Pareto-Improving Trading Clubs Without Income Transfers

Kazumichi Iwasa
Kobe University

Raymond Riezman
University of Iowa

Koji Shimomura
Kobe University

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Abstract

We show that, if the standard revenue and expenditure functions exist, any group of countries can engage in a Pareto-improving non-discriminatory tariff reform *without income transfers*, if (i) there are more than two tradable goods and (ii) the initial tariff vectors of the member countries satisfy the non-proportionality condition. We then show that if these two conditions hold then countries can form a *Pareto-optimal* customs union. Depending on initial conditions, transfers may be necessary for the customs union to be *Pareto-improving*.

Keywords: customs unions, transfer payments

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Correspondence: Raymond Riezman, Department of Economics, University of Iowa, Iowa City, IA 52242; 319-335-1956 (Fax); raymond-riezman@uiowa.edu (email).

1 Introduction

Three decades have passed since Kemp and Wan (1976) published one of the most important papers on customs union theory. The Kemp-Wan theorem states that starting from any initial equilibrium, if inter-country transfers are allowed, there always exists a set countries that can form a customs union that is Pareto-improving. Thus, starting from any initial equilibrium one can successively apply the Kemp-Wan theorem, enlarging the customs union, until free trade is reached.

The Kemp-Wan paper has spawned a large literature that has extended their result in many directions. Kowalczyk and Sjostrom (1994) show how inter-country transfer payments, calculated using the Shapley value, can be used to facilitate trade liberalization in a setting of multilateral trade negotiations. Konishi, Kowalczyk, and Sjostrom (2003) prove an interesting result showing that starting from any initial equilibrium if customs unions are required to have no effects on non-member countries (be Kemp-Wan customs unions), then one can find a set of inter-country transfers that will lead all countries to choose free trade. That is, free trade is in the core of a Kemp-Wan customs union game.

Ohyama (2002) and Panagariya and Krishna (2002) get results for free trade areas rather than customs unions. Again, their results rely on the use of inter-country transfer payments[1]. Raimondos-Moller and Woodland (2004b) consider non-discriminatory tariff reforms in trading clubs and show that there exist such reforms which if accompanied by inter-country transfers within club members produce a Pareto improvement.

A series of papers Richardson (1995), Kemp and Shimomura (2001), and Raimondos-Moller and Woodland (2004a) explore the issue of what happens

[1] Their work requires that each member country keep its trade with the rest of the world fixed whereas Kemp-Wan requires only that the aggregate trade vector of all member countries is fixed with respect to the rest of the world. For a more complete discussion see Kemp (2007).

when some or all tariffs are set optimally in a Kemp-Wan framework.

All these contributions as well as the Kemp-Wan paper rely crucially on the existence of inter-country transfer payments within customs unions, free trade areas or trading clubs. Our paper takes a different approach. We ask how far the Kemp-Wan Pareto-improving result can hold *without* inter-country transfers.

Specifically, we show that for any trading club if the initial tariff vectors of member countries satisfy a non-proportionality condition in an initial tariff-ridden world equilibrium then a Pareto-improving non-discriminatory tariff reform is possible. We also show that a Pareto-optimal customs union, which does not harm the rest of the world, is always possible with no inter-country transfers allowed. However, this customs union need not be Pareto-improving for the member countries.

Before formally deriving the main results, let us outline what we do. In Figure 1 we consider a trading club formed by Country 1 and Country 2 which agree to implement non-discriminatory tariff reform. Point F is a pair of the initial utility levels of the two countries; the dashed line CED is the utility possibility frontier under the following constraints:

Constraint (i) Tariffs to outside countries are adjusted to keep the international price vector unchanged (a la Kemp-Wan) so that the welfare of non-member countries is unchanged.

Constraint (ii) Income transfers are not allowed.

The solid line AEB is the utility possibility frontier when income transfers between the two countries are allowed and Constraint (i) holds. Thus, one could describe any point on the solid curve as Pareto-optimal.[2] Clearly, the initial utility pair is never above the dashed line CED, although it is not necessarily

[2]Strictly speaking, any point on AEB should be called *restricted* Pareto-optimal, since the frontier is derived subject to Constraint (i). For brevity, however, we omit the expression "restricted" in what follows.

strictly below. Our first main result is to show that the initial utility pair is strictly below the dashed line if initial tariff vectors of Country 1 and Country 2 are not proportional to each other. Thus, as is depicted by the arrow FF' in Figure 1, there is a Pareto-improving non-discriminatory tariff reform possible between Country 1 and Country 2 without income transfers. Apparently, *the non-proportionality condition* is very mild. Hence, we can say that Pareto-improving non-discriminatory tariff reforms without inter-country transfers are generally possible.

Now, let us outline the second main result. Since the Pareto-optimal frontier AEB is derived under milder constraints than the dashed line *CED*, the former must lie above the latter. Our second main result is that there exists at least one point which is shared by the two curves, as is shown by point E. Since any point on AEB is Pareto-optimal, the marginal rates of substitution have to be the same between Country 1 and Country 2. Therefore, the two countries can get to point E by choosing a common tariff vector equal to the difference between the initial international price of each good in terms of the numeraire good and the marginal rate of substitution between them. One can think of point E as a pair of utilities that is established when the two countries form a customs union. Therefore, it follows that, under the non-proportionality condition, two countries can form a Pareto-optimal customs union without harming the rest of the world.

If the initial pair of utility levels is within the rectangular area $OMES$ in Figure 1, then even if inter-country transfers are unavailable, the two countries can set a common tariff vector which makes both countries better off without harming the rest of the world. This second result implies that for certain initial utility levels (i.e., within the rectangular area $OMES$ in Figure 1) a Kemp-Wan Pareto-improving customs union is possible without inter-member transfers.

Section 2 sets the model. Section 3 and Section 4 derives the two main results. Section 5 provides a diagrammatic explanation of them. Section 6 concludes.

2 The Model and Equilibrium Conditions

In our model there are N countries and the rest of the world. We consider a non-discriminatory tariff reform by a trading club that consists of these N countries. The number of tradable goods is $M+1$, including the numeraire denoted as good 0. Let

$$\bar{p} \equiv \begin{pmatrix} \bar{p}_1 \\ \vdots \\ \bar{p}_M \end{pmatrix} \text{ and } \bar{\tau}^n \equiv \begin{pmatrix} \bar{\tau}_1^n \\ \vdots \\ \bar{\tau}_M^n \end{pmatrix} \neq 0$$

be the initial international equilibrium price vector of the non-numeraire goods and the tariff vector of Country n, $n = 1, ..., N$, imposed by the government of Country n.[3]

Next, define $E^n(p_0, p, \bar{u}^n)$ as the aggregate expenditure function of Country n and $F^n(p_0, p)$ is the aggregate revenue function of Country n. We impose two equilibrium conditions. The first is a material balance condition.

$$S(\bar{p}) = \sum_{n=1}^{N} \{E_p^n(1, \bar{p} + \bar{\tau}^n, \bar{u}^n) - F_p^n(1, \bar{p} + \bar{\tau}^n)\} \tag{1}$$

where $S(\bar{p})$ is the excess supply vector from the rest of the world. Material balance requires that net excess demand of the N countries must be equal to the excess supply from the rest of the world.

The second equilibrium condition is a balance of payments condition.

$$\begin{aligned} & E^n(1, \bar{p} + \bar{\tau}^n, \bar{u}^n) - F^n(1, \bar{p} + \bar{\tau}^n) \\ = \; & (\bar{\tau}^n)^T \left[E_p^n(1, \bar{p} + \bar{\tau}^n, \bar{u}^n) - F_p^n(1, \bar{p} + \bar{\tau}^n)\right], \; n = 1, ..., N, \end{aligned} \tag{2}$$

[3] We assume here that specific tariffs are used and that no tariff is charged on the numeraire good. In the appendix, we show that there is no substantial difference if we consider the tariff imposed on the numeraire good or ad valorem tariffs.

The superscript T denotes the transpose of a column vector to which it is attached. Equation (2) is a standard balance of payments condition. It requires that for all N countries expenditure equals revenue from production plus tariff revenue.

For convenience, in what follows, we write $E^n(1, \bar{p}+\bar{\tau}^n, \bar{u}^n)$ and $F^n(1, \bar{p}+\bar{\tau}^n)$ as $E^n(\bar{p}+\bar{\tau}^n, \bar{u}^n)$ and $F^n(\bar{p}+\bar{\tau}^n)$.

3 The First Main Result: Pareto-improving Non-discriminatory Reform

We now turn to establishing our first result, namely that under certain conditions one can achieve a Pareto-improving tariff reform without using international transfer payments. First, we totally differentiate (1) with respect to τ^n, $n = 1, ..., N$, and u^1 around the initial tariff-ridden equilibrium, we have

$$\begin{aligned} 0 &= \sum_{n=1}^{N} [E^n_{pp}(\bar{p}+\bar{\tau}^n, \bar{u}^n) - F^n_{pp}(\bar{p}+\bar{\tau}^n)] d\tau^n \\ &\quad + E^1_{pu}(\bar{p}+\bar{\tau}^1, \bar{u}^1) du^1 \end{aligned} \tag{3}$$

Doing the same for (2) we get

$$\begin{aligned} & [E^1_u(1, \bar{p}+\bar{\tau}^1, \bar{u}^1) - \left(\bar{\tau}^1\right)^T E^1_{pu}(1, \bar{p}+\bar{\tau}^1, \bar{u}^1)] du^1 \\ = & \left(\bar{\tau}^1\right)^T [E^1_{pp}(\bar{p}+\bar{\tau}^1, \bar{u}^1) - F^1_{pp}(\bar{p}+\bar{\tau}^1)] d\tau^1 \end{aligned} \tag{4}$$

$$0 = (\bar{\tau}^n)^T [E^n_{pp}(\bar{p}+\bar{\tau}^n, \bar{u}^n) - F^n_{pp}(\bar{p}+\bar{\tau}^n)] d\tau^n, \quad n = 2, ..., N, \tag{5}$$

Solving (4) for du^1 we get

$$du^1 = \frac{\left(\bar{\tau}^1\right)^T [E^1_{pp}(\bar{p}+\bar{\tau}^1, \bar{u}^1) - F^1_{pp}(\bar{p}+\bar{\tau}^1)] d\tau^1}{E^1_u(\bar{p}+\bar{\tau}^1, \bar{u}^1) - \left(\bar{\tau}^1\right)^T E^1_{pu}(\bar{p}+\bar{\tau}^1, \bar{u}^1)}, \tag{6}$$

We then substitute it into (3), to obtain

$$\begin{aligned}
& -\sum_{n=2}^{N}[E^n_{pp}(\bar{p}+\bar{\tau}^n,\bar{u}^n)-F^n_{pp}(\bar{p}+\bar{\tau}^n)]d\tau^n \\
= & \left[I_M+\frac{E^1_{pu}(\bar{p}+\bar{\tau}^1,\bar{u}^1)\left(\bar{\tau}^1\right)^T}{E^1_u(\bar{p}+\bar{\tau}^1,\bar{u}^1)-\left(\bar{\tau}^1\right)^T E^1_{pu}(\bar{p}+\bar{\tau}^1,\bar{u}^1)}\right] \\
& \times[E^1_{pp}(\bar{p}+\bar{\tau}^1,\bar{u}^1)-F^1_{pp}(\bar{p}+\bar{\tau}^1)]d\tau^1, \qquad (7)
\end{aligned}$$

where I_M is the $M\times M$ identity matrix. Pre-multiplying $(\bar{\tau}^1)^T$ to both sides of (7) we get,

$$\begin{aligned}
& (\bar{\tau}^1)^T\left\{-\sum_{n=2}^{N}[E^n_{pp}(\bar{p}+\bar{\tau}^n,\bar{u}^n)-F^n_{pp}(\bar{p}+\bar{\tau}^n)]d\tau^n\right\} \\
= & \left[(\bar{\tau}^1)^T+\frac{(\bar{\tau}^1)^T E^1_{pu}(\bar{p}+\bar{\tau}^1,\bar{u}^1)(\bar{\tau}^1)^T}{E^1_u(\bar{p}+\bar{\tau}^1,\bar{u}^1)-(\bar{\tau}^1)^T E^1_{pu}(\bar{p}+\bar{\tau}^1,\bar{u}^1)}\right] \\
& \times[E^1_{pp}(\bar{p}+\bar{\tau}^1,\bar{u}^1)-F^1_{pp}(\bar{p}+\bar{\tau}^1)]d\tau^1 \\
= & \frac{(\bar{\tau}^1)^T[E^1_{pp}(\bar{p}+\bar{\tau}^1,\bar{u}^1)-F^1_{pp}(\bar{p}+\bar{\tau}^1)]d\tau^1}{E^1_u(\bar{p}+\bar{\tau}^1,\bar{u}^1)-(\bar{\tau}^1)^T E^1_{pu}(\bar{p}+\bar{\tau}^1,\bar{u}^1)}E^1_u(\bar{p}+\bar{\tau}^1,\bar{u}^1) \qquad (8)
\end{aligned}$$

Combining (6) and (8), we obtain

$$du^1=\frac{(\bar{\tau}^1)^T\left\{-\sum_{n=2}^{N}[E^n_{pp}(\bar{p}+\bar{\tau}^n,\bar{u}^n)-F^n_{pp}(\bar{p}+\bar{\tau}^n)]d\tau^n\right\}}{E^1_u(\bar{p}+\bar{\tau}^1,\bar{u}^1)} \qquad (9)$$

Note that this simply means if tariff adjustments for the N trading club countries, $d\tau^n, n=1,\ldots,N$, satisfy the material balance condition (7), then the change in the trade vector of Country 1 which originates from $d\tau^1$ must be opposite to the sum of that of Country $n, n=2,\ldots,N$, and du^1 is positive when the tariff adjustments yield an increase in the tariff revenue of Country 1.

The next step is to introduce the tariff-adjustment formulas for the N trading club countries which satisfies both the material balance condition and the budget constraint for Country $n, n=2,\ldots,N$. Let

$$\begin{aligned}
\tau^1(\varepsilon_2,\ldots,\varepsilon_N) &\equiv \bar{\tau}^1+\left\{\Delta_1[E^1_{pp}(\bar{p}+\bar{\tau}^1,\bar{u}^1)-F^1_{pp}(\bar{p}+\bar{\tau}^1)]\right\}^{-1}\left(-\sum_{n=2}^{N}\Upsilon^n\varepsilon_n\right) \\
\tau^n(\varepsilon_n) &\equiv \bar{\tau}^n+[E^n_{pp}(\bar{p}+\bar{\tau}^n,\bar{u}^n)-F^n_{pp}(\bar{p}+\bar{\tau}^n)]^{-1}\Upsilon^n\varepsilon_n,\ n=2,...,N,
\end{aligned}$$

where Υ^n is an orthogonal vector of $\bar{\tau}^n$, i.e., $(\bar{\tau}^n)^T \Upsilon^n = 0$, ε_n, $n = 2, ..., N$, are scalars, and

$$\Delta_1 \equiv I_M + \frac{E^1_{pu}(\bar{p}+\bar{\tau}^1, \bar{u}^1)(\bar{\tau}^1)^T}{E^1_u(\bar{p}+\bar{\tau}^1, \bar{u}^1) - (\bar{\tau}^1)^T E^1_{pu}(\bar{p}+\bar{\tau}^1, \bar{u}^1)}$$

These formulas describe how the tariffs of the member countries of the trading club are determined. The above tariff adjustments clearly satisfy the material balance condition (7), since totally differentiating $\tau^1(\varepsilon_2, \ldots, \varepsilon_N)$ and $\tau^n(\varepsilon_n)$ yields

$$d\tau^1 = \left\{\Delta_1[E^1_{pp}(\bar{p}+\bar{\tau}^1, \bar{u}^1) - F^1_{pp}(\bar{p}+\bar{\tau}^1)]\right\}^{-1}\left(-\sum_{n=2}^{N}\Upsilon^n d\varepsilon_n\right) \quad (10)$$

$$d\tau^n = [E^n_{pp}(\bar{p}+\bar{\tau}^n, \bar{u}^n) - F^n_{pp}(\bar{p}+\bar{\tau}^n)]^{-1}\Upsilon^n d\varepsilon_n, \quad n = 2, ..., N \quad (11)$$

On the other hand, substituting (11) into (5), we see, for $n = 2, ..., N$,

$$\begin{aligned} 0 &= (\bar{\tau}^n)^T [E^n_{pp}(\bar{p}+\bar{\tau}^n, \bar{u}^n) - F^n_{pp}(\bar{p}+\bar{\tau}^n)]d\tau^n \\ &= (\bar{\tau}^n)^T \Upsilon^n d\varepsilon_n, \end{aligned}$$

which holds, due to the definition of Υ^n, whatever value $d\varepsilon_n$, $n = 2, ..., N$, takes on. That is, the budget constraint always holds under the above tariff adjustments.

Next, we formulate the condition on initial tariffs we need to get our results.

Non-Proportionality Condition: We say that tariff vector $\bar{\tau}^1$satisfies the non-proportionality condition if the following holds:

Consider a linear sub-space as follows

$$\Theta(s) \equiv \{\theta \in R^M : \theta = \sum_{n=1, n\neq s}^{N} \Upsilon^n x_n,\ x_n \in R,\ n = 1, 2, ..., N,\ \text{is a scalar}\}$$

As long as there is a $\theta^* \in \Theta(1)$ *such that* $(\bar{\tau}^1)^T\theta^* \neq 0$ *then* $\bar{\tau}^1$*satisfies the non-proportionality condition.*

Using (9) and (11), we obtain the following proposition.

Proposition 1: *Assume that* Δ_1 *and* $[E^n_{pp}(\bar{p}+\bar{\tau}^n, \bar{u}^n) - F^n_{pp}(\bar{p}+\bar{\tau}^n)], n = 1, ..., N,$ *are nonsingular and that for any* $n = 1, ..., N,$ *the tariff vector* $\bar{\tau}^n$ *is non-zero. If* $\bar{\tau}^1$ *satisfies the non-proportionality condition then it is possible for the trading club countries to make a Pareto-improving tariff adjustment.*

Proof: (9) and (11) together imply that

$$\begin{aligned} sign[du^1] &= -sign\left[(\bar{\tau}^1)^T \sum_{n=2}^{N} \Upsilon^n d\varepsilon_n\right] \\ &= -sign[(\bar{\tau}^1)^T \theta^*] \end{aligned}$$

Since Θ is a linear subspace, $\theta^* \in \Theta$ means $-\theta^* \in \Theta$. It follows that as long as $(\bar{\tau}^1)^T\theta^* \neq 0$, we can make du^1 positive by choosing an appropriate $(d\varepsilon_2, ..., d\varepsilon_N)$.■

Proposition 1 says that if we consider a trading club in which the member countries' initial tariffs are not proportional to each other then the trading club can implement a tariff reform that makes all trading club members better off without making the rest of the world worse off.[4] This result tells us that when initial tariffs are not proportional then there is room for tariff reform in the spirit of Kemp-Wan that does not require the use of transfer payments.

4 The Second Main Result: Pareto-optimal Customs Unions Without Income Transfers

We next turn to consideration of customs unions. We want to determine whether the use of transfer payments is necessary for the Kemp-Wan customs union result. We first formulate a constrained maximization problem in which the N countries jointly maximize a weighted sum of utilities by choosing an appropriate set of tariff vectors $(\tau^1, ..., \tau^N)$ subject to the condition that no country is hurt

[4] If the member countries' initial tariffs are proportional to each other then the tariff adjustments (10) and (11) have no effect on the tariff revenue of Country 1, hence Country 1's welfare, as well as that of Country $n, n = 2, \ldots, N$.

compared to their initial welfare levels. Formally, the constrained maximization problem is

$$\max_{u^n, \tau^n, n=1,\ldots,N} U = \sum_{n=1}^{N} a_n u^n$$

subject to

$$u^i \geq \bar{u}^i, \quad i = 1, ..., N \tag{12}$$

$$S(\bar{p}) = \sum_{n=1}^{N} [E_p^n(\bar{p} + \tau^n, u^n) - F_p^n(\bar{p} + \tau^n)] \tag{13}$$

$$\begin{aligned} E^n(\bar{p} + \tau^n, u^n) - F^n(\bar{p} + \tau^n) &= (\tau^n)^T [E_p^n(\bar{p} + \tau^n, u^n) - F_p^n(\bar{p} + \tau^n)], \\ n &= 1, \ldots, N \end{aligned} \tag{14}$$

Lemma: *Suppose that the numeraire good is indispensable in the sense that for any given q and u*

$$E_0^n(0, q, u) = \infty, \; n = 1, \ldots, N,$$

then the feasible set that satisfies the constraints (12)-(14) is bounded and closed.

Proof: See Appendix.

This lemma allows us to apply the Weierstras theorem to the above constrained maximization problem. That is, the solution to this problem must exist. Moreover, Proposition 1 implies that if the initial tariff vectors, $\bar{\tau}^n$, $n = 1, .., N$, are not proportional, then for small adjustments of tariffs (10) and (11) we have

$$\sum_{n=1}^{N} a_n u^n > \sum_{n=1}^{N} a_n \bar{u}^n,$$

which means that the solution is interior.

The problem we seek to solve does not require Pareto improvement for all countries but a much weaker condition, namely that the net external trade vector of the customs union is constant. This problem is solved with the following constrained optimization problem. Given these preliminary results, we now solve the main problem. We formulate the Lagrangian.

$$L = \sum_{n=1}^{N} a_n u^n + \Delta^T \{S(\bar{p}) - \sum_{n=1}^{N} [E_p^n(\bar{p} + \tau^n, u^n) - F_p^n(\bar{p} + \tau^n)]\}$$
$$+ \sum_{n=1}^{N} \lambda_n \{(\tau^n)^T [E_p^n(\bar{p} + \tau^n, u^n) - F_p^n(\bar{p} + \tau^n)] + F^n(\bar{p} + \tau^n) - E^n(\bar{p} + \tau^n, u^n)\}, \tag{15}$$

where $\Delta \equiv (\delta_1, ..., \delta_M)^T$. If there is an interior optimal solution, it has to satisfy the necessary conditions for optimality.

$$\begin{aligned} \frac{\partial L}{\partial u^n} &= a_n - \Delta^T E_{up}^n(\bar{p} + \tau^n, u^n) \\ &\quad -\lambda_n \left[E_u^n(\bar{p} + \tau^n, u^n) - (\tau^n)^T E_{up}^n(\bar{p} + \tau^n, u^n)\right] \\ &= 0, \quad n = 1, ..., N \end{aligned} \tag{16}$$

$$\begin{aligned} \left[\frac{\partial L}{\partial \tau^n}\right]^T &= -\Delta^T [E_{pp}^n(\bar{p} + \tau^n, u^n) - F_{pp}^n(\bar{p} + \tau^n)] \\ &\quad +\lambda_n (\tau^n)^T [E_{pp}^n(\bar{p} + \tau^n, u^n) - F_{pp}^n(\bar{p} + \tau^n)] \\ &= [\lambda_n (\tau^n)^T - \Delta^T][E_{pp}^n(\bar{p} + \tau^n, u^n) - F_{pp}^n(\bar{p} + \tau^n)] \\ &= 0, \quad n = 1, ..., N \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial L}{\partial \Delta} &= S(\bar{p}) - \sum_{n=1}^{N} [E_p^n(\bar{p} + \tau^n, u^n) - F_p^n(\bar{p} + \tau^n)] \\ &= 0 \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\partial L}{\partial \lambda_n} &= (\tau^n)^T [E_p^n(\bar{p} + \tau^n, u^n) - F_p^n(\bar{p} + \tau^n)] \\ &\quad +F^n(\bar{p} + \tau^n) - E^n(\bar{p} + \tau^n, u^n) \\ &= 0, \quad n = 1, ..., N \end{aligned} \tag{19}$$

First, from (17) and the nonsingularity of the substitution matrix for each $n = 1, ..., N$, we see that

$$\lambda_n (\tau^n)^T = \Delta^T, \quad n = 1, ..., N \tag{20}$$

It follows from (16) that

$$a_n - \lambda_n E_u^n(\bar{p} + \tau^n, u^n) = 0, \quad n = 1, ..., N$$

or

$$\lambda_n = \frac{a_n}{E_u^n(\bar{p} + \tau^n, u^n)} > 0$$

Thus, we have Proposition 2.

Proposition 2: *If there is an interior solution for the above problem, it satisfies the proportionality condition*

$$\frac{a_1}{E_u^1(\bar{p} + \tau^1, u^1)}\tau^1 = ... = \frac{a_n}{E_u^n(\bar{p} + \tau^n, u^n)}\tau^n = ... = \frac{a_N}{E_u^N(\bar{p} + \tau^N, u^N)}\tau^N \quad (21)$$

The existence of an interior solution is ensured if

$$\textit{for any } n = 1, ..., N, \textit{ there is } \theta^*(n) \in \Theta(n) \textit{ such that } (\bar{\tau}^n)^T \theta^*(n) \neq 0. \quad (22)$$

Now, consider the following mapping from $\Omega \equiv \{a \equiv (a_1, ..., a_N) : \sum_n^N a_n = 1 \text{ and } a_n \geq 0, n = 1, ..., N\}$ into itself

$$f^n(a) \equiv \frac{E_u^n(\bar{p} + \tau^n(a), u^n(a))}{\sum_j^N E_u^j(\bar{p} + \tau^j(a), u^j(a))}, \quad n = 1, ..., N, \; a \in \Omega$$

Since Ω is a convex and compact set and $f^n(a)$, $n = 1, ..., N$, are continuous in a, we can apply the Brouwer Fixed Point theorem to assert the existence of the fixed point $a^* \equiv (a_1^*, ..., a_N^*) \in \Omega$ such that

$$a_n^* = \frac{E_u^n(\bar{p} + \tau^n(a^*), u^n(a^*))}{\sum_j^N E_u^j(\bar{p} + \tau^j(a^*), u^j(a^*))}, \quad n = 1, ..., N$$

It then follows that

$$\frac{a_1^*}{E_u^1(\bar{p} + \tau^1(a^*), u^1(a^*))} = ... = \frac{a_n^*}{E_u^n(\bar{p} + \tau^n(a^*), u^n(a^*))} = ... = \frac{a_N^*}{E_u^N(\bar{p} + \tau^N(a^*), u^N(a^*))}$$

Using the proportionality condition (21) we conclude

$$\tau^1 = ... = \tau^n = ... = \tau^N \quad (23)$$

Thus, N countries set a common tariff vector. We can now state the second main result of the paper.

Proposition 3: *Suppose that there exists an initial tariff-ridden equilibrium and that a subset of the countries forms a customs union. We show that, given the existence of the standard revenue and expenditure functions, the countries can form a Pareto-optimal customs union without income transfers, if (i) there are more than two tradable goods and (ii) the initial tariff vectors of the member countries satisfy the non-proportionality condition.*

Remark: Proposition 3 just asserts that a *Pareto-optimal* customs union can be formed without income transfers among member countries. However, it does not ensure that the union is *Pareto-improving* (as a Kemp-Wan customs union is) compared with the initial tariff-ridden equilibrium. The proposition guarantees that there exists a customs union without income transfers that results in a Pareto-optimal pair of utilities like point E in Figure 1. However, the customs union will be *Pareto-improving* only if the initial utilities are within the rectangular area $OMES$ in Figure 1. If the initial equilibrium were instead a point like G in Figure 1 then transfer payments are *necessary* for a *Pareto-improving* customs union.

5 A Diagrammatic Exposition

In this section we provide a diagrammatic exposition of the main results to clarify them and to provide some intuition. For purposes of illustration, we consider a specific case, i.e., a pure exchange world economy with $N = 2$ and $M = 2$.

Figure 2 is the 3-dimensional Edgeworth box. O_1 and O_2 are the origins of Country 1 and Country 2, respectively. Point W which is located inside the box denotes the initial endowment point. For example, $(\bar{x}_1^1, \bar{x}_2^1, \bar{x}_0^1) = (O_1G,\ GJ,\ J\overline{W})$ is Country 1's initial endowments of Good 1, Good 2 and Good 0.

Next consider Figure 3. The surface SQR is the international price plane that contains the initial endowment point $\overline{W}$. On this surface we have $p_1(x_1 - \bar{x}_1) + p_2(x_2 - \bar{x}_2) + (x_0 - \bar{x}_0) = 0$. The consumption points of both countries have to be on this plane.

Figure 4 depicts an indifference surface of Country 1. The intersection of the indifference surface and the international price plane is the closed curve $ACFBD$ on the plane.

Figure 5 is derived when we see the international price plane from above. Imagine a plane that is tangent to the indifference surface at point F. The segment lFl' is the intersection of the tangent plane and the international price plane. Point F, $(x_{F1}^1, x_{F2}^1, x_{F0}^1)$, is the consumption point of Country 1. Hence, the slope of the line lFl' gives us the relative domestic prices in Country 1. Denoting the direct utility function of Country 1 by $u_1 = U^1(x_1^1, x_2^1, x_0^1)$, we see that the tangent plane is

$$\frac{U_1^1}{U_0^1}(x_1^1 - x_{F1}^1) + \frac{U_2^1}{U_0^1}(x_2^1 - x_{F2}^1) + (x_0^1 - x_{F0}^1) = 0$$

If point F is the consumption point, the marginal rates of substitution have to equal to the domestic prices. That is,

$$\frac{U_1^1}{U_0^1} = p_1 + \bar{\tau}_1^1 \quad \text{and} \quad \frac{U_2^1}{U_0^1} = p_2 + \bar{\tau}_2^1$$

Therefore, the equation for segment lFl' is

$$\bar{\tau}_1^1(x_1^1 - x_{F1}^1) + \bar{\tau}_2^1(x_2^1 - x_{F2}^1) + (x_0^1 - x_{F0}^1) = 0$$

Clearly, the segment FW measures the net trade vector of Country 1.

A parallel argument can be made for Country 2. In Figure 6, the vectors WF_1 and WF_2 are the trade vectors of Country 1 and Country 2, respectively.

So, the vector F_1F_2 denotes the net trade vector of countries 1 and 2, with the rest of the world. Figure 6 illustrates the pre-customs union world equilibrium.

Now, we illustrate how a customs union produces a Pareto improvement. First, solve the constrained maximization problem formulated in the previous section giving all the utility weight to Country 1, i.e., $a_1 = 1$ and $a_2 = 0$. Figure 7 illustrates the solution to this problem. The segment $\bar{l}_2F_1\bar{l}'_2$ is parallel to $l_2F_2l'_2$ and the dashed indifference curve is just a parallel shift of the indifference curve $\bar{u}_2$. The smaller closed curve u_1 is depicted in such a way to be tangent to the dashed indifference curve at point $\bar{F}_1$. Notice that line $\bar{F}_1\bar{F}_2$ that is parallel and equal to F_1F_2. This is because in the constrained optimization problem net trade of the customs unions countries to the rest of the world does not change. That is, if both countries adjust their tariff rates from $l'_1F_1l_1$ and $l_2F_2l'_2$ to $l^*_1\bar{F}_1l'^*_1$ and $l^*_2\bar{F}_2l'^*_2$, the consumption points change from F_1 and F_2 to $\bar{F}_1$ and $\bar{F}_2$ and respectively. Note that since the trade vector with the rest of the world is unchanged by the tariff adjustments, international prices do not change. Therefore, the rest of the world is not hurt by the tariff adjustment, and Country 1 is better off without hurting Country 2. Also note that the new tariff lines, $l^*_1\bar{F}_1l'^*_1$ and $l^*_2\bar{F}_2l'^*_2$, are parallel with each other. That is the ratio τ^1_1/τ^1_2 at $\bar{F}_1$ is equal to τ^2_1/τ^2_2 at $\bar{F}_2$. Thus, what happens is that the customs union adjusts tariffs such that trade with the rest of the world is unchanged and within the customs union, agents in both countries face the same domestic prices. This is the same as in Kemp-Wan Theorem, the difference being that we do not rely on transfer payments to ensure that all customs union members do at least as well in the post-customs union equilibrium.

A similar argument can be made for the case in which all utility weight is place on Country 2, $a_1 = 0$ and $a_2 = 1$. This case is illustrated in Figure 8, where the consumption points move from (F_1, F_2) to $(\tilde{F}_1, \tilde{F}_2)$, which means that Country 2 is better off while Country 1 and the rest of the world have the same utility as before the tariff adjustment.

It is clear that by changing the (a_1, a_2) from $(0, 1)$ to $(1, 0)$ keeping $a_1 + a_2 = 1$, we derive a locus of the solutions of the constrained maximization problem for which both member countries have higher utility. Consumption points like (F_1^0, F_2^0) in Figure 9 have the property that both countries are better off without hurting the rest of the world.

Figure 10 illustrates the case in which customs unions cannot produce welfare improvement. Here domestic prices are the same ($l_1 F_1 l_1'$ and $l_2 F_2 l_2'$ are parallel) at the pre-union equilibrium. It is clear from the diagram, that it is impossible to make Pareto-improving tariff adjustments without using transfer payments. This case illustrates the relationship between our result and the Kemp-Wan theorem. Customs unions can improve welfare without using transfer payments, unless member countries have the same domestic prices. In this case transfer payments are required for a customs union to produce a Pareto improvement.

6 Concluding Remarks

We have shown that a *Pareto-improving* non-discriminatory tariff reform is possible without income transfers if the number of goods is more than two and the pre-union tariff vectors of member countries satisfy the non-proportionality condition. In addition, under the same conditions a *Pareto-optimal* customs union always exists, however, transfers may be required for a *Pareto-improving* customs union.

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APPENDIX 1: Proof of Lemma 1

First of all, let us denote $q^n \equiv \bar{p} + \tau^n$, $n = 1, ..., N$, and define the set:

$$
\begin{aligned}
\Gamma &\equiv \{(q^1, ..., q^N) \geq 0 : {}^{\exists}(u^1, ..., u^N) \geq (\bar{u}^1, .., \bar{u}^N) \text{ such that for } n = 1, ..., N, \\
0 &\geq E^n(1, q^n, u^n) - F^n(1, q^n) \ - (q^n - \bar{p})^T [E_q^n(1, q^n, u^n) - F_q^n(1, q^n)], \text{ and} \\
0 &= S(\bar{p}) - \sum_s^N [E_q^s(1, q^s, u^s) - F_q^s(1, q^s)] \ \}
\end{aligned}
$$

Let us prove that Γ is bounded. Suppose it is not. Then, there are $(q_*^1, ..., q_*^N)$ in Γ and $\{\varepsilon_i\}_{i=1}^{i=\infty}$, with $\varepsilon_1 < \varepsilon_2 < ...$, and $\lim_{i\to\infty} \varepsilon_i = \infty$, such that $(\varepsilon_i q_*^1, ..., \varepsilon_i q_*^N) \in \Gamma$ for ${}^{\forall}i$. Thus, there is $\{u_i^n\}_{i=1}^{i=\infty}$ such that for $n = 1, ..., N$, and $i = 1, 2, 3, ...$

$$
\begin{aligned}
0 &\geq E^n(1, \varepsilon_i q_*^n, u_i^n) - F^n(1, \varepsilon_i q_*^n) \ - (\varepsilon_i q_*^n - \bar{p})^T [E_q^n(1, \varepsilon_i q_*^n, u_i^n) - F_q^n(1, \varepsilon_i q_*^n)] \\
0 &= S(\bar{p}) - \sum_{s=1}^N [E_q^s(1, \varepsilon_i q_*^s, u_i^s) - F_q^s(1, \varepsilon_i q_*^s))]
\end{aligned}
$$

Since both expenditure and revenue functions are homogeneous of degree one in all $m + 1$ prices,

$$
\begin{aligned}
0 &\geq \sum_s^N \{E^s(1, \varepsilon_i q_*^s, u_i^s) - F^s(1, \varepsilon_i q_*^s) \ - (\varepsilon_i q_*^s - \bar{p})^T [E_q^s(1, \varepsilon_i q_*^s, u_i^s) - F_q^s(1, \varepsilon_i q_*^s)]\} \\
&= \sum_s^N \{E^s(1, \varepsilon_i q_*^s, u_i^s) - F^s(1, \varepsilon_i q_*^s) \ - (\varepsilon_i q_*^s)^T [E_q^s(1, \varepsilon_i q_*^s, u_i^s) - F_q^s(1, \varepsilon_i q_*^s)]\} \\
&\quad + (\bar{p})^T \sum_s^N [E_q^s(1, \varepsilon_i q_*^s, u_i^s) - F_q^s(1, \varepsilon_i q_*^s)] \\
&= \sum_s^N [E_0^s(1, \varepsilon_i q_*^s, u_i^s) - F_0^s(1, \varepsilon_i q_*^s)] + (\bar{p})^T S(\bar{p}) \\
&= \sum_s^N \left[E_0^s\left(\frac{1}{\varepsilon_i}, q_*^s, u_i^s\right) - F_0^s\left(\frac{1}{\varepsilon_i}, q_*^s\right)\right] + (\bar{p})^T S(\bar{p}),
\end{aligned}
$$

where $[E_0^s - F_0^s]$ is the partial derivative of $[E^s - F^s]$ with respect to the price of Good 0, and therefore the net import of Good 0 of Country s. Since all goods are assumed to be normal, it follows from $u_i^n \geq \bar{u}^n$, for $n = 1, ..., N$, and

$i = 1, 2, 3, ...,$ that

$$\begin{aligned} 0 &\geq \sum_{s}^{N} \left[E_0^s \left(\frac{1}{\varepsilon_i}, q_*^s, u_i^s \right) - F_0^s \left(\frac{1}{\varepsilon_i}, q_*^s \right) \right] + (\bar{p})^T S(\bar{p}) \\ &\geq \sum_{s}^{N} \left[E_0^s \left(\frac{1}{\varepsilon_i}, q_*^s, \bar{u}^s \right) - F_0^s \left(\frac{1}{\varepsilon_i}, q_*^s \right) \right] + (\bar{p})^T S(\bar{p}) \end{aligned}$$

Since Good 0 is assumed to be indispensable, $\lim_{i \to \infty} \varepsilon_i = \infty$ means that

$$\begin{aligned} & \lim_{i \to \infty} \sum_{s}^{N} \left[E_0^s \left(\frac{1}{\varepsilon_i}, q_*^s, \bar{u}^s \right) - F_0^s \left(\frac{1}{\varepsilon_i}, q_*^s \right) \right] \\ = & \lim_{i \to \infty} \sum_{s}^{N} E_0^s \left(\frac{1}{\varepsilon_i}, q_*^s, \bar{u}^s \right) \\ = & \infty \end{aligned}$$

Therefore, for a sufficiently large i, we have

$$0 \geq \sum_{s=1}^{N} \left[E_0^s \left(\frac{1}{\varepsilon_i}, q_*^s, \bar{u}^s \right) - F_0^s \left(\frac{1}{\varepsilon_i}, q_*^s \right) \right] + (\bar{p})^T S(\bar{p}) >> 0,$$

a contradiction. Therefore Γ is bounded.

By definition of Γ, for any $Q \equiv (q^1, ..., q^N) \in \Gamma$ there is at least one $(u^1(Q), ..., u^N(Q))$ such that for $n = 1, ..., N$, $u^n(Q) \geq \bar{u}^n$,

$$\begin{aligned} & 0 \geq E^n(1, q^n, u^n(Q)) - F^n(1, q^n) \ - (q^n - \bar{p})^T [E_q^n(1, q^n, u^n(Q)) - F_q^n(1, q^n)], \\ & \text{and} \quad 0 = S(\bar{p}) - \sum_{s}^{N} [E_q^s(1, q^s, u^s(Q)) - F_q^s(1, q^s))] \end{aligned}$$

Consider the set $\tilde{\Gamma} \equiv \{(Q, u^1(Q), ..., u^N(Q)) : Q \in \Gamma\}$. Since $\max u^n \geq u^n(Q) \geq \bar{u}^n$, $\tilde{\Gamma}$ is bounded. Since $\tilde{\Gamma}$ contains the feasible set and satisfies (12)-(14), the feasible set is bounded. That the feasible set is closed is obvious.■

APPENDIX 2: Cases of specific tariffs and ad valorem tariffs imposed on all goods

We show that (i) if tariffs are imposed on not only non-numeraire goods but also the numeraire good, then the non-proportionality condition holds in a modified sense; (ii) whether specific or ad valorem tariffs do not make a substantial difference.

Specific tariffs

We consider the case where a tariff is possibly imposed on the numeraire good. Then, the material balance condition (1) becomes

$$S(\bar{p}) = \sum_{n=1}^{N} G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n), \tag{24}$$

where $G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \equiv E_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) - F_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n)$. On the other hand, the budget constraint of Country n becomes

$$\begin{aligned} G^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) &= \bar{\tau}_0^n G_{p_0}^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \\ &\quad +(\bar{\tau}^n)^T G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \end{aligned} \tag{25}$$

Note that, due to the properties of the expenditure and the aggregate revenue functions, $G^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n)$ is linearly homogeneous in $1+\bar{\tau}_0^n$ and $\bar{p}+\bar{\tau}^n$. Therefore, it follows that the identity

$$\begin{aligned} G^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) &= (1+\bar{\tau}_0^n) G_{p_0}^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \\ &\quad +(\bar{p}+\bar{\tau}^n)^T G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \end{aligned} \tag{26}$$

holds.[5] Combining (25) and (26) together, we obtain

$$G_{p_0}^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) = -(\bar{p})^T G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \tag{27}$$

The substitution of (27) into (25) yields.

$$\begin{aligned} G^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) &= -\bar{\tau}_0^n (\bar{p})^T G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \\ &\quad +(\bar{\tau}^n)^T G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \\ &= \left[\bar{\tau}^n - \bar{\tau}_0^n \bar{p}\right]^T G_p^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n) \end{aligned} \tag{28}$$

[5] The identity is often called the Euler condition.

Since the function $G^n(1+\bar{\tau}_0^n, \bar{p}+\bar{\tau}^n, \bar{u}^n)$ is linearly homogeneous in $1+\bar{\tau}_0^n$ and $\bar{p}+\bar{\tau}^n$ (i.e., homogeneous of degree one in these price terms), the partial derivatives with respect to the price terms are homogeneous of degree zero. Therefore, (24) and (28) can be rewritten to

$$\begin{aligned} S(\bar{p}) &= \sum_{n=1}^{N} G_p^n\left(1, \frac{\bar{p}+\bar{\tau}^n}{1+\bar{\tau}_0^n}, \bar{u}^n\right) && (29) \\ G^n\left(1, \frac{\bar{p}+\bar{\tau}^n}{1+\bar{\tau}_0^n}, \bar{u}^n\right) &= \frac{[\bar{\tau}^n - \bar{\tau}_0^n \bar{p}]^T}{1+\bar{\tau}_0^n} G_p^n\left(1, \frac{\bar{p}+\bar{\tau}^n}{1+\bar{\tau}_0^n}, \bar{u}^n\right) && (30) \end{aligned}$$

Finally, let

$$\Lambda^n \equiv \frac{\bar{\tau}^n - \bar{\tau}_0^n \bar{p}}{1+\bar{\tau}_0^n}$$

Then, since

$$\frac{\bar{p}+\bar{\tau}^n}{1+\bar{\tau}_0^n} = \bar{p} + \Lambda^n,$$

(29) and (30) become

$$\begin{aligned} S(\bar{p}) &= \sum_{n=1}^{N} G_p^n(1, \bar{p}+\Lambda^n, \bar{u}^n) && (31) \\ G^n(1, \bar{p}+\Lambda^n, \bar{u}^n) &= (\Lambda^n)^T G_p^n(1, \bar{p}+\Lambda^n, \bar{u}^n) && (32) \end{aligned}$$

From the formal point of view, (31) and (32) are the same as (1) and (2), if Λ^n is replaced by $\bar{\tau}^n$. Therefore, we can make formally the same calculations as in the main text, considering the tariff change taking the specific form $d\Lambda^n$ in stead of $d\tau_0^n$ and $d\tau^n$:

$$\begin{aligned} d\Lambda^1 &= \left[\tilde{\Delta}_1 G_{pp}^1(1, \bar{p}+\Lambda^1, \bar{u}^1)\right]^{-1} \left(-\sum_{n=2}^{N} \tilde{\Upsilon}^n d\varepsilon_n\right), && (33) \\ d\Lambda^n &= \left[G_{pp}^n(1, \bar{p}+\Lambda^n, \bar{u}^n)\right]^{-1} \tilde{\Upsilon}^n d\varepsilon_n, \quad n = 2, \ldots, N, && (34) \end{aligned}$$

yield

$$du^1 = \frac{(\Lambda^1)^T \left(-\sum_{n=2}^{N} \tilde{\Upsilon}^n d\varepsilon_n\right)}{E_u^1(\bar{p}+\Lambda^1, \bar{u}^1)}, \qquad (35)$$

where $\tilde{\Upsilon}^n$ is an orthogonal vector of Λ^n, i.e., $(\Lambda^n)^T\tilde{\Upsilon}^n = 0$, and

$$\tilde{\Delta}_1 \equiv I_M + \frac{E^1_{pu}(\bar{p}+\Lambda^1,\bar{u}^1)(\Lambda^1)^T}{E^1_u(\bar{p}+\Lambda^1,\bar{u}^1) - (\Lambda^1)^T E^1_{pu}(\bar{p}+\Lambda^1,\bar{u}^1)} \tag{36}$$

Therefore, we can conclude that it is possible to form a Pareto-improving trading club, if the following *modified non-proportionality condition,*

$$\text{There exists some } n \text{ such that } \Lambda^1 \neq k\Lambda^n \text{ for } {}^{\forall}k \in R$$

or

$$\frac{\bar{\tau}^1 - \bar{\tau}^1_0\bar{p}}{1+\bar{\tau}^1_0} \neq k\frac{\bar{\tau}^n - \bar{\tau}^n_0\bar{p}}{1+\bar{\tau}^n_0} \quad \text{for } {}^{\forall}k \in R \tag{37}$$

holds in the pre-club tariff-ridden world equilibrium.[6]

Note that if the numeraire good is freely traded (i.e., $\bar{\tau}^1_0 = \bar{\tau}^n_0 = 0$), then the modified non-proportionality condition is equivalent to the original non-proportionality condition. However, if the government of each country imposes a tariff on not only the non-numeraire good but also the numeraire good, the above modified non-proportionality condition is the one that makes possible the formation of a Pareto-improving trading club.

Ad valorem tariffs

Next, let us consider the case of ad valorem tariffs. To do so, we shall use the $M \times M$ diagonal matrix as follows

$$[\bar{t}^n] \equiv \begin{bmatrix} \bar{t}^n_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \bar{t}^n_M \end{bmatrix} \tag{38}$$

[6] It can be easily shown from (34) that $(\Lambda^n)^T\tilde{\Upsilon}^n d\varepsilon_n = 0$ yields

$$\bar{\tau}^n_0\left[G^n_{p_0p}(1,\bar{p}+\Lambda^n,\bar{u}^n)\right]^T d\Lambda^n + (\bar{\tau}^n)^T G^n_{pp}(1,\bar{p}+\Lambda^n,\bar{u}^n)d\Lambda^n = 0.$$

Clearly, the first term is the change in the tariff revenue of Country n originated from good 0 and the second term is the one originated from good $m \geq 1$, so this equation means that the tariff revenue of Country n is not affected by $d\Lambda^n$. Therefore, if the modified non-proportionality condition doesn't hold then the tariff adjustments (33) and (34) have no effect on the tariff revenue of Country 1, hence Country 1's welfare (see (35)). Thus, the modified non-proportionality condition has the same meaning of the original one.

Making use of this matrix as well as the the $M \times M$ identity matrix I_M, we can describe the budget constraint of Country n as

$$G^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) = \bar{t}_0^n G_{p_0}^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) + (\bar{p})^T [\bar{t}^n] \, G_p^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) \quad (39)$$

Note that $\bar{p}_0 = 1$. On the other hand, the Euler condition in the present case is

$$G^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) = (1+\bar{t}_0^n)G_{p_0}^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) + (\bar{p})^T(I_M + [\bar{t}^n])G_p^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) \quad (40)$$

Combining (39) and (40), we have

$$G_{p_0}^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) = -(\bar{p})^T G_p^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n)$$

The substitution of (**??**) into (39) yields

$$\begin{aligned} &G^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) \\ &= (\bar{p})^T(-\bar{t}_0^n I_M + [\bar{t}^n])G_p^n(1+\bar{t}_0^n, (I_M + [\bar{t}^n])\bar{p}, \bar{u}^n) \end{aligned} \quad (41)$$

Due to the linear homogeneity of the function $G^n(.)$ in price terms, we can rewrite (41) as

$$\begin{aligned} &G^n\left(1, \left(\frac{I_M + [\bar{t}^n]}{1+\bar{t}_0^n}\right)\bar{p}, \bar{u}^n\right) \\ &= (\bar{p})^T\left(\frac{-\bar{t}_0^n I_M + [\bar{t}^n]}{1+\bar{t}_0^n}\right) G_p^n\left(1, \left(\frac{I_M + [\bar{t}^n]}{1+\bar{t}_0^n}\right)\bar{p}, \bar{u}^n\right) \end{aligned} \quad (42)$$

Let

$$\Xi^n \equiv \left(\frac{-\bar{t}_0^n I_M + [\bar{t}^n]}{1+\bar{t}_0^n}\right)\bar{p}$$

Then, since

$$\left(\frac{I_M + [\bar{t}^n]}{1+\bar{t}_0^n}\right)\bar{p} = \bar{p} + \Xi^n$$

(42) becomes

$$G^n(1, \bar{p} + \Xi^n, \bar{u}^n) = (\Xi^n)^T G_p^n(1, \bar{p} + \Xi^n, \bar{u}^n),$$

which is formally the same as (32). The modified non-proportionality condition in the case of ad valorem tariffs is that there exists some n such that

$$\left(\frac{-\bar{t}_0^1 I_M + \left[\bar{t}^1\right]}{1+\bar{t}_0^1}\right)\bar{p} \neq k\left(\frac{-\bar{t}_0^n I_M + \left[\bar{t}^n\right]}{1+\bar{t}_0^n}\right)\bar{p} \quad \text{for } {}^{\forall}k \in R \tag{43}$$

If the numeraire good is freely traded $(\bar{t}_0^1 = \bar{t}_0^n = 0)$, (43) can be rewritten as

$$[\bar{t}^1]\bar{p} \neq k[\bar{t}^n]\bar{p} \quad \text{for } {}^{\forall}k \in R$$

Since $[\bar{t}^n]$ is the diagonal matrix, $[\bar{t}^1]\bar{p} \neq k[\bar{t}^n]\bar{p}$ is identical to $(\bar{t}_1^1, \ldots, \bar{t}_M^1) \neq k(\bar{t}_1^n, \ldots, \bar{t}_M^n)$, which looks close to the original non-proportionality condition.

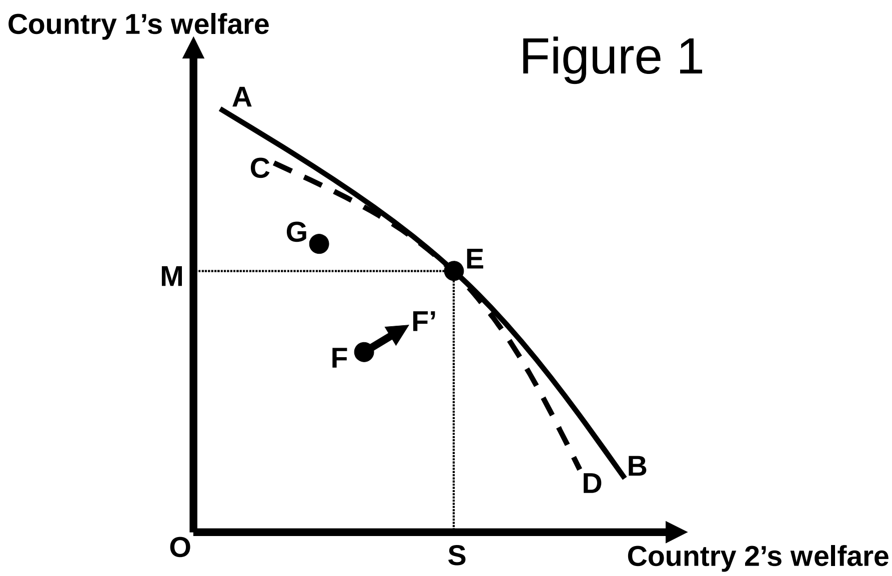
Country 1's welfare
Figure 1
A
C
G
E
M
F'
F
B
D
O
S
Country 2's welfare

Figure 1:

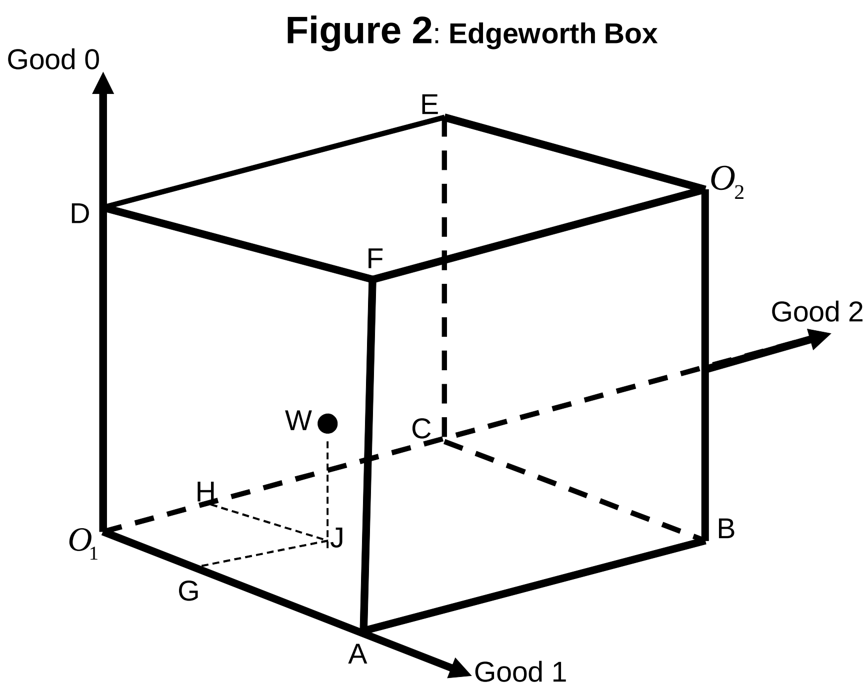

Figure 2:

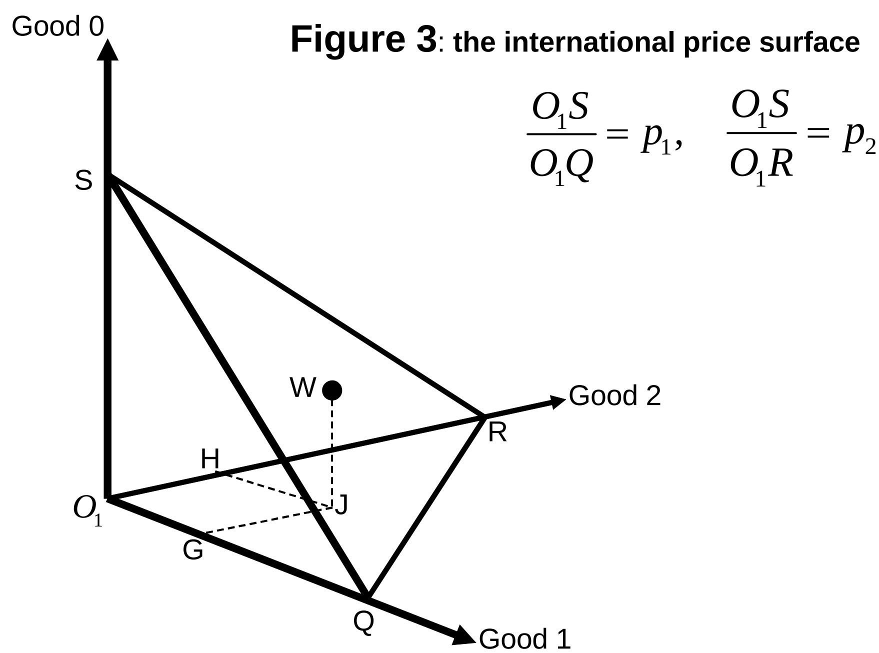
Figure 3: the international price surface
$\frac{O_1S}{O_1Q} = p_1, \quad \frac{O_1S}{O_1R} = p_2$
Good 0
S
W
Good 2
R
H
O_1
J
G
Q
Good 1

Figure 3:

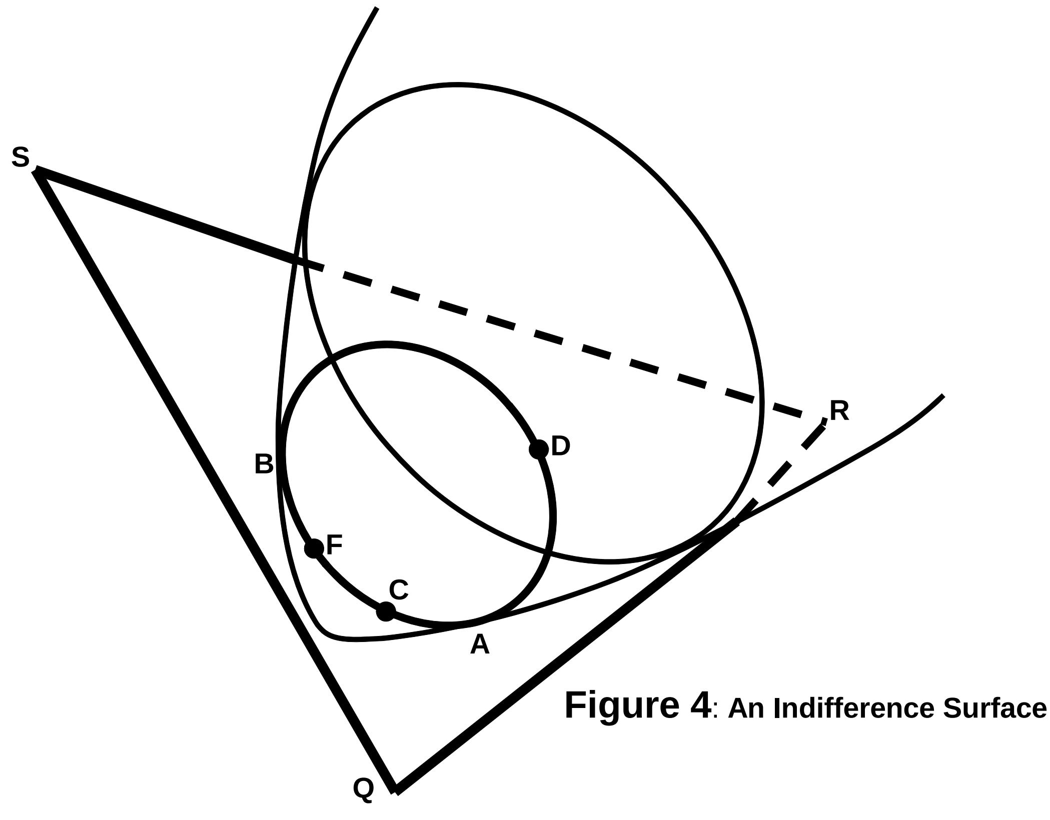

Figure 4:

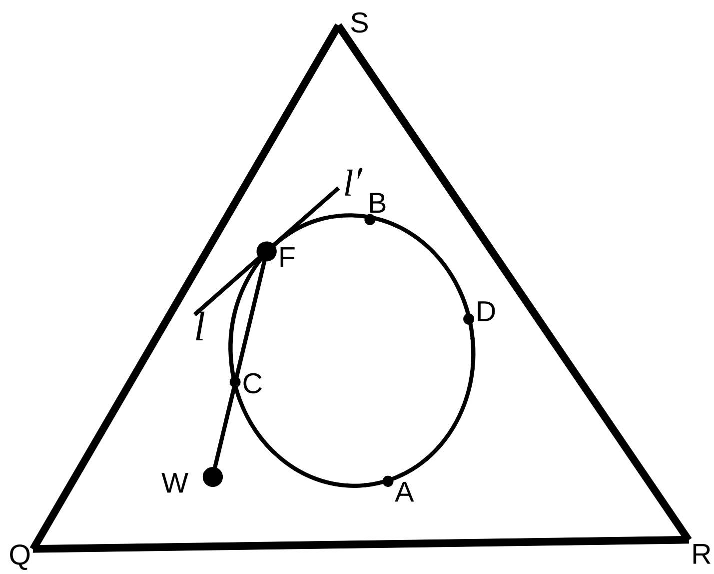

Figure 5: F: Country 1's consumption point

WF: Country !'s trade vector

lFl' :The tariff ratio imposed by Country 1

Figure 5:

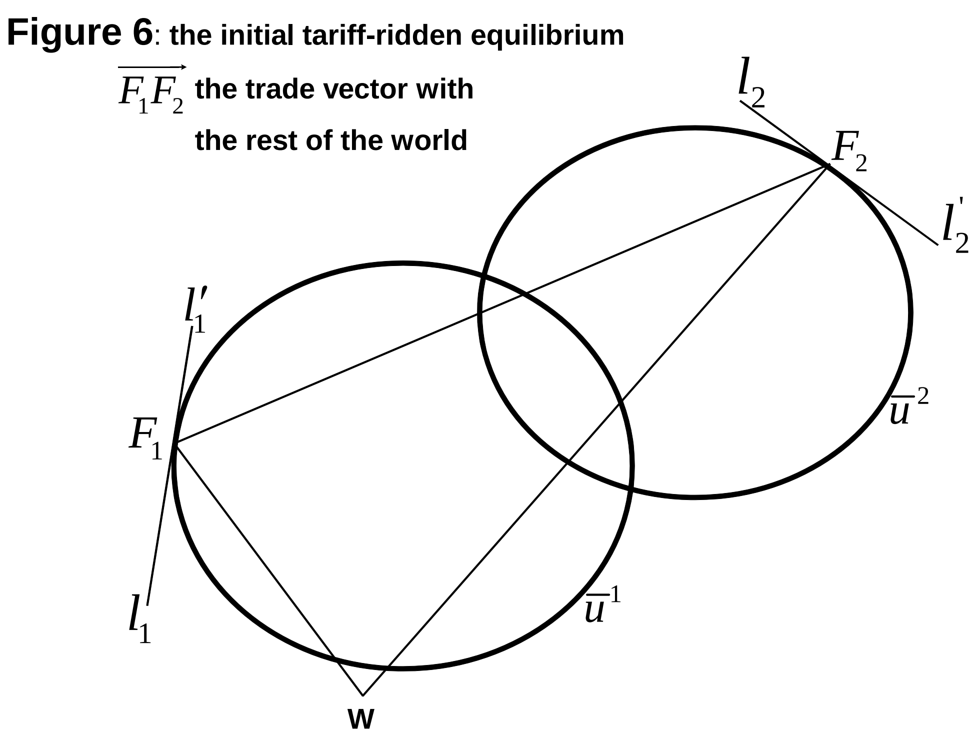
Figure 6: the initial tariff-ridden equilibrium
$\overrightarrow{F_1F_2}$ the trade vector with
the rest of the world
l_2
F_2
l_2'
l_1'
F_1
$\bar{u}^2$
l_1
$\bar{u}^1$
W

Figure 6:

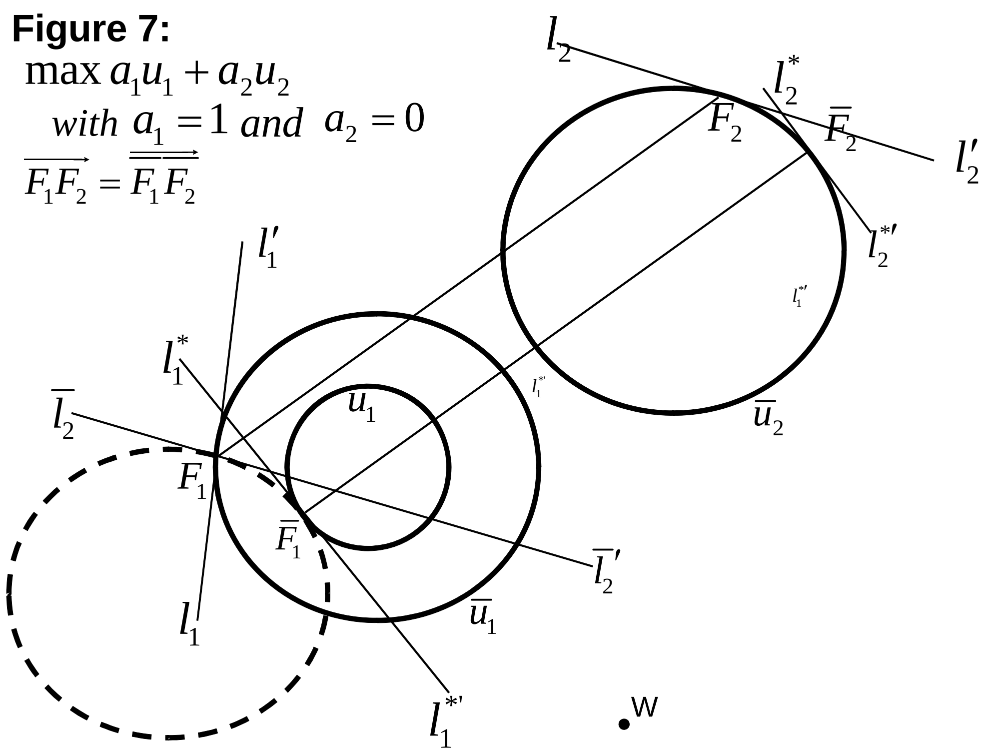

Figure 7:
max $a_1u_1 + a_2u_2$
with $a_1 = 1$ and $a_2 = 0$
$\overrightarrow{F_1F_2} = \overrightarrow{\bar{F}_1\bar{F}_2}$
l_2
l_2^*
F_2
$\bar{F}_2$
l_2'
$l_2^{*\prime}$
l_1'
l_1^*
u_1
$\bar{u}_2$
$\bar{l}_2$
F_1
$\bar{F}_1$
$\bar{l}_2'$
$\bar{u}_1$
l_1
$l_1^{*\prime}$
W

Figure 7:

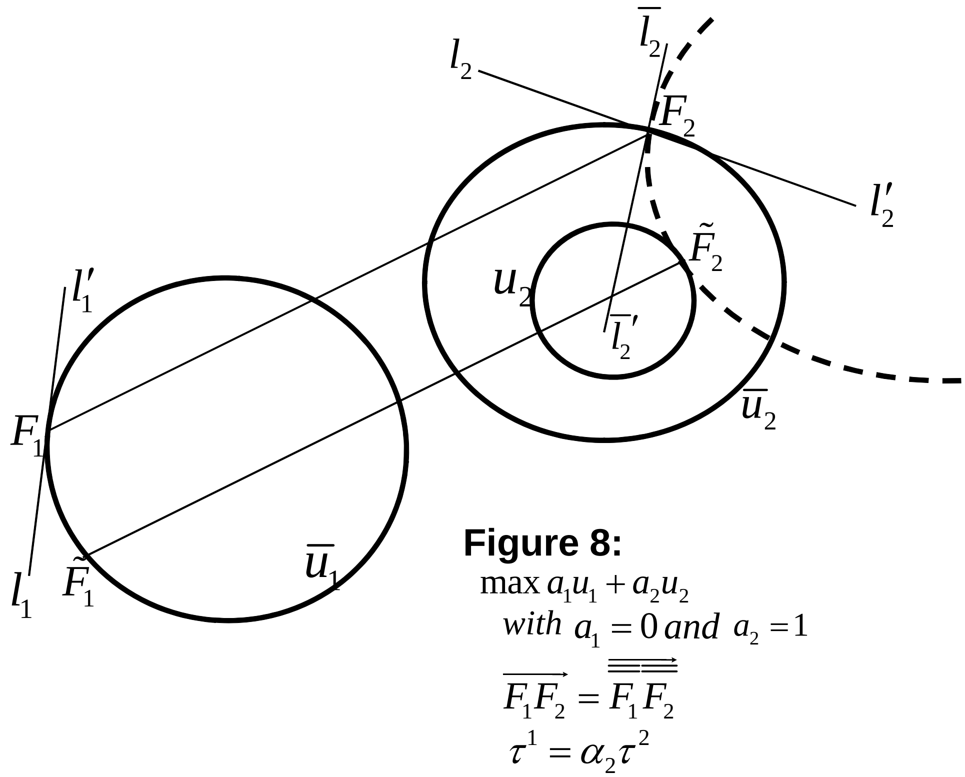
$\bar{l}_2$
l_2
F_2
l_2'
$\tilde{F}_2$
u_2
$\bar{l}_2'$
l_1'
F_1
$\bar{u}_2$
l_1
$\tilde{F}_1$
$\bar{u}_1$
Figure 8:
$\max a_1 u_1 + a_2 u_2$
with $a_1 = 0$ *and* $a_2 = 1$
$\overrightarrow{F_1 F_2} = \overrightarrow{\tilde{F}_1 \tilde{F}_2}$
$\tau^1 = \alpha_2 \tau^2$

Figure 8:

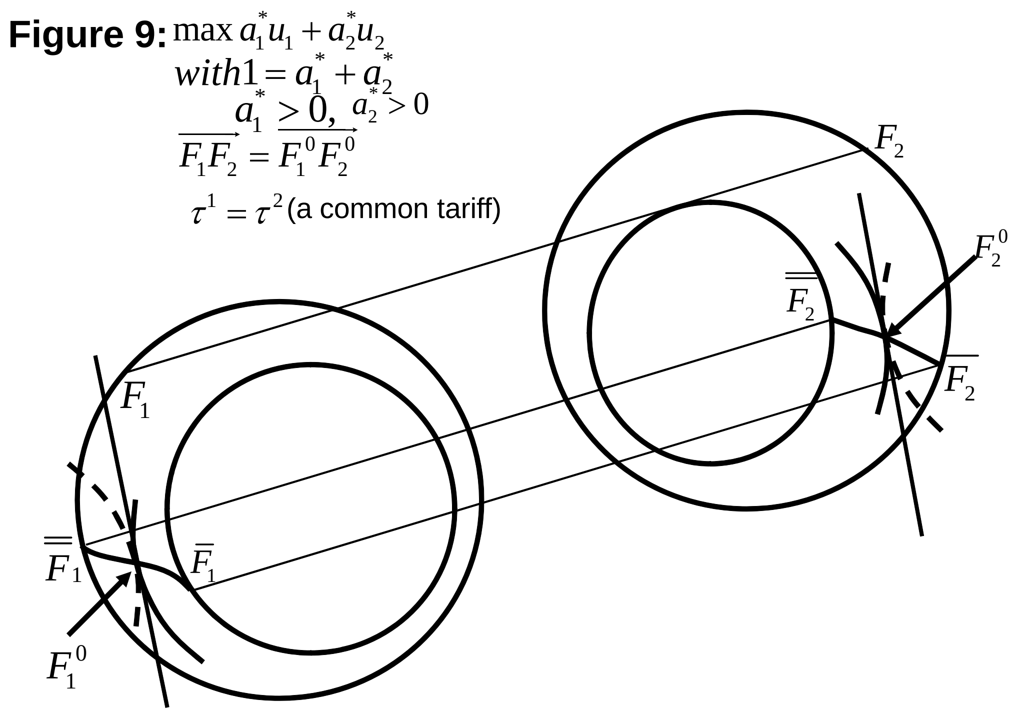
Figure 9: $\max a_1^* u_1 + a_2^* u_2$
$with 1 = a_1^* + a_2^*$
$a_1^* \geq 0, \; a_2^* > 0$
$\overrightarrow{F_1 F_2} = \overrightarrow{F_1^0 F_2^0}$
$\tau^1 = \tau^2$ (a common tariff)
F_2
F_2^0
$\overline{\overline{F}}_2$
$\overline{F}_2$
F_1
$\overline{\overline{F}}_1$
$\overline{F}_1$
F_1^0

Figure 9:

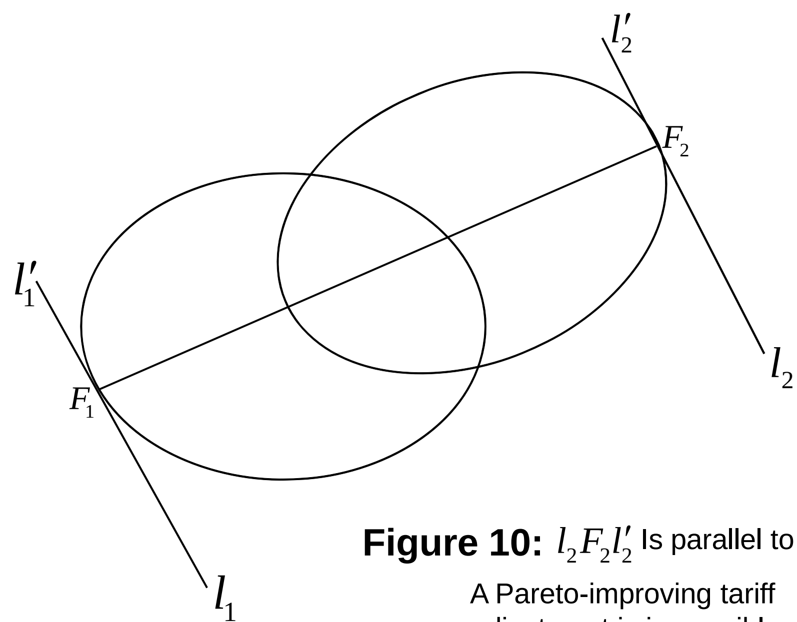

Figure 10: $l_2 F_2 l_2'$ Is parallel to $l_1 F_1 l_1'$

A Pareto-improving tariff adjustment is impossible without income transfers.

Figure 10: