A New Perspective on Rational Expectations

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Abstract: A central problem of rational expectations equilibrium (REE) theory is that a REE may fail to exist. This failure is known since Kreps (1977). The influential papers of Radner (1979) and Allen (1981) were able to establish that a REE exists only generically. We show that this undesirable feature is related to the assumption that individuals are expected utility maximizers (Bayesians). If individuals have ambiguity aversion in the form of the maximin expected utility (MEU) model introduced by Gilboa and Schmeidler (1989), then a REE exists *universally* and not only generically. To prove this, we provide a suitable generalization of REE, which we call maximin rational expectations equilibrium (MREE). MREE allocations need not to be measurable with respect to the private information of each individual and with respect to the information that the equilibrium prices generate, as it is in the case of the Bayesian REE. We also prove that a MREE is efficient and incentive compatible. These results are false for the Bayesian REE.

Keywords: Rational Expectations, Ambiguity Aversion, Maximin Expected Utility.

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1 Introduction

In economics, dissatisfaction with the reigning Bayesian paradigm is as old as the paradigm itself. Indeed, important criticisms of Savage (1954)'s expected utility theory go back to Allais (1953), Ellsberg (1961) and others. However, no criticism has substantial impact without an alternative. To this date, the most successful alternative to Bayesianism is the Maximin Expected Utility (MEU) model introduced by Gilboa and Schmeidler (1989), which generated a huge literature on ambiguity aversion. In recent years, ambiguity aversion models have led to interesting applications in finance, macroeconomics, game theory and mechanism design, as we briefly discuss in Section 1.2 below. The purpose of this paper is to show that ambiguity aversion also has an important impact on the way we understand *rational expectations equilibrium (REE)*.

Rational expectations are important whenever the market's participants have relevant private information. Indeed, as pointed out by Hayek (1945), a central feature of markets is the transmission of information through prices. Given that prices contain information, market participants should (and do) use such information when making decisions. The incorporation of the prices information into the agents' decisions leads to the rational expectations framework. Unfortunately, however, if individuals are expected utility maximizers (Bayesian), a REE may fail to exist. This fact was established by Kreps (1977), through an influential and well-known example, which we revisit in Section 1.1 below.[4] This failure of existence is an extremely undesirable feature, because it hampers the understanding of markets where private information is relevant, which encompasses virtually all economically relevant markets. This led to considerable efforts to overcome this difficult. In seminal papers, Radner (1979) and Allen (1981) prove the *generic* existence of REE when individuals are Bayesians.[5]

Our main result is that a REE exists *universally* (not generically) when the agents have MEU preferences. For this, we reformulate the Bayesian rational expectations equilibrium of Radner (1979) and Allen (1981) for MEU individuals. Specifically, in our setup, agents maximize their maximin expected utility conditioning on their own private information and also on the information the equilibrium prices have generated. In this setting the resulting maximin REE may not be measurable with respect to the private information of each individual and also with respect to the information that the equilibrium prices generate (contrary to the Bayesian REE). Nonetheless, market clearing occurs for every state of nature. Furthermore, we show that the maximin REE is incentive compatible and efficient. These results are false for the Bayesian REE.[6,7]

The following reexamination of the financial example introduced by Kreps (1977) clarifies our results.

[4] Green (1977) also presented a different non-existence example of the rational expectations equilibrium.

[5] For a history of rational expectations equilibrium, see Grossman (1981).

[6] See Glycopantis and Yannelis (2005, p. 31 and Example 9.1.1 p. 43).

[7] An attempt to introduce non-expected utility into general equilibrium theory was previously made by de Castro and Yannelis (2008). Specifically, de Castro and Yannelis (2008) showed that by replacing the Bayesian (subjective expected utility) by the maximin expected utility, the conflict between efficiency and incentive compatibility ceases to exist. In this paper, we continue this line of research by introducing non-expected utility into the rational expectations equilibrium.

1.1 Kreps' example

Kreps (1977) provides a simple financial example that allows us to understand the heart of our contribution. He assumes that there are two assets: a riskless asset that costs and pays 1 and a risky asset that is sold at period $t = 1$ by the price $p(\omega) \in \mathbb{R}_+$ and pays $V(\omega)$ in period $t = 2$. Here, ω denotes the state of the world. There are two individuals, both with utility $U(c) = -e^{-c}$ for the consumption of c units at $t = 2$. Individual 1 knows whether $V(\omega)$ is distributed according to a normal with mean m_1 and variance σ^2 or according to a normal with mean m_2 and variance σ^2. Let s_1 denote the first distribution and s_2, the second. That is, individual 1 knows which distribution s_j $(j = 1, 2)$ governs $V(\omega)$. On the other hand, individual 2 only knows that the distribution governing $V(\omega)$ is in the set $S \equiv \{s_1, s_2\}$, but he can infer s once he observes the prices.[8] To complete the description, assume that individual i is endowed with k_{ij} units of the risky asset if $s_j \in \{s_1, s_2\}$. Endowments of the riskless asset are constant and, therefore, ignored.

Now if an individual knows s and buys q units of the risky asset, his consumption will be $x(\omega) = -p(\omega) \cdot q + (q + k_{i\cdot}) \cdot V(\omega)$, leading to the expected utility:

$$u(s, x) = E_s \{-\exp[-(-p \cdot q + (q + k_{i\cdot}) \cdot V)]\}, \tag{1}$$

where E_s denotes expectation with respect to $s \in \{s_1, s_2\}$. As natural, we assume that the price $p(\omega)$ depends only on s and write $p(\omega) = p_j$ if $s = s_j$, $j = 1, 2$. Given the normality of the risky asset returns, we have for $j = 1, 2$:

$$u(s_j, x) = -\exp\left[-(q + k_{ij})(m_j - p_j) + \frac{\sigma^2}{2}(q + k_{ij})^2\right], \tag{2}$$

which leads to the following optimal quantity if the individual knows which s obtains:

$$q_{ij} = \frac{m_j - p_j}{\sigma^2} - k_{ij}, \quad \text{for } i = 1, 2 \text{ and } s = s_j, j = 1, 2. \tag{3}$$

Let us consider the case in which *both individuals are Bayesian*. If individual 2 is uniformed, that is, $p_1 = p_2$, then he considers a mixture of normals (s_1 and s_2). In any case, his optimal choice, although not given by (3), is a *single quantity* $q_{21} = q_{22}$. Kreps first observes that if $m_1 \neq m_2$ and $k_{1j} = 0$, for $j = 1, 2$ then prices cannot be uninformative, that is, we cannot have $p_1 = p_2$. Indeed, in this case $q_{11} \neq q_{12}$, but since $q_{2j} = -q_{1j}$, this would imply $q_{21} \neq q_{22}$, contradicting the previous observation.

Thus, assume that $p_1 \neq p_2$ and individual 2 is informed, that is, all choices are given by (3). Kreps notes that if $m_1 = 4$, $m_2 = 5$, $k_{21} = 2$, $k_{22} = 4$ and $\sigma^2 = 1$, then $p_1 = p_2 = 3$, which contradicts $p_1 \neq p_2$. This contradiction shows that no rational expectations equilibrium exists.

Let us now observe what happens with our MEU formulation. Under full information, there is no ambiguity and the individuals' behaviors are exactly as above. However, in the case that 2 is uniformed ($p_1 = p_2$), then he faces ambiguity and takes the worst-case scenario in his evaluation. He is, therefore, indifferent among a set of

[8] Nothing changes in our analysis if we assume that individual 2 considers all convex combinations of s_1 and s_2 as possible.

different quantities q_{ij}—in particular, he is indifferent among quantities that promises utilities above the minimum between the two states.[9] Which among his equally good quantities will be selected? It is standard to think that a Walrasian auctioneer selects the quantity that clears the market, but the information about the quantity chosen by the Walrasian auctioneer is available to the individual only after all choices are made and, therefore, cannot affect his behavior. This means that the restriction $q_{21} = q_{22}$ used above no longer holds. He could receive different quantities on different states. In the example above, this shows that $p_1 = p_2 = 3$ would be an equilibrium with $q_{21} = -1$ and $q_{22} = -2$.[10]

As the reader has noticed, the crucial property is the individual's indifference among many bundles. This indifference between allocations leads to an important departure from the Bayesian case. Early works, such as Dow and Werlang (1992), have explored this indifference. Note also that the property described that the individual does not know how many units he will actually receive is not an artifact of our model. In real world markets, this is almost always true. Once a trader submits an order, especially big ones, he does not know how many actual units will be traded and, when he learns that, the trade is already completed. In *dark pools*, this separation between the price and the volume information is even more pronounced, and our model and above discussion seems even more relevant.

Remark 1.1 Notice that in the original Kreps' model, information is transmitted not only through prices, but also through quantities. In contrast, our formulation takes this channel out of the picture. Notice that transmission through quantities is never emphasized as a desirable characteristics of a rational expectations model—transmission through prices is. The fact that quantities play an important role in the original Kreps' example seems more of a disadvantage than a feature that should be incorporated in rational expectations models.[11]

1.2 Ambiguity in Economics

Our paper belongs to the growing literature that applies ambiguity aversion to revisit old puzzles and facts that were not well understood under the Bayesian framework, but could be successfully explained using ambiguity aversion. Our contribution shows a new feature of ambiguity aversion that highlights the usefulness of these models established by previous papers in many different applications.

For instance, Caballero and Krishnamurthy (2008) used a MEU model to study flight to quality episodes, which are an important source of financial and macroeconomic instability. Given the repeated occurrence of such crises and their economic impact, this is an important topic of investigation. Their MEU model is able to explain crisis regularities such as market-wide capital immobility, liquidity hoarding and agents' disengagement from risk.

[9] Note that he is indifferent taking in account the information that he has when making decisions. Obviously, he is *not* indifferent ex post.

[10] Note that the individual is indifferent between $q_2 = (-1, q_{22})$ and $(-1, q'_{22})$ as long as $q_{22}, q'_{22} \geqslant -3$.

[11] This is perhaps the reason why Kreps' example was reformulated to avoid this aspect. See, for instance, example 19.H.3 of Mas-Colell, Whinston, and Green (1995, p. 722).

Epstein and Schneider (2007) consider portfolio choices and the effect of changes in confidence due to learning for Bayesian and ambiguity averse agents. They show that ambiguity aversion induces more stock market participation and investment in comparison to Bayesian individuals. A variation on this topic is pursued in Epstein and Schneider (2008), that assumes that investors perceive a range of signal precisions, and evaluate financial decisions with respect to worst-case scenarios. As a result, good news is discounted, while bad news is taken seriously. This implies that expected excess returns are thus higher when information quality is more uncertain. They are also able to provide an explanation to the classic question in finance of why stock prices are so much more volatile than measures of the expected present value of dividends. The recent work by Epstein and Schneider (2010) discusses how ambiguity aversion models have implications for portfolio choice and asset pricing that are very different from those of the standard Bayesian model. They also show how this can explain otherwise puzzling features of the data.

Ozsoylev and Werner (2011) also analyzed information in financial asset markets with ambiguity averse investors. They show the possibility of illiquid markets where arbitrageurs choose not to trade in a rational expectations equilibrium. Also, small informational shocks may have relatively large effects on asset prices. Condie and Ganguli (2011a) also studied rational expectations equilibria with ambiguity averse decision makers. They show that partial revelation can be robust in a MEU model, while Condie and Ganguli (2011b) established full revelation for almost all sets of beliefs for Choquet expected utility with convex capacities. Their papers are very related to ours; we discuss them further in Section 7 below.

Perhaps one of the more interesting set of implications of ambiguity aversion models has been obtained by Ju and Miao (2012). They calibrated a smooth ambiguity model that matches the mean equity premium, the mean risk-free rate and the volatility of the equity premium. Their model also allows to explain many curious facts previously observed in the data, such as the procyclical variation of price-dividend ratios, the countercyclical variation of equity premia and equity volatility, the leverage effect and the mean reversion of excess returns. All these results hinge crucially on the pessimistic behavior of ambiguity averse agents.

Ilut and Schneider (2012) use ambiguity aversion to study business cycle fluctuations in a DSGE model. They show that a loss of confidence about productivity works like "unrealized" bad news. This time-varying confidence can explain much of business cycle fluctuations.

Hansen and Sargent (2012) define three types of ambiguity, depending on how the models of a planner and agents differ. All these variations are departures from the standard rational expectations Bayesian model, where planner and agents share exactly the same model. They compute a robust Ramsey plan and an associated worst-case probability model for each of three types of ambiguity and examine distinctive implications of these models.

1.3 Organization of the paper

The paper is organized as follows: in Section 2 we describe the economic model and define the two sets of preferences that we consider in our paper. In Section 3 we define and compare the standard Bayesian REE and our maximin REE (MREE). Section 4 establishes the existence of MREE. Efficiency and Incentive Compatibility of MREE are established in Section 5. In Section 6, we discuss how our model can deal with the full generality of Gilboa and Schmeidler (1989)'s model. Section 7 further discusses related literature. Some concluding remarks and open questions are collected in Section 8. The appendix (Section 9) collects longer proofs.

2 Model—Differential Information Economy

2.1 Differential information economy

We consider an exchange economy under uncertainty and asymmetrically informed agents. The uncertainty is represented by a measurable space $(S, \mathcal{F})$, where S is a finite set of possible states of nature and $\mathcal{F}$ is the algebra of all the events. Let $\mathbb{R}^{\ell}_{+}$ be the commodity space and I be a set of n agents, i.e., $I = \{1, \ldots, n\}$. **A differential information exchange economy** $\mathcal{E}$ is the following collection:

$$\mathcal{E} = \{(S, \mathcal{F});\ (\mathcal{F}_i, u_i, e_i)_{i \in I}\},$$

where for all $i \in I$

- $\mathcal{F}_i$ is a partition of S, representing the **private information** of agent i. The interpretation is as usual: if $s \in S$ is the state of nature that is going to be realized, agent i observes $\mathcal{F}_i(s)$, the unique element of $\mathcal{F}_i$ containing s.[12] By an abuse of notation, we still denote by $\mathcal{F}_i$ the algebra generated by the partition $\mathcal{F}_i$.

- a **random utility function** (or state dependent utility) representing his (ex post) preferences:

$$\begin{array}{rccc} u_i: & S \times \mathbb{R}^{\ell}_{+} & \to & \mathbb{R} \\ & (s, x) & \to & u_i(s, x). \end{array}$$

  We assume that for all $s \in S$, $u_i(s, \cdot)$ is continuous.

- a **random initial endowment** of physical resources represented by a function $e_i : S \to \mathbb{R}^{\ell}_{+}$.

For some results (but not for our existence Theorem 4.1), we will need the following:

Assumption 2.1 *For each $i \in I$, $e_i(\cdot)$ is $\mathcal{F}_i$-measurable.*

We discuss this assumption, the interpretation of the above economy and its timing in Section 2.4 below.[13]

[12] But see also Section 6 below, where we discuss the interpretation of S as sets of probabilities.

[13] An alternative assumption is that the trade, that is, $x_i(\cdot) - e_i(\cdot)$ is measurable. This means that the individual makes a decision to buy specific quantities of the assets. This can be required, but as we have argued in the introduction, this is not always realistic. Moreover, an ambiguous averse individual will be indifferent among many bundles, making the specification arbitrary.

A **price** p is a function from S to $\mathbb{R}^\ell_+ \setminus \{\mathbf{0}\}$. In order to introduce the rational expectation notions in Section 3, we need the following notation. Let $\sigma(p)$ be the smallest sub-algebra of $\mathcal{F}$ for which $p(\cdot)$ is measurable and let $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ denote the smallest algebra containing both $\mathcal{F}_i$ and $\sigma(p)$.

A function $x : I \times S \to \mathbb{R}^\ell_+$ is said to be a **random consumption vector or allocation**. For each i, the function $x_i : S \to \mathbb{R}^\ell_+$ is said to be an allocation of agent i, while for each s, the vector $x_i(s) \in \mathbb{R}^\ell_+$ is a bundle of agent i in state s. We denote by L_i the set of all i's allocations, moreover let $\bar{L}_i$ and L_i^{REE} be the following sets:

$$\bar{L}_i = \{x_i \in L_i : \ x_i(\cdot) \text{ is } \mathcal{F}_i\text{−measurable}\}. \tag{4}$$
$$L_i^{REE} = \{x_i \in L_i : \ x_i(\cdot) \text{ is } \mathcal{G}_i\text{−measurable}\}. \tag{5}$$

Clearly, for each agent $i \in I$, since any $\mathcal{F}_i$-measurable allocation is also $\mathcal{G}_i$-measurable, it follows that $\bar{L}_i \subseteq L_i^{REE} \subseteq L_i$, and hence $\bar{L} \subseteq L^{REE} \subseteq L$, where $L = \prod_{i\in I} L_i$, $\bar{L} = \prod_{i\in I} \bar{L}_i$ and $L^{REE} = \prod_{i\in I} L_i^{REE}$.

An allocation x (i.e., $x \in L$) is said to be **feasible** if

$$\sum_{i\in I} x_i(s) = \sum_{i\in I} e_i(s) \quad \text{for all } s \in S.$$

We will describe the agents' preferences below. The above structure, including each agent's preference, is common knowledge for all agents.

2.2 Bayesian expected utility

We define now the (Bayesian or subjective expected utility) interim expected utility. To this end, we assume that each individual $i \in I$ has a known probability π_i on $\mathcal{F}$, such that $\pi_i(s) > 0$ for any $s \in S$. For any partition $\Pi \subset \mathcal{F}$ of S and any allocation $x_i : S \to \mathbb{R}^\ell_+$, agent i's **interim expected utility** function with respect to Π at x_i in state s is given by

$$v_i(x_i|\Pi)(s) = \sum_{s'\in S} u_i(s', x_i(s'))\pi_i(s'|s),$$

where

$$\pi_i(s'|s) = \begin{cases} 0 & \text{for} \quad s' \notin \Pi(s) \\ \frac{\pi_i(s')}{\pi_i\left(\Pi(s)\right)} & \text{for} \quad s' \in \Pi(s). \end{cases}$$

We can also express the interim expected utility as follows

$$v_i(x_i|\Pi)(s) = \sum_{s'\in\Pi(s)} u_i(s', x_i(s'))\frac{\pi_i(s')}{\pi_i\left(\Pi(s)\right)}. \tag{6}$$

Notice that the interim expected utility function v_i is well defined since we have assumed that for each $i \in I$ and $s \in S$, $\pi_i(s) > 0$, therefore $\pi_i(\Pi(s)) > 0$. Also note that this is just the conditional expected utility, where the conditioning is on $\Pi(s)$.

In the applications below, the partition Π will be agent-dependent, being the original private information partition $\mathcal{F}_i$ or, more frequently, the partition generated also by the prices, $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$.

2.3 Maximin Expected Utility

For many years, the Bayesian approach has been the only approach to attitude towards uncertainty. More recently, the ambiguity approach has been flourishing. One of the most used ambiguity models is the Maximin Expected Utility introduced by Gilboa and Schmeidler (1989). According to this model, each decision maker chooses the action that maximizes the minimum of the expected utility over a set $\mathcal{C}$ of possible priors. This approach reduces to the Bayesian one whenever the set of priors $\mathcal{C}$ is a singleton. We will consider conditional preferences in the Gilboa-Schmeidler form, where the set $\mathcal{C}$ depends on the private information that the decision maker might possess.

More specifically, let $\Pi \subset \mathcal{F}$ be a partition of S representing the information available to individual i. If the state is s, so that i knows that $\Pi(s)$ obtains, then $\mathcal{C}_i^s$ will be the set of all probabilities with support contained on $\Pi(s)$. In this case, the maximin utility of agent i with respect to Π at x_i in state s is:

$$\underline{u}_i^{\Pi}(s, x_i) = \min_{\mu \in \mathcal{C}_i^s} E_\mu \left[u_i(\cdot, x_i(\cdot))\right] = \min_{s' \in \Pi(s)} u_i(s', x_i(s')). \tag{7}$$

Whenever for each agent i the partition Π is his private information partition $\mathcal{F}_i$, then we do not use the superscript, i.e.,

$$\underline{u}_i(s, x_i) = \min_{s' \in \mathcal{F}_i(s)} u_i(s', x_i(s')).$$

On the other hand, when we deal with the notion of rational expectations equilibrium (according to which agents take into account also the information that the equilibrium prices generate), then for each agent i the partition Π is $\mathcal{G}_i$ and the maximin utility is defined as

$$\underline{u}_i^{REE}(s, x_i) = \min_{s' \in \mathcal{G}_i(s)} u_i(s', x_i(s')), \text{ where } \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p).$$

This model seems to be a particular case of Gilboa-Schmeidler's MEU, because we specify as the set of probabilities the one containing all probabilities with support in the element of the partition. However, as we discuss in Section 6, our theory can accommodate the full generality of Gilboa-Schmeidler's preferences.

Remark 2.1 For our purposes, it is enough to describe only the conditional preferences, as Condie and Ganguli (2011a) have done. These conditional preferences could be obtained through Bayesian update of each prior, in the way that is now widespread in papers considering ambiguity aversion. See for instance Epstein and Schneider (2003).This would complicate notation without any gain, since we consider only interim choices.

2.4 Timing and Budget Sets

We can specify the timing of the economy as follows. There are three periods: ex ante ($t = 0$), interim ($t = 1$) and ex post ($t = 2$). Although consumption takes place only at the ex post stage, the other events occur as follows:

- At $t = 0$, the state space, the partitions, the structure of the economy and the price functional $p : S \to \mathbb{R}^{\ell}_{+} \setminus \{\mathbf{0}\}$ are common knowledge. This stage does not play any role in our analysis and it is assumed just for a matter of clarity.
- At $t = 1$, each individual learns his private information $\mathcal{F}_i(s)$ and the prevailing prices $p(s) \in \mathbb{R}^{\ell}_{+} \setminus \{\mathbf{0}\}$. Therefore, he learns $\mathcal{G}_i(s)$, where $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. With this information, the individual plans how much he will consume, $x_i(s)$. Note, however, that his actual consumption (as his endowment) may be contingent to the final state of the world, not yet known by the individual. The agent only knows that one of the states $s' \in \mathcal{G}_i(s)$ obtains, but not exactly which. Therefore, he needs to make sure that he will be able to pay his consumption plan $x_i(s')$ for all $s' \in \mathcal{G}_i(s)$, that is, $p(s') \cdot x_i(s') \leqslant p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i(s')$.
- At $t = 2$, individual i receives and consumes his entitlement $x_i(s)$.

The interpretation of this model is that the plan that the individual makes at the interim stage ($t = 1$) serves as the channel through which his information is passed to the system, or to the "Walrasian auctioneer," if one prefers. This is necessary for the purpose of aggregation of information among the individuals and to guarantee the feasibility of the final allocations.

A particular case of the above specification is the model in which endowments are private information measurable (the individual knows his endowment), as in Allen (1981, p. 1179). Our model certainly allows this case, but our main result does not require it. Therefore, we refrain from imposing this condition in our general framework. This allows us to cover situations in which is more natural to assume that individuals *do not* know their endowments. For instance, in labor markets, workers may fail to be completely informed of their abilities. Another example: someone has stored corn in a barn, but does not know how much of it survived the appetite of the barn's rats.

Note also that the consumption plan $x_i(\cdot)$ does not need to be private information measurable, as it is usually assumed in these models (see Radner (1979, 1982)). We have already discussed this in the context of financial markets (see the end of Section 1.1), but it is also reasonable in many other situations. For example, assume that you visit a restaurant in an exotic country for the first time. Although you know how much you have in cash and the price that you will have to pay for your meal, you will not know exactly what you will eat (or its quality) until the meal is actually served to you. Yet another example: you may know what you contracted and how much is the premium for your insurance, but do not know how good their services will be in the event that you fill a claim.

Note that the above discussion leads to the following budget set:

$$B_i(s,p) = \{y_i \in L_i \ : \ p(s') \cdot y_i(s') \le p(s') \cdot e_i(s') \ \text{ for all } s' \in \mathcal{G}_i(s)\}. \tag{8}$$

We will use it in the next sections.

3 Maximin REE vs the Bayesian REE

This section defines the standard Bayesian rational expectations equilibrium (REE), followed by the maximin REE (MREE). We compare the two notions in Section 3.3 and establish further properties of the MREE in Section 3.4.

3.1 Bayesian rational expectations equilibrium (REE)

In this section, we consider a differential information economy in which all market participants have preferences represented by the Bayesian interim expected utility function given by (6).

According to the notion of rational expectations equilibrium, agents make their consumption decision taking into account not only their private information, but also the information generated by the equilibrium price. Thus, agents' preferences are represented by (6) where for any $i \in I$, $\Pi = \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, if s is the realized state of nature, each agent i receives the information signal $\mathcal{G}_i(s)$, the unique element of the partition $\mathcal{G}_i$ containing s. With this information, agent trades. In the second period, once consumption takes place, the state of nature is only incompletely and differently observed by agents. Indeed, if s occurs, each individual i does not know which state belonging in the event $\mathcal{G}_i(s)$ has occurred. Hence, i asks to consume the same bundle in those states he is not able to distinguish, which means that allocations are required to be $\mathcal{G}_i$-measurable.

The notion below is due to Radner (1979) and Allen (1981).

Definition 3.1 *A price p and a feasible allocation x are said to be a* **Bayesian rational expectations equilibrium** *(REE) for the economy $\mathcal{E}$ if*

(i) *for all $i \in I$, the allocation $x_i(\cdot)$ is $\mathcal{G}_i$-measurable;*

(ii) *for all $i \in I$ and for all $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;*

(iii) *for all $i \in I$ and for all $s \in S$,*

$$v_i(x_i|\mathcal{G}_i)(s) = \max_{y_i \in B_i(s,p) \cap L_i^{REE}} v_i(y_i|\mathcal{G}_i)(s),$$

where $B_i(s,p)$ was defined by (8) and L_i^{REE} was defined by (5).

Note that the maximization is done over a budget set that is more restricted than $B_i(s,p)$, because we require that the acts are $\mathcal{G}_i$-measurable. This definition does not seem to be exactly the one given by Radner (1968), who requires that the sum of prices are not exceeded, that is,[14]

$$\sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot y_i(s') \leq \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s'). \tag{9}$$

This difference is only apparent, because the above definition is equivalent to Radner (1968)'s, as the following lemma establishes.

[14] The reader can understand the justification throughout the following example: if the price to receive an ice-cream in a hot day is $\$1$ and this price in a not-hot day is $\$0.50$, then the price to receive the ice-cream irrespective of the temperature should be $\$1.50$.

Lemma 3.1 *Let assumption 2.1 hold. Given s, the following conditions are equivalent:*

(i) $y_i \in B_i(s,p) \cap L_i^{REE}$*;*

(ii) y_i *is* $\mathcal{G}_i$*-measurable and satisfies (9);*

(iii) y_i *is* $\mathcal{G}_i$*-measurable and*

$$p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s). \tag{10}$$

Proof. If $y_i \in L_i^{REE}$, that is, y_i is $\mathcal{G}_i$-measurable, then $p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s)$ is equivalent to $p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i(s)$, which establishes the equivalence of (i) and (iii).

$(ii) \Leftrightarrow (iii)$: Assume that y_i is $\mathcal{G}_i$-measurable. Since $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, then $p(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$, as well as $y_i(\cdot)$, because $y_i \in L_i^{REE}$. Furthermore, since for all $i \in I$, $e_i(\cdot)$ is $\mathcal{F}_i$-measurable and $\mathcal{F}_i \subseteq \mathcal{G}_i$, then $e_i(\cdot)$ is $\mathcal{G}_i$-measurable. Therefore, for all $i \in I$ and all $s \in S$

$$\begin{aligned} \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot y_i(s') &= p(s) \cdot y_i(s) \, |\mathcal{G}_i(s)| \quad \text{and} \\ \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s') &= p(s) \cdot e_i(s) \, |\mathcal{G}_i(s)| \, , \end{aligned}$$

where $|\mathcal{G}_i(s)|$ is the number of states in the event $\mathcal{G}_i(s)$. Hence, for all $i \in I$, $s \in S$ and $y_i \in L_i^{REE}$,

$$p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s) \quad \Leftrightarrow \quad \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot y_i(s') \leq \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s').$$

□

The Bayesian REE is an interim concept since agents maximize conditional expected utility based on their own private information and also on the information that equilibrium prices have generated. The resulting allocation clears the market for every state of nature.

It is by now well known that a Bayesian rational expectations equilibrium (REE), as introduced in Allen (1981), may not exist. It only exists in a generic sense and not universally. Moreover, it fails to be fully Pareto optimal and incentive compatible and it is not implementable as a perfect Bayesian equilibrium of an extensive form game (Glycopantis, Muir, and Yannelis (2005)).

3.2 Maximin REE

In this section, we consider a differential information economy in which all market participants have preferences represented by the maximin utility function given by (7), where for any $i \in I$, $\Pi = \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Again, in the second period, the state of nature is only incompletely and differently observed by agents, with the usual interpretation: if s occurs, each individual i does not know which state belonging in the event $\mathcal{G}_i(s)$ has occurred. According to the maximin expected utility, any individual $i \in I$ considers the worst possible scenario, that is the lowest possible bound of happiness. Thus, he does not ask to consume the same bundle in those states he is not able to distinguish, but to consume the bundle in the event $\mathcal{G}_i(s)$ that maximizes his lowest bound of happiness (i.e., his maximin expected utility). Formally, if $s \in S$ is realized, each agent $i \in I$ maximizes $\underline{u}_i^{REE}(s, x_i)$ subject[15] to $p(s') \cdot x_i(s') \leq p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i(s)$ (which implies that $\sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot x_i(s') \leq \sum_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s')$). We are now able to define the notion of a maximin rational expectations equilibrium (MREE).

Definition 3.2 *A price p and a feasible allocation x are said to be a* **maximin rational expectations equilibrium** *(MREE) for the economy $\mathcal{E}$ if:*

(i) *for all $i \in I$ and for all $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;*

(ii) *for all $i \in I$ and for all $s \in S$, $\underline{u}_i^{REE}(s, x_i) = \max_{y_i \in B_i(s,p)} \underline{u}_i^{REE}(s, y_i)$.*

Condition (ii) indicates that each individual maximizes his maximin utility conditioned on his private information and the information the equilibrium prices have generated, subject to the budget constraint.

Either a Bayesian REE or a MREE are said to be (i) *fully revealing* if the equilibrium price reveals to each agent all states of nature, i.e., $\sigma(p) = \mathcal{F}$; (ii) *non revealing* if the equilibrium price reveals nothing, that is it is a constant function across states, i.e., $\sigma(p) = \{\emptyset, S\}$; finally (iii) *partially revealing* if the equilibrium price reveals some but not all states of nature, i.e., $\{\emptyset; S\} \subset \sigma(p) \subset \mathcal{F}$.

3.3 Relationship between the Bayesian REE and the maximin REE

We denote by $REE(\mathcal{E})$ and $MREE(\mathcal{E})$ respectively the set of Bayesian rational expectations equilibrium allocations and the set of maximin rational expectations equilibrium allocations of the economy $\mathcal{E}$.

We first notice that whenever the equilibrium price p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$, since $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, it follows that $\mathcal{G}_i = \mathcal{F}$ for each agent $i \in I$. Thus, for each state $s \in S$ and each agent $i \in I$, $\mathcal{G}_i(s) = \{s\}$, and hence $v_i(x_i|\mathcal{G}_i)(s) = u_i(s, x_i(s))$ as well as $\underline{u}_i^{REE}(s, x_i) = u_i(s, x_i(s))$. Moreover, $\mathcal{G}_i$-measurability assumption on Bayesian REE allocations plays no role, i.e., $L = L^{REE}$. Therefore, fully revealing Bayesian REE and fully revealing maximin REE coincide, i.e., $REE_{FR}(\mathcal{E}) = MREE_{FR}(\mathcal{E})$[16].

[15] Notice that if Assumption 2.1 holds, since $p(\cdot)$ is $\mathcal{G}_i$-measurable for any $i \in I$, it follows that for all $s \in S$ and any allocation $x_i \in L_i$

$$p(s') \cdot x_i(s') \leq p(s') \cdot e_i(s') \text{ for all } s' \in \mathcal{G}_i(s) \Leftrightarrow \max_{s' \in \mathcal{G}_i(s)} p(s') \cdot x_i(s') \leq \max_{s' \in \mathcal{G}_i(s)} p(s') \cdot e_i(s').$$

This means that any agent pays the highest value to achieve the lowest possible bound of his satisfaction.

[16] The pedix "FR" means that we are considering only fully revealing equilibria. Thus, $REE_{FR}(\mathcal{E})$ and $MREE_{FR}(\mathcal{E})$ are respectively the set of fully revealing Bayesian REE and fully revealing maximin REE of the economy $\mathcal{E}$.

We now observe that such an equivalence is not true in general. To this end, we consider a variant of Kreps's example (see Kreps (1977)).

Consider an economy with two states, two agents and two goods. Endowments are identical and positive. Preferences are state-dependent and such that in state one (two), the agent type one (two) prefers good one relatively more. In a differential information economy in which the preferences of all agents are represented by Bayesian expected utility function (see (6)), since the setup is symmetric, the full information equilibrium price is the same in both states.

Now suppose that agent one can distinguish the states but agent two cannot. There cannot be a fully revealing Bayesian REE: it would have to coincide with the full information equilibrium, and that equilibrium has a constant price across states, which is not compatible with revelation. Also, there cannot be a non revealing equilibrium. In a non revealing equilibrium with equal prices across states, demand of the uninformed agent would have to be the same across states. But demand of the informed agent would be different across states, and therefore there will not be market clearing. Note that a key reason for the nonexistence of a non revealing equilibrium is that the demand of the uninformed agent is measurable with respect to his private information.

On the other hand, if we impose maximin evaluation of plans, then we can have a non revealing equilibrium. In such an equilibrium, the uninformed agent two puts probability one on the worse of the two states, and zero on the better one. He is thus indifferent between any two consumption bundles in the better state - his optimal demand is a correspondence. Therefore, we can select an element from the correspondence to clear the market. Note that the allocation is then typically not measurable with respect to the uninformed agent's information and this overcomes the non-existence.

Below, we explicitly consider again Kreps's example and show that while the Bayesian REE does not exist, a maximin rational expectations equilibrium does exist. From this we can conclude that MREE and REE are two different solution concepts.

Example 3.3 (Kreps[17]) There are two agents, two commodities and two equally probable states of nature $S = \{s_1, s_2\}$. The primitives of the economy are:

$$
\begin{aligned}
e_1 &= \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_1 = \{\{s_1\}, \{s_2\}\};\\
e_2 &= \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_2 = \{\{s_1, s_2\}\}.
\end{aligned}
$$

The utility functions of agents 1 and 2 in states s_1 and s_2 are given as follows

$$
\begin{aligned}
u_1(s_1, x_1, y_1) &= \log x_1 + y_1 \qquad u_1(s_2, x_1, y_1) = 2\log x_1 + y_1\\
u_2(s_1, x_2, y_2) &= 2\log x_2 + y_2 \qquad u_2(s_2, x_2, y_2) = \log x_2 + y_2.
\end{aligned}
$$

It is well known that for the above economy, a Bayesian rational expectations equilibrium does not exist (see Kreps (1977)). However we will show below that a maximin rational expectations equilibrium does exist.

[17] We are grateful to T. Liu and L. Sun for having checked the computations in Example 3.3.

The information generated by the equilibrium price can be either $\{\{s_1\},\{s_2\}\}$ or $\{\{s_1,s_2\}\}$. In the first case, the MREE coincides with the Bayesian REE, therefore it does not exist. Thus, let us consider the case $\sigma(p)=\{\emptyset,S\}$, i.e., $p^1(s_1)=p^1(s_2)=p$ and $p^2(s_1)=p^2(s_2)=q$.

Since for each s, $\mathcal{G}_1(s)=\{s\}$, agent one solves the following constraint maximization problems:

Agent 1 in state s_1:

$$\max_{x_1(s_1),y_1(s_1)} \log x_1(s_1)+y_1(s_1) \quad \text{subject to}$$

$$px_1(s_1)+qy_1(s_1)\leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(s_1)=\frac{q}{p} \quad y_1(s_1)=\frac{3}{2}\frac{p}{q}+\frac{1}{2}.$$

Agent 1 in state s_2:

$$\max_{x_1(s_2),y_1(s_2)} 2\log x_1(s_2)+y_1(s_2) \quad \text{subject to}$$

$$px_1(s_2)+qy_1(s_2)\leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(s_2)=\frac{2q}{p} \quad y_1(s_2)=\frac{3}{2}\frac{p}{q}-\frac{1}{2}.$$

Agent 2 in the event $\{s_1,s_2\}$ maximizes

$$\min\{2logx_2(s_1)+y_2(s_1); logx_2(s_2)+y_2(s_2)\}.$$

Therefore, we can distinguish three cases:

I Case: $2logx_2(s_1)+y_2(s_1)>logx_2(s_2)+y_2(s_2)$. In this case, agent 2 solves the following constraint maximization problem:

$\max\ logx_2(s_2)+y_2(s_2)$ subject to $px_2(s_1)+qy_2(s_1)\leq\frac{3}{2}(p+q)$ and $px_2(s_2)+qy_2(s_2)\leq\frac{3}{2}(p+q)$. Thus,

$$x_2(s_2)=\frac{q}{p} \quad y_2(s_2)=\frac{3}{2}\frac{p}{q}+\frac{1}{2}.$$

From feasibility it follows that $p=q$, and

$$(x_1(s_1),y_1(s_1))=(1,2) \quad (x_1(s_2),y_1(s_2))=(2,1)$$

$$(x_2(s_1),y_2(s_1))=(2,1) \quad (x_2(s_2),y_2(s_2))=(1,2).$$

Notice that $2logx_2(s_1)+y_2(s_1)=2log2+1>log1+2=logx_2(s_2)+y_2(s_2)$.

II Case: $2logx_2(s_1)+y_2(s_1)<logx_2(s_2)+y_2(s_2)$. In this case, agent 2 solves the following constraint maximization problem:

max $2logx_2(s_1)+y_2(s_1)$ subject to $px_2(s_1)+qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2)+qy_2(s_2) \leq \frac{3}{2}(p+q)$ Thus,

$$x_2(s_1) = \frac{2q}{p} \quad y_2(s_1) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1,2) \quad (x_1(s_2), y_1(s_2)) = (2,1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Clearly, as noticed above, $2log2 + 1 > log1 + 2$. Therefore, in the second case there is no maximin rational expectations equilibrium.

III Case: $2logx_2(s_1)+y_2(s_1) = logx_2(s_2)+y_2(s_2)$. In this case, agent 2 solves one of the following two constraint maximization problems:

max $logx_2(s_2)+y_2(s_2)$ or max $2logx_2(s_1)+y_2(s_1)$ subject to $px_2(s_1)+qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2)+qy_2(s_2) \leq \frac{3}{2}(p+q)$. In both cases, from feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1,2) \quad (x_1(s_2), y_1(s_2)) = (2,1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Hence, since $2logx_2(s_1)+y_2(s_1) = 2log2+1 > log1+2 = logx_2(s_2)+y_2(s_2)$, there is no maximin rational expectations equilibrium in the third case.

Therefore, we can conclude that the unique maximin REE allocation is given by

$$(x_1(s_1), y_1(s_1)) = (1,2) \quad (x_1(s_2), y_1(s_2)) = (2,1)$$

$$(x_2(s_1), y_2(s_1)) = (2,1) \quad (x_2(s_2), y_2(s_2)) = (1,2).$$

Observe that the maximin REE bundles are not $\mathcal{F}_i$-measurable.

Remark 3.4 It should be noted that in the above example, whenever agents maximize a Bayesian (subjective) expected utility as Kreps showed, the Bayesian REE either revealing or non revealing does not exist. However, allowing agents to maximize a non expected utility, i.e., the maximin utility, we showed that a maximin rational expectations equilibrium exists. The example makes it clear that the Bayesian choice of optimization seems to impose a functional restriction on the utility functions which does not allow agents to achieve the desired outcome. The functional form of the maximin utility seems to be achieving what we want agents to accomplish, i.e., to reach an equilibrium outcome. As we will see in the next section, this outcome is incentive compatible and efficient.

Remark 3.5 As we have already observed, the maximin rational expectations equilibrium allocations may not be $\mathcal{G}_i$-measurable. However, if we assume strict quasi concavity and $\mathcal{F}_i$-measurability of the random utility function of each agent, then the resulting maximin REE allocations will be $\mathcal{G}_i$-measurable, as the following proposition indicates.

Proposition 3.6 *Let assumption 2.1 hold. Let (p, x) be a maximin REE (MREE) and $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$ for all $i \in I$. Assume that for all i, (i) $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable for all $y \in \mathbb{R}^\ell_+$ and (ii) $u_i(s, \cdot)$ is strictly quasi concave for all $s \in S$. Then $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$.*

A similar proposition can be proved for the Bayesian rational expectations equilibrium, that is whenever the utility functions are private information measurable and strictly quasi concave, from the uniqueness of the maximizer, we obtain that the equilibrium allocations must be private information measurable. In other words, if the utility functions are private information measurable and strictly quasi concave, condition (i) in Definition 3.1 is automatically satisfied.

It was shown in Example 3.3 that the maximin and the Bayesian REE are not comparable. We have already observed that in the special case of fully revealing equilibrium prices, both concepts coincide. We show below that the same holds whenever the utility functions are $\mathcal{F}_i$-measurable. Note that in Example 3.3, utility functions are not $\mathcal{F}_i$-measurable and therefore Example 3.3 does not fulfill the assumptions of Proposition 3.7 below.

Proposition 3.7 *Let assumption 2.1 hold. Assume that for all $i \in I$ and for all $y \in \mathbb{R}^\ell_+$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable. If (p, x) is a Bayesian REE, then (p, x) is a MREE. The converse is also true if $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$.*

Remark 3.8 The above proposition remains true if we replace the $\mathcal{G}_i$-measurability of the allocations by the strict quasi concavity of the random utility functions. This follows by combining Propositions 3.6 and 3.7.

3.4 Properties of a maximin rational expectations equilibrium

In this section we investigate some basic properties of a maximin rational expectations equilibrium.

The first property of a MREE regards the equilibrium price p. We show that under certain assumptions the equilibrium price is strictly positive in each state of nature, i.e., $p(\omega) \gg 0$ for all $s \in S$.

Recall that in a complete information economy, if utility functions are strictly monotone, the equilibrium price is strictly positive. We prove the same for MREE prices. Notice that, typically in differential information economies an additional assumption is needed:

for each state $s \in S$, there exists an agent $i \in I$ such that $\{s\} \in \mathcal{F}_i$. (*)

This assumption is not strong—see for example Angeloni and Martins-da Rocha (2009) and Correia-da Silva and Hervés-Beloso (2011). It implies that $\bigvee_{i \in I} \mathcal{F}_i = \mathcal{F}$, which is used by Allen (1981) to guarantee that $\mathcal{F}$ contains no superfluous events about which no trader has information, and therefore cannot affect anyone's consumption decisions. The converse is not true: in particular in a differential information economy with three states of nature $S = \{a, b, c\}$ and two agents $I = \{1, 2\}$, with $\mathcal{F}_1 = \{\{a, b\}, \{c\}\}$ and $\mathcal{F}_2 = \{\{a, c\}, \{b\}\}$, it is true that $\mathcal{F}_1 \vee \mathcal{F}_2 = \{\{a\}, \{b\}, \{c\}\}$, but $\{a\} \notin \mathcal{F}_i$ for any $i \in \{1, 2\}$.

Proposition 3.9 *If $u_i(s, \cdot)$ is strictly monotone for each $i \in I$ and each $s \in S$ and (p, x) is a maximin rational expectations equilibrium, then $p(s) \gg 0$ for any $s \in S$.*

We now show a second property of a MREE: if the utility functions are private information measurable, then for each agent $i \in I$, the maximin utility at any MREE allocation is constant in each event of the partition $\mathcal{G}_i$.

Proposition 3.10 *Let assumption 2.1 hold. Assume that $u_i(\cdot, x)$ is $\mathcal{F}_i$-measurable for all $i \in I$ and $x \in \mathbb{R}^\ell_+$. If (p, x) is a maximin rational expectations equilibrium, then for all i and s, $\underline{u}_i^{REE}(s, x_i) = u_i(s', x_i(s'))$ for all $s' \in \mathcal{G}_i(s)$, that is the minimum in the event $\mathcal{G}_i(s)$ is obtained in each state s' of the event.*

Notice that if (p, x) is a fully revealing Maximin REE, Proposition 3.10 is trivially satisfied even if utility functions are not private information measurable.

4 Existence of a maximin rational expectations equilibrium

In this section, we prove the existence of a maximin rational expectations equilibrium. It should be noted that under the assumptions, which guarantee that a maximin rational expectations equilibrium exists, the Bayesian REE need not exist. In studies of rational expectations equilibria, it is common to appeal to an artificial family of complete information economies (see e.g., Radner (1979); Allen (1981); Einy, Moreno, and Shitovitz (2000), De Simone and Tarantino (2010)). Given a differential information economy $\mathcal{E}$ described in Section 2, since S is finite, there is a finite number of complete information economies $\{\mathcal{E}(s)\}_{s \in S}$. For each fixed s in S, the complete information economy $\mathcal{E}(s)$ is given as follows:

$$\mathcal{E}(s) = \{(u_i(s), e_i(s))_{i \in I}\},$$

where $I = \{1, \ldots, n\}$ is still the set of n agents, and for each $i \in I$, $u_i(s) : \mathbb{R}^\ell_+ \to \mathbb{R}$ and $e_i(s) \in \mathbb{R}^\ell_+$ represent respectively the utility function and the initial endowment of agent i. Let $W(\mathcal{E}(s))$ be the set of competitive equilibrium allocations of $\mathcal{E}(s)$.

We prove that the set of maximin REE allocations contains all the selections from the competitive equilibrium correspondence of the associated family of complete information economies. From the existence of a competitive equilibrium in each complete information economy $\mathcal{E}(s)$, we deduce the existence of a maximin REE. A related result has been shown by Einy, Moreno, and Shitovitz (2000) and De Simone and Tarantino (2010) but under the additional private information measurability assumption on the utility functions (see also Corollary 4.6).

Theorem 4.1 (Existence) *If for any $i \in I$ and $s \in S$ the function $u_i(s, \cdot)$ is strictly monotone and concave and $e_i(s) \gg 0$, then there exists a maximin rational expectations equilibrium in $\mathcal{E}$, i.e., $MREE(\mathcal{E}) \neq \emptyset$.*

Remark 4.2 In order to prove the existence of a maximin rational expectations equilibrium, we have shown that it contains the nonempty set of ex post competitive equilibria. In the example below, we show that such an inclusion is strict, that is, there may exist a maximin rational expectations equilibrium which is not a competitive equilibrium in some complete information economy $\mathcal{E}(s)$.

Example 4.3 Consider a differential information economy with three states of nature, $S = \{a, b, c\}$, two goods, $\ell = 2$ (the first good is considered as numerarie) and two agents, $I = \{1, 2\}$ whose characteristics are given as follows:

$$\begin{array}{cccc} u_i(a,x,y) = \sqrt{xy} & u_i(b,x,y) = \sqrt{xy} & u_i(c,x,y) = log(xy) & \text{for all } i = 1,2 \\ e_1(a) = e_1(b) = (2,1) & e_1(c) = (1,2) & e_2(a) = e_2(c) = (1,2) & e_2(b) = (2,1) \\ \mathcal{F}_1 = \{\{a,b\};\{c\}\} & & \mathcal{F}_2 = \{\{a,c\};\{b\}\}. & \end{array}$$

Notice that the initial endowment is private information measurable, while the utility functions are not.

The set W of ex post competitive equilibrium has only one element, i.e.,

$$\begin{array}{ccc} (p(a),q(a)) = (1,1) & (x_1(a),y_1(a)) = \left(\frac{3}{2},\frac{3}{2}\right) & (x_2(a),y_2(a)) = \left(\frac{3}{2},\frac{3}{2}\right) \\ (p(b),q(b)) = (1,2) & (x_1(b),y_1(b)) = (2,1) & (x_2(b),y_2(b)) = (2,1) \\ (p(c),q(c)) = \left(1,\frac{1}{2}\right) & (x_1(c),y_1(c)) = (1,2) & (x_2(c),y_2(c)) = (1,2)\,. \end{array}$$

Clearly, this equilibrium is also a fully revealing maximin rational equilibrium, since $(p(a), q(a)) \neq (p(b), q(b)) \neq (p(c), q(c))$ and hence $\mathcal{G}_i = \sigma(p, q) = \{\{a\}, \{b\}, \{c\}\}$ for any $i = 1, 2$. However, it is not unique. Indeed, the set $MREE(\mathcal{E})$ contains the following further element:

$$\begin{array}{ccc} (p(a),q(a)) = \left(1,\frac{1}{2}\right) & (x_1(a),y_1(a)) = \left(\frac{5}{4},\frac{5}{2}\right) & (x_2(a),y_2(a)) = \left(\frac{7}{4},\frac{1}{2}\right) \\ (p(b),q(b)) = (1,2) & (x_1(b),y_1(b)) = (2,1) & (x_2(b),y_2(b)) = (2,1) \\ (p(c),q(c)) = \left(1,\frac{1}{2}\right) & (x_1(c),y_1(c)) = (1,2) & (x_2(c),y_2(c)) = (1,2)\,. \end{array}$$

This is a partially revealing equilibrium, since $(p(a), q(a)) = (p(c), q(c)) \neq (p(b), q(b))$ and hence $\sigma(p, q) = \{\{a, c\}, \{b\}\}$, that is $\mathcal{G}_1 = \{\{a\}, \{b\}, \{c\}\}$, while $\mathcal{G}_2 = \mathcal{F}_2$. Notice that the equilibrium allocations are not $\mathcal{G}_i$-measurable.

Remark 4.4 Let Assumption 2.1 hold. If for any $i \in I$ and $s \in S$ $e_i(s) \gg 0$, the function $u_i(s, \cdot)$ is strictly monotone and strictly concave, and for any $y \in \mathbb{R}^{\ell}_{+}$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable, then from Remark 3.8 and Theorem 4.1 it follows that there exists a Bayesian REE in $\mathcal{E}$.

Remark 4.5 Notice that in Example 3.3, where the Bayesian REE does not exist, not all the above assumptions of Remark 4.4 are satisfied. In particular, the random utility functions are not $\mathcal{F}_i$-measurable. Hence, the Kreps's example of the nonexistence of a Bayesian REE does not contradict Remark 4.4.

Corollary 4.6 *Let Assumption 2.1 hold and the aggregate initial endowment be strictly positive for every state. If for any $i \in I$ and $s \in S$ the function $u_i(s, \cdot)$ is monotone and strictly quasi-concave, and for any $y \in \mathbb{R}^{\ell}_{+}$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable, then $W = REE(\mathcal{E}) = MREE(\mathcal{E})$.*

Remark 4.7 Observe that the non-existence problem of a Bayesian REE is deeply linked to the private information measurability of the allocations. Indeed, if we consider a Bayesian REE (p, x) but removing from Definition 3.1 the private information measurability constraint (i.e., condition (i)) and also consider in the optimization problem the interim budget set (not the pointwise), then we end up with the following notion which coincides in the Kreps's example 3.3 with the maximin REE:

1. for all i and for all s, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$;
2. for all i and for all s, $v_i(x_i|\mathcal{G}_i)(s) = \max_{y_i \in B_i(s,p)} v_i(y_i|\mathcal{G}_i)(s)$; where

$$B_i(s,p) = \{y_i \in L_i \ : \ p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s') \text{ for all } s' \in \mathcal{G}_i(s)\}.$$

3. $\sum_{i \in I} x_i(s) = \sum_{i \in I} e_i(s) \quad$ for all $s \in S$.

It is easy to show that if for all $i \in I$ and $s \in S$, $u_i(s, \cdot)$ is monotone[18], then the above REE notion coincides with the ex post competitive equilibrium, and therefore it exists. However, the above notion does not provide any new insights, since it is "equivalent" with the ex post competitive equilibrium. Moreover, from Theorem 4.1 and Example 4.3, one can easily deduce that any Bayesian REE without measurability constraints on allocations is a maximin REE but the reverse is not true. Hence, whenever we drop the private information measurability constraint, the equilibrium exists, but it may not be incentive compatible, as shown by Glycopantis, Muir, and Yannelis (2005). This conflict does not arise anymore with the maximin utility functions. In fact, a maximin rational expectations equilibrium exists and it is incentive compatible.

5 Efficiency and Incentive Compatibility of Maximin REE

This section discusses two important properties of maximin REE, namely, efficiency and incentive compatibility.

5.1 Efficiency of the maximin REE

We now define the notion of maximin and ex post Pareto optimality and we will exhibit conditions which guarantee that any maximin REE is maximin efficient and ex post Pareto optimal.

[18]Monotonicity assumption of the utility functions guarantees that if p is an equilibrium price according to the above definition, then $p(s) > 0$ for any $s \in S$.

Definition 5.1 *A feasible allocation x is said to be ex post efficient (or ex post Pareto optimal) if there does not exist an allocation $y \in L$ such that*

$$\begin{array}{ll} (i) & u_i(s, y_i(s)) > u_i(s, x_i(s)) \quad for\ all\ i \in I\ and\ for\ all\ s \in S \\ (ii) & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad for\ all\ s \in S. \end{array}$$

Definition 5.2 *A feasible allocation x is said to be maximin efficient (or maximin Pareto optimal) with respect to information structure*[19] Π*, if there does not exist an allocation $y \in L$ such that*

$$\begin{array}{ll} (i) & \underline{u}_i^{\Pi_i}(s, y_i) > \underline{u}_i^{\Pi_i}(s, x_i) \quad for\ all\ i \in I\ and\ for\ all\ s \in S, \\ (ii) & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad for\ all\ s \in S. \end{array}$$

Proposition 5.3 Any maximin efficient allocation x (with respect to any information structure) is ex post Pareto optimal. The converse may not be true.

We are now ready to exhibit the conditions under which any MREE is maximin efficient and hence ex post Pareto optimal (see Proposition 5.3).

Theorem 5.4 *Let assumption 2.1 hold and $u_i(s, \cdot)$ be monotone for each $s \in S$ and each $i \in I$. Let (p, x) be a maximin rational expectations equilibrium. If one of the following conditions holds true:*

1. *$u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable for each $i \in I$ and $t \in \mathbb{R}^\ell_+$;*
2. *there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{G}_i(\bar{s})$ for all $i \in I$;*
3. *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*
4. *the $n - 1$ agents are fully informed.*

then x is maximin Pareto optimal with respect to $\mathcal{G} = (\mathcal{G}_i)_{i \in I}$, and hence ex post efficient.

We now show that if none of the above conditions of Theorem 5.4 is satisfied, then the maximin REE is not maximin efficient.

Example 5.5 Consider a differential information economy with three states of nature, $S = \{a, b, c\}$, two goods, $\ell = 2$ (the first good is considered as numerarie) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{ccc} e_1(a) = e_1(b) = (2,1) & e_1(c) = (3,1) & \mathcal{F}_1 = \{\{a,b\};\{c\}\} \\ e_2(a) = e_2(c) = (1,2) & e_2(b) = (2,2) & \mathcal{F}_2 = \{\{a,c\};\{b\}\} \\ e_3(b) = e_3(c) = (2,1) & e_3(a) = (3,1) & \mathcal{F}_3 = \{\{a\};\{b,c\}\}. \\ u_1(a,x,y) = \sqrt{xy} & u_1(b,x,y) = log(xy) & u_1(c,x,y) = \sqrt{xy}, \\ u_2(a,x,y) = log(xy) & u_2(b,x,y) = \sqrt{xy} & u_2(c,x,y) = \sqrt{xy}, \\ u_3(a,x,y) = \sqrt{xy} & u_3(b,x,y) = \sqrt{xy} & u_3(c,x,y) = log(xy). \end{array}$$

[19] An information structure Π is simply a vector $(\Pi_1, \ldots, \Pi_i, \ldots, \Pi_n)$, where for each $i \in \{1, \ldots, n\} = I$, Π_i is a partition of S. If $\Pi_i = \mathcal{F}_i$ for each $i \in I$, then the information structure is the initial private information.

Consider the following maximin rational expectations equilibrium

$$\begin{array}{llll} (p(a),q(a)) = \left(1,\frac{3}{2}\right) & (x_1(a),y_1(a)) = \left(\frac{7}{4},\frac{7}{6}\right) & (x_2(a),y_2(a)) = \left(2,\frac{4}{3}\right) & (x_3(a),y_3(a)) = \left(\frac{9}{4},\frac{3}{2}\right) \\ (p(b),q(b)) = \left(1,\frac{3}{2}\right) & (x_1(b),y_1(b)) = \left(\frac{7}{4},\frac{7}{6}\right) & (x_2(b),y_2(b)) = \left(\frac{5}{2},\frac{5}{3}\right) & (x_3(b),y_3(b)) = \left(\frac{7}{4},\frac{7}{6}\right) \\ (p(c),q(c)) = \left(1,\frac{3}{2}\right) & (x_1(c),y_1(c)) = \left(\frac{9}{4},\frac{3}{2}\right) & (x_2(c),y_2(c)) = \left(2,\frac{4}{3}\right) & (x_3(c),y_3(c)) = \left(\frac{7}{4},\frac{7}{6}\right), \end{array}$$

and notice that it is a non revealing equilibrium, since $(p(a),q(a)) = (p(b),q(b)) = (p(c),q(c))$ and hence $\sigma(p,q) = \{\{a,b,c\}\}$, that is $\mathcal{G}_i = \mathcal{F}_i$ for any $i \in I$. Moreover, notice that Assumption 2.1 holds and the utility functions are monotone, but none of the above conditions of Theorem 5.4 is satisfied. We now show that the equilibrium allocation is not maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i\in I}$. Indeed, consider the following feasible allocation

$$\begin{array}{llll} (t_1(a),z_1(a)) & = & \left(\frac{20}{12},\frac{13}{12}\right) & (t_2(a),z_2(a)) = \left(\frac{25}{12},\frac{16}{12}\right) \quad (t_3(a),z_3(a)) = \left(\frac{27}{12},\frac{19}{12}\right) \\ (t_1(b),z_1(b)) & = & \left(\frac{22}{12},\frac{14}{12}\right) & (t_2(b),z_2(b)) = \left(\frac{30}{12},\frac{21}{12}\right) \quad (t_3(b),z_3(b)) = \left(\frac{20}{12},\frac{13}{12}\right) \\ (t_1(c),z_1(c)) & = & \left(\frac{28}{12},\frac{18}{12}\right) & (t_2(c),z_2(c)) = \left(\frac{23}{12},\frac{15}{12}\right) \quad (t_3(c),z_3(c)) = \left(\frac{21}{12},\frac{15}{12}\right), \end{array}$$

and notice that,

$$\begin{aligned} \underline{u}_1^{REE}(a,t_1,z_1) &= \underline{u}_1^{REE}(b,t_1,z_1) = \min\{\sqrt{\frac{260}{144}}, log\frac{308}{144}\} = log\frac{308}{144} > log\frac{49}{24} = \\ \min\{\sqrt{\frac{49}{24}}, log\frac{49}{24}\} &= \underline{u}_1^{REE}(a,x_1,y_1) = \underline{u}_1^{REE}(b,x_1,y_1), \end{aligned}$$

$$\underline{u}_1^{REE}(c,t_1,z_1) = u_1(c,t_1(c),z_1(c)) = \sqrt{\frac{504}{144}} > \sqrt{\frac{27}{8}} = u_1(c,x_1(c),y_1(c)) = \underline{u}_1^{REE}(c,x_1,y_1),$$

$$\begin{aligned} \underline{u}_2^{REE}(a,t_2,z_2) &= \underline{u}_2^{REE}(c,t_2,z_2) = \min\{log\frac{400}{144}, \sqrt{\frac{345}{144}}\} = log\frac{400}{144} > log\frac{8}{3} = \\ \min\{log\frac{8}{3}, \sqrt{\frac{8}{3}}\} &= \underline{u}_2^{REE}(a,x_2,y_2) = \underline{u}_2^{REE}(c,x_2,y_2), \end{aligned}$$

$$\underline{u}_2^{REE}(b,t_2,z_2) = u_2(b,t_2(b),z_2(b)) = \sqrt{\frac{630}{144}} > \sqrt{\frac{25}{6}} = u_2(b,x_2(b),y_2(b)) = \underline{u}_2^{REE}(b,x_2,y_2),$$

$$\underline{u}_3^{REE}(a,t_3,z_3) = u_3(a,t_3(a),z_3(a)) = \sqrt{\frac{513}{144}} > \sqrt{\frac{27}{8}} = u_3(a,x_3(a),y_3(a)) = \underline{u}_3^{REE}(a,x_3,y_3),$$

$$\underline{u}_3^{REE}(b,t_3,z_3) = \underline{u}_3^{REE}(c,t_3,z_3) = \min\{\sqrt{\frac{260}{144}}, log\frac{315}{144}\} = log\frac{315}{144} > log\frac{49}{24} =$$

$$\min\{\sqrt{\frac{49}{24}}, log\frac{49}{24}\} = \underline{u}_3^{REE}(b,x_3,y_3) = \underline{u}_3^{REE}(c,x_3,y_3).$$

Hence, the equilibrium allocation (x, y) is not maximin Pareto optimal with respect to the information structure $\mathcal{G} = (\mathcal{G}_i)_{i\in I}$.

Remark 5.6 Notice that in Kreps's example (Example 3.3), one of the two agents is fully informed, hence condition 4 of the above theorem is satisfied. This guarantees that the unique maximin rational expectations equilibrium (MREE) is maximin Pareto optimal and hence ex post efficient.

Remark 5.7 Whenever $u_i(\cdot, x)$ is $\mathcal{F}_i$-measurable for each $i \in I$ and each $x \in \mathbb{R}^\ell_+$, also Bayesian REE is ex post efficient because it coincides with the ex post competitive equilibrium (see Einy, Moreno, and Shitovitz (2000) and De Simone and Tarantino (2010)). However, one cannot obtain our Theorem 5.4 from theirs, since a maximin rational expectations equilibrium may not coincide with a Bayesian REE or an ex post competitive equilibrium allocation.

5.2 Incentive compatibility in rational expectations equilibrium

We now recall the notion of coalitional incentive compatibility of Krasa and Yannelis (1994).

Definition 5.8 *An allocation x is said to be coalitional incentive compatible (CIC) if the following does not hold: there exists a coalition C and two states a and b such that*

$$\begin{array}{lll} (i) & \mathcal{F}_i(a) = \mathcal{F}_i(b) & for\ all\ i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^\ell_+ & for\ all\ i \in C,\ and \\ (iii) & u_i(a, e_i(a) + x_i(b) - e_i(b)) > u_i(a, x_i(a)) & for\ all\ i \in C. \end{array}$$

In order to explain what incentive compatibility means in an asymmetric information economy, let us consider the following two examples[20].

Example 5.9 Consider an economy with two agents, three equally probable states of nature, denoted by a, b and c, and one good per state denoted by x. The primitives of the economy are given as follows:

$$\begin{array}{lll} u_1(\cdot, x_1) = \sqrt{x_1}; & e_1(a,b,c) = (20, 20, 0); & \mathcal{F}_1 = \{\{a,b\};\{c\}\}. \\ u_2(\cdot, x_2) = \sqrt{x_2}; & e_2(a,b,c) = (20, 0, 20); & \mathcal{F}_2 = \{\{a,c\};\{b\}\}. \end{array}$$

[20]The reader is also referred to Krasa and Yannelis (1994), Koutsougeras and Yannelis (1993) and Podczeck and Yannelis (2008) for an extensive discussion of the Bayesian incentive compatibility in asymmetric information economies.

Consider the following risk sharing (Pareto optimal) redistribution of initial endowment:

$$\begin{aligned} x_1(a,b,c) &= (20,10,10) \\ x_2(a,b,c) &= (20,10,10). \end{aligned}$$

Notice that the above allocation is not incentive compatible. Indeed, suppose that the realized state of nature is a, agent 1 is in the event $\{a,b\}$ and he reports c, (observe that agent 2 cannot distinguish between a and c). If agent 2 believes that c is the realized state of nature as agent 1 has claimed, then he gives him ten units. Therefore, the utility of agent 1, when he misreports, is $u_1(a, e_1(a)+x_1(c)-e_1(c)) = u_1(a, 20+10-0) = \sqrt{30}$ which is greater than $u_1(a, x_1(a)) = \sqrt{20}$, the utility of agent 1 when he does not misreport. Hence, the allocation $x_1(a,b,c) = (20,10,10)$ and $x_2(a,b,c) = (20,10,10)$ is not incentive compatible. Similarly, one can easily check that when a is the realized state of nature, agent 2 has an incentive to report state b and benefit.

In order to make sure that the equilibrium contracts are stable, we must insist on a coalitional definition of incentive compatibility and not an individual one. As the following example shows, a contract which is individual incentive compatible may not be coalitional incentive compatible and therefore may not be viable.

Example 5.10 Consider an economy with three agents, two goods and three states of nature $S = \{a,b,c\}$. The primitives of the economy are given as follows: for all $i = 1,2,3$, $u_i(\cdot, x_i, y_i) = \sqrt{x_i y_i}$ and

$$\begin{array}{rl} \mathcal{F}_1 = \{\{a,b,c\}\}; & e_1(a,b,c) = ((15,0);(15,0);(15,0)). \\ \mathcal{F}_2 = \{\{a,b\},\{c\}\}; & e_2(a,b,c) = ((0,15);(0,15);(0,15)). \\ \mathcal{F}_3 = \{\{a\},\{b\},\{c\}\}; & e_3(a,b,c) = ((15,0);(15,0);(15,0)). \end{array}$$

Consider the following redistribution of the initial endowments:

$$\begin{aligned} \mathbf{x}_1(a,b,c) &= ((8,5),(8,5),(8,13)) \\ \mathbf{x}_2(a,b,c) &= ((7,4),(7,4),(12,1)) \\ \mathbf{x}_3(a,b,c) &= ((15,6),(15,6),(10,1)). \end{aligned} \tag{11}$$

Notice that the only agent who can misreport either state a or b to agents 1 and 2 is agent 3. Clearly, agent 3 cannot misreport state c since agent 2 would know it. Thus, agent 3 can only lie if either state a or state b occurs. However, agent 3 has no incentive to misreport since he gets the same consumption in both states a and b. Hence, the allocation (11) is individual incentive compatible, but we will show that it is not coalitional incentive compatible. Indeed, if c is the realized state of nature, agents 2 and 3 have an incentive to cooperate against agent 1 and report b (notice that agent 1 cannot distinguish between b and c). The coalition $C = \{2,3\}$ will now be better off, i.e.,

$$
\begin{aligned}
u_2(c, e_2(c) + \mathbf{x}_2(b) - e_2(b)) &= u_2(c, (0,15) + (7,4) - (0,15)) \\
= u_2(c, (7,4)) &= \sqrt{28} > \sqrt{12} = u_2(c, \mathbf{x}_2(c)) \\
u_3(c, e_3(c) + \mathbf{x}_3(b) - e_3(b)) &= u_3(c, (15,0) + (15,6) - (15,0)) \\
= u_3(c, (15,6)) &= \sqrt{90} > \sqrt{10} = u_3(c, \mathbf{x}_3(c)).
\end{aligned}
$$

In Example 5.9 we have constructed an allocation which is Pareto optimal but it is not individual incentive compatible; while in Example 5.10 we have shown that an allocation, which is individual incentive compatible, need not be coalitional incentive compatible.

In view of Examples 5.9 and 5.10, it is easy to understand the meaning of Definition 5.8. An allocation is coalitional incentive compatible if no coalition of agents C can cheat the complementary coalition (i.e., $I \setminus C$) by misreporting the realized state of nature and make all its members better off. Notice that condition (i) indicates that coalition C can only cheat the agents not in C (i.e., $I \setminus C$) in the states that the agents in $I \setminus C$ cannot distinguish. If $C = \{i\}$ then the above definition reduces to individual incentive compatibility.

5.3 Maximin Incentive Compatibility

In this section, we will prove that the maximin rational expectations equilibrium is incentive compatible. To this end we need the following definition of maximin coalitional incentive compatibility, which is an extension of the Krasa and Yannelis (1994) definition to incorporate maximin preferences (see also de Castro and Yannelis (2011)).

Definition 5.11 *A feasible allocation x is said to be maximin coalitional incentive compatible (MCIC) with respect to information structure Π, if the following does not hold: there exists a coalition C and two states a and b such that*

$$
\begin{array}{lll}
(i) & \Pi_i(a) = \Pi_i(b) & for\ all\ i \notin C, \\
(ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^\ell_+ & for\ all\ i \in C,\ and \\
(iii) & \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) & for\ all\ i \in C,
\end{array}
$$

where for all $i \in C$,

$$
(*) \qquad y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \textit{if } s = a \\ x_i(s) & \textit{otherwise}. \end{cases}
$$

According to the above definition, an allocation is said to be maximin coalitional incentive compatible if it is not possible for a coalition to misreport the realized state of nature and have a distinct possibility of making its members better off in terms of maximin utility. Obviously, if $C = \{i\}$ then the above definition reduces to individual incentive compatibility.

Remark 5.12 Example 5.9 shows that an efficient allocation may not be incentive compatible in the Krasa-Yannelis sense. We now show that it is not the case in our maximin sense. Precisely, if agents take into account the worse possible state that can occur, then the allocation $x_i(a,b,c) = (20,10,10)$ for $i = 1,2$ in Example 5.9, is maximin incentive compatible. Indeed, if a is the realized state of nature, agent 1 does not have an incentive to report state c and benefit, because when he misreports he gets:

$$\underline{u}_1(a,y_1) = \min\{u_1(a, e_1(a)+x_1(c)-e_1(c)); u_1(b, x_1(b))\} = \min\{\sqrt{30}, \sqrt{10}\} = \sqrt{10}.$$

When agent 1 does not misreport, he gets:

$$\underline{u}_1(a,x_1) = \min\{u_1(a,x_1(a)); u_1(b,x_1(b))\} = \min\{\sqrt{20}, \sqrt{10}\} = \sqrt{10}.$$

Consequently, agent 1 does not gain by misreporting. Similarly, one can easily check that agent 2, when a is the realized state of nature, does not have an incentive to report state b and benefit. Indeed, if the realized state of nature is a, agent 2 is in the event $\{a,c\}$. If agent 2 reports the false event $\{b\}$ then his maximin utility does not increase since

$$\begin{aligned} \underline{u}_2(a,y_1) &= \min\{u_2(a, e_2(a) + x_2(b) - e_2(b)); u_2(c, x_2(c))\} \\ &= \min\{\sqrt{20+10-0}, \sqrt{10}\} = \sqrt{10} \\ &= \min\{\sqrt{20}, \sqrt{10}\} = \underline{u}_2(a,x_2). \end{aligned}$$

Remark 5.13 Observe that Definition 5.11 implicity requires that the members of the coalition C are able to distinguish between a and b; i.e., $a \notin \Pi_i(b)$ for all $i \in C$. One could replace condition (i) by $\Pi_i(a) = \Pi_i(b)$ if and only if $i \notin C$.

Proposition 5.14 *If x is CIC, then it is also maximin CIC. The converse may not be true.*

5.4 The maximin rational expectations equilibrium is maximin incentive compatible

Proposition 5.15 *Any maximin rational expectations equilibrium is maximin coalitional incentive compatible.*

Proof: Let (p,x) be a maximin rational expectations equilibrium. Since agents take into account the information generated by the equilibrium price p, the private information of each individual i is given by $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for each agent $i \in I$, $\Pi_i = \mathcal{G}_i$ and $\underline{u}_i^{\Pi_i} = \underline{u}_i^{REE}$. Assume on the contrary that (p,x) is not maximin CIC. This means that there exists a coalition C and two states $a, b \in S$ such that

$$\begin{array}{lll} (i) & \mathcal{G}_i(a) = \mathcal{G}_i(b) & \text{for all } i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^\ell_+ & \text{for all } i \in C, \text{ and} \\ (iii) & \underline{u}_i^{REE}(a, y_i) > \underline{u}_i^{REE}(a, x_i) & \text{for all } i \in C, \end{array}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Notice that condition (i) implies that $p(a) = p(b)$, meaning that the equilibrium price is partially revealing.[21] Clearly, if p is fully revealing, since for any $i \in I$, $\mathcal{G}_i = \mathcal{F}$, then there does not exist a coalition C and two states a and b such that $\mathcal{G}_i(a) = \mathcal{G}_i(b)$ for all $i \notin C$. Therefore, any fully revealing MREE is maximin coalitional incentive compatible. On the other hand, since (p, x) is a maximin rational expectations equilibrium, it follows from (iii) that for all $i \in C$ there exists a state $s_i \in \mathcal{G}_i(a)$ such that

$$p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i) \geq p(s_i) \cdot x_i(s_i).$$

By the definition of y_i, it follows that for all $i \in C$, $s_i = a$, that is $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot [x_i(b) - e_i(b)] > 0$. Furthermore, since $p(a) = p(b)$ it follows that $p(b) \cdot x_i(b) > p(b) \cdot e_i(b)$. This contradicts the fact that (p, x) is a maximin rational expectations equilibrium. □

Corollary 5.16 *Any maximin rational expectations equilibrium is maximin individual incentive compatible.*

Remark 5.17 It should be noted that the maximin rational expectations equilibrium in Krep's example (Example 3.3) is coalitional incentive compatible. Indeed if state s_1 occurs and agent 1 announces s_2, then

$$u_1(s_1, e_1^1(s_1)+x_1(s_2)-e_1^1(s_2), e_1^2(s_1)+y_1(s_2)-e_1^2(s_2)) = log2+1 < 2 = u_1(s_1, x_1(s_1), y_1(s_1)).$$

On the other hand, if state s_2 occurs and agent 1 announces s_1, then

$$u_1(s_2, e_1^1(s_2)+x_1(s_1)-e_1^1(s_1), e_1^2(s_2)+y_1(s_1)-e_1^2(s_1)) = 2 < 2log2+1 = u_1(s_2, x_1(s_2), y_1(s_2)).$$

Therefore, the unique maximin rational expectations equilibrium in Example 3.3 is maximin CIC.

Someone could debate on the fact that in the proof of Theorem 5.15 we have considered the algebra $\mathcal{G}_i$ and not $\mathcal{F}_i$. We now extend the above result to the private algebra $\mathcal{F}_i$.

Remark 5.18 Clearly, any non revealing maximin rational expectations equilibrium is (private)[22] maximin CIC, simply because $\mathcal{G}_i = \mathcal{F}_i$ for all $i \in I$, and hence the result follows from Proposition 5.15. Example 5.20 below shows that a fully revealing maximin REE may not be (private) maximin CIC. This suggests that a weaker notion of maximin CIC is needed.

[21] Notice that for all i, $\sigma(p) \subseteq \mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Thus, for all i, $p(\cdot)$ is $\mathcal{G}_i$-measurable. Therefore, condition (i) implies that $p(a) = p(b)$.

[22] By "private" we mean that for each agent i, the partition we consider in Definition 5.11 is the initial private information $\mathcal{F}_i$.

Definition 5.19 *A feasible allocation x is said to be weak maximin coalitional incentive compatible (weak MCIC) with respect to information structure Π, if the following does not hold: there exists a coalition C and two states a and b such that*

$$\begin{array}{ll} (I) & \Pi_i(a) = \Pi_i(b) \quad \textit{for all } i \notin C, \\ (II) & u_i(a, x_i(a)) = u_i(a, x_i(b)) \quad \textit{for all } i \notin C, \\ (III) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} \quad \textit{for all } i \in C, \textit{ and} \\ (IV) & \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) \quad \textit{for all } i \in C, \end{array}$$

where for all $i \in C$,

$$(*) \qquad y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \textit{if } s = a \\ x_i(s) & \textit{otherwise.} \end{cases}$$

Clearly, any maximin CIC allocation is also weak maximin CIC, but the converse may not be true as shown by the following example.

Example 5.20 We consider the Example 3.1 in Glycopantis, Muir, and Yannelis (2005) that we recall below.[23] There are two agents $I = \{1, 2\}$, two commodities and three states of nature $S = \{a, b, c\}$. The primitives of the economy are given as follows

$$\begin{array}{llll} e_1(a) = e_1(b) = (7,1) & e_1(c) = (4,1) & \mathcal{F}_1 = \{\{a,b\},\{c\}\} & u_1(\cdot, x_1, y_1) = \sqrt{x_1 y_1} \\ e_2(b) = e_2(c) = (1,7) & e_2(a) = (1,10) & \mathcal{F}_2 = \{\{a\},\{b,c\}\} & u_2(\cdot, x_2, y_2) = \sqrt{x_2 y_2}. \end{array}$$

In this economy the unique (Bayesian) REE is the following:

$$\begin{array}{lll} (p_1(a), p_2(a)) = \left(1, \frac{8}{11}\right) & (x_1(a), y_1(a)) = \left(\frac{85}{22}, \frac{85}{16}\right) & (x_2(a), y_2(a)) = \left(\frac{91}{22}, \frac{91}{16}\right) \\ (p_1(b), p_2(b)) = (1, 1) & (x_1(b), y_1(b)) = (4, 4) & (x_2(b), y_2(b)) = (4, 4) \\ (p_1(c), p_2(c)) = \left(1, \frac{5}{8}\right) & (x_1(c), y_1(c)) = \left(\frac{37}{16}, \frac{37}{10}\right) & (x_2(c), y_2(c)) = \left(\frac{43}{16}, \frac{43}{10}\right). \end{array}$$

Notice that (p, x) is a fully revealing (Bayesian) REE and hence it is also a maximin REE. Moreover, x is weak (private) maximin CIC, but it is not (private) maximin CIC. Indeed, take $C = \{2\}$ and the two states a and b, and observe that

$$\begin{array}{rcl} \mathcal{F}_1(a) & = & \mathcal{F}_1(b) \\ (e_2^1(a) + x_2(b) - e_2^1(b), e_2^2(a) + y_2(b) - e_2^2(b)) & = & (1 + 4 - 1, 10 + 4 - 7) = (4, 7) \gg 0 \\ u_2(a, e_2^1(a) + x_2(b) - e_2^1(b), e_2^2(a) + y_2(b) - e_2^2(b)) & = & \sqrt{28} > \sqrt{\frac{91^2}{352}} = u_2(a, x_2(a), y_2(a)). \end{array}$$

Hence, x is not (private) maximin CIC, but there does not exist two states s_1 and s_2 and an agent i, such that

$$\begin{array}{rcl} \mathcal{F}_i(s_1) & = & \mathcal{F}_i(s_2) \\ \sqrt{x_i(s_1) y_i(s_1)} & = & \sqrt{x_i(s_2) y_i(s_2)}. \end{array}$$

Therefore, x is weak (private) maximin coalitional incentive compatible.

[23] We thank Liu Zhiwei for having suggested this example us.

Proposition 5.21 *Let Assumption 2.1 hold and $u_i(s, \cdot)$ is monotone for each $s \in S$ and each $i \in I$. Let (p, x) be a maximin rational expectations equilibrium. If one of the following conditions holds true:*

1. *$u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable[24] for any $i \in I$ and any $y \in \mathbb{R}^{\ell}_{+}$;*
2. *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*

then x is weak (private) maximin coalitional incentive compatible.

Remark 5.22 Although in Kreps's example, the utility functions are not private information measurable, the unique maximin rational expectations equilibrium is (private) maximin coalitional incentive compatible, since the equilibrium price p is non revealing (see Remarks 5.17 and 5.18). On the other hand, in Example 5.20 both hypotheses of Proposition 5.21 are satisfied and the unique maximin REE is weak (private) maximin CIC. However, as it has been already observed, it is not (private) maximin CIC.

6 Alternative interpretation: S as set of probabilities

Throughout the paper, we stuck to the usual interpretation of S as the set of states of the world, that is, once s is known, there is no more uncertainty or risk to be faced: everything is defined.[25] In the standard notation, this would correspond to $S = \Omega$. However, in this section, we discuss an alternative interpretation of our model, in which we see S as a set of probabilities ($S = \Delta(\Omega) = \Theta$).

The idea of extending results from a set of states $S = \Omega$ to a set of probabilities $S = \Theta$ is certainly not new. In some sense, this is exactly what Gilboa and Schmeidler (1989) did when they extended Wald's maximin criterion (which was usually understood to be applied to a set of states) to sets of probabilities. Analogously, we can understand that the introduction of the smooth model by Klibanoff, Marinacci, and Mukerji (2005), henceforth KMM, followed the same pattern. That is, KMM's smooth model can be seen as an expected utility model applied over probabilities. In both cases, the obtained model is more general than the initial one and allows for richer phenomena.

Moreover, the usual understanding in statistics is exactly to consider the set of states S as set of probabilities. For example, Berger (1985, p. 3) describes the "states of nature" as follows:

> The unknown quantity θ which affects the decision process is commonly called the *state of nature*. In making decisions, it is clearly important to consider what the possible states of nature are. The symbol Θ will be used to denote the set of all possible states of nature. (...) The probability distribution of [a random variable] X will, of course, depend upon the

[24] Notice that the measurability assumption of utility functions is not too strong when we deal with coalitional incentive compatibility notions (see for example Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994), Angeloni and Martins-da Rocha (2009) where the utility functions are assumed to be state independent, and therefore $\mathcal{F}_i$-measurable.)

[25] There is an exception: in the example described in Section 1.1, S is a set of probabilities.

unkown state of nature θ. Let $P_\theta(A)$ or $P_\theta(X \in A)$ denote the probability of the event $A(A \subset \mathcal{X})$, when θ is the true state of nature. (...)

The discussion below will show that this standard interpretation in statistics is also useful in illuminating rational expectation models with ambiguity. Indeed, it allows us to describe our model as capturing a fully general system of conditional Gilboa-Schmeidler's MEU preferences, as we will show now.

Let Ω be a set of states and let $\mathbb{I}$ denote a partition of Ω. We will focus attention to a single decision maker and refrain from using subscripts i and we will also focus only on the interim stage, when the individual knows that the event $\mathbb{I}(\omega)$ obtains. Suppose that the individual has a system of measurable conditional Gilboa-Schmeidler's MEU preferences $(\succcurlyeq_\omega)_{\omega\in\Omega}$.[26] That is, for each ω there is a set $\mathcal{C}^\omega$ of probabilities over Ω with support contained in $\mathbb{I}(\omega)$ and a utility function $\tilde{u} : \mathbb{R}^\ell_+ \to \mathbb{R}$ such that[27]

$$x \succcurlyeq_\omega y \iff \min_{\mu\in\mathcal{C}^\omega} E_\mu \left[\tilde{u}(x(\cdot))\right] \geq \min_{\mu\in\mathcal{C}^\omega} E_\mu \left[\tilde{u}(y(\cdot))\right], \tag{12}$$

for every $x : \mathbb{I}(\omega) \to \mathbb{R}^\ell_+$ and $y : \mathbb{I}(\omega) \to \mathbb{R}^\ell_+$.

Now, define $S = \cup_\omega \mathcal{C}^\omega$, so that an element $s \in S$ is a probability distribution over Ω. Define a partition Π of S as follows: $\Pi(s) = \mathcal{C}^\omega$ whenever $s \in \mathcal{C}^\omega$. This definition is sound because $\mathcal{C}^\omega = \mathcal{C}_{\omega'}$ if $\omega' \in \mathbb{I}(\omega)$, and it indeed defines a partition because $\mathcal{C}^\omega \cap \mathcal{C}^{\omega'} = \emptyset$ if $\omega' \notin \mathbb{I}(\omega)$.[28] Then we can rewrite (12) as:

$$x \succcurlyeq_\omega y \iff \min_{s'\in\Pi(s)} u(s', x(s')) \geq \min_{s'\in\Pi(s)} u(s', y(s')),$$

where $u(s', x(s'))$ denotes $E_{s'}[\tilde{u}(x(\omega))]$. This gives exactly our maximin model (7).[29]

There are, of course, a few caveats. First, we have used a finite S, while Gilboa-Schmeidler's MEU applies to a general space. However, in the Gilboa-Schmeidler's MEU (12), each $\mathcal{C}^\omega$ is compact in the weak* topology. If we add the minor assumption that the partition $\mathbb{I}(\cdot)$ is finitely-based (as Epstein and Schneider (2003) do, for instance, when they work with conditional MEU preferences), then S will be compact. Since u is an expectation with respect to s, u will also be continuous on this topology. Fortunately, our results can be extended to a setting where S is compact and u is continuous, which solves this issue.

Second, notice that the acts $x : \mathbb{I}(\omega) \to \mathbb{R}^\ell_+$ are not defined on S as we considered in our model. However, for each $x : \mathbb{I}(\omega) \to \mathbb{R}^\ell_+$ we can define an act on S corresponding to it, in such a way that the choices over acts defined on $\mathbb{I}(\omega)$ and over S will be equivalent.[30] Although some issues with the interpretation of our results can arise with this formulation, the important point is our theory still applies.

[26] We say that the conditional preferences are measurable if $\succcurlyeq_\omega = \succcurlyeq_{\omega'}$ whenever $\omega' \in \mathbb{I}(\omega)$.

[27] Note that $\tilde{u}$ is state-independent in the standard Gilboa-Schmeidler setting.

[28] This comes from the fact that $\mathcal{C}^\omega$ has support contained in $\mathbb{I}(\omega)$ and $\mathbb{I}(\omega)$ and $\mathbb{I}(\omega')$ are disjoint if they are not equal.

[29] See below a detailed discussion of how to extend this construction for n agents.

[30] For instance, fix one of the inverse functions of $\tilde{u} : \mathbb{R}^\ell_+ \to \mathbb{R}$, that is, a function $h : \mathbb{R} \to \mathbb{R}^\ell_+$ satisfying $\tilde{u}(h(\tilde{u}(a))) = \tilde{u}(a)$. Then given $x : \mathbb{I}(\omega) \to \mathbb{R}^\ell_+$, define $\hat{x} : S \to \mathbb{R}^\ell_+$ by $\hat{x}(s) \equiv h\left(E_s\left[\tilde{u}(x(\omega)\right]\right)$.

A case of interest occurs when each individual's information is given by a signal. Since this case is important, we will detail the translation here.[31] Assume that each agent receives a signal $t_i \in T_i$, where T_i is a finite set and let Γ denote a set of payoff-relevant states of nature. Given signal t_i, agent i has (a set of) beliefs $\mathcal{B}_{t_i}$ about Γ, that is, $\mathcal{B}_{t_i} \subset \Delta(\Gamma)$. Define $S_i \equiv \cup_{t_i \in T_i}\{(t_i, \mu) : \mu \in \mathcal{B}_{t_i}\}$. It is obvious that $\Pi_i \equiv (\{t_i\} \times \mathcal{B}_{t_i})_{t_i \in T_i}$ defines a partition of S_i and that individual i knows what element of the partition of S_i obtains once he learns t_i. Let $S \equiv \prod_{i=1}^n S_i$ and let $\mathcal{F}_i$ be the partition of S that is known by individual i, that is, the set of all states $s = (s_1, ..., s_n) \in S$ such that $s_i = (\bar{t}_i, \mu) \in \{\bar{t}_i\} \times \mathcal{B}_{t_i}$ and individual i received the signal $t_i = \bar{t}_i$. Furthermore, for each $x : \Gamma \to \mathbb{R}^\ell_+$ and $s = (s_1, ..., s_n)$ such that $s_i = (t_i, \mu) \in \{t_i\} \times \mathcal{B}_{t_i}$, let $u(s, x(s))$ denote $E_\mu[\tilde{u}(x(\gamma))]$. The rest of the construction is as above and leads to a model that agrees with our basic assumptions.

7 Related literature

To the best of our knowledge, no universal existence and incentive compatible results have been obtained for rational expectations equilibria. It is well known by now that the Bayesian REE as formulated by Radner (1979), Allen (1981) and Grossman (1981) exists only generically and it may not be incentive compatible or efficient.

The description of differential information via a partition of the state space was used by Radner (1968) and Allen (1981). In contrast, Radner (1979) and Condie and Ganguli (2011a) use a model based on signals, similar to the one described above. Allen and Jordan (1998, p. 7-8) discuss the reinterpretation of this kind of model in Allen (1981)'s partition model. In particular, Radner (1979) and Condie and Ganguli (2011a) fix a "state-dependent utility" in the terminology of Allen (1981) and specify various economies by the appropriate notion of conditional beliefs. Radner (1979) describes signals as providing information on the conditional probability distribution over a set of states. All information in Radner (1979) is obtained by knowing everyone's joint signal.

As such, the partitions observable by traders are over the space of joint signals as opposed to the state space over which consumption occurs. Radner calls these consumption states the "payoff-relevant part of the environment" (top of page 659). If an individual receives signal t_i then he knows that the joint signal is in the set of joint signals for which he receives the signal t_i. This imposes additional structure on the types of partitions over the signal space that agents observe.

For Radner (1979) and Condie and Ganguli (2011a), the random utility function of investors is then the expected utility functional with beliefs that arise out of the information known about the joint signal.[32] Since the decisions are made after observing signals, Condie and Ganguli (2011a) do not focus on the updating rule and simply say that there is some method of defining updated MEU beliefs conditional on knowing the partition over joint signals. For either MEU or EU types, these (sets of) beliefs

[31] In particular, this is the setting used by Radner (1979) and Condie and Ganguli (2011a).

[32] In our discussion of the Kreps's example in the introduction (see Section 1.1), we have done exactly this.

are measurable with respect to the joint-signal knowledge that the trader's have (either from prices or their signal).

In the models of Condie and Ganguli and Radner, the trades of individuals are measurable with respect to the partition that prices and signals generate over the space of joint signals. In Radner, this takes a particular form in that prices are fully-revealing so the partition generated over the space of joint signals by prices differentiates all joint signals (i.e., it is the finest partition over joint signals). As such, equilibrium trades are necessarily measurable with respect to the signal/price partition. By extension, in Condie and Ganguli (2011a) equilibrium trades (when an equilibrium exists) must be measurable with respect to the signal/price partition.

Moreover, Condie and Ganguli do not show that non-revealing equilibria (or more generally partially-revealing equilibria) exist generically (i.e., over a set of parameters with probability one), just that they exist for a set of parameters that has positive Lebesgue measure when parameterized. For these partially-revealing equilibria, their proof is constructive and shows that these can be constructed such that trades are measurable with respect to the signal/price partition over the joint signal space.

Correia-da Silva and Hervés-Beloso (2009) proved an existence theorem for a Walrasian equilibrium for an economy with asymmetric information, where agents' preferences are represented by maximin expected utility functions. Their MEU formulation is in the ex-ante sense. This seems to be the first application of the MEU to the general equilibrium existence problem with asymmetric information. However, they do not consider the issue of incentive compatibility or the REE notion. Since, they work with the ex-ante maximin expected utility formulation, their results have no bearing on ours.

Efficiency results for continuum economies have been obtained by Laffont (1985). In particular, Laffont (1985) has tried to employ a framework where the Law of Large Numbers (LLN) is applicable and has shown that a partial revealing rational expectations equilibrium may not be ex post efficient (Proposition 4.2). The same has been proved by Einy, Moreno, and Shitovitz (2000, Example 4.2); see also De Simone and Tarantino (2010)). On the other hand, a fully revealing rational expectations equilibrium is ex post efficient (Proposition 2.3 in Laffont (1985)), but it may not be ex ante efficient neither interim efficient (Propositions 3.1 and 3.2 in Laffont (1985)).

8 Concluding remarks and open questions

We introduced a new rational expectations equilibrium notion which abandons the Bayesian (subjective expected utility) formulation. Our new rational expectations equilibrium notion is formulated in terms of the maximin expected utility. In particular, in our framework agents maximize maximin expected utility instead of Bayesian expected utility. Furthermore, the resulting equilibrium allocations need not to be measurable with respect to the private information and the information the equilibrium prices have generated as in the case of the Bayesian REE. Our new notion exists universally (and not generically), it is Pareto efficient and incentive compatible. These results are false for the Bayesian REE (see Kreps (1977) and Glycopantis and Yannelis (2005)).

In view of the several examples in this paper, it seems that the private information measurability of allocations in the definition of the REE creates problems. Recall that in the ex ante expected utility case (e.g., Radner (1968) and Yannelis (1991)) the role of the private information measurability of allocations is two-fold.

First it highlights the relevance of asymmetric information. If this condition is relaxed, then agents behave as they have symmetric information and the information partition does not influence the payoff of each player. Hence the asymmetric information in the Radner (1968) model is modeled by the private information measurability of allocations. In contract, in our MEU modeling the asymmetry of information is captured by the definition of the MEU itself. Specifically priors are defined on the events of each partition of each agents and therefore the MEU itself models the information asymmetry. Consequently there is no need to assume that allocations are private information measurable as it is the case with the Bayesian modeling of Radner.

Second, in the one good case the private information measurability of allocations becomes a necessary and sufficient condition to ensure that trades are incentive compatible (e.g., Krasa and Yannelis (1994)), and in the multi good case it is a sufficient condition to ensure incentive compatibility. Thus, the private information measurability seems to be a desirable assumption in the ex ante case as it ensures that private information Pareto optimal allocations are incentive compatible.

However, this is not the case with the Bayesian REE as it is not necessarily incentive compatible (Glycopantis, Muir, and Yannelis (2005)). Also, in the ex ante case as we mentioned above, the private information measurability amounts to asymmetric information but in the interim stage, (e.g., REE case), the interim expected utility is automatically private information measurable as it is conditioned on the event in the private information of each agent, thus constant on the individual's event. Hence, the asymmetric information in the interim case enters the model via the interim utility function of each agent. By also imposing the private information measurability on allocations we end up with an existence of equilibrium problem as the Kreps's example indicates.

In a general equilibrium model with asymmetric information, it is possible that the MEU choice does not reflect pessimistic behavior, but rather incentive compatible behavior. If an agent plays against the nature (e.g., Milnor game), since, nature is not strategic, it makes sense to view the MEU decision making as reflecting pessimistic behavior. However, when you negotiate the terms of a contract under asymmetric information and the other agents have an incentive to misreport the state of nature and benefit, then the MEU provides a mechanism to prevent others from cheating you. This in not pessimism, but incentive compatibility. It is exactly for this reason that the MEU solves the conflict between efficiency and incentive compatibility (see for example de Castro and Yannelis (2011)). This conflict seems to be inherent in the Bayesian analysis (see Example 5.9 in Section 5.2).

We hope that our new formulation of the REE will find useful applications in many areas and especially in macro economic general equilibrium models.

We conclude this paper with some open questions.

Throughout we have used the assumption that there is a finite number of states. It is an open question if the main existence theorem can be extended to infinitely many states of nature of the world and even to an infinite dimensional commodity space. This is also the case for the theorems on incentive compatibility and efficiency.

In Glycopantis, Muir, and Yannelis (2005) it was shown that the Bayesian REE is not implementable as a perfect Bayesian equilibrium of an extensive form game. We conjecture that a new definition of perfect maximin equilibrium can be introduced, which will be compatible with the implementation of the maximin REE. What reinforces this conjecture is the fact that incentive compatible equilibrium notions, i.e., private core (Yannelis (1991)) and private value allocations (Krasa and Yannelis (1994)) are implementable as a perfect Bayesian equilibrium. Since, the maximin REE is also maximin incentive compatible, we believe that such a conjecture should be true.

It is also of interest to know if the results of this paper could be extended to a continuum of agents.

Based on the Bayesian expected utility formulation, Sun, Wu, and Yannelis (2012) show that with a continuum of agents, whose private signals are independent conditioned on the macro states of nature, a REE universally exists, it is incentive compatible and efficient. These results have been obtained by means of the law of large numbers. It is of interest to know if the theorems of this paper can be extended in such a framework which makes the law of large numbers applicable.

Furthermore, it is of interest to know under what conditions the core-value-Walras equivalence theorems hold for the maximin expected utility framework.

9 Appendix

9.1 Proofs of Section 3

Proof of Proposition 3.6: Assume on the contrary that there exists an agent $i \in I$ and two states $a, b \in S$ such that $a \in \mathcal{G}_i(b)$ and $x_i(a) \neq x_i(b)$. Consider $z_i(s) = \alpha x_i(a) + (1-\alpha)x_i(b)$ for all $s \in \mathcal{G}_i(b)$, where $\alpha \in (0,1)$, and notice that z_i is constant in the event $\mathcal{G}_i(b)$. Moreover,

$$\underline{u}_i^{REE}(b, z_i) = \min_{s \in \mathcal{G}_i(b)} u_i(s, z_i(s)) = \min_{s \in \mathcal{G}_i(b)} u_i(s, \alpha x_i(a) + (1-\alpha)x_i(b))$$

Since $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable for all $y \in \mathbb{R}^\ell_+$, from strict quasi concavity of u_i it follows that

$$\begin{aligned}
\underline{u}_i^{REE}(b, z_i) = u_i(b, \alpha x_i(a) + (1-\alpha)x_i(b)) &> \min\{u_i(b, x_i(a)), u_i(b, x_i(b))\} \\
= \min\{u_i(a, x_i(a)); u_i(b, x_i(b))\} &\geq \min_{s \in \mathcal{G}_i(b)} u_i(s, x_i(s)) \\
&= \underline{u}_i^{REE}(b, x_i).
\end{aligned}$$

Since (p, x) is a maximin rational expectations equilibrium it follows that $z_i \notin B_i(b, p)$, that is, there exists a state $s_i \in \mathcal{G}_i(b)$ such that

$$p(s_i) \cdot z_i(s_i) > p(s_i) \cdot e_i(s_i) \;\Rightarrow\; \alpha p(s_i) \cdot x_i(a) + (1-\alpha) p(s_i) \cdot x_i(b) > p(s_i) \cdot e_i(s_i).$$

Moreover, since $p(\cdot)$ and $e_i(\cdot)$ are $\mathcal{G}_i$-measurable and $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$ for all $s \in S$ (see condition (i) in Definition 3.2), it follows that $p(s_i) \cdot e_i(s_i) > p(s_i) \cdot e_i(s_i)$, which is a contradiction. □

Proof of Proposition 3.7: All we need to show is that the maximin utility and the (Bayesian) interim expected utility coincide. Since for all $i \in I$ and for all $y \in \mathbb{R}^\ell_+$, $u_i(\cdot, y)$ is $\mathcal{F}_i$-measurable and $\mathcal{F}_i \subseteq \mathcal{G}_i$, then $u_i(\cdot, y)$ is $\mathcal{G}_i$-measurable.

Moreover, since for each $i \in I$, $x_i(\cdot)$ is $\mathcal{G}_i$-measurable it follows that for all $i \in I$ and $s \in S$, both maximin and interim utility function are equal to the ex-post utility function. That is,

$$\underline{u}_i^{REE}(s, x_i) = \min_{s' \in \mathcal{G}_i(s)} u_i(s', x_i(s')) = u_i(s, x_i(s)) \tag{13}$$

and

$$v_i(x_i|\mathcal{G}_i)(s) = \sum_{s' \in \mathcal{G}_i(s)} u_i(s', x_i(s')) \frac{\pi_i(s')}{\pi_i\left(\mathcal{G}_i(s)\right)} = u_i(s, x_i(s)). \tag{14}$$

From (13) and (14) it follows that for all i and s, $\underline{u}_i^{REE}(s, x_i) = v_i(x_i|\mathcal{G}_i)(s)$. Therefore, we can conclude that if (p, x) is a Bayesian REE, then (p, x) is a MREE; the converse is also true if $x_i(\cdot)$ is $\mathcal{G}_i$-measurable for all $i \in I$. □

9.2 Proofs of Section 3.4

Proof of Proposition 3.9: For each $s \in S$, let

$$H(s) = \{h \in \{1, \ldots, \ell\} \ : \ p^h(s) = 0\},$$

and let

$$\bar{S} = \{s \in S \ : \ H(s) \neq \emptyset\}.$$

Since (p, x) is a maximin REE, we consider the information generated by the equilibrium price, that is the algebra $\sigma(p)$. Clearly, $H(\cdot)$ is $\sigma(p)$-measurable[33], because $p(s_1) = p(s_2)$ whenever $\sigma(p)(s_1) = \sigma(p)(s_1)$. Moreover, since for any $i \in I$, $\sigma(p)$ is coarser than $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$, it follows that

$$\text{for all } i \in I \quad H(\cdot) \ \text{ is } \mathcal{G}_i - \text{measurable.} \tag{15}$$

Now, assume on the contrary that $\bar{S}$ is non empty and let $\bar{s} \in \bar{S}$. Hence, $H(\bar{s}) \neq \emptyset$, i.e., there exists at least a "free" good h such that $p^h(\bar{s}) = 0$. Define the following allocation: for each $i \in I$,

$$z_i^h(s) = \begin{cases} x_i^h(s) + K & \text{if } s \in \mathcal{G}_i(\bar{s}) \text{ and } h \in H(s) \\ x_i^h(s) & \text{otherwise,} \end{cases}$$

where $K > 0$.

Notice that for any $i \in I$ and $s \in \mathcal{G}_i(\bar{s})$, since $H(s) = H(\bar{s}) \neq \emptyset$ (see (15)), from the strict monotonicity it follows that $u_i(s, z_i(s)) > u_i(s, x_i(s))$, and hence for any $i \in I$

$$\underline{u}_i^{REE}(\bar{s}, z_i) > \underline{u}_i^{REE}(\bar{s}, x_i).$$

Since (p, x) is a maximin REE, for each $i \in I$ there exists a state $s_i \in \mathcal{G}_i(\bar{s})$ such that

$$p(s_i) \cdot [z_i(s_i) - e_i(s_i)] > 0.$$

From (15), it follows that $H(s_i) = H(\bar{s}) \neq \emptyset$, and therefore

$$\begin{aligned} 0 &< p(s_i) \cdot [z_i(s_i) - e_i(s_i)] = \\ \sum_{h \in H(s_i)} p^h(s_i)[x_i^h(s_i) + K - e_i^h(s_i)] &+ \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] = \\ 0 &+ \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] = \\ \sum_{h \in H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] &+ \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] = \\ p(s_i) \cdot [x_i(s_i) - e_i(s_i)] &\leq 0. \end{aligned}$$

This is a contradiction, hence $p(s) \gg 0$ for each $s \in S$. □

[33]We mean that $H(s_1) = H(s_2)$ if $\sigma(p)(s_1) = \sigma(p)(s_2)$.

Proof of Proposition 3.10: Let (p, x) be a maximin rational expectations equilibrium and define for each agent $i \in I$ and state $s \in S$ the following set:

$$M_i(s) = \left\{ s' \in \mathcal{G}_i(s) : \ \underline{u}_i^{REE}(s, x_i) = u_i(s', x_i(s')) \right\}.$$

Clearly, since S is finite, for all $i \in I$ and $s \in S$, the set $M_i(s)$ is nonempty, i.e., $M_i(s) \neq \emptyset$. Moreover, if $s' \in \mathcal{G}_i(s) \setminus M_i(s)$ it means that $\underline{u}_i^{REE}(s, x_i) < u_i(s', x_i(s'))$. Thus, we want to show that for all $i \in I$ and $s \in S$, $M_i(s) = \mathcal{G}_i(s)$.

Assume on the contrary that there exists an agent $j \in I$ and a state $\bar{s} \in S$ such that $\mathcal{G}_j(\bar{s}) \setminus M_j(\bar{s}) \neq \emptyset$. Notice that

$$\underline{u}_j^{REE}(\bar{s}, x_j) < u_j(s, x_j(s)) \quad \text{for any } s \in \mathcal{G}_j(\bar{s}) \setminus M_j(\bar{s}).$$

Fix $s' \in \mathcal{G}_j(\bar{s}) \setminus M_j(\bar{s})$ and define the following allocation

$$y_j(s) = \begin{cases} x_j(s) & \text{if } s \in G_j(\bar{s}) \setminus M_j(\bar{s}) \\ x_j(s') & \text{if } s \in M_j(\bar{s}). \end{cases}$$

Since the utility functions are assumed to be private information measurable, it follows that $u_j(s, y_j(s)) > \underline{u}_j^{REE}(\bar{s}, x_j)$ for any $s \in \mathcal{G}_j(\bar{s})$, and hence $\underline{u}_j^{REE}(\bar{s}, y_j) > \underline{u}_j^{REE}(\bar{s}, x_j)$. Recall that (p, x) is a maximin REE, therefore there exists $s \in \mathcal{G}_j(\bar{s})$ such that $p(s) \cdot y_j(s) > p(s) \cdot e_j(s)$. If $s \in M_j(\bar{s})$, thus $p(s) \cdot x_j(s') > p(s) \cdot e_j(s)$. Since $p(\cdot)$ and $e_j(\cdot)$ are both $\mathcal{G}_j$-measurable, it follows that $p(s') = p(s)$ and $e_j(s') = e_j(s)$. This implies that $p(s') \cdot x_j(s') > p(s') \cdot e_j(s')$, which is clearly a contradiction. On the other hand, if $s \in G_j(\bar{s}) \setminus M_j(\bar{s})$, thus we have that $p(s) \cdot x_j(s) > p(s) \cdot e_j(s)$ which is a contradiction as well. Therefore, for each $i \in I$ and $s \in S$, $M_i(s) = \mathcal{G}_i(s)$. □

9.3 Proofs of Section 4

Proof of Theorem 4.1: Let W be the following set[34]:

$$W = \{x \in L \mid x(\cdot, s) \in W(\mathcal{E}(s)) \text{ for all } s \in S\},$$

and notice that the assumptions guarantee that W is non empty. So, let $x \in W$. Since, for each s, $x(\cdot, s) \in W(\mathcal{E}(s))$, then there exists a price vector $p(s) \in \mathbb{R}^\ell_{++}$ such that $(p(s), x(\cdot, s))$ is a competitive equilibrium for the economy $\mathcal{E}(s)$. Consider now the functions $\hat{p} : S \to \mathbb{R}^\ell_+$ and $\hat{x} : I \times S \to \mathbb{R}^\ell_+$ such that for all $s \in S$ and $i \in I$, $\hat{p}(s) = p(s)$ and $\hat{x}_i(s) = x(i, s)$. First, notice that $\hat{x}$ is feasible in the economy $\mathcal{E}$ since so is $x(\cdot, s)$ in the economy $\mathcal{E}(s)$ for each s, and $\hat{p}$ is a price function since from the monotonicity of utility functions, it follows that $p(s) \gg 0$ for each state s. Consider the algebra generated by $\hat{p}$ denoted by $\sigma(\hat{p})$, and for each agent i let $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(\hat{p})$. We show that $(\hat{p}, \hat{x})$ is a maximin rational expectations equilibrium for $\mathcal{E}$. Clearly, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$ for all i and s, hence $\hat{x}_i \in B_i(s, \hat{p})$ for all i and s. It remains

[34] An element of W is said to be **ex post competitive equilibrium allocation**.

to prove that $\hat{x}_i$ maximizes $\underline{u}_i^{REE}$ on B_i. Assume, on the contrary, that there exists an alternative allocation $y \in L$ such that for some agent i and some state s,

$$\underline{u}_i^{REE}(s, y_i) > \underline{u}_i^{REE}(s, \hat{x}_i), \text{ and} \tag{16}$$

$y_i \in B_i(s, \hat{p})$, that is

$$p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s') \quad \text{for all } s' \in \mathcal{G}_i(s). \tag{17}$$

Since S is finite, from (16) it follows that there exists a state $\bar{s} \in \mathcal{G}_i(s)$ such that

$$u_i(\bar{s}, y_i(\bar{s})) \geq \underline{u}_i^{REE}(s, y_i) > \underline{u}_i^{REE}(s, \hat{x}_i) = u_i(\bar{s}, x(i, \bar{s})).$$

Since $(p(\bar{s}), x(\cdot, \bar{s}))$ is a competitive equilibrium for $\mathcal{E}(\bar{s})$, it follows that, $p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s})$, which clearly contradicts (17). Thus, $W \subseteq MREE(\mathcal{E})$, and the nonemptiness of W implies the existence of a maximin rational expectations equilibrium. □

Proof of Corollary 4.6: The equivalence between W, the set of competitive equilibria in each $\mathcal{E}(s)$, and $REE(\mathcal{E})$, the set of Bayesian rational expectations equilibrium has been proved by Einy, Moreno, and Shitovitz (2000) (see also De Simone and Tarantino (2010) for an extension to an infinite dimensional commodity space). Moreover, we have observed that under such assumptions, any Bayesian REE is also a maximin rational expectations equilibrium and vice versa (see Remark 3.8), i.e., $REE(\mathcal{E}) = MREE(\mathcal{E})$; therefore the conclusion. □

9.4 Proofs of Section 5.1

Proof of Proposition 5.3 Let x be a maximin efficient allocation and assume, on the contrary, that there exists an alternative allocation y such that

$$\begin{array}{ll} (i) & u_i(s, y_i(s)) > u_i(s, x_i(s)) \quad \text{for all } i \in I \text{ and for all } s \in S \\ (ii) & \sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad \text{for all } s \in S. \end{array}$$

Thus, for each agent $i \in I$ whatever his information partition is Π_i, it follows from (i) above that $\underline{u}_i^{\Pi_i}(s, y_i) > \underline{u}_i^{\Pi_i}(s, x_i)$ for each state s. Hence, a contradiction since x is maximin Pareto optimal. In order to show that the converse may not be true, consider an economy with two agents, three states of nature, $S = \{a, b, c\}$, and two goods, such that

$$\begin{aligned} u_i(a, x_i, y_i) &= \sqrt{x_i y_i} \quad u_i(b, x_i, y_i) = log(x_i y_i) \quad u_i(c, x_i, y_i) = x_i^2 y_i \quad \text{for all } i = 1, 2. \\ e_1(a) &= (2, 1) \quad e_2(a) = e_1(b) = e_2(b) = e_1(c) = e_2(c) = (1, 2) \\ \Pi_1 &= \{\{a, c\}, \{b\}\} \quad \Pi_2 = \{\{a\}, \{b, c\}\}. \end{aligned}$$

Consider the following feasible allocation:

$$\begin{aligned}
(x_1(a), y_1(a)) &= \left(3, \frac{1}{3}\right) \quad (x_2(a), y_2(a)) = \left(0, \frac{8}{3}\right) \\
(x_1(b), y_1(b)) &= (1, 2) \quad (x_2(b), y_2(b)) = (1, 2)\,, \\
(x_1(c), y_1(c)) &= (2, 1) \quad (x_2(c), y_2(c)) = (0, 3)\,.
\end{aligned}$$

Notice that it is ex post efficient, since if on the contrary there exists (t, z) such that

$$u_i(s, t_i(s), z_i(s)) > u_i(s, x_i(s), y_i(s)) \quad \text{for all } i \in I \text{ and all } s \in S,$$

in particular,

$$\begin{cases} log(t_1(b)z_1(b) > log2 \\ log(t_2(b)z_2(b)) > log2 \\ t_1(b) + t_2(b) = 2 \\ z_1(b) + z_2(b) = 4, \end{cases}$$

then[35]

$$\begin{cases} z_1(b) > \frac{2}{t_1(b)} \\ (2 - t_1(b))(2t_1(b) - 1) > t_1(b). \end{cases}$$

This implies that $(t_1(b) - 1)^2 < 0$, which is impossible. Thus, the above allocation is ex post Pareto optimal, but it is not maximin efficient with respect to the information structure Π, since it is (maximin) blocked by the following feasible allocation:

$$\begin{aligned}
(t_1(a), z_1(a)) &= \left(\frac{5}{4}, \frac{5}{2}\right) \quad (t_2(a), z_2(a)) = \left(\frac{7}{4}, \frac{1}{2}\right) \\
(t_1(b), z_1(b)) &= \left(1, \frac{8}{3}\right) \quad (t_2(b), z_2(b)) = \left(1, \frac{4}{3}\right) \\
(t_1(c), z_1(c)) &= \left(\frac{3}{4}, 2\right) \quad (t_2(c), z_2(c)) = \left(\frac{5}{4}, 2\right).
\end{aligned}$$

Indeed,

$$\begin{aligned}
\underline{u}_1^{\Pi_1}(a, t_1, z_1) &= \underline{u}_1^{\Pi_1}(c, t_1, z_1) = \min\{\sqrt{\frac{25}{8}}, \frac{9}{8}\} = \frac{9}{8} > 1 = \min\{1, 4\} = \underline{u}_1^{\Pi_1}(c, x_1, y_1) = \underline{u}_1^{\Pi_1}(a, x_1, y_1) \\
\underline{u}_1^{\Pi_1}(b, t_1, z_1) &= u_1(b, t_1(b), z_1(b)) = log\frac{8}{3} > log2 = u_1(b, x_1(b), y_1(b)) \\
\underline{u}_2^{\Pi_2}(a, t_2, z_2) &= u_2(a, t_2(a), z_2(a)) = \sqrt{\frac{7}{8}} > 0 = u_2(a, x_2(a), y_2(a)) = \underline{u}_2^{\Pi_2}(a, x_2, y_2) \\
\underline{u}_2^{\Pi_2}(b, t_2, z_2) &= \underline{u}_2^{\Pi_2}(c, t_2, z_2) = \min\{log\frac{4}{3}, \frac{25}{8}\} = log\frac{4}{3} > 0 = \min\{log2, 0\} = \underline{u}_2^{\Pi_2}(c, x_2, y_2) = \underline{u}_2^{\Pi_2}(b, x_2, y_2).
\end{aligned}$$

□

[35] Clearly, $(t_i(b), z_i(b)) \gg (0, 0)$ for each $i = 1, 2$, because they belong into the domain of log.

Proof of Theorem 5.4: Let (p, x) be a maximin rational expectations equilibrium, so since agents take into account the information that the equilibrium price generates, the private information of each individual i is $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. Assume on the contrary that there exists an alternative allocation $y \in L$ such that

$$\begin{array}{ll} (i) & \underline{u}_i^{REE}(s, y_i) > \underline{u}_i^{REE}(s, x_i) \quad \text{for all } i \in I \text{ and for all } s \in S, \\ (ii) & \displaystyle\sum_{i \in I} y_i(s) = \sum_{i \in I} e_i(s) \quad \text{for all } s \in S. \end{array}$$

I CASE: $u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable for each $i \in I$ and each $t \in \mathbb{R}^\ell_+$.

Fix an agent $i \in I$ and a state $s \in S$. From (i) it follows that $\underline{u}_i^{REE}(s, y_i) > \underline{u}_i^{REE}(s, x_i)$. Since x is a maximin REE, for some $\bar{s} \in \mathcal{G}_i(s)$,

$$p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s}).$$

Thus, from (ii) we can deduce that there exists at least one agent $j \in I \setminus \{i\}$ such that

$$p(\bar{s}) \cdot y_j(\bar{s}) < p(\bar{s}) \cdot e_j(\bar{s}).$$

Hence, since $p(\cdot)$ and $e_j(\cdot)$ are $\mathcal{G}_j$-measurable, it follows that

$$p(s) \cdot y_j(\bar{s}) < p(s) \cdot e_j(s) \qquad \text{for all } s \in \mathcal{G}_j(\bar{s}). \tag{18}$$

Define the allocation[36] z_j as follows:

$$z_j(s) = y_j(\bar{s}) + \frac{\mathbf{1} p(s) \cdot [e_j(s) - y_j(\bar{s})]}{\sum_{h=1}^{\ell} p^h(s)} \qquad \text{for any } s \in \mathcal{G}_j(\bar{s}),$$

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \ldots, 1)$. Notice that $z_j(\cdot)$ is constant in the event $\mathcal{G}_j(\bar{s})$ and $p(s) \cdot z_j(s) = p(s) \cdot e_j(s)$ for any $s \in \mathcal{G}_j(\bar{s})$. Therefore, since (p, x) is a maximin REE and $u_j(\cdot, x)$ is $\mathcal{F}_j$-measurable, from the monotonicity of $u_j(\bar{s}, \cdot)$ and (i), it follows that

$$\underline{u}_j^{REE}(\bar{s}, x_j) \geq \underline{u}_j^{REE}(\bar{s}, z_j) = u_j(\bar{s}, z_j(\bar{s})) > u_j(\bar{s}, y_j(\bar{s})) \geq \underline{u}_j^{REE}(\bar{s}, y_j) > \underline{u}_j^{REE}(\bar{s}, x_j),$$

a contradiction.

II CASE: there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{G}_i(\bar{s})$ for all $i \in I$.

Since for each $i \in I$, $\{\bar{s}\} = \mathcal{G}_i(\bar{s})$; from (i) it follows that $\underline{u}_i^{REE}(\bar{s}, y_i) = u_i(\bar{s}, y_i(\bar{s})) > u_i(\bar{s}, x_i(\bar{s})) = \underline{u}_i^{REE}(\bar{s}, x_i)$ for all $i \in I$. Hence, since (p, x) is a MREE, for each agent i there exists at least one state $s_i \in \mathcal{G}_i(\bar{s}) = \{\bar{s}\}$ (that is $s_i = \bar{s}$ for all $i \in I$) such that $p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s})$. Therefore,

$$\sum_{i \in I} p(\bar{s})[y_i(\bar{s}) - e_i(\bar{s})] > 0,$$

[36] Notice that for any $s \in \mathcal{G}_j(\bar{s})$, $\sum_{h=1}^{\ell} p^h(s) > 0$, because $p(s) \in \mathbb{R}^\ell_+ \setminus \{0\}$ for any $s \in S$.

which contradicts (ii).

III CASE: p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$.

Since p is fully revealing, for each agent $i \in I$ and state $s \in S$, $\mathcal{G}_i(s) = \{s\}$. Thus, from the above case, x is maximin efficient.

IV CASE: $n - 1$ agents are fully informed.

Since (p, x) is a MREE, from (i) it follows that for any state $s \in S$ and any agent $i \in I$ there exists at least one state $s_i \in \mathcal{G}_i(s)$ such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$. Let j be the unique not fully informed agent, and consider the state s_j for which $p(s_j) \cdot y_j(s_j) > p(s_j) \cdot e_j(s_j)$. Since each agent $i \neq j$ is fully informed, it follows that $\mathcal{G}_i(s_j) = \{s_j\}$ for all $i \neq j$. Thus,

$$p(s_j) \cdot y_i(s_j) > p(s_j) \cdot e_i(s_j) \quad \text{for all } i \in I.$$

Hence,

$$\sum_{i \in I} p(s_j) \cdot y_i(s_j) > \sum_{i \in I} p(s_j) \cdot e_i(s_j),$$

which is a contradiction. □

9.5 Proofs of Section 5.2

Before proving Proposition 5.14 the following lemma is needed.

Lemma 9.1 *Condition (iii) and $(*)$ in the Definition 5.11, imply that for all $i \in C$,*

$$u_i(a, x_i(a)) = \min_{s \in \Pi_i(a)} u_i(s, x_i(s)) = \underline{u}_i^{\Pi_i}(a, x_i),$$

and

$$u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad for\ all\ s \in \Pi_i(a) \setminus \{a\}.$$

Proof: Assume, on the contrary, there exists an agent $i \in C$ and a state $s_1 \in \Pi_i(a) \setminus \{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = \min_{s \in \Pi_i(a)} u_i(s, x_i(s)) = u_i(s_1, x_i(s_1))$.

Notice that

$$\underline{u}_i^{\Pi_i}(a, y_i) = \min\{u_i(a, e_i(a) + x_i(b) - e_i(b)); \min_{s \in \Pi_i(a) \setminus \{a\}} u_i(s, x_i(s))\}.$$

If, $u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $u_i(a, y_i(a)) \leq u_i(s_1, x_i(s_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This contradicts (iii). On the other hand, if there exists $s_2 \in \Pi_i(a) \setminus \{a\}$ such that $u_i(s_2, x_i(s_2)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $\underline{u}_i^{\Pi_i}(a, y_i) = u_i(s_2, x_i(s_2)) \leq u_i(s_1, x_i(s_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This again contradicts

(iii). Thus, for each member i of C, there does not exist a state $s \in \Pi_i(a) \setminus \{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = u_i(s, x_i(s))$. This means that

$$u_i(a, x_i(a)) = \min_{s \in \Pi_i(a)} u_i(s, x_i(s)) = \underline{u}_i^{\Pi_i}(a, x_i),$$

and

$$u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \Pi_i(a) \setminus \{a\}. \qquad \square$$

Proof of Proposition 5.14: Let x be a CIC and assume on the contrary that there exists a coalition C and two states a and b such that[37]

$$\begin{array}{ll} (i) & \mathcal{F}_i(a) = \mathcal{F}_i(b) \quad \text{for all } i \notin C, \\ (ii) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell \quad \text{for all } i \in C, \text{ and} \\ (iii) & \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) \quad \text{for all } i \in C, \end{array}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Notice that from (iii) and Lemma 9.1 it follows that for all $i \in C$,

$$u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)).$$

Hence x is not CIC, which is a contradiction. For the converse, we construct the following counterexample. Consider the economy, described in Example 5.9, with two agents, three states of nature, denoted by a, b and c, and one good per state denoted by x. Assume that

$$\begin{array}{lll} u_1(\cdot, x_1) = \sqrt{x_1}; & e_1(a,b,c) = (20, 20, 0); & \mathcal{F}_1 = \{\{a,b\}; \{c\}\}. \\ u_2(\cdot, x_2) = \sqrt{x_2}; & e_2(a,b,c) = (20, 0, 20); & \mathcal{F}_2 = \{\{a,c\}; \{b\}\}. \end{array}$$

Consider the allocation

$$\begin{aligned} x_1(a,b,c) &= (20, 10, 10) \\ x_2(a,b,c) &= (20, 10, 10). \end{aligned}$$

We have already noticed that such an allocation is not Krasa-Yannelis incentive compatible (see Example 5.9), but it is maximin CIC (see Remark 5.12). $\square$

Proof of Proposition 5.21: Let (p, x) be a maximin REE[38] and assume on the contrary that there exist a coalition C and two states $a, b \in S$ such that

[37]Instead of $\mathcal{F}_i$, we can use any structure Π_i. This means that if in the Definition 5.8 we use Π_i instead of $\mathcal{F}_i$, then we would have maximin CIC with respect to Π_i. The proof is the same with the obvious adaptations.

[38]If (p, x) is a non revealing MREE, then the proposition holds true without no additional assumptions on utility functions (see Remark 5.18).

$$
\begin{array}{lll}
(I) & \mathcal{F}_i(a) = \mathcal{F}_i(b) & \text{for all } i \notin C, \\
(II) & u_i(a, x_i(a)) = u_i(a, x_i(b)) & \text{for all } i \notin C, \\
(III) & e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} & \text{for all } i \in C, \text{ and} \\
(IV) & \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) & \text{for all } i \in C,
\end{array}
$$

where for all $i \in C$,

$$
y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}
$$

Assume that for any $i \in I$ $u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable for each $t \in \mathbb{R}^{\ell}_{+}$. Observe that if p is partially revealing and $\mathcal{G}_i(a) \setminus \{a\} \neq \emptyset$ for some agent i in C, then the allocation x is (private) maximin coalitional incentive compatible and hence weak (private) maximin CIC. Indeed, from Lemma 9.1 and condition (IV), it follows that

$$
\underline{u}_i^{REE}(a, x_i) = \underline{u}_i(a, x_i) = u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \mathcal{F}_i(a) \setminus \{a\}.
$$

In particular the above inequality holds for all $s \in \mathcal{G}_i(a) \setminus \{a\}$, and this contradicts Proposition 3.10. Moreover, if for some agent $i \notin C$, $\mathcal{G}_i(a) = \mathcal{G}_i(b)$, then it follows that $p(a) = p(b)$, and hence p is partially revealing. However, even if utility functions are not private information measurable, we can conclude that x is (private) maximin coalitional incentive compatible and hence weak (private) maximin CIC. In fact, from (IV) and Lemma 9.1, it follows that for all $i \in C$,

$$
\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i).
$$

Therefore, since (p, x) is a maximin REE, from the definition of the allocation y, it follows that for each $i \in C$, $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot x_i(b) > p(a) \cdot e_i(b)$, which is a contradiction because $p(a) = p(b)$.

Thus, let us assume that $\mathcal{G}_i(a) = \{a\}$ for all $i \in C$ and $\mathcal{G}_i(a) \neq \mathcal{G}_i(b)$ for any $i \notin C$. Again from (IV) and Lemma 9.1, it follows that for all $i \in C$,

$$
\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i),
$$

while from (II) it follows that for all $i \notin C$,

$$
\begin{aligned}
\underline{u}_i^{REE}(a, y_i) &= \min\{\min_{s \in \mathcal{G}_i(a) \setminus \{a\}} u_i(s, x_i(s)), u_i(a, y_i(a))\} \\
&= \min\{\min_{s \in \mathcal{G}_i(a) \setminus \{a\}} u_i(s, x_i(s)), u_i(a, x_i(b))\} \\
&= \min\{\min_{s \in \mathcal{G}_i(a) \setminus \{a\}} u_i(s, x_i(s)), u_i(a, x_i(a))\} \\
&= \underline{u}_i^{REE}(a, x_i).
\end{aligned}
$$

Moreover, y is feasible. Indeed, for each state $s \neq a$, y is feasible because so is x. On the other hand, if $s = a$, then

$$\sum_{i \in I} y_i(a) = \sum_{i \in I} e_i(a) + \sum_{i \in I} x_i(b) - \sum_{i \in I} e_i(b) = \sum_{i \in I} e_i(a).$$

Hence, there exists a feasible allocation y such that

$$\underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i) \quad \text{for all } i \in I \text{ and all } s \in S,$$

with a strict inequality for each $i \in C$ in state a. Since x is a maximin REE and $\mathcal{G}_i(a) = \{a\}$ for all $i \in C$, it follows that

$$p(a) \cdot y_i(a) > p(a) \cdot e_i(a) \quad \text{for any } i \in C.$$

Moreover, since y is feasible, there exists at least one agent $j \notin C$ such that

$$p(a) \cdot y_j(a) < p(a) \cdot e_j(a).$$

Notice that

$$p(s) \cdot y_j(a) < p(s) \cdot e_j(s) \qquad \text{for all } s \in \mathcal{G}_j(a), \tag{19}$$

because $p(\cdot)$ and $e_j(\cdot)$ are $\mathcal{G}_j$-measurable. Define the allocation[39] z_j as follows:

$$z_j(s) = y_j(a) + \frac{\mathbf{1}p(s) \cdot [e_j(s) - y_j(a)]}{\sum_{h=1}^{\ell} p^h(s)} \qquad \text{for any } s \in \mathcal{G}_j(a),$$

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \ldots, 1)$. Notice that $z_j(\cdot)$ is constant in the event $\mathcal{G}_j(a)$ and $p(s) \cdot z_j(s) = p(s) \cdot e_j(s)$ for any $s \in \mathcal{G}_j(a)$. Therefore, since (p, x) is a maximin REE and $u_j(\cdot, x)$ is $\mathcal{F}_j$-measurable, from the monotonicity of $u_j(a, \cdot)$, it follows that

$$\underline{u}_j^{REE}(a, x_j) \geq \underline{u}_j^{REE}(a, z_j) = u_j(a, z_j(a)) > u_j(a, y_j(a)) \geq \underline{u}_j^{REE}(a, y_j) = \underline{u}_j^{REE}(a, x_j),$$

a contradiction. Assume now that the equilibrium price p is fully revealing; hence $\mathcal{G}_i(a) = \{a\}$ for any $i \in I$. From (IV) and Lemma 9.1 it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i),$$

and hence

$$p(a) \cdot y_i(a) > p(a) \cdot e_i(a) \quad \text{for any } i \in C.$$

while from (II) it follows that for all $i \notin C$,

$$\underline{u}_i^{REE}(a, y_i) = u_i(a, x_i(b)) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i).$$

Since, we have already observed that y is feasible, we conclude that for some agent $j \notin C$,

$$p(a) \cdot y_j(a) < p(a) \cdot e_j(a).$$

[39] Notice that for any $s \in \mathcal{G}_j(a)$, $\sum_{h=1}^{\ell} p^h(s) > 0$, because $p(s) \in \mathbb{R}_+^{\ell} \setminus \{0\}$ for any $s \in S$.

Define the following bundle[40]

$$z_j(a) = y_j(a) + \frac{\mathbf{1}p(a) \cdot [e_j(a) - y_j(a)]}{\sum_{h=1}^{\ell} p^h(a)},$$

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \dots, 1)$. Notice that $p(a) \cdot z_j(a) = p(a) \cdot e_j(a)$ and

$$\underline{u}_j^{REE}(a, z_j) = u_j(a, z_j(a)) > u_j(a, y_j(a)) = \underline{u}_j^{REE}(a, y_j) = \underline{u}_j^{REE}(a, x_j),$$

contradicts the fact that x is a maximin REE allocation. □

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[40]Notice that $\sum_{h=1}^{\ell} p^h(a) > 0$, because $p(s) \in \mathbb{R}_+^{\ell} \setminus \{0\}$ for any $s \in S$.

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