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A Model of Latent Symmetry in Cross Price Elasticities*

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Abstract

This paper develops the Latent Symmetric Elasticity Structure (LSES), a market share price elasticity model which allows elasticities to be decomposed into two components: a symmetric substitution index revealing the strength of competition between brand pairs, and a brand-specific coefficient revealing the overall impact of a brand on its competitors. An application of the model to unconstrained cross price elasticities shows that brand-price competition in one market is well-represented by a LSES model in which brand substitutability and elasticity asymmetry are related to average price level.

The analysis and interpretation of cross price elasticities is a major topic in marketing science. Recent work has focused upon the development of improved estimation methodologies (Cooper and Nakanishi 1988, Krishnamurthi and Raj 1988, Shugan 1988, Kamakura and Russell 1989, Bucklin and Srinivasan 1991) and upon the derivation of theoretically appealing cross elasticity constraints (Russell and Bolton 1988, Allenby 1989, Blattberg and Wisniewski 1989).

From a brand management perspective, cross price elasticities are important for two reasons. First, the relative magnitudes of cross elasticities provide insights into market structure, the substitutability of brands as perceived by consumers (Shocker, Stewart and Zahorik 1990). For example, cross elasticities are useful in predicting the draw pattern expected when a brand implements a price promotion (Bucklin and Srinivasan 1991). Second, the amount of asymmetry in cross elasticities provides a measure of the strength of a brand with respect to interbrand price competition (Kamakura and Russell 1991). In particular, recent research argues that asymmetry favors brands which are perceived by consumers to be high quality (Allenby and Rossi 1991, Blattberg and Wisniewski 1989).

The challenge facing the manager in an applied setting is obtaining an understanding of the role of these two components in a particular market. Currently, two descriptive tools for analyzing elasticities are available. Allenby (1989), drawing upon the Generalized Extreme Value choice model, has proposed criteria which can be used to create a market structure map highlighting brand substitutability. Cooper (1988) has proposed a factor analytic procedure which yields a

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visual representation of the elasticity matrix. This representation tends to be more complex than Allenby's map, because Cooper's procedure is attempting to reproduce all information in the elasticity matrix (i.e., asymmetry as well as brand substitutability).

This paper builds upon this research stream by describing a class of elasticity models which is useful in studying the structure of brand price competition. These models, which we call Latent Symmetric Elasticity Structures (LSES), allow an elasticity matrix to be decomposed into two elements: a symmetric substitution index which reveals the strength of competition between brand pairs, and a brand-specific coefficient which reveals the overall impact of a brand on its competitors (i.e., the pattern of asymmetry). In contrast to earlier approaches, LSES models provide a flexible – yet parsimonious – way of modeling elasticities which separates the two major interpretations of price elasticities: market structure and competitive strength. Because the parameters in LSES models are explicitly linked to either brand substitutability or elasticity asymmetry, the underlying determinants of brand price competition may be easily studied.

We first discuss the properties of LSES models. We show, in particular, that many previous elasticity models – including Allenby's (1989) model – are LSES models. We then develop a matrix decomposition procedure which reveals the extent to which an observed elasticity matrix obeys LSES constraints. To illustrate the value of the LSES approach in studying brand competition, we decompose a matrix of unconstrained price elasticities estimated by Carpenter et al. (1988). The analysis reveals that cross price elasticities in this market conform to a LSES model in which brand substitutability and elasticity asymmetry are strongly influenced by average price level.

1. Latent symmetric elasticity structures

The Latent Symmetric Elasticity Structure (LSES) is a model of market share price elasticities in which cross elasticity asymmetry is constrained to follow a special pattern. We begin by describing the LSES assumptions and then relate the model to previous work on inter-brand price competition.

1.1. The LSES model

Let e_{ij} be the percentage change in MS_i, the market share of brand i, with respect to a one percent change in p_j, the price of brand j. A matrix of cross price elasticities conforms to the LSES model if for any two distinct brands i and j

$$e_{ii} = C(j)S(i,j) \tag{1}$$

where $C(j) > 0$, $S(i,j) > 0$, and $S(i,j) = S(j,i)$. For reasons which will become clear subsequently, we refer to $C(j)$ as the *clout factor* associated with brand j, and to $S(i,j)$ as the *substitution index* for the brand pair (i,j).

It is important to notice that the underlying symmetry of LSES elasticities is not apparent upon inspection of the cross elasticity matrix. Because the clout factors $C(j)$ are arbitrary positive numbers, neither the matrix of cross elasticities e_{ij} nor the matrix of derivatives $\delta(MS_i)/\delta(p_j)$ need be symmetric. In this sense, the symmetry of the substitution indices $S(i,j)$ is latent.

1.2. *Clout factors and brand substitutability*

To motivate the interpretations of $C(j)$ and $S(i,j)$, it is useful to distinguish between two types of cross elasticities. Let c denote all competitive brands different from brand j. Then, elasticities of the form e_{cj} report the impact of price changes of brand j upon its competitors, while elasticities of the form e_{jc} report the impact of competitive price changes upon the share of brand j. For this reason, we define e_{cj} as the clout elasticities of brand j, and e_{jc} as the receptivity elasticities of brand j.

Given this terminology, the definition of $C(j)$ as a clout factor is straightforward. According to equation 1, the size of e_{cj} is directly proportional to the size of $C(j)$. Thus, any increase in $C(j)$ increases the magnitude of all clout elasticities of brand j.

The interpretation of $S(i,j)$ follows from the fact that the relative impact of brand j's price changes upon competitors k and c is given by the ratio

$$e_{kj}/e_{cj} = S(k,j)/S(c,j). \tag{2}$$

That is, brand j has a greater impact upon competitive brands c for which $S(c,j)$ is large. Because $S(i,j)$ is also assumed to be symmetric, it is natural to define the index as a measure of brand substitutability.

These definitions lead to an interesting explanation for the magnitude of a brand's own price elasticity e_{jj}. Because market shares sum to one, market share elasticities always obey the constraint $\Sigma_k MS_k e_{kj} = 0$. Using equation 1, $e_{jj} = C(j)S(j,j)$ where

$$S(j,j) = [\Sigma_c MS_c S(c,j)]/MS_j \tag{3}$$

and the summation runs over all competitive brands c different from j. Thus, adjusting for brand-specific factors ($C(j)$ and MS_j), a brand which is highly substitutable (large $S(c,j)$) with respect to large share competitors will tend to have a large own price elasticity.

1.3. Brand positioning

The substitution index allows us to define market structure in terms of cross price elasticities. We state that brands i and j have *equivalent positioning* if

$$S(i,k) = S(j,k) \tag{4}$$

for every competitive brand k different from i and j. Interpreting S(i,j) as a measure of brand substitutability, it is clear that equivalent positioning means that brands and i and j have the same degree of substitutability with respect to any third brand k.

Combining equations 1 and 4, we learn that the cross elasticities of brands with equivalent positioning satisfy

$$e_{ki} = [C(i)/C(j)]e_{kj} \tag{5}$$

and

$$e_{ik} = e_{jk} \tag{6}$$

for each third brand k. These relationships, which are similar to cross elasticity restrictions developed by Allenby (1989), mean that brands with equivalent positioning have proportional clout elasticities and identical receptivity elasticities. Intuitively, brands with equivalent positioning differentially draw sales from the same set of competitors during a price promotion (equation 5) and also respond in the same fashion to competitive price promotions (equation 6).

1.4. Relationship to previous research

The LSES model may be viewed as a generalization of previous work in interbrand price competition. Examples of these earlier models are summarized in table 1. Using LSES terminology, each model assumes that the clout factor C(j) depends upon market share, but differs with respect to the pattern of the substitution indices S(i,j).

This observation suggests that the major differences in previous studies are differences in assumptions about brand substitutability. For example, the ACREP model of Russell and Bolton (1988) and the differential effects model of Cooper and Nakanishi (1988) implicitly assume an unstructured market by setting S(i,j) equal to a constant for all brand pairs. Allenby (1989) generalizes these models by assuming that brands can be categorized into a small number of submarkets within which submarket members have equivalent LSES positioning. This logic leads to a matrix of substitution indices which is block diagonal.

Table 1. Examples of LSES models

Model	Assumptions	Clout factor	Substitution index ($i \neq j$)
Russell and Bolton (1988) [ACREP Model]	Simple share attraction model	$\beta MS_j p_j$	$S(i, j) = 1$
Cooper and Nakanishi (1988) [Differential Effects Model]	Restricted version of MCI model[a]	$\beta_j MS_j$	$S(i, j) = 1$
Allenby (1989)	Population of GEV choice consumers	βMS_j	$S(i, j) = S(a_i, a_j)$ where a_i is the index of the submarket of brand i.
Kamakura and Russell (1989)	Population of Logit choice consumers	$\beta MS_j p_j$	$S(i, j) = \phi_{ij}/MS_i MS_j$ where ϕ_{ij} is a generalized measure of the aggregate probability of switching between brands i and j.
Bucklin and Srinivasan (1991)	Population of Logit choice consumers	$\beta MS_j p_j$	Same as above.[b]

[a]The unrestricted MCI model does not impose LSES constraints.
[b]Bucklin and Srinivasan (1991) estimate ϕ_{ij} using survey data. In contrast, the Kamakura and Russell (1989) analysis is based upon scanner data.

The models of Kamakura and Russell (1989) and Bucklin and Srinivasan (1991) can be viewed as alternate generalizations in which the substitution indices depend upon a market measure of the tendency for consumers to switch between brands. Although the elasticity matrix conforms to a LSES model in each case, the pattern of the substitution indices is left unconstrained. Thus, in contrast to Allenby (1989), each brand cannot be exclusively assigned to one submarket (i.e., no two brands must have equivalent LSES positioning).

In reviewing the past literature, it is also important to point out that the general MCI (Multiplicative Competitive Interaction) model of Cooper and Nakanishi (1988) – often termed the Fully Extended Attraction model – is not a LSES model. However, because the LSES model is nested inside the Fully Extended Attraction model, it is possible that elasticity matrices estimated using general Cooper-Nakanishi procedures will conform to a LSES representation. We examine this issue in the empirical work reported subsequently.

2. A matrix decomposition for LSES models

Although examples of LSES models can be found in the marketing literature, elasticity estimates constructed using general regression procedures need not match LSES constraints. Here, we present a simple method of matrix decomposition which reveals the extent to which a matrix of elasticities can be described using a LSES model.

2.1. Matrix version of LSES

Let E be a matrix of market share elasticities with typical elements e_{ij}. Notice that if E is described by a LSES model, then

$$E = SC \tag{7}$$

where S is a symmetric matrix of substitution indices S(i,j) and C is a diagonal matrix containing the clout factors C(j) along the diagonal.

2.2. General decomposition

We first display a general method of matrix decomposition which can be used to represent *any* matrix of market share elasticities. Let D be any diagonal matrix with positive elements along the diagonal. Then $E = RD$ where $R = ED^{-1}$.

By carrying out a singular value decomposition of R, we can write $R = ULV'$ where L is a diagonal matrix of eigenvalues, and U and V are orthonormal matrices of (row and column) eigenvectors. Because $U'U = I$ (the identity matrix), we can always write

$$E = RD = S^*TD \tag{8}$$

where $S^* = ULU'$ is symmetric and $T = UV'$ is orthogonal ($T^{-1} = T'$). Notice that the decomposition in equation 8 depends upon the choice of the diagonal matrix D. However, if we find a matrix D such that T is an identity matrix, we have found a LSES representation for the elasticities.

2.3. Choosing the D matrix

The LSES model constraints imply that $R = ED^{-1}$ is symmetric. Let d_j be the j-th diagonal element of D^{-1} and let r_{ij} be a typical element of R. Then, $r_{ij} = d_j e_{ij}$

and the latent symmetry assumption of LSES is equivalent to $d_i e_{ji} = d_j e_{ij}$. To select the values for d_j, we minimize the fitting function

$$Q = \Sigma_{j>i}[d_i e_{ji} - d_j e_{ij}]^2 \tag{9}$$

subject to the side conditions $\Sigma_j d_j = 1$ and $d_j > 0$. These side conditions are necessary both to prevent the trivial solution $d_j = 0$ and to make the solution unique. Because equation 9 (with side conditions) corresponds to a quadratic programming problem, standard software (e.g., the IML procedure of the SAS statistical language) can be used to choose a D matrix which minimizes violations of the LSES latent symmetry assumption.

2.4. *Assessing fit of LSES*

Let $c_j^* = 1/d_j^*$ where d_j^* is the solution corresponding to the minimum of equation 9. Define C^* as the diagonal matrix containing c_j^*. Using C^* and the singular value decomposition of $R = E[C^*]^{-1}$, we obtain from equation 8 the elasticity matrix decomposition $E = S^*TC^*$ where S^* is symmetric and T is orthogonal. Because this decomposition reproduces the observed elasticities *exactly,* we can assess the fit of the LSES model in two ways. First, we can examine the deviation of T from the identity matrix. Second, we can construct the approximation

$$E^* = S^*C^* \tag{10}$$

and examine the correspondence between E and E^*. If these checks show that LSES constraints are approximately valid, we interpret S^* as a matrix of substitution indices S(i,j) and the diagonal elements of C^* as clout factors C(j).

3. Application

To illustrate our approach, we analyze estimates of cross price elasticities for eight national brands in an Australian nondurable product category (Carpenter et al. 1988). These data are interesting for two reasons. First, detailed information on brand characteristics is available. Second, the elasticities are estimated using the fully-elaborated MCI model (Cooper and Nakanishi, 1988), a general market share response model which *does not* impose LSES constraints. Thus, by applying our decomposition approach to these elasticities, we are conducting an exploratory analysis of the validity of the LSES latent symmetry assumption.

3.1. Data description

The eight brands, listed in table 2, differ in terms of form (labeled wet (W) or dry (D)), manufacturer (labeled 1, 2 or 3) and price level. Brands also differ in terms of advertising spending and positioning. These positioning platforms can be described as economy (E), tangible benefits (T) or image (I).

3.2. Fit to LSES constraints

The LSES matrix decomposition was applied to the matrix of market share elasticities in Table 3. The minimum value of the Q criterion (equation 9) is .00218, suggesting a good fit. The resulting decomposition E = S*TC* (also shown in Table 3) demonstrates that T is nearly an identity matrix, again suggesting that the elasticities can be represented by a LSES model. To assess the fit of the model in more concrete terms, we constructed a LSES forecast of the elasticities by inserting the substitution indices and clout factors of table 3 into equation 10. The correlation between the actual and forecasted elasticities is .989.

3.3. Interpretation of clout factors

Because the LSES model fits these data well, we can use the clout factors C(j) to examine the determinants of asymmetry in price response. To understand the

Table 2. Brand Summary

Brand[a]	Relative share	Price	Adv. Spending[b]
Premium Dry			
TD1	9.6	2.14	$25.6
ID1	21.4	1.79	$34.8
TD2	9.6	1.87	$29.4
ID2	10.5	1.87	$42.4
Premium Wet			
TW2	16.7	1.81	$50.0
Economy			
ED1	11.5	1.48	$8.1
ED2	5.6	1.51	—
ED3	15.2	1.59	$14.3

[a]Code to brand names:
E = Economy, I = Image, T = Tangible benefits; D = Dry, W = Wet; 1, 2, or 3 = Name of firm.
[b]in thousands

Table 3. LSES Decomposition of elasticities

Brand	TD1	ID1	TD2	ID2	TW2	ED1	ED2	ED3
Market Share Price Elasticities (E Matrix)[a]								
TD1	−2.04	.59	.58	.57	.43	.12	−.01	.09
ID1	.62	−2.76	.43	.43	.35	.30	.15	.19
TD2	.91	.63	−3.40	.57	.46	.28	.14	.21
ID2	.82	.58	.52	−3.07	.42	.26	.12	.19
TW2	.34	.36	.25	.24	−1.60	.21	.06	.08
ED1	−.06	.57	.33	.33	.36	−4.47	.60	.55
ED2	.05	.54	.33	.33	.34	.67	−4.18	.46
ED3	.10	.32	.16	.16	.13	.29	.13	−1.58
T Matrix								
TD1	1.00	−.00	.00	.00	.00	.01	−.00	.01
ID1	.00	1.00	−.02	−.01	−.01	.00	−.02	−.00
TD2	−.00	.02	1.00	.00	−.00	−.01	.01	.00
ID2	−.00	.01	−.00	1.00	−.00	−.00	.00	.00
TW2	−.00	.01	.00	−.00	1.00	−.02	−.01	−.01
ED1	−.01	.00	.01	.00	.02	1.00	−.01	.00
ED2	.00	.02	−.01	−.00	.01	.01	1.00	.02
ED3	−.01	.00	−.00	−.00	.01	−.00	−.02	1.00
Substitution Indices (S* Matrix)[b]								
TD1	−12.52	3.71	5.58	5.02	2.22	.08	.08	.52
ID1	3.71	−17.20	4.00	3.68	2.03	3.82	3.15	1.57
TD2	5.58	4.00	−32.88	5.04	2.50	3.59	2.86	1.45
ID2	5.02	3.68	5.04	−27.14	2.26	3.28	2.60	1.31
TW2	2.22	2.03	2.50	2.26	−8.74	2.17	1.71	.61
ED1	.08	3.82	3.59	3.28	2.17	−61.43	9.71	3.81
ED2	.08	3.15	2.86	2.60	1.71	9.71	−72.65	3.02
ED3	.52	1.57	1.45	1.31	.61	3.81	3.02	−10.80
Clout Factors (Diagonal of C*)[b]								
	.163	.161	.103	.113	.183	.073	.058	.146

[a]Elasticities are expressed as the percentage change in the market share of the row brand with respect to a one percent change in the price of the column brand.

[b]Substitution indices and clout factors have been rescaled so that clout factors sum to one.

pattern of asymmetry, we regressed the logarithm of the clout factor (from table 3) on the logarithms of the brand's market share and price (from table 2):

$$\log(C(j)) = \underset{(.546)}{-4.834} + \underset{(.157)}{.672} \log(MS_j) + \underset{(.561)}{1.862} \log(p_j). \tag{11}$$

The standard errors, shown in parentheses, indicate that all coefficients are statistically different from zero; the adjusted R^2 is .83.

This regression, which is suggested by the form of the clout factors in table 1, shows that clout in this market is positively related to both market share and average price. As explained by Russell and Bolton (1988), the market share sum constraint (i.e., market shares sum to one) logically implies that clout cross elasticities of large share brands should be generally larger than the clout cross elasticities of small share brands. Because a brand's clout factor determines the scale of its clout cross elasticities (equation 1), a positive association between C(j) and MS_j is expected.

The association with average price, however, is not a logical consequence of market shares or of market share price elasticities. One possible explanation is the price tier theory of Blattberg and Wisniewski (1989). Briefly stated, this theory is based upon the notion that consumer perceptions of product quality are positively correlated with the brand's average price. When a high quality brand reduces price, price-sensitive consumers will trade up (i.e., switch from a low price brand to a high price brand). However, switching in the reverse direction is not expected. Even if a low quality brand reduces price, consumers who typically buy high price brands will not trade down.

Consequently, the price tier theory predicts that, controlling for differences in market share, the asymmetry in cross-price elasticities should favor the high price brand. That is, if H is a high price brand and L is a low price brand (and both have equal market shares), the cross elasticity reporting the impact of the price changes of H on the share of L (e_{LH}) should be larger than the cross elasticity reporting the impact of the price changes of L on the share of H (e_{HL}). In the LSES model, this is always true if the high price brand has the larger clout factor ($C(H) > C(L)$). The empirical results summarized in equation 11 are consistent with this prediction. Controlling for differences in market share, brands with higher average price do have larger clout factors.

3.4. Interpretation of substitution indices

In contrast to the clout factors, the matrix of symmetric substitution indices S^* is useful in understanding overall pattern of brand competition. Because the diagonal elements are redundant (see equation 3), we focus on the off-diagonal elements ($S(i,j)$, $i \neq j$). In order to study the structure implied by these indices, we created a metric multi-dimensional scaling (MDS) map by defining inter-brand distance to be a linear function of $-S(i,j)$. The eigenvalues of the MDS procedure suggest a three dimensional solution. For simplicity, we display the first two dimensions in figure 1.

The interpretation of figure 1 follows from the observation that brands which are located near one another on the MDS map must have large substitution indices. That is, adjusting for differences in clout, proximity on the map implies that brands should have a strong impact on one another during a price promotion. For this reason, the relative magnitudes of a brand's clout cross elasticities should be *inversely* related to the distance between the brand and each of its competitors (equation 2). For example, the location of brand TD1 suggests that its clout cross elasticities should be in the approximate rank order [ID2 = TD2] > ID1 > TW2 > ED3 > [ED1 = ED2]. The Carpenter et al. estimates of TD1 clout elasticities (first column of E matrix in table 3) correspond well with this prediction.

The substitution indices also provide a way of understanding the overall determinants of brand substitutability. For these data, the dominant attribute appears to be price level. Notice that the placement of brands on the horizontal axis accurately reproduces each brand's average price: the simple correlation between average price and the horizontal axis coordinate is .983. Theoretically, this relationship is important because it suggests that consumers define brand substitutability on the basis of price level – a key assumption of the Blattberg-Wisniewski (1989) price tier theory.

From a managerial perspective, the arrangement of brands on the horizontal

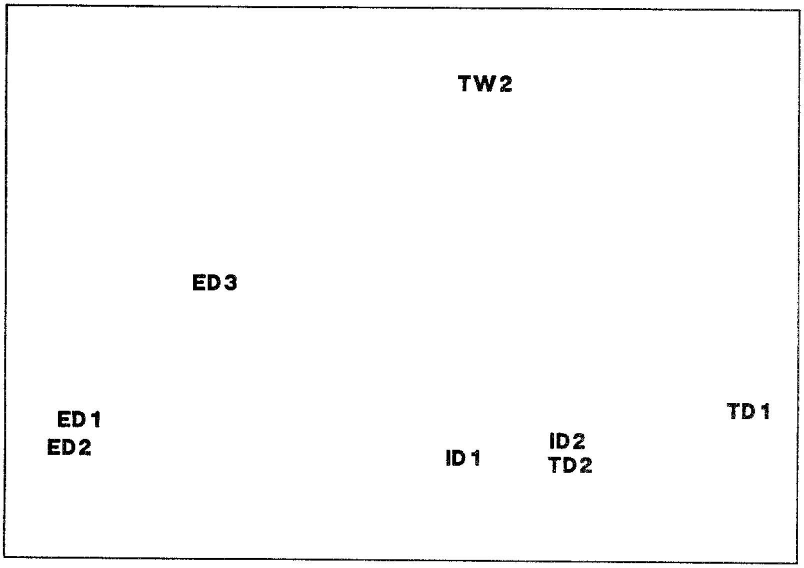

Figure 1. Map of substitution indices.

axis implies that a brand's price promotion will have most impact on competitors with similar average prices. The clear exception to this rule is brand TW2, a premium product in wet form – the only one of the eight brands not in dry form. Brand TW2 is located at the point of the horizontal axis corresponding to its average price, but uses the vertical axis to move away from other premium brands. Thus, as might be expected, product form plays a role in brand substitutability.

The role of advertising in determining product substitutability is not clear. Because high price brands also advertise more (table 1), advertising spending is positively related (correlation coefficient = .742) to the brand's location on the horizontal axis of the MDS map. However, aside from the economy versus premium distinction, emphasis on image rather than tangible benefits does not yield meaningful differences in terms of product substitutability. For example, ID2 (image) and TD2 (tangible benefits) share the same location on the map.

3.5. Summary

The LSES decomposition for this market provides strong evidence that price tiers largely govern the structure of brand price competition. Looking across brands, asymmetry favors brands with both high market share and high price. Looking within one brand, price promotions have most impact upon competitors (of the same product form) with similar average prices.

4. Conclusions

This research develops the Latent Symmetric Elasticity Structure (LSES). The model, which is a generalization of earlier work on inter-brand price competition, assumes that cross elasticity asymmetry can be removed by rescaling the columns of the elasticity matrix. Because LSES elasticities can be decomposed into brand-specific clout factors and inter-brand substitution indices, interpretation of the structure of brand price competition is greatly simplified.

Using a matrix decomposition procedure, we showed that cross elasticity asymmetry in one product category is well-described by LSES constraints. However, the descriptive value of the LSES model in other settings remains an empirical issue. To this end, it would be useful both to develop a statistical test for the validity of LSES constraints and to construct an estimation procedure which fits the general LSES model to scanner data. Currently, models are available to estimate LSES elasticities, but only if the researcher is willing to make assumptions about the structure of the substitution indices.

Substantively, we provided evidence in one product category that clout is related to market share and price level, and that brand substitutability is related to price level. These findings are broadly consistent with the notion that the pattern of brand price competition is determined by consumer perceptions of product quality (Allenby and Rossi 1991, Blattberg and Wisniewski 1989).

The LSES model provides a structured approach to the study of cross price elasticities. Given the widespread availability of scanner data, models of this sort have the potential to generate new insights into the nature of brand price competition.

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