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Incentive compatibility with interdependent preferences

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We introduce new concepts of incentive compatibility and efficiency for a differential information economy with interdependent preferences. Using the new definitions, we provide theorems on the consistency of efficiency and incentive compatibility.

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1 Introduction

It is well known that there is a conflict between efficiency and incentive compatibility in equilibrium theory with asymmetric information (see for example Glycopantis and Yannelis 2005, p. vi). One way to solve this conflict is to define an efficiency notion making the assumption that all allocations are private information measurable as in Yannelis (1991), i.e., allocations are measurable with respect to the σ-algebra $\mathcal{F}_i$ that the private information partition of each agent i generates. Once the private information measurability assumption on allocations is imposed, then any private efficient allocation is coalitional incentive compatible and a fortiori individual incentive compatible (Krasa and Yannelis 1994; Koutsougeras and Yannelis 1993; Hahn and Yannelis 1997). In fact, more is true, the private information measurability assumption is a necessary and sufficient condition for incentive compatibility in the one-good case and of course sufficient for more than one good. It is worth pointing out that in the above papers the decision making (utility maximization) of each agent is based on the subjective expected utility (Bayesian expected utility) and this assumption obviously imposes a certain functional form on the utility function of each agent.

Recently, De Castro and Yannelis (2010), De Castro, Pesce, and Yannelis (2011), Correia da Silva and Hervés Beloso (2009) have replaced the subjective expected utility by the

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To the memory of Lionel W. McKenzie. He was the first to introduce in a rigorous way interdependent preferences in general equilibrium theory (see McKenzie 1955).

maximin[1] and at the same time they have dropped the private information measurability assumption (see also Correia da Silva and Hervés Beloso 2011). Surprisingly, the maximin functional form of the utility function of each agent enables one to show that any efficient allocation is coalitional incentive compatible.

All the above results have been obtained for selfish preferences (utility functions), i.e., each agent derives utility from his/her own consumption only. The questions that this paper addresses are the following. How can one define incentive compatibility and efficiency notions with interdependent preferences (i.e., externalities)? Can one prove that efficient allocations are incentive compatible with either Bayesian or maximin interdependent preferences?

The main purpose of this paper is to provide an affirmative answer to the above question.

The paper proceeds as follow: in Section 2 the asymmetric information economy is introduced, as well as the definitions of Bayesian and maximin utility functions. In Sections 3 and 4 we introduce notions of efficiency and α-core, respectively. In Section 5 we introduce notions of incentive compatibility, and we prove that any interim Pareto optimal allocation with interdependent preferences is coalitional incentive compatible. From this, we deduce as corollaries that any ex ante Pareto optimal allocation with interdependent preferences and any α-core allocation is coalitional incentive compatible. We also derive related results by using maximin utility functions. Finally, Section 6 contains some conclusions.

2 Economy

Let $(\Omega, \mathcal{F})$ be a finite measurable space describing the exogenous uncertainty. The set Ω is finite and represents the possible states of nature, while $\mathcal{F}$ is the algebra of all the events. Let I be the finite set of agents, i.e., $I = \{1, \dots, n\}$, and $\mathbb{R}_+^\ell$ be the commodity space. An allocation is a function $x : I \times \Omega \to \mathbb{R}_+^\ell$ such that for each agent i and state of nature ω, $x_i(\omega) \in \mathbb{R}_+^\ell$ represents the bundle of agent i at state ω. A differential information economy is the following set

$$\mathcal{E} = \{(\Omega, \mathcal{F}); (\mathcal{F}_i, e_i, q_i, u_i)_{i\in I}\},$$

where for each agent i,

1. $\mathcal{F}_i$ is a measurable partition[2] of $(\Omega, \mathcal{F})$ denoting the private information of agent i;
2. $e_i : \Omega \to \mathbb{R}_+^\ell$ is the state-dependent initial endowment, which is assumed to be $\mathcal{F}_i$-measurable;
3. $q_i : \Omega \to \mathbb{R}_{++}$ is the prior of agent i;
4. $u_i : \mathbb{R}_+^{\ell n} \to \mathbb{R}$ is the utility function[3] such that for each $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}_+^{\ell n}$, with $x_i \in \mathbb{R}_+^\ell$ for each $i \in I$, $u_i(x_1, x_2, \dots, x_n) \in \mathbb{R}$.

[1] See Hayashi (2011) for an axiomatic study of preferences.

[2] By an abuse of notation we will still denote by $\mathcal{F}_i$ the algebra that the partition $\mathcal{F}_i$ generates.

[3] We consider state-independent utility function. This is not a strong assumption when we deal with incentive compatibility notions (see for example Koutsougeras and Yannelis 1993; Krasa and Yannelis 1994).

Notice that for each agent i the above utility function u_i depends not only on his/her own bundle, but also on the bundles of each other. For this reason we call it interdependent utility function, i.e., we allow for consumption externality.

On the other hand, usually the utility of each individual depends only on what he/she consumes, i.e., $\tilde{u}_i : \mathbb{R}^\ell_+ \to \mathbb{R}$. In this case, we call $\tilde{u}_i$ selfish utility function.

The field of events discernable by every individual is the "coarse" field $\bigwedge_{i \in I} \mathcal{F}_i$, which is the largest algebra contained in each $\mathcal{F}_i$. While, agents by pooling their information, discern the events in the "fine" field $\bigvee_{i \in I} \mathcal{F}_i$, which denotes the smallest algebra containing all $\mathcal{F}_i$. For each agent i and each state of nature ω, we denote by $\mathcal{F}_i(\omega)$ the unique element of the partition $\mathcal{F}_i$ containing the state ω.

For each $i \in I$, define the following set

$$L_i = \{x_i : \Omega \to \mathbb{R}^\ell_+ \text{ such that } x_i(\cdot) \text{ is } \mathcal{F}_i\text{-measurable}\},$$

and let $L = \prod_{i \in I} L_i$. For each coalition S, we denote by $L_S = \prod_{i \in S} L_i$ and therefore $L = L_S \times L_{I \setminus S}$.

An allocation x is said to be a private allocation or a privately measurable allocation if $x \in L$, i.e., $x_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in I$. An allocation x is said to be feasible if

$$\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all} \quad \omega \in \Omega.$$

As usual for any allocation x, we can write $x = (x_{-i}, x_i)$, where

$$x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n).$$

Moreover, if S is a coalition of agents, that is if $S \subseteq I$, then $(y^S, x^{I \setminus S})$ denotes the vector z, where $z_i = y_i$ if $i \in S$ and $z_i = x_i$ if $i \notin S$. If $S = I$, then we write x to mean x^I.

2.1 Bayesian expected utility function

We define the (Bayesian or subjective expected utility) ex ante and interim expected utility. For each agent i and for any private allocation x_i, agent i's ex ante selfish expected utility function is given by

$$\tilde{V}_i(x_i) = \sum_{\omega \in \Omega} \tilde{u}_i(x_i(\omega)) q_i(\omega).$$

For any private allocation x_i, agent i's interim selfish expected utility function with respect to $\mathcal{F}_i$ at x_i in state ω is given by

$$\tilde{v}_i(x_i | \mathcal{F}_i)(\omega) = \sum_{\omega' \in \Omega} \tilde{u}_i(x_i(\omega')) q_i(\omega' | \omega),$$

where

$$q_i(\omega' | \omega) = \begin{cases} 0 & \text{for} \quad \omega' \notin \mathcal{F}_i(\omega) \\ \dfrac{q_i(\omega')}{q_i(\mathcal{F}_i(\omega))} & \text{for} \quad \omega' \in \mathcal{F}_i(\omega) \end{cases}.$$

We can also express the interim expected utility using conditional probability as

$$\tilde{v}_i(x_i|\mathcal{F}_i)(\omega) = \sum_{\omega' \in \mathcal{F}_i(\omega)} \tilde{u}_i(x_i(\omega')) \frac{q_i(\omega')}{q_i(\mathcal{F}_i(\omega))}.$$

Notice that whenever the utility function is state-independent (this holds true also if $u_i(\cdot, x)$ is $\mathcal{F}_i$-measurable) the interim (selfish) expected utility function coincides with the ex post one. This may be no longer true with interdependent preferences, since u_i may depend on bundles which are not $\mathcal{F}_i$-measurable.

Similarly, we may define the ex ante and the interim interdependent preferences simply by replacing for all $\omega \in \Omega$, $x_i(\omega) \in \mathbb{R}^\ell_+$ with the whole vector $x(\omega) = (x_1(\omega), \ldots, x_n(\omega)) \in \mathbb{R}^{\ell n}_+$, i.e.,

$$V_i(x) = \sum_{\omega \in \Omega} u_i(x(\omega)) q_i(\omega),$$

and

$$v_i(x|\mathcal{F}_i)(\omega) = \sum_{\omega' \in \mathcal{F}_i(\omega)} u_i(x(\omega')) \frac{q_i(\omega')}{q_i(\mathcal{F}_i(\omega))}.$$

2.2 Maximin utility function

The maximin selfish utility of each agent i with respect to $\mathcal{F}_i$ at an allocation $x_i : \Omega \to \mathbb{R}^\ell_+$ in state ω is given by

$$\underline{\tilde{u}}_i(\omega, x_i) = \min_{\omega' \in \mathcal{F}_i(\omega)} \tilde{u}_i(x_i(\omega')).$$

Similarly, we may define the maximin interdependent preferences by considering in the domain the bundles of all agents, i.e.,

$$\underline{u}_i(\omega, x) = \min_{\omega' \in \mathcal{F}_i(\omega)} u_i(x(\omega')).$$

3 Pareto optimality

In this section we discuss different notions of efficiency[4] in differential information economies. When agents are asymmetrically informed, different concepts of Pareto optimality can be introduced depending on the degree of private information used by all the agents. The ex ante efficiency is defined at the stage where every agent has private information, but no state is yet realized. For the interim notions, every agent knows his/her private information event which contains the realized state. When agents have complete information, we define the notions of ex post efficiency. It should be noted that, contrary to the ex ante efficiency notion, the interim as well as the ex post notions depend on the state of nature.

[4] We use "Pareto optimal" and "efficient" as synonyms.

Moreover, the definitions introduced below can be considered as a generalization of the ordinary ones, when preferences are not interdependent (selfish).

3.1 Bayesian Pareto optimality with interdependent utility

Definition 1 *A private feasible allocation $x \in L$ is said to be* interim Pareto optimal *if there does not exist a private feasible allocation $y \in L$ such that*

$$v_i(y|\mathcal{F}_i)(\omega) \geq v_i(x|\mathcal{F}_i)(\omega) \qquad \text{for all } i \in I \text{ and for all } \omega \in \Omega, \tag{1}$$

with a strict inequality for at least one agent and at least one state.

Definition 2 *If in Definition 1 we replace the inequality (1) with*

$$V_i(y) \geq V_i(x) \qquad \text{for all } i \in I, \tag{2}$$

the private feasible allocation x is said to be ex ante Pareto optimal *for the economy $\mathcal{E}$. If there does not exist a feasible allocation y (not necessarily privately measurable) such that*

$$u_i(y(\omega)) \geq u_i(x(\omega)) \qquad \text{for all } i \in I \text{ and for all } \omega \in \Omega, \tag{3}$$

with a strict inequality for at least one agent and at least one state, then the feasible allocation x (which is not necessarily privately measurable) is said to be ex post Pareto optimal.

It can be easily checked that any ex ante efficient allocation is interim Pareto optimal, as the following lemma states.

Lemma 1 *Any ex ante efficient allocation is interim Pareto optimal.*

Notice that an ex ante as well as an interim efficient allocation may not be ex post Pareto optimal, since the feasible allocation y in (3) may not be privately measurable contrary to the one in (1) and (2).

Definition 3 *If in the Definitions 1 and 2 the allocation y is assumed to be $\bigwedge_{i\in I} \mathcal{F}_i$-measurable, then the private feasible allocation x is said to be respectively coarse interim and* coarse ex ante *Pareto optimal (see for example Definition 2 in Koutsougeras and Yannelis 1993). While, if y is $\bigvee_{i\in I} \mathcal{F}_i$-measurable, then x has the adjective fine (see for example Definition 3.3 in Koutsougeras and Yannelis 1993).*

Clearly, since for each $i \in I$, any $\bigwedge_{i\in I} \mathcal{F}_i$-measurable allocation is $\mathcal{F}_i$-measurable and a fortiori $\bigvee_{i\in I} \mathcal{F}_i$-measurable, thus any fine efficient allocation is (private) efficient which is also coarse efficient.

3.2 Maximin Pareto optimality with interdependent utility

In this section we define the notion of maximin efficiency with selfish preferences and we introduce the related notion of Pareto optimality with interdependent preferences.

Definition 4 *A feasible allocation x is said to be* maximin selfish efficient *if there does not exist a feasible allocation y such that*

$$\underline{\tilde{u}}_i(\omega, y_i) \geq \underline{\tilde{u}}_i(\omega, x_i) \quad \text{for all } i \in I \text{ and for all } \omega \in \Omega,$$

with a strict inequality for at least one agent and at least one state.

Notice that, contrary to the Bayesian notion, the feasible allocations x and y may not be privately measurable.

The natural extension of maximin Pareto optimality to the case of interdependent preferences is the following.

Definition 5 *A feasible allocation x is said to be* maximin efficient *if there does not exist a feasible allocation y such that*

$$\underline{u}_i(\omega, y) \geq \underline{u}_i(\omega, x) \quad \text{for all } i \in I \text{ and for all } \omega \in \Omega,$$

with a strict inequality for at least one agent and at least one state.

4 α-Core notions

We now recall the notion of α-core introduced by Yannelis (1991) (see also Aumann 1961).

Definition 6 *We say that a private feasible allocation $x \in L$ is an* interim α-core *allocation if it is not true that there exist $S \subseteq I$ and $y^S \in L_S$ with $\sum_{i \in S} y_i = \sum_{i \in S} e_i$, such that for any $z^{I \setminus S} \in L_{I \setminus S}$ with $\sum_{i \notin S} z_i = \sum_{i \notin S} e_i$,*

$$v_i((y^S, z^{I \setminus S})|\mathcal{F}_i)(\omega) \geq v_i(x|\mathcal{F}_i)(\omega) \qquad \text{for all } i \in S \text{ and } \omega \in \Omega,$$

with a strict inequality for at least one agent and at least one state.

Clearly if we replace the interim utility v_i with the ex ante one V_i, the allocation x is said to be ex ante α-core , *while if we replace v_i with the ex post u_i and x, y, z are not required to be privately measurable, then x is said to be an* ex post α-core *allocation.*

Remark 1 *Notice that there are several other deterministic α-core notions that can be found in the literature (see for example Scarf 1971; Holly 1994; Martins-da-Rocha and Yannelis 2011). All these notions can be extended to the differential information framework which is used in this paper.*

Remark 2 *Obviously, any α-core allocation is efficient, simply by considering $S = I$.*

Moreover we can simply define the notion of maximin α-core as follows.

Definition 7 *We say that a feasible allocation x is a* maximin α-core *allocation if it is not true that there exist $S \subseteq I$ and y^S with $\sum_{i \in S} y_i = \sum_{i \in S} e_i$, such that for any $z^{I \setminus S}$ with $\sum_{i \notin S} z_i = \sum_{i \notin S} e_i$,*

$$\underline{u}_i(\omega, y^S, z^{I \setminus S}) \geq \underline{u}_i(\omega, x) \qquad \text{for all } i \in S \text{ and } \omega \in \Omega,$$

with a strict inequality for at least one agent and at least one state.

Notice that, contrary to Definition 6, for the concept of maximin α-core, the allocations x, y and z are not assumed to be privately measurable. Moreover, with standard arguments, one can easily prove that any maximin α-core allocation is maximin efficient.

5 Incentive compatibility

5.1 Bayesian incentive compatibility

We now recall the notion of coalitional incentive compatibility of Krasa and Yannelis (1994) and Koutsougeras and Yannelis (1993).

Definition 8 *A private feasible allocation x is said to be coalitional incentive compatible (CIC) if the following is not true: there exist a coalition S and two states a and b such that*

(i) $\mathcal{F}_i(a) = \mathcal{F}(a) \in \bigcap_{i\in S} \mathcal{F}_i$ *for all* $i \in S$,
(ii) $b \in \mathcal{F}_i(a)$ *for all* $i \notin S$,
(iii) $e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+}$ *for all* $i \in S$, *and*
(iv) $\tilde{u}_i(e_i(a) + x_i(b) - e_i(b)) > \tilde{u}_i(x_i(a))$ *for all* $i \in S$.

According to the above definition, an allocation x is coalitional incentive compatible if it is not possible for a coalition S, whose members agree on whether a state has occurred, to become better off by announcing a false state b, which agents not in S cannot distinguish from the true state a.

Remark 3 *Observe that Definition 8 implicitly requires that the members of the coalition S are able to distinguish between a and b; i.e., $a \notin \mathcal{F}_i(b)$ for all $i \in S$. One could replace condition (ii) with $\mathcal{F}_i(a) = \mathcal{F}_i(b)$ if and only if $i \notin S$.*

It is well known that any (Bayesian) Pareto optimal allocation is (Bayesian) coalitional incentive compatible. The converse holds only in one good per state economies (see Koutsougeras and Yannelis 1993; Krasa and Yannelis 1994).

The main goal of this paper is to show the related result in the case of interdependent preferences. In other words, we will prove that any efficient allocation is coalitional incentive compatible. We first define the notion of incentive compatibility with interdependent preferences.

Definition 9 *A private feasible allocation x is said to be* interdependent coalitional incentive compatible *if the following is not true: there exist a coalition S and two states $a, b \in \Omega$, such that*

(i) $\mathcal{F}_i(a) = \mathcal{F}(a) \in \bigcap_{i\in S} \mathcal{F}_i$ *for all* $i \in S$,
(ii) $b \in \mathcal{F}_i(a)$ *for all* $i \notin S$,
(iii) $e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+}$ *for all* $i \in S$,
(iv) $u_i(e^S(a) + x^S(b) - e^S(b), \cdot) > u_i(x^S(a), \cdot)$ *for all* $i \in S$,
(v) $u_i(e^S(a) + x^S(b) - e^S(b), \cdot) = u_i(x^S(a), \cdot)$ *for all* $i \notin S$.

Remark 4 *Notice that thanks to condition (ii) and private measurability of the allocation x, it follows from (iv) and (v) that*

$$\begin{aligned} &\text{(iv)}\ \ u_i(e^S(a)+x^S(b)-e^S(b), x^{I\setminus S}(b)) > u_i(x^S(a), x^{I\setminus S}(a)) \quad \textit{for all } i \in S,\\ &\text{(v)}\ \ u_i(e^S(a)+x^S(b)-e^S(b), x^{I\setminus S}(b)) = u_i(x^S(a), x^{I\setminus S}(a)) \quad \textit{for all } i \notin S. \end{aligned}$$

Moreover, notice that with selfish preferences, condition (v) is trivially satisfied and hence Definition 9 reduces to Definition 8.

Contrary to the case when the preferences of each individual depend only on his/her own bundle, in the interdependent preferences case, we need to add condition (v), in order to guarantee that the members in the complementary coalition are not able to catch the lying agents in S. Indeed, it may be the case that, even if $b \in \mathcal{F}_i(a)$ for all $i \notin S$, that is $e_i(a) = e_i(b)$ and $x_i(a) = x_i(b)$, since $b \notin \mathcal{F}_i(a)$ for all $i \in S$, there may be an agent $i \in I \setminus S$ such that $u_i(e^S(a)+x^S(b)-e^S(b), x^{I\setminus S}(b)) \neq u_i(x^S(a), x^{I\setminus S}(a))$, that is i might be able to understand that agents in S are lying.

The importance of condition (v) in Definition 9 is underlined by the example below.

Example 1 Consider a two person economy, with one good and three equally probable states denoted by $\Omega = \{a, b, c\}$. Agents's characteristics are given below:

$$\begin{aligned} &e_1(a,b,c) = (2,1,1), \quad \mathcal{F}_1 = \{\{a\},\{b,c\}\}, \quad u_1(x) = x_1;\\ &e_2(a,b,c) = (1,1,0), \quad \mathcal{F}_2 = \{\{a\},\{b\},\{c\}\}, \quad u_2(x) = 2x_1 + x_2 + x_3;\\ &e_3(a,b,c) = (2,2,0), \quad \mathcal{F}_3 = \{\{a,b\},\{c\}\}, \quad u_3(x) = x_3 - x_1. \end{aligned}$$

Notice that the following $\mathcal{F}_i$-measurable allocation

$$\begin{aligned} &x_1(a,b,c) = (1,1,1),\\ &x_2(a,b,c) = (1,0,0),\\ &x_3(a,b,c) = (3,3,0) \end{aligned}$$

is (interdependent) interim efficient. Moreover, if a is the realized state, agents 1 and 2 have an incentive to lie and report b. Indeed,

$$\begin{aligned} u_1(e_1(a)+x_1(b)-e_1(b), e_2(a)+x_2(b)-e_2(b), x_3(b)) &= u_1(2,0,3) = 2\\ &> 1 = u_1(1,1,3)\\ &= u_1(x^I(a)), \quad \text{and} \end{aligned}$$

$$\begin{aligned} u_2(e_1(a)+x_1(b)-e_1(b), e_2(a)+x_2(b)-e_2(b), x_3(b)) &= u_2(2,0,3)\\ &= 4+0+3 = 7\\ &> 6 = 2+1+3\\ &= u_2(1,1,3)\\ &= u_2(x^I(a)). \end{aligned}$$

On the other hand, even if $b \in \mathcal{F}_3(a)$ and $u_3(x(a)) = u_3(x(b)) = 3 - 1 = 2$, the third guy realizes that the coalition S is lying since he/she gets less than what he/she expects to obtain in states a or b, i.e.,

$$\begin{aligned} u_3(e_1(a)+x_1(b)-e_1(b), e_2(a)+x_2(b)-e_2(b), x_3(b)) &= u_3(2,0,3) = 3-2 = 1 \\ &< 3-1 = 2 = u_3(1,1,3) \\ &= u_3(x^I(a)) = u_3(x^I(b)). \end{aligned}$$

Therefore, the coalition $S = \{1, 2\}$ cannot cheat.

This example suggests that in the case of interdependent preferences, we also need to assume that members in the complementary coalition get the same when the others lie, as condition (v) in Definition 9 requires.

We are now able to prove that with such a definition, although utilities of agents depend on the bundles of anybody, any interim Pareto optimal allocation is coalitional incentive compatible, as the next theorem states.

Theorem 1 *Any interim Pareto optimal allocation is interdependent coalitional incentive compatible.*

PROOF: Assume on the contrary that there exists an interim Pareto optimal allocation x which is not interdependent coalitional incentive compatible. Then, there exist a coalition S and two states $a, b \in \Omega$, such that

(i) $\mathcal{F}(a) = \mathcal{F}_i(a) \in \bigcap_{i\in S}\mathcal{F}_i$ for all $i \in S$,
(ii) $b \in \mathcal{F}_i(a)$ for all $i \notin S$,
(iii) $e_i(a) + x_i(b) - e_i(b) \in I\!R^{\ell}_{+}$ for all $i \in S$,
(iv) $u_i(e^S(a) + x^S(b) - e^S(b), \cdot) > u_i(x^S(a), \cdot)$ *for all* $i \in S$,
(v) $u_i(e^S(a) + x^S(b) - e^S(b), \cdot) = u_i(x^S(a), \cdot)$ *for all* $i \notin S$.

Since x is feasible, it follows that for all $\omega \in \Omega$,

$$\sum_{i\in S}[x_i(\omega) - e_i(\omega)] = -\sum_{i\notin S}[x_i(\omega) - e_i(\omega)];$$

in particular

$$\begin{aligned} \sum_{i\in S}[x_i(a) - e_i(a)] &= -\sum_{i\notin S}[x_i(a) - e_i(a)], \\ \sum_{i\in S}[x_i(b) - e_i(b)] &= -\sum_{i\notin S}[x_i(b) - e_i(b)]. \end{aligned}$$

Moreover, since $b \in \mathcal{F}_i(a)$ for all $i \notin S$, it follows that $e_i(a) = e_i(b)$ and $x_i(a) = x_i(b)$ for all $i \notin S$. This means that

$$\sum_{i\notin S}[x_i(a) - e_i(a)] = \sum_{i\notin S}[x_i(b) - e_i(b)],$$

and hence

$$\sum_{i\in S}[x_i(a) - e_i(a)] = \sum_{i\in S}[x_i(b) - e_i(b)]. \tag{4}$$

Define the following allocation

$$y_i(\omega) = \begin{cases} e_i(\omega) + x_i(b) - e_i(b) & \text{if } i \in S \text{ and } \omega \in \mathcal{F}(a) \\ x_i(\omega) & \text{otherwise} \end{cases}.$$

Clearly since for all $i \in I$, $x_i(\cdot)$ and $e_i(\cdot)$ are $\mathcal{F}_i$-measurable, so is $y_i(\cdot)$. Moreover, notice that y is feasible. Indeed, if $\omega \notin \mathcal{F}(a)$, it follows from the feasibility of x that so is y. On the other hand if $\omega \in \mathcal{F}(a)$, then from (4) and (i), we have that

$$\begin{aligned}\sum_{i\in I} y_i(\omega) &= \sum_{i\in S} e_i(\omega) + \sum_{i\in S}[x_i(b) - e_i(b)] + \sum_{i\notin S} x_i(\omega) \\ &= \sum_{i\in S} e_i(a) + \sum_{i\in S}[x_i(a) - e_i(a)] + \sum_{i\notin S} x_i(\omega) \\ &= \sum_{i\in S} x_i(a) + \sum_{i\notin S} x_i(\omega) = \sum_{i\in S} x_i(\omega) + \sum_{i\notin S} x_i(\omega) \\ &= \sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega).\end{aligned}$$

Clearly, for each $\omega \notin \mathcal{F}(a)$ and for each $i \in I$, $u_i(y(\omega)) = u_i(x(\omega))$. On the other hand if $\omega \in \mathcal{F}(a)$, then from (iv) it follows that for each $i \in S$

$$\begin{aligned}u_i(y(\omega)) = u_i(e^S(\omega) + x^S(b) - e^S(b), x^{I\setminus S}(\omega)) &= u_i(e^S(a) + x^S(b) \\ &\quad - e^S(b), x^{I\setminus S}(\omega)) \\ &> u_i(x^S(a), x^{I\setminus S}(\omega)) \\ &= u_i(x^S(\omega), x^{I\setminus S}(\omega)) \\ &= u_i(x(\omega)).\end{aligned}$$

Similarly, from (v) it follows that for each $i \notin S$, $u_i(y(\omega)) = u_i(x(\omega))$. This means that $u_i(y(\omega)) \geq u_i(x(\omega))$ for all $i \in I$ and $\omega \in \Omega$, which a strict inequality for each $i \in S$ and $\omega \in \mathcal{F}(a)$. Therefore, for each agent i and each state ω,

$$\begin{aligned}v_i(y|\mathcal{F}_i)(\omega) &= \sum_{\omega'\in\mathcal{F}_i(\omega)} u_i(y(\omega'))\frac{q_i(\omega')}{q_i\left(\mathcal{F}_i(\omega)\right)} \\ &\geq \sum_{\omega'\in\mathcal{F}_i(\omega)} u_i(x(\omega'))\frac{q_i(\omega')}{q_i\left(\mathcal{F}_i(\omega)\right)} = v_i(x|\mathcal{F}_i)(\omega),\end{aligned}$$

with a strict inequality for each $i \in S$ and $\omega \in \mathcal{F}(a)$. This contradicts the fact that x is interim Pareto optimal. □

Corollary 1 *Any ex ante Pareto optimal allocation is interdependent coalitional incentive compatible.*

PROOF: It directly follows from Lemma 1 and Theorem 1. □

Corollary 2 *Any α-core allocation is interdependent coalitional incentive compatible.*

PROOF: It follows from Remark 2 and Theorem 1. □

5.2 Maximin incentive compatibility

We now recall an extension of the Krasa and Yannelis (1994) definition to incorporate maximin preferences (see also De Castro and Yannelis 2010).

Definition 10 *A feasible allocation x is said to be maximin coalitional incentive compatible (MCIC) if the following is not true: there exist a coalition S and two states a and b such that*

(i) $b \in \mathcal{F}_i(a)$ *for all* $i \notin S$,
(ii) $e_i(a) + x_i(b) - e_i(b) \in I\!R^{\ell}_{+}$ *for all* $i \in S$,
(iii) $\underline{\tilde{u}}_i(a, y_i) > \underline{\tilde{u}}_i(a, x_i)$ *for all* $i \in S$,

where for all $i \in S$,

$$(*) \qquad y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise} \end{cases}.$$

Notice that the allocation y may not be privately measurable, i.e., for some agent i the allocation y_i may not be $\mathcal{F}_i$-measurable. According to the above definition, an allocation is said to be maximin coalitional incentive compatible if it is not possible for a coalition to misreport the realized state of nature and have a distinct possibility of making its members better off in terms of maximin utility. Obviously, if $S = \{i\}$ then the above definition reduces to individual incentive compatibility.

It is shown in De Castro and Yannelis (2010) that any (selfish) coalitional incentive compatible allocation is also maximin CIC. The converse may not be true. Moreover, it is well known that private efficient allocations are coalitional incentive compatible and therefore they are individual incentive compatible. For the maximin preferences the same result holds but we can dispense with the private information measurability assumption of allocations (see De Castro and Yannelis 2010; De Castro, Pesce, and Yannelis 2011).

5.3 Maximin CIC

We now introduce a notion of CIC with maximin interdependent preferences.

Definition 11 *A feasible allocation x is said to be* maximin interdependent coalitional incentive compatible *if the following is not true: there exist a coalition S and two states a and b such that*

(i) $b \in \mathcal{F}_i(a)$ *for all* $i \notin S$,
(ii) $e_i(a) + x_i(b) - e_i(b) \in I\!R^{\ell}_{+}$ *for all* $i \in S$,
(iii) $\underline{u}_i(a, y) > \underline{u}_i(a, x)$ *for all* $i \in S$,
(iv) $\underline{u}_i(a, y) = \underline{u}_i(a, x)$ *for all* $i \notin S$,

where for each agent $i \in I$,

$$y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise} \end{cases}.$$

Notice that for each $i \notin S$, since $e_i(a) = e_i(b)$ it comes out that $y_i(a) = x_i(b)$.

Theorem 2 *Any maximin (interdependent) efficient allocation is maximin interdependent coalitional incentive compatible.*

PROOF: Let x be a maximin (interdependent) efficient allocation (not necessarily privately measurable) and assume by the way of contradiction that it is not maximin interdependent coalitional incentive compatible, that is, there exist a coalition S and two states a and b such that

(i) $b \in \mathcal{F}_i(a)$ for all $i \notin S$,
(ii) $e_i(a) + x_i(b) - e_i(b) \in I\!R^{\ell}_{+}$ for all $i \in S$,
(iii) $\underline{u}_i(a, y) > \underline{u}_i(a, x)$ for all $i \in S$,
(iv) $\underline{u}_i(a, y) = \underline{u}_i(a, x)$ for all $i \notin S$,

where for each agent $i \in I$,

$$y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise} \end{cases}.$$

Notice that from (iii) and definition of y, it follows that for each $i \in S$, $\underline{u}_i(\omega, y) = \underline{u}_i(\omega, x)$ for each $\omega \notin \mathcal{F}_i(a)$ (because $\omega \neq a$ for all $\omega \notin \mathcal{F}_i(a)$). Moreover, for each $i \in S$,

$$\underline{u}_i(\omega, y) = \underline{u}_i(a, y) > \underline{u}_i(a, x) = \underline{u}_i(\omega, x)$$

for each $\omega \in \mathcal{F}_i(a)$. With similar arguments one can show that for all $i \notin S$, $\underline{u}_i(\omega, y) = \underline{u}_i(\omega, x)$ for any $\omega \in \Omega$.

Therefore there exists an allocation y such that $\underline{u}(\omega, y) \geq \underline{u}_i(\omega, x)$ for all $i \in I$ and for all $\omega \in \Omega$, with a strict inequality for each $i \in S$ and at least in state a. We need to show that y is feasible[5]. Clearly, if $\omega \neq a$, y is trivially feasible since it coincides with the feasible allocation x. On the other hand, if $\omega = a$, then

$$\sum_{i\in I} y_i(a) = \sum_{i\in I} e_i(a) + \sum_{i\in I} [x_i(b) - e_i(b)] = \sum_{i\in I} e_i(a).$$

This completes the proof. □

Corollary 3 *Any maximin α-core allocation is maximin interdependent coalitional incentive compatible.*

PROOF: Since any maximin α-core allocation is maximin efficient, the corollary comes from Theorem 2. □

[5] Notice that whenever agents have maximin preferences the private measurability assumption on the allocations is not required anymore.

6 Conclusions

We introduced new notions of efficiency and incentive compatibility with interdependent preferences for economies with asymmetric information. We showed that in this very general framework, either with Bayesian or maximin preferences, any efficient allocation is incentive compatible.

We didn't prove any existence results, however one can adopt the arguments in Yannelis (1991) to show that efficient individually rational allocations exist. Although, all our results have been proved for a finite number of states of nature, we conjecture that they can be extended to an infinite number of states.

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