

Human Capital Investment and Debt Constraints*

David Andolfatto
Simon Fraser University
dandolfa@sfu.ca

Martin Gervais
The University of Western Ontario
gervais@uwo.ca

September 13, 2005

Abstract

When young individuals face binding debt constraints, their human capital investments will be insufficiently financed by private creditors. If generations overlap, then a well-designed fiscal policy may be able to improve human capital investments by replacing missing capital markets with an intergenerational transfer scheme. The optimal (balanced budget) fiscal policy in this context entails the joint provision of an education subsidy for the young and a pension program for the old, financed with a tax on those in their peak earning years. We demonstrate, however, that the desirability of such a cradle-to-grave policy depends crucially on the assumption of an exogenous debt constraint. If debt constraints arise endogenously for reasons of limited commitment, then the optimal (balanced budget) fiscal policy looks radically different. Furthermore, we find that cradle-to-grave type policy interventions may actually lead to lower levels of human capital investment as altered default incentives induce private creditors to contract the supply of student loans by an amount greater than the subsidy. In some cases, the constrained-optimal policy entails zero intervention. These results highlight the importance of taking seriously the reasons for why debt constraints exist.

Journal of Economic Literature Classification Numbers: E62; H52; I28; J24

Keywords: Education; Fiscal Policy; Debt Constraint

*We would like to thank an anonymous referee, Igor Livshits as well as seminar participants at the 2003 Midwest Macro conference in Chicago, the 2003 SED conference in Paris and the 2004 CPEG conference in Toronto. Both authors gratefully acknowledge financial support from the Social Sciences and Humanities Research Council of Canada. The second author gratefully acknowledges financial support from the CIBC human capital and productivity project and the Economic Policy Research Institute, both located at the University of Western Ontario.

1 Introduction

Government programs to subsidize the human capital investments of young adults are common in many countries. In the United States, for example, students enrolled in post-secondary institutions during the 2001–2002 academic year received nearly \$90 billion in financial aid of various forms.¹ The justification behind such programs presumably rests on the belief that left on their own, a significant number of individuals are prone to under-invest in their human capital. The prominent reason given for such under-investment is the presence of debt constraints that arise owing to the inalienability of human capital—an institutional feature that makes it difficult (if not impossible) to collateralize loans with securities backed by claims to future labor earnings.

The question we pursue in this paper is a theoretical one.² In particular, there is more than one way in which to model a debt constraint. In this paper, we are interested in examining whether the standard justification for policy intervention is robust to a reasonable perturbation in the way one views the operation of private credit markets. We find that policy implications are highly sensitive to the way in which one models the source of financial market imperfections.

The standard way in which to model a debt constraint is simply to assume that individuals face an exogenous borrowing limit (e.g., Aiyagari (1994), Huggett (1993)). The specification of an exogenous debt constraint essentially boils down to assuming that optimal behavior on the part of creditors is invariant to the structure of the economy. In particular, lending practices are assumed to be invariant to policy changes that may affect the incentive structure for debt repayment. Such a specification has the virtue of simplicity and, for some applications, may turn out to be a relatively innocuous assumption. However, from a theoretical perspective, it makes more sense to think of debt limits as being determined by creditors as a part of their optimal lending practices and that these practices may change in response to various policy regimes (e.g., Krueger and Perri (2001)). Empirically, there is considerable evidence to suggest that lending practices do vary across policy regimes (e.g., Gropp et al.

¹See *Trends in Student Aid*, published by the College Board.

²The issue concerning the empirical relevance of debt constraints in limiting the accumulation of human capital is the subject of a current debate in the literature; see, for example, Kane (1994), Card (2001), and Cameron and Heckman (1998, 2001).

(1997) and Pagano (2001)). Accordingly, a careful theoretical treatment concerning the effects and desirability of government education subsidies should be performed in the context of a model that endogenizes the debt constraint.

Because we are concerned primarily with policies directed at subsidizing the human capital investment expenditure of young adults, we adopt as a framework of analysis a deterministic overlapping generations model with endogenous human capital formation. We endogenize the debt constraint in a manner suggested by Kehoe and Levine (1993, 2000). In this setup, debt constraints arise owing to the inalienability of certain types of assets (primarily human capital, but also various government entitlements). We assume that such assets are beyond the reach of private creditors, but not necessarily beyond the reach of the government; in particular, individuals are not free to default on their current and future tax obligations. Unsecured private credit is extended only to the extent that the act of default imposes some costs. Following Kehoe and Levine, we assume that the act of default precludes any subsequent access to financial markets, inhibiting one's ability to smooth consumption over the remainder of the life-cycle. In this environment, the amount of credit extended to some individuals may be less than the amount that would be extended if debt contracts could be costlessly enforced. In these circumstances, the cost and benefit of any act of default is exactly balanced. As in Krueger and Perri (2001), these costs and benefits are influenced by the structure of fiscal policy. For example, a generous government pension plan may have the unintended consequence of increasing the incentive to default on student debt, leading to a contraction in the supply of private credit financing human capital expenditure in the earlier stages of the life-cycle.

The subject of our investigation is related to that of Boldrin and Montes (2005), who examine the state's role in mitigating the adverse consequences of debt constraints that inhibit human capital formation. These authors demonstrate that when generations overlap, credit market failures can in principle be circumvented by an appropriately designed fiscal policy that features the *joint* provision of public education and public pensions financed by a tax on those individuals in their peak earning years. This policy in effect replaces missing private credit markets with an intergenerational transfer scheme that mimics the behavior of a well-functioning credit market. Their analysis provides a theoretical rationale for the 'cradle to grave' policies that are so commonly observed (in varying degrees) across many countries. Unfortunately, this

rather strong conclusion appears to rest almost entirely on their maintained assumption of an exogenous debt constraint.

In this paper, we investigate the effects and desirability of the Boldrin and Montes ‘welfare state’ policy when debt constraints are endogenous. We find that the way in which endogenous human capital formation interacts with the endogenous debt constraint renders individual choice sets non-convex. In particular, human capital investments can only feasibly be financed at relatively high levels if they are to satisfy the no-default conditions implied by the endogenous debt constraint. Intuitively, a middle-aged doctor is unlikely to default on the debt accumulated during medical school, since any act of default would imply a precipitous decline in retirement living standards. For this reason, students in medical school should have access to relatively large quantities of credit. In contrast, middle-aged individuals in relatively low-skilled professions may very well find it in their interest to default on debts accumulated in earlier stages of the life-cycle, especially in the presence of inalienable government pension entitlements. For this reason, intermediate levels of human capital investment may not be feasible, leaving only very high or very low levels of human capital investment possible. This non-convexity opens up the possibility for multiple solutions to a young person’s choice problem, in which case *ex ante* identical individuals may exhibit *ex post* heterogeneity.

By way of a numerical example, we demonstrate how behavior in our model can differ substantially from what one would predict in a model with exogenous debt constraints as the level of education subsidies (and pensions) is increased in a cradle-to-grave type of policy. Over some range of intervention, behavior is qualitatively similar to the exogenous debt constraint model. In particular, increasing the subsidy to education increases the resources available to the young, leading to higher levels of human capital formation and higher levels of welfare. It is also possible, however, to have more generous subsidies increase human capital formation and *reduce* economic welfare, owing to the way in which the credit market responds to policy and how this response inhibits consumption smoothing. Finally, it is also possible that more generous education subsidies actually end up reducing human capital investments along with welfare. In these latter cases, the government subsidy does not compensate for the contraction in private lending.

We also find that with endogenous debt constraints the ‘optimal’ fiscal policy takes the form of an intergenerational transfer scheme that entails taxing the young and old, and redistributing the proceeds to those in their peak earning years. The logic underpinning this argument is simple. A credible commitment to tax the old implies that individuals must save (accumulate financial assets) during their peak earning years if they wish to smooth consumption over the later stages of their life-cycle. Such saving is made possible in part by the generous transfer that individuals receive from the government (via the young and old) during their peak earning years. Since financial assets are not exempt from seizure by private creditors, the cost of defaulting on private debts accumulated in the periods approaching middle age (e.g., student loans) now becomes excessively costly, so that a rational individual would never choose to default. To the extent that private creditors understand these incentives, they should be more than willing to extend credit to students (up to what is budget feasible), so that debt-constraints on the young become non-binding.

Our work is closely related to Lochner and Monge (2002), who also study an overlapping generations model with an endogenous debt constraint.³ The focus of their study and the model employed differ from ours along a number of dimensions. Of particular interest is the manner in which they endogenize the debt constraint. In their setup, an individual in default is (temporarily) precluded from borrowing and earns a lower rate of return on any planned saving. In addition, creditors may garnishee a fraction of earnings. This latter assumption turns out to be important because it allows at least some fraction of human capital to be used as collateral. Thus, an education subsidy is more likely to relax the debt constraint and promote human capital formation, since creditors can lay some claim on the higher levels of human capital. In contrast, if creditors cannot garnishee any significant fraction of human capital (as in our approach), an education subsidy is more likely to tighten the debt constraint since higher levels of human capital typically increase default incentives.

The paper is organized as follows. In Section 2, we present the basic model and characterize the allocations that would arise under two regimes: complete markets and exogenous debt constraints. In this section, we confirm the logic provided by Boldrin

³See also de la Croix and Michel (2004), who characterize equilibria in a model similar to ours both under a fixed interest rate and in a closed economy.

and Montes (2005) and show how the complete market allocation can be implemented by an intergenerational transfer scheme when individuals face exogenous borrowing limits. In section 3, we endogenize the debt constraints and characterize equilibria in general terms. Unfortunately, even in the simple framework we employ, not much can be said generally about how the economy reacts to policy. In part, this is due to the non-convexity that arises when debt constraints and human capital accumulation are endogenous. For this reason, we resort to a numerical example to demonstrate how the economy may be expected to respond to an increase in the generosity of a tax-financed education/pension policy. A number of interesting possibilities are recorded. Section 4 provides a brief conclusion and some suggestions for further research.

2 Basics

The economy is populated by a constant population of 3-period-lived individuals with preferences defined over lifetime consumption profiles (c_1, c_2, c_3) represented by the utility function:

$$U = \sum_{j=1}^3 \beta^{j-1} u(c_j), \quad \beta > 0, \quad (1)$$

where β is a discount factor and $u(\cdot)$ is increasing, strictly concave and satisfies the Inada conditions.

Each person is born with an endowment (w_1, w_2, w_3) which can be interpreted as basic labor income. In addition, each person has the opportunity to invest in a capital project when young, which we interpret as education. This technology takes x units of output when young and returns $f(x)$ units of output when middle-aged, where $f(\cdot)$ is increasing, strictly concave, with $f(0) = 0$. For simplicity, we assume that human capital depreciates fully as one enters the final period of life.

2.1 Complete Market Allocation

Assume that individuals are free to save or borrow at an exogenous (gross) real rate of interest $R = \beta^{-1} > 1.0$. Let a_j denote an individual's net financial asset position

in the j^{th} period of life. Then, by definition we have:

$$\begin{aligned} c_1 + x + a_2 &= w_1 + a_1; \\ c_2 + a_3 &= w_2 + f(x) + Ra_2; \\ c_3 + a_4 &= w_3 + Ra_3. \end{aligned}$$

Assume that individuals begin life with zero financial wealth, so that $a_1 = 0$. If individuals cannot die in debt ($a_4 \geq 0$), then the equations above imply a life-time budget constraint:

$$c_1 + x + \frac{c_2}{R} + \frac{c_3}{R^2} \leq w_1 + \frac{w_2 + f(x)}{R} + \frac{w_3}{R^2}. \quad (2)$$

Throughout the analysis, we restrict attention to steady state allocations. The complete market allocation (CMA) features $c_j^* = c^*$ and an x^* satisfying:

$$\begin{aligned} c^* &= \left[\frac{w_1 + \beta(w_2 + f(x^*)) + \beta^2 w_3 - x^*}{1 + \beta + \beta^2} \right]; \\ R &= f'(x^*). \end{aligned}$$

Note that the CMA also maximizes the utility of a representative young agent (out of the set of stationary allocations). For this reason, we will also refer to the CMA as an ‘optimal’ allocation.

2.2 Exogenous Debt Constraints

Assume that individuals are prevented from borrowing an amount greater than a , so that:

$$\begin{aligned} a_2 &\geq -a; \\ a_3 &\geq -a, \end{aligned} \quad (3)$$

where $0 \leq a \leq w_3/R$ is a parameter. Then one of four possible outcomes may arise depending on which combination of constraints in (3) bind. Since student loan programs are largely based on the presumption of binding debt constraints afflicting the young, the most natural case to consider here is when $a_2 = -a$ and $a_3 > -a$, which is likely to be the case when the earnings profile displays a ‘hump-shaped’

pattern. In this case, individuals wish to accumulate debt when young and save when middle-aged. To prevent the second constraint from binding, we assume that earnings in the late stage of the life-cycle are sufficiently low. In addition, in order to ensure that the young wish to borrow, assume that they have no earnings. These assumptions on the endowment profile are summarized as follows:

$$w_2 > w_3 > w_1 = 0. \quad (4)$$

Let $(c_1^0, c_2^0, c_3^0, x^0)$ denote the equilibrium allocation that arises when only the young are debt-constrained. This allocation is characterized as follows:

$$\begin{aligned} u'(c_1^0) &= u'(c_2^0) + \mu_2^0; \\ c_2^0 &= c_3^0; \\ c_1^0 + x^0 &= a; \\ \beta u'(c_2^0) [f'(x^0) - R] &= \mu_2^0; \\ a + \beta c_2^0 + \beta^2 c_3^0 &= \beta(w_2 + f(x^0)) + \beta^2 w_3, \end{aligned}$$

where $\mu_2^0 > 0$ is the value of the Lagrange multiplier associated with the debt constraint $a_2 \geq -a$.

A few observations are in order here. First, observe that $c_1^0 < c_2^0 = c_3^0$. That is, the debt constraint inhibits consumption smoothing over the life-cycle. Second, observe that $x^0 < x^*$, so that the debt constraint also inhibits the acquisition of human capital, leading to lower lifetime wealth. Finally, note that the condition $c_1^0 + x^0 = a$ implies that individuals face a ‘mechanical’ one-for-one trade-off between consumption and investment when young. In other words, the amount that a creditor is willing to lend (a) to students is by assumption invariant to how the student chooses to use the borrowed funds (e.g., beer versus books).

2.3 Optimal Policy

In order to facilitate a direct comparison with the results of Boldrin and Montes (2005), we restrict attention to government policies that balance the budget on a period-by-period basis.⁴

⁴This restriction arises more naturally in Boldrin and Montes (2005) since they employ a general equilibrium model. In the context of our small open economy, this restriction prevents the govern-

Let τ_j denote the lump-sum transfer of resources accruing to age- j individuals. The government budget constraint implies that $\sum_{j=1}^3 \tau_j = 0$. Furthermore, if a fiscal policy is to implement the CMA, the policy must induce a present value budget constraint that corresponds to the CMA scenario. This latter fact imposes the restriction that $\sum_{j=1}^3 \beta^{j-1} \tau_j = 0$. If we let $s \equiv \tau_1$, then these two restrictions imply the following structure for any optimal fiscal policy:

$$\begin{aligned}\tau_1 &= s; \\ \tau_2 &= -(1 + R)s; \\ \tau_3 &= Rs.\end{aligned}\tag{5}$$

The structure of an optimal fiscal policy (in this environment) necessarily entails transfers that accrue to the young and old, financed by taxes on the middle-aged ($s > 0$). This is the key insight developed in Boldrin and Montes (2005): an optimal policy requires the joint provision of public education and public pensions.

For any given level of s , let $c_j(s)$, for $j = 1, 2, 3$ and $x(s)$ denote optimal individual behavior. Associated with this behavior are the desired net asset positions $a_j(s)$, for $j = 2, 3$ that must satisfy $a_j(s) \geq -a$. We are assuming a parameterization that yields $a_2(0) = -a$ and $a_3(0) > -a$ (only the young are debt-constrained in a laissez-faire world).

As long as the young remain debt-constrained, their first-period decisions are constrained by:

$$c_1 + x = a + s.$$

Clearly, higher levels of s increase the resources available for the young, so that the debt constraint binds less severely with higher levels of s . It is easy to show that $c_1(s)$ and $x(s)$ are increasing in s , so that more generous education subsidies promote the formation of human capital. For a subsidy level equal to $\underline{s} = (c^* - c_1^0) + (x^* - x^0) > 0$, the debt constraint becomes slack (and remains slack for $s > \underline{s}$).

On the other hand, note that higher levels of s also serve to reduce the after-tax income of agents in their middle age and increase the after-tax income of agents

ment from borrowing from foreigners. In fact, if the government could not commit to making good on foreign obligations, it would be in its incentive here to default on any level of foreign debt (and use the stolen funds to operate a pay-as-you go pension program).

in their old age. Standard consumption smoothing arguments imply that $a_3(s)$ is a decreasing function of s . For sufficiently generous levels of s , the debt constraint facing middle-aged individuals may begin to bind. Let \bar{s} denote such an s ; i.e., $a_3(\bar{s}) = 0$. Note that depending on parameters, \underline{s} may be either larger or smaller than \bar{s} .

Proposition 1 *Assume that $\underline{s} \leq \bar{s}$. Then there exists a range of fiscal policies $s \in [\underline{s}, \bar{s}]$ that implement the CMA.*

To prove this proposition, consider the ‘minimalist’ intervention $\underline{s} = (c^* - c_1^0) + (x^* - x^0) > 0$. Now, consider the resources that are available to the young:

$$c_1 + x = a + \underline{s}.$$

Substituting the definition of \underline{s} into the equation above and using the fact that $a = c_1^0 + x^0$ implies that $c_1 + x = c_1^* + x^*$. In other words, the choices (c_1^*, x^*) are now feasible for the young. Furthermore, since by construction individuals face the lifetime budget constraint (2), the solution to their choice problem corresponds to the CMA. Subsidy levels in the range $s \in (\underline{s}, \bar{s}]$ serve only to alter private net asset positions $a_2(s)$, $a_3(s)$, but otherwise leaves the allocation unchanged. That is, increasing s in this range results in a dollar-for-dollar increase in the saving of young individuals, which they use to pay for the higher taxes they will face when middle-aged. A similar argument applies to the savings of the middle-aged. For the case in which $\underline{s} > \bar{s}$, there is no fiscal policy capable of implementing the CMA, although an optimal policy will still, in general, entail some government intervention.

In either case, the optimal fiscal policy improves welfare by effectively replacing the missing private debt markets with an intergenerational transfer scheme. In order to leave the present value budget constraint unchanged, these intergenerational transfers must ‘look like’ loans to individuals. In particular, the subsidy to the young imitates a loan that is repaid (with interest) when middle-aged in the form of higher taxes. Likewise, a part of these higher taxes constitute ‘forced savings’ that are meant to augment retirement income when old.

3 Endogenous Debt Constraints

The analysis above provides some justification for some of the ‘cradle-to-grave’ policies favored by so many governments around the world. From this perspective, the issue is not whether to intervene or not, but how to best choose a level of intervention $s > 0$. What we wish to argue below is that this rather sweeping claim hinges critically on the assumed exogeneity of debt constraints.

3.1 The Credit Market

We assume that creditors will rationally lend up to what they may feasibly be expected to recover. In practice, creditors can sometimes garnishee earnings and/or seize other assets belonging to debtors in the event of default. Thus, the ability to collateralize loans is an important aspect determining the supply of credit. But even if loans can not be collateralized (say, because human capital is inalienable), unsecured loans may still be repaid to the extent that any act of default imposes other costs on debtors.

In this paper, we will follow Kehoe and Levine (1993, 2000) and assume that conditional on default, debtors are forever excluded from accessing financial markets again. One way of thinking about this is that private creditors are able to garnishee 100% of any future savings contemplated by the transgressor (making the act of saving following an act of default irrational), but are able to garnishee 0% of any future wages or government transfers (human capital and government pensions are inalienable). On the other hand, debtors are not free to discharge public debt (outstanding tax obligations).

In the context of the present model then, any allocation will have to respect the following ‘no-default’ conditions:

$$\begin{aligned} u(c_2) + \beta u(c_3) &\geq u(w_2 + f(x) - (1 + R)s) + \beta u(w_3 + Rs); \\ u(c_3) &\geq u(w_3 + Rs). \end{aligned} \tag{6}$$

As before, four cases may arise depending on the combination of debt constraints that bind in any given situation.

3.2 Individual Choice Problem

Conditional on some level of s satisfying (5), a representative young agent chooses (c_j, x) to maximize (1) subject to the lifetime budget constraint (2) and the no-default conditions (6). Formally, the problem under study takes the following form:

$$v(s) = \max_{y \in \Gamma(s)} h(y) \quad (7)$$

where $y \equiv (x, c_1, c_2, c_3) \in Y \subseteq R_+^4$ and $s \in S \subseteq R$. The function $h : Y \rightarrow R$ is given by:

$$h(y) \equiv \sum_{j=1}^3 \beta^{j-1} u(c_j),$$

and the constraint correspondence $\Gamma : S \rightarrow Y$ consists of all values of y satisfying:

$$\begin{aligned} \beta [w_2 + f(x)] + \beta^2 w_3 - x - c_1 - \beta c_2 + \beta^2 c_3 &\geq 0; \\ u(c_2) + \beta u(c_3) - u(c_2^D(x, s)) - \beta u(c_3^D(s)) &\geq 0; \\ u(c_3) - u(c_3^D(s)) &\geq 0; \end{aligned} \quad (8)$$

where $c_2^D(x, s) \equiv w_2 + f(x) - (1+R)s$ and $c_3^D(s) \equiv w_3 + Rs$ represent the consumption allocations associated with default. Unfortunately, one cannot guarantee the convexity of $\Gamma(s)$ under general conditions. Non-convexities may arise owing to the way in which endogenous debt constraints interact with the endogenous accumulation of human capital.⁵ We now explore the reasons for why this is so.

First note that there exists a $\tilde{s} = \frac{w_2 - w_3}{1+2R}$ such that at $x = 0$, we have $c_2^D(0, \tilde{s}) = c_3^D(\tilde{s})$. Similarly, for any $s \geq \tilde{s}$, at $x = 0$ we have $c_2^D(0, s) < c_3^D(s)$. Since c_2^D is increasing in x , it follows that there exists an \hat{x} such that: $c_2^D(\hat{x}, s) = c_3^D(s)$. That is, $\hat{x}(s)$ represents the level of human capital (conditional on s) that would provide perfect consumption smoothing from middle to old age in the default allocation. Clearly, $\hat{x}(s)$ is well defined for all $s \geq \tilde{s}$ and is an increasing function of s .

Recall that the only way in which creditors may punish those in default is to exclude them from further participation in the financial market. For this punishment to be costly to the defaulter, his default allocation must feature consumption variability

⁵Note that for endowment economies (e.g., an economy in which x is exogenous), the issue of non-convexity of the constraint set does not arise.

in the sense that $c_2^D \neq c_3^D$. In other words, the no-default restrictions prevent human capital allocations in the neighborhood of $\hat{x}(s)$ from satisfying the (middle-aged) no-default condition (i.e., the second equation in (8)).

Figure 1 plots the set of feasible human capital levels as a function of s . The restriction that x cannot lie in the neighborhood of $\hat{x}(s)$ cuts a ‘path’ through the set of feasible human capital allocations, potentially leaving the constraint set non-convex over a wide range of s . This non-convexity opens up the possibility that multiple solutions may exist for some s , leaving the value function $v(s)$ non-differentiable at such points. The intuition for this possibility is that for low values of human capital, the default allocation features low consumption when middle-aged relative to old, which provides an incentive for individuals to pay back their loan. Similarly, for high values of human capital, consumption when middle-aged is high relative to consumption when old, which again provides an incentive for individuals to pay back their loan. However, for intermediate values of human capital, the default allocation features relatively good consumption smoothing between middle-aged and old, making the default allocation relatively attractive. These intermediate levels of human capital are therefore not feasible. Although lifetime wealth is low for low levels of human capital, consumption smoothing is relatively good. On the other hand, although individuals have higher wealth when human capital is high, consumption smoothing is relatively bad because consumption when young is relatively low. For some value of s , the two types of allocation can give individuals the same level of utility, that is, both allocations are solutions to the consumer’s problem.

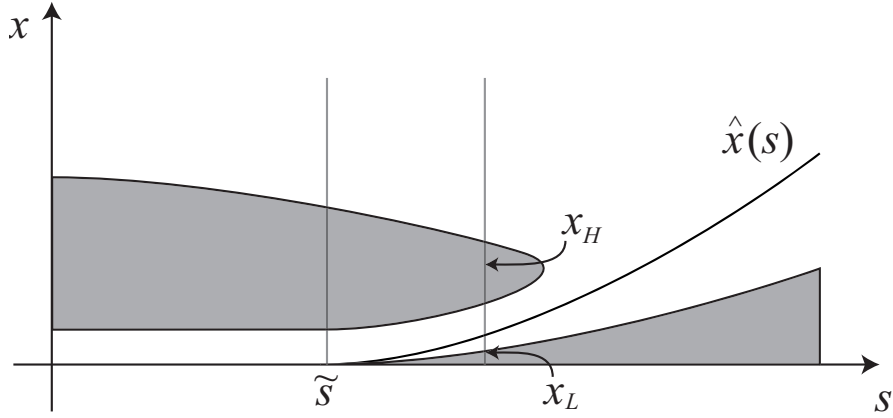
Let $y(s)$ denote a solution to the individual’s choice problem conditional on some s satisfying (5). Assume that in the laissez-faire state ($s = 0$), the following holds true: (1) the solution $y(0)$ exists and is unique; and (2) only the young are debt constrained; i.e.,

$$\begin{aligned} u(c_2(0)) + \beta u(c_3(0)) &= u(w_2 + f(x(0)) + \beta u(w_3)); \\ c_3(0) &> w_3. \end{aligned}$$

This is the analog to what we assumed earlier in the exogenous debt constraint scenario. In particular, note that since only the young are debt-constrained, we must have $c_1(0) < c_2(0) = c_3(0)$. In addition, $x(0) < x^*$.

Since $w_2 > w_3$, such an equilibrium must necessarily feature $c_2^D(x(0), 0) > c_3^D(0)$

Figure 1: Constraint Correspondence



and $c_3(0) > c_3^D(0)$. That is, the act of default must carry the prospect of a precipitous decline in mid to old-age consumption. This cost must be such that the benefit to default, measured by the consumption gain $c_2^D(x(0), 0) > c_2(0)$, makes the middle-aged individual just indifferent between defaulting or not.

3.3 Evaluating the Welfare State

In this section, we restrict our attention to exogenously determined levels of $s \geq 0$ that satisfy (5). Here we are interested in examining how different levels of intervention may affect economic behavior and the welfare of individuals. Because little can be said about behavior in general terms, we restrict our attention to an example that is suggestive of how policy can influence behavior when debt constraints are endogenous.

3.3.1 Parameterization

Functional forms are given by:

$$\begin{aligned} u(c) &= (1 - \sigma)^{-1} c^{(1-\sigma)}; \\ f(x) &= \theta^{-1} x^\theta, \end{aligned}$$

where $\sigma = 2.0$ and $\theta = 0.5$. Assuming that each period is of twenty years in length, the discount factor is chosen to yield an annualized real rate of interest equal to one

percent; i.e., $\beta = (0.99)^{20} = 0.8179 = R^{-1}$. The lifetime endowment process is given by $(w_1, w_2, w_3) = (0.0, 1.0, 0.5)$.

For this parameterization, we are able to solve for the individual's policy correspondence over a wide range of S . The parameterization is such that for $s = 0$, only the young are debt-constrained. Figures 2-4 display the effects of different policies over the range $s \in S = [0.0, 1.2]$. For expositional purposes, it will be useful to divide S into three sets: $A = [0.0, 0.2]$; $B = [0.20, 0.65]$; and $C = [0.65, 1.2]$. These three sets correspond to low, moderate and high subsidy levels, respectively. A separate subsection (Endogenous Heterogeneity) is devoted to a special case that occurs at the point $s = 0.20$.

3.3.2 Low Subsidy Levels

Consider the effects of increasing s over the range A . When the debt constraint is modeled exogenously, it follows that debt constrained students will respond to an education subsidy by increasing their human capital formation monotonically with increases in s . This result turns out not to be robust to the way in which the debt constraint is modeled. With an endogenous debt constraint, higher levels of s may also increase the incentive to default. Rational creditors would then respond by restricting the supply of loans to the young, thereby making the debt constraint bind more tightly. In fact, a one dollar education subsidy may very well lead to a reduction in private credit by more than one dollar, leaving the young with even less resources than prior to the intervention. A more generous education subsidy may therefore lead to lower levels of human capital formation. Figure 2 shows that this is exactly what happens in this parameterization.

Figure 3 shows that the consumption of the young decreases with s , consistent with the fact that the young have fewer available resources. Note that as the middle-aged are not debt-constrained, individuals can perfectly smooth their consumption in the later stages of the life-cycle. Consumption for the middle-aged and old increases initially with s and then declines somewhat. This behavior results from two forces that work in opposite directions as the level of the subsidy increases. First, as human capital declines, the middle-aged have fewer resources ($f(x)$ is lower). Second, as creditors restrict the supply of loans (to the young), the middle-aged have a smaller

loan to repay, leaving them with more resources. The second effect dominates initially and the first effect dominates subsequently. Even over the range where middle and old-age consumption increases with s , we see from Figure 4 that welfare declines with s . Welfare declines for two reasons: (1) the decline in human capital formation results in lower lifetime wealth; and (2) the degree of lifetime consumption smoothing deteriorates for higher levels of s .

3.3.3 Moderate Subsidy Levels

Let us now consider the effects of increasing s over the range B . From Figure 2 we see that individuals consume their autarkic allocation ($a_2 = a_3 = 0$). As the education subsidy rises, so does the level of human capital formation. From Figure 3, we see that consumption for the young and old is increasing, while consumption for the middle-aged is decreasing. Figure 4 reveals that welfare is increasing in s over this range. Over this range of s , the model's qualitative implications correspond to those that would emerge in a model of exogenous debt constraints.

3.3.4 High Subsidy Levels

Consider now the effects of increasing s over the range C . Over this range, human capital formation rises with s up to a point at which $x = x^*$. Increasing s beyond this point has no further effect on human capital, but induces the young to begin saving a portion of their generous transfer (once they are no longer debt-constrained).

The generous transfer also implies a high level of pension income, which accounts for why old-age consumption continues to increase with s (note that the middle aged are debt-constrained here and so cannot borrow against the prospect of higher future pension income). The higher tax burden associated with a high subsidy level results in further declines in consumption for the middle-aged. The consumption of the young rises modestly at first and then declines to a point at which $c_1 = c_2$ (again when the young are no longer debt-constrained). Note, however, that consumption smoothing over the later periods of life continues to deteriorate with further increases in s .

From Figure 4, observe that welfare declines in s even as human capital formation increases in s . Hence, the model provides an important caveat in terms of how poli-

cymakers should interpret the welfare consequences of increasing education subsidies. In particular, finding evidence that education subsidies have increased human capital formation is not, in itself, sufficient to show that the program has been welfare improving.

Evaluating the welfare function $v(s)$ over the entire range of $S = [0.0, 1.2]$, we see that for this parameterization, the optimal level of intervention is $s = 0$. Notice that a local maximum also occurs at an s that defines the borders of the sets B and C . For different parameterizations, an interior s may be optimal over subsidy levels $s \geq 0$. Although we do not have a general proof, our numerical experiments suggest that from a *laissez-faire* equilibrium in which the young are borrowing constrained, no level of $s \geq 0$ implements the CMA.

3.3.5 Endogenous Heterogeneity

As we remarked earlier, the interaction of endogenous debt constraints with endogenous human capital formation potentially induces a non-convexity in the constraint set. When a non-convexity is present, optimal choices may be characterized by policy correspondences so that even identical agents may optimally make different choices, giving rise to *ex post* heterogeneity. In the parameterization considered here, there is indeed a subsidy level for which multiple solutions exist.

We now consider that subsidy level $s = 0.20$, where the policy correspondence contains two distinct solutions. One allocation features a relatively high level of human capital (x_H) but relatively poor consumption smoothing. In this allocation, only the young are debt-constrained. The other allocation features a relatively low level of human capital (x_L) and a relatively high degree of consumption smoothing. In this allocation, both the young and middle-aged are debt-constrained. Both allocations x_L and x_H yield precisely the same utility payoff (associated with the kink in the value function displayed in Figure 4).

To develop some intuition for the multiple solutions that exist at $s = 0.20$, refer back to Figure 1, which depicts the relative positions of x_H and x_L in the constraint set. However, convex combinations of these two allocations are not feasible. Consider, for example, the allocation associated with human capital level

$x' = \lambda x_H + (1 - \lambda)x_L > \hat{x}$ for some $\lambda \in (0, 1)$. Remember that x_H is chosen such that a middle-aged individual precisely balances the cost (reduced consumption smoothing) and benefit (increased resources) associated with defaulting so that they are just indifferent between defaulting or not. Moving from x_H to x' implies fewer resources for the defaulting agent. However, this cost is more than offset by the improvement in consumption smoothing afforded by an allocation x' (which is closer than x_H to \hat{x}). An allocation associated with x' will therefore not be possible, since it would necessarily trigger a default.

Consider now an allocation associated with human capital level x'' ; i.e., a convex combination of x_H and x_L such that $x_L < x'' < \hat{x}$. The allocation x'' now increases the incentive to default for two reasons: (1) a middle-aged individual has more resources that are beyond the reach of private creditors; and (2) such an allocation actually improves consumption smoothing in the later stages of the life-cycle (i.e., again, note that x'' is closer than x_L to \hat{x}). Consequently, an allocation associated with x'' is not feasible, since it would necessarily trigger a default.

The multiplicity of solutions in this environment suggests a theory of income distribution and heterogenous career choices even under circumstances in which individuals do not differ in any fundamental sense. Banks may be willing to extend student loans to young people who choose medical school or law school (x_H) but not to those who choose secretarial school or art school (x_L). Banks understand that doctors and lawyers are unlikely to default on their loans, since banks would be able to garnishee 100% of their (non-pension) retirement savings (effectively, precluding their future participation in the financial market). In this way, the act of default offers the prospect of a precipitous decline in old-age living standards for those who choose high human capital career paths. The cost of such a path is in the form of lower consumption levels in the early stage of the adult life-cycle (i.e., during the schooling period). In contrast, those who choose relatively low-skill career paths cannot be punished in the same way, so that banks rationally refuse to extend such individuals unsecured credit.

3.4 Optimal Policy Revisited

In the analysis above, we restricted our attention to $s \geq 0$ (i.e., subsidies to the young and old financed with a tax on the middle aged). This restriction was motivated primarily by two facts. First, it seems to correspond to empirical observation; and second, the optimal policy derived by Boldrin and Montes (2005) satisfied this restriction. However, one should note that there is nothing in principle (at least, in our theoretical setup) that restricts $s \geq 0$. In the present context, $s < 0$ would correspond to a policy that taxed the young and old, using the proceeds to finance a transfer to the middle aged. Surprisingly, we find that the optimal policy in fact takes this form when debt constraints are endogenous. Below, we explain the economics underlying this result.

The first thing to note is that debtors will not default if the cost inflicted by such a decision can be made sufficiently high. The costs that private creditors can impose on defaulters is limited by the inalienability of human capital (and government transfers). For example, consider the case of a middle-aged individual who enters the period with $a_2 < 0$ (i.e., he is in debt to private creditors). This individual has the option to default on private debt. The punishment for this action is exclusion from further participation in the financial market, so that $a_3 = 0$. In this event, the individual can expect to consume the following amount in old age:

$$c_3^D(s) = w_3 + Rs.$$

Here, we see clearly how government policy can affect the incentive to default (and hence the supply of credit). In particular, a generous (and inalienable) public pension plan reduces the cost to defaulting on private debt. To the extent that private creditors understand this incentive, the supply of credit may contract with an expansion in s . However, note that there is nothing, in principle, that prevents setting s to a negative number. Such a policy would entail a tax on the young and old, together with a transfer to the middle-aged. Consider, for example, setting the subsidy level to any s ‘sufficiently close’ to s^* , where:

$$s^* \equiv -\frac{w_3}{R}. \tag{9}$$

This policy promises to tax away almost all earnings that are expected to accrue in old age. To the extent that such a policy can be made credible, the punishment for default

when middle-aged can then be made arbitrarily large (recall the Inada assumptions on the utility function). Furthermore, individuals must save for old-age consumption, guaranteeing that the second no-default constraint does not bind. It follows that neither of the no-default conditions in (6) will bind under any circumstance. Intuitively, the only way to provide for consumption in old-age is to privately accumulate assets. To protect such assets from seizure by creditors, individuals understand that they must not default when they have the opportunity to do so (when they are middle-aged). Creditors, understanding the nature of the incentives in place, are then willing to extend all loans (to the young) that are budget feasible. The following proposition states this result, which holds under any specification of preferences, technology, and endowments that satisfy the conditions given earlier in section 2:

Proposition 2 *There exists a policy $s \in [s^* - \varepsilon, s^*)$ for some arbitrarily small $\varepsilon > 0$ that can implement the CMA. Such policy entails an intergenerational transfer of resources away from the young and old to the middle-aged.*

3.4.1 Discussion

The optimal policy described above appears to be radically different from what we typically observe. In reality, most economies are characterized by publicly funded education and pension programs, so that $s > 0$. In contrast, our optimal policy calls for a tax on the young and old, with subsidies accruing to those in their peak earning years. How are we to interpret this discrepancy between theory and reality?

One interpretation is that the Kehoe and Levine (1993) debt-constraint specification is wrong. If this is in fact the case, then one is left wondering how else one might model the presence of debt constraints. One alternative is to simply assume that debt constraints are exogenous. As Boldrin and Montes (2005) have shown, the optimal policy intervention in this case resembles actual policy interventions quite closely. But as we have remarked above, the notion of an exogenous debt constraint is theoretically unappealing. Furthermore, there is evidence suggesting that lending practices do respond to changes in policy regime (e.g., changes in bankruptcy law).

Another interpretation is that the Kehoe–Levine debt-constraint specification is correct, but that the model developed here that embeds this specification is somehow

not quite right. In fact, there are at least two important drawbacks to our theoretical formulation when it comes to the issue of implementation. First, we have assumed above that the government can credibly commit to the optimal policy. A part of this optimal policy requires the government to transfer resources to the middle-aged, who then use these resources to pay back private creditors. If creditors do not believe that the government will act in the prescribed way, then they will be unwilling to extend credit to the young in the first place. But even if the government could commit to such a policy, our analysis features one other potential drawback. That is, by focussing on steady state allocations, we have glossed over issues relating to the political feasibility of implementing our ‘optimal’ policy from some arbitrary initial condition. In particular, note that since the optimal policy entails a heavy tax on the old, the initial old may be placed in extremely dire straights (depending on their initial level of assets). To the extent that the older generation carries significant political weight, they may reasonably be expected to block the implementation of our suggested optimal policy.

4 Conclusion

Intergenerational transfer schemes that subsidize the young and old are a prominent component of many government policies. Such policies are motivated at least in part by the perception that debt-constraints prohibit the young (especially those from poorer families) from attaining the efficient level of human capital investment. Such a view appears to have some theoretical foundation. In particular, Boldrin and Montes (2005) demonstrate that an education subsidy can be thought of as a loan to be repaid in the form of higher taxes during one’s peak earning years. To align incentives correctly while balancing the government budget, one must implement the education subsidy jointly with a public pension program.

Unfortunately, this strong policy conclusion appears to rest heavily on the questionable assumption of an exogenous debt constraint. In particular, the prescription for policy appears to be dramatically different when the debt constraint is determined endogenously by the optimal lending practices of creditors. Arbitrary interventions may even lead to a reduction in human capital investment and economic welfare. The

basic conclusion here is that any government intervention intended to replace missing private financial markets should be designed with a clear view as to why these markets are missing in the first place (rather than simply assuming their non-existence without explanation). As different policy conclusions are likely to follow from different hypotheses concerning the functioning of private credit markets, future research should be directed toward identifying more precisely the fundamental source of credit market imperfections.

References

- Aiyagari, S. R. (1994). Uninsurable idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics* 109(3), 659–684.
- Boldrin, M. and A. Montes (2005). The intergenerational state: Education and pensions. *The Review of Economic Studies*, Forthcoming.
- Cameron, S. V. and J. J. Heckman (1998). Life cycle schooling and dynamic selection bias: Models and evidence for five cohorts of american males. *Journal of Political Economy* 106(2), 262–333.
- Cameron, S. V. and J. J. Heckman (2001). The dynamics of educational attainment for black, hispanic, and white males. *Journal of Political Economy* 109(3), 455–499.
- Card, D. (2001). Estimating the return to schooling: Progress on some persistent econometric problems. *Econometrica* 69(5), 1127–1160.
- de la Croix, D. and P. Michel (2004). Education and growth with endogenous debt constraints. Unpublished manuscript.
- Gropp, R., J. K. Scholz, and M. J. White (1997). Personal bankruptcy and credit supply and demand. *Quarterly Journal of Economics* 112(1), 217–251.
- Huggett, M. (1993). The risk-free rate in heterogenous-agent, incomplete insurance economies. *Journal of Economic Dynamics and Control* 17(5–6), 953–969.
- Kane, T. J. (1994). College entry by blacks since 1970: The role of college costs, family background, and the returns to education. *Journal of Political Economy* 102(5), 878–911.
- Kehoe, T. J. and D. K. Levine (1993). Debt constrained asset markets. *Review of Economic Studies* 60(4), 865–888.
- Kehoe, T. J. and D. K. Levine (2000). Liquidity constrained markets versus debt constrained markets. *Econometrica* 69(3), 575–598.
- Krueger, D. and F. Perri (2001). Risk sharing: Private insurance markets or redistributive taxes? Unpublished manuscript.

Lochner, L. and A. Monge (2002). Endogenous credit constraints and human capital formation. NBER Working Paper No. 8815.

Pagano, M. (2001). *Defusing Default: Incentives and Institutions*. Washington: Johns Hopkins University Press.

Figure 2: Human capital and Assets as a function of s

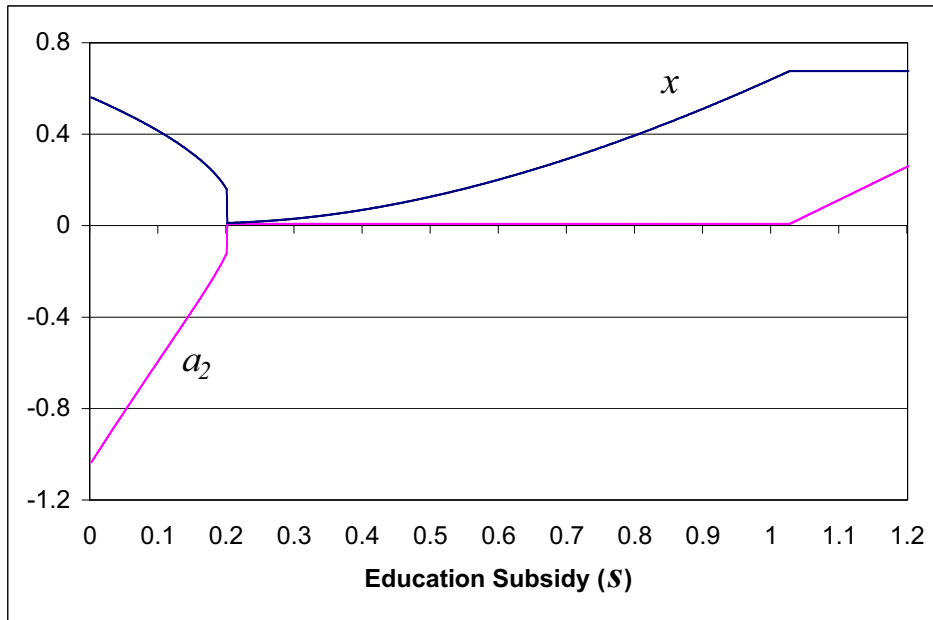


Figure 3: Consumption as a function of s

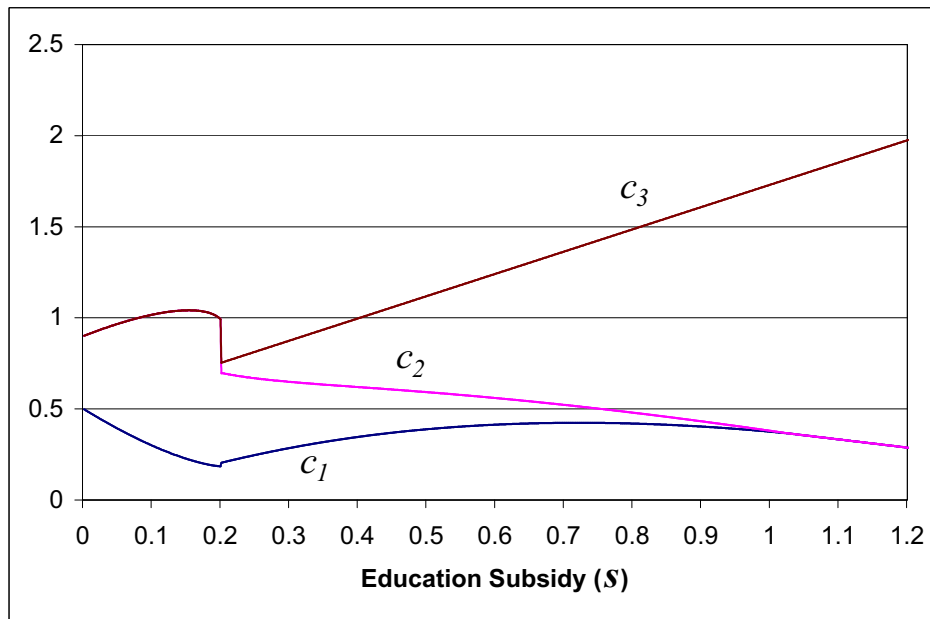


Figure 4: Lifetime Utility as a function of s : $v(s)$

