Ambiguity aversion solves the conflict between efficiency and incentive compatibility*

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Abstract

The conflict between Pareto optimality and incentive compatibility, that is, the fact that some Pareto optimal (efficient) allocations are not incentive compatible is a fundamental fact in information economics, mechanism design and general equilibrium with asymmetric information. This important result was obtained assuming that the individuals are expected utility maximizers. Although this assumption is central to Harsanyi's approach to games with incomplete information, it is not the only one reasonable. In fact, a huge literature criticizes EU's shortcomings and propose alternative preferences. Thus, a natural question arises: does the mentioned conflict extend to other preferences? We show that when individuals have (a special form of) maximin expected utility (MEU) preferences, then any efficient allocation is incentive compatible. Conversely, only MEU preferences have this property. We also provide applications of our results to mechanism design and show that Myerson-Satterthwaite's negative result ceases to hold in our MEU framework.

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1 Introduction

One of the fundamental problems in mechanism design and equilibrium theory with asymmetric information is the conflict between efficiency and incentive compatibility. That is, there are allocations that are efficient but not incentive compatible. This important problem was alluded to in early seminal works by Wilson (1978), Myerson (1979), Holmstrom and Myerson (1983), and Prescott and Townsend (1984). Since incentive compatibility and efficiency are some of the most important concepts in economics, this conflict generated a huge literature and became a cornerstone of the theory of information economics, mechanism design and general equilibrium with asymmetric information.

It is a simple but perhaps important observation, that this conflict was predicated on the assumption that the individuals were expected utility maximizers (EUM), that is, they would form Bayesian beliefs about the type (private information) of the other individuals and seek the maximization of the expected utility with respect to those beliefs. Since the Bayesian paradigm has been central to most of economics, this assumption seemed not only natural, but the only one worth pursuing.

The Bayesian paradigm is not immune to criticism, however, and many important papers have discussed its problems; e.g. Allais (1953), Ellsberg (1961) and Kahneman and Tversky (1979) among others. The recognition of those problems have led decision theorists to propose many alternative models, beginning with Bewley (1986, 2002), Schmeidler (1989) and Gilboa and Schmeidler (1989), but extending in many different models. For syntheses of these models, see Maccheroni, Marinacci, and Rustichini (2006), Cerreia, Maccheroni, Marinacci, and Montrucchio (2008) and Ghirardato and Siniscalchi (2010).

The fact that many different preferences have been considered leads naturally to the following questions: Does the conflict between efficiency and incentive compatibility extend to other preferences? Is there any preference under which there is no such conflict? Does the set of efficient and incentive compatible allocations increases or reduces if a different preference is considered other than EU? The purpose of this article is to answer these questions.

We show that (a special form of) the maximin expected utility (MEU) introduced by Gilboa and Schmeidler (1989) has the remarkable property that all efficient (Pareto optimal) allocations are also incentive compatible. This property is probably best understood in a simple example, based on Myerson and Satterthwaite (1983)'s setting.

Myerson-Satterthwaite example. A seller values the object as $v \in [0,1]$ and a buyer values it as $t \in [0,1]$. Both values are private information. An allocation will be efficient in this case if trade happens if and only if $t \geqslant v$. Under the Bayesian paradigm, that is, the assumption that both seller and buyer are expected utility maximizers (EUM), Myerson and Satterthwaite (1983) have proved that there is no incentive compatible, individual rational mechanism (without subsidies) that would achieve ex post efficiency in this situation.

Consider now the following simple mechanism: the seller places an ask a and the buyer, a bid b. If the bid is above the ask, they trade at $p = \frac{a+b}{2}$; if it below, there is no trade. Therefore, if they negotiated at price p, the (ex post) profit for the seller will be $p - v$, and for the buyer, $t - p$; if they do not negotiate, both get zero. By Myerson and Satterthwaite (1983)'s result mentioned above, if the individuals are EUM, this mechanism does not always lead to efficient allocations. The problem is that this mechanism would be efficient if and only if both seller and buyer report truthfully, that is, $a = v$ and $b = t$, but these choices are not incentive compatible if the individuals are EUM. Now, we will show that $a = v$ and $b = t$ are incentive compatible choices if both seller and buyer have maximin preferences, which we will define now.

For this, we depart from the Bayesian assumption used by Harsanyi and *do not* assume that each player form a prior about the distribution of the other player's value. Indeed, in most real-life cases it would be hard to accept that a buyer knows the definitive distribution generating the seller's value (and vice-versa). In sum, both buyer and seller are in a case of uncertainty. Classical statistics has a well known prescription for these cases: Wald (1950)'s maximin criterion. Adopting such criterion, we assume that each individual considers the worst-case scenario for each action, and chooses the action that leads to the best worst-case outcome.

Now recall that $a = v$ and $b = t$ are incentive compatible if buyer and seller do not have any incentive to choose a different action. If the buyer chooses $b = t$, the worst-case scenario is to end up with zero (either by buying by $p = t$ or by not trading). Can she do better than this? If she chooses $b > t$, the worst-case scenario is to buy by $p > t$, which leads to a (strict) loss. If she considers $b < t$, the worst-case scenario is to get zero (it always possible that there is no trade). Therefore, neither $b < t$ nor $b > t$ is better (by the maximin criterion) than $b = t$ and she has no incentive to deviate. The argument for the seller is analogous.

Note that our notions of efficiency and incentive compatibility are completely standard. The only difference from the classic framework is the preference considered. Also, although the individuals are pessimistic, they achieve the best possible outcome, even from an EU point of view, that is, the outcome is efficient.[1] Thus, our results have a clear implication on the problem of adverse selection and the "market for lemons" (see Akerlof (1970)).

Another interesting property of these preferences is that the set of efficient allocations is not small. At least in the case of one-good economies, the set of efficient allocations under maximin preferences includes all allocations that are incentive compatible and efficient for EU individuals. This result seems somewhat surprising, since other papers have indicated that ambiguity may actually be bad for efficiency, limiting trading opportunities. See for instance Mukerji (1998) and related comments in section 8.

It is important to know if there are other preferences that present no conflict between incentive compatibility and efficiency. Since maximin preferences may seem somewhat restrictive, it is important to know if other preferences can have the strong property that all efficient allocations are incentive compatible. We answer this question in the negative: only the maximin preferences considered in this paper have this property (Theorem 4.3). In other words, we show that all efficient (first-best) allocations are incentive compatible under maximin preferences and only these preferences have this property in general.

The paper is organized as follows. In section 2, we describe the setting and introduce definitions and notation. Section 3 shows that in economies with (a special form of) MEU preferences, all Pareto optimal allocations are incentive compatible. Section 4 establishes that the maximin preferences are the only preferences where efficiency and incentive compatibility are fully consistent. We illustrate how our results can be cast in the mechanism design perspective in section 5. Section 6 establishes that the set of efficient and incentive compatible allocations in the EU setting are also MEU efficient. Section 8 reviews the relevant literature and section 9 discusses future directions of research. An appendix collects some technical proofs.

[1] The reader may be concerned with the multiplicity of equilibria in this example. Indeed, to choose $b < t$ could also be an equilibrium. However, the multiplicity of equilibria is also possible in the standard Bayesian framework and is also a concern there, specially in issues related to implementation. Since this issue is not restricted to our framework and a large part of the mechanism design literature does not discuss it, we will follow the standard practice and leave further discussions to future work.

2 Preliminaries

The set $I = \{1, ..., n\}$ represents the set of individuals in the economy. Each agent $i \in I$ observes a signal in some finite set of possible signals, $t_i \in T_i$. The restriction to finite signals is not crucial and is assumed here just for simplicity. Write $T = T_1 \times \cdots \times T_n$.[2] A vector $t = (t_1, \ldots, t_i, \ldots, t_n)$ represents the vector of all types. T_{-i} denotes $\Pi_{i \neq j}^n T_j$ and, similarly, t_{-i} denotes $(t_1, \ldots, t_{i-1}, t_{t+1}, \ldots, t_n)$. Occasionally, it will be convenient to write the vector $t = (t_1, \ldots, t_i, \ldots, t_j, \ldots, t_n)$ as (t_i, t_j, t_{-i-j}).

For clarity, it is useful to specify the following periods (timing structure) for information and decision makings by the individuals:

1. *Ex-ante*: contracts are negotiated.

2. *Interim*: types are privately known by each individual. Then, individuals announce their types (truthfully or not).

3. *Ex post*: contracts are executed and consumption takes place.

Now, it remains to clarify the preferences of the agents with respect to the goods to be consumed.

2.1 Goods and allocations

Each individual cares about an outcome (e.g. consumption bundle) $b \in \mathcal{B}$. The set of bundles $\mathcal{B}$ is assumed to be a topological vector space. To fix ideas, the reader may find it useful to identify $\mathcal{B}$ with $\mathbb{R}_+^\ell$, for some $\ell \in \mathbb{N}$. We assume that there is a continuous function $u_i : T \times \mathcal{B} \to \mathbb{R}$ such that $u_i(t, b)$ represents individual i's utility for consuming b when types $t \in T$ are realized. The *ex post* preference on $\mathcal{B}$ depending on $t \in T$ is denoted by $\succcurlyeq^t$ and defined by:

$$a \succcurlyeq_i^t b \iff u_i(t, a) \geqslant u_i(t, b), \forall a, b \in \mathcal{B}. \tag{1}$$

[2] Although this specification is not essential for our development, we could fix ideas and assume that this type space corresponds to "payoff types," that is, types associated to the payoff uncertainty. This is the most common kind of types spaces in the mechanism design literature (see Bergemann and Morris (2005)). It is also possible to add another set of payoff relevant states S to parametrize the uncertainty, as in Morris (1994). Since this would slightly complicate our notation and offer no new insights, we refrain from doing this straightforward extension. Our theory can also be developed for partition models, as we have done in the first version of this paper, de Castro and Yannelis (2008), and also in section 7.4 below.

A particular case of interest below is the one-good economy:

Definition 2.1 (One-good economy) *We say that the economy is one-good if* $\mathcal{B} \subseteq \mathbb{R}$*, and for every* $i \in I$ *and* $t \in T$*,* $a \mapsto u_i(t, a)$ *is strictly increasing.*

For future discussion, it will be useful to define private values.

Definition 2.2 (Private values) *We say that we have private values if the utility function of agent* i *depends on* t_i *but not on* t_j *for* $j \neq i$*, that is,* $u_i(t_i, t_{-i}, a) = u_i(t_i, t'_{-i}, a)$ *for all* i, t_i, t_{-i}, t'_{-i} *and* a*.*

We are most interested in the *interim* stage, that is, the moment in which each individual knows her type t_i, but not the types of other individuals $(t_j, j \neq i)$, that is, the *interim* stage. Since individuals do not know others' types, Harsanyi assumes that they form Bayesian beliefs about these other types. Finally, he assumes that there is an *ex ante* stage, where the types of all players are drawn according to those beliefs (with the additional assumption that those beliefs come from a common prior).

This paper departs from Harsanyi's assumption of Bayesian beliefs and assumes that individual i has an *interim* preference $\succcurlyeq_i^{t_i}$ (depending on t_i) defined by:

$$a \succcurlyeq_i^{t_i} b \iff \min_{t_{-i} \in T_{-i}} u_i(t_i, t_{-i}, a) \geqslant \min_{t_{-i} \in T_{-i}} u_i(t_i, t_{-i}, b). \tag{2}$$

It is easy to see that this is a maximin preference and it belongs to the class of preferences studied for modeling *complete ignorance*. See for instance Milnor (1954) and Luce and Raiffa (1989). Therefore, our model differs from Harsanyi's in the sense that, instead of assuming that the individuals have Bayesian beliefs about other individuals' types, they are completely ignorant about the distribution of those types and adopt pessimist views.[3] Although there is a large literature criticizing the Bayesian framework (see references on section 8), we do not need to assume that maximin preferences are *descriptively* better than Bayesian preferences in games with incomplete information. Rather, our objective here is to report some properties of this maximin preference. For further discussion on these preferences and how it relates to Harsanyi's approach, see section 7.

As it turns out, the definition in (2) is yet not satisfactory, because at the interim stage, in general the individual faces a set of bundles instead of a single bundle—the actual bundle to be consumed in the ex post stage is determined depending on

[3] The situation would be more properly described as one of "partial ignorance," because the agent is informed of his type. See more discussion about this point on section 7.

other individuals' types. That is, individual i has to compare individual allocations $f : T \to \mathcal{B}$ and $g : T \to \mathcal{B}$ and the preference has to be defined on these functions rather than on bundles. In this case, equation (2) should be changed to:[4]

$$f \succcurlyeq_i^{t_i} g \iff \min_{t_{-i}, t'_{-i} \in T_{-i}} u_i(t_i, t_{-i}, f(t_i, t'_{-i})) \geqslant \min_{t_{-i}, t'_{-i} \in T_{-i}} u_i(t_i, t_{-i}, g(t_i, t'_{-i})). \tag{3}$$

For each function $f : T \to \mathcal{B}$, it is convenient to define:

$$\underline{f}(t_i) \equiv \min_{t_{-i}, t'_{-i} \in T_{-i}} u_i(t_i, t_{-i}, f(t_i, t'_{-i})). \tag{4}$$

We can also define an ex ante preference $\succcurlyeq_i$ correspondent to this interim preference. This step is not essential for our results, however, and can be made in the same way that Harsanyi did, that is, by assuming that there is a measure μ_i generating t_i. Following the above notation, the *ex ante* preference would be:

$$f \succcurlyeq_i g \iff \int_{T_i} \underline{f}(t_i) \mu_i(dt_i) \geqslant \int_{T_i} \underline{g}(t_i) \mu_i(dt_i). \tag{5}$$

We will assume throughtout the paper that μ_i puts positive probability on all types on T_i, that is, $\mu_i(\{t_i\}) > 0, \forall t_i \in T_i$. In this case, the ex ante preference $\succcurlyeq_i$ and the interim preference $\succcurlyeq_i^{t_i}$ will agree for all types t_i. See also a "construction" of these preferences from another perspective in section 7.4.

It is useful to observe that the preference just defined is an instance of the Maximin Expected Utility (MEU) preferences defined by Gilboa and Schmeidler (1989). To see this, let Δ_i denote the set of measures π on $T_i \times T_{-i} \times T_{-i}$. For $\pi \in \Delta$, let $\pi|_{T_i}$ denote the marginal of π in T_i. Define, for each i, the following set:

$$\mathcal{P}_i \equiv \{\pi \in \Delta : \pi|_{T_i} = \mu_i\}. \tag{6}$$

Then, the preference defined by (5) is equivalently defined by:

$$\begin{aligned} f \succcurlyeq_i g \iff \quad & \min_{\pi \in \mathcal{P}_i} \int_{T_i \times T_{-i} \times T_{-i}} u_i(t_i, t_{-i}, f(t_i, t'_{-i}))\, d\pi(t_i, t_{-i}, t'_{-i}) \\ & \geqslant \min_{\pi \in \mathcal{P}_i} \int_{T_i \times T_{-i} \times T_{-i}} u_i(t_i, t_{-i}, g(t_i, t'_{-i}))\, d\pi(t_i, t_{-i}, t'_{-i}), \end{aligned}$$

[4]The reader may think that the most natural definition of the preference would involve the min with respect to only one t_{-i}, that is, compare $\min_{t_{-i} \in T_{-i}} u_i(t_i, t_{-i}, f(t_i, t_{-i}))$. However, if one remembers the "complete ignorance" motivation cited above, definition (8) could be considered more natural. In any case, under the *private values assumption, these two definitions are equivalent*. Our results require (8) only for the general (interdependent values) case.

which is easily seen to be a particular case of Gilboa and Schmeidler's MEU.[5]

Finally, we will adopt the usual notation for the strict and symmetric part of the preference defined above. That is, we will write $f \succ_i g$ if $f \succcurlyeq_i g$ but it is not the case that $g \succcurlyeq_i f$ and we will write $f \sim_i g$ if $f \succcurlyeq_i g$ and $g \succcurlyeq_i f$.

We will always assume that the preferences are as described above, unless otherwise explicitly stated.

2.2 Allocations and endowments

An individual allocation is a function $f : T \to \mathcal{B}$. Each individual has an *initial endowment* $e_i : T \to \mathcal{B}$. We assume that individual i's endowment depends only on t_i and not on the types of other individuals, that is, we have the following:

Assumption 2.3 (Private information measurability of the endowments) *For every $i \in I$, $t_i \in T_i$ and $t_{-i}, t'_{-i} \in T_{-i}$, the endowments satisfy: $e_i(t_i, t_{-i}) = e_i(t_i, t'_{-i})$, that is, we assume that e_i is $\mathcal{F}_i$-measurable.*[6]

This assumption is almost always assumed in the literature regarding general equilibrium with asymmetric information, no-trade, auctions and mechanism design. In the latter, endowments are usually assumed to be constant with respect to types (as in Morris (1994)) or not explicitly considered. Note that if endowments are constant, assumption 2.3 is automatically satisfied. In auctions, the players are assumed to be buyers or sellers with explicit fixed endowments, which again implies assumption 2.3. Even when the endowments may vary with types, as in Jackson and Swinkels (2005), where the private information is given by (e_i, v_i), i.e., endowments and values, assumption 2.3 is still satisfied, because the endowment depends only on player i's private information. Note also that since we allow interdependent values, the *ex post value* of the endowment may vary across all states. The assumption is about only the *quantity* endowed, not values. In this sense, it may be considered a mild and natural assumption.

An *allocation* is a profile $x = (x_i)_{i \in I}$, where x_i is an individual allocation for individual i. An allocation is *feasible* if $\sum_{i \in I} x_i(t) = \sum_{i \in I} e_i(t)$, for every t. Unless otherwise explicitly defined, all allocations considered in this paper will be feasible.

[5] This is a particular case of the maximin expected utility axiomatized by Gilboa and Schmeidler (1989) because we require $\mathcal{P}_i$ to have the format given by (17), while the set $\mathcal{P}_i$ in Gilboa and Schmeidler (1989) has to be only compact and convex.

[6] $\mathcal{F}_i$ denotes the (σ)-algebra generated by the partition $\cup_{t_i \in T_i} \{t_i\} \times T_{-i}$.

2.3 Incentive compatibility

Our definition of incentive compatibility is standard. Formally, an allocation $x = (x_i)_{i\in I}$ is incentive compatible provided that there is no individual i and type t_i that could benefit from reporting t_i'' instead of his true type t_i', assuming that all other individuals report truthfully.

Before formalizing this notion, we would like to call attention to two aspects. First, when deciding to make a false report, the individual is at the interim stage and, therefore, makes all comparisons with respect to his interim preference. Second, note that in the case that individual i reports t_i'' instead of his true type t_i', he will receive the allocation $e_i\left(t_i', t_{-i}\right) + x_i\left(t_i'', t_{-i}\right) - e_i\left(t_i'', t_{-i}\right)$ instead of $x_i\left(t_i', t_{-i}\right)$, because $x_i\left(t_i'', t_{-i}\right) - e_i\left(t_i'', t_{-i}\right)$ is the trade that i is entitled to receive at the state $\left(t_i'', t_{-i}\right)$. Therefore, we have the following:

Definition 2.4 *An allocation x is incentive compatible (IC) if there is no i, t_i', t_i'' such that $\left[e_i\left(t_i', \cdot\right) + x_i\left(t_i'', \cdot\right) - e_i\left(t_i'', \cdot\right)\right] \succ_i^{t_i'} x_i\left(t_i', \cdot\right)$. For the maximin preference, this last condition can be expressed as:*

$$\begin{aligned} &\min_{t_{-i}, t'_{-i} \in T_{-i}} u_i\left(t_i', t_{-i}, e_i\left(t_i', t'_{-i}\right) + x_i\left(t_i'', t'_{-i}\right) - e_i\left(t_i'', t'_{-i}\right)\right) \\ &> \min_{t_{-i}, t'_{-i} \in T_{-i}} u_i\left(t_i', t_{-i}, x_i\left(t_i', t'_{-i}\right)\right). \end{aligned} \tag{7}$$

2.4 Notions of efficiency

Given the ex post, interim and ex ante preferences defined above, the following definitions are standard:

Definition 2.5 *Consider a feasible allocation $x = (x_i)_{i\in I}$ and let $\succcurlyeq_i, \succcurlyeq_i^{t_i}$ and $\succcurlyeq_i^{t}$ represent respectively the ex ante, interim and ex post preferences of agent $i \in I$, as defined above (see section 2.1). We say that x is:*

1. ex post efficient *if there is no feasible allocation $y = (y_i)_{i\in I}$ such that $y_i(t) \succcurlyeq_i^t x_i(t)$ for every i and $t \in T$, with strict preference for some i and t.*

2. interim efficient *if there is no feasible allocation $y = (y_i)_{i\in I}$ such that $y_i \succcurlyeq_i^{t_i} x_i$ for every i and $t_i \in T_i$, with strict preference for some i and t_i.*

3. ex ante efficient *if there is no feasible allocation $y = (y_i)_{i\in I}$ such that $y_i \succcurlyeq_i x_i$ for every i, with strict preference for some i.*

Let E_A, E_I and E_P denote, respectively, the sets of ex ante, interim and ex post efficient allocations.

Let $\mathcal{A}$ denote the set of allocations $a : T \to \mathcal{B}$ and $D_A(x), D_I(x)$ and $D_P(x)$ denote, respectively, the set of ex post, interim and ex ante deviations of $x \in \mathcal{A}$. That is, $D_A(x)$ is the set of those $y \in \mathcal{A}$ that satisfy the property defined in the item 1 above. Thus, $E_A = \{x : D_A(x) = \emptyset\}$. Analogous statements hold for $D_I(x)$, $D_P(x)$, E_I and E_A.

Holmstrom and Myerson (1983) note that $E_A \subset E_I \subset E_P$ for Bayesian preferences. However, in our maximin expected utility setting we have the following:

Proposition 2.6 *$E_A \subsetneq E_I$ but we may have $E_P \not\subset E_I$, $E_I \not\subset E_P$ and $E_A \not\subset E_P$.*

Proof. See the appendix. ∎

The fact that the inclusion $E_I \subset E_P$ may fail for maximin preferences is, however, not essential. First, it holds in one-good economies. Second, we could require the ex post efficiency together with the interim and the ex ante efficiency. We clarify both issues in the sequel.

Lemma 2.7 *In an one-good economy, $E_I \subset E_P$.*

Proof. Suppose that $x \in E_I \setminus E_P$. Then there exists y, j, t' such that $y_i \succsim_i^t x_i$ for all $i \in I, t \in T$ and $y_j \succ_j^{t'} x_j$. Since utilities are strictly increasing, we have $\sum_{i \in I} y_i(t') > \sum_i x_{i \in I}(t) = \sum_{i \in I} e_i(t)$, that is, y is not feasible. ∎

Now, let us consider the requirement of strong efficiency.

Definition 2.8 *We say that an allocation x is strongly efficient if $x \in E_A \cap E_I \cap E_P$. The set of strongly efficient allocations is denoted $E \equiv E_A \cap E_I \cap E_P$.*

We are interested in the following:

Proposition 2.9 *There exist strongly efficient allocations, that is, $E \neq \emptyset$.*

Proof. Trivially, there exists $x \in E_A$. By Proposition 2.6, $x \in E_I$. If $x \notin E_P$, then there exists ex post efficient y such that $y_i \succsim_i^t x_i$ for all i, t (and it improves upon x at least for one i, t). But then this implies that $y_i \succsim_i^{t_i} x_i$ and $y_i \succsim_i x_i$ also hold for all i, t_i. Since $x \in E_A \cap E_I$, $y \in E = E_A \cap E_I \cap E_P$. ∎

In this paper, we will be most concerned with the interim efficiency (which is equivalent to ex ante efficiency by the proposition above), because the interim stage is the one that most interests us, as discussed above. More explicitly, we consider the following:

Definition 2.10 *An allocation x is* maximin efficient *if it is* interim efficient *in the sense of definition 2.5, that is, there is no feasible allocation $y = (y_i)_{i\in I}$ such that $y_i \succcurlyeq_i^{t_i} x_i$ for every i and $t_i \in T_i$, with strict preference for some i and t_i, where $\succcurlyeq_i^{t_i}$ is the maximin preference, i.e.,*

$$f \succcurlyeq_i^{t_i} g \iff \min_{t_{-i},t'_{-i}\in T_{-i}} u_i(t_i,t_{-i},f(t_i,t'_{-i})) \geqslant \min_{t_{-i},t'_{-i}\in T_{-i}} u_i(t_i,t_{-i},g(t_i,t'_{-i})). \tag{8}$$

3 Efficiency and incentive compatibility

The conflict between efficiency and incentive compatibility, that is, the fact that some (or even all) efficient allocations may fail to be incentive compatible for expected utility preferences is well-known. For an illustration of this conflict, see the appendix. In this section, we show that such conflict does not exist if the preferences are maximin. That is, we show that maximin efficiency implies incentive compatibility.

Theorem 3.1 *If $x = (x_i)_{i\in I}$ is a maximin efficient allocation, then x is incentive compatible.*

It should be noted that the interim efficiency required in the definition of maximin efficiency (see definition 2.10) is the most natural for the above result, since the incentive compatibility condition is an interim notion. In other words, the theorem above maintains a parallelism of timing (interim) between premise and conclusion.

Proof of Theorem 3.1: Suppose that x is not incentive compatible. This means that there exists an individual i and types t'_i, t''_i such that:

$$\begin{aligned}&\min_{t_{-i},t'_{-i}\in T_{-i}} u_i\left(t'_i,t_{-i},e_i\left(t'_i,t'_{-i}\right)+x_i\left(t''_i,t'_{-i}\right)-e_i\left(t''_i,t'_{-i}\right)\right)\\ &>\min_{t_{-i},t'_{-i}\in T_{-i}} u_i\left(t'_i,t_{-i},x_i\left(t'_i,t'_{-i}\right)\right).\end{aligned} \tag{9}$$

We will prove that x cannot be maximin Pareto optimal by constructing another feasible allocation $y = (y_i)_{i \in I}$ that Pareto improves upon x. For this, define

$$y_j\left(t_i, t_{-i}\right) = \begin{cases} x_j\left(t_i, t_{-i}\right), & \text{if } t_i \neq t_i' \\ e_j\left(t_i', t_{-i}\right) + x_j\left(t_i'', t_{-i}\right) - e_j\left(t_i'', t_{-i}\right), & \text{if } t_i = t_i' \end{cases} \tag{10}$$

To see that $\left(y_j\right)_{j \in I}$ is feasible, it is sufficient to consider what happens when $t_i = t_i'$:

$$\begin{aligned} \sum_{j \in I} y_j\left(t_i', t_{-i}\right) &= \sum_{j \in I} e_j\left(t_i', t_{-i}\right) + \sum_{j \in I} x_j\left(t_i'', t_{-i}\right) - \sum_{j \in I} e_j\left(t_i'', t_{-i}\right) \\ &= \sum_{j \in I} e_j\left(t_i', t_{-i}\right), \end{aligned}$$

because $\sum_{j \in I} x_j\left(t_i'', t_{-i}\right) = \sum_{j \in I} e_j\left(t_i'', t_{-i}\right)$, from the feasibility of x_j at $\left(t_i'', t_{-i}\right)$.

From (9) and (10), we have $y_i \succ_i^{t_i'} x_i$ and $y_i \sim_i^{t_i} x_i$ for any $t_i \neq t_i'$. It remains to prove that $y_j \succcurlyeq_j^{t_j} x_j$ for any $j \neq i$ and t_j. The fact that e_j depends only on t_j implies that $e_j\left(t_i', t_{-i}\right) = e_j\left(t_i'', t_{-i}\right)$ for all $t_{-i} \in T_{-i}$. Then, for every $t_{-i} \in T_{-i}$,

$$y_j\left(t_i', t_{-i}\right) = e_j\left(t_i', t_{-i}\right) + x_j\left(t_i'', t_{-i}\right) - e_j\left(t_i'', t_{-i}\right) = x_j\left(t_i'', t_{-i}\right). \tag{11}$$

For each $t_j \in T_j$, define $X_j^{t_j}$ as the set $\{x_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}$ and $Y_j^{t_j} \equiv \{y_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}$. Fix a $t = (t_i, t_j, t_{-i-j})$. If $t_i \neq t_i'$, the definition (10) of y_j implies that $y_j(t) = x_j(t) \in X_j(t_j)$. If $t_i = t_i'$, (11) gives $y_j\left(t_i', t_{-i}\right) = x_j\left(t_i'', t_{-i}\right) \in X_j(t_j)$. Thus, $Y_j(t_j) \subset X_j(t_j)$, for all $t_j \in T_j$. Therefore,

$$\underline{y}_j\left(t_j\right) = \min_{t_{-j}, y \in Y_j^{t_j}} u_j(t_j, t_{-j}, y) \geqslant \min_{t_{-j}, x \in X_j^{t_j}} u_j(t_j, t_{-j}, x) = \underline{x}_j\left(t_j\right). \tag{12}$$

This shows that $y_j \succcurlyeq_j^{t_j} x_j$ for all $j \neq i$ and $t_j \in T_j$. Thus, y is a Pareto improvement upon x, that is, x is not maximim efficient. ∎

The reader can observe that the only place where we used the specific definition of the interim preference as the minimum was to conclude (12). Indeed if we were to use other preferences (in particular the expected utility preferences), this step would not go through.

Maximin efficiency in fact implies coalitional incentive compatibility, which is a strictly stronger notion. For this consider the following definition, which corresponds to a definition introduced by Krasa and Yannelis (1994) for the partition model.

Definition 3.2 (Blocking Coalition) *A nonempty set $C \subset I$ is a blocking coalition to an allocation $x = (x_i)_{i \in I}$ if there exist two profiles $t'_C = (t'_i)_{i \in C}$ and $t''_C = (t''_i)_{i \in C}$ and transfers $\tau_C = (\tau_i)_{i \in C} \in \mathcal{B}^{|C|}$ such that*

$$[e_i(t'_i, \cdot) + x_i(t''_i, \cdot) - e_i(t''_i, \cdot) + \tau_i] \succ_i^{t'_i} x_i(t'_i, \cdot),$$

for all $i \in C$, where $\succ_i^{t'_i}$ denotes the strict maximin interim preference at type t'_i.

Definition 3.3 *An allocation x is* coalitional incentive compatible (CIC) *if there is no blocking coalition to x.*

Thus, the following result is actually a stronger version of Theorem 3.1:

Theorem 3.4 *If x is maximin efficient, x is coalitional incentive compatible.*

Proof. It is enough to adapt Theorem 3.1's proof, substituting i by the blocking coalition C. ∎

4 Necessity of maximin preferences

From the results presented in section 3, a natural question is: does any other preference also have the property of no conflict between efficiency and incentive compatibility? In this section, we answer this question in the negative, that is, we show that if the preferences satisfy some reasonably weak properties (which we will define in a moment) but are not maximin, then there are allocations which are efficient but not incentive compatible. In other words, only maximin preferences present no conflict between efficiency and incentive compatibility in this sense. For formalizing these results, we need some definitions.

For this section, assume that $\mathcal{B} = \mathbb{R}^{\ell}_{+}$, for some $\ell \in \mathbb{N}$. Let $\succsim$ be a preference over the set $\mathcal{F}$ of all functions $f : T \to \mathcal{B}$.

Definition 4.1 *We say that $\succsim$ is:*

1. *complete if for every f, g, $f \succsim g$ or $g \succsim f$.*
2. *transitive if for every f, g, h, $f \succsim g$ and $g \succsim h$ imply $f \succsim h$.*
3. *monotonic if $f(\omega) \geqslant (\gg) g(\omega), \forall \omega \in \Omega$ implies $f \succsim (\succ) g$.*[7]

[7]By $f(\omega) \gg g(\omega)$ we mean that all coordinates of $f(\omega)$ are strictly above all coordinates of $g(\omega)$. We write $f \succ g$ if $f \succsim g$ but it is not the case that $g \succsim f$.

4. *continuous if for all* $f, g, h \in \mathcal{L}$, *the sets* $\{\alpha \in [0,1] : \alpha f + (1-\alpha) g \succsim h\}$ *and* $\{\alpha \in [0,1] : h \succsim \alpha f + (1-\alpha) g\}$ *are closed.*

We wish to state results about preferences $\{\succsim_i\}_{i \in I}$, such that they agree with the maximin preference $\succcurlyeq$ on constant acts and such that each type is important for the ex ante preference. We will also require that the (ex post) utility function has a convex image. These mild conditions are summarized in the following:

Definition 4.2 *We say* $\succsim_i$ *is* adequate *if the following conditions hold:*

- *it agrees* ex post *with the maximin preferences* $\succcurlyeq_i$, *that is, if* $f(t) = a$ *and* $g(t) = a'$ *for all* $t \in T$, *then,*[8]

$$f \succsim_i g \iff f \succcurlyeq_i g \iff u_i(t, a) \geqslant u_i(t, a'). \tag{13}$$

- *the image of the function* $a \mapsto u_i(t, a)$ *is an interval* $I \subseteq \mathbb{R}$, *for every* $t \in T$;

- *if* $f \succsim_i^{t_i} g$ *for all* $t_i \in T_i$ *and there is* $t_i' \in T_i$ *such that* $f \succ_i^{t_i'} g$, *then* $f \succ g$.

In this section, we want to prove the following:

Theorem 4.3 *Let* 1 *and* 2 *be individuals with preferences* $\succsim_1$ *and* $\succsim_2$, *respectively, which are adequate, complete, transitive, monotonic and continuous. If at least one of these preferences is not maximin, then there exists an allocation* $x = (x_1, x_2)$ *which is Pareto optimal but not incentive compatible.*

This Theorem's proof can be easily extended to the following:

Theorem 4.4 *Let* $I = \{1, ..., N\}$ *be a set of individuals with preferences* $\succsim_i$, *for* $i = 1, ..., N$, *which are adequate, complete, transitive, monotonic and continuous. If at least one of these preferences is not maximin, then there exists an allocation* $x = (x_i)_{i \in I}$ *which is Pareto optimal but not coalitionally incentive compatible.*

For establishing these results, we begin by providing a new "axiomatization" of maximin preferences, which will be useful in the proof of the above results.[9] This is done in section 4.1 below. The proofs of all results in this section are included in the appendix.

[8] Remember that $\succcurlyeq_i$ denotes the ex ante maximin preference.

[9] Some purist decision theorists may oppose the qualification of Theorem 4.7 below as an "axiomatization" of maximin preferences on the grounds that one of its assumptions— namely, "supported by minimal prices," introduced below— is not a standard axiom in the decision theory tradition. In this case, it would be better to read "characterization" whenever we write "axiomatization."

4.1 Characterization of Maximin preferences

For providing an axiomatization of maximin preferences, it is convenient to adopt a slightly more abstract setting than the rest of the paper and consider the standard setup of decision under complete ignorance. Thus, in this section let the finite set W represent the alternatives w about which a decision maker is completely ignorant. The decision maker has a preference $\succsim$ over the set $\mathcal{F}$ of all real valued functions $f : W \to I \subseteq \mathbb{R}$, with $\succ$ denoting, as before, its strict part. We can identify $\mathcal{F}$ with (a subset of) the Euclidean space $\mathbb{R}^{|W|}$ and use its Euclidian norm, topology, etc. Recall that $\succsim$ is maximin if for all $f, g \in \mathcal{F}$,

$$f \succsim g \iff \min_{w \in W} f(w) \geqslant \min_{w \in W} g(w).$$

Our main assumption depends on the set of supporting probabilities. As usual, let $\Delta(W)$ denote the set of probabilities in W.

Definition 4.5 *Fix a preference $\succsim$. For each $f \in \mathcal{F}$, the set of supporting probabilities at f is*

$$S_f^{\succsim} \equiv \{p \in \Delta(W) : g \succsim f \Rightarrow p \cdot g \geqslant p \cdot f\}.$$

Whenever $\succsim$ is clear from the context, we will write S_f instead of $S_f^{\succsim}$.

Definition 4.6 $\succsim$ *is* supported by minimal prices *if for all $f \in \mathcal{F}$ and $p \in S_f^{\succsim}$,*

$$f(w') > \min_{w \in W} f(w) \implies p(w') = 0. \tag{14}$$

We have the following characterization:

Theorem 4.7 *A preference $\succsim$ is maximin if and only if it is complete, transitive, monotonic, continuous and supported by minimal prices.*

Proof. See the appendix. ∎

The following result is used in the proof of theorems 4.3 and 4.4 and might be of interest in its own.

Proposition 4.8 *Suppose that $\succsim$ is complete, transitive, monotonic and continuous. If $\succsim$ is not maximin, there exists*

$$h \in \mathcal{E} \equiv \{f \in \mathcal{F} : \exists w' \in W \text{ such that } f(w') > \min_{w \in W} f(w)\},$$

such that for every $g \neq h$ satisfying $h \geqslant g$, we have $h \succ g$.

Proof. See the appendix. ∎

5 A mechanism design perspective

It is natural to ask what is the relevance of the above results from a mechanism design perspective. This section clarifies this issue. We begin by translating the usual mechanism design setting into our framework. The set of individuals and their information is exactly as we described before and there is a mechanism designer who wants to implement an efficient allocation. Instead of initial endowments, the mechanism design literature uses to consider only initial levels of utility, to inform whether it is individually rational or not to participate in the mechanism. Of course, this is made only for simplicity and in many cases, endowments could be explicitly defined. In the sequel, we consider separately the two cases.

5.1 Case with explicit initial endowments

Suppose that the individuals have initial endowments $e = (e_i)_{i \in I}$. The mechanism designer wants to find a mechanism that implements a feasible allocation $x = (x_i)_{i \in I}$. Here, we are concern with efficient allocations. In particular, allocations that are efficient in the strongest sense.[10] A mechanism is incentive compatible if no individual has an interest of misreporting his information (see definition 2.4). A mechanism is *budget balanced* if it can be implemented for any report by the agents, without the need of extra goods.

Theorem 5.1 *Suppose that $x = (x_i)_{i \in I}$ is a strongly efficient feasible allocation and each agent $i \in I$ has a maximin preference as defined in section 2.1. Then, there exists a mechanism that implements x and is incentive compatible and budget balanced.*

Moreover, if x dominates (ex ante, interim, ex post) the initial endowment e for each consumer, then the mechanism is also (ex ante, interim, ex post) individually rational.

Proof. Let us define a simple mechanism that implements the strongly efficient allocation x. The space of messages for individual i is just the set of types T_i. The mechanism simply implements the transfers that are supposed to occur at the reported types. That is, if agents report the profile of types $t' = (t'_1, ..., t'_n)$, while their true types are $t = (t_1, ..., t_n)$ then individual i's final allocation will be $e_i(t) + x_i(t') - e_i(t')$, since $x_i(t') - e_i(t')$ is transfer supposed to occur if the types are t'. Now the fact that this mechanism is budget balanced comes from the

[10] See definition 2.8.

fact that x is feasible, that is, $\sum_{i\in I}(x_i(t') - e_i(t')) = 0$ for every $t' \in T$. This mechanism is incentive compatible by Theorem 3.1.

Note that the initial endowments mark the reference point for each individual. Therefore, besides their role of defining feasibility, they also define whether it is individually rational for each individual to participate in the mechanism. If the outcome will be better for the individual, then it is in his interest to participate in the mechanism. Thus, the claim in the second paragraph is straightforward. ∎

5.2 Public outcomes

In some models, as in d'Aspremont and Gérard-Varet (1979), the individuals may care about the whole allocation, that is, the set of bundles is $\mathcal{B} = O \times \mathbb{R}$, where $O \subset \mathbb{R}^\ell_+$ indicates the set of *possible physical outcomes*, as in the set of all possible public projects. The last component of $\mathcal{B}$, namely $\mathbb{R}$, refers to monetary transfers among the n individuals. The (ex post) utility is given by: $u_i(t,(a,\tau_i)) = v_i(t_i,a) + \tau_i$, where $\tau_i \in \mathbb{R}$ and $a \in O \subset \mathbb{R}^\ell_+$. In this case, it is natural to consider *outcome* efficiency instead of the normal efficiency.

Definition 5.2 *We say that $a^* \in O$ is outcome efficient if:*

$$\sum_{i\in I} v_i(t_i,a^*) = \sup_{a\in O}\sum_{i\in I} v_i(t_i,a).$$

Note that the setting above is slightly more general than in the rest of the paper (at least in one direction), since the consumers may care not only about their own consumption (a_i) but above the entire a. Indeed, in this setup we do not need even to refer to individual consumptions. In other words, it is possible to consider externalities in this setup. The following simple result establishes the connection between outcome efficiency and (strong) Pareto efficiency. Since we are not considering endowments here, we need to substitute the feasibility constraint by a condition of the type $\sum_{i\in I}\tau_i = c$, for some $c \in \mathbb{R}$. Although the actual c is not important, we will focus on the case of budget balance, that is, $c = 0$. Therefore, it is straightforward to adapt for this setup the notion of strong efficiency introduced in definition 2.8.

Lemma 5.3 *a^* is outcome efficient if and only if there exists $\tau = (\tau_i)_{i\in I} \in \mathbb{R}^n$ such that (a^*,τ) is strongly Pareto efficient and $\sum_{i\in I}\tau_i = 0$.*

Proof. Let (a^*, τ) be strongly efficient, with $\sum_{i\in I} \tau_i = 0$ but a^* is not outcome efficient. Then there exists $o \in O$ such that

$$r \equiv \sum_{i\in I} v_i(t_i, o) = \max_{a\in O} \sum_{i\in I} v_i(t_i, a) > \sum_{i\in I} v_i(t_i, a^*) \equiv s.$$

Then, for any $\tau \in \mathbb{R}^n$, the allocation (a^*, τ) is dominated by (o, τ'), where

$$\tau_i' = v_i(t_i, a^*) + \tau_i - v_i(t_i, o) + \frac{r - s}{n}.$$

Indeed, $v_i(t_i, o) + \tau_i' > v_i(t_i, a^*) + \tau_i$ and $\sum_{i\in I} \tau_i' = \sum_{i\in I} \tau_i$. This shows that there is no $\tau \in \mathbb{R}^n$ such that (a^*, τ) is Pareto optimal.

Conversely, assume that a^* is outcome efficient. Let (o, τ) be a strongly efficient allocation with $\sum_{i\in I} \tau_i = 0$. By the first part of the proof, o is outcome efficient, that is, $\sum_{i\in I} v_i(t_i, o) = \sum_{i\in I} v_i(t_i, a^*)$. Define $\tau_i' \equiv v_i(t_i, o) + \tau_i - v_i(t_i, a^*)$. Then for each i, (a^*, τ_i') is indifferent (ex post, interim and ex ante) to (o, τ_i). Therefore, (a^*, τ') is strongly efficient and $\sum_{i\in I} \tau_i' = 0$. ∎

In a sense, this result shows that when we have monetary transfers, we just need to worry about outcome efficiency, instead of Pareto efficiency. This justifies the focus on this definition of efficiency, usually considered in the mechanism design literature.

Now, we consider decision rules, which are functions $d : T \to O$ from types to outcomes. We say that d is outcome efficient if $d(t)$ is outcome efficient for every $t \in T$. A mechanism $m = (d, \tau)$ consists of a decision rule $d : T \to O$ and transfers $\tau : T \to \mathbb{R}^n$. A mechanism m is budget balanced if $\sum_{i\in I} \tau(t) = 0$ for all $t \in T$ and it is incentive compatible if there is no individual i and types t_i, t_i' such that

$$\min_{t_{-i}\in T_{-i}} \left[v_i\left(t_i, d(t_i', t_{-i})\right) + \tau(t_i', t_{-i})\right] > \min_{t_{-i}\in T_{-i}} \left[v_i\left(t_i, d(t_i, t_{-i})\right) + \tau(t_i, t_{-i})\right].$$

Finally, we say that d is incentive compatible if there exist transfers $\tau : T \to \mathbb{R}^n$ such that (d, τ) is incentive compatible.

Theorem 5.4 *Assume that individuals have private values and maximin preferences.*[11] *If the decision rule $d : T \to O$ is outcome efficient, then it is incentive compatible.*

[11] The assumption of private values is not important for the result. It is used just to simplify the minimizations. It can be extended to interdependent values as we did in Theorem 3.1.

Proof. By Lemma 5.3, we can find $\tau : T \to \mathbb{R}^n$ such that (d,τ) is strongly efficient. Suppose that (d,τ) is not incentive compatible. This means that there exists an individual i and types t'_i, t''_i such that:

$$\min_{t_{-i}\in T_{-i}} \left[v_i\left(t'_i, d(t''_i, t_{-i})\right) + \tau_i(t''_i, t_{-i})\right] > \min_{t_{-i}\in T_{-i}} \left[v_i\left(t'_i, d(t'_i, t_{-i})\right) + \tau_i(t'_i, t_{-i})\right]. \tag{15}$$

Let T'_{-i} denote the set of those $t'_{-i} \in T_{-i}$ that realize the minimum for t'_i, that is:

$$v_i\left(t'_i, d(t'_i, t'_{-i})\right) + \tau_i(t'_i, t'_{-i}) = \min_{t_{-i}\in T_{-i}} \left[v_i\left(t'_i, d(t'_i, t_{-i})\right) + \tau_i(t'_i, t_{-i})\right].$$

Define $d' : T \to O$ and $\tau' : T \to \mathbb{R}^n$ as follows: if $t_i \neq t'_i$ or $t_{-i} \notin T'_{-i}$, put $d'(t_i, t_{-i}) = d(t_i, t_{-i})$ and $\tau'(t_i, t_{-i}) = \tau(t_i, t_{-i})$; otherwise, define:

$$d'(t'_i, t_{-i}) = d(t''_i, t_{-i});\ \text{and}\ \tau'(t'_i, t_{-i}) = \tau(t''_i, t_{-i}).$$

Since $\sum_{j\in I} \tau'_j(t_i, t_{-i}) = \sum_{j\in I} \tau_j(t_i, t_{-i}) = 0$ if $t_i \neq t'_i$ or $t_{-i} \notin T'_{-i}$, and $\sum_{j\in I} \tau'_j(t'_i, t_{-i}) = \sum_{j\in I} \tau_j(t''_i, t_{-i}) = 0$ otherwise, then $\sum_{j\in I} \tau'_j(t) = 0$ for every $t \in T$.

For each $j \neq i$, define the set of utilities achieved by individual j with type t_j:

$$U_j(t_j) \equiv \{v_j(t_j, d(t_j, t_{-j})) + \tau_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}.$$

Observe that we used (d,τ) in the definition of $U_j(t_j)$. Thus, unless $t_i = t'_i$ and $t_{-i} \in T'_{-i}$, we have

$$v_j(t_j, d'(t_j, t_{-j})) + \tau'_j(t_j, t_{-j}) = v_j(t_j, d(t_j, t_{-j})) + \tau_j(t_j, t_{-j}) \in U_j(t_j).$$

Now consider $t' = (t'_i, t'_{-i})$, where $t'_{-i} = (t_j, t'_{-i-j}) \in T'_{-i}$, $t_j \in T_j$. Then,

$$v_j(t_j, d'(t_j, t'_i, t'_{-i-j})) + \tau'_j(t_j, t'_i, t'_{-i-j}) = v_j(t_j, d(t_j, t''_i, t'_{-i-j})) + \tau_j(t_j, t''_i, t'_{-i-j}).$$

Note, however, that $(t''_i, t'_{-i-j}) = t_{-j}$ for some $t_{-j} \in T_{-j}$. Therefore, for any $j \neq i$, $t_j \in T_j$ and $t_{-j} \in T_{-j}$,

$$v_j(t_j, d'(t_j, t_{-j})) + \tau'_j(t_j, t_{-j}) \in U_j(t_j).$$

This allows us to obtain the following inequality:

$$\begin{aligned}
& \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j, d(t_j, t_{-j})\right) + \tau_j(t_j, t_{-j})\right] \\
= \quad & min_{u_j\in U(t_j)} u_j \\
\leqslant \quad & \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j, d'(t_j, t_{-j})\right) + \tau'_j(t_j, t_{-j})\right].
\end{aligned}$$

This shows that (d', τ') is not *interim* worse than (d,τ) for any $j \neq i$. On the other hand, (15) shows that (d', τ') is strictly (interim) better for i. Therefore, (d,τ) cannot be strongly efficient. ∎

5.3 Myerson-Satterthwaite setup

It is now straightforward to apply Theorem 5.4 to the Myerson-Satterthwaite setup described in the introduction. We can in fact be a little bit more general. Suppose that there are n buyers (individuals 1,2,...,n) and m sellers (individuals $n+1$, ..., $n+m$) of indivisible objects. Individual $i \in I \equiv \{1, ..., n+m\}$ has valuation t_i for the object.[12] Since the objects are indivisible, we consider: $O = \{o \in \{0,1\}^{n+m} : \sum_{i \in I} o_i = m\}$. Then, an allocation (decision) rule $d : T \to O$ is outcome efficient if for every $t \in T$, $d_i(t) = 1$ if and only if t_i is among the m highest valuations. Let us denote by d^* an outcome efficient allocation rule. It is easy to see that d^* is essentially unique (there is room for multiplicity only in the way that ties are broken). Then we have the following:

Corollary 5.5 *Let d^* be an outcome efficient allocation rule. There are transfers $\tau : T \to \mathbb{R}^n$ such that (d^*, τ) is incentive compatible, budget balanded and individually rational.*

6 How do Bayesian and maximin efficiency compare?

From the fact that all maximin efficient allocations are incentive compatible, the reader may wonder whether maximin efficiency is not an excessively strong requirement, which could explain our results. In this section, we address this question in particular cases. For one-good economies, we show that whenever an allocation is Bayesian efficient and incentive compatible, then it is also maximin efficient. That is, the set of maximin efficient allocations is at least as large as the set of Bayesian efficient and incentive compatible allocations. For economies with numéraire, studied in section 5.2, we show that maximin strong efficiency is equivalent to Bayesian strong efficiency.

However, the formal statement of these results require a clarification of the relationship between the maximin and the Bayesian preferences. For this, it is useful to recall how we defined the ex ante maximin preference, at the end of section 2.1—see discussion after equation (4). There, we assumed the existence of a measure μ_i generating the types t_i for each agent $i \in I$. In fact, μ_i could be consider just the marginal of agent i's belief π_i over T_i, if this agent were Bayesian. In other words, we consider two economies:

[12]We could describe a Jackson and Swinkels (2005) double auction model, where each individual can be a buyer or seller and can have endowments of multiple units. This setup is essentially the one already covered by Theorem 3.1 and it does not seem necessary to reconsider it here.

- a maximin economy, exactly as described and studied up to now, which will denote (in this section only) by $\mathcal{E}^M$; and
- a Bayesian economy, where the agents have Bayesian preferences $\succsim_i$, defined by ex post utility functions $u_i : T \times \mathcal{B} \to \mathbb{R}$ (the same as in the maximin economy) and the priors π_i, satisfying $\pi_i|_{T_i} = \mu_i$, that is,

$$f \succsim_i g \iff \int_T u_i(t, f(t))\, \pi_i(dt) \geqslant \int_T u_i(t, g(t))\, \pi_i(dt).$$

  The interim and ex post preferences are defined in the usual way. This variant will be denoted by $\mathcal{E}^B$.

6.1 One-good economies

Our result for one-good economies is the following:

Theorem 6.1 *Consider a one-good economy with private values. If x is an interim efficient allocation in $\mathcal{E}^B$ which is also coalitionally incentive compatible,*[13] *it is an interim efficient allocation in $\mathcal{E}^M$, i.e., x is maximin Pareto optimal. The reverse is not true.*

Proof. See the appendix. ∎

This result shows that the maximin preferences do not destroy efficient and incentive compatible outcomes. To the contrary, any incentive compatible outcome that is efficient under a Bayesian preference will be also efficient under the corresponding maximin preference.

6.2 Economies with numéraire

Now, consider an economy as described in section 5.2, that is, the set of bundles is $\mathcal{B} = O \times \mathbb{R}$, where $O \subset \mathbb{R}^\ell_+$ indicates the set of *possible physical outcomes*. The last part of $\mathcal{B}$ ($\mathbb{R}$) refers to monetary transfers among the n individuals. The (ex post) utility is given by: $u_i(t, (a, \tau_i)) = v_i(t_i, a) + \tau_i$, where $\tau_i \in \mathbb{R}$ and $a \in O \subset \mathbb{R}^\ell_+$. Recall that an allocation is strongly efficient if it is ex ante,

[13] See definitions 3.2 and 3.3 for the definition of coalitionally incentive compatible in the maximin setting. The definition in the Bayesian setting is practically the same: the only difference is that we consider the interim Bayesian preference instead of the interim maximin preference.

interim and ex post efficient. In this section, we will consider two variations of this concept: Bayesian strongly efficient and maximin strongly efficient, if the agents have Bayesian and maximin preferences, respectively. The following result shows that maximin strong efficiency is the same as Bayesian strong efficiency:

Proposition 6.2 *In the economy with transfers described above, an allocation is Bayesian strongly efficient allocation if and only if it is also maximin strongly efficient.*

Proof. In Lemma 5.3, we show that (a^*, τ) is maximin strongly Pareto efficient for some $\tau = (\tau_i)_{i \in I} \in \mathbb{R}^n$ satisfying $\sum_{i \in I} \tau_i = 0$ if and only if a^* is outcome efficient. An examination of the proof of that Lemma shows that it establishes the same equivalence for Bayesian strong efficiency. Therefore, the two concepts are equivalent to outcome efficiency of a^*. ∎

Despite the fact that strong efficiency agree for the two kind of preferences, incentive compatibility does not. That is, a strong efficient allocation will be incentive compatible under maximin preferences but will not be incentive compatible under Bayesian preferences in general.

7 Maximin preferences—a reassessment

As we emphasized previously, it is not important for our contribution whether or not maximin preferences are realistic or representative of actual market participants' preferences. This is ultimately an empirical question, which goes beyond the scope of this paper.

However, we recognize that the pessimism exhibited in the maximin preferences may suggest that they are unrealistic. Despite being able to explain the Ellsberg paradox, for instance, some readers may find this pessimism excessive. Such skeptical readers may then ask what is the contribution of our exercise after all. Notwithstanding the fact that unrealistic assumptions are useful (and overly used) in economic theory,[14] such a skeptical reader can learn that the conflict between efficiency and incentive compatibility is so severe as to disappear only under (what he considers) unacceptably restrictive conditions on the preferences. From this

[14] Some important papers in economics, as Modigliani and Miller (1958), have presented striking conclusions under unrealistic assumptions. Thus, the fact that a fundamentally important economic conclusion depends on restrictive assumptions can be of general economic interest.

point of view, our contribution highlights the difficulties associated to obtaining incentive compatibility and reminds that incentive compatibility can (should?) be studied with respect to more general preferences.[15] This obvious direction of future research is further discussed in section 9 below.

These remarks clarify that the skepticism with respect to the *descriptive* power of maximin preferences do not strip this paper of relevant content. However, the objective of this section is to revisit the *normative* reasons for the dismissal of maximin preferences and reassess its suitability for the economic environment studied in this paper. For this, we begin by discussing Harsanyi's approach to the problem of incomplete information in games in section 7.1. We show that maximin preferences could be seen as a reasonable alternative to Harsanyi's approach. Then, in section 7.2 we discuss the motivation of maximin preferences through games and observe that its classic justification is suitable for the economic environment studied in this paper.

7.1 Uncertainty in games and Harsanyi's approach

In a seminal paper, Harsanyi (1967-8) proposed an approach to treat games of incomplete information that begins with a parametrization of the utility functions and beliefs of the game's participants, through "types." Assuming also common priors, he turns games of incomplete information into tractable objects. Later, Mertens and Zamir (1985) developed a rigorous construction of Harsanyi's types. See also Böge and Eisele (1979). In sum, Harsanyi's approach is composed of the following main ideas:

1. The uncertainty of the game is described in terms of "types" of players;
2. "Each player is assumed to know his own actual type but to be in general *ignorant* about the other players' actual types;"[16]
3. "In dealing with incomplete information, every player i will use the Bayesian approach. That is, he will assign a subjective joint probability distribution P_i to all variables unknown to him—or at least to all unknown independent variables, i.e., to all variables not depending on the players' own strategy

[15]Our point concerns a non-expected utility preference and therefore is not covered by previous works that considered risk aversion, even in extreme forms. It is well-known that ambiguity aversion has distinct implications from risk aversion. For a short and direct discussion about the maximin criterion and risk aversion, see Binmore (2008, p. 52-3). See also Bodoh-Creed (2010).

[16]Harsanyi (1967-8, Part I, p. 172). Emphasis added.

> choices. Once this has been done he will try to maximize the mathematical expectation of his own payoff x_i in terms of this probability distribution P_i. This assumption will be called the *Bayesian hypothesis*."[17]

In our model, we keep the first two points above, while we drop the third, that is, the Bayesian hypothesis. Note that Harsanyi describes each participant as "ignorant" about the others' types. If they are *ignorant*, they could use any of the criterion proposed for decision under ignorance, among which there is the maximin criterion.[18] Therefore, our change in the basic model is actually a minor (and natural) departure from Harsanyi's: instead of the Bayesian assumption that, in face of uncertainty, everybody has a prior, we consider an alternative decision criterion that was studied in the literature since the 1940's (see below).

A couple of remarks are in order. First, some authors (as Luce and Raiffa (1989), discussed below) refer to situations where the maximin criterion is applied or suggested as situations of "complete ignorance," while the Bayesian criterion is used in situations of "partial ignorance." Although this terminology suggests a scope of applicability, it does not offer clear boundaries. In a classic paper about decision criterion for situations under "complete ignorance," Milnor (1954, p. 49) notes that a situation of partial ignorance could be reformulated to one of complete ignorance.[19] We illustrate how this reformulation can be done in section 7.4 below, when we use the "ignorance" perspective to revisit the famous Ellsberg's paradox.

Another observation is that the construction of types as executed by Mertens and Zamir (1985) already presumes Bayesian beliefs. In this case, it would not be clear how to construct hierarchies of "beliefs" if the preferences are not Bayesian. Fortunately, Epstein and Wang (1996) already solved this problem, extending Mertens and Zamir (1985)'s construction to general preferences.

These brief comments are collected here to point out the opportunity of revisiting Harsanyi's approach. We still know very little about how the classical criteria

[17] Harsanyi (1967-8, Part I, p. 167). Emphasis as in the original.

[18] Milnor (1954) consider three other: minimax regret, the principle of insufficient reason and the Hurwicz criterion. See more discussion on this topic below.

[19] For readers' convenience, we transcribe here the quote: " Our basic assumption that the player has absolutely no information about Nature may seem too restrictive. However, such no information games may be used as a normal form for a wider class of games in which certain types of partial information are allowed. For example if the information consists of bounds for the probabilities of the various states of Nature, then by considering only those mixed strategies for Nature which satisfy these bounds, we construct a new game having no information. Unfortunately in practice partial information often occurs in vague, non-mathematical forms which are difficult to handle." Milnor (1954, p. 49).

for decision under ignorance and different preferences interact with Harsanyi's approach to games. This paper is but an initial exploration of this matter.

7.2 Maximin preferences and games

In their classic book about game theory, Luce and Raiffa (1989, p. 275) classify individual decision making "according to whether it is being carried out under conditions of certainty, risk, or uncertainty." Then they make the interesting observation that most of their book concerns a "very particular context of uncertainty known as a game."[20] From this, Luce and Raiffa (1989, p. 279) discuss different criteria for decision under complete ignorance. Among them, they present the maximin criterion and comment:

> The maximin principle can be given another interpretation which, although often misleading in our opinion, is sufficiently prevalent to warrant some comment. According to this view the decision problem is a two-person zero-sum game where the decision maker plays against a diabolical Miss Nature. The maximin strategy is then a best retort against nature's minimax strategy, i.e., against the "least favorable" *a priori* distribution nature can employ. We recall that in a two-person zero-sum game the maximin strategy makes good sense from various points of view: it maximizes 1's security level; and it is good against player 2's minimax strategy, which there is reason to suspect 2 will employ since it optimizes his security level and, in turn, it is good against 1's maximin strategy. In a game against nature, however, such a cyclical reinforcing effect is completely lacking.

Note how Luce and Raiffa dismiss the parallel with games because nature could hardly be accepted as strategic. In the cases we study, *there are* strategic agents reporting the types instead of a non strategic nature. Therefore, this dismissal does not directly apply. Indeed, Luce and Raiffa (1989, p. 307) briefly considers the case when the opponent is again a strategic player instead of the "diabolical Miss Nature" and observes that most of the axioms that characterize maximin preferences make (even more) sense in this situation. However, they refrain from re-evaluating the criterion in this case.

[20] That games can be considered a "context of uncertainty" already suggests how models of decision under uncertainty and incomplete information games are deeply related. This observation is, of course, not novel at all and it has been previously explored in detail. See for instance, Tan and Werlang (1988), who reduce any game to a decision problem under uncertainty.

Wald (1950, p.27), who advocates for the use of the maximin criterion on normative grounds, explores this connection:

> The analogy between the decision problem and a two-person game seems to be complete, except for one point. Whereas the experimenter wishes to minimize the risk $r(F, \delta)$, we can hardly say that Nature wishes to maximize $r(F, \delta)$. Nevertheless, since Nature's choice is unknown to the experimenter, it is perhaps not unreasonable for the experimenter to behave as if Nature wanted to maximize the risk. But, even if one is not willing to take this attitude, the theory of games remains of fundamental importance for the problem of statistical decisions, since as will be seen in Chapter 3, it leads to basic results concerning admissible decision functions and complete classes of decision functions.

Note again that Wald's main concern is the interpretation of Nature as a strategic player.

It should also be noted that a particular case of our model—namely, the one-good economy with constant aggregate endowment—literally corresponds to the case of a zero-sum game. In this case, the maximin preferences just encapsulate the natural equilibrium solution for this game. Of course, this justification is suitable only under conditions that lead to zero sum games. Whether people learn these choices in this kind of games and misapply them in other situations or whether it is possible to offer a more fundamental and general justification for maximin preferences can be themes of future research.

7.3 Criticism of the maximin criterion

Most common objections to maximin preferences concern variations on the following example. Two actions are to be chosen, where payoffs (in utils) depend on two situations (1 and 2) and are given in the following table.[21]

actions	situation 1	situation 2
a_1	x	100
a_2	1	1

By the maximin criterion, action a_2 will be strictly preferred to a_1 whenever $x < 1$. In this case, the individual ignores the possible upside of getting $100 >>$

[21] Luce and Raiffa (1989, p.279) discusses this example with $x = 0$.

1 if the actual situation is 2. In a game "against nature," this seems indeed an unreasonable choice if x is sufficiently close to 1. However, if situations 1 and 2 correspond to actions of a *strategic* player in a zero-sum game, this choice is not only more reasonable but it is in fact predicted by our game theory concepts. As we discussed above, all situations analyzed in this paper could be seen as an interaction between strategic agents, which makes this objection less relevant for our purposes.

7.4 Ignorance, Non-measurability and the Ellsberg's Paradox

One of the most repeated justifications for using maximin expected utility preferences (MEU) or other ambiguity averse preference is the Ellsberg's paradox. In this section, we revise and reframe it in the language of the asymmetric information literature. That is, we describe Ellsberg's paradox in terms of ignorance. Then, we use this asymmetric information language to "construct" MEU preferences in subsection 7.4.1.[22] Throughout this section, it will be more convenient to describe the decision maker information through partitions rather than types.

Consider an urn with *three balls*, one of which is *red*, and the other two are either *black or yellow*, but the exact composition is unknown. We will drawn a ball from this urn and we offer two different pair of bets for an individual to choose. In the first pair, it is offered the choice between the act[23] f_1 that pays $\$1$ if the red ball is drawn and zero otherwise and the act f_2 that pays $\$1$ if the ball is black and zero otherwise. In the second pair, the choice is between an act f_3 that pays $\$1$ if the ball is either red or yellow and zero otherwise and the act f_4 that pays $\$1$ if the ball is either black or yellow and zero otherwise. To summarize, f_i is given, for $i = 1, ..., 4$ as follows:

$$f_1(\omega) = \begin{cases} 1, & \omega = R \\ 0, & \text{otherwise} \end{cases} \qquad f_2(\omega) = \begin{cases} 1, & \omega = B \\ 0, & \text{otherwise} \end{cases}$$
$$f_3(\omega) = \begin{cases} 1, & \omega \in \{R, Y\} \\ 0, & \text{otherwise} \end{cases} \qquad f_4(\omega) = \begin{cases} 1, & \omega \in \{B, Y\} \\ 0, & \text{otherwise} \end{cases}$$

Most individuals exhibit preferences as: $f_1 \succ f_2$ and $f_4 \succ f_3$. This is called the Ellsberg Paradox because there is no expected utility that can rationalize this

[22] The way we frame this analysis is somewhat unusual, but it is just a matter of interpretation. This interpretation—or rather reinterpretation—is not central to this paper and none of our results depend on it.

[23] "Acts" is the terminology used by Savage (1954, 1972) for bets.

choice. To see this, assume w.l.o.g. $u(0) = 0, u(1) = 1$. The first preference would imply $\pi(\{R\}) > \pi(\{B\})$, while the second

$$\pi(\{B, Y\}) = \pi(\{B\}) + \pi(\{Y\}) > \pi(\{R, Y\} = \pi(\{R\}) + \pi(\{Y\}),$$

that is, $\pi(\{B\}) > \pi(\{R\})$ and these implications contradict each other.

Now, let's formulate this example in the asymmetric information terminology. Let $\Omega = \{R, B, Y\}$, corresponding to the color of a ball (red, black, yellow) to be extracted from an urn. For simplicity, let us assume that the utility index of the individual is $u(x) = x$. The agent's information about the state of the nature is described by the algebra generated by the following partition: $\mathcal{F} = \{\{R\}, \{B, Y\}\}$, and let us assume that, based on the information that the decision maker receives, his belief is defined $\mu : \mathcal{F} \to [0, 1]$ is given by $\mu(\{R\}) = \frac{1}{3}$ and $\mu(\{B, Y\}) = \frac{2}{3}$. Therefore, the acts $f_1 = 1_{\{R\}}$ and $f_4 = 1_{\{B,Y\}}$ are $\mathcal{F}$-measurable, while the acts $f_2 = 1_{\{B\}}$ and $f_3 = 1_{\{R,Y\}}$ *are not $\mathcal{F}$-measurable* since the events $\{B\}$ and $\{R, Y\}$ are not elements of $\mathcal{F}$. Thus, while $U(f_1) = \int u(f_1)\, d\mu = \mu(\{R\}) = \frac{1}{3}$ and $U(f_4) = \int u(f_4)\, d\mu = \mu(\{B, Y\}) = \frac{2}{3}$, the integrals $U(f_2) = \int u(f_2)\, d\mu$ and $U(f_3) = \int u(f_3)\, d\mu$ are not defined. Therefore, the individual is unable to compare act f_1 with f_2 (and f_4 with f_3). In other words, this preference is *incomplete*, because it does not obey the completeness axiom, which requires that either $f_1 \succcurlyeq f_2$ or $f_2 \succcurlyeq f_1$. However, in the above example we have forced the individual to make a choice. This means that the individual had to find a way to complete her preferences.

From this perspective, we see that the decision maker's ignorance translates into incomplete preferences. The need of completing preferences in situations of ignorance is a problem that goes back at least to the 50's. Returning to this issue, Binmore (2008, Chapter 9) discusses the following criteria: Wald's minimax,[24] Savage's minimax regret, the principle of insufficient reason and the Hurwicz criterion. He notes that Savage prescribed his expected utility to be used in "small worlds", which are worlds about which the decision maker knows enough to be capable of evaluating the odds. Thus, the need of the extension of the preference arises as long as the decision maker faces a "large world", that is, a world in which she cannot properly evaluate the likelihood of possible outcomes.

Now, of course a modeler could assume that the decision maker actually attributes probabilities to all events (a position known as "Bayesian doctrine").

[24]The term minimax (instead of maximin) used by Wald (1950) comes from the fact that instead of maximization of utility, he models the objective as minimization of "risk", as it is usual in statistics.

However, the choices obtained in the Ellsberg's paradox show that this is not consistent with the way in which many people make choices. The impossibility of accommodating both the assumption of expected utility defined for all events and the choices in the Ellsberg's paradox, motivated the ambiguity aversion literature to reject the expected utility framework and consider other forms of preferences.

In the following subsection we show how MEU preferences can solve the Ellsberg paradox.[25]

7.4.1 Incomplete Expected Utility Preferences and Maximin Completion

Assume that the agents have standard Expected Utility preferences, as usually described in the asymmetric information literature, with a subtle caveat that we will explain in a moment. For each agent $i \in I$, $(\Omega, \mathcal{F}_i, \mu_i)$ is a probability space. Then, the preference $\succcurlyeq_i^\circ$ of the individual i is described as:[26,27]

$$f \succcurlyeq_i^\circ g \iff \int_\Omega u_i(f(\omega))\mu_i(d\omega) \geqslant \int_\Omega u_i(g(\omega))\mu_i(d\omega), \forall f, g \in \mathcal{L}_i. \tag{16}$$

The preferences above described are just expected utility preferences that take in account the private information of each individual. As such, these preferences are incomplete. To see this, it is sufficient to observe that the preference is capable of comparing only $\mathcal{F}_i$-measurable acts. If h is not $\mathcal{F}_i$-measurable, its integral $\int h \, d\mu_i$ is not defined and, therefore, it is not possible for individual i to compare h with any other act. In other words: neither $f \succcurlyeq_i^\circ h$ nor $h \succcurlyeq_i^\circ f$ hold for any act f, which is the same as saying that the preference is *incomplete*.[28] This was

[25] That MEU preferences solve the Ellsberg paradox is, of course, well-known since Gilboa and Schmeidler (1989).

[26] We use the notation $\succcurlyeq_i^\circ$ for this incomplete expected utility preference and reserve the more standard $\succcurlyeq_i$ for the complete preferences given in (18) below . Note that μ_i is just a partial probability, that is, a probability restricted only to some events (those in $\mathcal{F}_i$).

[27] Although Ω is finite, the integral $\int f \, d\mu_i$ of some function $f : \Omega \to \mathbb{R}$ *is not* equal to $\sum_{\omega \in \Omega} f(\omega)\mu_i(\{\omega\})$. The reason is that $\mu_i(\{\omega\})$ is defined only if $\{\omega\} \in \mathcal{F}_i$. The correct definition of the integral would be as follows. Let agent i's partition be $\mathcal{F}_i \equiv \{A_1, ..., A_n\}$ and fix any $\omega_k \in A_k$. If $f : \Omega \to \mathbb{R}$ is $\mathcal{F}_i$-measurable, then $f(\omega) = f(\omega_k)$ for any $\omega \in A_k$. Then, $\int f \, d\mu_i = \sum_{k=1}^n f(\omega_k)\mu_i(A_k)$. From this, we see that the integral notation is simpler than the sum notation, and this is the reason why we write integrals.

[28] Although Savage's original theory considers complete preferences, Kopylov (2007) has shown that completeness is not essential at all for an axiomatization of expected utility. That is, Savage's expected utility theory can be developed in such a way that the probability is defined only in a restricted class of events, exactly as we do here. Lehrer (2008) also presents an axiomatization of partially defined probabilities.

exemplified in the discussion of the Ellsberg paradox above. We solve the problem of incompleteness by adopting the MEU completion.

Let Δ denote the set of measures $\pi : \mathcal{F} \to [0,1]$. Define, for each i, the following set:

$$\mathcal{P}_i \equiv \{\pi \in \Delta : \pi(A) = \mu_i(A), \forall A \in \mathcal{F}_i\}. \tag{17}$$

Thus, $\mathcal{P}_i$ is the set of all extensions of μ_i to from $\mathcal{F}_i$ to $\mathcal{F}$, that is, the set of all probability measures defined in $\mathcal{F}$ that agree with μ_i in the events that individual i is informed about. It should be noted that this represents *partial ignorance*. The individual is “completely” ignorant only inside of its partition, but has sufficient knowledge as to attribute probabilities to the elements of the partition.

Let $\mathcal{L}$ denote the set of all acts $f : \Omega \to \mathbb{R}^\ell_+$. Then, the maximin preference $\succcurlyeq_i$ extends $\succcurlyeq_i^\circ$ from $\mathcal{L}_i$ to the set of all acts, $\mathcal{L}$:

$$f \succcurlyeq_i g \iff \min_{\pi \in \mathcal{P}_i} \int_\Omega u_i(f(\omega))\pi(d\omega) \geqslant \min_{\pi \in \mathcal{P}_i} \int_\Omega u_i(g(\omega))\pi(d\omega), \forall f, g \in \mathcal{L}. \tag{18}$$

It is also not difficult to see that if $f : \Omega \to \mathbb{R}^\ell_+$ and $g : \Omega \to \mathbb{R}^\ell_+$ are $\mathcal{F}_i$-measurable, then

$$f \succcurlyeq_i^\circ g \iff f \succcurlyeq_i g. \tag{19}$$

Indeed, if f is $\mathcal{F}_i$-measurable, it is constant on any event in $\mathcal{F}_i$ and therefore, for every $\pi \in \mathcal{P}_i$,

$$\int_\Omega u_i(f(\omega))\pi(d\omega) = \int_\Omega u_i(f(\omega))\mu_i(d\omega).$$

Now, we can return to the Ellsberg's urn example and show how the choices described in section 7.4 are represented by MEU preferences. The following example clarifies the issue:[29]

Example 7.1 (Ellsberg's Experiment) *See section 7.4 for a description of the Ellsberg's thought experiment. Given the partition: $\mathcal{F} = \{\{R\}, \{B, Y\}\}$, the set of probabilities defined by (17) is:*

$$\mathcal{P}_i \equiv \{\pi \in \Delta : \pi(\{R\}) = \frac{1}{3}; \pi(\{B, Y\}) = \frac{2}{3}\}.$$

[29] As clarified before, it is well known that MEU preferences can rationalize Ellsberg's paradox.

Let us assume that $0 = u(0) < u(1) = 1$. *Thus,*

$$\begin{aligned}
U(f_1) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{R\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{R\}) = \frac{1}{3}; \\
U(f_2) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{B\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{B\}) = 0; \\
U(f_3) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{R,Y\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{R,Y\}) = \frac{1}{3}; \\
U(f_4) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{B,Y\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{B,Y\}) = \frac{2}{3}.
\end{aligned}$$

This implies $f_1 \succ f_2$ *and* $f_4 \succ f_3$*, exactly as in the Ellsberg's thought experiment.*[30]

8 Discussion of the Related Literature

8.1 General Equilibrium with Asymmetric Information

It is well known that in a finite economy with asymmetric information once people exhibit standard expected utility, then it is not possible in general to find allocations which are Pareto optimal and also incentive compatible; see for an example the appendix. The key issue is the fact that, in a finite economy each agent's private information has an impact and therefore an agent will take advantage of this private informational effect to influence the equilibrium allocation to favor herself. This is what creates the incentive compatibility problem. To get around this problem, Yannelis (1991) imposes the private information measurability condition, and in this case indeed, any ex ante private information Pareto optimal allocation is incentive compatible (see Krasa and Yannelis (1994), (Koutsougeras and Yannelis 1993) and Hahn and Yannelis (1997) for an extensive discussion of the private information measurability of allocations). In fact, the private information measurability is not only sufficient for proving that ex ante efficient allocations are incentive compatible, but it is also necessary in the one-good case.

It is useful to try to understand why measurability was used to solve the problem of the conflict between efficiency and incentive compatibility. If an agent

[30] Note that f_2 and f_3 are not $\mathcal{F}$- measurable and therefore, could not be compared using the expected utility preference $\succcurlyeq_i^\circ$. Now, the preference $\succcurlyeq_i$ is complete and we can compare every acts, including the non-measurable ones. In this sense, the maximin criterion completes the preference relation.

trades a non-measurable contract, this means that the contract makes promises depending on conditions that she cannot verify. Therefore, other agents may have an incentive to cheat her and do not deliver the correct amount in those states. This possibility is exactly the failure of incentive compatibility. To the contrary, if she insists to trade only measurable contracts (allocations), then she cannot be cheated and incentive compatibility is preserved.

However, the requirement of private information measurability raises two main concerns. First, it is an exogenous, theoretical requirement, which may be difficult to justify in real economies. The second concern, which is more relevant, is that the private information measurability restriction may lead to reduced efficiency and in certain cases even to no-trade. Thus, on the one hand, the private information measurability restriction implies incentive compatibility, but on the other hand, it reduces efficiency. To the contrary, the maximin expected utility allows for trade and results in a Pareto efficient outcome which is also incentive compatible.

Different solutions to the conflict between efficiency and incentive compatibility for the standard (Bayesian) expected utility for replica economies have been proposed by Gul and Postlewaite (1992) and McLean and Postlewaite (2002). Those authors impose an "informational smallness" condition and show the existence of incentive compatible and Pareto optima allocations in an approximate sense for a replica economy. The informational smallness can be viewed as an approximation of the idea of perfect competition and as a consequence only approximate results can be obtained in this replica economy framework. Sun and Yannelis (2007) and Sun and Yannelis (2008) formulate the idea of perfect competition in an asymmetric information economy with a continuum of agents. In this case each individual's private information has negligible influence and as a consequence of the negligibility of the private information, they are able to show that any ex ante Pareto optimal allocation is incentive compatible. The above results are obtained in the set up of standard (Bayesian) expected utilities and they are only approximately true in large but finite economies.

Subsequently to the completion of this paper, de Castro, Pesce, and Yannelis (2010) revisited the Kreps (1977)'s example of the non-existence of the rational expectation equilibrium. They showed that there is nothing wrong with the rational expectation equilibrium notion other than the assumption that agents are expected utility maximizers. Using the maximin preferences studied here, de Castro, Pesce, and Yannelis (2010) recomputed the Kreps' example and showed that the rational expectation equilibrium not only exists, but it is also unique, efficient and incentive compatible.

Another related paper is Morris (1994). He departures from the Milgrom and Stokey (1982) no-trade theorem, which requires the common prior assumption, and shows that the incentive compatibility requirement allows for obtaining equivalent no-trade theorems under assumptions weaker than the common prior assumption. In this context, no trade theorems may be interpreted as a loss of efficiency created by the constraint of incentive compatibility.

Correia-da Silva and Hervés-Beloso (2009) used a MEU for a general equilibrium model with uncertain deliveries, and proved the existence of a new equilibrium concept, which they called prudent equilibrium. Although they considered MEU preferences, their focus was different and did not consider the incentive compatibility studied here.

8.2 Decision Theory

The maximin criterion has a long history. It was proposed by Wald (1950) and Rawls (1971), and axiomatized by Milnor (1954), Maskin (1979), Barbera and Jackson (1988), Nehring (2000) and Segal and Sobel (2002). Binmore (2008, Chapter 9) presented an interesting discussion of the principle, making the connection of the large worlds of Savage (1972). Gilboa and Schmeidler (1989) generalized at the same time the maximin criterion (see footone 5) and Bayesian preferences by allowing for multiple priors. Bewley (2002) introduced a model of decision under incomplete information. His model also included the preference $\succcurlyeq_i^\circ$ described in subsection 7.4.1, as a special case.

The approach discussed in section 7.4 is very much related to Gilboa, Maccheroni, Marinacci, and Schmeidler (2010). They consider decision makers who have two preferences. One of these preferences is incomplete and corresponds to the part of her preference that she can justify for third persons. They call this preference objective and model it as a Bewley incomplete preference. The other preference corresponds to a subjective preference, where the decision maker cannot be proven wrong and this is modeled as a maximin expected utility preference.

In a recent paper, Gul and Pesendorfer (2009) proposed an axiomatization of preferences that allowed them to deal with unmeasurable acts, but our models of behavior differ. Note also, that their focus was not the asymmetric information, as it is in this paper. Lehrer (2008) axiomatized a model with partial probabilities. Our preferences are a particular case of his, although our presentation and motivation is quite different from his. Rigotti, Shannon, and Strzalecki (2008), de Castro and Chateauneuf (2010), characterized conditions for ex ante efficiency for convex preferences (the first) and MEU preferences (the second). Kajii and

Ui (2009) and Martins-da Rocha (2010) characterized interim efficiency for MEU and Bewley preferences, but do not mention incentive compatibility issues.

Mukerji (1998) used a model with ambiguity to analyze the problem of investment holdup and incomplete contracts in a model with *moral hazard*. Interestingly, he obtained results that go in the opposite direction than those obtained here: in the moral hazard model that he considered, ambiguity makes harder to obtain incentive compatibility, not easier as we proved for our general equilibrium with asymmetric information model.[31] The connection between ambiguity and information has been addressed before by Mukerji (1997) and Ghirardato (2001). With respect to efficiency and incentive compatibility, Haller and Mousavi (2007) presented evidence that ambiguity improves the second-best in a simple Rothschild and Stiglitz (1976)'s insurance model.

The analysis of games with ambiguity averse players has also a limited literature. Klibanoff (1996) considered games where players have MEU preferences. Salo and Weber (1995), Lo (1998) and Ozdenoren (2000, Chapter 4) analyzed auctions where players have ambiguity aversion. More recently, Bose, Ozdenoren, and Pape (2006) and Bodoh-Creed (2010) studied optimal auction mechanisms when individuals have MEU preferences, while Lopomo, Rigotti, and Shannon (2009) investigated mechanisms for individuals with Bewley's preferences. However, none of these papers have uncovered the property of no conflict between efficiency and incentive compatibility for the maximin preferences considered here.

9 Concluding Remarks and Open questions

We showed that maximin preferences present no conflict between incentive compatibility and efficiency. Our MEU preferences are not only sufficient for any efficient allocation to be incentive compatible but they are also necessary. Additionally, this paper provides an axiomatization of the maximin preferences. Applications of our results to mechanism design were given. Finally, we applied our results to the Myerson-Satterthwaite's setup and showed that their negative result does not hold in our framework. We close now by discussing some open questions and directions of future research.

It is of interest to know the incentive compatibility properties for all uncertainty averse preferences (as defined by Cerreia, Maccheroni, Marinacci, and Montrucchio (2008)). In other words, fixing a profile of uncertainty averse prefer-

[31] We are grateful to Sujoy Mukerji for bringing this paper to our attention.

ences, we would like to know how close the sets of efficient and incentive compatible allocations are. Or yet: how close are the set of second-best outcomes (that is, outcomes that are efficient subject to being incentive compatible) and first-best (just efficient) outcomes?

In an earlier version of this paper, we introduced notions of maximin core and maximin perfect equilibrium. It is natural to investigate these concepts in more detail. Also, we have not pursued the issue of implementation. It is our conjecture that in view of the inherent efficiency and incentive compatibility of the new equilibrium notions, one should be able to show that they are implementable as a maximin perfect equilibrium and thus provide non cooperative foundations for the maximin core and maximin value.

Finally, it would be interesting to study an evolutionary model of populations of agents with different preferences. Will a society formed only by maximin agents outperform societies formed by individuals with diverse preferences? What happens if some mutations lead to Bayesian subjects inside this maximin society?

In sum, we hope this paper stimulates new venues of investigation.

A Proofs

For the examples below, it will be convenient to use a concise notation for the allocations. Consider two-individual economies, with set of types $T_1 = \{U, D\}$ and $T_2 = \{L, R\}$. The allocation $x = (x_1, x_2)$ will be represented by:

x_1	L	R
U	$x_1(U,L)$	$x_1(U,R)$
D	$x_1(D,L)$	$x_1(D,R)$

and

x_2	L	R
U	$x_2(U,L)$	$x_2(U,R)$
D	$x_2(D,L)$	$x_2(D,R)$

where $x_i(t_1, t_2) \in \mathcal{B}$. Sometimes, we will write the above in just one table and often omit the types in the columns and rows. For example, an allocation $x = (x_1, x_2) \in \mathcal{B} \times \mathcal{B} = \mathbb{R}^2_+$ will appear as:

(x_1, x_2)		
	$(5,3)$	$(6,1)$
	$(2,5)$	$(3,4)$

A.1 Proofs for results in section 2

Proof of Proposition 2.6.

Assume that $x \in E_A \setminus E_I$. Then there exists y, j, t_j such that $y_i \succcurlyeq_i^{t_i} x_i$ for all $i \in I, t_i \in T_i$ and $y_j \succ_j^{t_j} x_j$. Since $\mu_i(\{t_i\}) > 0$, this implies that $y_i \succcurlyeq_i x_i$, for all i and $y_j \succ_j x_j$ for some j, that is, $y \in D_A(x)$, which contradicts $x \in E_A$.

Now we offer counterexamples for the other inclusions.

- $E_I \not\subset E_A$. Let $n = 2$, $\mathcal{B} = \mathbb{R}_+$, $T_i = \{t_i', t_i''\}$, $u_i(t, a) = a$, for $i = 1, 2$ and any $t \in T$. Put $\mu_1(\{t_1'\}) = 0.3$ and $\mu_2(\{t_2'\}) = 0.6$. Consider the allocations $x = (x_1, x_2)$ and $y = (y_1, y_2)$ defined as follows:

(x_1, x_2)	t_2'	t_2''
t_1'	$(2, 2)$	$(2, 2)$
t_1''	$(3, 3)$	$(2, 2)$

and

(y_1, y_2)	t_2'	t_2''
t_1'	$(1, 3)$	$(2, 2)$
t_1''	$(3, 3)$	$(3, 1)$

  Thus, $\underline{x}_1(t_1') = \underline{x}_1(t_1'') = 2$; $\underline{y}_1(t_1') = 1; \underline{y}_1(t_1'') = 3$, which implies that $y_1 \succ_1 x_1$ because $\mu_1(\{t_1'\}) = 0.3 < \mu_1(\{t_1''\}) = 0.7$. On the other hand, $\underline{x}_2(t_2') = 2; \underline{x}_2(t_2'') = 2$; $\underline{y}_2(t_2') = 3; \underline{y}_2(t_2'') = 1$, which implies $y_2 \succ_2 x_2$ because $\mu_2(\{t_2'\}) = 0.6 > \mu_2(\{t_2''\}) = 0.4$. Therefore, $y \in D_A(x)$, that is, $x \notin E_A$. Now suppose that there is z such that $z \in D_I(x)$, that is, $z_i \succcurlyeq_i^{t_i} x_i$, $\forall i, t_i \in T_i$ and $z_j \succ_j^{t_j} x_j$ for some $j \in I$. This means that $\underline{z}_1(t_1'), \underline{z}_1(t_1''), \underline{z}_2(t_2'), \underline{z}_2(t_2'') \geqslant 2$ and at least one of these inequalities has to be strict. Observe that this requires $z_1(t_1, t_2) \geqslant 2$ and $z_2(t_1, t_2) \geqslant 2$, for any $(t_1, t_2) \in T_1 \times T_2$. But then feasibility implies $z_1(t_1, t_2) = z_2(t_1, t_2) = 2$, for any $(t_1, t_2) \neq (t_1'', t_2')$. In turn, this implies that none of the inequalities $\underline{z}_1(t_1'), \underline{z}_1(t_1''), \underline{z}_2(t_2'), \underline{z}_2(t_2'') \geqslant 2$ can be strict. Therefore, $z \notin D_I(x)$, which is a contradiction that shows $x \in E_I$.

- $E_P \not\subset E_I$. Consider that $n = 2$, $\mathcal{B} = \mathbb{R}_+$ and $u_1(t_1, t_2, a) = u_2(t_1, t_2, a) = a$, where $T_1 = T_2 = \{1, -1\}$. Let $e_1(t) + e_2(t) = 1$ for all t. Consider the allocation $x = (x_1, x_2)$ defined by:

(x_1, x_2)	$t_2 = 1$	$t_2 = -1$
$t_1 = 1$	$(1, 0)$	$(0, 1)$
$t_1 = -1$	$(0, 1)$	$(1, 0)$

  Then x is feasible and ex post efficient, that is, $x \in E_P$. However, $x \notin E_I$. Indeed, consider the deviation $y = (y_1, y_2)$ defined by $y_1(t) = \frac{1}{2} = y_2(t)$. This satisfies: $y_i \succ_i^{t_i} x_i, i = 1, 2$ because:

$$\frac{1}{2} = \min_{t_{-i}' \in \{1,-1\}} u_i(t_i, y_i(t_i, t_{-i}')) > \min_{t_{-i}' \in \{1,-1\}} u_i(t_i, x_i(t_i, t_{-i}')) = 0.$$

This shows that $E_P \not\subset E_I$.

- $E_I \not\subset E_P$ and $E_A \not\subset E_P$. Let $n = 2$, $\mathcal{B} = \mathbb{R}^2_+$, $T_1 = T_2 = \{1, 2\}$, $u_i(t, (a_1, a_2)) = a_1 a_2$ and $e_i(t) = (t_i, t_i)$, for $i = 1, 2$. Consider the following allocation:

(x_1, x_2)	$t_2 = 1$	$t_2 = 2$
$t_1 = 1$	$((1,1),(1,1))$	$((1.5,1.5),(1.5,1.5))$
$t_1 = 2$	$((1.5,1.5),(1.5,1.5))$	$((3,1),(1,3))$

In this case, we have $\underline{x}_i(1) = 1$; $\underline{x}_i(2) = 2.25$, $i = 1, 2$, which are the best possible levels for both players (it is not possible to improve these minima for both players). Therefore, $x = (x_1, x_2)$ is interim efficient and ex ante efficient. However, it is clearly not ex post efficient, because we can define $y_i(t) = x_i(t)$ for all $t \neq (2, 2)$ and $y_i(2, 2) = (2, 2)$, $i = 1, 2$ and this is clearly better than $(x_1(2, 2), x_2(2, 2)) = ((3, 1), (1, 3))$. This shows that $E_I \not\subset E_P$ and $E_A \not\subset E_P$. ∎

A.2 Example of the conflict between efficiency and incentive compatibility

This section provides an example to illustrate the conflict between efficiency and incentive compatibility when agents have expected utility preferences. There are two agents, with ex post utilities $u_i(t, a) = a$ for all $t \in T$, where $T_1 = \{U, D\}$ and $T_2 = \{L, R\}$. The priors at each pair of profile of types $\pi_1(\{(t_1, t_2)\})$ and $\pi_2(\{(t_1, t_2)\})$ are given by the following tables:[32]

π_1	L	R
U	$1/3$	$1/3$
D	$1/6$	$1/6$

and

π_2	L	R
U	$1/6$	$1/3$
D	$1/6$	$1/3$

For simplicity, we assume that $e_i(t) = 2$, for all $i \in I$ and $t \in T$. Then, the following allocation is (strongly) efficient:

(x_1, x_2)	L	R
U	$(4, 0)$	$(2, 2)$
D	$(2, 2)$	$(0, 4)$

[32]It is possible to construct analogous examples with common priors.

Observe that the negotiated trade $z = (z_1, z_2)$ is given by:

(z_1, z_2)	L	R
U	$(2, -2)$	$(0, 0)$
D	$(0, 0)$	$(-2, 2)$

Transfers (in the amount of 2) occur only at types (U, L) and (D, R). However, this allocation is not incentive compatible, because when $t_1 = D$ individual 1 has an incentive to misreport $t'_1 = U$ and end up with the better consumption (x'_1):

$$\overset{x'_1}{\begin{array}{|c|c|}\hline 4 & 2 \\ \hline 4 & 2 \\ \hline\end{array}} = \overset{e_1}{\begin{array}{|c|c|}\hline 2 & 2 \\ \hline 2 & 2 \\ \hline\end{array}} + \overset{z'_1}{\begin{array}{|c|c|}\hline 2 & 0 \\ \hline 2 & 0 \\ \hline\end{array}} \succ_1 \overset{x_1}{\begin{array}{|c|c|}\hline 4 & 2 \\ \hline 2 & 0 \\ \hline\end{array}} = e_1 + \overset{z_1}{\begin{array}{|c|c|}\hline 2 & 0 \\ \hline 0 & -2 \\ \hline\end{array}}$$

This incentive to misreport is exactly the failure of incentive compatibility in the Bayesian setup.

A.3 Proofs of results in section 4

Proof of Theorem 4.7.

Necessity: If $\gtrsim$ is maximin, it is clearly complete, transitive, monotonic and continuous. Therefore, it is sufficient to show that $\gtrsim$ is supported by minimal prices. For an absurd, assume the contrary.

Thus, there exist $f : W \to I \subseteq \mathbb{R}$, $p \in S_f$ and $w' \in W$ such that $p(w') > 0$ and $f(w') > m \equiv \min_{w \in W} f(w)$. Let $\bar{p} \equiv \sum_{w \in W} p(w) > 0$ and choose $\epsilon > 0$ such that $\bar{p}\epsilon < p(w')\,[f(w') - m]$. Define $g : \Omega \to \mathbb{R}_+$ by:

$$g\,(w) = \begin{cases} f\,(w) + \epsilon, & \text{if } w \neq w' \\ m + \epsilon, & \text{if } w = w' \end{cases}$$

It is clear that $\min_w g(w) \geqslant m + \epsilon > m$, which implies that $g \succ f$. However,

$$p \cdot g = p \cdot f + \bar{p}\epsilon - p(w')\,[f(w') - m] < p \cdot f,$$

which contradicts $p \in S_f$.

Sufficiency: Assume that $\gtrsim$ is complete, transitive, monotonic, continuous and supported by minimal prices. For each $f \in \mathcal{F}$, define $m_f \in \mathcal{F}$ by $m_f(w') \equiv \min_{w \in W} f(w)$ for all $w' \in W$. Since $\gtrsim$ is complete and transitive, it is sufficient to show that $f \sim m_f$ for every $f \in \mathcal{F}$. For each $\alpha \in [0, 1]$, define $f^\alpha(w) =$

$\alpha f(w) + (1-\alpha)m_f(w)$. Note that $m_{f^\alpha} = m_f$ and that for every $\alpha \in (0,1]$, $\{w \in W : f^\alpha(w) = \min_{w\in W} f^\alpha(w)\} = \{w \in W : f(w) = \min_{w\in W} f(w)\} \equiv M_f$.

By monotonicity, $f \gtrsim f^\alpha \gtrsim m_f, \forall \alpha \in [0,1]$. It is sufficient to show that $f \sim f^\alpha$ for every $\alpha \in (0,1]$, since this would imply $f \sim m_f$ by continuity and the fact that $f^{1/k} \to m_f$. Suppose then that there is $\alpha \in (0,1]$ such that $f \succ f^\alpha$. If $p \in S_{f^\alpha}$ then $p \cdot f > p \cdot f^\alpha$. However, since $\gtrsim$ is supported by minimal prices,

$$p \cdot f^\alpha = \sum_{w \in M_f} p(w) f^\alpha(w) = \sum_{w \in M_f} p(w) f(w) = p \cdot f,$$

which contradicts $p \cdot f > p \cdot f^\alpha$. The contradiction establishes the result. ∎

Proof of Proposition 4.8.

Since $\gtrsim$ is not maximin, there exists $f \in \mathcal{E}$ such that $f \succ m_f$. Let $W = \{w_1, w_2, ..., w_K\}$. We will define functions $f_k, g_k^\alpha : W \to \mathbb{R}_+$, for $k = 1, 2, ..., K$ and $\alpha \in [0,1]$. The definition of f_k will be recursive. Let $f_1 \equiv f$ and suppose that f_k is defined satisfying $f_k \sim f$. Define g_k^α as follows:

$$g_k^\alpha(w) = \begin{cases} f_k(w), & \text{if } w \neq w_k \\ \alpha f_k(w) + (1-\alpha) m_f(w), & \text{if } w = w_k \end{cases}$$

The set $A_k = \{\alpha \in [0,1] : g_k^\alpha \sim f_k\}$ contains 1 and is closed. Moreover, by monotonicity and continuity, there is the smallest $\alpha_k \in A_k$. Define f_{k+1} as $g_k^{\alpha_k}$. Then by definition, for $k = 1, ..., K$,

$$f_{k+1} \sim f_k \sim f \text{ and } f_{k+1} \leqslant f_k. \tag{20}$$

We claim that $h \equiv f_{K+1}$ satisfies the properties in the statement above.

Indeed, suppose that there is a $g \neq h$ satisfying $h \geqslant g$, such that $h \sim g$. Since $g \neq h$, the set $\{k : g(w_k) < h(w_k)\}$ is non-empty. Let k be the largest element of this set. Observe that $f_{k+1} \sim h$, $f_{k+1} \geqslant h \geqslant g$ and $h(w_j) = f_k(w_j) = g_k^\alpha(w_j)$ for every $j < k$ and $\alpha \in [0,1]$. Since $g_k^{\alpha_k}(w_k) = f_{k+1}(w_k) \geqslant h(w_k) > g(w_k)$, there exists $\alpha < \alpha_k$ such that $g(w_k) < g_k^\alpha(w_k) < g_k^{\alpha_k}(w_k) = f_{k+1}(w_k)$. However, by definition of α_k, for any $\alpha < \alpha_k$, $f_{k+1} \succ g_k^\alpha$. It is easy to see that $g_k^\alpha \geqslant g$ and, therefore, $g_k^\alpha \gtrsim g$. But then $h \sim f_{k+1} \succ g_k^\alpha \gtrsim g$, which contradicts $h \sim g$, thus concluding the proof. ∎

The proof of Theorem 4.3 will require the following:

Lemma A.1 (Alternative for corner allocations) *Let the preferences $\{\succsim_i\}_{i\in I}$ be adequate. Suppose that $x = (x_j)_{j\in I}$ is a i-corner allocation, that is, $x_j(\omega) = 0 \in \mathcal{B} = \mathbb{R}^\ell_+$ for all $\omega \in \Omega$ and all $j \neq i$. Then one (and only one) of the following alternatives is true:*

1. *x is an ex ant efficient allocation;*
2. *there exists $z : T \to \mathcal{B}$ and $j \neq i$ such that:*
   (a) *$z \succ_j 0$;*
   (b) *$z \geqslant 0$;*
   (c) *$x_i \geqslant z$;*
   (d) *$x_i - z \sim_i x_i$.*

Proof. It is easy to see that if there exists z satisfying the conditions above, it is possible to transfer z to individual j, strictly improving j and without making any individual worse off; therefore x is not ex ante efficient. Conversely, if x is not ex ante efficient, then there exists a Pareto improving $y = (y_j)_{j\in I}$ satisfying $y_k \succsim_k x_k$, for all $k \in I$ and $y_j \succ_j x_j$ for some j. Fix such j. Of course, this j cannot be i, since i already has all the endowment of the economy and cannot be strictly better by a feasible transfer. Therefore, define $z \equiv y_j \succ_j x_j = 0$, which gives (a) above. Since $y_k \geqslant 0$ for all k, then we also have (b). This also allows to conclude that $\sum_{k\in I} x_k = x_i = \sum_{k\in I} y_k \geqslant y_j = z$, which establishes (c). For the same reason, $x_i \geqslant y_i + y_j = y_i + z$, that is, $x_i - z \geqslant y_i$. Since $z \geqslant 0$, we have $x_i \succsim_i x_i - z$. By monotonicity, $x_i - z \succsim_i y_i$. On the other hand, the fact that y is Pareto improving gives $y_i \succsim_i x_i$. Transitivity then establishes (d). ∎

Proof of Theorem 4.3. Suppose that individual 1's preference is not maximin, that is, there exists some type t_1' such that $\succsim_1^{t_1'}$ is not maximin. Let I be the image of the function $a \mapsto u_i(t, a)$ for each $t \in T$ and let $\bar{e} = (1, 1, ..., 1) \in \mathcal{B} = \mathbb{R}^\ell_+$ be the unitary bundle. Then, for each $\alpha \in I$, there exists $\lambda(t_2) \in \mathbb{R}_+$ such that $u_1(t_1, t_2, \lambda(t_2)\bar{e}) = \alpha$. Let $E = \{\lambda\bar{e} : \lambda \in \mathbb{R}_+\}$. Thus, given a function $f : \{t_1'\} \times T_2 \to I \subseteq \mathbb{R}$, we can find for each $t_2 \in T_2$ a bundle $f^{u_1}(t_1', t_2) \in E$ such that :

$$u_1(t_1', t_2, f^{u_1}(t_1', t_2)) = f(t_1', t_2).$$

Let $W = \{t_1'\} \times T_2$ and define $\succeq^*$ over functions $f : W \to I \subseteq \mathbb{R}$ by:

$$f \succeq^* g \iff f^{u_1} \succsim_1^{t_1'} g^{u_1}.$$

By Proposition 4.8, there exists

$$f(t_1', \cdot) \in \mathcal{E} = \{f : W \to I : \exists w' \in W \text{ such that } f(w') > \min_{w \in W} f(w)\}$$

such that for every $g(t_1', \cdot) \neq f(t_1', \cdot)$ satisfying $f(t_1', \cdot) \geqslant g(t_1', \cdot)$, we have $f(t_1', \cdot) \succ^* g(t_1', \cdot)$. By the definition of $\succeq^*$, f^{u_1} and $\gtrsim_1$'s properties, $\forall g : T \to \mathcal{B}$,

$$f^{u_1} \geqslant g \Rightarrow f^{u_1} \gtrdot_1^{t_1'} g. \tag{21}$$

Let $M_f \equiv \{t_2 : u_1(t_1', t_2, f^{u_1}(t_1', t_2)) = \min_{t_2 \in T_2} u_1(t_1', t_2, f^{u_1}(t_1', t_2))\}$. Fix $t_2' \in M_f$ and define: $e_1(t_1', \cdot) = f^{u_1}(t_1', t_2')$. For any $t_2 \in T_2$, define $e_2(\cdot, t_2) \equiv f^{u_1}(t_1', t_2) - e_1(t_1', t_2)$. By the definition of $f^{u_1}(t_1', \cdot)$, $e_2(\cdot, t_2) \geqslant 0$. Note also that $e_2(\cdot, t_2') = 0$. Now, for $t_1 \neq t_1'$, define $e_1(t_1, t_2) = 0$ and $f^{u_1}(t_1, t_2) = e_2(t_1, t_2)$. It is easy to see that $(f^{u_1}, 0)$ is then a feasible 1-corner allocation.

Let $z : T \to \mathcal{B}$ be such that $z \geqslant 0$. Monotonicity implies then that $f^{u_1} \gtrsim_1^{t_1} g \equiv f^{u_1} - z$ for all $t_1 \in T_1$ and (21) implies that $f^{u_1} \gtrdot_1^{t_1'} g$. Since $\gtrsim_1$ is adequate, $f^{u_1} \gtrdot_1 g$. Therefore, there is no z satisfying all the assumptions in item 2 of Lemma A.1, which implies that the i-corner allocation $(f^{u_1}, 0)$ is (ex ante) efficient.[33]

On the other hand, since $f(t_1, \cdot) \in \mathcal{E}$, there is a type $t_2'' \notin M_f$ such that

$$f(t_1', t_2'') > f(t_1', t_2') \Rightarrow f^{u_1}(t_1', t_2'') - f^{u_1}(t_1', t_2') \gg 0. \tag{22}$$

Then, if individual 2 is of type t_2'', he has an incentive to report t_2'. Indeed, if $t_2 = t_2''$ and individual 2 reports t_2' instead of t_2'', he will consume, for any $t_1 \in T_1$,

$$\begin{aligned} e_2(t_1, t_2'') - e_2(t_1, t_2') &= [f^{u_1}(t_1', t_2'') - e_1(t_1', t_2'')] - [f^{u_1}(t_1', t_2') - e_1(t_1', t_2')] \\ &= f^{u_1}(t_1', t_2'') - f^{u_1}(t_1', t_2') \gg 0, \end{aligned}$$

where the first equality comes from the definition of $e_2(\cdot, t_2)$, the second comes from the definition of $e_1(t_1', \cdot)$ and the inequality comes from (22). Since individual 2's allocation under $(f, 0)$ is always zero and the preference is monotonic, he would be strictly better off. Thus, the allocation is not incentive compatible. ∎

Proof of Theorem 4.4

The proof above works by substituting individual 1 by i and individual 2 by a coalition of all individuals other than i. ∎

[33] We have defined the initial endowments here only to make the example completely specified. The allocation $(f^{u_1}, 0)$ is of course not individually rational, but this is a side issue.

Proof of Theorem 6.1

Let $\succsim_i$ and $\succcurlyeq_i$ denote respectively the Bayesian and the Maximin preferences. Assume that x is Bayesian Pareto optimal and coalitional incentive compatible. We claim that x_j is $\mathcal{F}_j$-measurable for each $j \in I$.

We establish this claim by contradiction. Suppose that x is incentive compatible but x_j is not $\mathcal{F}_j$-measurable for some $j \in I$, that is, suppose that there exist $t_{-j}, t'_{-j} \in T_{-j}$ such that $x_j(t_j, t_{-j}) \neq x_j(t_j, t'_{-j})$. Without loss of generality, we may assume that $x_j(t_j, t_{-j}) > x_j(t_j, t'_{-j})$. Since e_j is $\mathcal{F}_j$-measurable, $e_j(t_j, t'_{-j}) = e_j(t_j, t_{-j})$. Therefore

$$x_j(t_j, t_{-j}) - e_j(t_j, t_{-j}) > x_j(t_j, t'_{-j}) - e_j(t_j, t'_{-j}). \tag{23}$$

Let $C \equiv I \setminus \{j\}$. From feasibility of x and (23), we have:

$$\begin{aligned}
\sum_{i \in C} [x_i(t_j, t_{-j}) - e_i(t_j, t_{-j})] &= -[x_j(t_j, t_{-j}) - e_j(t_j, t_{-j})] \\
&< -\left[x_j(t_j, t'_{-j}) - e_j(t_j, t'_{-j})\right] \\
&= \sum_{i \in C} \left[x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j})\right].
\end{aligned}$$

Thus,

$$\delta \equiv \sum_{i \in C} \left[x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j}) - x_j(t_j, t_{-j}) + e_j(t_j, t_{-j})\right] > 0.$$

For each $i \in C$, let

$$\tau_i \equiv -x_i(t_j, t'_{-j}) + e_i(t_j, t'_{-j}) + x_i(t_j, t_{-j}) - e_i(t_j, t_{-j}) + \frac{\delta}{n-1},$$

so that $\sum_{i \in C} \tau_i = 0$ and

$$e_i(t_j, t_{-j}) + x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j}) + \tau_i > x_i(t_j, t_{-j}).$$

By the monotonicity of u_i, we can conclude that for all $i \in C$,

$$u_i\left(t_i, e_i(t_j, t_{-j}) + x_i(t_j, t'_{-j}) - e_i(t_j, t'_{-j}) + \tau_i\right) > u_i\left(t_i, x_i(t_j, t_{-j})\right),$$

which contradicts the assumption that x is coalitionally incentive compatible. This establishes the claim that x_j is $\mathcal{F}_j$-measurable.

Now, assume that x is not maximin Pareto optimal. This means that there exists a feasible allocation y such that $y_j \succcurlyeq_j^{t_j} x_j$ for all $j \in I, t_j \in T_j$ and there is $i \in I, t_i' \in T_i$ such that $y_i \succ_i^{t_i} x_i$, that is, $\underline{y}_i(t_i') > \underline{x}_i(t_i')$. Since x_i is $\mathcal{F}_i$-measurable, this implies that $u_i(t_i', y_i(t_i', t_{-i})) > u_i(t_i', x_i(t_i', t_{-i}))$ for every t_{-i}. The monotonicity of u_i now gives $y_i(t_i', t_{-i}) > x_i(t_i', t_{-i})$. Similarly, $y_j \succcurlyeq_j^{t_j} x_j$ and the fact that x_j is $\mathcal{F}_j$-measurable imply that $y_j(t_i', t_{-i}) \geqslant x_j(t_i', t_{-i})$ for all $j \neq i$. But then, $\sum_{i \in I} y_i(t_i', t_{-i}') > \sum_{i \in I} x_i(t_i', t_{-i}') = \sum_{i \in I} e_i(t_i', t_{-i}')$ and y is not feasible, which is a contradiction.

The counterexample for the reverse implication is based on the example $E_I \not\subset E_A$ given in the proof of Proposition 2.6. Since we did not have to specify the full Bayesian beliefs at that example, we repeat the example here with this specification. There are two individuals, $\mathcal{B} = \mathbb{R}_+$, $T_i = \{t_i', t_i''\}$, $u_i(t, a) = a$, for $i = 1, 2$ and any $t \in T$. The Bayesian beliefs $\mu_i(\{(t_1, t_2)\})$ of individual i for the event $\{(t_1, t_2)\}$ are defined by the following:

$\mu_1(\cdot)$	t_2'	t_2''
t_1'	0.15	0.15
t_1''	0.35	0.35

and

$\mu_2(\cdot)$	t_2'	t_2''
t_1'	0.3	0.2
t_1''	0.3	0.2

Note that the numbers in each table add up to one. Consider the allocations $x = (x_1, x_2)$ and $y = (y_1, y_2)$ defined as follows:

(x_1, x_2)	t_2'	t_2''
t_1'	$(2, 2)$	$(2, 2)$
t_1''	$(3, 3)$	$(2, 2)$

and

(y_1, y_2)	t_2'	t_2''
t_1'	$(1, 3)$	$(2, 2)$
t_1''	$(3, 3)$	$(3, 1)$

It was shown in the proof of Proposition 2.6 that x is maximin efficient. Repeating the arguments given in the proof of Proposition 2.6, it is easy to see that y is a Pareto improvement upon x for the Bayesian preferences defined above. Thus, the converse does not hold. ∎

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