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RESEARCH ARTICLE

Incentive compatibility under ambiguity

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Abstract

The paper examines notions of incentive compatibility in an environment with ambiguity-averse agents. In particular, we propose the notion of maxmin transfer coalitional incentive compatibility, which is immune to coalitional manipulations and thus more stable than the individual incentive compatibility condition. The main result characterizes the set of allocations that satisfy the maxmin transfer coalitional incentive compatibility condition. We show that an allocation satisfies maxmin transfer coalitional incentive compatibility if and only if it is maxmin interim efficient. This result extends that of De Castro and Yannelis (J Econ Theory 177:678–707, 2018) in the sense that ambiguity not only resolves the conflict between efficiency and incentive compatibility, it also accommodates stability. Furthermore, this result is false in a finite economy where agents are subjective expected utility maximizers.

Keywords Efficiency · Incentive compatibility · Transfer coalitional incentive compatibility · Maxmin preferences

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1 Introduction

This paper studies notions of incentive compatibility in a finite exchange economy where agents are ambiguity-averse, or more specifically, agents are maxmin expected utility maximizers *à la* Gilboa and Schmeidler (1989).

It is well known that when agents are subjective expected utility maximizers, it is generally impossible for Pareto efficient allocations to be individually incentive compatible, i.e., there is a conflict between efficiency and individual incentive compatibility (see, for example, Holmström and Myerson (1983)). However, this conflict ceases to exist if agents are maxmin expected utility maximizers as shown recently in De Castro and Yannelis (2018).

But what is the right notion of incentive compatibility when agents are maxmin expected utility maximizers? An important observation is that allocations that are individually incentive compatible may not be immune to coalitional manipulations. Therefore, an individually incentive compatible allocation may not be stable or viable. Example 1 in the paper presents a situation where three agents face an allocation that is individually incentive compatible, but not coalitionally incentive compatible. Specifically, two agents can form a coalition, and by misreporting their information and making side payments to each other, they can become better off at the expense of the third agent. Such an allocation is susceptible to coalitional manipulations, and thus may not be viable.

The main purpose of this paper is to resolve this problem by introducing what we believe to be the right incentive compatibility notion and to explore the relationship between incentive compatibility notions and efficiency notions in an asymmetric information economy with ambiguity-averse agents.

We propose the condition of maxmin transfer coalitional incentive compatibility. This condition requires that an allocation cannot be improved upon by any coalition's misreporting of information and redistribution of wealth. It is stronger and more stable than the maxmin individual incentive compatibility condition considered by De Castro and Yannelis (2018).

Our main result characterizes the set of allocations that satisfy the maxmin transfer coalitional incentive compatibility condition. In fact, we show that an allocation satisfies maxmin transfer coalitional incentive compatibility if and only if it is maxmin interim efficient. This result extends that of De Castro and Yannelis (2018) in the sense that ambiguity not only resolves the conflict between efficiency and incentive compatibility, it also accommodates stability. Furthermore, this result is false in a finite economy where agents are subjective expected utility maximizers as shown in Sect. 4.2.

The usefulness of our main result is shown by applying it to a few general equilibrium solution concepts in an asymmetric information economy with ambiguity-averse agents. For example, we consider adaptations of maxmin core and maxmin value allocations introduced by De Castro et al. (2011) and Angelopoulos and Koutsougeras (2015). They satisfy the maxmin transfer coalitional incentive compatibility condition. This contrasts with the Bayesian implementation literature, e.g., Palfrey and Srivastava (1987), which shows that in the presence of asymmetric information, efficient allocations and core allocations generally fail to satisfy the Bayesian incentive compatibility condition.

The paper proceeds as follows. In Sect. 2, we set up the model. In Sect. 3, we define two notions of incentive compatibility and present an example of an individually incentive compatible allocation that is susceptible to coalitional manipulations. Section 4 characterizes the set of allocations satisfying maxmin transfer coalitional incentive compatibility. Section 5 applies the main result to a few general equilibrium solution concepts. Section 6 concludes.

2 Setup

2.1 Ambiguous asymmetric information economy

We consider an n-agent exchange economy with asymmetric information. The set of all agents is denoted by I. The state of nature is modeled by a finite state space Ω. There are l goods in the economy.

Agents' characteristics are given by $\{(\mathcal{F}_i, \mu_i, e_i, u_i)_{i\in I}\}$ as follows.

1. Each agent i's **private information partition** is denoted by $\mathcal{F}_i$, which is a partition of the state space Ω. For each $\omega \in \Omega$, let $\mathcal{F}_i(\omega)$ be the event in $\mathcal{F}_i$ that contains the state ω. Namely, when ω is realized, agent i knows that the true state is in $\mathcal{F}_i(\omega)$, but she cannot tell the exact state in $\mathcal{F}_i(\omega)$ that is realized.
2. In the ex-ante stage, each agent i evaluates her private information to be realized in the interim stage with a probability distribution. Let $\Delta(\mathcal{F}_i)$ denote the set of all distributions over $\mathcal{F}_i$. For agent i, let $\mu_i \in \Delta(\mathcal{F}_i)$ represent agent i's **prior distribution over the private information partition**.
3. For each $i \in I$, let $e_i : \Omega \to \mathbb{R}^l_+$ represent agent i's **initial endowment**. Let the vector e denote the profile $(e_i)_{i\in I}$.
4. For each $i \in I$, the **utility function** $u_i : \mathbb{R}^l_+ \times \Omega \to \mathbb{R}$ describes agent i's ex-post payoff. In particular, $u_i(a_i, \omega)$ denotes agent i's ex-post payoff at ω when she consumes a_i.

The structure of characteristics $\{(\mathcal{F}_i, \mu_i, e_i, u_i)_{i\in I}\}$ is common knowledge between all agents. However, each agent acquires additional information in the interim stage by observing her private information event.

We impose a few assumptions on the characteristics throughout this paper.

First, we assume that agents know the exact state that is realized if they pool their private information events.

Assumption 1 For all $\omega \in \Omega$, $\bigcap\limits_{i\in I} \mathcal{F}_i(\omega) = \{\omega\}$.

The assumption implies that for any profile of private information events $(E_i \in \mathcal{F}_i)_{i\in I}$, either there exists a unique state $\omega \in \Omega$ such that $\bigcap\limits_{i\in I} E_i = \{\omega\}$ or $\bigcap\limits_{i\in I} E_i = \emptyset$.[1]

[1] See Palfrey and Srivastava (1989) for a proof.

In particular, if for any $(E_i \in \mathcal{F}_i)_{i \in I}$, there exists a unique state $\omega \in \Omega$ such that $\bigcap_{i \in I} E_i = \{\omega\}$, then the private information structure $(\mathcal{F}_i)_{i \in I}$ is said to be **diffuse**. In this case, an event E_i in private information partition $\mathcal{F}_i$ can be interpreted as a type of agent i, and a state can be interpreted as a profile of types of all agents.

Second, we impose a full support assumption on each μ_i.

Assumption 2 For all $i \in I$ and $E_i \in \mathcal{F}_i$, $\mu_i(E_i) > 0$.

According to this assumption, every agent believes that each event in her private information partition occurs with positive probability. This assumption is not crucial for the main result of the paper (Theorem 1). However, with this assumption, we can verify the incentive compatibility conditions for a few ex-ante notions discussed in Sect. 5.

Third, it is assumed that e is private information measurable in the following sense.

Assumption 3 For each i, $E_i \in \mathcal{F}_i$, and $\omega, \omega' \in E_i$, $e_i(\omega) = e_i(\omega')$.

This assumption means that no agent can acquire further information by observing her initial endowment, or that the information contained by the stochastic initial endowment of an agent i is accounted for by her private information partition $\mathcal{F}_i$. This assumption is satisfied in the special case that e is state-independent.

Fourth, the utility functions are assumed to be private information measurable.

Assumption 4 For all $i \in I$, $a_i \in \mathbb{R}^l_+$, $E_i \in \mathcal{F}_i$, and $\omega, \omega' \in E_i$, it holds that $u_i(a_i, \omega) = u_i(a_i, \omega')$.

This assumption requires that given an agent's consumption and private information event, the private information events observed by other agents do not affect this agent's ex-post payoff. It is equivalent to the commonly seen private valuation assumption in a type model, i.e., a model with diffuse private information. The assumption is satisfied in the special case that utility functions are state-independent.

2.2 Coalitions

A nonempty set of agents in I is said to be a **coalition**. The set I is also called the **grand coalition**. For each agent i, let $\sigma(\mathcal{F}_i)$ denote the σ-algebra generated by the partition $\mathcal{F}_i$. For a coalition $S \subseteq I$, let $\wedge_{i \in S}\sigma(\mathcal{F}_i)$ denote the largest σ-algebra that is contained in $\sigma(\mathcal{F}_i)$ for all $i \in S$ and represent the **common knowledge information** of agents in the coalition. Let $\mathcal{F}_{\wedge S}$ denote the partition of Ω that generates $\wedge_{i \in S}\sigma(\mathcal{F}_i)$. The within-coalition communication required to form the common knowledge information structure is weak. Each agent in S does not need to acquire any additional information from other members in the coalition. Similarly, let $\vee_{i \in S}\sigma(\mathcal{F}_i)$ denote the smallest σ-algebra that contains $\sigma(\mathcal{F}_i)$ for all $i \in S$. This notation is used to capture the **pooled information** of agents in the coalition. We let $\mathcal{F}_{\vee S}$ denote the partition of Ω that generates $\vee_{i \in S}\sigma(\mathcal{F}_i)$. Assuming that agents pool their information in the within-coalition communication is typically a strong assumption when each member possesses private information.[2] Because of this, we mainly look at the common knowledge information structure when a coalition is formed.

2.3 Maxmin expected utility

Let x be a mapping $x : \Omega \to \mathbb{R}_+^{nl}$ that describes the state-contingent consumption of each agent. Such a mapping is called an **allocation**. Agent i's component of x, denoted by x_i, is the state-contingent consumption of agent i. Let L denote the set of all allocations. Notice that $e \in L$. Given an allocation $x \in L$ and initial endowment $e \in L$, let the vector $z_i^x = x_i - e_i$ represent agent i's **net trade**. Denote the profile of all agents' net trade by $z^x = (z_i^x)_{i \in I}$. Free disposal is not allowed in the paper. Thus, an allocation $x \in L$ is said to be **feasible** if

$$\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega), \forall \omega \in \Omega.$$

The above expression can be equivalently written as

$$\sum_{i \in I} z_i^x(\omega) = 0, \forall \omega \in \Omega.$$

This paper adopts a version of the maxmin expected utility of Gilboa and Schmeidler (1989) to model agents' preferences. In Gilboa and Schmeidler (1989), a decision maker may have little information about the distribution of the state, and thus forms a nonempty, compact, and convex set of priors over the state space. This decision maker evaluates her ex-ante utility based on the worst-case distribution in the set of priors. In our model with asymmetric information, we assume that each agent knows the distribution of her private information to be realized in the interim stage, but knows nothing about the distribution of her opponents' private information. This is reasonable when agents know too little about others to form a subjective belief. For each $i \in I$, let Δ_i be the set of all probability measures $\pi \in \Delta(\Omega)$ that are consistent with μ_i over $\mathcal{F}_i$, or formally,

$$\Delta_i = \{\pi \in \Delta(\Omega) \mid \pi(E_i) = \mu_i(E_i), \forall E_i \in \mathcal{F}_i\}.$$

The set Δ_i represents agent i's **multiple-prior set**. Notice that Δ_i is a nonempty, compact, and convex set of distributions.

We postulate that each agent has maxmin expected utility with multiple-prior set Δ_i. Hence, an agent i's ex-ante maxmin expected utility to evaluate allocation x is defined by

$$\min_{\pi \in \Delta_i} \sum_{\omega \in \Omega} u_i(x_i(\omega), \omega)\pi(\omega).$$

[2] For example, an agent with finer information partition than other members of the coalition may wish to cheat other members by manipulating her private information.

As a worst-case distribution $\pi \in \Delta(\mathcal{F}_i)$ imposes all the weight on the worst states within each $E_i \in \mathcal{F}_i$, it is easy to establish the following equivalent expression of the ex-ante maxmin expected utility:

$$\min_{\pi\in\Delta_i}\sum_{\omega\in\Omega} u_i(x_i(\omega),\omega)\pi(\omega) = \sum_{E_i\in\mathcal{F}_i}\min_{w\in E_i} u_i\big(x_i(\omega),\omega\big)\mu_i(E_i). \tag{1}$$

With prior-by-prior updating, the interim payoff of agent i with private information $E_i \in \mathcal{F}_i$ is given by

$$\min_{w\in E_i} u_i\big(x_i(\omega),\omega\big).$$

Hence, expression (1) decomposes the ex-ante maxmin expected utility into a weighted sum of interim payoffs.

We would like to remark on two features of the preferences adopted by this paper. First, each agent i does not impose any restriction on possible distributions of others' private information. Hence, in the interim stage, an agent has the Wald-type maxmin preference, which means that she only takes into consideration the worst states in her private information event (see Wald (1945)). This feature is essential for our main result in Sect. 4. Second, each agent i has a prior over her private information partition. This is mainly a simplifying assumption that allows us to connect the interim maxmin expected utility and the ex-ante one. We discuss how to relax this assumption after introducing two efficiency notions in the next subsection.

2.4 Efficiency

This paper considers two notions of efficiency under maxmin preferences. We begin by introducing the maxmin ex-ante Pareto efficiency. A maxmin ex-ante Pareto efficient allocation is one that cannot be dominated by another feasible allocation in the ex-ante stage.

Definition 1 A feasible allocation $x \in L$ is said to be **maxmin ex-ante Pareto efficient** if there does not exist a feasible allocation $y \in L$ such that the following two conditions hold:

1. for all $i \in I$,

$$\sum_{E_i\in\mathcal{F}_i}\min_{w\in E_i} u_i\big(y_i(\omega),\omega\big)\mu_i(E_i) \geq \sum_{E_i\in\mathcal{F}_i}\min_{w\in E_i} u_i\big(x_i(\omega),\omega\big)\mu_i(E_i);$$

2. the strict inequality holds for some $i \in I$.

Following the interim domination notion of Holmström and Myerson (1983), an allocation y dominates x in the interim stage if y makes every agent weakly better off under every private information event, and strictly improves the payoff of some agent under some private information event. The formal definition is presented as follows.

Definition 2 A feasible allocation $x \in L$ is said to be **maxmin interim Pareto efficient** if there does not exist a feasible allocation $y \in L$ such that the following two conditions are satisfied:

1. for all $i \in I$ and $E_i \in \mathcal{F}_i$
$$\min_{w \in E_i} u_i\big(y_i(\omega), \omega\big) \geq \min_{w \in E_i} u_i\big(x_i(\omega), \omega\big);$$
2. the strict inequality holds for some $i \in I$ and $E_i \in \mathcal{F}_i$.

Remark 1 We remark that if a feasible allocation $x \in L$ is maxmin ex-ante Pareto efficient, then it is also maxmin interim Pareto efficient. To show this, suppose by way of contradiction that a maxmin ex-ante Pareto efficient allocation x is not maxmin interim Pareto efficient, then there exists a feasible allocation $y \in L$ such that for all $i \in I$ and $E_i \in \mathcal{F}_i$,

$$\min_{\omega \in E_i} u_i\big(y_i(\omega), \omega\big) \geq \min_{\omega \in E_i} u_i\big(x_i(\omega), \omega\big), \tag{2}$$

and the strict inequality holds for some $i \in I$ and $E_i \in \mathcal{F}_i$. For each $i \in I$, by making a weighted sum of the above inequalities over all her private information events, we have

$$\sum_{E_i \in \mathcal{F}_i} \min_{w \in E_i} u_i\big(y_i(\omega), \omega\big)\mu_i(E_i) \geq \sum_{E_i \in \mathcal{F}_i} \min_{w \in E_i} u_i\big(x_i(\omega), \omega\big)\mu_i(E_i). \tag{3}$$

By Assumption 2, the strict inequality in expression (3) holds when i is such that the strict inequality in expression (2) holds. Hence, we obtain that y Pareto dominates x in the ex-ante stage, a contradiction.

For the result in Remark 1 to hold, the assumption that each agent has a unique prior over her private information partition can be generalized. The recursive multiple-prior model can accommodate this result. For example, consider a state space $\Omega = \{\omega_1, \omega_2, \omega_3\}$ where agent i's information partition is given by $\{\{\omega_1, \omega_2\}, \{\omega_3\}\}$. Let a multiple-prior set $\tilde{\Delta}_i$ be the convex hull of the following four distributions $(0.75, 0, 0.25)$, $(0, 0.75, 0.25)$, $(0.25, 0, 0.75)$, and $(0, 0.25, 0.75)$. As distributions in $\tilde{\Delta}_i$ may not agree on sets $\{\omega_1, \omega_2\}$ and $\{\omega_3\}$, the preference is not consistent with the one we adopt in this paper. However, it is easy to see that one can express the ex-ante payoff in terms of interim payoffs:

$$\begin{aligned} &\min_{\pi \in \tilde{\Delta}_i} \sum_{\omega \in \Omega} u_i(x_i(\omega), \omega)\pi(\omega) \\ &= \min_{\bar{\pi} \in [0.25, 0.75]} \Big(\bar{\pi} \min_{\omega \in \{\omega_1, \omega_2\}} u_i\big(x_i(\omega), \omega\big) + (1 - \bar{\pi}) \min_{\omega \in \{\omega_3\}} u_i\big(x_i(\omega), \omega\big)\Big). \end{aligned}$$

Notice that the weights over agent i's interim payoffs are interior in $[0, 1]$. When all agents have similar preferences, it is easy to see that the result in Remark 1 still holds.

Remark 2 We remark that a maxmin interim Pareto efficient allocation may not be maxmin ex-ante Pareto efficient.

To see this, consider a one-good economy with $I = \{1, 2\}$ and $\Omega = \{\omega_1, \omega_2\}$. Each agent's private information partition, prior over private information partition, utility function, and initial endowment are given as follows.

$$\begin{array}{llll} \mathcal{F}_1 = \{\{\omega_1\}, \{\omega_2\}\}, & \mu_1(\{\omega_1\})=\mu_1(\{\omega_2\})=0.5, & u_1(a_1, \omega)=\sqrt{a_1}, & e_1=(2, 0), \\ \mathcal{F}_2 = \{\{\omega_1\}, \{\omega_2\}\}, & \mu_2(\{\omega_1\})=\mu_2(\{\omega_2\})=0.5, & u_2(a_2, \omega)=\sqrt{a_2}, & e_2=(0, 2). \end{array}$$

Notice that e_i is a vector describing agent i's initial endowment at states ω_1 and ω_2 respectively. This complete information setup can be viewed as a degenerated model with asymmetric information. It is easy to see that the allocation e is maxmin interim Pareto efficient. However, in the ex-ante stage, it is Pareto dominated by the fully insured allocation x defined by $x_i(\omega) = 1$ for all $i \in I$ and $\omega \in \Omega$.

3 Incentive compatibility

When agents sign a contract prescribing feasible allocation $x \in L$, the final distribution of resources depends on the state, which can be revealed if agents truthfully report their private information. However, individuals may have the incentive to manipulate private information. Also, groups may have the incentive to make side payments to each other and jointly manipulate private information. Thus, we explore notions of incentive compatibility so that an allocation is not susceptible to individual or coalitional manipulations.

Since members of a coalition S may make side payments to each other, we first define a feasible **within-coalition redistribution** as a profile of mappings $\tau = (\tau_i : \Omega \to \mathbb{R}^l)_{i \in S}$ such that $\sum\limits_{i \in S} \tau_i(\omega) = 0$ for all $\omega \in \Omega$. Note that the only feasible within-coalition redistribution τ for a singleton $S = \{i\}$ is degenerate, i.e., $\tau_i(\omega) = 0$ for all $\omega \in \Omega$.

A **deception** for agent i is a mapping $\alpha_i : \mathcal{F}_i \to \mathcal{F}_i$. When agent i observes the event $\mathcal{F}_i(\omega)$, she can report $\alpha_i(\mathcal{F}_i(\omega))$ to the rest of the agents.[3] Specifically, let the identity mapping $\alpha_i^* : \mathcal{F}_i \to \mathcal{F}_i$ denote the trivial deception, i.e., truthful reporting by agent i.

We want to explore what happens when agents in S follow the profile of deceptions $\alpha_S = (\alpha_i)_{i \in S}$ and those out of S follow $\alpha_{-S}^* = (\alpha_i^*)_{i \notin S}$, i.e., truthful reporting. Given a feasible allocation $x \in L$, a coalition S, and a feasible within-coalition redistribution τ, a profile of deceptions $(\alpha_S, \alpha_{-S}^*)$ is said to be **compatible** at ω with respect to ω' if the following two conditions are satisfied:

1. undetectability: $[\bigcap\limits_{i \in S} \alpha_i(\mathcal{F}_i(\omega))] \cap [\bigcap\limits_{i \notin S} \alpha_i^*(\mathcal{F}_i(\omega))] = \{\omega'\}$;

[3] The present paper allows an agent to choose different deceptions for different allocations. Hence, one can also parameterize a deception with respect to the underlying allocation by defining a deception as a mapping $\alpha_i : \mathcal{F}_i \times L \to \mathcal{F}_i$. As the underlying allocation $x \in L$ is clear in Definitions 3 and 4, we follow Palfrey and Srivastava (1989) and Hahn and Yannelis (2001) to drop the L component for simplicity.

2. feasibility: $e_i(\omega) + z_i^x(\omega') + \tau_i(\omega') \in \mathbb{R}^l_+$ for all $i \in S$.

When $[\bigcap_{i \in S} \alpha_i(\mathcal{F}_i(\omega))] \cap [\bigcap_{i \notin S} \alpha_i^*(\mathcal{F}_i(\omega))] = \{\omega'\}$, we slightly abuse the notation and denote $(\alpha_S, \alpha^*_{-S})(\omega) = \omega'$. Note that when $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω', for $i \notin S$, it holds that $e_i(\omega) + z_i^x(\omega') = e_i(\omega) + x_i(\omega') - e_i(\omega') = x_i(\omega') \in \mathbb{R}^l_+$ by Assumption 3.

If a social planner wishes to design a mechanism to implement the allocation x, the mechanism should specify the outcomes induced by all profiles of deceptions that are not compatible. For example, in Glycopantis et al. (2003), the outcome of incompatible reports is no trade; in Palfrey and Srivastava (1989), if incompatible reports occur, each agent's initial endowment is fully appropriated and destroyed by the social planner. As this paper does not formally look into the problem of implementation, we focus on compatible reports when defining the notions of incentive compatibility.

Below we consider the maxmin individual incentive compatibility condition. Since the only feasible within-coalition redistribution τ for a singleton coalition is degenerate, we omit the redistribution in the following definition. When x satisfies the maxmin individual coalitional incentive compatibility condition, no individual can come up with a profitable deception that is compatible with other agents' truthful reporting.

Definition 3 A feasible allocation $x \in L$ is said to satisfy the **maxmin individual incentive compatibility** condition if there does not exist an agent $i \in I$, a set $E_i \in \mathcal{F}_i$, and a deception $\alpha_i : \mathcal{F}_i \to \mathcal{F}_i$, such that

1. for each $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that $(\alpha_i, \alpha^*_{-i})$ is compatible at ω with respect to ω';
2. $\min_{w \in E_i} u_i\Big(e_i(\omega) + z_i^x\big((\alpha_i, \alpha^*_{-i})(\omega)\big), \omega\Big) > \min_{w \in E_i} u_i\big(x_i(\omega), \omega\big)$.

The individual incentive compatibility condition defined above does not consider coordination within coalitions. Below we introduce the notion of maxmin transfer coalitional incentive compatibility. When x satisfies the maxmin transfer coalitional incentive compatibility condition, it should not be common knowledge to any group S that there exists a feasible within-coalition redistribution τ and a profile of deceptions α_S to improve over x. Otherwise, such a group would form a deceiving coalition.

Definition 4 A feasible allocation $x \in L$ is said to satisfy the **maxmin transfer coalitional incentive compatibility** condition if there does not exist a coalition S, a common knowledge information event $E \in \mathcal{F}_{\wedge S}$, a feasible within-coalition redistribution τ, and a profile of deceptions $(\alpha_i : \mathcal{F}_i \to \mathcal{F}_i)_{i \in S}$, such that

1. for each state $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω';
2. for each $i \in S$ and $\omega \in E$,

$$\begin{aligned}&\min_{w' \in \mathcal{F}_i(\omega)} u_i\Big(e_i(\omega') + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega')\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega')\big), \omega'\Big)\\ &\geq \min_{w' \in \mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big);\end{aligned}$$

3. there exists some $i \in S$ and $\omega \in E$ for which the above inequality holds strictly.

Notice that in the above definition, we do not require the profile of deceptions α_S to be different from α_S^*. Hence, if a group of agents facing $x \in L$ can benefit from redistribution and truthful reporting, then x violates the maxmin transfer coalitional incentive compatibility condition.

Remark 3 If an allocation $x \in L$ satisfies the maxmin transfer coalitional incentive compatibility condition, then it also satisfies maxmin individual incentive compatibility. This is because by restricting the coalitions to be singletons, the maxmin transfer coalitional incentive compatibility condition degenerates to the maxmin individual incentive compatibility condition.

In the following example, the allocation x satisfies maxmin individual incentive compatibility. However, it violates the maxmin transfer coalitional incentive compatibility condition.

Example 1 In a one-good economy with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and $I = \{1, 2, 3\}$, each agent's private information partition, prior over private information partition, utility function, and initial endowment are given as follows.

$$\begin{array}{llll}
\mathcal{F}_1=\{\{\omega_1,\omega_2\},\{\omega_3,\omega_4\}\}, & \mu_1(\{\omega_1,\omega_2\})=\mu_1(\{\omega_3,\omega_4\})=0.5, & u_1(a_1,\omega)=\sqrt{a_1}, & e_1=(2,2,2,2),\\
\mathcal{F}_2=\{\{\omega_1,\omega_3\},\{\omega_2,\omega_4\}\}, & \mu_2(\{\omega_1,\omega_3\})=\mu_2(\{\omega_2,\omega_4\})=0.5, & u_2(a_2,\omega)=\sqrt{a_2}, & e_2=(2,2,2,2),\\
\mathcal{F}_3=\{\{\omega_1,\omega_2,\omega_3,\omega_4\}\}, & \mu_3(\{\omega_1,\omega_2,\omega_3,\omega_4\})=1, & u_3(a_3,\omega)=\sqrt{a_3}, & e_3=(2,2,2,2).
\end{array}$$

Suppose agents sign a contract prescribing allocation x defined as follows.

$$\begin{aligned}
x_1 &= (4, 0, 0, 4),\\
x_2 &= (1, 3, 3, 1),\\
x_3 &= (1, 3, 3, 1).
\end{aligned}$$

We claim that x satisfies the maxmin individual incentive compatibility condition but does not satisfy maxmin transfer coalitional incentive compatibility.

To establish the maxmin individual incentive compatibility condition, we can consider agent 1 with private information $E_1 = \{\omega_1, \omega_2\}$ for example. Suppose she unilaterally deviates from truthful reporting and adopts a deception α_1 defined by $\alpha_1(\{\omega_1, \omega_2\}) = \alpha_1(\{\omega_3, \omega_4\}) = \{\omega_3, \omega_4\}$. Thus, $(\alpha_1, \alpha^*_{\{2,3\}})(\omega_1) = \omega_3$, $(\alpha_1, \alpha^*_{\{2,3\}})(\omega_2) = \omega_4$, $(\alpha_1, \alpha^*_{\{2,3\}})(\omega_3) = \omega_3$, and $(\alpha_1, \alpha^*_{\{2,3\}})(\omega_4) = \omega_4$. Also, $e_1(\omega) + z_1^x((\alpha_1, \alpha^*_{\{2,3\}})(\omega)) \in \mathbb{R}_+$ for all $\omega \in \Omega$. Hence, for each $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that $(\alpha_1, \alpha^*_{\{2,3\}})$ is compatible at ω with respect to ω'. However, since

$$\begin{aligned}
&\min_{\omega \in \{\omega_1,\omega_2\}} u_1\Big(e_1(\omega) + z_1^x\big((\alpha_1, \alpha^*_{\{2,3\}})(\omega)\big), \omega\Big)\\
&= \min\{\sqrt{2 + (0-2)}, \sqrt{2 + (4-2)}\} = 0\\
&= \min_{\omega \in \{\omega_1,\omega_2\}} u_1(x_1(\omega), \omega) = \min\{\sqrt{4}, \sqrt{0}\} = 0,
\end{aligned}$$

such a misreport is not profitable. Similarly, we can verify that there is no other way for agent 1 or agent 2 to profit from misreporting unilaterally. In addition, agent 3's private information partition is $\{\Omega\}$ and thus she can only truthfully report.

To see that x does not satisfy the maxmin transfer coalitional incentive compatibility condition, consider the coalition $S = \{1, 2\}$, its only common knowledge information event $E = \Omega$, deceptions defined by $\alpha_1(E_1) = \{\omega_1, \omega_2\}$ for all $E_1 \in \mathcal{F}_1$ and $\alpha_2(E_2) = \{\omega_1, \omega_3\}$ for all $E_2 \in \mathcal{F}_2$, and feasible within-coalition redistribution τ defined by $\tau_1(\omega) = -1.5$ and $\tau_2(\omega) = 1.5$ for all $\omega \in \Omega$. For each $\omega \in \Omega$ and $i \in \{1, 2\}$, $(\alpha_{\{1,2\}}, \alpha_3^*)(\omega) = \omega_1$ and $e_i(\omega) + z_i^x((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)) + \tau_i((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)) \in \mathbb{R}_+$. This implies that for all $\omega \in \Omega$, $(\alpha_{\{1,2\}}, \alpha_3^*)$ is compatible at ω with respect to ω_1. Under this coordination, agent 3 can only consume 1 unit of good regardless of the true state, and agents 1 and 2 jointly consume 2 extra units when the true state is ω_2 or ω_3. To formally verify that the manipulation is profitable for S, we demonstrate below that each agent in S earns a strictly higher interim maxmin expected utility under all private information events.

Agent 1 is better off under this coordination regardless of the event she observes, as

$$
\begin{aligned}
&\min_{w\in\{\omega_1,\omega_2\}} u_1\Big(e_1(\omega)+z_1^x\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big)+\tau_1\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big), \omega\Big)\\
&= \min\{\sqrt{2+(4-2)-1.5}, \sqrt{2+(4-2)-1.5}\} = \sqrt{2.5} > \min_{w\in\{\omega_1,\omega_2\}} u_1\big(x_1(\omega), \omega\big)\\
&= \min\{\sqrt{4}, \sqrt{0}\} = 0,\\
&\min_{w\in\{\omega_3,\omega_4\}} u_1\Big(e_1(\omega)+z_1^x\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big)+\tau_1\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big), \omega\Big)\\
&= \min\{\sqrt{2+(4-2)-1.5}, \sqrt{2+(4-2)-1.5}\} = \sqrt{2.5} > \min_{w\in\{\omega_3,\omega_4\}} u_1\big(x_1(\omega), \omega\big)\\
&= \min\{\sqrt{0}, \sqrt{4}\} = 0.
\end{aligned}
$$

Similarly, agent 2 is better off under both events, as

$$
\begin{aligned}
&\min_{w\in\{\omega_1,\omega_3\}} u_2\Big(e_2(\omega)+z_2^x\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big)+\tau_2\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big), \omega\Big)\\
&= \min\{\sqrt{2+(1-2)+1.5}, \sqrt{2+(1-2)+1.5}\} = \sqrt{2.5} > \min_{w\in\{\omega_1,\omega_3\}} u_2\big(x_2(\omega), \omega\big)\\
&= \min\{\sqrt{1}, \sqrt{3}\} = 1,\\
&\min_{w\in\{\omega_2,\omega_4\}} u_2\Big(e_2(\omega)+z_2^x\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big)+\tau_2\big((\alpha_{\{1,2\}}, \alpha_3^*)(\omega)\big), \omega\Big)\\
&= \min\{\sqrt{2+(1-2)+1.5}, \sqrt{2+(1-2)+1.5}\} = \sqrt{2.5} > \min_{w\in\{\omega_2,\omega_4\}} u_2\big(x_2(\omega), \omega\big)\\
&= \min\{\sqrt{3}, \sqrt{1}\} = 1.
\end{aligned}
$$

Hence, x fails to satisfy the maxmin transfer coalitional incentive compatibility condition.

3.1 Relationship to the literature

Below we discuss the connection of our notion of coalitional incentive compatibility with the existing ones in the literature.

In Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994), Glycopantis et al. (2001, 2003), and De Castro et al. (2011), a coalition S can profit by misreporting the realized state of nature to the complementary coalition. An essential difference of our maxmin transfer coalitional incentive compatibility is that in the present paper a deception is defined as a policy, i.e., for every private information event, whereas in these papers it is defined only conditionally on a certain event that contains the true state. Hahn and Yannelis (1997, 2001) define the deception as a policy as well. A coalitional manipulation happens in Hahn and Yannelis (1997, 2001) when there is a state at which the manipulation is profitable, whereas a coalitional manipulation happens in our paper when it is common knowledge for agents in the coalition that the manipulation is profitable.

A branch of the literature on general equilibrium with asymmetric information focuses on private information measurable allocations. A feasible allocation $x \in L$ is said to be **private information measurable** if for all $i \in I$, $E_i \in \mathcal{F}_i$, and $\omega, \omega' \in E_i$, it holds that $x_i(\omega) = x_i(\omega')$. Starting with Yannelis (1991), this literature uncovers interesting connections between the above discussed coalitional incentive compatibility notions and private information measurability of allocations. For example, Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994), and Glycopantis et al. (2003) show that in a one good economy, an allocation satisfies a coalitional incentive compatibility condition if and only if it is private information measurable. When there are more goods, Hahn and Yannelis (1997) establish that every private information measurable and Pareto efficient allocation satisfies another version of coalitional incentive compatibility condition. Now, we further establish a relationship between our maxmin transfer coalitional incentive compatibility condition and two related notions in the literature by focusing on private information measurable allocations.

One can consider the following two variants of Definitions 4.1 and 4.2 of Koutsougeras and Yannelis (1993) and Definition 1 of Krasa and Yannelis (1994).

Definition 5 A feasible allocation x is said to satisfy the **transfer coalitional incentive compatibility** condition if there does not exist a coalition S, states $a, b \in \Omega$, and a net-trade vector $t_i \in \mathbb{R}^l$ for all $i \in S$ with $\sum_{i \in S} t_i = 0$ such that

1. $a \in \mathcal{F}_i(b)$ for all $i \notin S$;
2. $e_i(a) + x_i(b) - e_i(b) + t_i \in \mathbb{R}^l_+$ for all $i \in S$;
3. $u_i(e_i(a) + x_i(b) - e_i(b) + t_i, a) > u_i(x_i(a), a)$ for all $i \in S$.

When x satisfies transfer coalitional incentive compatibility, it is impossible for any coalition to cheat the complementary coalition by making side payments within the coalition and misreporting the state which cannot be observed by the complementary coalition. This transfer coalitional incentive compatibility condition is slightly stronger than the one in Krasa and Yannelis (1994), which entails the nonexistence of two *different* states $a, b \in \Omega$, a coalition S, and a net-trade vector $t_i \in \mathbb{R}^l$ for all $i \in S$

with $\sum_{i \in S} t_i = 0$ such that the above three requirements are satisfied. In Definition 5, an allocation x fails to satisfy transfer coalitional incentive compatibility even if a coalition can benefit from following a feasible within-coalition redistribution and truthfully reporting the state.

Definition 6 A feasible allocation x is said to satisfy the **weak transfer coalitional incentive compatibility** condition if there does not exist a coalition S, states $a, b \in \Omega$, and a net-trade vector $t_i \in \mathbb{R}^l$ for all $i \in S$ with $\sum_{i \in S} t_i = 0$ such that

1. $\mathcal{F}_i(a) \in \mathcal{F}_{\wedge S}$ for all $i \in S$;
2. $a \in \mathcal{F}_i(b)$ for all $i \notin S$;
3. $e_i(a) + x_i(b) - e_i(b) + t_i \in \mathbb{R}^l_+$ for all $i \in S$;
4. $u_i(e_i(a) + x_i(b) - e_i(b) + t_i, a) > u_i(x_i(a), a)$ for all $i \in S$.

According to the above definition, when x satisfies weak transfer coalitional incentive compatibility, it is impossible for members of a coalition S to agree on an event in $\mathcal{F}_{\wedge S}$ that has occurred and to cheat the complementary coalition.

The following two propositions show that when x is private information measurable, Definition 5 implies Definition 4, which further implies Definition 6. Proposition 1 below relies on additional assumptions on utility functions. Given vectors $a_i, b_i \in \mathbb{R}^l_+$, we say $a_i > b_i$ if each dimension of a_i is weakly larger than b_i and $a_i \neq b_i$. In particular, we say a utility function $u_i(\cdot, \omega)$ is strictly monotone in private consumption if $a_i > b_i$ implies that $u_i(a_i, \omega) > u_i(b_i, \omega)$.

Proposition 1 *Suppose utility functions are continuous and strictly monotone in private consumption. If a feasible allocation $x \in L$ is private information measurable and satisfies the transfer coalitional incentive compatibility condition, then x satisfies the maxmin transfer coalitional incentive compatibility condition.*

Proof We prove by contrapositive. Suppose x does not satisfy the maxmin transfer coalitional incentive compatibility condition. Thus, there exists a coalition S, a common knowledge information event $E \in \mathcal{F}_{\wedge S}$, a feasible within-coalition redistribution τ, and a profile of deceptions $(\alpha_i : \mathcal{F}_i \to \mathcal{F}_i)_{i \in S}$, such that

1. for each state $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω';
2. for each $i \in S$ and $\omega \in E$,

$$
\begin{aligned}
&\min_{w' \in \mathcal{F}_i(\omega)} u_i\Big(e_i(\omega') + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega')\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega')\big), \omega'\Big) \\
&\geq \min_{w' \in \mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big);
\end{aligned}
$$

3. there exists some $i \in S$ and $\omega \in E$ for which the above inequality holds strictly.

Let $j \in S$ denote an agent and a denote a state in E for which the strict inequality holds. Define $b = (\alpha_S, \alpha^*_{-S})(a)$. Thus, it is clear that $b \in \mathcal{F}_i(a)$ for all $i \notin S$. Equivalently, we have $a \in \mathcal{F}_i(b)$ for all $i \notin S$.

From Assumption 4 and the fact that $b = (\alpha_S, \alpha^*_{-S})(a)$, we have that

$$
\begin{aligned}
& u_i\big(e_i(a) + x_i(b) - e_i(b) + \tau_i(b), a\big) \\
& \quad \geq \min_{w' \in \mathcal{F}_i(a)} u_i\Big(e_i(\omega') + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega')\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega')\big), \omega'\Big) \\
& \quad \geq \min_{w' \in \mathcal{F}_i(a)} u_i\big(x_i(\omega'), \omega'\big) = u_i\big(x_i(a), a\big) \qquad (4)
\end{aligned}
$$

for all $i \in S$. Note that the second inequality holds strictly for agent j. The last equality follows from Assumption 4 and the fact that x is private information measurable.

Since u_j is strictly monotone and $x_j(a) \in \mathbb{R}^l_+$, the fact that expression (4) is strict for j implies that $e_j(a) + x_j(b) - e_j(b) + \tau_j(b)$ is a nonzero vector in $\mathbb{R}^l_+$. Also, by continuity of u_j, there exists $\epsilon \in (0, 1)$ such that

$$
u_j\big(\epsilon[e_j(a) + x_j(b) - e_j(b) + \tau_j(b)], a\big) > u_j\big(x_j(a), a\big). \qquad (5)
$$

For each $i \in S$, define

$$
t_i = \begin{cases} \tau_j(b) - (1-\epsilon)[e_j(a) + x_j(b) - e_j(b) + \tau_j(b)] & \text{for } i = j, \\ \tau_i(b) + \frac{1-\epsilon}{|S|-1}[e_j(a) + x_j(b) - e_j(b) + \tau_j(b)] & \text{for } i \neq j. \end{cases}
$$

It is easy to verify that $\sum_{i \in S} t_i = 0$.

Since $e_j(a) + x_j(b) - e_j(b) + \tau_j(b)$ is a nonzero vector in $\mathbb{R}^l_+$, we have $t_i > \tau_i(b)$ for all $i \in S$ with $i \neq j$. Thus, for each $i \in S$ with $i \neq j$,

$$
e_i(a) + x_i(b) - e_i(b) + t_i > e_i(a) + x_i(b) - e_i(b) + \tau_i(b) \in \mathbb{R}^l_+.
$$

In addition, notice that

$$
\begin{aligned}
& e_j(a) + x_j(b) - e_j(b) + t_j \\
& \quad = e_j(a) + x_j(b) - e_j(b) + \tau_j(b) - (1-\epsilon)[e_j(a) + x_j(b) - e_j(b) + \tau_j(b)] \\
& \quad = \epsilon[e_j(a) + x_j(b) - e_j(b) + \tau_j(b)] \in \mathbb{R}^l_+. \qquad (6)
\end{aligned}
$$

Hence, we have verified that $e_i(a) + x_i(b) - e_i(b) + t_i \in \mathbb{R}^l_+$ for all $i \in S$.

Expressions (5) and (6) show that

$$
u_j\big(e_j(a) + x_j(b) - e_j(b) + t_j, a\big) > u_j\big(x_j(a), a\big).
$$

In addition, for all $i \in S$ with $i \neq j$,

$$
\begin{aligned}
u_i\big(e_i(a) + x_i(b) - e_i(b) + t_i, a\big) & > u_i\big(e_i(a) + x_i(b) - e_i(b) + \tau_i(b), a\big) \\
& \geq u_i\big(x_i(a), a\big),
\end{aligned}
$$

where the strict inequality follows from strict monotonicity of u_i and the fact that $t_i > \tau_i(b)$, and the weak inequality follows from expression (4).

To this end, we have established that x does not satisfy the transfer coalitional incentive compatibility condition. □

Proposition 2 *If a feasible allocation $x \in L$ is private information measurable and satisfies the maxmin transfer coalitional incentive compatibility condition, then it satisfies the weak transfer coalitional incentive compatibility condition.*

Proof We prove by contrapositive. Suppose that a private information measurable feasible allocation $x \in L$ violates the weak transfer coalitional incentive compatibility condition. Thus, there exists a coalition S, states $a, b \in \Omega$, and a net-trade vector $t_i \in \mathbb{R}^l$ for all $i \in S$ with $\sum_{i \in S} t_i = 0$ such that

1. $\mathcal{F}_i(a) \in \mathcal{F}_{\wedge S}$ for all $i \in S$;
2. $a \in \mathcal{F}_i(b)$ for all $i \notin S$;
3. $e_i(a) + x_i(b) - e_i(b) + t_i \in \mathbb{R}^l_+$ for all $i \in S$;
4. $u_i(e_i(a) + x_i(b) - e_i(b) + t_i, a) > u_i(x_i(a), a)$ for all $i \in S$.

Let E denote the common knowledge event $E \in \mathcal{F}_{\wedge S}$ such that $E = \mathcal{F}_i(a)$ for all $i \in S$.

For each $i \in S$, define $\alpha_i = \alpha_i^*$ and $\tau_i : \Omega \to \mathbb{R}^l$ as follows:

$$\tau_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) + t_i - x_i(\omega) & \text{for all } i \in S \text{ and } \omega \in E, \\ 0 & \text{elsewhere.} \end{cases}$$

We claim that τ is feasible. To see this, for each $\omega \in E$, $\sum_{i \in S} \tau_i(\omega) = \sum_{i \in S} [e_i(a) + x_i(b) - e_i(b) + t_i - x_i(\omega)] = \sum_{i \in S} [e_i(a) + x_i(b) - e_i(b) + t_i - x_i(a)] = \sum_{i \in S} [e_i(a) - x_i(a)] - \sum_{i \in S} [e_i(b) - x_i(b)] + \sum_{i \in S} t_i = -\sum_{i \notin S} [e_i(a) - x_i(a)] + \sum_{i \notin S} [e_i(b) - x_i(b)] = \sum_{i \notin S} [e_i(b) - e_i(a)] + \sum_{i \notin S} [x_i(a) - x_i(b)] = 0$, where the second equality follows from private information measurability of x and the fact that $\omega \in E = \mathcal{F}_i(a)$ for all $i \in S$, the fourth equality follows from feasibility of x and the fact that $\sum_{i \in S} t_i = 0$, and the sixth equality follows from private information measurability of x, Assumption 3, and the fact that $a \in \mathcal{F}_i(b)$ for all $i \notin S$. Also, for each $\omega \notin E$, $\sum_{i \in S} \tau_i(\omega) = 0$.

Notice that for each $\omega \in \Omega$, $(\alpha_S, \alpha^*_{-S})(\omega) = \omega$. Also, for each $i \in S$ and $\omega \in \Omega$, $e_i(\omega) + z_i^x(\omega) + \tau_i(\omega) = e_i(\omega) + x_i(\omega) - e_i(\omega) + \tau_i(\omega) = e_i(\omega) + x_i(\omega) - e_i(\omega) + e_i(a) + x_i(b) - e_i(b) + t_i - x_i(\omega) = e_i(a) + x_i(b) - e_i(b) + t_i \in \mathbb{R}^l_+$. Thus, for each $\omega \in \Omega$, $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω.

For each $i \in S$ and $\omega \in E$,

$$\begin{aligned} &\min_{w' \in \mathcal{F}_i(\omega)} u_i\Big(e_i(\omega') + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega')\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega')\big), \omega'\Big) \\ &\quad = u_i(e_i(a) + x_i(b) - e_i(b) + t_i, a) > u_i(x_i(a), a) = \min_{w' \in \mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big), \end{aligned}$$

where the equalities rely on the private information measurability of x, Assumption 4, and the definition of τ, and the strict inequality follows from the supposition that x violates the weak transfer coalitional incentive compatibility condition.

To this end, we have established that x does not satisfy the maxmin transfer coalitional incentive compatibility condition. □

It is worth mentioning that the private information measurability constraint on the allocation x is solely imposed to study the relationship between our maxmin transfer coalitional incentive compatibility condition and related notions in the literature. Out of Sect. 3.1, we do not impose the private information measurability constraint on an allocation x when studying its incentive compatibility properties.

4 Main result

4.1 Characterization

The following theorem provides a characterization of allocations satisfying the maxmin transfer coalitional incentive compatibility condition. In fact, maxmin interim Pareto efficiency is the necessary and sufficient condition for an allocation to satisfy maxmin transfer coalitional incentive compatibility.

Theorem 1 *A feasible allocation $x \in L$ satisfies the maxmin transfer coalitional incentive compatibility condition if and only if it is maxmin interim Pareto efficient.*

Proof First, we demonstrate the **if** direction. Let $x \in L$ be a feasible maxmin interim Pareto efficient allocation. Suppose by way of contradiction that x does not satisfy the maxmin transfer coalitional incentive compatibility condition. Then, there exists a coalition S, a common knowledge information event $E \in \mathcal{F}_{\wedge S}$, a feasible within-coalition redistribution τ, and a profile of deceptions α_S, such that

1. for each state $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω';
2. for each $i \in S$ and $\omega \in E$,

$$\begin{aligned}&\min_{w' \in \mathcal{F}_i(\omega)} u_i\Big(e_i(\omega') + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega')\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega')\big), \omega'\Big)\\ &\geq \min_{w' \in \mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big);\end{aligned}$$

3. there exists some $i \in S$ and $\omega \in E$ for which the above inequality holds strictly.

Define

$$y_i(\omega) = \begin{cases} e_i(\omega) + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega)\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big) & \text{for } i \in S \text{ and } \omega \in E,\\ e_i(\omega) + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega)\big) & \text{for } i \notin S \text{ and } \omega \in E,\\ x_i(\omega) & \text{for } i \in I \text{ and } \omega \notin E. \end{cases}$$

The allocation y is feasible. To see this, for each $\omega \in E$,

$$\begin{aligned}\sum_{I} y_i(\omega) &= \sum_{i\in S}[e_i(\omega) + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega)\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big)] \\ &\quad + \sum_{i\in S^c}[e_i(\omega) + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega)\big)] \\ &= \sum_{i\in I}[e_i(\omega) + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega)\big)] = \sum_{i\in I} e_i(\omega),\end{aligned}$$

where the first equality comes from the definition of y, the second follows from the feasibility of τ, and the third is a result of the feasibility of x. For each $\omega \notin E$, we have

$$\sum_{i\in I} y_i(\omega) = \sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega),$$

where the equalities come from the definition of y and the feasibility of x again.

Now we show that y Pareto improves upon x in the interim stage.

For each $i \in S$ and $\omega \in E$, the definition of y implies that

$$\min_{w'\in\mathcal{F}_i(\omega)} u_i\big(y_i(\omega'), \omega'\big) \geq \min_{w'\in\mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big) \tag{7}$$

and the strict inequality holds for some $i \in S$ and $\omega \in E$.

For $i \in S$ and $\omega \notin E$, the definition of y implies that

$$\min_{w'\in\mathcal{F}_i(\omega)} u_i\big(y_i(\omega'), \omega'\big) = \min_{w'\in\mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big). \tag{8}$$

For all $i \notin S$ and $\omega \in \Omega$, define $Y_i(\omega) = \{y_i(\omega')|\omega' \in \mathcal{F}_i(\omega)\}$ and $X_i(\omega) = \{x_i(\omega')|\omega' \in \mathcal{F}_i(\omega)\}$. Now, we show that $Y_i(\omega) \subseteq X_i(\omega)$ for all $i \notin S$ and $\omega \in \Omega$. To see this, for $i \notin S$ and $\omega \in E$,

$$y_i(\omega) = e_i(\omega) + x_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big) - e_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big) = x_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big) \in X_i(\omega),$$

where the first equality follows from the definition of y and the second follows from Assumption 3 and the fact that $i \notin S$. The claim that $x_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big) \in X_i(\omega)$ comes from the definition of $X_i(\omega)$ and the fact that $i \notin S$. Also, for $i \notin S$ and $\omega \notin E$, $y_i(\omega) = x_i(\omega) \in X_i(\omega)$. Thus, we have verified that $Y_i(\omega) \subseteq X_i(\omega)$ for all $i \notin S$ and $\omega \in \Omega$. Then for all $i \notin S$ and $\omega \in \Omega$,

$$\begin{aligned}\min_{w'\in\mathcal{F}_i(\omega)} u_i\big(y_i(\omega'), \omega'\big) &= \min_{\bar{y}_i\in Y_i(\omega)} u_i\big(\bar{y}_i, \omega\big) \geq \min_{\bar{x}_i\in X_i(\omega)} u_i\big(\bar{x}_i, \omega\big) \\ &= \min_{w'\in\mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big).\end{aligned} \tag{9}$$

Notice that the two equalities follow from Assumption 4 and the definition of sets $Y_i(\omega)$ and $X_i(\omega)$. The inequality follows from the minimization operation and the fact that $Y_i(\omega) \subseteq X_i(\omega)$.

The above argument shows that y is weakly more profitable for each agent $i \in I$ and each private information event she holds. In addition, y is strictly more profitable for some agent-event pair. Hence, the feasible allocation y Pareto dominates x in the interim stage, a contradiction.

To verify the **only if** direction, suppose by way of contradiction that x satisfies maxmin transfer coalitional incentive compatibility but not maxmin interim Pareto efficiency. Then there exists a feasible allocation $y \in L$ such that for all $i \in I$ and $E_i \in \mathcal{F}_i$,

$$\min_{w \in E_i} u_i\big(y_i(\omega), \omega\big) \geq \min_{w \in E_i} u_i\big(x_i(\omega), \omega\big) \tag{10}$$

and the strict inequality holds for some $i^* \in I$ and $E^*_{i^*} \in \mathcal{F}_{i^*}$.

Consider the grand coalition I. Let E be the set in $\mathcal{F}_{\wedge I}$ that contains $E^*_{i^*}$. We define $\tau_i(\omega) = y_i(\omega) - x_i(\omega)$ for all $i \in I$ and $\omega \in \Omega$. It is easy to see that $(\tau_i)_{i \in I}$ is a feasible within-coalition redistribution since

$$\sum_{i \in I} \tau_i(\omega) = \sum_{i \in I} y_i(\omega) - \sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega) - \sum_{i \in I} e_i(\omega) = 0$$

for all $\omega \in \Omega$.

Consider the profile of deceptions $\alpha = \alpha^*$. Notice that for each ω and $i \in I$, $\alpha(\omega) = \omega$ and $e_i(\omega) + z_i^x(\alpha(\omega)) + \tau_i(\alpha(\omega)) = y_i(\omega) \in \mathbb{R}^l_+$. Thus, for each $\omega \in \Omega$, $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω. Furthermore, by expression (10), for all $i \in I$ and $\omega \in E$,

$$\min_{w' \mathcal{F}_i(\omega)} u_i\big(e_i(\omega') + z_i^x(\alpha(\omega')) + \tau_i(\alpha(\omega')), \omega'\big) \geq \min_{w' \mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big)$$

and the strict inequality holds for $i^* \in I$ and every $\omega \in E^*_{i^*} \subseteq E$.

Hence, x does not satisfy the maxmin transfer coalitional incentive compatibility condition, a contradiction. □

Recall Remark 3, the "if" direction in Theorem 1 implies one of the main results of De Castro and Yannelis (2018).

Corollary 1 *[De Castro and Yannelis (2018)] A maxmin Pareto efficient allocation $x \in L$ satisfies the maxmin individual incentive compatibility condition.*

Remark 4 Maxmin interim Pareto efficiency does not characterize maxmin individual incentive compatibility, as a feasible allocation satisfying maxmin individual incentive compatibility may not be maxmin interim Pareto efficient. This can be seen from allocation x in Example 1. In the example, we have verified that x satisfies maxmin individual incentive compatibility. Nevertheless, the allocation x is not maxmin interim Pareto efficient, since x is Pareto dominated by e in the interim stage.

Under the Bayesian framework, the requirement of incentive compatibility often reduces efficiency. Our Theorem 1 reinforces the implication of De Castro and Yannelis (2018) that ambiguity resolves the conflict between efficiency and incentive compatibility. In addition, we identify the version of incentive compatibility condition that coincides with maxmin interim Pareto efficiency: the maxmin transfer coalitional incentive compatibility condition, which is stronger and more stable than maxmin individual incentive compatibility. Hence, under the maxmin framework, the more demanding requirement of maxmin transfer coalitional incentive compatibility can be understood as one of efficiency. This implies that efficiency, incentive compatibility, and stability go hand in hand in the maxmin framework.

4.2 Discussion

In this subsection, we discuss two underlying assumptions that are needed for Theorem 1 to hold.

The first key assumption is the Wald-type maxmin preference setup. De Castro and Yannelis (2018) have established that the maxmin preference is the only preference under which all efficient allocations are individually incentive compatible. Under the Bayesian framework where we replace the maxmin expected utility with the subjective expected utility, Pareto efficiency does not imply individual incentive compatibility (and therefore does not imply transfer coalitional incentive compatibility). This means that Theorem 1 fails when agents do not have maxmin preferences.

To see this, we consider the following example under two frameworks. Under maxmin preferences, maxmin interim Pareto efficiency is consistent with maxmin transfer coalitional incentive compatibility. However, under a subjective expected utility (Bayesian) framework, the allocation x is ex-ante and interim Pareto efficient, but fails to satisfy Bayesian individual incentive compatibility.

Example 2 Consider a one-good economy with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and $I = \{1, 2\}$. Each agent's private information partition, prior over private information partition, utility function, and initial endowment are given as follows.

$$
\begin{array}{llll}
\mathcal{F}_1=\{\{\omega_1,\omega_2\},\{\omega_3,\omega_4\}\}, & \mu_1(\{\omega_1,\omega_2\})=\mu_1(\{\omega_3,\omega_4\})=0.5, & u_1(a_1,\omega)=\sqrt{a_1}, & e_1=(3,3,1,1),\\
\mathcal{F}_2=\{\{\omega_1,\omega_3\},\{\omega_2,\omega_4\}\}, & \mu_2(\{\omega_1,\omega_3\})=\mu_2(\{\omega_2,\omega_4\})=0.5, & u_2(a_2,\omega)=\sqrt{a_2}, & e_2=(3,1,3,1).
\end{array}
$$

The allocation $x \in L$ defined below gives both agents equal consumption.

$$
\begin{aligned}
x_1 &= (3, 2, 2, 1),\\
x_2 &= (3, 2, 2, 1).
\end{aligned}
$$

Framework 1. Maxmin Expected Utility

We claim that under the maxmin expected utility framework, allocation e is maxmin ex-ante Pareto efficient (and thus maxmin interim Pareto efficient) and satisfies maxmin transfer coalitional incentive compatibility (and thus maxmin individual incentive compatibility).

To show that e is maxmin ex-ante Pareto efficient, we assume by way of contradiction that there are constant numbers c_1, c_2, c_3, and c_4 such that the allocation $x' \in L$ defined below Pareto dominates e in the ex-ante stage. Notice that by adjusting constants c_1 to c_4, x' can be used to denote any feasible allocation.

$$x'_1 = (3 + c_1, 3 + c_2, 1 + c_3, 1 + c_4),$$
$$x'_2 = (3 - c_1, 1 - c_2, 3 - c_3, 1 - c_4).$$

We discuss two cases. First, suppose $\min\{c_1, c_2\} \geq 0$. We know that $x'_2(\omega_1) = 3 - c_1 \leq 3$ and $x'_2(\omega_2) = 1 - c_2 \leq 1$. Hence, under x', agent 2's ex-ante maxmin expected utility cannot be higher than that under e. The only way for allocation x' to Pareto dominate e in the ex-ante stage is that agent 1's ex-ante maxmin expected utility under x' should be higher than that under e. Then, we must have $\min\{c_1, c_2\} > 0$ or $\min\{c_3, c_4\} > 0$. However, this will result in a decrease in agent 2's ex-ante maxmin expected utility, contradicting the supposition that x' Pareto dominates e in the ex-ante stage. Second, suppose $\min\{c_1, c_2\} < 0$. One must have $\min\{c_3, c_4\} > 0$ in order for allocation x' to at least weakly improve agent 1's ex-ante maxmin expected utility compared to e. The fact that $\min\{c_3, c_4\} > 0$ implies that agent 2's ex-ante maxmin expected utility is strictly lower under x' than under e, contradicting the supposition that x' Pareto dominates e in the ex-ante stage. Both cases lead to contradictions, implying that e is maxmin ex-ante Pareto efficient. Then by Remark 1, e is also interim Pareto efficient.

To establish that e satisfies maxmin transfer coalitional incentive compatibility, notice that the only coalitions in this problem are $\{1\}$, $\{2\}$, and I. The net trade associated with e is always zero for both agents. Hence, the allocation e is immune to unilateral misreporting of $\{1\}$ or $\{2\}$. Notice that e is maxmin interim Pareto efficient, and that the only common knowledge information event of I is $E = \Omega$. We thus know that there does not exist a profitable deviation of the grand coalition either. Therefore, e satisfies maxmin transfer coalitional incentive compatibility.

We also claim that allocation x is neither ex-ante maxmin Pareto efficient, nor interim maxmin Pareto efficient. It also violates the maxmin individual incentive compatibility condition, and thus the maxmin transfer coalitional incentive compatibility condition.

It is easy to check that x is Pareto dominated by e in both the ex-ante stage and the interim stage. Hence, x is neither ex-ante nor interim maxmin Pareto efficient.

To see that x does not satisfy maxmin individual incentive compatibility, consider agent 1, an event $E_1 = \{\omega_1, \omega_2\} \in \mathcal{F}_1$, and the deception α_1 defined by $\alpha_1(\{\omega_1, \omega_2\}) = \alpha_1(\{\omega_3, \omega_4\}) = \{\omega_3, \omega_4\}$. Then it is easy to check that for each $\omega \in \Omega$, there exists ω' such that (α_1, α_2^*) is compatible at ω with respect to ω'. As

$$\begin{aligned} &\min_{\omega \in E_1} u_1\Big(e_1(\omega) + z_1^x\big((\alpha_1, \alpha_2^*)(\omega)\big), \omega\Big) = \min\{\sqrt{3+(2-1)}, \sqrt{3+(1-1)}\} = \sqrt{3} \\ &\quad > \min_{\omega \in E_1} u_1\big(x_1(\omega), \omega\big) = \min\{\sqrt{3}, \sqrt{2}\} = \sqrt{2}, \end{aligned}$$

x does not satisfy maxmin individual incentive compatibility.

Framework 2. Subjective Expected Utility

Suppose that agents hold subjective beliefs instead of ambiguous beliefs and that the subjective beliefs are generated by a common prior $\pi(\omega) = 0.25$ for all $\omega \in \Omega$. Assume that agents use the subjective expected utility to evaluate their payoffs in the ex-ante stage and the interim stage.

We claim that x is Bayesian interim Pareto efficient (defined by replacing the maxmin expected utility with the subjective expected utility in Definition 2), but does not satisfy the Bayesian individual incentive compatibility condition (defined by replacing the maxmin expected utility with the subjective expected utility in Definition 3).

To establish that x is Bayesian interim Pareto efficient, suppose by way of contradiction that a feasible allocation x'' Pareto dominates x in the interim stage. Then

1. for all $i \in I$ and $E_i \in \mathcal{F}_i$,

$$\sum_{w \in E_i} 0.5 u_i\big(x_i''(\omega), \omega\big) \geq \sum_{w \in E_i} 0.5 u_i\big(x_i(\omega), \omega\big);$$

2. the strict inequality holds for some $i \in I$ and $E_i \in \mathcal{F}_i$.

A weighted sum of the above inequalities implies that

$$\sum_{i \in I} \sum_{w \in \Omega} 0.25 u_i(x_i''(\omega), \omega) > \sum_{i \in I} \sum_{w \in \Omega} 0.25 u_i(x_i(\omega), \omega),$$

or equivalently,

$$\begin{aligned}\sum_{i \in I} \sum_{w \in \Omega} 0.25 \sqrt{x_i''(\omega)} > \sum_{i \in I} \sum_{w \in \Omega} 0.25 \sqrt{x_i(\omega)} \\ = 2[0.25\sqrt{3} + 0.25\sqrt{2} + 0.25\sqrt{2} + 0.25\sqrt{1}] \\ = 0.5\sqrt{3} + \sqrt{2} + 0.5,\end{aligned}$$

contradicting the feasibility of x''.

To establish that x is not Bayesian individually incentive compatible, consider agent 1, an event $E_1 = \{\omega_1, \omega_2\} \in \mathcal{F}_1$, and a deception α_1 defined by $\alpha_1(\{\omega_1, \omega_2\}) = \alpha_1(\{\omega_3, \omega_4\}) = \{\omega_3, \omega_4\}$. Then it is easy to check that for each $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that (α_1, α_2^*) is compatible at ω with respect to ω'. Notice that

$$\begin{aligned}&\sum_{\omega \in E_1} 0.5 u_1\Big(e_1(\omega) + z_1^x\big((\alpha_1, \alpha_2^*)(\omega)\big), \omega\Big) \\ &= 0.5\sqrt{3+(2-1)} + 0.5\sqrt{3+(1-1)} = 1 + 0.5\sqrt{3} \\ &> \sum_{\omega \in E_1} 0.5 u_1\big(x_1(\omega), \omega\big) = 0.5\sqrt{3} + 0.5\sqrt{2}.\end{aligned}$$

As a result, x is not Bayesian individually incentive compatible and thus violates the Bayesian transfer coalitional incentive compatibility condition (defined by replacing the maxmin expected utility in Definition 4 with the subjective expected utility).

The second essential underlying assumption for Theorem 1 is that a coalition deviates when it is profitable to do so under the coalition's common knowledge information structure. The focus on common knowledge information is important to establish the equivalence between maxmin interim Pareto efficiency and maxmin transfer coalitional incentive compatibility.

It is also possible that agents within a coalition communicate with each other to acquire additional information about the state. For example, the finest form of communication is that agents in a coalition S truthfully pool private information with each other. Under the pooling information assumption, the following alternative incentive compatibility notion can be considered.

Definition 7 A feasible allocation $x \in L$ is said to satisfy the **maxmin fine transfer coalitional incentive compatibility** condition if there does not exist a coalition S, a pooled information event $E \in \mathcal{F}_{\vee S}$, a feasible within-coalition redistribution τ, and a profile of deceptions $(\alpha_i : \mathcal{F}_i \to \mathcal{F}_i)_{i \in S}$ such that

1. for each $\omega \in \Omega$, there exists $\omega' \in \Omega$ such that $(\alpha_S, \alpha^*_{-S})$ is compatible at ω with respect to ω';
2. for each $i \in S$ and $\omega \in E$,

$$\min_{w \in E} u_i\Big(e_i(\omega) + z_i^x\big((\alpha_S, \alpha^*_{-S})(\omega)\big) + \tau_i\big((\alpha_S, \alpha^*_{-S})(\omega)\big), \omega\Big) \geq \min_{w \in E} u_i\big(x_i(\omega), \omega\big);$$

3. there exists some $i \in S$ and $\omega \in E$ for which the above inequality holds strictly.

In the above definition, agents in S truthfully pool their information and form the information structure $\mathcal{F}_{\vee S}$. Knowing that a pooled information event $E \in \mathcal{F}_{\vee S}$ has happened, each agent i makes decisions with the worst state in E.

If we adopt such a notion of incentive compatibility, the characterization in Theorem 1 fails. Specifically, neither maxmin fine transfer coalitional incentive compatibility nor maxmin interim Pareto efficiency implies the other. To see this, we consider the following example.

Example 3 Consider a two-good economy with $I = \{1, 2\}$ and $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. Each agent's information partition, prior over private information partition, and utility function are given as follows.

$$\begin{array}{lll}
\mathcal{F}_1 = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}, & \mu_1(\{\omega_1, \omega_2\}) = \mu_1(\{\omega_3, \omega_4\}) = 0.5, & u_1(a_1, \omega) = \min\{a_1^1, a_1^2\}, \\
\mathcal{F}_2 = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}, & \mu_2(\{\omega_1, \omega_3\}) = \mu_2(\{\omega_2, \omega_4\}) = 0.5, & u_2(a_2, \omega) = \min\{a_2^1, a_2^2\}.
\end{array}$$

Let the initial endowment e and the contract x be given as follows.

$$\begin{array}{ll}
e_1 = ((2, 3), (2, 3), (4, 3), (4, 3)), & x_1 = ((3, 3), (1, 1), (3, 1), (4, 4)), \\
e_2 = ((4, 3), (2, 3), (4, 3), (2, 3)), & x_2 = ((3, 3), (3, 5), (5, 5), (2, 2)).
\end{array}$$

We claim that the maxmin interim Pareto efficient allocation e does not satisfy maxmin fine transfer coalitional incentive compatibility. In addition, the allocation x

satisfies maxmin fine transfer coalitional incentive compatibility, but is not maxmin interim Pareto efficient.

To see that e is maxmin interim Pareto efficient, we assume by way of contradiction that a feasible allocation y Pareto dominates e in the interim stage. Suppose agent 1 with private information $\{\omega_1, \omega_2\}$ is strictly better off under y than e. Then we must have $y_1^1(\omega_2) > 2$. However, by feasibility of y, one must have $y_2^1(\omega_2) < 2$, which means that y makes agent 2 with private information $\{\omega_2, \omega_4\}$ worse off compared to e, a contradiction. Adopting a similar argument, we can verify that y cannot make any agent with any private information strictly better off without hurting the other agent. Hence, e is interim maxmin Pareto efficient.

To see that e does not satisfy maxmin fine transfer coalitional incentive compatibility, consider $S = \{1, 2\}$. As $\mathcal{F}_{\vee S}$ contains only singletons, by pooling their private information, agents in S know the state that has occurred. Consider the pooled information event $E = \{\omega_1\} \in \mathcal{F}_{\vee S}$, a redistribution τ satisfying $\tau_1(\omega) = (1, 0)$ and $\tau_2(\omega) = (-1, 0)$ for all $\omega \in \Omega$, and a profile of deceptions α^*. It is easy to see that for each $\omega \in \Omega$, α^* is compatible at ω with respect to ω. Under event E, the manipulation is strictly more profitable for agent 1 and weakly more profitable for agent 2, since

$$
\begin{aligned}
&u_1\Big(e_1(\omega_1) + z_1^e\big(\alpha(\omega_1)\big) + \tau_1\big(\alpha(\omega_1)\big), \omega_1\Big) \\
&\quad = \min\{2 + (2 - 2) + 1, 3 + (3 - 3) + 0\} = 3 \\
&\quad > u_1\big(e_1(\omega_1), \omega_1\big) = \min\{2, 3\} = 2, \\
&u_2\Big(e_2(\omega_1) + z_2^e\big(\alpha(\omega_1)\big) + \tau_2\big(\alpha(\omega_1)\big), \omega_1\Big) \\
&\quad = \min\{4 + (4 - 4) - 1, 3 + (3 - 3) + 0\} = 3 \\
&\quad \geq u_2\big(e_2(\omega_1), \omega_1\big) = \min\{4, 3\} = 3.
\end{aligned}
$$

Note that agents' maxmin expected utility degenerates to ex-post utility as E is a singleton. Hence, allocation e violates the maxmin fine transfer coalitional incentive compatibility condition.

To see that x is not interim Pareto efficient, notice that it is Pareto dominated by e in the interim stage. Following the standard approach, we can verify that x satisfies maxmin individual incentive compatibility. Also, notice that at each state of nature, $x(\omega)$ is Pareto efficient and thus the grand coalition does not have the incentive to deviate after members in it pool their private information. Hence, x satisfies the maxmin fine transfer coalitional incentive compatibility condition.

5 Applications

In this section, we present a few general equilibrium solution concepts under the maxmin expected utility framework. Then, we show that these solution concepts satisfy the maxmin transfer coalitional incentive compatibility condition.

5.1 Core

Core notions are important stable notions in the sense that no coalition can redistribute its initial endowments in a way that makes agents in the coalition better off. Under asymmetric information and the Bayesian framework, it has been established by Palfrey and Srivastava (1987) that generally, core notions are not incentive compatible. De Castro et al. (2011) and Moreno-García and Torres-Martínez (2020) extend the core notions to environments with ambiguity-averse agents. By adopting the maxmin framework and following the spirit of ex-ante and interim domination of Holmström and Myerson (1983), we consider two core notions that are adaptations of the ones in De Castro et al. (2011). The two notions satisfy maxmin transfer coalitional incentive compatibility.

Definition 8 A feasible allocation $x \in L$ is said to be a **maxmin ex-ante core** allocation if there does not exist a coalition S and an allocation $y \in L$ such that

1. $\sum\limits_{i \in S} y_i(\omega) = \sum\limits_{i \in S} e_i(\omega)$ for all $\omega \in \Omega$;
2. for all $i \in S$,
$\sum\limits_{E_i \in \mathcal{F}_i} \min_{w \in E_i} u_i\big(y_i(\omega), \omega\big)\mu_i(E_i) \geq \sum\limits_{E_i \in \mathcal{F}_i} \min_{w \in E_i} u_i\big(x_i(\omega), \omega\big)\mu_i(E_i)$;
3. the strict inequality holds for some $i \in S$.

Definition 9 A feasible allocation $x \in L$ is said to be a **maxmin interim core** allocation if there does not exist a coalition S, a common knowledge information event $E \in \mathcal{F}_{\wedge S}$, and a feasible allocation $y \in L$ such that

1. $\sum\limits_{i \in S} y_i(\omega) = \sum\limits_{i \in S} e_i(\omega)$ for all $\omega \in \Omega$;
2. for all $i \in S$ and $\omega \in E$,
$$\min_{\omega' \in \mathcal{F}_i(\omega)} u_i\big(y_i(\omega'), \omega'\big) \geq \min_{\omega' \in \mathcal{F}_i(\omega)} u_i\big(x_i(\omega'), \omega'\big);$$
3. the strict inequality holds for some $i \in S$ and $\omega \in E$.

Corollary 2 *A maxmin ex-ante core allocation satisfies the maxmin transfer coalitional incentive compatibility condition. A maxmin interim core allocation satisfies the maxmin transfer coalitional incentive compatibility condition.*

Proof By setting $S = I$, we can see that when an allocation $x \in L$ is a maxmin ex-ante core allocation, it is maxmin ex-ante Pareto efficient. By Remark 1, x is also maxmin interim Pareto efficient. According to Theorem 1, it satisfies the maxmin transfer coalitional incentive compatibility condition.

By setting $S = I$, we know that when an allocation $x \in L$ is a maxmin interim core allocation, it is maxmin interim Pareto efficient. By Theorem 1, x satisfies the maxmin transfer coalitional incentive compatibility condition. □

5.2 Value

The Shapley value is a widely used solution concept in economic theory that measures each agent's marginal contribution to coalitions that she is a member of. A value allocation is a fair notion in the sense that agents obtain resources that reflect their Shapley values. Angelopoulos and Koutsougeras (2015) extend this idea to an asymmetric information environment with maxmin preferences. In this paper, we consider the maxmin ex-ante value allocation and the maxmin interim value allocation, and then verify the maxmin transfer coalitional incentive compatibility for the two solution concepts.

We first define the maxmin ex-ante value allocation. Given a weight vector $\lambda \in \mathbb{R}^n_+$, define an ex-ante characteristic function $V_\lambda : 2^I \to \mathbb{R}$ of a transferable utility game by $V_\lambda(\emptyset) = 0$ and for each coalition $S \subseteq I$,

$$V_\lambda(S) = \max_{x \in L} \left\{ \sum_{i \in S} [\lambda_i \sum_{E_i \in \mathcal{F}_i} \min_{\omega \in E_i} u_i(x_i(\omega), \omega)\mu_i(E_i)] | \sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega), \forall \omega \in \Omega \right\}.$$

For each $\lambda \in \mathbb{R}^n_+$ and disjoint coalitions $S^1, S^2 \subseteq I$, it is easy to see that $V_\lambda(S^1 \cup S^2) \geq V_\lambda(S^1) + V_\lambda(S^2)$. Given $\lambda \in \mathbb{R}^n_+$, the ex-ante Shapley value of agent i is defined as

$$Sh_i(V_\lambda) = \sum_{S \ni i} \frac{(|S| - 1)!(|I| - |S|)!}{|I|!} [V_\lambda(S) - V_\lambda(S \backslash \{i\})].$$

Definition 10 A feasible allocation $x \in L$ is said to be a **maxmin ex-ante value** allocation if there exists a weight vector $\lambda \in \mathbb{R}^n_+$ such that

$$\lambda_i \sum_{E_i \in \mathcal{F}_i} \min_{\omega \in E_i} u_i(x_i(\omega), \omega)\mu_i(E_i) = Sh_i(V_\lambda), \forall i \in I.$$

To introduce the maxmin interim value allocation, one needs to consider a state-dependent weight vector $\lambda(\omega) \in \mathbb{R}^n_+$ for each $\omega \in \Omega$. Given the state $\omega \in \Omega$ and weight vector $\lambda(\omega)$, define the interim characteristic function $V_{\lambda,\omega} : 2^I \to \mathbb{R}$ by $V_{\lambda,\omega}(\emptyset) = 0$ and for each coalition $S \subseteq I$,

$$V_{\lambda,\omega}(S) = \max_{x \in L} \left\{ \sum_{i \in S} [\lambda_i(\omega) \min_{\omega' \in \mathcal{F}_i(\omega)} u_i(x(\omega'), \omega')] | \sum_{i \in S} x_i(\omega') = \sum_{i \in S} e_i(\omega'), \forall \omega' \in \Omega \right\}.$$

One can show that for each state $\omega \in \Omega$, weight vector $\lambda(\omega) \in \mathbb{R}^n_+$, and disjoint coalitions $S^1, S^2 \subseteq I$, it holds that $V_{\lambda,\omega}(S^1 \cup S^2) \geq V_{\lambda,\omega}(S^1) + V_{\lambda,\omega}(S^2)$. Given $\omega \in \Omega$ and $\lambda(\omega) \in \mathbb{R}^n_+$, the interim Shapley value of agent i under state ω is defined as

$$Sh_i(V_{\lambda,\omega}) = \sum_{S \ni i} \frac{(|S| - 1)!(|I| - |S|)!}{|I|!} [V_{\lambda,\omega}(S) - V_{\lambda,\omega}(S \backslash \{i\})].$$

Definition 11 A feasible allocation $x \in L$ is said to be a **maxmin interim value** allocation if for $\omega \in \Omega$, there exists a weight vector $\lambda(\omega) \in \mathbb{R}^n_+$ such that

$$\lambda_i(\omega) \min_{\omega' \in \mathcal{F}_i(\omega)} u_i(x_i(\omega'), \omega') = Sh_i(V_{\lambda,\omega}), \forall i \in I.$$

Subsequently, we demonstrate that both notions defined above satisfy the maxmin transfer coalitional incentive compatibility condition. We say that a vector is in $\mathbb{R}^n_{++}$ if each dimension of the vector is positive.

Corollary 3 *If there exists a weight vector $\lambda \in \mathbb{R}^n_{++}$ such that x is a maxmin ex-ante value allocation, then x satisfies the maxmin transfer coalitional incentive compatibility condition. If for each $\omega \in \Omega$ there exists a weight function $\lambda(\omega) \in \mathbb{R}^n_{++}$ such that x is a maxmin interim value allocation, then x satisfies the maxmin transfer coalitional incentive compatibility condition.*

Proof Given a maxmin ex-ante value allocation $x \in L$ under weight vector $\lambda \in \mathbb{R}^n_{++}$, we want to show that x is maxmin interim Pareto efficient. Suppose by way of contradiction that a feasible allocation $y \in L$ Pareto dominates x in the ex-ante stage. Then

$$\begin{aligned}\sum_{i \in I}\left[\lambda_i \sum_{E_i \in \mathcal{F}_i} \min_{\omega \in E_i} u_i(y_i(\omega), \omega)\mu_i(E_i)\right] &> \sum_{i \in I}\left[\lambda_i \sum_{E_i \in \mathcal{F}_i} \min_{\omega \in E_i} u_i(x_i(\omega), \omega)\mu_i(E_i)\right] \\ &= \sum_{i \in I} Sh_i(V_\lambda) = V_\lambda(I),\end{aligned}$$

where the first equality follows from the definition of maxmin ex-ante vallue allocation, and the second equality follows from the definition of $Sh_i(V_\lambda)$. However, as y is feasible, the above expression contradicts the definition of $V_\lambda(I)$. Hence, x has to be maxmin ex-ante Pareto efficient. By Remark 1 and Theorem 1, x satisfies the maxmin interim efficiency condition and thus maxmin transfer coalitional incentive compatibility.

Given a maxmin interim value allocation $x \in L$ under weight vectors $(\lambda(\omega) \in \mathbb{R}^n_{++})_{\omega \in \Omega}$, we want to show that x is maxmin interim Pareto efficient. Suppose by way of contradiction that a feasible allocation $y \in L$ Pareto dominates x in the interim stage. Then for all $i \in I$ and $E_i \in \mathcal{F}_i$,

$$\min_{\omega \in E_i} u_i(y_i(\omega), \omega) \geq \min_{\omega \in E_i} u_i(x_i(\omega), \omega),$$

and the strict inequality holds for some $i \in I$ and $E_i \in \mathcal{F}_i$. Now let i^* and $E^*_{i^*} \in \mathcal{F}_{i^*}$ be an agent-event pair for which the strict inequality holds. Then pick any state $\omega^* \in E^*_{i^*}$. Hence, we have

$$\sum_{i\in I}[\lambda_i(\omega^*)\min_{\omega\in\mathcal{F}_i(\omega^*)} u_i(y_i(\omega),\omega)] > \sum_{i\in I}[\lambda_i(\omega^*)\min_{\omega\in\mathcal{F}_i(\omega^*)} u_i(x_i(\omega),\omega)]$$
$$= \sum_{i\in I} Sh_i(V_{\lambda,\omega^*}) = V_{\lambda,\omega^*}(I).$$

Again, the first and second equalities come from the definitions of maxmin interim value allocation and $Sh_i(V_{\lambda,\omega^*})$ respectively. As y is feasible, this expression contradicts the definition of $V_{\lambda,\omega^*}(I)$. Hence, x is maxmin interim Pareto efficient. By Theorem 1, x satisfies the maxmin transfer coalitional incentive compatibility condition. □

5.3 Maxmin rational expectations equilibrium

One can also study the incentive compatibility properties of non-cooperative solution concepts under the maxmin framework. It is well known that the rational expectations equilibrium of Radner (1979) is neither efficient nor incentive compatible. Recent papers, e.g., Sun et al. (2012), Qin and Yang (2020), and Huang (2020), adopt alternative assumptions under which the notion becomes efficient or Bayesian incentive compatible. Instead of following the Bayesian approach, De Castro et al. (2020) introduce the maxmin rational expectations equilibrium so that each agent maximizes her maxmin expected utility conditioned on her private information and also the information generated by the price subject to an interim budget constraint. The welfare and incentive compatibility properties of the maxmin rational expectations equilibrium have been studied by De Castro et al. (2020), Liu (2014, 2016), and Glycopantis and Yannelis (2018) among others.

A price vector p is a nonzero function from Ω to $\mathbb{R}^l_+$. We denote the smallest σ-algebra of Ω for which p is measurable by $\sigma(p)$. For each $i \in I$, let $\mathcal{G}_i$ denote the partition of Ω that generates σ-algebra $\sigma(p) \vee \sigma(\mathcal{F}_i)$. Furthermore, let $\mathcal{G}_i(\omega)$ denote the set in $\mathcal{G}_i$ that contains ω. Given a price vector p and state $\omega \in \Omega$, let $B_i(p,\omega)$ defined below be agent i's interim budget set when ω is realized:

$$B_i(p;\omega) = \{y_i : \Omega \to \mathbb{R}^l_+ | y_i(\omega')\cdot p(\omega') \le e_i(\omega')\cdot p(\omega'), \forall \omega' \in \mathcal{G}_i(\omega)\}.$$

Definition 12 A price vector p and a feasible allocation x constitute a **maxmin rational expectations equilibrium** if

1. for each $i \in I$ and $\omega \in \Omega$, $x_i(\omega)\cdot p(\omega) \le e_i(\omega)\cdot p_i(\omega)$;
2. for each $i \in I$ and $\omega \in \Omega$,

$$\min_{\omega'\in\mathcal{G}_i(\omega)} u_i(x_i(\omega'),\omega') \ge \min_{\omega'\in\mathcal{G}_i(\omega)} u_i(y_i(\omega'),\omega'), \forall y_i \in B_i(p;\omega).$$

De Castro et al. (2020) prove that the maxmin rational expectations equilibrium allocation is maxmin interim Pareto efficient. Therefore, by our Theorem 1, the maxmin rational expectations equilibrium allocation satisfies the maxmin transfer coalitional incentive compatibility condition. We omit the details and refer interested readers to De Castro et al. (2020).

6 Concluding remarks

This paper studies incentive compatibility notions in an environment with ambiguity-averse agents. In particular, we propose the notion of maxmin transfer coalitional incentive compatibility, which is immune to coalitional manipulations. We show that an allocation satisfies maxmin transfer coalitional incentive compatibility if and only if it is maxmin interim Pareto efficient. This characterization shows that efficiency, incentive compatibility, and stability go hand in hand in the maxmin expected utility framework.

As a final remark, it may be of interest to explore the implications of our main result on implementation theory. Implementation theory has three branches, partial implementation, full implementation, and weak implementation. The literature focusing on incentive compatible direct mechanisms mostly follows the concept of partial implementation, which requires the existence of at least one equilibrium leading to consistent outcomes with the social choice function. The full implementation literature requires the existence of a mechanism such that the set of equilibria coincide with a social choice set or social choice correspondence (see, for example, Lombardi and Yoshihara (2013, 2019)). Weak implementation requires the set of equilibria to be a subset of the social choice set (see Pram (2020) for a discussion). A few papers, e.g., Bose and Renou (2014), Liu (2016), De Castro et al. (2017a, b), Guo (2019), and Guo and Yannelis (2020a), have established positive results on partial implementation or full implementation of efficient allocations under the maxmin expected utility framework. These papers do not consider coalitional manipulations though. A few other papers, e.g., Tian (1999), Guo and Yannelis (2020b), and Guo (2020), embed coalition structures into implementation theory without taking advantage of the connection between efficiency and incentive compatibility established in the current paper. Given the equivalence between maxmin interim Pareto efficiency and maxmin transfer coalitional incentive compatibility, one may extend the coalitional implementation concept of Hahn and Yannelis (2001) to the maxmin expected utility framework and investigate the implementability of efficient allocations.

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