be lower relative to autarky.*

Proof. See the Appendix. ■

Putting Propositions 6 and 7 together the intuition is straightforward. When $\gamma + \delta < 1$ the marginal utility of income is decreasing in income. We know that when the equilibrium free trade price is below the autarky price inequality increases. What happens in this case is that trade transfers income from agents with low endowments of human capital to agents with relatively high endowments. But, given that agents marginal utility of income is decreasing in income, the absolute value of the welfare losses of those agents with low endowments are higher than the welfare gains of those agents with high endowments rich. In contrast, when $\gamma + \delta > 1$ the marginal utility of income is increasing in income. When the equilibrium free trade price is above the autarky price inequality decreases. In this case, trade transfers income from agents with high endowments to agents with relatively low endowments. But given that agent's marginal utility of income is increasing the absolute value of the welfare losses of those agents with high endowments are higher than the welfare gains of those agents with low endowments.

4 Trade and Political Economy Equilibrium

In the previous section we showed that welfare results depend critically on whether or not there is redistribution of income to compensate those agents who suffer losses under free trade. In this section, we demonstrate that such policies might be ruled out in a political economy equilibrium. In addition, we are going to show that it is possible that the majority might vote for *trade without redistribution* even when trade reduces aggregate welfare. We adopt a very simple political economy model and assume that majority voting decides (a) the choice between autarky and trade, and (b) any redistribution policies.[9] Our work follows the median-voter approach to trade policy that

[9] Implicitly, in the text we have assumed that when both votes are available they take place simultaneously. However, the results remain the same in the case of as sequential voting procedure.

was first employed by Mayer (1984) in his classic work on endogenous tariff formation.

We completely characterize the political economy equilibria for the case of diminishing marginal utility of income ($\theta < 1$)[10] and prices in the interval $(1, 2)$. Our proposition characterizes equilibria when redistribution is not on the political agenda. We then discuss how the results would change when redistribution is available. Let h^m denote the human capital endowment of the median voter; i.e. $F_A(h^m) = 0.5$.

Remember that when the marginal utility of income is diminishing if $\underline{p} < p_T < p_A$ uncompensated trade reduces social welfare. We need to consider three cases. The first case is when $1 < p_T < \underline{p} < p_A < 2$. We know that the welfare of all those agents with human capital endowments such that $h > p_A$ is higher under trade and the welfare of all agents with human capital endowments such that $h < p_T$ is lower under trade. Since utility is weakly monotonic in endowments it implies that for those agents, with human capital endowments such that $p_T < h < p_A$ there exists a threshold level of endowment h_1 such that the welfare of all agents with human capital endowments such that $p_T < h < h_1$ is lower under trade and the welfare of all agents with human capital endowments such that $h_1 < h < p_A$ is higher under trade.

The second case is when $1 < \underline{p} < p_T < p_A < 2$. As in the previous case, there exists a threshold level of endowment h_2 such that the welfare of all agents with human capital endowments such that $p_T < h < h_2$ is lower under trade and the welfare of all agents with human capital endowments such that $h_2 < h < p_A$ is higher under trade.

The last case is when $1 < \underline{p} < p_A < p_T < 2$. Now, the welfare of all those agents with human capital endowments such that $h < p_A$ is higher under trade and the welfare of all agents with human capital endowments such that $h > p_T$ is lower under trade. Using a similar argument as above we can show that there exists a threshold level of income h_3 such that the welfare of all agents with human capital endowments such that $p_A < h < h_3$ is higher under trade and the welfare of all agents with human capital endowments such that $h_3 < h < p_T$ is lower under trade.

When the political agenda does not include the option of redistribution we have the following proposition:

Proposition 8 *Characterization of Politico-Economic Equilibria without redistribution for $\theta < 1$.*

[10] Similar results can be obtained when $\theta \geqslant 1$.

Let $1 < p_T < \underline{p}$
If $h_1 > h^m$ *then Autarky*
If $h_1 < h^m$ *then Trade (Social welfare increases)*
Let $\underline{p} < p_T < p_A$
If $h_2 > h^m$ *then Autarky*
If $h_2 < h^m$ *then Trade (Social welfare decreases)*
Let $p_A < p_T < 2$
If $h_3 > h^m$ *then Trade (Social welfare increases)*
If $h_3 < h^m$ *then Autarky*

Now suppose the possibility of redistribution is included in the political agenda. In the above three cases that result in Autarky, we will get trade with redistribution chosen and in these cases welfare increases. In the three above cases in which free trade is the equilibrium, it means that the majority of voters are better off under trade and that majority will vote for trade, but against redistribution. In those three cases, since redistribution will be voted down, the outcome is unchanged by introducing the possibility of redistribution. So, introducing the availability of redistribution always leads to free trade, and in some, but not all cases, to welfare improvement.[11]Interestingly, there is the possibility that free trade without redistribution is chosen and social welfare falls.

5 Conclusion

In this paper, we have assumed that the distribution of human capital is exogenous. One obvious extension would be to allow for endogenous accumulation of skills. This can be accomplished by considering an economy in which 2-period lived agents spend their first period of their lives investing in skill accumulation while during the second period produce, trade and consume. In such a model, the agents' investment in skills will depend on their expectations about both government policies and the trade regime. Because of the associated costs with skill accumulation, underemployment of human capital becomes a much more serious issue.

There are two types of government policies that would be worthwhile to consider; namely redistribution policies and educational subsidies. There is a growing literature that examines issues related to the relationship between skill accumulation and income inequality but the majority of the work in

[11]Mayer (1984) resticted his analysis to the case of a constant marginal utility of income and thus in his model uncompensated trade is always welfare increasing.

this area has ignored government policies. Two exceptions are Deardoff (1997) and Janeba (2000). However both papers focus on the optimality of government policies ignoring their potential implementation in systems where decisions are not taken by a social planner but rely on a majority rule.

Another possible extension is to consider the problem that governments face when they decide how to allocate a fixed budget for investments in human capital accumulation. In this case government policies completely determine the distribution of human capital (there is no initial distribution to begin with) which in turn will determine the patterns of trade and post-trade income distribution.

A third extension would be to apply our model to immigration issues. As it stands our model cannot explain immigration because we obtain factor price equalization.[12] However, by adding a third factor, say physical capital, that is complimentary to human capital factor price equalization might fail. Our analysis suggests that immigration or emigration of agents will affect both welfare and income distribution.

6 APPENDIX

6.1 Proof of Proposition 6

Using the demand functions that we derived in example 1, we find that we can write the indirect utility function V of an agent with income h who trades at price p as

$$V = c\frac{h^{\gamma+\delta}}{p^{\delta}}$$

where $c = A\left(\frac{\gamma}{\gamma+\delta}\right)^{\gamma}\left(\frac{\delta}{\gamma+\delta}\right)^{\delta}$. Notice that the income of an agent who produces the primary commodity is equal to p.

Then, using (2), social welfare is given by:

$$p^{\gamma}\int_{1}^{p} f(h)dh + p^{-\delta}\int_{p}^{h_{MAX}} h^{\gamma+\delta} f(h)dh$$

[12] In Ishikawa (1996) immigration is possible because national economies of scale with respect to human capital imply that an individual's efficiency units change with migration and factor price equalization obtains only in terms of efficiency units.

The f.o.c. condition for a minimum is given by:

$$p^{-1}\left(\gamma p^{\gamma}F(p)-\delta p^{-\delta}\int_{p}^{h_{MAX}}h^{\gamma+\delta}f(h)dh\right)=0$$

Notice that the s.o.c. is also satisfied. Rearranging the above expression we find that if the social welfare minimizing price is given by the solution of the following equation::

$$p^{\gamma+\delta}=\frac{\delta}{\gamma}\frac{\int_{p^a}^{h_{MAX}}h^{\gamma+\delta}f(h)dh}{F(p)}$$

The proof is completed by adding the observation that unless $\gamma+\delta=1$ the solution of the above equation will not be equal to p^a.

6.2 Proof of Proposition 7

Let $\theta=\gamma+\delta$ and $k=\frac{\delta}{\gamma}$. From proposition 6 we know that the price that minimizes social welfare is given by the solution to the following equation

$$p^{\theta}=k\frac{\int_{p}^{2}h^{\theta}dh}{p-1}$$

In order to prove the proposition we need to show that this price increases with θ, i.e. the marginal utility of income. The reason that this step is sufficient follows from (a) proposition 6, where we have shown that when $\theta=1$ the minimum is attained at the autarky price, and (b) the continuity of the social welfare function with respect to p. After solving the integral and rearranging the above expression we get

$$p^{1+\theta}\left(1+\frac{k}{1+\theta}\right)-p^{\theta}=k\frac{1}{1+\theta}2^{\theta+1} \tag{A1}$$

The left-hand side of (A1), denoted by L, is strictly increasing in p while the right-hand side, denoted by R, is independent of p. Then to complete the proof we need to show that $\frac{dL}{d\theta}<\frac{dR}{d\theta}$. Now,

$$\frac{dL}{d\theta}=p^{\theta+1}\left(1+\frac{k}{1+\theta}\right)\log p-p^{\theta+1}\frac{k}{(1+\theta)^2}-p^{\theta}\log p$$

and

$$\frac{dR}{d\theta}=-\frac{k}{(1+\theta)^2}2^{\theta+1}+\frac{k}{1+\theta}2^{\theta+1}\log 2$$

thus

$$\frac{dR}{d\theta} - \frac{dL}{d\theta} = \frac{k}{1+\theta}\left(2^{\theta+1}\log 2 - p^{\theta+1}\log p\right) - \frac{k}{(1+\theta)^2}(2^{\theta+1} - p^{\theta+1}) - p^{\theta}(p-1)\log p \quad (A2)$$

From (A1) we find that

$$\frac{k}{1+\theta} = \frac{p^{\theta}(p-1)}{2^{\theta+1} - p^{\theta+1}}$$

We can substitute this expression in (A2) to get

$$\begin{aligned}\frac{dR}{d\theta} - \frac{dL}{d\theta} &= \frac{p^{\theta}(p-1)}{2^{\theta+1} - p^{\theta+1}}\left(2^{\theta+1}\log 2 - p^{\theta+1}\log p\right) - \frac{1}{1+\theta}p^{\theta}(p-1) - \\ &\quad p^{\theta}(p-1)\log p \\ &= \frac{2^{\theta+1}\log 2 - p^{\theta+1}\log p}{2^{\theta+1} - p^{\theta+1}} - \frac{1}{1+\theta} - \log p \\ &= (1+\theta)2^{\theta+1}(\log 2 - \log p) - (2^{\theta+1} - p^{\theta+1})\end{aligned}$$

But this last expression is monotonically decreasing in p for $1 < p < 2$ and it is equal to 0 for $p = 2$ which completes the proof.

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