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Implications of Direct and Indirect Range Restriction for Meta-Analysis Methods and Findings

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Range restriction in most data sets is indirect, but the meta-analysis methods used to date have applied the correction for direct range restriction to data in which range restriction is indirect. The authors show that this results in substantial undercorrections for the effects of range restriction, and they present meta-analysis methods for making accurate corrections when range restriction is indirect. Applying these methods to a well-known large-sample empirical database, the authors estimate that previous meta-analyses have underestimated the correlation between general mental ability and job performance by about 25%, indicating that this is potentially an important methodological issue in meta-analysis in general.

Keywords: range restriction, indirect range restriction, bias corrections, data analysis

In many research situations, such as educational and employment selection, researchers have data only from a restricted population and yet must attempt to estimate parameters of the unrestricted population. For example, the validity of the Graduate Record Examination (GRE) for predicting performance in graduate school can be estimated only with samples of students admitted to the graduate program (the restricted sample). However, the goal is to estimate the validity of the GRE when used in the population of applicants to the graduate program. Because of range restriction, the population of admitted students typically has higher mean GRE scores and a smaller standard deviation (*SD*) of scores. To estimate the validity in the applicant population from the observed validity in the incumbent population of admitted students, one must correct for the effects of range restriction on GRE scores. In such a situation, if applicants have been selected directly on test scores top down, we have what is called *direct* or *explicit range restriction.* On the other hand, if students have been selected on some other variable that is correlated with GRE scores (such as a composite of undergraduate grade point and letters of recommendation), then the range restriction is said to be indirect. In this article, we show that corrections for both direct and indirect range restriction are more complicated than is generally realized and are often erroneously applied.

Building on the earlier work of Pearson (1903), Thorndike (1949) presented corrections for direct and indirect univariate range restriction—that is, equations for correcting for range restriction when restriction has occurred on only one variable. His Case II correction for direct range restriction is widely used, as described later in the Research Domains With Direct Range Restriction section. His Case III correction for indirect range restriction produced by direct restriction on a third, known variable can rarely be used because its use requires considerable information on the third variable, and in most research, this information is unknown. Later in the Research Domains With Indirect Range Restriction section, we present a correction method that does not require that information. Corrections for multivariate range restriction are also available (Lawley, 1943; Johnson & Ree, 1994; Ree, Caretta, Earles, & Albert, 1994). Multivariate range restriction corrections are straightforward extensions of the univariate correction, and they correct for both the direct and the indirect range restriction simultaneously. However, to use this procedure one must know the intercorrelations of the independent variable measures in both the restricted and unrestricted populations and must know both what tests are used in the selection composite and what the selection ratio is. This information is rarely available outside the military testing context. (For an overview of this and related issues in range restriction, see Sackett & Yang, 2000.) Because the range restriction corrections used in research are almost always univariate corrections, the focus of this article is on univariate range restriction.

Developments presented in Mendoza and Mumford's (1987) article suggested that meta-analysis procedures (Callender & Osburn, 1980; Hunter & Schmidt, 1990a; Raju, Burke, Normand, & Langlois, 1991; Schmidt & Hunter, 1977) and range correction

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methods in general have used suboptimal procedures in correcting for range restriction when range restriction is indirect (the most common case). As shown later in the Research Domains With Direct Range Restriction section, corrections for direct range restriction (where it does exist) have also frequently been suboptimal because of issues related to measurement error. Because it is mathematically easier to understand direct range restriction, researchers have typically learned to correct for direct restriction and have then applied the correction equation for direct restriction to cases of both direct and indirect range restriction (Cohen & Cohen, 1983, p. 70). However, when the predictor is not perfectly measured (which is virtually always the case), the key equation is not the same, and much of the underlying rationale is also different. The resulting implications for meta-analysis findings are important in many areas but are particularly important in the areas of educational and employment selection. For example, it is likely that there has been considerable underestimation of the predictive validity of selection procedures. The implications are likewise important for other research areas in which range restriction is a factor. For example, the participants in many studies are volunteers. Volunteering can be a self-selection mechanism that can result in indirect range restriction on many traits and other characteristics relevant to the research. The appropriate correction for such indirect range restriction can result in a very different pattern of study findings than correction based on the traditional but inappropriate correction for direct range restriction (or that resulting when no correction for range restriction is made). The method we present later in the Research Domains With Indirect Range Restriction section can be used to correct for such indirect range restriction.

Mendoza and Mumford (1987) focused on the differences in the mathematics of direct and indirect range restriction at the level of the individual study. This article differs from their work in that it explicates the implications of these differences for meta-analysis methods. To establish the concept of simple statistical artifacts, we first briefly discuss meta-analysis in domains with no range restriction. The next two sections examine the role of the complex statistical artifacts created by range restriction. The first of these examines complex artifacts in domains with direct range restriction and shows that accurate corrections require a specific but little known sequencing of corrections for range restriction and measurement error. This demonstration implies that widely used current practices should be reconsidered, both in corrections of individual correlations and in meta-analysis. The final section (Research Domains With Indirect Range Restriction) examines domains with indirect range restriction; it presents a new method for correcting for indirect range restriction and describes how this method can be used in meta-analysis.

In the derivations in this article, it is necessary to present equations in terms of fully corrected correlations; that is, it is necessary to include corrections not only for range restriction and for measurement error in the dependent variable (performance measure), but also for measurement error in the independent variable (predictor). It becomes apparent, especially in the Research Domains With Indirect Range Restriction section, that this is necessary to explicate structural properties of the correction processes and to produce correct final estimates. However, we are well aware that such true score correlations do not estimate operational validities in applied settings and that the correction for measurement error in the predictor is not made in such validity estimates. We show in each case how final operational estimates are obtained, and the results of our empirical analysis are presented in terms of operational validities, not true score correlations. We also note that the methods presented here do not directly address the issue of sampling error. Random sampling error effects and corrections in meta-analysis can be separated from the systematic effects of the artifacts of range restriction and measurement error, and are so separated in this article to facilitate the presentation. However, it is understood that the final meta-analysis methods do include corrections for sampling error (as described in Hunter & Schmidt, 2004).

Recent literature includes numerous articles on the distinctions between, and properties of, fixed effects versus random effects models in meta-analysis (Field, 2001; Hedges & Vevea, 1998; Hall & Brannick, 2002; Hunter & Schmidt, 2000; Schmidt & Hunter, 2003; Schmidt, Oh, & Hayes, 2006). In general, random effects models appear to be more appropriate for research data (Field, 2001; Hunter & Schmidt, 2000; National Research Council, 1992). The meta-analysis models considered in this article are all random effects models. They are also models that correct for statistical and measurement artifacts, such as range restriction and measurement error, that distort study results. These models fall into two classes: (a) those that use statistical methods based on distributions of artifacts (e.g., reliability coefficients) to make these corrections (e.g., see Callender & Osburn, 1980; Hunter & Schmidt, 2004, chap. 4; Raju & Burke, 1983; Schmidt & Hunter, 1977) and (b) those that correct each study finding (correlation or *d* value) individually for these artifacts and then perform the meta-analysis on the resulting corrected statistics (correlations or *d* values; e.g., see Hunter & Schmidt, 2004, chap. 3; Raju, Burke, Normand, & Langlois, 1991).

Research Domains With No Range Restriction

When there is no range restriction, artifacts are *simple artifacts* and their effects on study correlations are linear; that is, they can be represented as multipliers. Each artifact multiplier can be computed separately from other artifacts, because each artifact depends on a different study imperfection and has a causal structure independent of that for other artifacts. We show in the two following sections that this is not true in the case of range restriction. Measurement error is an example of a simple artifact. Measurement error is random at the level of individual participant scores but has a systematic effect at the level of population statistics. In particular, the correlation between measures of two variables is reduced in size to the extent that there is measurement error in the independent variable and the dependent variable (Lord & Novick, 1968; Schmidt & Hunter, 1996, 1999). Other simple artifacts of this sort include artificial dichotomization of quantitative variables (Hunter & Schmidt, 1990b; MacCallum, Zhang, Preacher, & Rucker, 2002) and imperfect construct validity (systematic error of measurement; Hunter & Schmidt, 2004).

If several simple artifacts are present in a study, their effects combine in a simple multiplicative way. The order in which the artifacts are entered does not play a role. The final result is the same for any order.

Start with no artifacts, and let ρ = result for a methodologically perfect study.

Add Artifact 1: $\rho_1 = a_1\rho$ (e.g., $a_1 = \sqrt{r_{XX}}$, where r_{XX} is the independent variable reliability).

Add Artifact 2: $\rho_2 = a_2\rho_1 = a_2(a_1\rho) = a_1a_2\rho$ (e.g., $a_2 = \sqrt{r_{YY}}$, where r_{YY} is the dependent variable reliability).

Add Artifact 3: $\rho_3 = a_3\rho_2 = a_3(a_1a_2\rho) = a_1a_2a_3\rho$ (e.g., a_3 = attenuation factor for dichotomization of variable X; Hunter & Schmidt, 1990b).

The final result is as follows:

$$\rho_o \; = \; A\rho, \quad \text{where} \quad A = a_1a_2a_3. \tag{1}$$

For simple artifacts, the final result is an attenuation formula with the same form as that for a single artifact. The true effect size is multiplied by an overall artifact multiplier, A, which is the product of the multipliers for the individual artifacts. The population correlation can then be estimated by solving Equation 1 for ρ:

$$\rho \; = \; \rho_o/A. \tag{2}$$

When only simple artifacts are present, meta-analysis is quite straightforward and accurate, whether correlations are corrected individually or artifact distribution meta-analysis is used (Hunter & Schmidt, 1990a, 2004). This is not the case when there is range restriction.

Research Domains With Direct Range Restriction

This section demonstrates the little known fact that when range restriction is direct, accurate corrections for range restriction require not only use of the appropriate correction formula (Thorndike's, 1949, Case II formula), but also the correct sequencing of corrections for measurement error and range restriction.

Direct Range Restriction as a Single Artifact

Suppose that promotion from one level to another within a job classification is required to be based solely on a job knowledge test (X), a condition of direct range restriction. That is, people are promoted top down based solely on their job knowledge test scores. We want to estimate how well the knowledge test predicts job performance in the population of applicants for promotion, but we can get job performance data only for those who are actually promoted (incumbents), with the result that we get no data on people with low test scores. The usefulness of the knowledge test depends on the correlation between knowledge and performance in the applicant population, and so that is the correlation we want to estimate. However, our data provides only the correlation for the incumbent population.

The ratio of the observed standard deviations indexes how much the incumbent population is restricted in comparison with the applicant population: SD_{X_a} is the standard deviation on X in the applicant population (unrestricted SD), while SD_{X_i} is the standard deviation on X in the incumbent population (restricted SD). The comparison ratio is then $u_X = SD_{X_i}/SD_{X_a}$. Because the standard deviation is smaller in the restricted population, $u_X < 1.00$.

The attenuation formula for direct range restriction can be written in the same form as that for simple artifacts, but the multiplier is more complicated than for simple artifacts. Let ρ = correlation between X and Y in the applicant population, and let ρ_o = correlation between X and Y in the incumbent population. We then have the attenuation formula $\rho_o = a\rho$, where (Callender & Osburn, 1980)

$$a = \frac{u_X}{\sqrt{1 + (u_X^2 - 1)\rho^2}}\,. \tag{3}$$

The complication can be seen in the denominator of the multiplier a. The presence of ρ in the denominator means that the multiplier depends not only on the degree of restriction (u_X) but also on the level of correlation. *This distinguishes range restriction from the simple artifacts. For a simple artifact, the multiplier is determined entirely by the extent of the artifact. For range restriction, the multiplier is determined not only by the extent of the artifact but also by the size of the unrestricted correlation.*

Correction for Range Restriction

The simple reversal of the multiplication process that we saw in Equation 2 for simple artifacts works to correct for range restriction in principle but not in practice. The problem is that to compute the multiplier a, one must already know ρ. The conventional formula for correction for direct range restriction (Thorndike's, 1949, Case II) algebraically reverses the nonlinear algebra of the attenuation formula:

$$\rho \; = \; \alpha\rho_o, \quad \text{where} \quad \alpha \; = \; \frac{U_X}{\sqrt{1 + (U_X^2 - 1)\rho_o^2}}\,, \tag{4}$$

where $U_X = 1/u_X$. The formula for α is the same in algebraic form as the formula for attenuation due to range restriction; substitution of the parameter U_X for u_X makes it correct in the opposite direction. (Note that $\alpha = 1/a$, where a is defined in Equation 3).

Meta-Analysis for Range Restrictions as a Single Artifact

Range restriction is not a simple artifact, and thus the meta-analysis methods used for simple artifacts are not perfectly accurate for range restriction. The problem is the term with ρ_o in the denominator of the multiplier α in Equation 4. If that term is small, then the simple artifact methods provide a good approximation. If that term is large, the multiplicative method is less accurate. The problem term in Equation 3 is $(u_X^2 - 1)\rho^2$. This term is small if either u_X is close to 1 or if ρ^2 is small. The ratio u_X will be close to 1 if there is very little range restriction. The squared correlation ρ^2 will be small if ρ is modest. There are many research domains where these two conditions are probably met, but unfortunately both conditions can fail to be met in employment and educational selection. For example, general mental ability has a substantial correlation with job performance (Schmidt & Hunter, 1998), and range restriction on general mental ability is substantial in most samples of job incumbents. There are almost certainly other research areas in which these conditions are not met. The Taylor series methods of Raju and Burke (1983) and the interactive method of Schmidt, Gast-Rosenberg, and Hunter (1980) were derived to solve this problem under direct range restriction. These methods have been shown by means of computer simulation to provide good approximations when range restriction is direct unless the level of range restriction is very extreme (Law, Schmidt, & Hunter, 1994a, 1994b).

The fact that we have two populations means that for any parameter (and any estimate of that parameter), there will usually be two different values, one for each population. For both variables, the means and standard deviations will differ between the two populations. This duality is important for the consideration of artifacts other than range restriction because the artifact values might differ between the two populations. For example, consider the reliability of the dependent variable Y. Direct selection on X will produce indirect selection on Y. This means that the reliability of Y will be smaller in the incumbent population than in the applicant population. We return to this point later in the Research Domains With Indirect Range Restriction section.

Error of Measurement in the Independent Variable in Direct Range Restriction

Because the standard deviation of the predictor differs between the two populations, the reliability of X will differ between the two populations. However, there is a more serious problem that is widely ignored in practice. The substantive nature of the direct selection process makes the meaning of *reliability* unclear for the independent variable measure in the incumbent data (Mendoza & Mumford, 1987). As we show later, this fact implies that accuracy of range restriction correction requires a specific sequencing of range restriction and measurement error corrections. If we analyze applicant data, then we can make the usual measurement assumption that the true scores and errors are uncorrelated. For the incumbent (restricted) population, this assumption does not hold when range restriction is direct. Most researchers would assume that they could use incumbent data to compute the reliability of the independent variable in the incumbent group, but there is a problem in doing this: True scores and errors of measurement are negatively correlated in the incumbent data for the scores that the incumbents were selected on (Mendoza & Mumford, 1987). Using an example as a vehicle, we demonstrate this fact in Appendix A. In that example, the correlation between true scores and measurement errors is $-.47$. Because of this negative correlation, reliability cannot be estimated or even defined. For example, the conventional definition of reliability as the square of the correlation between observed score X and true score T (r^2_{XT}) does not hold in this context.

The solution to the problem created by correlated true and error scores is to consider attenuation due to measurement error in the independent variable before considering range restriction (Mendoza & Mumford, 1987; Mendoza, Stafford, & Stauffer, 2000; Stauffer & Mendoza, 2001). That is, because range restriction is *not* a simple artifact, it is critical to consider the order in which range restriction enters into the attenuation process. As shown later in this section, this order in turn determines the order in which corrections must be made in the disattenuation (correction) process in meta-analysis.

In the following presentation, we use the symbol a to denote the artifact multiplier (effect) of measurement error in the independent variable, and we use the symbol c to denote the effect of range restriction. The subscript a has a different meaning; it denotes the applicant (i.e., unrestricted) group, while the subscript i denotes the incumbent (i.e., restricted) group.

In the applicant population, we have the following:

$$\rho_{XP_a} = a\rho_{TP_a}, \tag{5}$$

where $a = \sqrt{r_{XX_a}}$ and r_{XX_a} = the reliability of predictor scores in the applicant (unrestricted) population; P = the true score underlying the dependent variable; and T = the true score underlying X.

In the incumbent population, we have the following:

$$\rho_{XP_i} = c\rho_{XP_a}, \tag{6}$$

where

$$c = u_X/\sqrt{[1 + (u_X^2 - 1)\,\rho^2_{XP_a}]} \tag{7}$$

$$= u_X/\sqrt{[1 + (u_X^2 - 1)\,r_{XX_a}\rho^2_{TP_a}]}.$$

The artifact multiplier for range restriction is complicated. The denominator contains not only ρ_{TP_a}, but also r_{XX_a}. That is, the value of the artifact multiplier depends not only on the extent of restriction (u_X) but also on the true effect size (ρ_{TP_a}) and the reliability of the independent variable in the unrestricted group (r_{XX_a}; Mendoza & Mumford, 1987; Stauffer & Mendoza, 2001).

Error of Measurement in the Dependent Variable in Direct Range Restriction

From a practical standpoint, the key question is how well the test predicts educational or job performance. No study measures performance perfectly. Most studies in the employment area use supervisor ratings of job performance, which contain considerable measurement error. For our discussion, we define *performance* as the job performance rating true score.

Job performance ratings have two reliabilities: one for the applicant population and one for the incumbent population. In research domains other than selection, the reliability is usually estimated in the full population. Thus, we would usually know the reliability of the dependent variable in the unrestricted population. If so, then we can compute the attenuation of the correlation by adhering to the order principle for the independent variable: Introduce range restriction last. Using the symbol b to denote the artifact multiplier (effect) of measurement error in the dependent variable, we have the following in the applicant population:

$$\rho_{TY_a} = b\rho_{TP_a}, \quad \text{where} \quad b = \sqrt{r_{YY_a}}. \tag{8}$$

In the incumbent population, we have the following:

$$\rho_{TY_i} = c\rho_{TY_a}, \tag{9}$$

where

$$c = u_X/\sqrt{[1 + (u_X^2 - 1)\rho^2_{TY_a}]} \tag{10}$$

$$= u_X/\sqrt{[1 + (u_X^2 - 1)r_{YY_a}\rho^2_{TP_a}]}. \tag{10'}$$

Again, the artifact multiplier for range restriction is complicated. The denominator not only has ρ_{TP_a} in it, but it has r_{YY_a}, too. That is, the value of the artifact multiplier depends not only on the extent of restriction (u_X) but also on the true effect size (ρ_{TP_a}) and the reliability of the dependent variable in the unrestricted group (r_{YY_a}).

Error of Measurement in Both Variables in Direct Range Restriction

For scientific reasons, we want to know the construct level correlation. Thus, we seek to eliminate the effects of measurement error in both the predictor and the criterion measure. The key to the attenuation formula is again to consider range restriction last.

In the applicant population, we have the following:

$$\rho_{XY_a} = ab\rho_{TP_a}, \quad \text{where} \quad a = \sqrt{r_{XX_a}} \text{ and } b = \sqrt{r_{YY_a}}. \tag{11}$$

In the incumbent population, we have the following:

$$\rho_{XY_i} = c\rho_{XY_a}, \tag{12}$$

where

$$c = u_X/\sqrt{[1 + (u_X^2 - 1)\rho_{XY_a}^2]}$$
$$= u_X/\sqrt{[1 + (u_X^2 - 1)r_{XX_a}r_{YY_a}\rho_{TP_a}^2]}.$$

The artifact multiplier for range restriction is now even more complicated. The denominator not only has ρ_{TP_a} in it, but it has both r_{XX_a} and r_{YY_a} as well. That is, the value of the artifact multiplier depends not only on the extent of restriction (u_X) but also on the true effect size (ρ_{TP_a}), the reliability of the independent variable (r_{XX_a}), and the reliability of the dependent variable (r_{YY_a}).

Meta-Analysis in Direct Range Restriction

Three research teams have contributed methods of meta-analysis that include range restriction: Hunter and Schmidt (1990a) and Schmidt and Hunter (1977); Callender and Osburn (1980); and Raju and Burke (1983). The model for meta-analysis for all three teams has been the model for direct range restriction. As long as corrections for range restriction and measurement error are sequenced in the correct order, there is no problem with that model if range restriction is indeed direct. In fact, these methods have been shown by means of computer simulation to be quite accurate under conditions of direct range restriction (Law et al., 1994a, 1994b). One problem is that corrections are not always appropriately sequenced. Another problem is that this model has been used for domains where the range restriction is indirect, leading to much larger inaccuracies than inappropriate sequencing of corrections. In the Research Domains With Indirect Range Restriction section, we show that attenuation Equations 11 and 12 do not hold for indirect range restriction.

Correction Order in Educational and Employment Selection

In educational selection, the dependent variable is usually grade point average (often first-year grade point average). In personnel selection, the dependent variable is usually job performance or some employment behavior such as training performance, accidents, theft, or turnover. Consider performance ratings. All research on performance ratings has been of necessity conducted on incumbents. For example, a recent review of interrater reliability findings (Viswesvaran, Ones, & Schmidt, 1996) found the average interrater reliability of multiscale rating scales to be .47. This is the incumbent reliability and should not be used in the attenuation formula presented as the attenuation model for meta-analysis. The applicant reliability is higher.

Equation 13 can be used to compute r_{YY_a} (Brogden, 1968; Schmidt, Hunter, & Urry, 1976):

$$r_{YY_a} = 1 - \frac{1 - r_{YY_i}}{1 - r_{XY_i}^2\left[1 - \dfrac{S_{X_a}^2}{S_{X_i}^2}\right]}, \tag{13}$$

where r_{XY_i} is the observed correlation between X and Y in the incumbent sample. (An equivalent formula is given in Callender & Osburn, 1980.) Consider a realistic case: Let $u_X = SD_{X_i}/SD_{X_a} = .70$, $r_{XY_i} = .25$, and $r_{YY_i} = .47$. Equation 13 then yields $r_{YY_a} = .50$. Hence the reliability of ratings of job performance would be .03 (6%) higher in the absence of range restriction. Equation 13 provides an estimate of r_{YY_a}, and this makes possible use of Equation 12. However, it is possible to develop a hybrid model that requires only an estimate of r_{YY_i}. This model is more convenient to use and is the one on which corrections in meta-analysis programs are based under direct range restriction.

In most research domains, the reliabilities in the unrestricted group are known. The conventional method (Equations 11 and 12) above works for these domains. For domains like educational and employment selection, the dependent variable reliability is known only for the incumbent population. In this case, we can analyze the data using a different model, one that introduces the dependent variable measurement error *after* range restriction. This is possible because the random measurement errors in the dependent variable (Y) come into being after the selection process and are not affected by the direct selection on the independent variable.

In the applicant population, we have the following:

$$\rho_{XP_a} = a\rho_{TP_a}, \quad \text{where} \quad a = \sqrt{r_{XX_a}}. \tag{14}$$

In the incumbent population, we have the following:

$$\rho_{XP_i} = c\rho_{XP_a}, \quad \text{where} \quad c = u_X/\sqrt{[1 + (u_X^2 - 1)\rho_{XP_a}^2]}, \tag{15}$$

and

$$\rho_{XY_i} = b\rho_{XP_i}, \quad \text{where} \quad b = \sqrt{r_{YY_i}}. \tag{16}$$

Correction for attenuation in this model proceeds in three steps. We start with ρ_{XY_i} and correct it for dependent variable unreliability using the incumbent reliability (r_{YY_i}). The corrected correlation is ρ_{XP_i}. This correlation is then corrected for (direct) range restriction. That correlation is ρ_{XP_a}, which is the estimate of operational (true) validity. If researchers desire to estimate the true score or construct level correlation, they then correct this value for predictor unreliability using the applicant reliability r_{XX_a}. The resulting correlation is the estimate of ρ_{TP_a} in the applicant population. Table 1 summarizes the steps in this process. It is important that corrections be made in this order; this has not always been the case in published meta-analyses. Hence, even when range restriction is direct and the appropriate correction formula for direct range restriction is used, corrections can be inaccurate because of inappropriate sequencing

Table 1
Steps in Correcting for Direct Range Restriction

Step	Purpose	Input	Output	Formula
1	Correcting for measurement error in Y (measure of construct P)	Correlation between X and Y in the restricted population: ρ_{XY_i} Reliability of Y in the restricted population: r_{YY_i}	Correlation between X and P in the restricted population: ρ_{XP_i}	$\rho_{XP_i} = \rho_{XY_i}/\sqrt{r_{YY_i}}$
2[a]	Correcting for the effect of direct range restriction on X	Correlation between X and P in the restricted population: ρ_{XP_i} Range restriction on X: u_X	Correlation between X and P in the unrestricted population: ρ_{XP_a}	$\rho_{XP_a} = \frac{U_X\rho_{XP_i}}{\sqrt{1 + (U_X^2 - 1)\rho_{XP_i}^2}}$ *Note.* $U_X = 1/u_X$
3	Correcting for measurement error in the X (measure of construct T)	Correlation between X and P in the unrestricted population: ρ_{XP_a} Reliability of X in the unrestricted population: r_{XX_a}	Correlation between T and P in the unrestricted population: ρ_{TP_a}	$\rho_{TP_a} = \rho_{XP_a}/\sqrt{r_{XX_a}}$

[a] The output for this step (ρ_{XP_a}) is the operational validity of measure X, so depending on research questions, the correction process may end here.

of range restriction and measurement error corrections or because of use of reliabilities estimated on inappropriate populations. Further, use of r_{XX} computed in the restricted group is an additional and separate error, as noted earlier.

Meta-Analysis Correcting Correlations Individually—Direct Range Restriction

As noted earlier, in addition to artifact-distribution-based meta-analysis, researchers can also conduct a meta-analysis that corrects each observed correlation individually for measurement error and range restriction. This procedure is used less frequently because primary studies often do not report u_X values and estimates of reliability for independent and dependent variables for the individual correlations reported in the study. However, when these data are reported, the procedure for correcting each individual study correlation is the one described here. With the symbolism described above, this three-step procedure can be combined into one equation:[1]

$$\rho_{TP_a} = \frac{U_X\rho_{XY_i}}{[r_{XX_a}(r_{YY_i} + U_X^2\rho_{XY_i}^2 - \rho_{XY_i}^2)]^{1/2}}, \tag{17}$$

where $U_X = 1/u_X$. The meta-analysis is then performed on the corrected correlations (the ρ_{TP_a} estimates) using the methods described in Hunter and Schmidt (2004, chap. 3). Schmidt and Le's (2004) program package includes a program that computes the corrected values in Equation 17 and performs meta-analysis on them. For applications in personnel selection, this program has a subroutine that converts the ρ_{TP_a} values (true score correlations) to ρ_{XP_a} values (operational validities).

Artifact Distribution Meta-Analysis—Direct Range Restriction

Artifact-distribution-based meta-analysis is used when reliabilities and range restriction values are only sporadically available for the correlations in the meta-analysis. Application of these methods based on the hybrid model presented here is discussed in Hunter and Schmidt (2004, chap. 4). Again, Schmidt and Le's (2004) program package includes a program that implements this meta-analysis method.

Research Domains With Indirect Range Restriction

Until recently the formula to correct range restriction that is used in the methods of Callender and Osburn (1980), Hunter and Schmidt (1990a), Hunter, Schmidt, and Jackson (1982), Raju and

[1] This can be shown as follows. Working back from Equation 14, we have the following:

$$\rho_{TP_a} = \rho_{XP_a}/\sqrt{r_{XX_a}}.$$

Equation 15, which is Thorndike's (1949) Case II formula showing ρ_{XP_i} as a function of ρ_{XP_a}, can be rewritten to show the reverse relationship, ρ_{XP_a} as a function of ρ_{XP_i}:

$$\rho_{XP_a} = U_X\rho_{XP_i}/\sqrt{[1 + (U_X^2 - 1)\rho_{XP_i}^2]}.$$

Solving Equation 16 for ρ_{XP_i}, we have the following:

$$\rho_{XP_i} = \rho_{XY_i}/\sqrt{r_{YY_i}}.$$

Now replacing the third equation above into the second equation and summarizing, we obtain the new equation showing ρ_{XP_a} as a function of ρ_{XY_i}:

$$\rho_{XP_a} = \frac{U_X\rho_{XY_i}}{\sqrt{r_{YY_i} + U_X^2\rho_{XY_i}^2 - \rho_{XY_i}^2}}.$$

Replacing the above equation into the first equation relating ρ_{TP_a} to ρ_{XP_a}, we have the desired equation:

$$\rho_{TP_a} = \frac{U_X\rho_{XY_i}}{[r_{XX_a}(r_{YY_i} + U_X^2\rho_{XY_i}^2 - \rho_{XY_i}^2)]^{1/2}}.$$

Burke (1983), Raju et al. (1991), and Schmidt and Hunter (1977) has been Thorndike's (1949) Case II formula (our Equation 4). This formula assumes direct range restriction (truncation) on the predictor. It has long been known that if range restriction is indirect rather than direct, this formula will undercorrect (e.g., see Linn, Harnisch, & Dunbar, 1981; Schmidt, Hunter, & Pearlman, 1981; Schmidt, Hunter, Pearlman, Hirsh, 1985, p. 751; Schmidt et al., 1993, p. 7). Until recently, no method was available that could be used to correct for the most common case of indirect range restriction. Hence, Thorndike's Case II correction formula was used by default. Although it was known that this formula undercorrected, it was apparently assumed that the undercorrection was modest (e.g., 5% or so). As it turns out, it is not.

Historically, a common context for validity studies in applied psychology is an organization that is considering moving to the use of a new test for future selection. A concurrent validation study is conducted: The test is given to incumbent workers (or students) at the same time that job (or academic) performance is measured. Those data are used to estimate the true effect size (operational validity) in the applicant group. Range restriction is indirect in all cases of this sort. In fact, range restriction is indirect in almost all validity studies in employment and education (Thorndike, 1949, p. 175). For example in the U.S. Department of Labor database on the General Aptitude Test Battery (Hunter, 1983; Hunter & Hunter, 1984), all of the 515 validity studies (425 based on job performance measures and 90 based on training success measures) were concurrent in nature, and hence range restriction was indirect in all studies. Range restriction is often indirect even in predictive studies: The tests to be validated are often given to *incumbents*, with the criterion measures being taken months or years later. In our research over the years, we have rarely seen a study (outside of military selection research) in which there was direct selection on the predictor(s) being validated. This is the only kind of study for which the Case II range correction formula for direct range restriction would not undercorrect.

If the Case II range correction formula undercorrects, then one might ask why this undercorrection has not shown up in computer simulation studies. As noted most recently by Hall and Brannick (2002), computer simulation studies have shown that these methods are quite accurate. The answer is that all published computer simulation studies have assumed (and programmed) only direct range restriction. Hence these simulation studies by definition cannot detect the undercorrection that occurs when range restriction is indirect.

In indirect range restriction, selection is based on variables other than the predictor itself. For example, suppose the organization considers a high school diploma critical to high job performance. If only high school graduates are hired, then the bottom 20% of the intelligence distribution will be strongly underrepresented, leading to a smaller standard deviation of intelligence test scores. Mendoza and Mumford (1987) noted that indirect range restriction leads to a reduction in the slope of the regression line of the dependent variable on test scores. Direct range restriction produces no such reduction. This is important because the Case II formula for range restriction assumes that the regression slope is the same in the two populations. If range restriction is indirect, this equality would hold only if the independent variable were perfectly measured. In such situations, the Case II correction formula would be accurate, but such a case is hypothetical because variables are never measured perfectly.

When range restriction is direct, people are selected on observed test scores, and so they are selected partly on their true scores and partly on their measurement errors. For indirect selection, scores on the test of interest are not used in selection and so errors of measurement in those test scores have no effect on the selection process. The impact of indirect selection is on the predictor true score T; there is no effect on the errors of measurement in the observed scores.

A Model for Indirect Range Restriction

The selection process is a decision process. The organization has certain information about applicants, and this information is converted into a judgment about the suitability of the applicants. The organization then hires or admits top down on the basis of these suitability judgments. The assumption is that an evaluation variable is implicitly constructed by the organization and used to make selection decisions (Linn, Harnish, & Dunbar, 1981). We call this variable *suitability* and denote it by S. The variables that go into S can be any combination of explicitly measured variables, subjectively assessed unmeasured variables (e.g., interview impressions and judgments), or any other information. The method used to combine these variables to produce S can be any linear or nonlinear function. For direct range restriction, the selection variable S is identical to the predictor observed score X, but for indirect range restriction, S can be very different from X.

There are five variables in our model: the selection variable S, the predictor (independent variable) true score T, the predictor observed score X, the criterion (dependent variable) true score P, and the criterion observed score Y. The path diagram for the applicant (unrestricted) population is presented in Figure 1. The causality depicted in Figure 1 is not substantive causality but rather the causality of range restriction. For example, S does not cause T in any substantive sense; rather (direct) range restriction on S causes (indirect) range restriction on T. In indirect range restriction, the selection process reduces the standard deviation for the predictor variable. This is represented in Figure 1 by an arrow from the selection variable S to the predictor variable. There is effectively no way that an organization could predict the errors of measurement in the predictor scores; that is, selection on S cannot be correlated with the measurement errors in X.

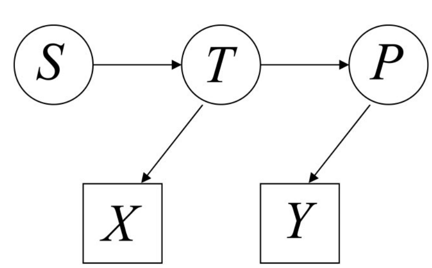

Figure 1. The path diagram for range restriction effects for the applicant population in indirect range restriction. S = the suitability composite used by the organization to select applicants; X = the predictor (e.g., test scores); T = true scores on the predictor X; Y = the measure of job performance; and P = the true score on Y.

Thus, in our model, we make the usual covariance analysis assumption that the arrow goes from construct to construct rather than from observed score to observed score. Hence there is an arrow from S to T. The arrow from T to X represents the predictor measurement process. Because errors of measurement do not enter the selection process, there is no arrow from S to X. The criterion variable is also represented by two variables: the true score P and the observed score Y. The model for the usual substantive theory represents the effect as an arrow from one true score to the other; that is, from T to P. The measurement process is represented by an arrow from P to Y.

Our model has an arrow from S to T and from T to P, so there is an indirect effect of selection on the criterion variable. Our model makes the assumption that there is no other arrow connecting S and P. This corresponds to the assumption that the selection process does not assess any characteristic that would affect the criterion variable for reasons other than those measured by the current predictor variable X. That is, all effects of selection on range restriction are assumed to be mediated by their effect on the predictor. If this assumption is met, our correction for indirect range restriction is identical to Thorndike's (1949) Case III correction. If this assumption is violated, our method for correcting for indirect range restriction will generally undercorrect, as can be seen logically from Figure 1. However, computer simulation studies indicate that the values produced under this condition are still more accurate (or less inaccurate) than those produced by the use of the correction for direct range restriction (Le, 2003). (Thorndike's, 1949, Case III formula does not make this assumption and does not undercorrect when there is a direct path from S to P. However, as discussed earlier, the information needed to apply the Case III formula is rarely available.)

Effects of Range Restriction on S

The most fundamental restriction is on the selection variable S. The organization hires top down on S, creating direct range restriction on S. The path coefficient from T to P is the correlation between T and P, and that is the target of formulas for correction for range restriction. This path coefficient is smaller in the incumbent population than in the applicant population.

Selection reduces the standard deviations of all five variables in the path diagram. The only comparison ratio computed in traditional research is the u ratio for the observed predictor score X, that is, u_X. Range restriction on the other variables is represented by the corresponding ratios of their restricted to unrestricted *SD*s: u_S, u_T, u_P, and u_Y. The strongest indirect range restriction is on the predictor true score T. The range restriction on T is caused directly by the range restriction on S. If the regression of T on S is linear, then we can compute u_T from u_S. The formulas that relate them can be derived for either applicant population or incumbent population and are derived in Appendix B.

The equations for restriction on T are as follows:

$$\text{Applicant population: } u_T^2 = \rho_{ST_a}^2 u_S^2 - \rho_{ST_a}^2 + 1; \tag{18}$$

$$\text{Incumbent population: } u_T^2 = u_S^2/(u_S^2 - \rho_{ST_i}^2 u_S^2 + \rho_{ST_i}^2). \tag{18a}$$

Similarly, the restriction on X is caused directly by restriction on T. The formulas that relate them are as follows:

$$\text{Applicant population: } u_X^2 = \rho_{TX_a}^2 u_T^2 - \rho_{TX_a}^2 + 1; \tag{19}$$

$$\text{Incumbent population: } u_X^2 = u_T^2/(u_T^2 - \rho_{TX_i}^2 u_T^2 + \rho_{TX_i}^2). \tag{19a}$$

The restriction on P is caused directly by restriction on T. The formulas that relate them are as follows:

$$\text{Applicant population: } u_P^2 = \rho_{TP_a}^2 u_T^2 - \rho_{TP_a}^2 + 1; \tag{20}$$

$$\text{Incumbent population: } u_P^2 = u_T^2/(u_T^2 - \rho_{TP_i}^2 u_T^2 + \rho_{TP_i}^2). \tag{20a}$$

The restriction on Y is caused directly by restriction on P. The formulas that relate them are as follows:

$$\text{Applicant population: } u_Y^2 = \rho_{PY_a}^2 u_P^2 - \rho_{PY_a}^2 + 1; \tag{21}$$

$$\text{Incumbent population: } u_Y^2 = u_P^2/(u_P^2 - \rho_{PY_i}^2 u_P^2 + \rho_{PY_i}^2). \tag{21a}$$

Estimation in Indirect Range Restriction

S is not observed and neither is u_S. Fortunately, we do not need this value for our purposes. The critical value is the restriction ratio for the predictor true score T. This is not observed, but it can be computed from the observed value for u_X. Equation 19 relating u_X to u_T can be rewritten as follows:

$$\text{Restriction on } X\text{: } u_X^2 = r_{XX_a} u_T^2 - r_{XX_a} + 1,$$

where r_{XX_a} is the applicant reliability of the predictor variable ($r_{XX_a} = \rho_{TX_a}^2$).

Solving this equation for u_T yields the following:

$$u_T^2 = [u_X^2 - (1 - r_{XX_a})]/r_{XX_a}. \tag{22}$$

This formula may appear strange at first, but it is important to remember that the smallest possible value for u_X is not zero. Although in the case of direct range restriction, u_X can be as small as zero (because SD_{X_i} can be zero and therefore $SD_{X_i}/SD_{X_a} = u_X = 0$), this is not true for indirect range restriction. For indirect range restriction, errors of measurement are not included in the selection process, so selection produces no change in the measurement error variance. Thus, the restricted variance of X is always at least as large as the measurement error variance of X, and the minimum value for u_X is $\sqrt{1 - r_{XX_a}}$.

As an example, consider the U.S. Employment Service (USES) database analyzed by Hunter (1983; Hunter & Hunter, 1984). The average value for u_X across the 425 studies that used job performance as the dependent variable was .67. The applicant population reliability (r_{XX_a}) of the General Aptitude Test Battery (Hunter & Hunter, 1984) measure of intelligence used was a constant .81. The average value of u_T is therefore as follows:

$$u_T^2 = [.67^2 - (1 - .81)]/.81 = .3196;$$

$$u_T = \sqrt{.3196} = .56.$$

The mean value of u_T of .56 is considerably smaller than the mean value of u_X ($\bar{u}_X = .67$). This means that taking u_X as a measure of range restriction leads to a consistent understatement of the extent of actual range restriction and an underestimation of the attenuating effects of range restriction. The finding for the example can be generalized: The value of u_T will always indicate more extreme range restriction than the value of u_X. That is, u_T is always smaller than u_X.

The Correlation Between S and T

The correlation between S and T is a measure of the extent to which the organization indirectly relies on T in their selection scheme when it selects on S. That correlation can be computed from the range restriction values for S and T, that is, from u_S and u_T. The extent of range restriction on the predictor true score depends on the extent of range restriction on S and on the size of the correlation between S and T in the applicant population. Equation 18 can be solved for ρ_{ST_a} as follows:

$$\rho^2_{ST_a} = (u_T^2 - 1)/(u_S^2 - 1) = (1 - u_T^2)/(1 - u_S^2). \qquad (23)$$

Consider the USES database example. If an organization selects the top 10% from a normal distribution on S, then the range restriction on S is $u_S = .41$ (cf. Schmidt et al., 1976). If its average range restriction on T is $u_T = .56$, then the correlation between S and T in the applicant population is

$$\rho_{ST_a} = \sqrt{(1 - u_T^2)/(1 - u_S^2)} = \sqrt{(1 - .56^2)/(1 - .41^2)}$$

$$= \sqrt{(.6864/.8319)} = .91.$$

If the average selection ratio for the organizations studied by the USES was .10, then $\rho_{ST_a} = .91$ would be the average correlation between suitability and intelligence. If the average selection ratio is smaller than 10%, the average correlation would be lower. (The actual average selection ratio for S in this data set is, of course, unknown.)

The Attenuation Model in Indirect Range Restriction

The easiest context in which to understand the attenuation model in indirect range restriction is concurrent validation. In a concurrent validity study, it is clear that selection precedes measurement on both X and Y. Thus, range restriction will be the first artifact in operation, and measurement errors are added later for the incumbent population when measures of predictor and criterion are administered.

Starting in the applicant population, range restriction occurs on T:

$$\rho_{TP_i} = c\rho_{TP_a}, \quad \text{where} \quad c = u_T/\sqrt{(u_T^2\rho^2_{TP_a} + 1 - \rho^2_{TP_a})}. \qquad (24)$$

Then in the incumbent population, measurement error produces attenuation. For measurement error in the predictor (independent variable),

$$\rho_{XP_i} = a\rho_{TP_i}, \text{ where } a = \sqrt{r_{XX_i}}.$$

For measurement error in the criterion (dependent variable),

$$\rho_{XY_i} = b\rho_{XP_i}, \text{ where } b = \sqrt{r_{YY_i}}.$$

For educational and employment selection research, this model is fortunate in that the needed dependent variable reliability information is that for the restricted population, and this reliability coefficient can be computed from the data on hand (i.e., the restricted data).

Predictor Measurement Error in Indirect Range Restriction

Our model indicates that incumbent reliability of the independent variable may be considerably lower than the applicant reliability. As shown below, the incumbent predictor reliability can be estimated from either range restriction on u_T or range restriction on u_X.

Estimating incumbent reliability from u_T. It is useful in deriving a formula to shift from the reliability to the square root of the reliability, which is the correlation ρ_{TX}. This correlation will change between populations due to range restriction on T, and the formula for the incumbent ρ_{TX} is the same range restriction formula as for the effect size ρ_{TP}.

In the applicant population, we have ρ_{TX_a}, and range restriction on T is indexed by u_T. In the incumbent population, we have ρ_{TX_i}:

$$\rho_{TX_i} = c\rho_{TX_a}, \text{ where } c = u_T/\sqrt{(u_T^2\rho^2_{TX_a} + 1 - \rho^2_{TX_a})}. \qquad (25)$$

The resulting reliability equation is as follows:

$$r_{XX_i} = \rho^2_{TX_i} = c^2\rho^2_{TX_a} = c^2 r_{XX_a}. \qquad (26)$$

In the case of the USES database, the applicant reliability is $r_{XX_a} = .81$, and so $\rho_{TX_a} = .90$. Range restriction on T causes ρ_{TX_i} to drop to .756, which means the incumbent reliability is only $(.756)^2 = .57$. That is, a test that has a reliability of .81 in the applicant population has only a modest reliability of .57 in the incumbent population.

Estimating incumbent reliability from u_X. This can be achieved with the formula relating reliability in two populations (cf. Nunnally & Bernstein, 1994; Schmidt, Le, & Ilies, 2003):

$$r_{XX_i} = 1 - \frac{S^2_{X_a}(1 - r_{XX_a})}{S^2_{X_i}} = 1 - U_X^2(1 - r_{XX_a}), \qquad (27)$$

where $U_X = 1/u_X$. Part 2 of Appendix B demonstrates that Equations 26 and 27 are equivalent. Equation 27 is more convenient to use. In cases in which the independent variable reliability estimates are from the restricted group, one can skip this step and go straight to Equation 28.

Meta-Analysis Correcting Each Correlation Individually—Indirect Range Restriction

When meta-analysis is performed on study correlations corrected individually for the effects of all three artifacts, the procedure is as follows: First, convert r_{XX_a} to r_{XX_i} using Equation 26 or 27 above. Second, correct the observed correlation in the restricted group (ρ_{XY_i}) with r_{XX_i} and r_{YY_i}:

$$\rho_{TP_i} = \rho_{XY_i}/(r_{XX_i}r_{YY_i})^{1/2}. \qquad (28)$$

Third, correct this value for range restriction using $U_T = 1/u_T$ in the equation for direct range restriction correction:

$$\rho_{TP_a} = \frac{U_T\rho_{TP_i}}{(1 + U_T^2\rho_{TP_i}^2 - \rho_{TP_i}^2)^{1/2}}. \quad (29)$$

This sequence of corrections is summarized in Table 2. This procedure relies on the fact that in the absence of measurement error, the formula for correcting for direct range restriction is accurate in cases in which range restriction is indirect. In this procedure, the measurement error is first removed and then the formula for correcting for direct range restriction is applied. After each study correlation is corrected in this manner, the meta-analysis is performed on the corrected correlations with the methods described in Hunter and Schmidt (2004, chap. 3). Schmidt and Le's (2004) program package includes a program that computes the values in Equation 29 and conducts a meta-analysis on these values. In the case of personnel selection, the program has a subroutine that converts ρ_{TP_a} values (true score correlations) to ρ_{XP_a} values (operational validities). However, this procedure frequently cannot be used because primary studies often do not include the necessary information on reliabilities and range restriction for individual correlations. In such cases, meta-analysis must be conducted with distributions of artifacts compiled from the studies that report them (Hunter & Schmidt, 2004, chap. 4).

Artifact Distribution Meta-Analysis: The Model for Indirect Range Restriction

The attenuation equations presented earlier provide a model for artifact distribution meta-analysis in the context of indirect range restriction. In the *Meta-Analysis for Indirect Range Restriction* section, we present a method for applying meta-analysis based on this new model.

The first step is a bare bones meta-analysis (Hunter & Schmidt, 1990a, 2004). This meta-analysis converts the mean and standard deviation of sample correlations (r_{XY_i}) into a mean and standard deviation for incumbent population attenuated correlations (ρ_{XY_i}) by correcting for sampling error. That is, the bare bones meta-analysis provides estimates of the mean and standard deviation of attenuated population correlations (ρ_{XY} for the incumbent populations). The purpose of subsequent steps in the meta-analysis is to remove the effects of three artifacts: range restriction, error of measurement in the dependent variable measure, and error of measurement in the independent variable measure.

For indirect range restriction, it is convenient if predictor reliability information is applicant data, so it can be used in estimating u_T, on the basis of Equation 22. On the other hand, criterion reliability information is easiest to use if it is incumbent reliability

Table 2
Steps in Correcting for Indirect Range Restriction

Step	Purpose	Input	Output	Formula
1	Correcting for measurement error in Y (measure of construct P)	Correlation between X and Y in the restricted population: ρ_{XY_i} Reliability of Y in the restricted population: r_{YY_i}	Correlation between X and P in the restricted population: ρ_{XP_i}	$\rho_{XP_i} = \rho_{XY_i}/\sqrt{r_{YY_i}}$
2	Estimating reliability of X in the restricted population: r_{XX_i} (if r_{XX_i} is already known from the data, proceed directly to Step 3)	Range restriction on X: u_X Reliability of X in the unrestricted population: r_{XX_a}	Reliability of X in the restricted population: r_{XX_i}	$r_{XX_i} = 1 - U_X^2(1 - r_{XX_a})$ *Note.* $U_X = 1/u_X$
3	Correcting for measurement error in X (measure of construct T)	Correlation between X and P in the restricted population: ρ_{XP_i} Reliability of X in the restricted population: r_{XX_i}	Correlation between T and P in the restricted population: ρ_{TP_i}	$\rho_{TP_i} = \rho_{XP_i}/\sqrt{r_{XX_i}}$
4	Estimating reliability of X in the unrestricted population: r_{XX_a} (if r_{XX_a} is already known from the data, proceed directly to Step 5)	Range restriction on X: u_X Reliability of X in the restricted population: r_{XX_i}	Reliability of X in the unrestricted population: r_{XX_a}	$r_{XX_a} = 1 - u_X^2(1 - r_{XX_i})$
5	Estimating range restriction on T: u_T	Range restriction on X: u_X Reliability of X in the unrestricted population: r_{XX_a}	Range restriction on T: u_T	$u_T = \sqrt{[u_X^2 - (1 - r_{XX_a})]/r_{XX_a}}$
6	Correcting for the effect of indirect range restriction	Range restriction on T: u_T Correlation between T and P in the restricted population: ρ_{TP_i}	Correlation between T and P in the unrestricted population: ρ_{TP_a}	$\rho_{TP_a} = \frac{U_T\rho_{TP_i}}{\sqrt{1 + (U_T^2 - 1)\rho_{TP_i}^2}}$ *Note.* $U_T = 1/u_T$
7[a]	Reintroducing measurement error in T to estimate the operational validity of X (as a measure of T): ρ_{XP_a}	Correlation between T and P in the unrestricted population: ρ_{TP_a} Reliability of X in the unrestricted population: r_{XX_a}	Correlation between X and P in the unrestricted population: ρ_{XP_a}	$\rho_{XP_a} = \rho_{TP_a}\sqrt{r_{XX_a}}$

[a] The output for this step (ρ_{XP_a}) is the operational validity of measure X, so depending on research questions, the correction process may end at Step 6.

data. Both of these conventions fit the usual data-gathering situation for educational and employment selection. That is, typically reliability information is available for the unrestricted population for the independent variable and for the restricted population for the dependent variable. The crucial value for indirect range restriction is u_T rather than u_X. If an empirical estimate of u_X is accompanied by the corresponding estimate of unrestricted predictor reliability in the applicant group, then the corresponding estimate of u_T can be computed for that source with Equation 22. (If values of u_X are given independently from values for the predictor reliability, approximate estimates of u_T can be obtained with the average unrestricted predictor reliability to convert each value of u_X to an estimated value for u_T.)

The input artifact data for the multiplicative method of meta-analysis is three sets of means and standard deviations: the means and standard deviations of (a) square roots of applicant independent variable reliability (ρ_{TX_a}), (b) square roots of incumbent dependent variable reliability (ρ_{PY_i}), and (c) range restriction (u_T). Continuing use of our earlier notation, we have

$$\rho_o = abc\rho_{TP_a},$$

where a is the multiplier for predictor measurement error, b is the multiplier for criterion measurement error, and c is a multiplier for range restriction.

The major problem for the multiplicative model lies in the relationship between the multipliers a and c. For direct range restriction, a is the simple quantity ρ_{TX_a}, and there is only a small correlation between a and c. For indirect range restriction, the formula for a is the same range restriction formula as for c, and there is a high correlation between a and c. This high correlation between a and c rules out use of Callender and Osburn's (1980) multiplicative method of analysis when range restriction is indirect—because the violation of the assumption of the independence of a, b, and c is extreme, as we show next.

For this purpose, we found it useful to express the attenuation equation in a different format from the *abc* multiplicative format. Let $q_X = \rho_{TX} = \sqrt{r_{XX}}$, and $q_Y = \rho_{PY} = \sqrt{r_{YY}}$. The incumbent attenuation formula can be written as

$$\rho_{XY_i} = q_{X_i} q_{Y_i} \rho_{TP_i}, \tag{30}$$

whereas the applicant attenuation formula is $\rho_{XY_a} = q_{X_a} q_{Y_a} \rho_{TP_a}$. All three of these component values will be different in these two populations. We seek an attenuation formula that relates ρ_{TP_a} to ρ_{XY_i}.

In the attenuation formula for measurement error in the incumbent population (Equation 30):

$$q_{X_i} = \frac{u_T q_{X_a}}{\sqrt{u_T^2 q_{X_a}^2 + 1 - q_{X_a}^2}}$$

and

$$\rho_{TP_i} = \frac{u_T \rho_{TP_a}}{\sqrt{u_T^2 \rho_{TP_a}^2 + 1 - \rho_{TP_a}^2}}.$$

This yields the following attenuation formula:

$$\rho_{XY_i} = q_{Y_i} \frac{u_T q_{X_a}}{\sqrt{u_T^2 q_{X_a}^2 + 1 - q_{X_a}^2}} \frac{u_T \rho_{TP_a}}{\sqrt{u_T^2 \rho_{TP_a}^2 + 1 - \rho_{TP_a}^2}}. \tag{31}$$

The reader should note the double occurrence of the range restriction expression, first for q_{X_i} and then for ρ_{TP_i}. As a result of this, the last two terms are so highly correlated that they rule out use of the multiplicative model.

Meta-Analysis for Indirect Range Restriction

The mean true effect size. Earlier computer simulation tests of the meta-analysis model for direct range restriction show that, given the correct sequencing of corrections, there is little error in estimating the mean effect size (e.g., see Law et al., 1994a, 1994b). The mean study population correlation ($\bar{\rho}_{XY_i}$) corrected for attenuation with the mean values for the artifact values provides an accurate estimate of $\bar{\rho}_{TP_a}$. There is no reason why this should not also hold true for indirect range restriction, and Le (2003) has shown by means of a computer simulation that this is indeed the case. We illustrate this procedure by use of the USES database originally analyzed by Hunter (1983) and Hunter and Hunter (1984). In that database, the mean study population correlation (pooling all data) is .26, the mean applicant square root of the predictor reliability (q_{X_a}) is .90, the mean incumbent square root of criterion reliability (q_{Y_i}) is .69, and the mean range restriction on T is $u_T = .56$.

For indirect range restriction, the correlation is attenuated for range restriction before measurement error. Corrections must be applied in the order opposite to the order in which the artifacts have attenuated the correlation. For indirect range restriction, that means that we first correct for measurement error and then correct for range restriction (see Table 2). The correction for criterion (dependent variable) measurement error is straightforward. The data on criterion reliability is incumbent data and can thus be used directly. To correct for criterion unreliability, we simply divide the mean study correlation by the mean square root of incumbent criterion reliability (q_{Y_i}), as noted earlier. For example, the mean reliability of performance ratings for incumbents on a multi-item rating instrument is .47 (Viswesvaran et al., 1996). Thus, the mean (q_{Y_i}) is $\sqrt{.47} = .69$. We divide the mean study correlation of .26 by .69 to get .38.

The correction for predictor measurement error is more complicated. The data on predictor reliability are applicant data. Thus, the applicant value of $\sqrt{.81} = .90$ must be converted to the incumbent value with the range restriction formula (i.e., with Equations 26 or 27). For the USES data, this reduces the applicant value of .90 to a considerably lower .76. This is the value used to correct the .38 obtained by correcting for criterion unreliability. The correlation corrected for both sources of error is $.38/.76 = .50$.

After we correct for measurement error, we must correct for range restriction using the mean value for u_T. For the USES data, this means that we correct the .50 using the mean value of u_T, which is .56. The correction formula used is Equation 29. The estimate of mean true score correlation is then .73. The mean true (operational) validity is then $\sqrt{.81}\,(.73) = .66$. Again, the steps in this process are summarized in Table 2.

Hunter (1983) analyzed the USES data using the direct range restriction model, an inappropriate model. The average true validity correlation was estimated to be .54, considerably smaller than the .66 obtained using the correct model. Other published estimates of the average true validity of selection procedures for predicting job or academic performance are likely to be underestimates for the same reason. That is, all other currently published analyses have inappropriately used the model for direct range restriction and hence have produced underestimates of mean operational validities.

The standard deviation of true effect sizes. The major differences between methods for the direct and indirect range restriction models are in the methods for estimating the standard deviation of effect sizes ($SD_{\rho_{TP_a}}$). As shown earlier, the multiplicative method is not accurate for the indirect range restriction model because of the high correlations among its component terms (see Equation 31). An adaptation of the interactive meta-analysis model for the direct range restriction condition (Law et al., 1994a, 1994b; Schmidt et al., 1980, 1993) to indirect range restriction (Schmidt & Le, 2004) has proven to provide acceptably accurate estimates of ($SD_{\rho_{TP_a}}$) for the indirect range restriction case (Le, 2003). This procedure is not based on the multiplicative model. In the case of direct range restriction, the interactions method (with certain refinements) has proven to be slightly more accurate than other methods (Law et al., 1994a, 1994b). Computer simulation results (Le, 2003) suggest it may also be the method of choice for artifact-distribution-based meta-analysis when range restriction is indirect. However, this method is very cumbersome to present and describe. We have therefore derived an alternative method based on the multivariate Taylor's series approach used by Raju and Burke (1983), and we now present that method.

As a mathematical expression, the attenuation model has four independent quantities: q_{X_a}, q_{Y_i}, u_T, and ρ_{TP_a}. Mathematically these are treated as variables, even though in the scientific context we would consider them parameters. As is the case with the other derivations presented, sampling error is not part of this model. Sampling error is addressed in a prior step in which it is subtracted out in a bare bones meta-analysis (see *The USES Database: An Empirical Example* section). The multivariate Taylor's series approximates the nonlinear function with a multivariate polynomial. The first level of approximation, a linear function of the four parameters, is used in this method of meta-analysis. This approximation has the desirable property that it breaks the variance of study population correlations into four terms, one for each independent variable. Within the approximation, these terms can be considered as variance components and approximate percentages of variance accounted for can be associated with them. (Such percentage breakdowns cannot be obtained for the actual nonlinear model.)

The linear approximation uses the deviation score of each parameter from its mean. Suppose we denote the deviation scores as x_1 through x_4. The linear approximation then takes the form

$$\rho_{XY_i} = c + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4. \quad (32)$$

The constant term c is the value of the nonlinear function evaluated at the mean values for all the parameters. To a close approximation, c is $\bar{\rho}_{XY_i}$; that is, the mean population attenuated correlation in the restricted population.

The four deviation scores x_1 through x_4 in Equation 32 represent q_{X_a}, q_{Y_i}, u_T, and ρ_{TP_a}. The multivariate Taylor's series approximates the nonlinear function with a multivariate polynomial (Raju & Burke, 1983). Specifically, variance of the observed (restricted) correlations ρ_{XY_i} can be broken down into four variance components:

$$\mathrm{var}_{\rho_{XY_i}} = b_1^2\mathrm{var}_{q_{X_a}} + b_2^2\mathrm{var}_{q_{Y_i}} + b_3^2\mathrm{var}_{u_T} + b_4^2\mathrm{var}_{\rho_{TP_a}}. \quad (33)$$

where b_1 = first order partial derivative of ρ_{XY_i} with respect to q_{X_a}; b_2 = first order partial derivative of ρ_{XY_i} with respect to q_{Y_i}; b_3 = first order partial derivative of ρ_{XY_i} with respect to u_T; and b_4 = first order partial derivative of ρ_{XY_i} with respect to ρ_{TP_a}.

Again, in connection with sampling error, note that $\mathrm{var}_{\rho_{XY_i}} = \mathrm{var}_{r_{XY_i}} - \mathrm{var}_e$, where $\mathrm{var}_{r_{XY_i}}$ is the variance of the observed correlations and var_e is the sampling error variance. Solving Equation 33 for $\mathrm{var}_{\rho_{TP_a}}$, we obtain the formula to estimate the desired variance of true score correlations:

$$\mathrm{var}_{\rho_{TP_a}} = [\mathrm{var}_{\rho_{XY_i}} - (b_1^2\mathrm{var}_{q_{X_a}} + b_2^2\mathrm{var}_{q_{Y_i}} + b_3^2\mathrm{var}_{u_T})]/b_4^2. \quad (34)$$

Appendix C presents formulas for the derivatives b_1, b_2, b_3, and b_4 and proofs for the derivations. Taking the square root of Equation 34, we have the standard deviation of the true score correlation:

$$SD_{\rho_{TP_a}} = \sqrt{\mathrm{var}_{\rho_{TP_a}}}. \quad (35)$$

The estimated standard deviation of true validities is then

$$SD_{\rho_{XP_a}} = \sqrt{r_{XX_a}}\, SD_{\rho_{TP_a}}.$$

Testing of this method through computer simulation shows that it is almost as accurate in estimating $SD_{\rho_{TP_a}}$ as the Taylor's series method applied to the direct range restriction model (Le, 2003). The slight drop in accuracy is due to the fact that the effect of range restriction is more nonlinear for indirect range restriction than for direct range restriction. Computer simulation results show that this Taylor's series method and the interactive method described earlier produce about equally accurate estimates of $SD_{\rho_{TP_a}}$ for the case of indirect range restriction (Le, 2003).

The USES Database: An Empirical Example

To contrast the models for direct and indirect range restriction, we use the USES database as analyzed by Hunter (1983) and Hunter and Hunter (1984). The data to be considered here are the data for the measure of general mental ability derived from the General Aptitude Test Battery in the prediction of job performance ratings (425 studies). The job families and the methods used to construct them are described in Hunter and Hunter. The job families are numbered from 1 (*most complex*) to 5 (*least complex*) on the basis of their requirements for information processing.

Table 3 presents results for four analyses. The columns under "Sample values" show the mean and standard deviation of observed sample correlations (r_{XY_i}) in each family. The columns under "Bare bones meta-analysis" present the bare bones meta-analysis on each family. These are the estimated means and standard deviations of study population correlations (ρ_{XY_i}). The standard deviations here are smaller because sampling error variance has been removed, but mean values are the same. The columns under "Hunter's original meta-analysis" present an update of Hunter's (1983) full meta-analysis results, with corrections for direct range restriction. These figures are somewhat different from those given in Hunter (1983) and Hunter and Hunter (1984), because in these results, the estimates of $SD_{\rho_{XP_a}}$ are corrected for variation across studies in range restriction, whereas Hunter did

Table 3
Validities From the U.S. Employment Service Database Presented at Four Levels of Analysis Accuracy (Cognitive Ability Predicting Job Performance Ratings)

Job family number	Sample values		Bare bones meta-analysis[a]		Hunter's original meta-analysis[b]		Current meta-analysis[c]	
	$\bar{r}$	SD_r	$\bar{\rho}_{XY_i}$	$SD_{\rho_{XY}}$	$\bar{\rho}_{XP_a}$	$SD_{\rho_{XP_a}}$	$\bar{\rho}_{XP_a}$	$SD_{\rho_{XP_a}}$
1	.32	.13	.32	.01	.58	.02	.73	.00
2	.32	.16	.32	.08	.58	.09	.74	.04
3	.26	.15	.26	.08	.49	.09	.66	.05
4	.21	.13	.21	.01	.40	.02	.56	.00
5	.13	.14	.13	.05	.25	.06	.39	.06

Note. Job Family 1 = complex set up jobs (2.5% of the workforce, 17 studies); Job Family 2 = managerial/ professional jobs (14.7% of the workforce, 36 studies); Job Family 3 = technican and skilled jobs (62.7% of the workforce, 151 studies); Job Family 4 = semiskilled jobs (17.7% of the workforce, 201 studies); Job Family 5 = unskilled jobs (2.4% of the workforce, 20 studies).
[a] These correlations are corrected only for sampling error. [b] Correlations corrected for direct range restriction and for measurement error in the criterion. These are operational validity estimates (see Hunter, 1983). [c] Correlations corrected for indirect range restriction and for measurement error in the criterion. These are operational validity estimates.

not make this correction in his earlier analysis. The effect of this is to decrease $SD_{\rho_{XP_a}}$ estimates slightly.

The final two data columns (under "Current meta-analysis") present the more accurate estimates produced by the model for indirect range restriction based on the Taylor's series approximation. In all cases, the mean for the indirect range restriction model is considerably higher than for the direct restriction model. It is clear that Hunter (1983) and Hunter and Hunter (1984) underestimated these correlations by a substantial margin. For example, consider the mean for medium complexity jobs (over half of all jobs in the U.S. economy). The correct model estimates this correlation to be .66, whereas Hunter (1983) estimated it as .49, a 26% underestimate. The values in Table 3 are for operational validities. (Values for true score [i.e., construct level] correlations can be obtained by multiplying all figures by $1/\sqrt{.81}$ or 1.11.) These findings are consistent with those of Schmidt, Oh, and Le (2006), which are based on four different independent databases.

The point here is not that general mental ability is a highly valid predictor of job performance; that is a well-known fact. The point is the *size* of the underestimation of the validity and the fact that underestimation by such large amounts is likely to have occurred not only in past validity and validity generalization studies but also in many other areas of industrial and organizational research. That is, these results suggest the hypothesis that the strength of many relationships in industrial and organizational psychology has been substantially underestimated. If so, this would have implications both for levels of practical utility and for the nature and evaluation of theories of performance and work behavior. For example, the extent to which job performance is determined by combinations of traits may have been seriously underestimated.

Related Questions

A reviewer raised the question of the proper range restriction correction when the focus is on incremental validity of individual predictors in a sequential (multiple hurdle) selection process. Research has shown that relations between selection methods and performance criteria are almost always linear (Coward & Sackett, 1990). An important implication of this is that compensatory selection models yield the highest selection utility. In a compensatory model, an applicant can make up for being low on one predictor by being high on another (or others). To implement a compensatory model, all applicants must be given all predictors. This means the desired validity estimate for each predictor is that in the initial (complete) applicant pool. The present article is cast in terms of using range restriction corrections to estimate such validities. If there are multiple predictors, then the applicant pool validity of each should be estimated and used along with the applicant pool predictor intercorrelations (computable without range restriction corrections) in a regression equation predicting job performance. The regression equation is the statistical implementation of the compensatory selection model.

All multiple hurdle selection models, by contrast, are noncompensatory. That is, any applicant who falls below the cut score on any predictor is rejected at that point, no matter how high his or her scores are on predictors previously administered (or would have been on those yet to come in the sequence). Although the explicit reasons for adopting multiple hurdle models usually center around administrative convenience, the noncompensatory feature means that use of multiple hurdle models (also called multiple cutoff models) is equivalent to making the false assumption of nonlinear relationships between predictors and criteria. Therefore, traditional selection utility models (Brogden, 1949; Schmidt, Hunter, McKenzie, & Muldrow, 1979) do not apply to multiple hurdle selection. In a multiple hurdle selection process, the incremental validity is its validity in the applicant group that has already been selected on the predictors preceding it in the sequence. This incremental validity depends on how stringent selection is on the previous predictors and on the order in which the predictor is given. The number of possible combinations of cutoff scores and orders of predictor administration is very large, making a complete assessment of incremental validity nearly impossible. In contrast, with the compensatory selection model, each predictor has a single zero order validity (the validity in the initial applicant pool), and the incremental contribution to job performance of each predictor over and above other predictors is represented by its partial regression weight in the regression equation. Sequential selection models are often used by employers to save testing time and testing costs, although the savings are almost always less than the loss in selection utility (Hunter & Schmidt, 1982). If the decision is made to use a sequential procedure, decisions about the sequencing should be based not only on each predictor's costs, but also on its initial validity (i.e., its validity in the initial applicant pool). Hence, even when a sequential procedure is used, one needs the validity estimates discussed in this article.

Another question concerns the relationship between estimates of true score correlations and operational validities. For the presentation in this article, we have assumed that the only difference between these two is that in the case of operational validity, there is no correction for measurement error in the independent (predictor) variable, whereas this correction is made in the case of the true score correlation (making it an estimate of the construct level correlation or construct validity). This assumption is correct if both estimates are assumed to have been made for the applicant population (as is the case here) or if the applicant population and other relevant populations have the same independent variable *SD*. However, if the operational validity is estimated for the applicant population and the construct level correlation is assumed to be estimated for a more general population (e.g., the U.S. workforce as a whole), the more general population may have a different (presumably larger) unrestricted independent variable *SD*, requiring a larger correction for range restriction. In this case, simply correcting the operational validity estimate for measurement error in the predictor would underestimate the true score correlation in the broader population. However, research by Sackett and Ostgaard (1994) has suggested that the underestimation would be minimal. Using a large database on a cognitive ability test and a large number of jobs, Sackett and Ostgaard compared job-specific applicant pool predictor *SD*s to predictor *SD*s in national norm (general workforce) data. On average, predictor *SD*s were only 8% larger in the national norm data, suggesting that true score correlations estimates on job-specific applicant samples underestimate true score correlations in the general workforce only slightly. Likewise, use of national norm *SD*s as estimates of unrestricted applicant pool *SD*s would lead to only slight overestimates of operational validities. It is important to remember that mean predictor scores can vary substantially across applicant pools for different jobs (as was indeed the case in the Sackett & Ostgaard's, 1994, data), while still allowing the mean applicant pool *SD* to be only slightly smaller than the overall workforce *SD*. By a well know theorem in analysis of variance, we know that if group means differ, the average within-group *SD* must be smaller than the overall *SD*. However, it is not well known that this difference in *SD*s can be quite small even when the subgroup means differ considerably. A related question concerns the effects of predictor *SD* differences on estimates of reliability. If reliability is estimated in a workforce norm group, and the *SD* in that group is larger than in the applicant group, the reliability estimate will be at least slightly overestimated relative to the applicant group (leading to slight undercorrection for measurement error). If the two *SD*s are known, Equation 27 can be used to determine the appropriate reliability in the applicant group. A more detailed discussion of the dependence of reliability values on group *SD*s can be found in Hunter and Schmidt (2004, chap. 3).

In some cases, there may be not only range restriction (direct or indirect) on the independent variable, but also direct range restriction on the dependent variable. Another possibility is that there is indirect range restriction on the dependent variable caused by something other than the range restriction on the independent variable. In such cases, all standard range correction procedures, including those presented in this article, will undercorrect. These special cases and procedures proposed for dealing with them are discussed in Hunter and Schmidt (2004, pp. 39–41). One might also raise the question of whether range restriction corrections can be applied to the *d* statistic (the standardized difference between mean groups). In some cases, range correction procedures, including those presented in this article, can be applied to the *d* statistic. A full discussion of such applications is beyond the scope of this article. Such a discussion is provided by Roth, Bobko, Switzer, and Dean (2001).

Discussion and Conclusions

When range restriction is direct, accurate corrections for range restriction require the correct sequencing of corrections for measurement error and range restriction. In particular, because reliability of the predictor cannot be determined in the restricted group data, any correction for measurement error in the predictor must be made after the correction for range restriction and must use the reliability in the unrestricted group. One first corrects for measurement error in the dependent variable (criterion), then for range restriction, and finally for measurement error in the predictor (if desired). This little known sequence and the reasons underlying it are explicated in detail in this article and are summarized (along with the relevant equations) in Table 1.

However, range restriction is usually indirect rather than direct. A major error in range restriction correction methods, both in individual studies and in meta-analysis methods, has been generalization of the model for direct range restriction to use in contexts of indirect range restriction. We have shown that this practice results in substantial negative biases in corrected values. This article has presented a method for correcting for indirect range restriction that can be used when the information needed to apply Thorndike's (1949) Case III correction for indirect range restriction is not available. The sequence of corrections is that one first corrects for measurement error in both variables using restricted sample reliability values and then corrects for range restriction using the Case II formula but using u_T rather than u_X as the index of range restriction. The article presents a detailed analytical derivation of this procedure. This sequence of corrections and the equations needed for it are summarized in Table 2. This procedure should be used whenever range restriction is indirect, including those cases in which the range restriction results from self-selection. A reviewer noted that some degree of self-selection may be nearly ubiquitous in research samples and asked how a researcher can determine when to apply this correction for indirect range restriction. This correction should be applied whenever the independent variable *SD*s are different in one's sample and the population of interest.

This method of correcting for indirect range restriction can be applied in meta-analysis methods as well as to correlations in individual studies. This article presented a Taylor-series-based method for conducting artifact distribution meta-analysis when range restriction is indirect. Computer simulations testing the accuracy of this model indicate that the method produces accurate estimates of SD_ρ (Le, 2003). There is also a version of the interactive model (Law et al., 1994a, 1994b; Schmidt et al., 1980; Schmidt & Le, 2004) adapted to the requirements of data shaped by indirect range restriction, and simulation tests indicate that this model is as accurate as the Taylor-series-based model (Le, 2003). This article also presented meta-analysis methods for correcting each correlation individually, methods that incorporate the new correction for indirect range restriction. Meta-analysis programs that incorporate both direct and indirect range restriction corrections are available (Schmidt & Le, 2004). Meta-analysis results produced when one corrects for indirect range restriction produce larger mean ($\bar{\rho}_{XP_a}$) values in comparison with those produced by application of the inappropriate correction for direct range restriction. The derivations, equations, and results presented in this article suggest that the use of the direct range restriction model may have led to considerable underestimation of the validity of measures used in educational and employment selection and, by extension, may have led to underestimation of important parameters in other areas of research. In view of the fact that range restriction is indirect in most social and behavioral science research domains subject to range restriction, this issue appears to be an important one, and the development of accurate estimation models for meta-analysis of such data therefore appears to be important.

References

Brogden, H. E. (1949). When testing pays off. *Personnel Psychology, 2,* 171–184.

Brogden, H. E. (1968). *Restriction in range.* Unpublished manuscript, Purdue University.

Callender, J. C., & Osburn, H. G. (1980). Development and test of a new model for validity generalization. *Journal of Applied Psychology, 65,* 543–558.

Cohen, J., & Cohen, P. (1983). *Applied multiple regression/correlation analysis for the behavioral sciences.* Hillsdale, NJ: Erlbaum.

Coward, W. M., & Sackett, P. R. (1990). Linearity of ability–performance relationships: A re-confirmation. *Journal of Applied Psychology, 75,* 297–300.

Field, A. P. (2001). Meta-analysis of correlation coefficients: A Monte Carlo comparison of fixed- and random-effects methods. *Psychological Methods, 6,* 161–180.

Hall, S. M., & Brannick, M. (2002). Comparison of two random effects methods of meta-analysis. *Journal of Applied Psychology, 87,* 337–389.

Hedges, L. V., & Vevea, J. L. (1998). Fixed- and random-effects models in meta-analysis. *Psychological Methods, 3,* 486–504.

Hunter, J. E. (1983). *Test validation for 12,000 jobs: An application of job classification and validity generalization analysis to the General Aptitude Test Battery* (USES Test Research Report No. 45). Washington, DC: U.S. Employment Service, U.S. Department of Labor.

Hunter, J. E., & Hunter, R. F. (1984). Validity and utility of alternative predictors of job performance. *Psychological Bulletin, 96,* 72–98.

Hunter, J. E., & Schmidt, F. L. (1982). Fitting people to jobs: Implications of personnel selection for national productivity. In E. A. Fleishman & M. D. Dunnette (Eds.), *Human performance and productivity: Vol. 1: Human capability assessment* (pp. 233–284). Hillsdale, NJ: Erlbaum.

Hunter, J. E., & Schmidt, F. L. (1990a). Dichotomization of continuous variables: The implications for meta-analysis. *Journal of Applied Psychology, 75,* 334–349.

Hunter, J. E., & Schmidt, F. L. (1990b). *Methods of meta-analysis: Correcting error and bias in research findings.* Beverly Hills, CA: Sage.

Hunter, J. E., & Schmidt, F. L. (2000). Fixed effects vs. random effects meta-analysis models: Implications for cumulative research knowledge. *International Journal of Selection and Assessment, 8,* 275–292.

Hunter, J. E., & Schmidt, F. L. (2004). *Methods of meta-analysis: Correcting error and bias in research findings* (2nd ed.). Thousand Oaks, CA: Sage.

Hunter, J. E., Schmidt, F. L., & Jackson, G. B. (1982). *Meta-analysis: Cumulating research findings across studies.* Beverly Hills, CA: Sage.

Johnson, J. T., & Ree, M. J. (1994). RANGEJ: A Pascal program to compute the multivariate correction for range restriction. *Educational and Psychological Measurement, 54,* 693–695.

Law, K. S., Schmidt, F. L., & Hunter, J. E. (1994a). Nonlinearity of range corrections in meta-analysis: A test of an improved procedure. *Journal of Applied Psychology, 79,* 425–438.

Law, K. S., Schmidt, F. L., & Hunter, J. E. (1994b). A test of two refinements in meta-analysis procedures. *Journal of Applied Psychology, 79,* 978–986.

Lawley, D. N. (1943). A note on Karl Pearson's selection formulae. *Proceedings of the Royal Society of Edinburgh, 62*(A), 28–30.

Le, H. A. (2003). *Correcting for indirect range restriction in meta-analysis: Testing a new meta-analytic method.* Unpublished doctoral dissertation, University of Iowa.

Linn, R. L., Harnisch, D. L., & Dunbar, S. B. (1981). Corrections for range restriction: An empirical investigation of conditions resulting in conservative corrections. *Journal of Applied Psychology, 66,* 655–663.

Lord, F., & Novick, M. (1968). *Statistical theories of mental test scores.* Reading, MA: Addison Wesley.

MacCallum, R. C., Zhang, S., Preacher, K. J., & Rucker, D. D. (2002). On the practice of dichotomization of quantitative variables. *Psychological Methods, 7,* 19–40.

Mendoza, J. L., & Mumford, M. (1987). Correction for attenuation and range restriction on the predictor. *Journal of Educational Statistics, 12,* 282–293.

Mendoza, J. L., Stafford, K. L., & Stauffer, J. M. (2000). Large-sample confidence intervals for validity and reliability coefficients. *Psychological Methods, 5,* 356–369.

National Research Council. (1992). *Combining information: Statistical issues and opportunities for research.* Washington, DC: National Academy of Sciences Press.

Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric theory* (3rd ed.). New York: McGraw-Hill.

Pearson, K. (1903). Mathematical contributions to the theory of evolution: XI. On the influence of natural selection on the variability and correlation of organs. *Royal Society Philosophical Transactions, Series A, 200,* 1–66.

Raju, N. S., & Burke, M. J. (1983). Two new procedures for studying validity generalization. *Journal of Applied Psychology, 68,* 382–395.

Raju, N. S., Burke, M. J., Normand, J., & Langlois, G. M. (1991). A meta-analysis approach. *Journal of Applied Psychology, 76,* 432–446.

Ree, M. J., Caretta, T. R., Earles, J. A., & Albert, W. (1994). Sign changes when correcting for range restriction: A note on Pearson's and Lawley's selection formulas. *Journal of Applied Psychology, 79,* 298–301.

Roth, P. L., Bobko, P., Switzer, F. S., III, & Dean, M. A. (2001). Prior selection causes biased estimates of standardized ethnic group differences: Simulation and analysis. *Personnel Psychology, 54,* 591–617.

Sackett, P. R., & Ostgaard, D. J. (1994). Job-specific applicant pool and national norms for cognitive ability tests: Implications for range restriction corrections in validation research. *Journal of Applied Psychology, 79,* 680–684.

Sackett, P. R., & Yang, H. (2000). Correction for range restriction: An expanded typology. *Journal of Applied Psychology, 85,* 112–118.
Schmidt, F. L., Gast-Rosenberg, I. F., & Hunter, J. E. (1980). Validity generalization results for computer programmers. *Journal of Applied Psychology, 65,* 643–661.
Schmidt, F. L., & Hunter, J. E. (1977). Development of a general solution to the problem of validity generalization. *Journal of Applied Psychology, 62,* 529–540.
Schmidt, F. L., & Hunter, J. E. (1996). Measurement error in psychological research: Lessons from 26 research scenarios. *Psychological Methods, 1,* 199–223.
Schmidt, F. L., & Hunter, J. E. (1998). The validity and utility of selection methods in personnel psychology: Practical and theoretical implications of 85 years of research findings. *Psychological Bulletin, 124,* 262–274.
Schmidt, F. L., & Hunter, J. E. (1999). Theory testing and measurement error. *Intelligence, 27,* 183–198.
Schmidt, F. L., & Hunter, J. E. (2003). History, development, evolution, and impact of validity generalization and meta-analysis methods, 1975–2001. In K. R. Murphy (Ed.), *Validity generalization: A critical review* (pp. 31–66). Mahwah, NJ: Erlbaum.
Schmidt, F. L., Hunter, J. E., McKenzie, R. C., & Muldrow, T. W. (1979). The impact of valid selection procedures on work force productivity. *Journal of Applied Psychology, 64,* 609–626.
Schmidt, F. L., Hunter, J. E., & Pearlman, K. (1981). Task differences and the validity of aptitude tests in selection: A red herring. *Journal of Applied Psychology, 66,* 166–185.
Schmidt, F. L., Hunter, J. E., Pearlman, K., & Hirsh, H. R. (1985). Forty questions about validity generalization and meta-analysis (with commentary by P. R. Sackett, M. L. Tenopyr, N. Schmitt, J. Kehoe, & S. Zedeck). *Personnel Psychology, 38,* 697–798.
Schmidt, F. L., Hunter, J. E., & Urry, V. W. (1976). Statistical power in criterion-related validity studies. *Journal of Applied Psychology, 61,* 473–485.
Schmidt, F. L., Law, K., Hunter, J. E., Rothstein, H. R., Pearlman, K., & McDaniel, M. (1993). Refinements in validity generalization procedures: Implications for the situational specificity hypothesis. *Journal of Applied Psychology, 78,* 3–13.
Schmidt, F. L., & Le, H. (2004). *Software for the Hunter–Schmidt meta-analysis methods.* Iowa City: Department of Management and Organizations, University of Iowa.
Schmidt, F. L., Le, H., & Ilies, R. (2003). Beyond alpha: An empirical examination of the effects of different sources of measurement error on reliability estimates for measures of individual differences constructs. *Psychological Methods, 8,* 206–224.
Schmidt, F. L., Oh, I., & Hayes, T. (2006). *Fixed versus random effects meta-analyses models in meta-analysis: An empirical investigation of differences in results in the psychological literature.* Manuscript submitted for publication.
Schmidt, F. L., Oh, I., & Le, H. (2006). Increasing the accuracy of corrections for range restriction: Implications for selection procedure validities and other research results. *Personnel Psychology, 59,* 281–305.
Stauffer, J. M., & Mendoza, J. L. (2001). The proper sequence for correction correlation coefficients for range restriction and unreliability. *Psychometrika, 66,* 63–68.
Thorndike, R. L. (1949). *Personnel selection.* New York: Wiley.
Viswesvaran, C., Ones, D. S., & Schmidt, F. L. (1996). Comparative analysis of the reliability of job performance ratings. *Journal of Applied Psychology, 81,* 557–560.

Appendix A

Demonstration That True Scores and Measurement Errors Are Correlated Under Direct Range Restriction

For simplicity, we consider an applicant reliability of .50. This makes the measurement error variance (V_e) the same size as the true score variance (V_T). That is, if we use the classical equation, $X = T + e$, for notation, then $V_e = V_T$. Accordingly, we have the following:

$$V_X = V_T + V_e = 2V_T = 2V_e.$$

Suppose we set the selection cutoff score at the median of the applicant population. That is, suppose we promote the top half of the applicants. If we delete the bottom half of a normal distribution, then the standard deviation in the restricted group (top half) is only 60% as large as the original standard deviation (Schmidt et al., 1976). That is, if the applicant standard deviation is 10 ($SD = 10$), then the incumbent standard deviation would be 6 ($SD = 6$). The variance is smaller by the square of .60 or .36. For example, if the applicant variance is 100, then the incumbent variance is 36.

At this point, the mathematics is made easier if we introduce a new variable that has no substantive meaning but is mathematically convenient. Denote it by B. We then compute its variance and its correlation with X:

$$B = T - e$$

Applicant population, $V_B = V_T + V_e$ and $r_{BX} = 0$.
Because the reliability is .50, the new variable B is uncorrelated with the old variable X.[A1] In the applicant population, both variables are normally distributed and independent (because they are uncorrelated).

Consider direct range restriction on X. B is independent of X. Thus, when we select on X, there will be no selection on B. Thus, in the incumbent population, the variance of B will be unchanged. We next use a procedure that allows the computation of the correlation between T and e in the incumbent population. Note that in our example, V_T and V_e are equal before selection, so selection operates symmetrically on them. That means that the two variances will still be equal after selection. In the incumbent population, we have the following:

$$V_e = V_T$$

$$V_X = V_T + V_e + 2\text{cov}_{Te} = 36$$

$$V_B = V_T + V_e - 2\text{cov}_{Te} = 100$$

Add variances:

$$V_X + V_B = 2V_T + 2V_e = 36 + 100 = 136$$

$$V_T + V_e = (1/2)2(V_T + V_e) = (1/2)136 = 68$$

[A1] This can be shown as follows:

$$r_{BX} = r_{(T+e)(T-e)} = \frac{\text{cov}_{TT} - \text{cov}_{ee}}{2(S_T^2 + S_e^2)} = \frac{S_T^2 - S_e^2}{2(S_T^2 + S_e^2)} = \frac{0}{2(S_T^2 + S_e^2)} = 0.$$

(*Appendixes continue*)

Because $V_e = V_T$ (above), we have $V_T + V_T = V_e + V_e = 68$

$$V_T = V_e = 68/2 = 34$$

Subtract variances:

$$V_X - V_B = 4\text{cov}_{Te} = 36 - 100 = -64$$

$$\text{cov}_{Te} = -16$$

Compute r_{Te}:

$$r_{Te} = \text{cov}_{Te}/(SD_T SD_e) = -16/(\sqrt{34}\sqrt{34}) = -16/34 = -.47.$$

To summarize the example, in the whole applicant population, the measurement errors are uncorrelated with true scores, but in the selected subpopulation of incumbents, the true scores and errors are correlated –.47. All procedures for estimating reliability assume that $r_{Te} = 0$. Hence, it is a problem to estimate or even define reliability in this context (Mendoza & Mumford, 1987).

Appendix B

Estimating Range Restriction in the Dependent Variable From Range Restriction on Its Independent Variable

We demonstrate the derivations of formulas to estimate the effect of range restriction on an independent variable X (u_X) on range restriction of a dependent variable Y (u_Y), using correlation between the variables either from the unrestricted population or from the restricted population.

The notation follows:

$$\rho = \rho_{XY} = \text{correlation between } X \text{ and } Y;$$

Subscript i = values estimated in the restricted population (e.g., ρ_i = correlation between X and Y in the restricted population); subscript a = values estimated in the unrestricted population (e.g., ρ_a = correlation between X and Y in the unrestricted population); S_{X_i} and S_{X_a} = standard deviations of variable X in the restricted population and unrestricted population, respectively; S_{Y_i} and S_{Y_a} = standard deviations of variable Y in the restricted population and unrestricted population, respectively. We have the following:

$$u_X = S_{X_i}/S_{X_a} = \text{range restriction in } X; \quad \text{(B1)}$$

$$u_Y = S_{Y_i}/S_{Y_a} = \text{range restriction in } Y. \quad \text{(B2)}$$

The two basic assumptions made by Pearson and Thorndike in deriving the formulas to correct for range restriction are: (a) the regression weight (for predicting/explaining dependent variable from independent variable) in the restricted population is the same as the regression weight in the unrestricted population, and (b) the standard error of estimate in the restricted population is the same as the standard error of estimate in the unrestricted population (Mendoza & Mumford, 1987). The first assumption can be demonstrated as follows:

$$\rho_i S_{Y_i}/S_{X_i} = \rho_a S_{Y_a}/S_{X_a}. \quad \text{(B3)}$$

The second assumption can be demonstrated as follows:

$$S_{Y_i}(1 - \rho_i^2)^{1/2} = S_{Y_a}(1 - \rho_a^2)^{1/2} \quad \text{(B4)}$$

Multiply both sides of Equation B3 for S_{X_i}/S_{Y_a} and simplify:

$$\rho_i S_{Y_i}/S_{Y_a} = \rho_a S_{X_i}/S_{X_a}. \quad \text{(B5)}$$

From Equations B1, B2, and B5, we have the following:

$$\rho_i u_Y = \rho_a u_X. \quad \text{(B6)}$$

Square both sides of Equation B6 and solve for ρ_i^2, and we have the following:

$$\rho_i^2 = \rho_a^2 u_X^2/u_Y^2. \quad \text{(B6}')$$

Divide both sides of Equation B4 by S_{Y_a}:

$$(S_{Y_i}/S_{Y_a})(1 - \rho_i^2)^{1/2} = (1 - \rho_a^2)^{1/2}. \quad \text{(B7)}$$

From Equations B2 and B7, we have the following:

$$u_Y(1 - \rho_i^2)^{1/2} = (1 - \rho_a^2)^{1/2}. \quad \text{(B8)}$$

Square both sides of Equation B8, and then solve it for u_Y^2:

$$u_Y^2 = (1 - \rho_a^2)/(1 - \rho_i^2). \quad \text{(B9)}$$

Equations B6′ and B9 above now can be used to derive the formulas of interest. To estimate u_Y from u_X and ρ_a, complete the following:

Replace Equation B6′ into Equation B9:

$$u_Y^2 = (1 - \rho_a^2)/(1 - \rho_a^2 u_X^2/u_Y^2). \quad \text{(B10)}$$

Expand and simplify Equation B10:

$$u_Y^2 - \rho_a^2 u_X^2 = 1 - \rho_a^2. \quad \text{(B11)}$$

Solving Equation B11 for u_Y^2, we have the formula of interest:

$$u_Y^2 = \rho_a^2 u_X^2 - \rho_a^2 + 1. \quad \text{(B12)}$$

To estimate u_Y from u_X and ρ_i, first, solve Equation B6′ for ρ_a^2:

$$\rho_a^2 = \rho_i^2 u_Y^2/u_X^2. \quad \text{(B13)}$$

Then replace Equation B13 into Equation B9:

$$u_Y^2 = (1 - \rho_i^2 u_Y^2/u_X^2)/(1 - \rho_i^2). \quad \text{(B14)}$$

Expand Equation B14:

$$u_X^2 u_Y^2 - u_X^2 u_Y^2 \rho_i^2 = u_X^2 - \rho_i^2 u_Y^2. \quad \text{(B15)}$$

Rearrange the components of Equation B15:

$$u_X^2 u_Y^2 - u_X^2 u_Y^2 \rho_i^2 + \rho_i^2 u_Y^2 = u_X^2. \quad \text{(B16)}$$

Solving B16 for u_Y^2, we have the formula of interest:

$$u_Y^2 = u_X^2/(u_X^2 - u_X^2 \rho_i^2 + \rho_i^2). \quad \text{(B17)}$$

Equivalence of Equations Estimating Restricted Reliability From Unrestricted Reliability

In this part of Appendix B, we show that the two equations introduced in the text to estimate restricted reliability from unrestricted reliability (Equations 26 and 27) are in fact equivalent. To do that, we demonstrate that Equation 27 can be derived from Equation 26 through algebraic transformation. The demonstration makes use of Equation B13 above. It can be seen that when Equation B13 is applied to the current situation, ρ_a^2 and ρ_i^2 represent the unrestricted r_{XX_a} and restricted reliability r_{XX_i}, respectively. Similarly, the squared range restriction ratios u_Y^2 and u_X^2 reflect squared range restriction on observed score (u_X^2) and true score (u_T^2), respectively. Equation B13 can then be rewritten as follows:

$$r_{XX_a} = r_{XX_i} u_X^2 / u_T^2. \tag{B18}$$

Solving B18 for u_T^2, we have the following:

$$u_T^2 = u_X^2 r_{XX_i} / r_{XX_a}. \tag{B19}$$

Replacing u_T^2 in Equation B19 above into Equation 26 in the text, we have the following:

$$r_{XX_i} = \frac{u_T^2 r_{XX_a}}{u_T^2 r_{XX_a} + 1 - r_{XX_a}} = \frac{(u_X^2 r_{XX_i}/r_{XX_a}) r_{XX_a}}{(u_X^2 r_{XX_i}/r_{XX_a}) r_{XX_a} + 1 - r_{XX_a}}$$

$$= \frac{u_X^2 r_{XX_i}}{u_X^2 r_{XX_i} + 1 - r_{XX_a}}. \tag{B20}$$

Equation B20 can be simplified by multiplying both sides by $\frac{u_X^2 r_{XX_i} + 1 - r_{XX_a}}{r_{XX_i}}$:

$$u_X^2 r_{XX_i} + 1 - r_{XX_a} = u_X^2. \tag{B21}$$

Solving Equation B21 for r_{XX_i}, we obtain Equation 27 in the text:

$$r_{XX_i} = 1 - \frac{1 - r_{XX_a}}{u_X^2} = 1 - U_X^2(1 - r_{XX_a}). \tag{B22}$$

Appendix C

Development of the Taylor's Series: Approximation Approach

This approach is based on Equation 31 in the text, which shows the observed correlation ρ_{XY_i} as the function of the following: (a) the square root of the unrestricted independent variable reliability ($q_{X_a} = \sqrt{r_{XX_a}}$), (b) the square root of the restricted dependent variable reliability ($q_{Y_i} = \sqrt{r_{YY_i}}$), (c) range restriction on true score $T(u_T)$, and (d) the true score correlation in the unrestricted population ρ_{TP_a}:

$$\rho_{XY_i} = q_{Y_i} \frac{u_T q_{X_a}}{\sqrt{u_T^2 q_{X_a}^2 + 1 - q_{X_a}^2}} \frac{u_T \rho_{TP_a}}{\sqrt{u_T^2 \rho_{TP_a}^2 + 1 - \rho_{TP_a}^2}}. \tag{C1}$$

As shown in the text, variance of the true score correlation ρ_{TP_a} can be estimated by the following equation:

$$\mathrm{var}_{\rho_{TP_a}} = [\mathrm{var}_{\rho_{XY_i}} - (b_1^2 \mathrm{var}_{q_{X_a}} + b_2^2 \mathrm{var}_{q_{Y_i}} + b_3^2 \mathrm{var}_{u_T} + \mathrm{var}_e)]/b_4^2, \tag{C2}$$

with b_1 = first order partial derivative of ρ_{XY_i} with respect to q_{X_a}; b_2 = first order partial derivative of ρ_{XY_i} with respect to q_{Y_i}; b_3 = first order partial derivative of ρ_{XY_i} with respect to u_T; b_4 = first order partial derivative of ρ_{XY_i} with respect to ρ_{TP_a}.

To simplify the presentation, the following notation is used:

$$P = \rho_{XY_i};$$

$$X = q_{X_a};$$

$$Y = q_{Y_i};$$

$$U = u_T;$$

$$\rho = \rho_{TP_a};$$

$$A = \frac{1}{\sqrt{u_T^2 \rho_{TP_a}^2 - \rho_{TP_a}^2 + 1}};$$

$$B = \frac{1}{\sqrt{u_T^2 q_{X_a}^2 - q_{X_a}^2 + 1}}.$$

Equation C1 above can be rewritten as follows:

$$P = XY\rho U^2 AB. \tag{C3}$$

The derivatives in Equation C2 can then be estimated from Equation C3 as follows.

Estimating b_1

The coefficient b_1 is the first order partial derivative of P with respect to X:

$$b_1 = dP/dX = Y\rho U^2 A[d(XB)/dX] = Y\rho U^2 A \frac{d\left(\frac{X}{\sqrt{U^2X^2 - X^2 + 1}}\right)}{dX}. \tag{C4}$$

Applying the formula for derivatives of the product to Equation C4, we have the following:

$$b_1 = Y\rho U^2 A \left\{ \frac{1}{\sqrt{U^2X^2 - X^2 + 1}} + X[2X(U^2 - 1)]\left[\frac{-1}{2\sqrt{(U^2X^2 - X^2 + 1)^3}}\right]\right\}. \tag{C5}$$

Simplifying and rearranging Equation C5, we have the needed formula to estimate b_1:

$$b_1 = Y\rho U^2 A[B - X^2(U^2 - 1)B^3] = \frac{P}{X} - PXB^2(U^2 - 1). \tag{C6}$$

Estimating b_2

As mentioned above, b_2 is first order partial derivative of P with respect to Y:

$$b_2 = dP/dY = X\rho U^2 AB = P/Y. \tag{C7}$$

(*Appendixes continue*)

Estimating b_3

The coefficient b_3 is the first order partial derivative of P with respect to U:

$$b_3 = dP/dU = XY\rho\left[\frac{d\left(\frac{U^2}{\sqrt{U^2X^2 - X^2 + 1}\,\sqrt{U^2\rho^2 - \rho^2 + 1}}\right)}{dU}\right]. \quad \text{(C8)}$$

Applying formula for derivatives of the product to Equation C8, we have the following:

$$b_3 = XY\rho\left\{2UAB + U^2A(2UX^2)\left[\frac{-1}{2\sqrt{(U^2X^2 - X^2 + 1)^3}}\right] + U^2B(2U\rho^2)\left[\frac{-1}{2\sqrt{(U^2\rho^2 - \rho^2 + 1)^3}}\right]\right\}. \quad \text{(C9)}$$

Simplifying and rearranging Equation C9, we have the needed formula to estimate b_3:

$$b_3 = XY\rho(2UAB - U^3X^2AB^3 - U^3\rho^2A^3B) = \frac{2P}{U} - PUX^2B^2 - PU\rho^2A^2. \quad \text{(C10)}$$

Estimating b_4

The coefficient b_4 is the first order partial derivative of P with respect to ρ:

$$b_4 = dP/d\rho = XYU^2B[d(XA)/d\rho] = XYU^2B\,\frac{d\left(\frac{\rho}{\sqrt{U^2\rho^2 - \rho^2 + 1}}\right)}{d\rho}. \quad \text{(C11)}$$

Applying formula for derivatives of the product to Equation C11, we have the following:

$$b_4 = XYU^2B\left\{\frac{1}{\sqrt{U^2\rho^2 - \rho^2 + 1}} + \rho[2\rho(U^2 - 1)]\left[\frac{-1}{2\sqrt{(U^2\rho^2 - \rho^2 + 1)^3}}\right]\right\}. \quad \text{(C12)}$$

Simplifying and rearranging Equation C12, we have the needed formula to estimate b_4:

$$b_4 = XYU^2B[A - \rho^2(U^2 - 1)B^3] = \frac{P}{\rho} - P\rho A^2(U^2 - 1). \quad \text{(C13)}$$

The variance of true score correlation can then be estimated by Equation C2. The values of b_1, b_2, b_3, and b_4 in Equation C2 are estimated by replacing the means of X, Y, U, and ρ in their respective distributions into Equations C6, C7, C10, and C13.

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