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Optimal Taxation in Life-Cycle Economies[1]

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We use a very standard life-cycle growth model, in which individuals have a labor-leisure choice in each period of their lives, to prove that an optimizing government will almost always find it optimal to tax or subsidize interest income. The intuition for our result is straightforward. In a life-cycle model the individual's optimal consumption–work plan is almost never constant and an optimizing government almost always taxes consumption goods and labor earnings at different rates over an individual's lifetime. One way to achieve this goal is to use capital and labor income taxes that vary with age. If tax rates cannot be conditioned on age, a nonzero tax on capital income is also optimal, as it can (imperfectly) mimic age-conditioned consumption and labor income tax rates. *Journal of Economic Literature* Classification Numbers: E62, H21.

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A classic problem in public finance concerns the optimal manner in which to finance a given stream of government purchases in the absence of lump-sum taxation. In the context of a standard neoclassical growth model with infinitely-lived individuals, [8, 18] establish that an optimal income tax policy entails taxing capital at confiscatory rates in the short run and setting capital income taxes equal to zero in the long run. While this policy remains optimal in a vast array of infinitely-lived agent models, little is known about the principles underlying optimal taxation in overlapping generations models.[3] This is an important limitation since the life-cycle model is widely used in applied work on dynamic fiscal policy ([5, 4], and many others, surveyed in [21]).

In this paper, we address the optimal (Ramsey) taxation problem within a standard life-cycle growth model in which individuals have a labor-leisure choice in each period of their finite lives.[4] We show that within this framework, an optimizing government will almost always tax consumption goods and labor earnings at different rates over an individual's lifetime. One way to achieve this goal is to use capital and labor income taxes that vary with age. If tax rates cannot be conditioned on age, a nonzero tax on capital income is still optimal, as it can (imperfectly) mimic age-conditioned consumption and labor tax rates. These results follow simply from the fact that in a life-cycle model an individual's optimal consumption–work plan is almost never constant over his/her lifetime, even in steady–state. In contrast, individual consumption and leisure must be constant in the steady–state of representative agent models since they are equal to aggregate consumption and leisure, which is the key difference between the two frameworks in terms of their implications for optimal capital income taxes. In fact, in the unlikely case where the life-cycle Ramsey economy does converge to a long-run allocation in which consumption and leisure are constant over individuals' lives, optimal taxation works as in infinitely-lived agent models and capital income taxes are zero.

The fundamental problem in setting optimal fiscal policy is that leisure cannot be taxed directly. The fact that in overlapping generations models leisure is, in general, not constant over the lifetime of individuals provides two potential roles for interest taxation that do not arise in infinitely-lived agent models. The first role is that interest taxation is one way to affect the implicit price of leisure over time. The clearest example arises when preferences are such that standard uniform commodity taxation, or its dynamic

[3] Exceptions include [2, 13].

[4] Atkinson and Sandmo [2] consider an economy in which individuals supply labor for only one period. Thus, they abstract from life-cycle issues in the labor supply decision which, according to our results, plays a crucial role. Besley and Jewitt [7] identify necessary and sufficient conditions for the optimality of uniform consumption taxation in a model where individuals only supply labor for one period.

equivalent, zero interest taxation, results hold.[5] In this case, the government taxes labor income more heavily when leisure is low, thereby making leisure relatively expensive when it is high. With an increasing leisure profile, this entails a labor income tax rate that declines with age. Evidently, this policy requires the government to rely upon age-dependent labor income taxes. If labor income tax rates cannot be conditioned on age, the government can (imperfectly) imitate this age-dependent tax policy by taxing interest income, as a positive interest tax makes leisure more expensive as individuals age.[6]

The second role for interest taxation arises when uniform commodity taxation is not optimal. Here, the government can affect individuals' leisure profile by taxing more heavily goods that are complementary with leisure [11]. In life-cycle models, where consumption and leisure generally move together over time, consumption should be taxed more heavily when it is relatively high. Since consumption tends to increase over an individual's lifetime, optimal consumption taxes tend to increase as an individual ages, that is, capital income taxes tend to be positive. In contrast, consumption and leisure are constant in the steady–state of infinitely-lived agent models. As a result the government in these models has no incentive to affect the relative price of leisure over time and a zero capital income tax is optimal.

We formulate a standard Ramsey problem in which the government maximizes a utilitarian welfare function defined as the discounted sum of successive generation's lifetime utility—as in [2, 12, 14, 26]—by choosing government debt as well as proportional taxes on consumption, labor income and capital income. Unlike previous studies of optimal taxation in overlapping generations models, we formulate the Ramsey problem using the primal approach, which leads to an explicit, analytical characterization of the Ramsey policy.[7] Under this approach, rather than choosing a sequence of tax rates, the government directly chooses an allocation subject to a series of constraints guaranteeing that the allocation is feasible and is consistent with consumers' optimization conditions.[8] We show that such an allocation can be decentralized as a competitive equilibrium only if the

[5] Atkinson and Stiglitz [3] show that a utility function that is weakly separable between consumption and leisure and homogeneous in consumption is sufficient for uniform commodity taxation to be optimal.

[6] Alvarez *et al.* [1] derive a similar result in a partial equilibrium setting. This finding is reminiscent of results in [17, 27], where the government taxes capital income when it is constrained to use tax rates that are independent of individuals' skills levels.

[7] Escolano [13] uses the dual approach, where the government chooses (age-independent) tax rates. Since characterizing optimal fiscal policy using this formulation of the government's problem is virtually impossible, the analysis in that paper is limited to numerical simulations.

[8] On the primal approach, see [3, 25]. This is the approach generally used to study optimal taxation in infinitely-lived agent models ([8, 24] and others).

government can use a full set of age-conditioned taxes. The need for age-dependent taxes is a natural implication of life-cycle behavior.[9] Additional restrictions need to be imposed in the formulation of the Ramsey problem to study optimal taxation under an age-independent tax system.

We show that, given a set of fiscal instruments, many fiscal policies can implement the same allocation. Consequently, a fiscal policy arrangement has to be evaluated as a whole rather than by considering each tax instrument in isolation. In our framework, consumption taxes and labor income taxes are equivalent. While this result may appear to contradict the findings of [5, 28], it is only so because these authors rule out certain fiscal instruments. The welfare gains associated with a switch from a wage to a consumption tax found in [5] are due to an implicit (nondistortionary) levy of the initial generations' capital which is not offset by changes in other fiscal instruments.

While [5] advocates the use of a consumption tax, it also shows that steady-state welfare is higher with an income tax, which of course taxes both wage and interest income, than it is with a pure wage tax. Their simulations thus suggest that the addition of some interest taxation to a pure wage tax system raises steady-state utility. Using their parameterization, we show that optimal capital income taxes in their economy are indeed positive and significant. We also confirm that this result, which contradicts conjectures in [19, 24], is robust to alternative preference specifications.[10]

The rest of the paper is organized as follows. The next section presents the economic environment. We formulate the Ramsey problem in Section 2. We show that under an age-dependent tax system, our framework allows for a natural formulation of the Ramsey problem in which the government chooses allocations rather that tax rates. We use this formulation to characterize optimal fiscal policies in Section 3. In Section 4 we further characterize optimal Ramsey taxation under alternative preference specifications and illustrate our results through numerical simulations. Section 5 concludes.

1. THE ECONOMY

We consider an economy populated by overlapping generations of identical individuals similar to that of [5]. Individuals live for an arbitrary

[9] This is another advantage of the primal approach. For instance, [13] does not discuss the role of age-dependent taxes which, as we show, underlie the principles of optimal taxation in life-cycle economies.

[10] Through numerical simulations, [15] also finds a role for capital income taxation in a life-cycle model with incomplete markets. Our paper shows that incomplete markets are not needed for this result. For studies where taxes are the outcome of a political process rather than chosen by a benevolent government, see [6, 22].

but finite number of periods and make a labor/leisure choice in each period. They also pay taxes that the government collects in order to finance public spending. The set of fiscal policy instruments available to the government consists of government debt as well as taxes on consumption, labor income, and capital income, where all tax rates can be age-conditioned.

Households

Individuals live $(J+1)$ periods, from age 0 to age J. At each time period a new generation is born and is indexed by its date of birth. At date 0, the generations alive are $-J, -J+1, \ldots, 0$. The population is assumed to grow at constant rate n per period; consequently, the share of age-j individuals in the population, μ_j, is time invariant and satisfies $\mu_j = \mu_{j-1}/(1+n)$, for $j = 1, \ldots, J$, where $\sum_{j=0}^{J} \mu_j = 1$.

Individuals are endowed with one unit of time in each period of their life. We assume that an age-j individual can transform one unit of time into z_j efficiency units of labor (for $j = 0, \ldots, J$). Individuals derive utility from consumption and leisure. We let $c_{t,j}$ and $l_{t,j}$, respectively, denote consumption and time devoted to work in period $(t+j)$ by an age-j individual born in period t. Since tax rates can be conditioned on age, the after-tax prices that individuals face in a given period also depend on age. Accordingly, the after-tax price of consumption that an age-j individual faces in period t is denoted $q_{t-j,j}$. Similarly, the after-tax prices of labor services and capital services are denoted $w_{t-j,j}$ and $r_{t-j,j}$, respectively.

The problem faced by an individual born at period $t \geqslant -J$ is to maximize lifetime utility subject to a sequence of budget constraints,

$$\max U(c_{t,j_0(t)}, \ldots, c_{t,J}, l_{t,j_0(t)}, \ldots, l_{t,J}), \tag{1}$$

$$\begin{gathered} q_{t,j} c_{t,j} + a_{t,j+1} = w_{t,j} z_j l_{t,j} + (1 + r_{t,j})\, a_{t,j}, \qquad j = j_0(t), \ldots, J, \\ a_{t,j_0(t)} \text{ given and equal to 0 if } t \geqslant 0, \end{gathered} \tag{2}$$

where $a_{t-j,j}$ denotes the total asset holdings of an age-j individual at date t. We assume that the utility function U is increasing in consumption and leisure, strictly concave, and satisfies standard Inada conditions. In the above problem, the age of individuals alive at date zero is denoted $j_0(t)$. For generations $t \geqslant 0$, $j_0(t) = 0$, so that in general $j_0(t) \equiv \max\{-t, 0\}$.[11]

Technology and Feasibility

At each date there is a unique produced good that can be used as capital or as a private or government consumption good. The technology to

[11] One can think of $j_0(t)$ as the first period of an individual's life which is affected by the switch in fiscal policy, which occurs at date zero.

produce goods is represented by a neoclassical production function with constant returns to scale,

$$y_t = f(k_t, l_t), \tag{3}$$

where y_t, k_t, and l_t denote the aggregate (per capita) levels of output, capital, and effective labor, respectively. Capital and labor services are paid their marginal products: before-tax prices of capital and labor are given by $\hat{r}_t = f_k(k_t, l_t) - \delta$ and $\hat{w}_t = f_l(k_t, l_t)$, where $0 < \delta < 1$ is the depreciation rate of capital.

Feasibility requires that total consumption plus investment be less than or equal to the aggregate output

$$c_t + (1+n)\, k_{t+1} - (1-\delta)\, k_t + g_t \leqslant y_t, \tag{4}$$

where c_t denotes aggregate private consumption at date t and g_t stands for date-t government consumption, and all aggregate variables are expressed in per capita terms. In particular, the date-t aggregate levels of consumption and labor input (expressed in efficiency units) are given by

$$c_t = \sum_{j=0}^{J} \mu_j c_{t-j,\,j},$$

$$l_t = \sum_{j=0}^{J} \mu_j z_j l_{t-j,\,j}.$$

The Government

To finance its exogenous stream of expenditures, we assume that the government has access to a set of fiscal policy instruments and a commitment technology to implement its fiscal policy. The set of instruments available to the government consists of government debt and proportional taxes on consumption, labor income, and capital income. At each date, the tax rates on factor services are allowed to depend on the age of the individual supplying the services. The date-t tax rates on capital and labor services supplied by an age-j individual (born in period $(t-j)$) are denoted by $\tau^k_{t-j,\,j}$ and $\tau^w_{t-j,\,j}$, respectively. Similarly, consumption taxes are allowed to depend on the age of the consumer, and we use $\tau^c_{t-j,\,j}$ to represent the date-t tax rate on the consumption of an age-j individual. In addition to consumption and factor income taxes, the government can issue debt (b_t) to match imbalances between expenditures and revenues in any given period. The government is assumed to tax the return on capital and debt at the same rate, so that debt and capital are perfect substitutes. The resulting government budget constraint at date $t \geqslant 0$ is given by

$$(1+\hat{r}_t)\, b_t + g_t = (1+n)\, b_{t+1} + \sum_{j=0}^{J} (q_{t-j,j} - 1)\, \mu_j c_{t-j,j}$$
$$+ \sum_{j=0}^{J} (\hat{r}_t - r_{t-j,j})\, \mu_j a_{t-j,j}$$
$$+ \sum_{j=0}^{J} (\hat{w}_t - w_{t-j,j})\, \mu_j z_j l_{t-j,j}, \tag{5}$$

where $q_{t,j} \equiv (1+\tau^c_{t_j})$, $w_{t_j} \equiv (1-\tau^w_{t,j})\, \hat{w}_{t+j}$, and $r_{t,j} \equiv (1-\tau^k_{t,j})\, \hat{r}_{t+j}$.[12]

The government takes individuals' optimizing behavior as given and chooses a fiscal policy to maximize social welfare, where social welfare is defined as the discounted sum of individual lifetime welfares [2, 26]. In other words, the government's objective is the maximization of

$$\sum_{t=-J}^{\infty} \gamma^t U^t,$$

where $0 < \gamma < 1$ is the intergenerational discount factor and U^t denotes the indirect utility function of generation t as a function of the government tax policy.

One property of this formulation of the government's objective is that it preserves the valuations individuals place on consumption at different dates. Consequently, an allocation solving the Ramsey problem is "constrained Pareto efficient" in the sense that it cannot be Pareto dominated by any other allocation that is a competitive equilibrium for some fiscal policy.

It is interesting to note that in this framework the individuals on whom the burden of a front-loading policy falls are different from (and unrelated to) those that benefit from lower distortionary taxes in the future. The government's desire to resort to confiscatory taxation of initial asset holdings is thus endogenously limited by intergenerational redistribution considerations. This is in sharp contrast with optimal taxation problems in infinitely-lived agent models, where upper bounds on feasible capital income tax rates need to be imposed to avoid initial confiscatory taxes [8, 9, 16, 18]. These bounds, however, determine the magnitude of the welfare gains achieved by switching to the taxes prescribed by the Ramsey problem. Indeed, with sufficiently high bounds a Pareto optimal equilibrium can be achieved.[13]

[12] Note that the producer price of consumption goods has been normalized at one.

[13] Although there is no need to impose such bounds in our framework, the government will nevertheless tax at date 0 the asset holdings that individuals accumulated in the past. In this way, the government collects taxes from generations born prior to date 0 while minimizing the distortionary impact of taxation.

2. THE RAMSEY PROBLEM

The Ramsey problem consists of choosing a set of taxes so that the resulting allocation, when prices and quantities are determined in competitive markets, maximizes a given welfare function. In this section, we show that there is an equivalent formulation of the Ramsey problem in which the government chooses allocations rather than tax rates. Before showing this equivalence, we define the set of allocations that the government can choose from and use this definition to eliminate redundant tax instruments from the tax system.

2.1. *Implementable Allocations and Fiscal Policy Instruments*

The set of allocations that the government can implement consists of the allocations chosen by individuals for any arbitrary fiscal policy and are formally defined below.

Definition 2.1 (Implementable Allocation). Let $\{g_t\}_{t=0}^{\infty}$ be a given sequence of government expenditures. Given initial aggregate endowments $\{k_0, b_0\}$ and initial individual asset holdings $\{a_{-j,j}\}_{j=1}^{J}$ such that

$$k_0 + b_0 = \sum_{j=1}^{J} \mu_j a_{-j,j},$$

an allocation $\{\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1}\}_{t=-J}^{\infty}$ is implementable if there exist a fiscal policy arrangement $\{\{q_{t,j}, r_{t,j}, w_{t,j}, \}_{j=j_0(t)}^{J}, b_{t+J+1}\}_{t=-J}^{\infty}$ and a sequence of asset holdings $\{\{a_{t,j}\}_{j=j_0(t)}^{J}\}_{t=-J}^{\infty}$ such that:

D2.1a. Given prices from the fiscal policy arrangement, $\{c_{t,j}, l_{t,j}, a_{t,j+1}\}_{j=j_0(t)}^{J}$ solves the consumer problem given by (1)–(2) for $t = -J, \ldots$.

D2.1b. Factor prices are competitive: $\hat{r}_t = f_k(k_t, l_t) - \delta$ and $\hat{w}_t = f_l(k_t, l_t)$, $t = 0, 1, \ldots$.

D2.1c. The government budget constraint (5) is satisfied at $t = 0, 1, \ldots$.

D2.1d. Aggregate feasibility (4) is satisfied at $t = 0, 1, \ldots$.

It is important to emphasize that the set of implementable allocations depends crucially on the set of fiscal policy instruments available to the government. An allocation that is implementable according to Definition 2.1 may not be implementable with age-independent taxes. For a given set of fiscal policy instruments, however, many different fiscal policies can implement the same allocation. For example, an age-dependent tax on consumption can be perfectly imitated by combining an age-independent tax on consumption and an age-dependent tax on capital income. Proposition 2.1 below establishes that even after eliminating age-dependent consumption taxes, a given allocation can still be implemented by a family of fiscal policies. The conditions under which two fiscal policies implement the same allocation are fairly intuitive: The real wage (P2.1b), the relative price of present versus future consumption (P2.1c), and the value of asset holdings (P2.1a) and (P2.1d), all in terms of the after tax price of consumption goods, must be equal across the two fiscal policies.

PROPOSITION 2.1. *Let* $\{g_t\}_{t=0}^{\infty}$ *be a given sequence of government expenditures and let* $(k_0, b_0, \{a_{-j,j}\}_{j=1}^{J})$ *be initial endowments such that*

$$k_0 + b_0 = \sum_{j=1}^{J} \mu_j a_{-j,j}.$$

If the fiscal policy $\{\{q_{t,j}, r_{t,j}, w_{t,j}\}_{j=j_0(t)}^{J}, b_{t+J+1}\}_{t=-J}^{\infty}$ *and the sequence of asset holdings* $\{\{a_{t,j}\}_{j=j_0(t)+1}^{J}\}_{t=-J}^{\infty}$ *implements the allocation* $\{\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1}\}_{t=-J}^{\infty}$, *then any other fiscal policy* $\{\{\tilde{q}_{t,j}, \tilde{r}_{t,j}, \tilde{w}_{t,j},\}_{j=j_0(t)}^{J}, \tilde{b}_{t+J+1}\}_{t=-J}^{\infty}$ *and sequence of asset holdings* $\{\{\tilde{a}_{t,j}\}_{j=j_0(t)+1}^{J}\}_{t=-J}^{\infty}$ *satisfying*

$$\frac{1+r_{t,j_0(t)}}{q_{t,j_0(t)}} = \frac{1+\tilde{r}_{t,j_0(t)}}{\tilde{q}_{t,j_0(t)}}, \qquad t=-J, ..., \tag{P2.1a}$$

$$\frac{w_{t,j}}{q_{t,j}} = \frac{\tilde{w}_{t,j}}{\tilde{q}_{t,j}}, \qquad t=-J, ..., \quad j=j_0(t), ..., J, \tag{P2.1b}$$

$$\frac{(1+r_{t,j+1})\, q_{t,j}}{q_{t,j+1}} = \frac{(1+\tilde{r}_{t,j+1})\, \tilde{q}_{t,j}}{\tilde{q}_{t,j+1}}, \qquad t=-J, ..., \quad j=j_0(t)+1, ..., J, \tag{P2.1c}$$

$$\frac{a_{t,j+1}}{q_{t,j}} = \frac{\tilde{a}_{t,j+1}}{\tilde{q}_{t,j}} \qquad t=-J, ..., \quad j=j_0(t)+1, ..., J, \tag{P2.1d}$$

also implements the allocation.

Proof. We need to show that any alternative fiscal policy and sequence of asset holdings which satisfy conditions (P2.1a)–(P2.1d) also satisfy conditions (D1a)–(D1d) in Definition 2.1. Note that conditions (D1b) and (D1d) (factor prices and feasibility) are trivially satisfied under the alternative fiscal policy.

Using the consumer's budget constraint under the initial fiscal policy and conditions (P2.1b) and (P2.1c), we obtain

$$q_{t,j} c_{t,j} + a_{t,j+1} = \tilde{w}_{t,j} \frac{q_{t,j}}{\tilde{q}_{t,j}} z_j l_{t,j} + (1+\tilde{r}_{t,j}) \frac{\tilde{q}_{t,j-1}}{\tilde{q}_{t,j}} \frac{q_{t,j}}{q_{t,j-1}} a_{t,j}.$$

Multiplying this expression by $\tilde{q}_{t,j}/q_{t,j}$ and using condition (P2.1d),

$$\tilde{q}_{t,j} c_{t,j} + \tilde{a}_{t,j+1} = \tilde{w}_{t,j} z_j l_{t,j} + (1+\tilde{r}_{t,j})\, \tilde{a}_{t,j}.$$

Thus, any allocation $\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}$ satisfying the consumer's budget constraints under the initial fiscal policy also satisfies the budget constraints under the alternative fiscal policy (with asset holdings for generation $t \geqslant -J$ given by $\{\tilde{a}_{t,j}\}_{j=j_0(t)+1}^{J}$). Since the converse is also true, the two budget sets are identical. Further, since consumers face the same decision problem under both fiscal policies, we conclude that $\{\{c_{t,j}, l_{t,j}, \tilde{a}_{t,j+1}\}_{j=j_0(t)}^{J}\}_{t=-J}^{\infty}$ solves the consumer problem when prices are given by the alternative fiscal policy. Condition (D1a) is thus satisfied.

The government budget constraint (5) must also be satisfied under the alternative fiscal policy. Using feasibility (4) and the fact that the production function (3) is homogeneous of degree one, the government budget constraint can be written as

$$\sum_{j=0}^{J} (1+\tilde{r}_{t-j,j})\, \mu_j \tilde{a}_{t-j,j} = (1+n)\, \tilde{a}_{t+1} + \sum_{j=0}^{J} \tilde{q}_{t-j,j} c_{t-j,j} - \sum_{j=0}^{J} \tilde{w}_{t-j,j} \mu_j z_j l_{t-j,j}.$$

Using conditions (P2.1a)–(P2.1d) the above expression is equivalent to

$$\sum_{j=0}^{J} (1+r_{t-j,j}) \frac{q_{t-j,j-1}}{q_{t-j,j}} \frac{\tilde{q}_{t-j,j}}{\tilde{q}_{t-j,j-1}} \mu_j a_{t-j,j} \frac{\tilde{q}_{t-j,j-1}}{q_{t-j,j-1}}$$
$$= (1+n)\, a_{t+1} \frac{\tilde{q}_{t-j,j}}{q_{t-j_j}} + \sum_{j=0}^{J} \tilde{q}_{t-j,j} c_{t,j} - \sum_{j=0}^{J} w_{t-j,j} \frac{\tilde{q}_{t-j,j}}{q_{t-j,j}} \mu_j z_j l_{t-j,j}.$$

Multiplying both sides of the previous expression by $q_{t-j,j}/\tilde{q}_{t-j,j}$, we obtain the government budget constraint under the initial fiscal policy. This constraint holds at all dates, by assumption, under the initial fiscal policy: We conclude that the government budget constraint also holds under the alternative fiscal policy, and condition (D1c) is also satisfied. ∎

An important implication of Proposition 2.1 is that a fiscal policy arrangement has to be evaluated as a whole rather than by considering each tax instrument in isolation. In particular, it is possible to eliminate either consumption taxes or labor income taxes from a given fiscal policy without affecting the allocation being implemented. These changes in fiscal policy require redefining the other fiscal instruments so that conditions (P2.1a)–(P2.1d) are satisfied. Then, a fiscal policy arrangement with no consumption taxes can implement the same allocations as a fiscal policy arrangement with no labor income taxes. This observation applies whether taxes are allowed to be conditioned on age or not; that is, it applies to both age-dependent and age-independent tax systems.[14]

[14] Under an age-independent tax system, the government has access to four instruments per period—government debt, consumption taxes, and labor and capital income taxes—one of which is redundant.

The reader may suspect that our equivalence result between labor income taxes and consumption taxes contradicts the findings of [28] and [5]. These authors show that there is an efficiency gain of switching from a labor income tax regime to a consumption tax regime. This efficiency gain arises because consumption taxes act like a lump sum tax on the people alive at the moment of the change in tax policy, and this lump sum tax is not offset by changes in other fiscal instruments. The initial increase in consumption taxes necessary to compensate for the revenue loss from the elimination of labor income taxes acts like a lump sum tax since it reduces the value, in terms of the after-tax price of consumption goods, of the asset holdings accumulated in the past $((1+\tilde{r}_{t,\,j_0(t)})/\tilde{q}_{t,\,j_0(t)})$. Because date-0 capital income taxes are maintained fixed, the condition (P2.1a) is not satisfied when labor income taxes are replaced with consumption taxes in the aforementioned papers.

Without any loss of generality, we set $\tau^c_{t,j}$ equal to zero for all t and j throughout the rest of the paper.

2.2. *Equivalent Ramsey Representations*

We now formulate a Ramsey problem in which the government chooses allocations (primal) rather than tax rates (dual) and show that both formulations are equivalent. We use the consumers' optimality conditions to construct a sequence of implementability constraints which guarantee that any allocation chosen by the government can be decentralized as a competitive equilibrium. In the next section, we use the primal approach to characterize optimal fiscal policies.

Let $p_{t,j}$ denote the Lagrange multiplier associated with the budget constraint (2) faced by an age-j individual born in period t. The necessary and sufficient conditions for a solution to the consumers' problem are given by (2) and

$$U_{c_{t,j}} - p_{t,j} = 0, \tag{6}$$

$$U_{l_{t,j}} + p_{t,j} w_{t,j} z_j \leqslant 0, \qquad \text{with equality if} \quad l_{t,j} > 0, \tag{7}$$

$$-p_{t,j} + p_{t,j+1}(1 + r_{t,j+1}) = 0, \tag{8}$$

$$a_{t,J+1} = 0, \tag{9}$$

$j = j_0(t), ..., J$, where $U_{c_{t,j}}$ and $U_{l_{t,j}}$ denote the derivative of U with respect to $c_{t,j}$ and $l_{t,j}$, respectively.[15]

The time-t implementability constraint is obtained by multiplying the budget constraints (2) by $p_{t,j}$, summing over $j \in \{j_0(t), ..., J\}$, and using

[15] The Inada conditions guarantee that consumption and leisure will be strictly positive in each period.

(6)–(8) to substitute out prices.[16] The implementability constraint associated with the cohort born in period t is

$$\sum_{j=j_0(t)}^{J} (U_{c_{t,j}} c_{t,j} + U_{l_{t,j}} l_{t,j}) = U_{c_{t,j_0(t)}} (1 + r_{t,j_0(t)})\, a_{t,j_0(t)}. \tag{10}$$

PROPOSITION 2.2. *An allocation* $\{\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1}\}_{t=-J}^{\infty}$ *is implementable with age-dependent taxes if and only if it satisfies feasibility* (4) *and the implementability constraint* (10).

Proof. By construction, implementable allocations satisfy feasibility and implementability. We now show that the converse is also true.

Suppose that $\{\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}, k_{t+J+1}\}_{t=-J}^{\infty}$ satisfies the feasibility and implementability constraints (4) and (10). Define before-tax prices as $\hat{r}_t \equiv f_k(k_t, l_t) - \delta$ and $\hat{w}_t \equiv f_l(k_t, l_t)$. Define the sequence of after-tax prices $\{\{w_{t,j}, r_{t,j+1}\}_{j=j_0(t)}^{J}\}_{t=-J}^{\infty}$ as

$$w_{t,j} \equiv -\frac{U_{l_{t,j}}}{z_j U_{c_{t,j}}},$$

$$r_{t,j+1} \equiv \frac{U_{c_{t,j}}}{U_{c_{t,j+1}}},$$

and let $p_{t,j} = U_{c_{t,j}} > 0$ for $j = j_0(t), ..., J$ and $t \geqslant -J$. Then, by construction, $\{c_{t,j}, l_{t,j}\}_{j=j_0(t)}^{J}$ satisfies the consumer's first order conditions (6)–(8) for all $t \geqslant -J$.

To show that the budget constraints (2) and the transversality condition (9) are satisfied, given $a_{t,j_0(t)}$ define recursively for $j = j_0(t), ..., J$

$$a_{t,j+1} = w_{t,j} z_j l_{t,j} + (1 + r_{t,j})\, a_{t,j} - c_{t,j},$$

and note that, given the definition of after-tax prices, the implementability constraint implies that $a_{t,J+1} = 0$ for all $t \geqslant -J$.

Finally, we need to show that the government budget constraint is satisfied. To do so, the budget constraint of the age-j individual born in period $(t-j)$ is multiplied by μ_j and the resulting equations are added for $j \in \{0, ..., J\}$, yielding

$$\sum_{j=0}^{J} \mu_j (c_{t-j,j} + a_{t-j,j+1}) = \sum_{j=0}^{J} \mu_j (w_{t-j,j} z_j l_{t-j,j} + (1 + r_{t-j,j})\, a_{t-j,j})$$

[16] The transversality condition (9) allows us to set $a_{t,J+1}$ equal to zero for all t.

or

$$c_t + (1+n)\, a_{t+1} = a_t + \sum_{j=0}^{J} \mu_j (w_{t-j,j} z_j l_{t-j,j} + r_{t-j,j} a_{t-j,j}). \tag{11}$$

Since the production function is homogeneous of degree one, we can write the feasibility constraint as

$$c_t + (1+n)\, k_{t+1} - (1-\delta)\, k_t + g_t = (\hat{r}_t + \delta)\, k_t + \hat{w}_t \sum_{j=0}^{J} \mu_j z_j l_{t-j,j}. \tag{12}$$

Combining Eqs. (11) and (12), we have

$$g_t - (1+\hat{r}_t)\, k_t + a_t = (1+n)(a_{t+1} - k_{t+1}) + \sum_{j=0}^{J} \mu_j z_j (\hat{w}_t - w_{t-j,j})\, l_{t-j,j}$$
$$- \sum_{j=0}^{J} \mu_j r_{t-j,j} a_{t-j,j}.$$

Adding $\hat{r}_t a_t$ on both sides of the previous expression and defining $b_t \equiv a_t - k_t$, the previous expression can be written as

$$(1+\hat{r}_t)\, b_t + g_t = (1+n)(b_{t+1}) + \sum_{j=0}^{J} \mu_j z_j (\hat{w}_t - w_{t-j,j})\, l_{t-j,j}$$
$$+ \sum_{j=0}^{J} \mu_j (\hat{r}_t - r_{t-j,j})\, a_{t-j,j}.$$

∎

Proposition 2.2 shows that a feasible allocation can be decentralized as a competitive equilibrium if and only if it satisfies the implementability constraints (10). It should be emphasized that an age-dependent tax system is essential for this proposition to hold: The marginal rates of substitution between present and future consumption and between consumption and leisure are not necessarily constant across generations alive at any given date, even for allocations that satisfy the implementability constraints. Consequently, an allocation can only be consistent with the consumers' first-order conditions if after-tax interest and wage rates are age-dependent.

Further restrictions need to be imposed on the Ramsey problem for an allocation to be implementable with age-independent taxes. First, the marginal rate of substitution between consumption and leisure needs to be constant across individuals of different ages. In particular, the consumer's first-order conditions (6)–(8) imply that $\tau^w_{t-j,j} = \tau^w_t$ if and only if

$$\frac{U_{l_{t-j,j}}}{U_{c_{t-j,j}} z_j} \leqslant \frac{U_{l_{t,0}}}{U_{c_{t,0}} z_0}, \qquad \text{with equality if} \quad l_{t-j,j} > 0, \qquad j = 1, ..., J.$$

Second, the marginal rate of substitution between present and future consumption also needs to be constant across individuals of different ages, and $\tau^k_{t-j,j} = \tau^k_t$ if and only if

$$\frac{U_{c_{t-j,j+1}}}{U_{c_{t-j,j}}} = \frac{U_{c_{t,1}}}{U_{c_{t,0}}}, \qquad j = 1, ..., J-1.$$

The above necessary conditions for age-independent taxes can be expressed in the compact way

$$R^1_{t-j,j} \leqslant 0, \qquad \text{with equality if} \quad l_{t,j} > 0, \qquad j = 1, ..., J, \tag{13}$$

$$R^2_{t-j,j} = 0, \qquad j = 1, ..., J-1, \tag{14}$$

where for all $t \geqslant 0$,

$$R^1_{t-j,j} \equiv \log U_{c_{t,0}} z_0 - \log U_{l_{t,0}} + \log U_{l_{t-j,j}} - \log U_{c_{t-j,j}} z_j, \qquad j = 1, ..., J,$$

$$R^2_{t-j,j} \equiv \log U_{c_{t,1}} - \log U_{c_{t,0}} + \log Uc_{t-j,j} - \log Uc_{t-j,j+1} \qquad j = 1, ..., J-1.$$

PROPOSITION 2.3. *An allocation* $\{\{c_{t,j}, l_{t,j}\}^J_{j=j_0(t)}, k_{t+J+1}\}^\infty_{t=-J}$ *is implementable with age-independent taxes if and only if it satisfies feasibility* (4) *and the implementability constraints* (10) *and* (13)–(14).

3. OPTIMAL FISCAL POLICY

In this section we show that the solution to the Ramsey problem generally features nonzero taxes on labor and capital income. In particular, and in contrast with infinitely-lived agent models, if the Ramsey allocation converges to a steady–state solution, optimal capital income taxes will in general be different from zero even in that steady–state. We begin our analysis by characterizing optimal fiscal policies under an age-dependent tax system. This characterization not only helps us to understand the principles underlying optimal taxation in overlapping generations economies, it also provides some intuition regarding how age-independent tax rates should be set.

Let $\gamma^t \lambda_t$ be the Lagrange multiplier associated with generation t's implementability constraint (10) and define the pseudo welfare function (W_t) to include generation t's implementability constraint

$$W_t = U_t + \lambda_t \sum_{j=j_0(t)}^{J} (U_{c_{t,j}} c_{t,j} + U_{l_{t,j}} l_{t,j}) - \lambda_t U_{c_{t,j_0(t)}} (1 + r_{t,j_0(t)}) \, a_{t,j_0(t)}. \tag{15}$$

The Ramsey problem with age-dependent taxes then consists of choosing an allocation to maximize the discounted sum of pseudo welfares

$$\max_{\{\{c_{t,j},\, l_{t,j}\}_{j=j_0(t)}^{J},\, k_{t+J+1}\}_{t=-J}^{\infty}} \sum_{t=-J}^{\infty} \gamma^t W_t$$

subject to feasibility (4) for $t = 0, \ldots$. Since government debt is unconstrained, the government budget constraint does not effectively constrain the maximization problem and therefore has been omitted from the Ramsey problem. Once a solution is found, the government budget constraint can be used to back out the level of government debt.

Let $\gamma^t \phi_t$ denote the Lagrange multiplier associated with the time-t feasibility constraint (4). The necessary conditions for a solution to the Ramsey problem with age-dependent taxes are

$$-\gamma^t \phi_t (1+n) + \gamma^{t+1} \phi_{t+1} (1 - \delta + f_{k_{t+1}}) = 0, \tag{16}$$

$$\gamma^t W_{c_{t,j}} - \gamma^{t+j} \phi_{t+j} \mu_j = 0, \tag{17}$$

$$\gamma^t W_{l_{t,j}} + \gamma^{t+j} \phi_{t+j} \mu_j z_j f_{l_{t+j}} \leqslant 0, \qquad \text{with equality if} \quad l_{t,j} > 0, \tag{18}$$

for $t = -J, \ldots,$ and $j = j_0(t), \ldots, J$, where $W_{c_{t,j}}$ and $W_{l_{t,j}}$ denote the derivatives of W_t with respect to $c_{t,j}$ and $l_{t,j}$, respectively.

We now derive the necessary conditions under which the Ramsey allocation features zero taxation of either labor or capital income by comparing the optimality conditions from the consumer's problem to those of the Ramsey problem. Combining Eqs. (17) and (18) for the nontrivial case of a positive labor supply implies

$$-\frac{W_{l_{t,j}}}{W_{c_{t,j}}} = \frac{(1+\lambda_t)\, U_{l_{t,j}} + \lambda_t U_{l_{t,j}} H^l_{t,j}}{(1+\lambda_t)\, U_{c_{t,j}} + \lambda_t U_{c_{t,j}} H^c_{t,j}} = z_j \hat{w}_{t+j}, \tag{19}$$

where

$$H^c_{t,j} = \frac{\sum_{i=j_0(t)}^{J} (U_{c_{t,i}, c_{t,j}} c_{t,i} + U_{l_{t,i}, c_{t,j}} l_{t,i})}{U_{c_{t,j}}}, \tag{20}$$

$$H^l_{t,j} = \frac{\sum_{i=j_0(t)}^{J} (U_{c_{t,i}, l_{t,j}} c_{t,i} + U_{l_{t,i}, l_{t,j}} l_{t,i})}{U_{l_{t,j}}}. \tag{21}$$

Since any optimal fiscal policy has to satisfy the consumer's optimality conditions, we can compare (19) to its analogue from the consumer's optimization problem. Combining Eqs. (6) and (7), assuming a positive labor supply, implies

$$-\frac{U_{l_{t,j}}}{U_{c_{t,j}}} = z_j w_{t,j} = z_j \hat{w}_{t+j}(1-\tau^w_{t,j}). \tag{22}$$

From Eqs. (19) and (22), the tax rate on labor income for an age-j individual born in period t is given by

$$\tau^w_{t,j} = \frac{\lambda_t(H^l_{t,j} - H^c_{t,j})}{1+\lambda_t+\lambda_t H^l_{t,j}}. \tag{23}$$

Since λ_t is, in general, different from zero, this tax rate on labor income will be equal to zero only if $H^l_{t,j} = H^c_{t,j}$.

Similarly, consider the ratio of Eq. (17) at ages j and $j+1$. Using (16),

$$\frac{W_{c_{t,j}}}{W_{c_{t,j+1}}} = \frac{(1+\lambda_t)\,U_{c_{t,j}} + \lambda_t U_{c_{t,j}} H^c_{t,j}}{(1+\lambda_t)\,U_{c_{t,j+1}} + \lambda_t U_{c_{t,j+1}} H^c_{t,j+1}} = 1+\hat{r}_{t+j+1}. \tag{24}$$

Equation (6) from the consumer's problem at ages j and $j+1$ together with (8) yields the individual counterpart of (24),

$$\frac{U_{c_{t,j}}}{U_{c_{t,j+1}}} = 1+r_{t,j+1}. \tag{25}$$

Dividing (24) by (25) we have

$$\frac{1+\hat{r}_{t+j+1}}{1+r_{t,j+1}} = \frac{1+\lambda_t+\lambda_t H^c_{t,j}}{1+\lambda_t+\lambda_t H^c_{t,j+1}}, \tag{26}$$

which implies that the tax rate on capital income is different from zero unless $H^c_{t,j} = H^c_{t,j+1}$.

These results are summarized in the following proposition.

PROPOSITION 3.1. *At each date, the optimal tax rate on labor income is different from zero unless $H^l_{t,j} = H^c_{t,j}$ and the optimal tax rate on capital income is different from zero unless $H^c_{t,j} = H^c_{t,j+1}$.*

Proposition 3.1 sheds some light on why the celebrated Chamley–Judd result on the optimality of not taxing capital income in the steady–state of infinitely-lived agent models does not extend to life-cycle economies. Since consumption and leisure are constant in the steady–state of infinitely-lived agent models, $H^c_{t,j}$ converges to a constant in the long run and, thus, zero capital income taxation is optimal regardless of the form of the utility function. In contrast, consumption and leisure are generally not constant over an individual's lifetime in overlapping generations models, even when the solution to the Ramsey problem converges to a steady–state. There is thus no reason to expect $H^c_{t,j} = H^c_{t,j+1}$ for t sufficiently large and, consequently, capital income taxes will generally not be equal to zero in the long run.

A natural question to ask is whether there exist restrictions under which optimal capital income tax rates are indeed zero. Propositions 3.2 and 3.3 illustrate two such cases below. In the first case, the economy is specified so that individuals are not characterized by life-cycle behavior; labor supply and consumption are independent of age.

PROPOSITION 3.2. *Assume that the utility function is additively separable across time and that individuals discount the future at a fixed rate* β. *If* (i) $z_j = z$, $j = 0, \ldots, J$, *and* (ii) $\gamma = \beta(1+n)$, *then it is not optimal to tax capital income in the long run, under both age-dependent and age-independent tax systems.*

The proof of the above proposition is trivial. Proposition 3.2 assumes that the utility function is additively separable across time, the age profile of labor productivity is flat, and the intergenerational discount factor γ is such that the steady-state return on capital coincides with individuals' rate of time preference.[17] Under these assumptions, individuals' optimization conditions imply that consumption and leisure do not depend on age in steady–state.[18] The absence of life-cycle behavior implies that the functions $H^c_{t,j}$ and $H^l_{t,j}$ do not depend on age for t sufficiently large. It follows from Proposition 3.1 that the optimal capital income tax rate in steady–state is zero. Moreover, when the economy is in steady–state, labor earnings are taxed at a constant rate over the life of individuals (see Eq. (23)). Consequently, Proposition 3.2 holds regardless of whether age-dependent taxes are available to the government.

The second case establishes a relation between the classic result on uniform commodity taxation and capital income taxation. In Proposition 3.3, we consider utility functions that are weakly separable between consumption and leisure and homothetic in consumption: i.e.,

$$U(c, l) = V(G(c), l), \tag{27}$$

where $c = (c_0, \ldots, c_J)$, $l = (l_0, \ldots, l_J)$, and $G(\cdot)$ is homothetic.

[17] The last observation follows from the fact that the steady-state version of Eq. (16) implies that the rate of time preference of individuals coincides with the marginal product of capital net of depreciation (or with the pre-tax interest rate), $f_k - \delta = \hat{r} = (1+n)/\gamma - 1 = 1/\beta - 1$ when $\gamma = \beta(1+n)$.

[18] Note that the conditions stated in Proposition 3.2 have empirically unappealing implications. They imply that individuals consume their labor earnings period by period, and since individuals do not save, the entire stock of capital is owned by the government.

PROPOSITION 3.3. *For utility functions of the form given by* (27), *the Ramsey problem prescribes zero taxes on capital income for time period 1 and thereafter provided labor income taxes can be age-conditioned.*

Proof. Since preferences are homothetic in consumption, we know that for every t [3, 10]

$$\sum_{i=j_0(t)}^{J} \frac{U_{c_{t,i}c_{t,j}} c_{t,i}}{U_{c_{t,j}}} = A_t, \qquad \text{for all} \quad j \in j_0(t), ..., J.$$

Thus,

$$\begin{aligned} H^c_{t,j} &= A_t + \sum_{i=j_0(t)}^{J} \frac{U_{l_{t,i}c_{t,j}} l_{t,i}}{U_{c_{t,j}}} \\ &= A_t + \sum_{i=j_0(t)}^{J} \frac{V^t_{1,2} G_{c_{t,j}} l_{t,i}}{V^t_1 G_{c_{t,j}}} \\ &= A_t + \frac{V^t_{1,2}}{V^t_1} \sum_{i=j_0(t)}^{J} l_{t,i}. \end{aligned}$$

The function $H^c_{t,j}$ is therefore independent of j, which implies that $H^c_{t,j} = H^c_{t,j+1}$. ∎

The conditions under which a zero tax rate on capital income is optimal, as stated in Proposition 3.3, are closely related to the conditions for uniform commodity taxation studied in the public finance literature. In static optimal commodity taxation problems, [3] shows that it is optimal to tax consumption goods uniformly when the utility function satisfies (27). Proposition 3.3 shows that the extension of this result to a dynamic framework implies that zero taxation of capital income is optimal not only in the long run, but also in the transition to the final steady–state as well. While this result is general in that it holds for all possible values of γ and profiles of labor productivity, it is restrictive in that it only holds under a specific class of utility functions and only if labor income taxes can be conditioned on age.

When labor income taxes cannot be conditioned on age, there is a role for interest taxation as a tax on capital income can imitate an age-dependent labor income tax. Labor supplied at different ages are different commodities; there is thus no reason to expect uniform taxation of labor earnings to be optimal. Even in cases where uniform leisure taxation would be first best optimal, the solution to the Ramsey problem does not prescribe uniform taxation of labor earnings. For instance, for utility functions of the form $U(c, l) = V(G(c), F(1-l))$ with $G(\cdot)$ and $F(\cdot)$ homothetic, it is optimal to tax leisure uniformly. However, since leisure cannot be taxed directly, it is second best optimal to tax labor supplied at different ages non-uniformly. When labor income taxes are not allowed to depend on age, the government can affect the way individuals substitute labor intertemporally by resorting to nonzero capital income taxes and use this margin to tax labor income more heavily at ages where it is relatively income inelastic.

We end our characterization of optimal fiscal policies by focusing on some steady–state properties of the Ramsey problem. The steady state solution, if it exists and if age-dependent taxes are available, is characterized by the following equations:

$$1-\delta+f_k=\frac{1+n}{\gamma}, \tag{28}$$

$$(1+\lambda)\,U_{c_j}+\lambda U_{c_j}H_j^c=\gamma^j\phi\mu_j, \qquad j=0, \ldots, J, \tag{29}$$

$$(1+\lambda)\,U_{l_j}+\lambda U_{l_j}H_j^l\leqslant \gamma^j\phi\mu_j z_j f_l, \qquad j=0, \ldots, J, \quad \text{with equality if} \quad l_j>0, \tag{30}$$

as well as by the feasibility and implementability constraints (4) and (10). Thus, a steady–state is characterized by $(2(J+1)+3)$ equations in $(2(J+1)+3)$ unknowns $(\{c^j, l^j\}_{j=0}^J, k, \phi, \lambda)$. Note that in steady–state the marginal product of capital (net of depreciation) equals the effective discount rate applied to different generations $(1+n)/\gamma-1$. This condition implies that the capital–labor ratio coincides with that of the first best allocation, the capital–labor ratio that would be achieved if the government had access to lump-sum taxation [26]. In other words, the steady-state capital–labor ratio has the modified golden rule property.[19]

The steady-state allocation is also independent of the transition path.[20] In particular, for each value of γ we can solve for the final steady–state path independent of the transition that leads to this steady–state. Associated with each value of γ, or with the final steady–state, there is an optimal amount of public debt which can be backed out of the government budget constraint. This amount of government debt is accumulated during the transition from the initial to the final steady–state allocation. The higher is γ, the lower are the accumulated government debt and the welfare of generations alive during the transition. In contrast, the transition path in infinitely-lived agent models determines the steady–state allocation of the Ramsey problem. In particular, the steady–state allocations in these models

[19] The steady-state capital–labor ratio of the Ramsey problem with age-independent taxes also has the modified golden-rule property since Eq. (28) is unaffected by the tax code restrictions. See the Appendix for details.

[20] This property holds under an age-independent tax system as well. See the Appendix for details.

depend on the exogenous bounds that have to be imposed on capital income tax rates during the transition.

4. SOME INTERESTING CASES

In a very influential paper, Auerbach, Kotlikoff, and Skinner (hereafter AKS) [5] advocate replacing income taxes with consumption taxes. They illustrate the gains of eliminating capital income taxation using numerical simulations of a life-cycle model. Yet, we show that Ramsey taxation in their economy features significant taxes on capital income. This finding also holds under alternative preference specifications.

4.1. *The AKS Economy*

We now parameterize the model presented in Section 2 in order to replicate the economy used by AKS in their study of tax reforms. We thus obtain an initial steady–state equilibrium that is identical in all respects to that considered by AKS. Next, we assume that taxes are set once and for all according to the principles of Ramsey taxation and that the equilibrium allocation converges to a steady–state. We then study the implications of Ramsey taxation for the steady–state tax rates on capital and labor income. Since the solution to the Ramsey problem depends crucially on the value of the intergenerational discount factor (γ), we study optimal taxation under a wide range of values for γ.

Parameterization. As in AKS [5], individuals live for 55 years ($J=54$) and the population grows at 1% per annum ($n=0.01$). The labor productivity profile is taken from [29] and normalized so that labor productivity is equal to one in the first year ($z_0=1$).[21] The utility function is specified as

$$U(c,l)=\sum_{j=j_0(t)}^{J}\beta^j\frac{[c_{t,j}^{\rho}+\theta(1-l_{t,j})^{\rho}]^{(1-\sigma)/\rho}}{1-\sigma}, \tag{31}$$

with intratemporal elasticity of substitution between consumption and leisure equal to 0.8 ($\rho=-0.25$), intertemporal elasticity of substitution equal to 0.25 ($\sigma=4$), and discount rate equal to 1.5% per period ($\beta=(1+0.015)^{-1}$). The parameter determining the intensity of leisure is set such that aggregate working time represents about 35% of total time ($\theta=1.5$).

The production function is given by $f(k,l)=Ak^{\alpha}l^{1-\alpha}$. Following AKS [5], the capital share of output is set to 25% ($\alpha=0.25$) and capital does not depreciate ($\delta=0$). The scaling constant A is chosen such that the pre-tax wage rate is equal to one ($A=0.9395$).

[21] The equation generating the productivity profile is $\log z_j=4.47+0.033(j+1)-0.00067(j+1)^2$ for $j=0,\ldots,54$.

Following AKS [5], we assume that the economy is initially in a steady–state featuring a proportional income tax rate of 30% ($\tau^w = \tau^k = 0.3$) and no government debt. The government budget constraint then implies that government spending is 30% of GDP. This parameterization of the economy generates a steady–state equilibrium with a capital-output ratio of 3.04 and a pre-tax interest rate of 8.22%. AKS emphasize that this parameterization leads to empirically plausible life-cycle profiles of consumption, asset holdings, and labor income.[22]

Findings. In order to compute Ramsey taxes we need to specify a value for the intergenerational discount factor (γ). We set $\gamma = 0.9333$ as our benchmark. This choice is motivated by the fact that under this value of γ the return on capital in the Ramsey equilibrium is equal to that of the AKS economy. This choice also leads to a plausible amount of government debt (about 30% of the GDP).

Table 1 presents (steady–state) tax rates for six values of γ. The most striking finding is that capital income taxes are well above zero in the AKS economy: the tax rate on capital income for our benchmark case is equal to 14.4%. As γ increases, the welfare weight on future generations increases. Higher values of γ thus lead to lower government debt and tax rates in the long run. For instance, with $\gamma = 0.97$ the government owns a large fraction of the capital stock (government debt is negative and more than three times larger than aggregate income in the economy) and the tax rates on capital and labor income are much lower than under the benchmark value of γ. Nevertheless, the tax rate on capital income is still above 4%. We thus conclude that there is a role for interest taxation in the AKS economy.

The efficiency gains of taxing capital income can best be understood by studying optimal taxation when taxes can be conditioned on age. Figure 1 illustrates the age profile of capital and labor income tax rates. Until retirement, the tax rate on capital income increases with age and the tax rate on labor income decreases with age. The shape of these profiles bears the Corlett and Hague [11] intuition: since leisure cannot be taxed directly, the government taxes commodities that are complementary with leisure. Since consumption and leisure are both increasing functions of age, the government taxes leisure indirectly by taxing consumption more heavily as individuals age. This is achieved with nonzero capital income taxes. Similarly, labor income taxes decrease with age, making leisure (relatively) more expensive as individuals age.

4.2. *Additively Separable Utility*

In this subsection we show that when labor income taxes cannot be conditioned on age, capital income taxation may be helpful to imitate

[22] See AKS [5] for more details.

TABLE 1

Optimal Taxation in the AKS Economy

γ	$\hat{r}$	τ^w	τ^k	b/y
0.9300	0.0860	0.4221	0.1515	0.4823
0.9333	0.0822	0.3999	0.1442	0.3235
0.9350	0.0802	0.3887	0.1396	0.2342
0.9400	0.0745	0.3543	0.1272	−0.0645
0.9500	0.0632	0.2833	0.1015	−0.8570
0.9700	0.0412	0.1469	0.0427	−3.7281

age-dependent labor income taxes. Our example shows that capital income taxes can be positive even when preferences are such that uniform commodity taxation is optimal. We consider utility functions of the form

$$U(c, l) = \sum_{j=j_0(t)}^{J} \beta^j (u(c_{t,j}) + v(l_{t,j})), \tag{32}$$

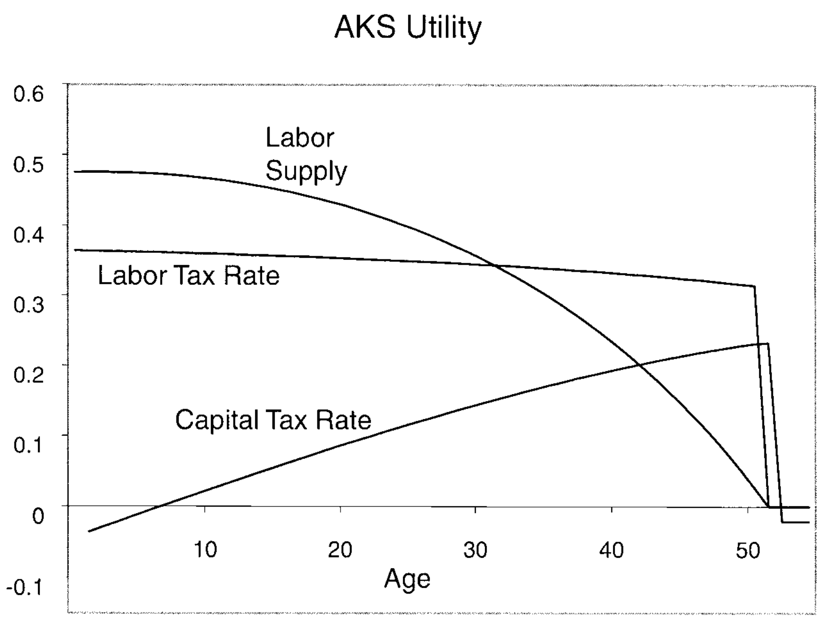

FIG. 1. Labor supply and tax rates over the lifetime of individuals.

where $u(\cdot)$ is homogeneous of degree $(1-\sigma)$ and $U(\cdot,\cdot)$ satisfies the Inada conditions. Since these preferences satisfy the conditions stated in Proposition 3.3, it follows that capital income taxes will not be used if labor income taxes can be conditioned on age. In this case, Proposition 4.1 establishes how labor income taxes should be set in steady–state.

PROPOSITION 4.1 (Age-Profile of Labor Income Taxes). *Let the utility function be of the form given by* (32). *In steady–state, the relative tax rates on labor income at different ages are inversely related to the relative income elasticities of labor supplied at those ages.*

Proof. Combining Eq. (17) at age j and Eq. (18) at age $j+1$ for the nontrivial case of positive labor supply, together with their counterparts from the consumers' problem (6) at age j, Eq. (7) at age $j+1$, and (8), the tax rate on labor income at age $j+1$ is given by

$$\tau^w_{j+1} = \frac{\lambda(H^l_{j+1} - H^c_j)}{1+\lambda+\lambda H^l_{j+1}}. \tag{33}$$

Using (23), it follows that

$$\frac{\tau^w_{j+1}/(1-\tau^w_{j+1})}{\tau^w_j/(1-\tau^w_j)} = \frac{H^l_{j+1} - H^c_j}{H^l_j - H^c_j}. \tag{34}$$

Now let m denote nonfactor income, and let $l_j(w, r, m)$ denote the supply of labor at age j, where $w \equiv \{w_j\}^J_{j=0}$ and $r \equiv \{r_j\}^J_{j=0}$. The first order condition for labor (7), assuming a positive supply at age j, can then be expressed as

$$-\beta^j v'(l_j(w, r, m)) = p_j(w, r, m)\, w_j z_j. \tag{35}$$

Applying the logarithmic operator on both sides of Eq. (35) and differentiating with respect to m implies that

$$\frac{v''(l_j)}{v'(l_j)} \frac{\partial l_j}{\partial m} = \frac{\partial p_j}{\partial m} \frac{1}{p_j}. \tag{36}$$

Under separable preferences $H^l_j = v''(l_j)\, l_{t,j}/v'(l_j)$ and since $(\partial p_j/\partial m)$ $(1/p_j) = (\partial p_i/\partial m)(1/p_i)$ for all (i, j), Eq. (36) implies that

$$\frac{H^l_{j+1}}{H^l_j} = \frac{\eta_{l_j}}{\eta_{l_{j+1}}},$$

where η_j is the income elasticity of l_j. It follows from (34) that the income from relatively inelastically supplied labor is to be taxed proportionally more than the income from relatively more elastically supplied labor. ∎

Proposition 4.1 can be viewed as an application of the public finance principle that necessities should be taxed more than luxuries [3] to a life-cycle framework: labor income should be taxed relatively more heavily when it is relatively more income-inelastic. It can also be shown that the income elasticity of the labor supply depends on the labor productivity profile as well as the discount rate of individuals relative to the intergenerational rate of time preferences (which determines the steady–state interest rate). To see this, notice that when $\gamma = \beta(1+n)$ and $z_j = z$ for all j, the steady–state consumption and labor supply are constant throughout individuals' lives. It then follows from (33) that labor taxes are age-independent, as these restrictions imply that the income elasticity of the labor supply is constant across ages. Any deviation from these restrictions will change the relative income elasticity of labor supplied at different ages, and age-dependent taxes will be used.

Example. The following example illustrates the role played by capital income taxes when taxes cannot be age-conditioned. We change the AKS parameterization such that both the intratemporal elasticity of substitution between consumption and leisure and the intertemporal elasticity of substitution are equal to one ($\rho = 0$ and $\sigma = 1$), producing a (limiting) utility function of the form given by (32) with $u(c_{t,j}) = \log c_{t,j}$ and $v(l_{t,j}) = \theta \log l_{t,j}$. We then set θ such that the aggregate amount of time devoted to work in steady–state matches that of the AKS economy ($\theta = 1.43$).[23] This parameterization produces a steady–state interest rate of 4.57%, suggesting a benchmark value of γ equal to 0.9659.

Table 2 reports the properties of Ramsey taxes under these preferences. It shows that capital income taxes can be quite large, even in the most favorable case against capital income taxation (additive separable utility): the optimal tax rate on capital income in the benchmark case is equal to 11%. Examining the age-profile of the tax rate on labor income, as illustrated in Fig. 2, is useful in understanding this result. Labor income tax increases slightly from age 0 to age 1, after which it decreases with age.[24]

[23] We also adjust the scaling constant in the production function to maintain the pre-tax wage rate in the initial steady–state equal to one. The initial tax code is the same as in the AKS economy ($\tau^w = \tau^k = 0.3$ and $b = 0$): government spending thus remains at 30% of the GDP.

[24] Labor income taxes tend to follow, with a lag, the shape of the labor productivity profile. This observation is consistent with Proposition 4.1. It is easy to show, *ceteris paribus*, that the income elasticity of labor supply at a particular age is inversely related to the labor productivity at that age. Similarly, since in our simulations the interest rate is higher than individuals' rate of time preference, labor is more income inelastic the lower the age of the individuals is.

TABLE 2

Optimal Taxation with Separable Utility

γ	$\hat{r}$	τ^w	τ^k	b/y
0.9550	0.0576	0.5636	0.1522	2.3456
0.9600	0.0521	0.5051	0.1346	1.8697
0.9650	0.0466	0.4426	0.1156	1.1698
0.9659	0.0457	0.4323	0.1105	1.0234
0.9700	0.0412	0.3769	0.0955	0.1730
0.9800	0.0306	0.2429	0.0583	−3.1918

The government thus wants to tax labor income more heavily when individuals are (relatively) young rather than old. Equivalently, the government wants to make the consumption of leisure relatively cheap when individuals are young. In the absence of age-conditioned labor income taxes, taxing capital income is an imperfect way of achieving this goal, as a positive capital income tax implies taxing leisure tomorrow more than today.

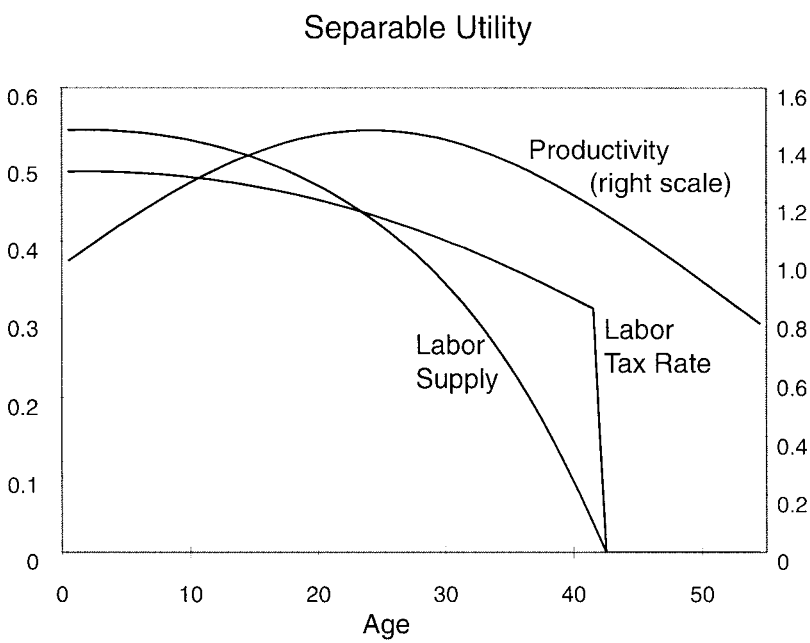

FIG. 2. Labor supply, productivity, and tax rate over the lifetime of individuals.

4.3. *Cobb–Douglas Utility*

The utility function considered in this subsection is one commonly used in applied studies in public finance and macroeconomics (for example, [9, 16, 24]). It consists of a general formulation of the utility function with a unitary intratemporal elasticity of substitution between consumption and leisure, a restriction on preferences that is necessary for any growth model to be consistent with a balanced growth path. Specifically, we consider utility functions of the form

$$U(c, l) = \sum_{j=j_0(t)}^{J} \beta^j u(c_{t,j})\, v(l_{t,j}), \tag{37}$$

where $u(\cdot)$ is homogeneous of degree $(1-\sigma)$ and $U(\cdot,\cdot)$ satisfies the Inada conditions. It is straightforward to show that optimal capital income taxes in this case are zero in the long run only under the very restrictive conditions stated in Proposition 3.2.

When the utility function is not separable across consumption and leisure, the uniform commodity taxation results no longer hold. Instead, the principles guiding the optimal manner in which to tax consumption and labor over the lifetime of individuals are stated in the following Proposition.

PROPOSITION 4.2 (Age-Profile of Optimal Taxes). *Assume that the utility function takes the form given by* (37) *with* $u(c) = (1-\sigma)^{-1} c^{1-\sigma}$ *and* $v(l) = (1-l)^{\theta(1-\sigma)}$. *In the steady–state,* (i) *the capital income tax at age* j *is positive if and only if* $l_{j+1} < l_j$ *and* (ii) *the labor income tax at age* j *is higher than at age* $j+1$ *if and only if* $l_{j+1} < l_j$.

Proof. (i) The definition of H_j^c (Eq. (20) in steady–state) under these preferences implies that $H_j^c = -\sigma - \eta/(1-l_j)$, where $\eta = \theta(1-\sigma)$. We can then rewrite Eq. (26) as

$$\frac{1+\hat{r}}{1+r_j} = \frac{1+\lambda+\lambda(-\sigma-\eta/(1-l_j))}{1+\lambda+\lambda(-\sigma-\eta/(1-l_{j+1}))}. \tag{38}$$

Note that $\tau_j^k > 0$ if and only if

$$\frac{1+\hat{r}}{1+r_j} = \frac{1+\hat{r}}{1+(1-\tau_j^k)\,\hat{r}} > 1. \tag{39}$$

From Eqs. (38) and (39), we obtain that $\tau_j^k > 0$ if and only if $l_{j+1} < l_j$.

(ii) Similarly, the definitions of H_j^c and H_j^l (Eqs. (20) and (21) in steady–state) under these preferences imply that $H_j^l - H_j^c = 1/(1-l_j)$. Equation (23) thus implies that

$$\tau_j^w/(1-\tau_j^w) = \frac{\lambda(H_j^l - H_j^c)}{1+\lambda+\lambda H_j^c}$$

$$= \frac{\lambda}{1+\lambda - l_j(1+\lambda(1-\sigma)(1+\theta)) - \lambda\sigma}.$$

It follows that the ratio $[\tau_j^w/(1-\tau_j^w)]/[\tau_{j+1}^w/(1-\tau_{j+1}^w)]$ is bigger than one if and only if $l_{j+1} < l_j$. ∎

Proposition 4.2 also bears on the Corlett and Hague intuition discussed earlier. Since leisure cannot be taxed directly, the first best solution is not achievable. However, the government can tax leisure indirectly by taxing more heavily commodities that are more complementary with leisure. Specifically, if leisure at age $j+1$ is higher than at age j, and if leisure and consumption move together, then consumption should be taxed more heavily at age $j+1$ than at age j; equivalently, capital income at age j should be taxed at a positive rate. Similarly, taxing labor income relatively more when it is high makes leisure relatively cheap when it is low.

An implication of the principle of optimal taxation developed in Proposition 4.2 is that capital income should not be taxed during retirement. This follows directly from the fact that the labor supply is constant during retirement.[25] Note, however, that leisure time during retirement is taxed indirectly by taxing the return on savings prior to retirement.

EXAMPLE. We illustrate the previous results with a numerical example. The AKS parameterization is modified by setting the intratemporal elasticity of substitution between consumption and leisure equal to one ($\rho = 0$). The (limiting) utility function is then of the form given by (37), with $u(c_{t,j}) = (1-\sigma)^{-1}(c_{t,j})^{1-\sigma}$ and $v(l_{t,j}) = (1-l_{t,j})^{\theta(1-\sigma)}$. We then set θ such that the steady–state aggregate amount of time devoted to work under these preferences matches that of the AKS economy ($\theta = 1.32$).[26] The interest rate in this initial steady–state is 12.4%, suggesting a benchmark value for γ equal to 0.8984.

Table 3 shows that capital income taxes are substantial under this parameterization: the tax rate on capital income is over 17% in the

[25] The same conclusion can be reached from the uniform commodity taxation intuition: during retirement, the utility function in Proposition 4.2 is effectively homothetic in the goods consumed.

[26] We again adjust the scaling constant in the production function so that the pretax wage rate is equal to one in the initial steady–state ($A = 1.04$).

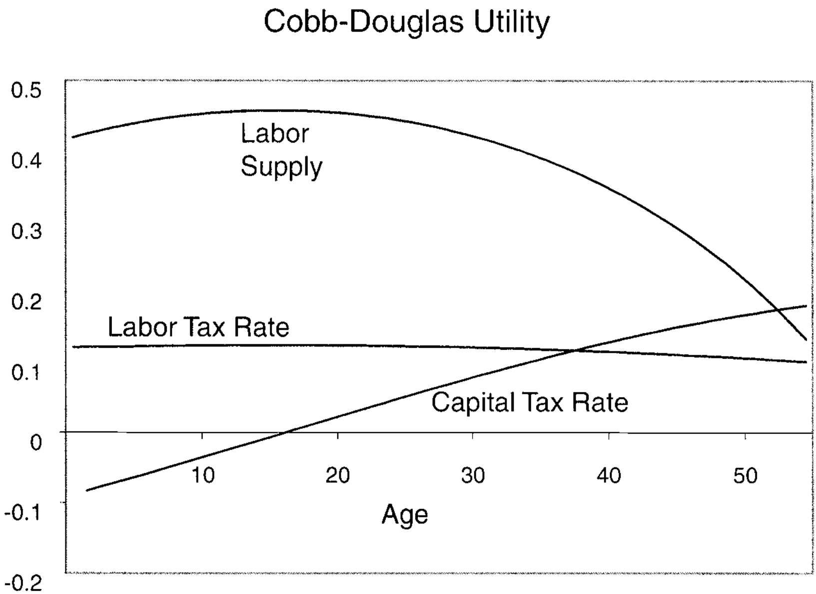

FIG. 3. Labor supply and tax rates over the lifetime of individuals.

benchmark case. Figure 3 illustrates how taxes vary with age under an age-dependent tax system. Following Proposition 4.2, capital income taxes are positive (negative) when the labor supply is decreasing (increasing), and labor income taxes follow the shape of the labor supply. As discussed previously, the government conditions both capital and labor income taxes on age in order to imitate a tax on leisure.

5. CONCLUSION

This paper studies optimal taxation in an overlapping generations economy similar to the one developed by [4, 5] to study fiscal policies. We show that uniform commodity taxation results are not likely to hold in this framework, giving rise to a role for interest taxation. Indeed, standard parameterizations of the model imply capital income tax rates well above zero.

The principles underlying optimal taxation of capital income in life-cycle economies relate to the Corlett–Hague intuition: optimal tax rates on capital are set in order to imitate a tax on leisure. A nonzero interest tax is efficient in life-cycle economies because consumption and leisure move

TABLE 3

Optimal Taxation with Cobb–Douglas Utility

γ	$\hat{r}$	τ^w	τ^k	b/y
0.8950	0.1285	0.3966	0.1808	0.1909
0.8984	0.1242	0.3800	0.1743	0.1194
0.9100	0.1099	0.3241	0.1494	−0.1731
0.9200	0.0978	0.2739	0.1274	−0.5076
0.9400	0.0745	0.1724	0.0827	−1.5108
0.9600	0.0521	0.0785	0.0450	−3.3115

together over the lifetime of individuals. And since leisure tends to increase with age, the tax rates on capital income tend to be positive. Furthermore, when preferences are such that uniform commodity taxation (or zero age-dependent capital income taxes) results hold, a positive capital income tax becomes optimal when the government cannot condition tax rates on age, as it imitates a labor income tax that declines with age.

An important drawback in our study, which permeates most of the literature on optimal taxation, is that the fiscal policies we consider are not time consistent [23]. Although this problem is not as acute in overlapping generations economies as it is in infinitely-lived agent models, a more satisfactory characterization of optimal taxation would focus on time-consistent policies.[27]

APPENDIX: AGE-INDEPENDENT TAXES

Under age-independent taxes, the Ramsey problem is given by

$$\max \sum_{t=-J}^{\infty} \gamma^t W_t,$$

where W_t is defined in (15), subject to feasibility (4) and the implementability constraints (13) and (14), to which we associate the Lagrange multipliers $\gamma^t \phi_t$, $\gamma^t \varepsilon^1_{t-j,j}$, and $\gamma^t \varepsilon^2_{t-j,j}$, respectively.

Since we only study this problem in steady–state, we only provide the first order conditions of this problem for $t \geqslant J$. These first-order conditions are as follows:

$$\gamma^t \phi_t (1+n) - \gamma^{t+1} \phi_{t+1} (1-\delta+f_{k_{t+1}}) = 0 \qquad \text{“}k_{t+1}\text{”}$$

$$\gamma^t W_{c_{t,0}} - \gamma^t \phi_t \mu_0 - \gamma^t \sum_{j=1}^{J} \varepsilon^1_{t-j,j} \partial_{c_{t,0}} R^1_{t-j,j} - \gamma^t \sum_{j=1}^{J-1} \varepsilon^2_{t-j,j} \partial_{c_{t,0}} R^2_{t-j,j} = 0 \qquad \text{“}c_{t,0}\text{”}$$

[27] Klein and Rios-Rull [20] study time-consistent optimal taxation in infinitely-lived agent models.

$$\gamma^{t-1}W_{c_{t-1,1}} - \gamma^t\phi_t\mu_1 - \gamma^t\varepsilon^1_{t-1,1}\partial_{c_{t-1,1}}R^1_{t-1,1} - \gamma^t\varepsilon^2_{t-1,1}\partial_{c_{t-1}1}R^2_{t-1,1}$$
$$-\gamma^{t-1}\sum_{j=1}^{J-1}\varepsilon^2_{t-1-j,j}\partial_{c_{t-1,1}}R^2_{t-1-j,j} = 0 \qquad \text{“}c_{t-1,1}\text{”}$$

$$\gamma^{t-j}W_{c_{t-j,j}} - \gamma^t\phi_t\mu_j - \gamma^t\varepsilon^1_{t-j,j}\partial_{c_{t-j,j}}R^1_{t-j,j} - \gamma^t\varepsilon^2_{t-j,j}\partial_{c_{t-j,j}}R^2_{t-j,j}$$
$$-\gamma^{t-1}\varepsilon^2_{t-j,j-1}\partial_{c_{t-jj}}R^2_{t-j,j-1} = 0 \qquad \text{“}c_{t-j,j}\text{”} \quad j = 2, \ldots, J-1$$

$$\gamma^{t-J}W_{c_{t-J,J}} - \gamma^t\phi_t\mu_J - \gamma^t\varepsilon^1_{t-J,J}\partial_{c_{t-J,J}}R^1_{t-J,J} - \gamma^{t-1}\varepsilon^2_{t-J,J-1}\partial_{c_{t-J,J}}R^2_{t-J,J-1} = 0$$
$$\text{“}c_{t-J,J}\text{”}$$

$$\gamma^t W_{l_{t,0}} + \gamma^t\phi_t\mu_0 z_0 f_{l_t} - \gamma^t\sum_{j=1}^{J}\varepsilon^1_{t-j,j}\partial_{l_{t,0}}R^1_{t-j,j} - \gamma^t\sum_{j=1}^{J-1}\varepsilon^2_{t-j,j}\partial_{l_{t,0}}R^2_{t-j,j} = 0$$
$$\text{“}l_{t,0}\text{”}$$

$$\gamma^{t-1}W_{l_{t-1,1}} + \gamma^t\phi_t\mu_1 z_1 f_{l_t} - \gamma^t\varepsilon^1_{t-1,1}\partial_{l_{t-1,1}}R^1_{t-1,1} - \gamma^t\varepsilon^2_{t-1,1}\partial_{l_{t-1,1}}R^2_{t-1,1}$$
$$-\gamma^{t-1}\sum_{j=1}^{J-1}\varepsilon^2_{t-1-j,j}\partial_{l_{t-1,1}}R^2_{t-1-j,j} = 0 \qquad \text{“}l_{t-1,1}\text{”}$$

$$\gamma^{t-j}W_{l_{t-j,j}} + \gamma^t\phi_t\mu_j z_j f_{l_t} - \gamma^t\varepsilon^1_{t-j,j}\partial_{l_{t-j,j}}R^1_{t-j,j} - \gamma^t\varepsilon^2_{t-j,j}\partial_{l_{t-j,j}}R^2_{t-j,j}$$
$$-\gamma^{t-1}\varepsilon^2_{t-j,j-1}\partial_{l_{t-j,j}}R^2_{t-j,j-1} = 0 \qquad \text{“}l_{t-j,j}\text{”} \quad j = 2, \ldots, J-1$$

$$\gamma^{t-J}W_{l_{t-J,J}} + \gamma^t\phi_t\mu_J z_J f_{l_t} - \gamma^t\varepsilon^1_{t-J,J}\partial_{l_{t-J,J}}R^1_{t-J,J} - \gamma^{t-1}\varepsilon^2_{t-J,J-1}\partial_{l_{t-J,J}}R^2_{t-J,J-1} = 0$$
$$\text{“}l_{t-J,J}\text{”}.$$

The steady–state Ramsey path is thus characterized by a system of $(4J+4)$ equations—the above $(2J+3)$ first-order conditions; R^1_j for $j = 1, \ldots, J$; R^2_j for $j = 1, \ldots, J-1$; and feasibility and implementability—in $(4J+4)$ unknowns $(\{c^j, l^j\}_{j=0}^{J}, \{\varepsilon^1_j\}_{j=1}^{J}, \{\varepsilon^2_j\}_{j=1}^{J-1}, k, \phi, \lambda)$.

Two properties of the steady–state are immediate. First, the steady–state is independent of the transition path. In particular, government debt is not part of the above system of equations and it can be obtained from the government budget constraint once a steady–state solution is found. Second, the steady–state capital–labor ratio has the modified golden rule property.

REFERENCES

1. Y. Alvarez, J. Burbidge, T. Farrell, and L. Palmer, Optimal taxation in a life cycle model, *Can. J. Econ.* **25** (Feb. 1992), 111–122.
2. A. B. Atkinson and A. Sandmo, Welfare implications of the taxation of savings, *Econ. J.* **90** (Sep. 1980), 529–549.

3. A. B. Atkinson and J. E. Stiglitz, "Lectures on Public Economics," McGraw–Hill, New York, 1980.
4. A. J. Auerbach and L. J. Kotlikoff, "Dynamic Fiscal Policy," Cambridge University Press, Cambridge, UK, 1987.
5. A. J. Auerbach, L. J. Kotlikoff, and J. Skinner, The efficiency gains from dynamic tax reform, *Int. Econ. Rev.* **24**, No. 1 (Feb. 1983), 83–100.
6. M. Bassetto, Political economy of taxation in an overlapping-generations economy, Discussion Paper 133, Federal Reserve Bank of Minneapolis, Dec. 1999.
7. T. Besley and I. Jewitt, Uniform taxation and consumer preferences, *J. Public Econ.* **58**, No. 1 (Sep. 1995), 73–84.
8. C. Chamley, Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* **54**, No. 3 (May 1986), 607–622.
9. V. V. Chari, L. J. Christiano, and P. J. Kehoe, Optimal fiscal policy in a business cycle model, *J. Polit. Economy* **102**, No. 4 (Aug. 1994), 617–652.
10. V. V. Chari and P. J. Kehoe, Optimal fiscal and monetary policy, *in* "Handbook of Macroeconomics," Vol. 1C, "Handbooks in Economics" (J. B. Taylor and M. Woodford, Eds.), Elsevier Science/North-Holland, Amsterdam, 1999.
11. W. Corlett and D. Hague, Complementarity and the excess burden of taxation, *Rev. Econ. Stud.* **21**, No. 1 (1953–1954), 21–30.
12. P. A. Diamond, National debt in a neoclassical model, *Amer. Econ. Rev.* **55**, No. 5, Part 1 (Dec. 1965), 1126–1150.
13. J. Escolano, Essays on Fiscal Policy, Intergenerational Transfers and Distribution of Wealth, Ph.D. dissertation, University of Minnesota, 1992.
14. C. Ghiglino and M. Tvede, Optimal policy in OG models, *J. Econ. Theory* **90**, No. 1 (Jan. 2000), 62–83.
15. S. Imrohoroglu, A quantitative analysis of capital income taxation, *Int. Econ. Rev.* **39**, No. 2 (May 1998), 307–328.
16. L. E. Jones, R. E. Manuelli, and P. E. Rossi, Optimal taxation in models of endogenous growth, *J. Polit. Economy* **101**, No. 3 (Jun. 1993), 485–517.
17. L. E. Jones, R. E. Manuelli, and P. E. Rossi, On the optimal taxation of capital income, *J. Econ. Theory* **73**, No. 1 (Mar. 1997), 93–117.
18. K. L. Judd, Redistributive taxation in a simple perfect foresight model, *J. Public Econ.* **28**, No. 1 (Oct. 1985), 59–83.
19. K. L. Judd, Optimal taxation and spending in general competitive growth models, *J. Public Econ.* **71**, No. 1 (Jan. 1999), 1–26.
20. P. Klein and J. V. Ríos-Rull, Time-consistent optimal fiscal policy, manuscript, Dec. 2000.
21. L. J. Kotlikoff, The A-K, model: It's past, present, and future, Working Paper 6684, National Bureau of Economic Research, Dec. 1998.
22. P. Krusell, V. Quadrini, and J. V. Ríos-Rull, Are consumption taxes really better than income taxes?, *J. Monet. Econ.* **37**, No. 3 (Jun. 1996), 475–503.
23. F. E. Kydland and E. C. Prescott, Rules rather than discression: The inconsistency of optimal plans, *J. Polit. Economy* **85**, No. 3 (Jun. 1977), 473–491.
24. R. E. Lucas Jr., Supply-side economy: An analytical review, *Oxford Econ. Papers* **42**, No. 2 (Apr. 1990), 293–316.
25. R. E. Lucas, Jr., and N. L. Stokey, Optimal fiscal and monetary policy in an economy without capital, *J. Monet. Econ.* **12**, No. 1 (Jul. 1983), 55–93.
26. P. A. Samuelson, The two-part golden rule deduced as the asymptotic turnpike of catenary motions, *Western Econ. J.* **6** (Jan. 1968), 85–89.
27. J. E. Stiglitz, Pareto efficient and optimal taxation and the new new welfare economics, *in* "Handbook of Public Economics," Vol. 2, "Handbooks in Economics" (A. J. Auerbach and M. S. Feldstein, Eds.), Chap. 15, Elsevier Science/North-Holland, Amsterdam, 1987.

28. L. H. Summers, Capital taxation and accumulation in a life cycle growth model, *Amer. Econ. Rev.* **71**, No. 4 (Sep. 1981), 533–544.
29. F. Welch, Effects of cohort size on earnings: The baby boom babies' financial bust, *J. Polit. Econ.* **87**, No. 5, Part 2 (Oct. 1979), S65–S97.