Executive stock and option valuation in a two state-variable framework:
Allowing optimal investment of outside wealth in the riskfree asset and the market portfolio

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Abstract
We provide a two state-variable discrete-time executive option valuation model that allows optimal investment of executive’s outside wealth in the riskfree asset and the market portfolio. Our model adopts Rubinstein (1994) rainbow option pricing grid and leads to several improvements over the existing one state-variable models that allow outside investment only in the riskfree asset. First, our model is consistent with portfolio theory and the capital asset pricing model. Second, it produces executive option values that are always lower than risk-neutral values. Third, it implies less subjective values as it gives significantly lower sensitivities of option values to the unobservable expected market risk premium, executive’s risk aversion, and executive’s diversification level. Fourth, we show that for common parameter values the executive values in our model are significantly lower than in the one state-variable model, but the company costs are not much lower. This suggests that the divergence between the executive value and the company cost is bigger than previously documented. Fifth, we show that our methodology can be easily applied to the valuation of American-style indexed strike price executive options.

Keywords: Executive stock options; executive compensation; restricted stock; option valuation.

JEL classifications: G11; G13; J33; J44; M52.

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1. Introduction

In recent years stock options have become the most significant part of compensation to senior executives and a significant part of compensation to other employees in U.S. firms. Using Compustat’s ExecuComp data, we estimate that new stock options granted by all listed firms during 2002 had a Black-Scholes value of $98 billion. This represented 1.2% of the year-end market value of these firms. While restricted stock has thus far been a smaller part of compensation than stock options, it is gaining in popularity, and together these two forms of compensation account for a significant part of new wealth created by U.S. firms. It is therefore not surprising that determining the executive value and the company cost of restricted stock and option grants has been a topic of considerable interest to the finance and accounting professions. This paper illustrates some important limitations of existing models and presents a new model that overcomes these limitations while being consistent with traditional portfolio theory.

Executive value of stock options and restricted stock is generally believed to be lower than their value to a diversified investor or shareholder. The diversified investor can hedge the option risk, which makes her option value equal the American option value derived in the risk-neutral Black-Scholes framework. She can also buy or sell stock openly in the market, which makes her stock value equal to its market price. However, the executive can neither hedge nor sell, which makes him undiversified, especially since he may hold additional amounts of company stock. His personal value is therefore lower, and it is usually determined by utility-based models that ask what minimum dollar amount would make him give up the stock options or the restricted stock.

Lambert, Larcker, and Verrecchia (1991), Huddart (1994), Kulatilaka and Marcus (1994), Carpenter (1998), and Hall and Murphy (2002) present versions of utility-based models that share many common characteristics. We refer to these models as the one state-variable models (henceforth the S1 models) as they consider the stock price as the only state-variable in determining executive values. We present some results and limitations of S1 models based on the Hall and Murphy paper as it may provide
the best current state of the art in S1 models. They assume that the executive has a power utility function and invests his outside wealth in the riskfree asset. Assuming further that the stock returns are lognormally distributed, they determine the certainty equivalent amount that would make the executive give up his options. Hall and Murphy illustrate several results with their derived option values; such as, why executives often argue that Black-Scholes values are too high, how this divergence is related to risk aversion and lack of diversification, why executives exercise options earlier than maturity, why virtually all options are granted at the money, why companies often re-set the exercise prices of underwater or out-of-the-money options, and why in absence of strong incentive effects stock options (and to a lesser extent restricted stock) are an inefficient form of executive compensation.

While the S1 model does an excellent job of explaining the direction of above results, the derived executive option values suffer from important limitations. First, we find that the executive option values are often too high, exceeding the option values derived using the traditional risk-neutral model (henceforth the RN model). For example, an executive holding $3.75 million of outside wealth, $1.25 million of company stock, and 10,000 at-the-money stock options assigns an S1 value of $17.05 to each option, which exceeds the RN value of $16.55 (other parameters: risk-aversion coefficient 2.0, stock price $30.00, stock volatility 30%, dividend yield 0%, time to maturity 10 years, European style exercise, riskfree rate 6%, stock beta 1.00, and market risk premium 6.5%).1 This finding is inconsistent with the traditional portfolio theory in which an individual not possessing superior information assigns a lower value than market value to any non-negligible stock holding in excess of its market weight. This overvaluation in S1 model arises because an investor constrained to hold only the riskfree asset with outside wealth overvalues any marginal holding of a risky asset with an expected return higher than the riskfree return.

As a second limitation of S1 models, we find that the derived executive option values are highly sensitive to assumptions of rmrf, or the expected market risk premium, which determines the expected stock returns. Using the same parameters as above, except $1.70 million of outside wealth and $3.30 million of company stock, an rmrf value of 3% gives an option value of $4.57 while an rmrf value of 8%

1 The calculations in this and the following paragraph are based on a spreadsheet provided on Professor Kevin Murphy’s web-site. We set his step size to 0.10.
gives an option value $8.83. This result is in sharp contrast with the RN model that gives an option value of $16.55 regardless of \( \text{rmrf} \) value. It also makes the empirical implementation of S1 model difficult in situations where the researcher must estimate the option values for hundreds of executives over several years with possible heterogeneous and time-varying beliefs concerning the unobservable \( \text{rmrf} \) value.

We argue that both the overvaluation of stock options (at least for not so undiversified executives and employees) and the high \( \text{rmrf} \) sensitivity of option prices reflect a common cause, which constitutes a third limitation of all S1 models. These models assume that an executive invests all his outside wealth in the riskfree asset. This assumption is inconsistent with portfolio theory, which implies that investors with finite risk-aversion should hold combinations of the riskfree asset and the market portfolio. Allowing optimal investments of outside wealth in the market portfolio helps in the following ways. First, starting from the optimal combination of only the riskfree asset and the market portfolio, an executive values a small incremental investment in any risky asset at its market value, and increasing investments in the same asset at less than its market value. Alternately stated, a nearly fully diversified executive values a stock option at nearly its risk-neutral value, and restricted stock at nearly its market value, while an undiversified executive attaches strictly lower values. In either case there is no violation of what clearly must be the upper bound on executive value. Second, the high \( \text{rmrf} \) sensitivity of S1 option values arises because an increase in \( \text{rmrf} \) increases the attractiveness of stock and options in this model, but has no effect on the attractiveness of outside wealth invested only in the riskfree asset. Now consider the alternate situation where the executive invests a proportion of his outside wealth in the market portfolio and is also free to choose the optimal proportion. An increase in \( \text{rmrf} \) in this case simultaneously increases the attractiveness of outside investments, first because the expected market return increases, and second because the proportion of outside wealth invested in the market portfolio increases. The combined result is a sharp reduction in the \( \text{rmrf} \) sensitivity of executive’s stock and option values.

Allowing market investment with outside wealth poses some challenge as it introduces two state variables in the valuation problem. Fortunately, Rubinstein (1994) provides a rainbow option pricing model that can be used to value contingent claims on two jointly lognormal state variables. We adopt and modify Rubinstein’s model to build a three-dimensional pyramidal grid and show that it can be used to
value all European and American style options within portfolios that are invested in the riskfree asset, the market portfolio, the company stock, and the stock options. Our model accommodates all complications related to the executive risk aversion, vesting period, and early exercise. In addition, we choose optimal proportions of the riskfree asset and the market portfolio with outside wealth by using a numerical search process. The resulting two state-variable executive option pricing model (henceforth the S2 model) corrects many limitations of previous executive option pricing models discussed above. We leave the full model and implementation details and the numerical results to later sections. Here we summarize our main contributions as follows:

1. We provide an executive option pricing model that is consistent with the portfolio theory and the capital asset pricing model (henceforth CAPM).\(^2\) It allows optimal investments in the riskfree asset and the market portfolio, and values forced investments in company stock and options at less than their market value determined by diversified and therefore risk-neutral investors. The one state-variable S1 model is a special case of the two state-variable S2 model when the proportion of market investment with outside wealth is exogenously set to zero. Restricted stock is also a special case of the S2 model when the option strike price is set to zero or an arbitrarily low value.

2. We show that the S2 values of stock options and restricted stock are uniformly and significantly lower than the S1 values for the same set of parameters (the executive’s risk aversion, the proportion of his non-option wealth invested in company stock, and the stock and market return distributions).

3. Although the S2 option values are significantly lower than the S1 option values, the average time to exercise is not much lower. Early exercise reduces the company cost, which is computed as what a

\(^2\) Recently some other papers also consider outside investments in the market portfolio, but their results do not apply to the valuation of a substantial block of American-style executive stock options with vesting restrictions. First, Kahl, Liu, and Longstaff (2003) study the optimal investment and consumption choices of executives who own illiquid or restricted stock besides the riskfree asset and the market portfolio in a continuous time framework. However, they do not consider options. Second, Hall and Knox (2002) analyze the incentive effects of underwater options and Holland and Elder (2003) analyze a financing explanation for options. Both consider European-style options and allow the same outside investments. Third, Ingersoll (2002) provides closed-form solutions for the general executive stock option values by using a pricing kernel approach. He presents a model in which the executive holds a combination of the company stock and optimal proportions of the riskfree asset and the market portfolio. By solving the optimization problem, he obtains the undiversified executive’s subjective values of the riskfree rate, the stock price, and the stock return. These subjective parameter values can be used to price options using standard Black-Scholes and binomial model techniques. However, as Ingersoll points out, the resulting option values are valid only for a marginal (or small) option holding. Unlike our model, his methodology does not apply to executives who hold substantial numbers of options.
diversified and risk-neutral investor would pay for the same option. Thus, for the same parameters, the S2 model shows a greater divergence between the company cost and the executive value of both stock options and restricted stock. This greater divergence requires that stock options and restricted stock must provide stronger incentive effects to executives to increase the firm value.

4. In most cases we show that the rmrf sensitivity of the S2 option value is a small fraction of the rmrf sensitivity of the S1 option value. The S2 option values are also less sensitive to the two remaining unobservable parameters, which are the executive’s risk aversion and the proportion of his non-option wealth invested in the company stock. The lower sensitivities lead to less subjective estimates of executive value. This helps the company accountants, the regulators, and the empirical researchers.

5. We show that the S2 model can be easily extended to compute the executive value and the company cost of American style options whose strike price is adjusted for changes in the market level according to any previously set formula. Such indexed strike price options are currently not very common, but their popularity may increase if there is shareholder activism to reward executives for their performance net of the market performance.³

The remaining paper proceeds as follows. Section 2 presents the model and the implementation procedure. Section 3 presents the valuation results, Section 4 presents the company cost results, and Section 5 presents the valuation of indexed strike price options. Section 6 discusses the pros and cons of our utility-based approach to estimating company cost of options versus the historical data approach recommended by the Financial Accounting Standards Board (FASB). Section 7 concludes.

2. Methodology

The traditional risk-neutral option pricing theory assumes that an investor can hedge the risk of a call option by short-selling a certain amount of the underlying stock. Based on this assumption, the RN value of a call option equals the present value of its expected cash flow under the risk-neutral distribution. To an outside investor who faces no constraints in short-selling the stock, the option value equals the RN value. However, the RN value overstates the option value to an undiversified risk-averse executive who

³ Johnson and Tian (2000) also provide a model of pricing indexed strike price options. However, they use risk-neutral parameters and allow only European style exercise.
cannot sell his options or short-sell the stock of his company. The executive value must take into consideration his unique portfolio of option and non-option wealth, his utility function, and the actual stock return distribution as follows.\(^4\)

2.1. The two state-variable model (the S2 model)

We estimate the executive value of options as the amount of outside wealth that gives him the same expected utility as the options. The executive has non-option wealth of \(W_0\), of which \(\alpha\) proportion is invested in the company stock and the rest is invested in outside wealth. He further optimally invests \(p\) proportion of his outside wealth in the market portfolio and the rest in the riskfree asset.\(^5\) When the options mature in \(T\) years, assuming no early exercise, his total wealth will equal

\[
W_T = W_0 \left[ \alpha e^{\delta T} \frac{S_T}{S_0} + (1 - \alpha) \left( p \frac{M_T}{M_0} + (1 - p)R_f^T \right) \right] + N_{opt} \text{Max}(S_T - X, 0). \tag{1}
\]

Here \(S_0\) is the current stock price, \(S_T\) is the stock price at year \(T\), \(M_0\) is the current market level, \(M_T\) is the market level at year \(T\), \(\delta\) is the continuous dividend yield on stock, \(R_f\) is the annual riskfree rate, \(N_{opt}\) is the number of options the executive holds, and \(X\) is the exercise price of options. Note that \(S_T\) excludes dividends paid between now and year \(T\). We assume the executive reinvests all dividends in the stock, thus ending up with stock wealth of \(e^{\delta T} W_0 \alpha S_T/S_0\) at year \(T\). The executive invests the market portfolio through an index fund that reinvests all dividend proceeds. Therefore, \(M_T\) already includes all dividends paid between today and year \(T\) (which is just a matter of notational convenience).

Executive options are generally exercisable after an initial vesting period. If the executive exercises his options at any time \(t < T\), we assume he invests the exercise proceeds in the riskfree asset.

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\(^4\) Bettis, Bizjak, and Lemmon (2001) suggest that executives sometimes hedge their option risk by entering into zero-cost collars and equity swaps in the over-the-counter market. However, Hall and Murphy (2002) suggest that such transactions are quite rare. Such transactions would undoubtedly kill the incentive effect of stock options.

\(^5\) We have assumed the traditional portfolio theory framework in which the market portfolio is efficient and all investors hold optimal (i.e., utility-maximizing) combinations of the riskfree asset and the market portfolio. In this framework the executive also invests his outside wealth in an optimal combination of the riskfree asset and the market portfolio. In a more realistic setting with heterogeneous beliefs and information, it is possible that the executive holds other investments (such as real estate, hedge funds, and commodities) that are excluded from the typical proxies for the market portfolio. Intuitively, a wider investment opportunity set may further increase the attractiveness of a dollar of outside wealth relative to a dollar (in market value) of his firm’s stock and options, which in turn may further decrease their executive value relative to the risk-neutral price. However, the modeling of option values with such an expanded investment opportunity set is beyond the scope of this paper.
and the market portfolio according to his optimal portfolio choice \( p \). In this case, the executive’s total wealth at year \( T \) is given by

\[
W_T = W_0 \alpha \frac{S_T}{S_0} e^{\delta T} + p [W_0 (1 - \alpha) \frac{M_t}{M_0} + N_{opt} \max(S_t - X, 0)] \frac{M_T}{M_t} + (1 - p) [W_0 (1 - \alpha) R_f + N_{opt} \max(S_t - X, 0)] R_f^{T-t}.
\] (2)

We define the executive’s utility function on his terminal wealth at year \( T \) as

\[
U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}.
\] (3)

The executive chooses \( p \) to maximize his expected utility.

\[
p^* = \arg \max_p [E(U(W_T))].
\] (4)

In the one state-variable S1 model, the executive can only invest his outside wealth in the risk-free asset. It becomes a special case of our S2 model where \( p \) is exogenously specified as 0.

Utility maximization also determines the executive’s option exercise decision. Once the options are vested, the executive exercises the options whenever the expected utility from exercising the options exceeds the expected utility from holding the options.

We estimate the executive value of an option as the amount of outside wealth equivalent, \( OWE \), that solves

\[
E[U(W_T^{OFE})] = E[U(W_T^{OPT})].
\] (5)

where \( W_T^{OFE} \) is given by

\[
W_T^{OFE} = W_0 \alpha e^{\delta T} \frac{S_T}{S_0} + [W_0 (1 - \alpha) + N_{opt} OWE] (p^* \frac{M_T}{M_0} + (1 - p^*) R_f^T).
\] (6)

\( W_T^{OPT} \) is given by Equation (1) if the options are not exercised prior to maturity, and it is given by Equation (2) if the options are exercised at any time before maturity. Solving Equation (5) numerically

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6 In general, the executive’s portfolio choice \( p \) at any time \( t \) may be different from his portfolio choice today. However, as explained later, we do not allow the executive to update \( p \) over time due to computational constraints.

7 We implicitly assume that the executive maintains a stable ownership of company stock. This is justified on two grounds. First, Ofek and Yermack (2000) show that executives with large ownership of their company stock respond to additional stock received by new stock grants or options exercise by selling equivalent amounts of previously owned stock. Second, many companies offer cashless exercise programs, where the executive pays nothing and receives the spread between the stock price and the exercise price.
for OWE gives the S2 model executive value of a stock option. We can easily extend this model to estimate the executive value of restricted stock by setting the exercise price arbitrarily close to zero.

In concept, our OWE is similar to the certainty equivalent amount used by Hall and Murphy and others. However, we prefer to use the term OWE, since in our case the outside investment opportunity set includes the uncertain market portfolio in addition to the certain riskfree asset.

2.2. The rainbow grid

The S2 model has two state variables: the stock price \( S_t \) and the market portfolio \( M_t \). Following a common practice, we adopt the CAPM to calculate expected stock returns and also assume that in any year \( t \) the log stock returns and the log market returns follow a joint normal distribution\(^9\)

\[
\begin{pmatrix}
\ln(M_t) - \ln(M_{t-1}) \\
\ln(S_t) - \ln(S_{t-1})
\end{pmatrix}
\sim N
\left( 
\begin{pmatrix}
\mu_M \\
\mu_S
\end{pmatrix},
\begin{pmatrix}
\sigma_M^2 & \sigma_{S,M} \\
\sigma_{S,M} & \sigma_S^2
\end{pmatrix}
\right)
\]

\[\mu_M = \ln(R_f + \text{rmrf}) - \frac{1}{2}\sigma_M^2 \]
\[\mu_S = \ln(R_f + \beta \cdot \text{rmrf}) - \delta - \frac{1}{2}\sigma_S^2 . \quad (7)\]

Here \( R_{M,t} \) is the annual market return in year \( t \), \( R_{S,t} \) is the annual stock return, \( \mu_M \) is the expected log market return, \( \mu_S \) is the expected log stock return, \( \sigma_M^2 \) is the variance of the log market return, \( \sigma_S^2 \) is the variance of log stock return, \( \sigma_{S,M} \) is the covariance between the log stock return and the log market return, \( \beta \) is the systematic risk of the stock, and \( \text{rmrf} \) is the executive’s expected annual market risk premium. Note that \( \mu_M \) includes dividend since we assume the executive invests the market portfolio through an index fund that reinvests all dividend proceeds.

To accommodate the two state variables and the early exercise feature of American style options, we adopt and modify the rainbow option pricing model of Rubinstein (1994). His model is designed to value options whose payoffs depend on the realization of two less than perfectly correlated assets.

\(^8\) Once again, a complete optimization would require that the optimal portfolio choice \( p^* \) in Equation (6) be different from the optimal portfolio choice in Equation (4). However, we make them equal due to computational constraints. Test results show that the two can deviate significantly for very large option holdings (as expected), but the resulting option values are not very different (perhaps due to flatness of the expected utility function in proximity of \( p^* \)).

\(^9\) Similar to other executive option pricing models, we also assume that the stock return distribution is independent of whether the executive receives cash or stock options. This assumption can be further justified on grounds that most companies grant options pursuant to some compensation policy in place, so the incentive effects of options may be largely included in the stock prices.
Rubinstein constructs a three-dimensional binomial grid to approximate the movements of the two assets. We use this rainbow grid for our two state variables. The mechanics are best explained in Figure 1. Panel A shows the first-period moves. The first state variable, the market level in our case, moves up by a factor of \( u \) or down by a factor of \( d \) with equal probability. Given that the market moves up, the second state variable, the stock price in our case, can move up by a factor of \( A \) or down by a factor of \( B \) with equal probability. Similarly, given that the market moves down, the stock can move up by a factor of \( C \) or down by a factor of \( D \) with equal probability. Thus, at the end of the first period, there are four nodes, and the probability of reaching any of these four nodes from the previous node is \( 1/4 \). Panel B shows the second-period moves. There are four possible moves from each of the four existing nodes. To make the grid recombine and evolve in a compact fashion, it is necessary to assume that \( AD = BC \). This leads to \( 3^2 = 9 \) nodes at the end of second period. In general, there are \( (t+1)^2 \) nodes after \( t \) periods. The probability of reaching a node with \( i \) up moves in the market portfolio and \( j \) up moves in the stock after \( t \) periods is given by

\[
Pr(t, i, j) = C_i C_j \left( \frac{1}{4} \right)^t \quad \text{where} \quad C_i = \frac{t!}{i!(t-i)!}.
\]

In Rubinstein’s original model, the six parameters \{\( u, d, A, B, C, D \)\} are determined so that the expected returns of the two assets equal the riskfree return net of their individual dividend yield. We modify this condition by setting the expected log annual market return to \( \mu_M \), and the expected log annual stock return to \( \mu_S \). Assuming \( h \) periods per year, and therefore \( h \cdot T \) periods in total, the expected market return per period is \( e^{\mu_M \Delta t} \), and the expected stock return per period is \( e^{\mu_S \Delta t} \), where \( \Delta t = 1/h \) is the time interval of one period. The six parameters are now determined by using the two expected return conditions, two variance conditions, one covariance condition, and one grid recombining condition as follows:

\[
\begin{align*}
    u &= \exp(\mu_M \Delta t + \sigma_M \sqrt{\Delta t}) \\
    d &= \exp(\mu_M \Delta t - \sigma_M \sqrt{\Delta t}) \\
    A &= \exp[\mu_S \Delta t + \sigma_S \sqrt{\Delta t}(\rho + \sqrt{1 - \rho^2})] \\
    B &= \exp[\mu_S \Delta t + \sigma_S \sqrt{\Delta t}(\rho - \sqrt{1 - \rho^2})] \\
    C &= \exp[\mu_S \Delta t - \sigma_S \sqrt{\Delta t}(\rho + \sqrt{1 - \rho^2})] \\
    D &= \exp[\mu_S \Delta t - \sigma_S \sqrt{\Delta t}(\rho - \sqrt{1 - \rho^2})].
\end{align*}
\]
Here $\rho$ is the correlation between the log stock return and the log market return.

2.3. The implementation procedure

The first step in implementation is to find $E(U(W_{T_{OPT}}))$ in Equation (5). This step is straightforward for European style options. We start by constructing a rainbow grid with $h \cdot T$ periods. Next, for each of the $(h \cdot T+1)^2$ terminal nodes, we calculate the terminal wealth $W_T$ by Equation (1), the corresponding utility by Equation (3), and the corresponding probability by Equation (8). Finally, we calculate $E(U(W_{T_{OPT}}))$ as the probability-weighted average utility of all terminal nodes.

For American style options, finding $E(U(W_{T_{OPT}}))$ requires much more computation. We first calculate the utility at each of the $(hT+1)^2$ terminal nodes assuming no early exercise. Then, starting backward from one period before last, we calculate at each node the expected utility from holding the option for one more period and the expected utility from early exercising. The expected utility from holding the option for one more period is the simple average of the expected utilities at each of the four following nodes, since the probability of reaching any following node from the current node is always $1/4$. The expected utility from early exercising is calculated in the following two steps. First, at all terminal nodes attainable from the current node, we calculate the terminal wealth $W_T$ by Equation (2), the corresponding utility, and the corresponding probability. Note that the corresponding probability is conditional on the probability of reaching the current node. Second, we calculate the expected utility from early exercising as the probability-weighted average utility of all attainable terminal nodes. We then compare the expected utility from holding the option for one more period to the expected utility from early exercising, and choose the higher one as the expected utility for this node. We repeat this process at each node until the initial node is reached, and the expected utility at the initial node is $E(U(W_{T_{OPT}}))$ in Equation (5). The overall procedure is similar to evaluating American style options in other contexts, but with one difference. For options with an initial vesting period, we only compare the expected utility from holding the option for one more period to the expected utility from early exercising at any node on or after the vesting date. The expected utility at any node before the vesting date is simply the expected utility from holding the option.
The next step in implementation is to find the optimal \( p \) that maximizes the expected utility. We use a grid search method by calculating the expected utility for 101 values of \( p \) from 0 to 1 in increments of 0.01, and choose the \( p \) that gives the highest expected utility.

The final step in implementation is to solve Equation (5) for \( OWE \). We solve this equation numerically. One point needs special mention. If the option wealth is small relative to the total wealth, the executive’s expected utility changes only slightly when the options are excluded. Therefore, to obtain accurate option values, it is necessary to specify very tight convergence criteria.\(^{10}\)

2.4. Computational issues

For the same number of periods, the rainbow grid used by the S2 model requires far more computation than the traditional one-dimensional grid. For \( N \) periods, the rainbow grid has \((N+1)\cdot(N+2)\cdot(2N+3)/6\) nodes. In addition, evaluating the early exercise decision for a node at period \( n \) requires calculating the expected utility across \((N-n+1)^2\) terminal nodes. As a result, it is computationally infeasible for us to have more than four periods per year for a ten-year option. Fortunately, given the high node-density of rainbow grid, that is enough periods for a good convergence of our results. In fact, results obtained using a grid with one period per year are not much different from the results obtained using a grid with four periods per year.\(^{11}\)

We need to introduce two imperfections in the portfolio optimization step due to computational constraints. First, the executive is not allowed to rebalance his outside portfolio. In an ideal world, the executive would choose the optimal \( p \) at every node of the grid. However, it is computationally infeasible given the large number of nodes in the rainbow grid.\(^{12}\) This imperfection makes the portfolio choice with outside wealth sub-optimal, which lowers the attractiveness of outside wealth. A related imperfection is to assume that the same optimal \( p \) describes the outside portfolio choice when the portfolio includes the options and when it includes the \( N_{opt} OWE \), which also lowers the attractiveness of outside wealth. Both of these imperfections work against us in finding the difference between the S2 executive option value

\(^{10}\) We use the \( nlp \) procedure in SAS to solve Equation (5). The convergence criteria are set to \( 10^{-25} \).

\(^{11}\) We use a grid with one period per year in determining the optimal \( p \). The optimal \( p \) is then used to calculate \( E(U(W_{T opt})) \) in Equation (5) in a grid with four periods per year.

\(^{12}\) For an option with 10 years to maturity, a rainbow grid with four periods per year has 23,821 nodes. Allowing the executive to rebalance at every individual node would lead to \( 10^{10.821} \) possible portfolio allocation strategies.
and the S1 executive option value. This difference would be bigger if the executive were allowed to rebalance his outside portfolio.

Second, the optimal \( p \) value is restricted to lie between 0 and 1, implying that the executive cannot short-sell the riskfree asset or the market portfolio. This restriction is necessary because of the first imperfection. When \( p \) is fixed over time, short-selling the riskfree asset or the market portfolio leads to a non-zero probability of negative terminal wealth, which is not defined under the power utility function. As a practical matter, this restriction does not seem to affect our results for reasonable parameter values. We later show that most optimal \( p \) values fall between 0 and 1 for reasonable parameters (see later Tables 1 and 2). Besides, a \( p \) value outside the range of 0 to 1 may not represent the portfolio choice of a typical executive. Certainly, with positive net supply of both the risky and riskfree assets in the economy, this cannot be the portfolio choice of an average investor.

3. Valuation results

3.1. The S2 model gives lower executive option values than the S1 model

Both the two state-variable S2 model and the one state-variable S1 model measure the executive option value by the amount of outside wealth that gives the executive the same expected utility as options. However, the S2 model allows the executive to optimally invest his outside wealth in a combination of the riskfree asset and the market portfolio, while the S1 model allows him to invest his outside wealth only in the riskfree asset. The investment opportunity set for outside wealth is therefore more attractive in the S2 model than in the S1 model. For any given option holding, it follows that the amount of outside wealth required to generate the same expected utility as options is smaller in the S2 model than in the S1 model. As a result, the S2 executive option value is smaller than the S1 executive option value.

Table 1 reports the simulation results for an executive who has 25,000 options and $5 million in non-option wealth. The options have an exercise price of $30, 10 years to maturity, and 3 years to vesting. The annual riskfree rate equals 5%, the annual standard deviations of stock and market returns equal 40% and 20%, the stock beta equals 1.0, and the annual dividend yield equals 1%. (These parameters are generally kept unchanged in the remaining paper, unless stated otherwise.) Table 1 has four panels that consider different combinations of the expected market risk premium, \( rmrf \), and the executive’s risk-
aversion, $\gamma$. Within panels, we report results for different values of the proportion of non-option wealth invested in company stock, $\alpha$, and the current stock price, $S_0$. Finally, each cell of the table reports the ratio of the S2 value to RN value, the optimal $p$ in the S2 model, and the ratio of the S1 value to RN value. The RN value itself is reported in a separate panel on top of these four panels.

Panel A reports the results for $rmrf = 6\%$ and $\gamma = 2$. It shows that the ratio of S2 to RN value is always less than or equal to the ratio of S1 to RN value. The two ratios are equal only when the optimal $p$ in the S2 model is zero, i.e., when the two models are identical. This happens when nearly all of the executive’s wealth is tied up in the company stock ($\alpha = 0.99$). Intuitively, when the executive is extremely undiversified, he is very averse to any additional risk, and invests all his outside wealth in the riskfree asset. A more diversified executive in the S2 model invests a positive proportion of his outside wealth in the market portfolio, and his S2 value is lower than his S1 value. In other words, the S1 value overstates the executive option value for all but extremely undiversified executives. This result holds for out-of-the-money options, at-the-money options, in-the-money options, and restricted stock.

The difference between the S2 and the S1 executive option values is economically significant. Here we discuss the results for at-the-money options (corresponding to a stock price of $30) held by an executive who has 50 percent of his non-option wealth invested in the company stock. The RN value for this option equals $15.70$. Given $rmrf$ and $\gamma$ values of $6\%$ and 2, Panel A further shows that the S2 value equals $0.457 \times 15.70 = 7.17$ while the S1 value equals $0.550 \times 15.70 = 8.64$. The difference between these two values equals $1.47$, which is 21 percent of the S2 value. This difference increases as $\alpha$ decreases, and it seems to be the highest for options that are not too far from the money.

Panel B reports the results for $rmrf = 6\%$ and $\gamma = 3$. The intuition of Panel A remains unchanged, but the differences between the S2 and the S1 values become narrower than in Panel A. This occurs because a more risk-averse executive invests a smaller proportion of his outside wealth in the market portfolio. For the same benchmark option chosen for illustration above, the S2 and the S1 values equal $0.356 \times 15.70 = 5.59$ and $0.403 \times 15.70 = 6.33$, and the difference equals 13 percent of the S2 value.

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13 Even for extremely undiversified executives, the S2 value would likely be lower than the S1 value if optimal $p$ were not constrained to lie between 0 and 1. However, as explained in Section 2.4, this is computationally infeasible.
Panel C next reports the results for \( rmrf = 8\% \) and \( \gamma = 2 \). A higher market risk premium combined with a moderate risk aversion now makes the S2 value lower than the S1 value in every single case considered in this panel. This occurs because the executive always chooses to invest some (in fact, the majority) proportion of his outside wealth in the market portfolio. For the benchmark option, the S2 and the S1 values equal $0.459 \times 15.70 = $7.21$ and $0.611 \times 15.70 = $9.59$, and the difference equals 33 percent of the S2 value. Finally, Panel D reports the results for \( rmrf = 8\% \) and \( \gamma = 3 \). As before, the differences between the S2 and the S1 values become narrower than in Panel C. For the benchmark option, the S2 and the S1 values now equal $0.356 \times 15.70 = $5.59$ and $0.441 \times 15.7 = $6.92$, and the difference equals 24 percent of the S2 value.

Figure 2 uses a bar chart to show the percentage difference between the S2 and S1 values (calculated as \( 100 \times (S1 - S2) / S2 \)) for selected combinations of the market risk premium, the proportion of non-option wealth invested in company stock, and the current stock price (relative to the fixed exercise price of $30). The difference between the two values is quite negligible when the proportion equals 0.99, but quite significant when the proportion equals 0.50 or 0.01. The combined results of Table 1 and Figure 2 illustrate that the one state-variable S1 model significantly overvalues the executive stock options and restricted stock.

3.2. Violation of the RN value upper bound

Since an executive is restricted from selling or hedging his options, he attaches lower value to the options than an unrestricted outside investor. The option value to an outside investor, or the RN value, imposes an upper bound that any estimated executive option value should not exceed. As noted by Hall and Murphy (2002), one limitation of the S1 model is that this upper bound is sometimes violated.

Table 1 shows that the S1 value exceeds the RN values when the company stock only makes up a small proportion of an executive’s total non-option wealth (e.g., \( \alpha = 0.01 \)). However, the S2 value is always lower than the RN value. Intuitively, this is because an investor who only holds riskfree asset is locally risk neutral. If the expected return of a risky asset is higher than the riskfree rate, she will always prefers $1 in the risky asset to $1 in the riskfree asset. In the S1 model, an executive’s outside wealth is invested in the riskfree asset only. He will also be locally risk neutral if he has little wealth in the
company stock. If the expected return (based on RN value) of an option is higher than the riskfree rate, 
the executive derives higher utility from $1 in the option (of RN value) than from $1 in the riskfree asset. 
Thus, he is willing to trade more than $1 of his outside wealth in riskfree asset for $1 in options (of RN 
value). In the S2 model, an executive’s outside wealth is optimally invested in the riskfree asset and the 
market portfolio. According to the CAPM, any investor’s optimal portfolio consists of only the riskfree 
asset and the market portfolio. Forcing the executive to hold options pushes him away from his optimal 
portfolio. Therefore, the executive always derives higher utility from $1 in the outside wealth than from 
$1 in the options (of RN value). Thus, he is willing to trade $1 in options (of RN value) for less than $1 in 
the outside wealth.

We prove the intuition rigorously in the Appendix in a simple one-period model for restricted 
stock, which is a European style option with zero exercise price. We show that in the one state-variable 
S1 model, as long as the company has positive $\beta$ and $rmrf$ is positive, a sufficiently diversified executive 
will value restricted stock at more than its market price. In the two state-variable S2 model, a less than 
fully diversified executive always values a restricted stock at less than its market price, and a fully 
diversified executive, like an outside investor, values the restricted stock at its market price. Numerical 
simulations in the one-period model show that this result also holds for options, although it is difficult to 
prove it mathematically due to the non-linear payoff structure of options.

In multi-period simulations, it is possible to have an S2 value higher than the RN value if $rmrf$ is 
very high, say 20%. This is caused by the numerical imperfections discussed in Section 2.4.\(^{14}\) When $rmrf$ 
is very high, the executive’s optimal portfolio involves short selling the riskfree asset and investing the 
proceeds in the market. However, our algorithm does not allow short sales, which makes the executive’s 
outside portfolio sub-optimal and less attractive than the stocks and options. As a result, the S2 value can 
be higher than the RN value when $rmrf$ is very high. However, for reasonable parameter values, the S2 
value is always lower than the RN value.

Figure 3 reports the range of $\alpha$, the proportion of non-option wealth invested in company stock, 
over which the S1 value of an option exceeds the RN value for given stock prices and the market risk

\(^{14}\) In single period simulations, we allow the executive to short sell the riskfree asset or the market portfolio to the 
extent that terminal wealth $W_T$ is positive at all four nodes.
premium \textit{rmrf}. Given a 6\% market risk premium, the S1 executive option value exceeds the RN value when \( \alpha \) is lower than 7.4, 6.9, and 5.2 percent for stock prices of $15, $30, and $45. Although not shown in the figure, the threshold levels are higher when the executive holds fewer options or the expected stock return is higher. Given the same market risk premium and a holding of 5,000 options, the corresponding \( \alpha \) thresholds are 9.6, 11.8, and 12.6 percent. If we increase the market risk premium to 8\% and the stock beta to 2.0, the \( \alpha \) thresholds increase to 29.5, 33.5, and 34.1 percent with a holding of 25,000 options.

The combined analysis of Table 1 and Figures 2 and 3 shows that the S1 values are generally too high and may even exceed the RN values for not so undiversified executives. This can be especially problematic when evaluating options held by junior executives and non-executive employees, who face fewer constraints and probably invest only a small proportion of their total wealth in their company’s stock. Notice that, for the same parameter values, the S2 value is never higher than the RN value.

3.3. \textit{The S2 executive value is less sensitive to unobservable parameters than the S1 executive value}

One drawback of all utility-based option valuation models is that the answer depends on several parameters that are only observable to the executive himself, and not to the company accountants, the regulators, the researchers, and the general public. In the present context, these parameters are the executive’s expected market risk premium \textit{rmrf}, the proportion \( \alpha \) of his non-option wealth invested in the company stock, and his risk aversion \( \gamma \). Rubinstein (1995) points out in a related context: “… Many of these variables are difficult to estimate. Indeed, a firm seeking to overvalue its options might report values almost double those reported by an otherwise similar firm seeking to undervalue its options.” The unobservable parameters also make the empirical testing with utility-based models difficult since the result depends on the choice of parameter values. For these reasons, a model less sensitive to the unobservable parameters should be preferred. Below we show that the two state-variable S2 executive option value is less sensitive to each of these unobservable parameters than the one state-variable S1 executive option value.

3.3.1. \textit{Sensitivity to the expected market risk premium \textit{rmrf} (which determines the expected stock return)}

Table 2 shows that the S2 value is much less sensitive to changes in \textit{rmrf} than the S1 value. The illustration assumes that the executive has 25,000 options, $5 million of non-option wealth divided evenly
between the company stock and the outside portfolio, and risk-aversion coefficient of 2.5. When the \( rmref \) increases from 2% to 10%, the ratio of the S2 value to RN value of at-the-money option changes from 0.380 to 0.402, which represents a 5.8 percent increase. In comparison, the ratio of the S1 value to RN value goes up from 0.380 to 0.565, which represents a 48.7 percent increase. The out-of-the-money option, the in-the-money option, and the restricted stock show similar patterns. Figure 4 plots the results of Table 2 and provides a visual comparison between the two models.\(^ {15} \)

The lower \( rmref \) sensitivity of the S2 values is comforting. It compares more favorably with the \( rmref \) sensitivity of the RN model than the S1 model. (Recall that the RN values are totally insensitive to changes in \( rmref \).) The reasons behind this result are as follows. In the S1 model, the executive can only invest his outside wealth in the riskfree asset, thus the attractiveness of outside wealth remains unchanged no matter how much \( rmref \) increases. However, as \( rmref \) increases, the CAPM implies that the expected stock return increases by a factor of \( \beta \). As long as the company has a positive \( \beta \), the expected return increases for both the stock and the option. Therefore, an increase in \( rmref \) increases the attractiveness of the option portfolio relative to the outside wealth. The executive now requires a greater amount of outside wealth to give up his options. This explains why the S1 value is very sensitive to changes in \( rmref \).

In the S2 model, the executive optimally allocates his outside wealth between the riskfree asset and the market portfolio. An increase in \( rmref \) now increases the expected return of the outside wealth in two ways. First, the expected market return increases, which increases the overall expected return of the outside wealth. Second, as the relative attractiveness of the market portfolio to the riskfree asset increases, the executive also invests a higher proportion of his outside wealth \( p \) in the market, which further increases the overall expected return of the outside wealth. This is apparent from Table 2, which shows that the optimal \( p \) increases with \( rmref \). The net result is that an increase in \( rmref \) makes both the option

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\(^ {15} \) Both Figure 3 and Table 2 show that the \( rmref \) sensitivity of S2 values is even lower when \( rmref \) lies between 4% to 10%. For example, the S2 value of at-the-money option changes by less than 1.0 percent over this range. This observation has two implications. First, this may be the more likely range of \( rmref \) values. Note that \( rmref \) is the arithmetic market risk premium, and the geometric market risk premium is lower by \( 0.5\sigma_2^2 = 0.02 \). Thus, an \( rmref \) value of 2% implies a geometric market risk premium of 0%. Second, note that the \( p \) proportion of outside wealth invested in the market portfolio takes the value 0.00 in the S2 model for an \( rmref \) value of 2%. This is the likely result of sub-optimally constraining the \( p \) values to lie between 0 and 1. As an experiment, we relax this constraint in a one-period model, which still does not result in negative wealth in any state. We find that the optimal \( p \) now becomes negative for an \( rmref \) value of 2%, and the overall \( rmref \) sensitivity is even lower.
portfolio and the outside portfolio more attractive, but their relative attractiveness does not change by much. This explains why the S2 model value is much less sensitive to changes in rmrf.

3.3.2. Sensitivity to the executive’s risk aversion $\gamma$

Figure 5 shows that the difference between the S2 value and the S1 value increases as the executive’s risk aversion $\gamma$ decreases, which leads to lower sensitivity of the S2 value to changes in $\gamma$. This is a direct result of the overvaluation of option values implied by the S1 model for low $\gamma$, but not for high $\gamma$. When $\gamma$ is low, an executive in the S2 model optimally invests a large proportion of his outside wealth in the market portfolio, while in the S1 model he can only invest his outside wealth in the riskfree asset. This makes the options too attractive relative to the outside wealth for the executive in the S1 model, leading the model to overestimate the option value. When $\gamma$ is high, even an executive in the S2 model invests almost all his outside wealth in the riskfree asset, similar to what he has to do in the S1 model. This is why the two values converge as $\gamma$ increases.

3.3.3. Sensitivity to the proportion $\alpha$ of non-option wealth invested in the company stock

Figure 6 shows that the difference between the S2 value and the S1 value increases as the proportion $\alpha$ of non-option wealth invested in the company stock decreases, which leads to lower sensitivity of the S2 value to changes in $\alpha$. This is also a direct result of the overvaluation of option values implied by the S1 model for low $\alpha$, but not for high $\alpha$. When $\alpha$ is low, then also an executive in the S2 model optimally invests a large proportion of his outside wealth in the market portfolio, while in the S1 model he can only invest his outside wealth in the riskfree asset. This again makes the options too attractive relative to the outside wealth for the executive in the S1 model, leading the model to overestimate the option value. When $\alpha$ is very low, Section 3.2 in fact showed that the S1 value even crosses the RN value upper bound. When $\alpha$ is high, an executive in the S2 model invests almost all his outside wealth in the riskfree asset, similar to what he has to do in the S1 model. This is why the two values converge as $\alpha$ increases.
4. Early exercise, company cost, and the required incentive effects of option compensation

The company cost of stock option grants is their opportunity cost to diversified and risk-neutral shareholder-investors. This may be quite different from their executive value. The company cost also differs from the RN value derived and discussed throughout this paper. Whereas the RN value represents the option value to risk-neutral investors assuming that these investors optimally exercise the option, the company cost represents the option value to the same investors assuming that the executives optimally exercise the option. Since the executive exercise policy is sub-optimal from the perspective of risk-neutral investors, the company cost must be lower than the RN value. The company cost must, however, be greater than the executive value, since the shareholder-investors are well diversified and require a lower return for bearing the unsystematic risk of stock options.

Kulatilaka and Marcus (1994), Huddart and Lang (1996), Carpenter (1998), Hall and Murphy (2002), and many others discuss the computation of the company cost versus the executive value of stock options. The difference between these two quantities represents the required incentive effects of option compensation.\(^{16}\) If option compensation does not motivate the executives to increase the company value by at least the difference amount, it may be more desirable to offer cash compensation unrelated to stock price performance.\(^{17}\) We now discuss how to compute the expected life and the company cost of executive options using the two state-variable S2 model. We present results to show that the one state-variable S1 model understates the difference between the executive value and the company cost.

4.1. The expected life of options

We use the expected life of options to summarize the early exercise decisions. The expected life is calculated as the probability-weighted average time between the grant date and the exercise date (or the maturity date in case of no exercise).\(^{18}\) Table 3 shows that the expected life in the S2 model is shorter than

\(^{16}\) The difference between the executive value and the company cost also follows from the principal-agent literature. Holmstrom (1979) shows that when a risk-neutral principal deals with a risk-averse agent in the presence of moral hazard, first-best outcomes are impossible, since they would require the principal to bear all the risk, leaving the agent unmotivated. Thus, in equilibrium the value to the agent (the executive) of the compensation will necessarily be lower than that to the principal (the shareholders).

\(^{17}\) Besides the incentive effects to increase firm value, there may be other benefits of option compensation that explain its popularity (for example, tax savings and accounting considerations).

\(^{18}\) We prefer to report “the expected life” instead of “the expected time to exercise” as some options may never be exercised.
in the S1 model. This is because the executive invests the exercise proceeds in his outside portfolio, and a more attractive outside portfolio in the S2 model induces the executive to exercise his options earlier than in the S1 model. The difference increases with the depth in the money (which causes early exercise to begin with), lower $\alpha$ (which causes the S1 model to overvalue the option and makes early exercise less attractive), and higher $rmrf$ (which results in a higher expected stock and option return and makes early exercise followed by investment in the riskfree asset less attractive in the S1 model).

4.2. The company cost and the incentive effects

Hall and Murphy (2002) estimate the company cost of stock options by using a barrier option methodology. We modify this methodology for our model. We start by constructing the stock price thresholds that trigger early exercise. At each period, and for each value of cumulative market return, we find the stock price above which an executive always exercises the option. We next estimate a barrier function at each period by regressing the stock price thresholds on the cumulative market returns.\footnote{In general, the barrier function can be a polynomial expansion of the cumulative market return. However, the second moment tends to capture the threshold oscillation caused by a coarse grid. This hurts the convergence of the company cost calculations. We therefore use a linear barrier function that results in smooth convergence.} We expect a positive regression coefficient for the following reason. If the market return has been high, the executive’s outside wealth is likely to account for a larger proportion of his total wealth. As a result, the executive is likely to be more diversified and more willing to hold on to his options, leading to a higher exercise threshold. The measured regression coefficients are consistent with our intuition. We next build a parallel risk-neutral grid and calculate the implied risk-neutral exercise barrier at each period and for each value of cumulative market return using the barrier function estimated in the previous step. We finally estimate the company cost of options as their risk-neutral value given the exogenous exercise barriers.

Table 3 also shows the executive value, the company cost, and the difference between the two as computed by using the two state-variable S2 model and the one state-variable S1 model for stock options. We present the results for different values of $\alpha$, depth in the money, and $rmrf$. In all cases, the company cost in the S2 model is slightly lower than in the S1 model. However, the executive value in the S2 model is substantially lower than in the S1 model. As a result, the difference between the two quantities in the S2 model is greater than in the S1 model. For example, the executive value is 54 percent lower than the
company cost in the S2 model for at-the-money options when $\alpha = 0.50$ and $rmrf = 6\%$. In comparison, the executive value is 47 percent lower than the company cost in the S1 model. The divergence between the executive value and the company cost is particularly acute when $\alpha = 0.10$ and $rmrf = 8\%$. For this choice of parameters, the executive value is 29 percent lower than the company cost in the S2 model, but 6 percent higher in the S1 model (which is counter-intuitive and results from violation of the RN value upper bounds discussed in Section 3.2). Overall, compared to the S1 model, the S2 model results show that stock options must provide greater benefits in the form of incentive effects to executives to increase the firm value, tax savings, and perceived accounting benefits to overcome a greater difference between the executive value and the company cost.

5. Valuation of indexed strike price options

Indexed options, whose strike (or exercise) price is indexed to a market or industry benchmark, have attracted some attention in recent years. These options can filter out the effect of market-wide or industry-wide trends in rewarding the executives on their stock price performance. Johnson and Tian (2000) describe a risk-neutral methodology for the valuation of indexed options. They derive closed-form solutions and describe the incentive effects of indexed strike price options, which differ significantly from the incentive effects of fixed strike price options. However, their methodology does not adjust for the executive’s subjective valuation arising from his inability to hedge the options. Their methodology also cannot value the general American style option with an initial vesting period.

Since the S2 methodology advanced in this paper is based on a two state-variable grid, we can easily extend it to estimate the RN value, the executive value, and the company cost of an American style option whose exercise price is any arbitrary function of the current market level. However, we cannot accommodate indexing to a benchmark other than this second state variable in our model. Table 4 shows the results for indexed strike options whose strike price equals $30$ multiplied by the ratio of the current market level at the time of exercise to the starting market level at the time of grant. Note the market’s dividend yield is excluded in determining the exercise price. But the executive’s investment in the market portfolio through an index fund does earn the dividend included return.
$30. Some other parameters are unchanged from before (the option has 10 years to maturity and 3 years to vesting, the executive has 25,000 options and $5 million of non-option wealth split evenly between company stock and outside investment, the annual riskfree rate equals 5%, the annual market risk premium equals 6%, the annual standard deviation of log market return equals 20%, and both the stock and the market dividend yields equal 1%).

The different rows in Table 4 show the results for a range of values of the correlation between log stock returns and log market returns, stock volatility, beta, and depth in the money. The RN value, the executive value, and the company cost are always lower for the indexed strike price options than for the fixed strike price options. The difference between executive values increases as the correlation increases. When the correlation equals 0.50 and the stock volatility equals 0.30, the RN value, the executive value, and the company cost for at-the-money options equal $9.04, $3.79, and $8.34 for indexed strike options, and $13.34, $6.11, and $11.68 for fixed strike options. The last column shows the difference between the executive value and the company cost. In the above case, the executive value is 55 percent less than the company cost for indexed strike options, and 48 percent less for fixed strike options. In general, the percent difference between the executive value and the company cost is greater for the indexed strike price options than for the fixed strike price options. The comparison becomes worse as the correlation increases.

6. Implications of our model for the FASB proposal on the accounting treatment of stock options

The accounting of stock option compensation has been a controversial issue in recent years. Currently, companies may choose to record their option grants at the intrinsic value or the fair value. The intrinsic value of an option grant is zero if the options are granted at the money, while the fair value is typically calculated using established option pricing models, such as the Black-Scholes model. In a recent exposure draft, the Financial Accounting Standards Board (FASB) proposes to require U.S. public companies to recognize stock options compensation at the fair value in their income statement. The FASB recommends calculating the fair value of option grants as the risk neutral value of the options using a lattice model, with adjustment to reflect the executive’s exercise behavior. From the company’s point of view, the cost of granting options should be calculated in the risk-neutral setting since it is the amount a
diversified outside investor is willing to pay for the options. However, because the executives are risk averse and undiversified, their personal value of the options can be substantially lower than the company cost. Therefore, the executive value estimated by our model is the appropriate measure in empirical works concerning the incentive effects of these options.

In addition, our model is also useful in estimating the company cost of option grants, since it helps us understand the option exercising patterns. The undiversified and risk-averse executives tend to exercise the options earlier than a diversified investor optimally does. This makes the company cost of the option grants lower than the risk neutral value estimated by the standard option pricing models. In this paper, we build a utility-based model to endogenously determine the optimal exercise behavior of the undiversified executives. This is the theoretically correct approach, and helps us understand how the various parameters affect the option exercising pattern. The disadvantage of this approach is that it requires several assumptions about the executive’s portfolio of option holdings, risk aversion, diversification level, and expected market risk premium. In the exposure draft, FASB recommends that companies should use historical exercising data in calculating the company cost of the options. This alternative approach requires fewer assumptions about the unobservable parameters. But it assumes the option exercising pattern is exogenously given with no relation to many parameters, such as the time to maturity and market conditions. A study combining our model with the historical option exercising data may improve the estimation of company cost of option grants. This would be a worthwhile research project subject to data availability.

7. Conclusions

In recent years the annual grants of executive and employee stock options have accounted for around one percent of the market value of all firms included in the ExecuComp database (estimates using their Black-Scholes values). Such large and recurrent option grants have attracted considerable attention from shareholders, accountants, regulators, and researchers. It is widely accepted that the executive value of stock options is much lower than the company cost incurred by diversified shareholders. How much lower has been the subject matter of several research articles. We identify several shortcomings of existing one state-variable models (for convenience of exposition referred to as the S1 models) that allow
investment of executive’s outside wealth only in the risk-free asset. We show that the one state-variable models lead to counter-intuitive valuation results and that their use may also be limited due to the high sensitivity of their results to certain unobservable model parameters. We then propose a new two state-variable model (the S2 model) that allows investment of executive’s outside wealth in combinations of the risk-free asset and the market portfolio. We show that our two state-variable model is consistent with traditional portfolio theory and the capital asset pricing model, accommodates all option features, is computationally tractable, and overcomes many of the shortcomings of the one state-variable models.

We use a three-dimensional binomial grid proposed by Rubinstein (1994) to model the movement of two less than perfectly correlated state variables, which in our case are the stock price and the market level. This grid can be used to value all dynamic portfolio strategies whose outcome depends on these two state variables in addition to the riskfree return. We next allow the executive to optimally allocate his non-company outside wealth between the riskfree asset and the market portfolio. This critical feature distinguishes our two state-variable model from all one state-variable models that allow the executive to invest his outside wealth only in the riskfree asset. Using theoretical arguments as well as simulation results, we show that our model has several desirable features that are lacking in previous models of determining the executive value and the company cost of stock options and restricted stock.

We finally summarize our main contributions. First and foremost, our model allows an outside investment opportunity set that is suggested by traditional portfolio theory. Second, our model produces executive option values that are always lower than risk-neutral values. Intuitively, the risk-neutral values must serve as an upper bound on executive values. Third, our model leads to significantly lower sensitivities of option values to the unobservable expected market risk premium (which determines the expected stock returns), executive risk aversion, and the proportion of executive’s non-option wealth invested in company stock. This helps the company accountants and regulators who desire less subjective values. This also helps the empirical researchers who typically deal with a large cross-section of executives over a long calendar period with possibly heterogeneous and time-varying beliefs about the expected market risk premium. Fourth, we show that for most common parameters the executive values in our two state-variable model are significantly lower than in the one state-variable models, but the
company costs are not that much lower. This suggests that the difference between the executive value and the company cost that must be surpassed by incentive effects to increase firm value, tax savings, and perceived accounting benefits is greater than previously documented. Fifth, we show that our methodology can be easily applied to the valuation of American style indexed strike price option with vesting restrictions. Such options may become more popular if there is shareholder activism to reward executives for their performance in excess of the market performance.
References


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Appendix

We consider a simplified version of the model in the text to prove that the S1 value can exceed the RN value whereas the S2 value cannot exceed the RN value. We illustrate this point for the simpler case of restricted stock. In particular, we consider the stock valuation problem for an executive whose portfolio consists of restricted company stock and unrestricted outside wealth. In the S1 model, the executive can only invest his outside wealth in the riskfree asset. In the S2 model, the executive optimally allocates his outside wealth between the riskfree asset and the market portfolio.

In this one-period model, the executive’s initial wealth $W_0$ consists of two components: outside wealth $W_{out}$ and restricted stock wealth $W_{stk}$ (computed as the number of shares of restricted stock multiplied by the market value)

$$W_0 = W_{out} + W_{stk}.$$  \hspace{1cm} (A.1)

The executive’s end-of-period wealth, $W_T$, is given by

$$W_T = W_{out} [R_f + p(R_m - R_f)] + W_{stk} R_s ,$$  \hspace{1cm} (A.2)

where $R_f$ is the riskfree rate, $R_m$ is the market return, $R_s$ is the stock return, and $p$ is the proportion of outside wealth invested in the market portfolio. Notice the restricted stock becomes unrestricted at the end of the holding period. The market return dynamics are given by

$$R_m = R_f + rmrf + \varepsilon_m ,$$

$$E(\varepsilon_m) = 0 , \quad Var(R_m) = E(\varepsilon_m^2) = \sigma_m^2 ,$$  \hspace{1cm} (A.3)

where $rmrf$ is the market risk premium, $\varepsilon_m$ is the unexpected market return with mean zero and standard deviation $\sigma_m$. The stock return dynamics are given by

$$R_s = R_f + \beta \cdot (R_m - R_f) + \varepsilon_s = R_f + \beta \cdot rmrf + \beta \cdot \varepsilon_m + \varepsilon_s ,$$

$$E(\varepsilon_s) = 0 , \quad E(\varepsilon_s^2) = (1 - \rho^2) \sigma_s^2 , \quad cov(\varepsilon_s, \varepsilon_m) = 0 ,$$

$$Var(R_s) = \beta^2 \cdot E(\varepsilon_m^2) + E(\varepsilon_s^2) = \sigma_s^2 , \quad \beta = \frac{\rho \sigma_s}{\sigma_m} .$$  \hspace{1cm} (A.4)
Here $\varepsilon_s$ is the unique risk of stock, $\rho$ is the correlation between $R_m$ and $R_s$, $\beta$ is the stock beta, and $\sigma_s$ is the standard deviation of stock returns. Notice these return dynamics are slightly different from our model in the text which assumes that the stock returns and market returns are joint-lognormally distributed.

The executive’s utility function is defined on his end-of-period wealth $W_T$ as follows

$$E[U(W_T)] = E\left(\frac{W_T^{1-\gamma}}{1-\gamma}\right).$$  

(A.5)

We value the executive’s restricted stock using the outside wealth equivalent (OWE) method. OWE measures how much outside wealth the executive is willing to give up for an increase in restricted stock grant in the market value of $1$. Notice this definition is also a little different from our model in the text, where OWE measures how much outside wealth the executive is willing to give up for one share of stock or one option. Using standard microeconomic principles,

$$OWE = \frac{\partial E(U)}{\partial E(U)}\frac{\partial W_{stk}}{\partial W_{out}}.$$  

(A.6)

If $OWE$ is greater than one, the executive is willing to give up outside wealth in the market value of more than $1$ for additional restricted stock in the market value of $1$. This means the executive will actually buy restricted company stocks at market price with his outside wealth. Intuitively, this should not happen. In the following sections, we show that $OWE$ can be greater than one in the S1 model, but $OWE$ is always less than or equal to one in the S2 model.

The two marginal utilities are given by

$$\frac{\partial E(U)}{\partial W_{stk}} = E\left(U'(W_T) \cdot \frac{\partial W_T}{\partial W_{stk}}\right) = E[U'(W_T)] \cdot (R_f + \beta \cdot rmrf) + \text{cov}(U'(W_T), R_s),$$  

(A.7)

$$\frac{\partial E(U)}{\partial W_{out}} = E\left(U'(W_T) \cdot \frac{\partial W_T}{\partial W_{out}}\right) = E[U'(W_T)] \cdot (R_f + p \cdot rmrf) + \text{cov}(U'(W_T), pR_m),$$  

(A.8)
which use the simple relationship \( E(xy) = E(x)E(y) + \text{Cov}(x, y) \). In the S1 model, the executive can only invest his unrestricted wealth in the riskfree asset, so \( p \) is set to zero. We can calculate the difference between marginal utility of restricted stock and marginal utility of outside wealth in this case as

\[
\frac{\partial E(U)}{\partial W_{stk}} - \frac{\partial E(U)}{\partial W_{out}} = E[U'(W_T)] \cdot \beta \cdot rmrf + \text{cov}(U'(W_T), R_s) .
\]  

(A.9)

If Equation (A.9) is greater than zero, \( OWE \) is greater than one, implying that the executive values restricted stock at more than its market value. Setting \( p \) to zero also gives us

\[
W_T = W_{out} R_f + W_{stk} R_s
\]  

(A.10)

For an executive with no restricted company stock, \( W_{stk} = 0 \). His terminal wealth \( W_T = W_{out} R_f \) is a constant, which implies that \( \text{cov}(U'(W_T), R_s) = 0 \). Thus, Equation (A.9) becomes

\[
\frac{\partial E(U)}{\partial W_{stk}} - \frac{\partial E(U)}{\partial W_{out}} = E[U'(W_T)] \cdot \beta \cdot rmrf .
\]  

(A.11)

Since the utility function is concave, its first order derivative is positive. Equation (A.11) is greater than zero if the company has positive \( \beta \) and \( rmrf \) is positive. This implies that the executive value of restricted stock exceeds its market value. However, as the executive’s restricted stock wealth \( W_{stk} \) increases, \( \text{cov}(U'(W_T), R_s) \) becomes more negative. For a sufficiently large \( W_{stk} \), Equation (A.9) becomes negative. For any \( W_{stk} \) lower than this critical point, the executive in the S1 model values restricted stock at more than its market value.

Now we will show that the executive value of restricted stock is never higher than its market value in the S2 model. In this model, the executive optimally allocates \( p \) proportion of his outside wealth in the market portfolio and the remaining \((1-p)\) proportion in the riskfree asset. First order condition gives

\[
\frac{\partial E(U)}{\partial p} = E \left( U'(W_T) \cdot \frac{\partial W_T}{\partial p} \right) = -W_{out} \left[ E[U'(W_T)] \cdot rmrf + \text{cov}(U'(W_T), R_m) \right] = 0 .
\]  

(A.12)

If the executive has non-zero outside wealth, we have
By substituting Equation (A.13) into Equations (A.7) and (A.8), we have

\[
\frac{\partial E(U)}{\partial W_{stk}} = E\left(U'(W_T) \frac{\partial W_T}{\partial W_{stk}}\right) = E[U'(W_T)] \cdot R_f + \text{cov}(U'(W_T), R_s - \beta R_m) \tag{A.14}
\]

\[
\frac{\partial E(U)}{\partial W_{out}} = E\left(U'(W_T) \frac{\partial W_T}{\partial W_{out}}\right) = E[U'(W_T)] \cdot R_f \tag{A.15}
\]

The difference between the two marginal utilities therefore equals

\[
\frac{\partial E(U)}{\partial W_{stk}} - \frac{\partial E(U)}{\partial W_{out}} = \text{cov}(U'(W_T), R_s - \beta R_m) = \text{cov}(U'(W_T), (1 - \beta) \cdot R_f + \epsilon_s)
\]

\[
= \text{cov}(U'(W_T), \epsilon_s) \tag{A.16}
\]

By Equations (A.2) and (A.4), \(\text{cov}(\epsilon_s, \epsilon_m) = 0\). \(W_T\) monotonically increases in \(\epsilon_s\) if \(W_{stk} > 0\). Since \(U'\) monotonically decreases in \(W_T\), \(U'\) becomes negatively correlated with \(\epsilon_s\). Thus, the right-hand side of Equation (A.16) is always less than or equal to zero. The equality only holds when \(W_{stk}\) is zero or \(\rho^2\) is one. When \(W_{stk}\) is zero, \(W_T\) is not correlated with \(\epsilon_s\). When \(\rho^2\) is one, the market and the stock are perfectly correlated, implying \(\epsilon_s = 0\).

Overall, in the S2 model the marginal utility of the outside wealth with a market value of $1 is always greater than or equal to the marginal utility of the restricted wealth with a market value of $1. The executive value of restricted stock is therefore always less than or equal to its market value. The equality holds only if the executive holds no restricted stock or the restricted stock is perfectly correlated with the market portfolio.
Table 1
Ratios of executive value to risk-neutral value for stock options and restricted stock in the S2 model (the two state-variable model) and the S1 model (the one state-variable model)

The executive stock option has an exercise price of $30, 10 years to maturity, and 3 years to vesting. The restricted stock has a restriction period of 10 years, and is treated as an option with zero exercise price. The log return of the underlying stock is normally distributed with mean $\ln(R_f + \beta \cdot rmrf) - \frac{\sigma_s^2}{2}$, and standard deviation $\sigma_s=40\%$. Here $\beta=1$ is the stock’s beta, $R_f - 1=5\%$ is the annual riskfree rate, and $rmrf=6\%$ or $8\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f + rmrf) - \frac{\sigma_m^2}{2}$, and standard deviation $\sigma_m=20\%$. The stock has an annual dividend yield of 1%, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5$ million of non-option wealth split between company stock and outside investment. $\alpha$ denotes the proportion of non-option wealth in company stock, and $\gamma$ denotes the executive’s relative risk-aversion coefficient. The top panel shows the RN value, which is the risk-neutral value of stock option estimated using the standard binomial model. The lower four panels report three ratios in each cell. The first ratio is the S2 value divided by the RN value, where the S2 value is the executive value of option assuming that he optimally allocates his outside wealth between the riskfree asset and the market portfolio, and the second ratio $p$ is the optimal investment in the market portfolio divided by the total outside wealth. The third ratio is the S1 value divided by the RN value, where the S1 value is the executive value of option assuming that he invests his entire outside wealth in the riskfree asset. Superscript † identifies cases where the S2 or the S1 value exceeds the RN value.

<table>
<thead>
<tr>
<th>Stock price ($)</th>
<th>Stock options</th>
<th>Restricted stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock price ($)</td>
<td>5.00</td>
<td>15.00</td>
</tr>
<tr>
<td>RN value ($)</td>
<td>0.68</td>
<td>5.34</td>
</tr>
</tbody>
</table>

**Panel A: rmrf = 6\%, \gamma = 2**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>S2/RN</th>
<th>$p$</th>
<th>S1/RN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>0.75</td>
<td>0.044</td>
<td>0.052</td>
<td>0.052</td>
</tr>
<tr>
<td>0.50</td>
<td>0.106</td>
<td>0.106</td>
<td>0.106</td>
</tr>
<tr>
<td>0.25</td>
<td>0.237</td>
<td>0.237</td>
<td>0.237</td>
</tr>
<tr>
<td>0.01</td>
<td>0.767</td>
<td>0.767</td>
<td>0.767</td>
</tr>
</tbody>
</table>

**Panel B: rmrf = 6\%, \gamma = 3**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>S2/RN</th>
<th>$p$</th>
<th>S1/RN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.75</td>
<td>0.012</td>
<td>0.042</td>
<td>0.054</td>
</tr>
<tr>
<td>0.50</td>
<td>0.042</td>
<td>0.128</td>
<td>0.190</td>
</tr>
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<td>0.25</td>
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<td>0.128</td>
<td>0.128</td>
</tr>
<tr>
<td>0.01</td>
<td>0.678</td>
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cont’d
Table 1 continued

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<th>Restricted stock</th>
</tr>
</thead>
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<td>RN value ($)</td>
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<td>15.00</td>
</tr>
<tr>
<td>α</td>
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<td>5.34</td>
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</table>

Panel C: rmrf = 8%, $\gamma = 2$

<table>
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<tr>
<th>α</th>
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<th>0.50</th>
<th>0.25</th>
<th>0.01</th>
</tr>
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<td>0.99</td>
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<td>0.053</td>
<td>0.122</td>
<td>0.260</td>
<td>0.787</td>
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<td>0.75</td>
<td>0.046</td>
<td>0.187</td>
<td>0.311</td>
<td>0.475</td>
<td>0.825</td>
</tr>
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<td>0.50</td>
<td>0.113</td>
<td>0.327</td>
<td>0.459</td>
<td>0.599</td>
<td>0.831</td>
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<tr>
<td>0.25</td>
<td>0.171</td>
<td>0.409</td>
<td>0.535</td>
<td>0.656</td>
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<tr>
<td>0.01</td>
<td>0.223</td>
<td>0.462</td>
<td>0.580</td>
<td>0.688</td>
<td>0.830</td>
</tr>
</tbody>
</table>

| S2/RN| 0.133| 0.51  | 0.436| 0.581| 0.825 |
| p   | 0.51  | 0.142 |

Panel D: rmrf = 8%, $\gamma = 3$

<table>
<thead>
<tr>
<th>α</th>
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<th>0.75</th>
<th>0.50</th>
<th>0.25</th>
<th>0.01</th>
</tr>
</thead>
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<td>0.014</td>
<td>0.046</td>
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</tr>
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<td>0.75</td>
<td>0.008</td>
<td>0.102</td>
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<td>0.361</td>
<td>0.762</td>
</tr>
<tr>
<td>0.50</td>
<td>0.030</td>
<td>0.221</td>
<td>0.356</td>
<td>0.505</td>
<td>0.779</td>
</tr>
<tr>
<td>0.25</td>
<td>0.052</td>
<td>0.296</td>
<td>0.434</td>
<td>0.569</td>
<td>0.778</td>
</tr>
<tr>
<td>0.01</td>
<td>0.072</td>
<td>0.345</td>
<td>0.480</td>
<td>0.603</td>
<td>0.775</td>
</tr>
</tbody>
</table>

| S2/RN| 0.054|
| p   | 0.054|

† Significant at the 5% level.
Table 2
The sensitivity of executive values to expected market risk premium in the S2 model (the two state-variable model) and the S1 model (the one state-variable model)

The executive stock option has an exercise price of $30, 10 years to maturity, and 3 years to vesting. The restricted stock has a restriction period of 10 years, and is treated as an option with zero exercise price. The log return of the underlying stock is normally distributed with mean $\ln(R_f + \beta \cdot rmrf) - \frac{\sigma_s^2}{2}$, and standard deviation $\sigma_s = 40\%$. Here $\beta = 1$ is the stock’s beta, $R_f = 5\%$ is the annual riskfree rate, and $rmrf$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f + rmrf) - \frac{\sigma_m^2}{2}$, and standard deviation $\sigma_m = 20\%$. The stock has an annual dividend yield of 1%, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5 million of non-option wealth split evenly between company stock and outside investment. The executive’s relative risk aversion is given by $\gamma = 2.5$. The RN value is the risk-neutral value of stock option estimated using the standard binomial model. Three ratios are reported for each value of market risk premium. The first ratio is the S2 value divided by the RN value, where the S2 value is the executive value of option assuming that he optimally allocates his outside wealth between the riskfree asset and the market portfolio, and the second ratio $p$ is the optimal investment in the market portfolio divided by the total outside wealth. The third ratio is the S1 value divided by the RN value, where the S1 value is the executive value of option assuming that he invests his entire outside wealth in the riskfree asset.

<table>
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<th>Stock price ($)</th>
<th>RN value ($)</th>
<th>Expected market risk premium</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>2%</td>
<td>4%</td>
<td>6%</td>
<td>8%</td>
<td>10%</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel A: Option</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.00 (OTM)</td>
<td>0.225</td>
<td>0.240</td>
<td>0.244</td>
<td>0.249</td>
<td>0.255</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.00 (ATM)</td>
<td>0.380</td>
<td>0.399</td>
<td>0.400</td>
<td>0.402</td>
<td>0.402</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45.00 (ITM)</td>
<td>0.458</td>
<td>0.478</td>
<td>0.481</td>
<td>0.480</td>
<td>0.479</td>
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<tr>
<td><strong>Panel B: Restricted stock</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.00</td>
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<td>0.372</td>
<td>0.378</td>
<td>0.380</td>
<td>0.382</td>
<td></td>
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</tr>
<tr>
<td>35.00</td>
<td>0.350</td>
<td>0.390</td>
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<td>0.482</td>
<td>0.534</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The executive value, the expected life, and the company cost of stock options in the S2 model (the two state-variable model) and the S1 model (the one state-variable model)

We present results for various combinations of $\alpha$, the proportion of executive’s non-option wealth invested in company stock, the stock price, and $rmrf$, the expected market risk premium. The RN value is the risk-neutral value of stock option estimated using the standard binomial model, the S2 model assumes that the executive optimally allocates his outside wealth between the riskfree asset and the market portfolio, and the S1 model assumes that his entire outside wealth is invested in the riskfree asset. In all cases, the option has an exercise price of $30, 10 years to maturity, and 3 years to vesting. The log return of the underlying stock is normally distributed with mean $\ln(R_f+\beta \cdot rmrf)-\sigma_s^2/2$, and standard deviation $\sigma_s=40\%$. Here $\beta=1$ is the stock’s beta, $R_f -1=5\%$ is the annual riskfree rate, and $rmrf=6\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f+rmrf)-\sigma_m^2/2$, and standard deviation $\sigma_m=20\%$. The stock has an annual dividend yield of 1%, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5$ million of non-option wealth split evenly between company stock and outside investment. The executive’s relative risk aversion is given by $\gamma=2.5$. The executive value is estimated as the amount of outside wealth the executive is willing to give up for an option. The expected life is the probability-weighted average time between the grant date and the exercise date (or the maturity date in case of no exercise). The company cost is estimated as the risk-neutral value of option given the executive’s sub-optimal exercise decisions.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Stock price ($)</th>
<th>RN value ($)</th>
<th>Executive value ($)</th>
<th>Expected life (years)</th>
<th>Company cost ($)</th>
<th>Difference (cost-value)/cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>15.00</td>
<td>5.34</td>
<td>0.36 0.37</td>
<td>7.72 7.73</td>
<td>3.91 3.90</td>
<td>0.91 0.91</td>
</tr>
<tr>
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<td>30.00</td>
<td>15.70</td>
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<td>0.80 0.79</td>
</tr>
<tr>
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<td>45.00</td>
<td>27.70</td>
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<td>23.56 23.56</td>
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</tr>
<tr>
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<td>15.00</td>
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<td>15.70</td>
<td>6.29 7.32</td>
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<tr>
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<td>0.28 0.05</td>
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<td>45.00</td>
<td>27.70</td>
<td>19.76 25.14</td>
<td>6.28 7.16</td>
<td>26.40 27.09</td>
<td>0.25 0.07</td>
</tr>
</tbody>
</table>

Panel A: $rmrf=6\%$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Stock price ($)</th>
<th>RN value ($)</th>
<th>Executive value ($)</th>
<th>Expected life (years)</th>
<th>Company cost ($)</th>
<th>Difference (cost-value)/cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>15.00</td>
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<td>6.11 7.38</td>
<td>26.43 27.32</td>
<td>0.26 -0.03</td>
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Panel B: $rmrf=8\%$
Table 4
Valuation of indexed strike price options using the S2 model (the two state-variable model)

This table reports the RN value, the executive value, and the company cost of indexed strike (or exercise) price options at grant date using the S2 model. The S2 model assumes that the executive optimally allocates his outside wealth between the riskfree asset and the market portfolio. The strike price of indexed options at any time $t$ equals $X(M_t/M_0)$, where $X=30$ is the exercise price of the fixed strike options, $M_t$ is the market index level at time $t$, and $M_0$ is the market index level at grant date. In this table both the stock price and the market index level exclude dividend reinvestment. Both types of options have 10 years to maturity and 3 years to vesting. The log return of the underlying stock is normally distributed with mean $\ln(R_f+\beta \cdot \text{rmrf})-\sigma_s^2/2$, and standard deviation $\sigma_s = 40\%$. Here $\beta$ is the stock’s beta, $R_f-1=5\%$ is the annual riskfree rate, and $\text{rmrf}=6\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f+\text{rmrf})-\sigma_m^2/2$, and standard deviation $\sigma_m = 20\%$. Both the stock and the market have an annual dividend yield of 1%. The executive has 25,000 options and $5$ million of non-option wealth split evenly between company stock and outside investment. The RN value is the risk-neutral value of stock option estimated using the standard binomial model. The executive value is estimated as the amount of outside wealth the executive is willing to give up for an option. The company cost is estimated as the risk-neutral value of the option given the executive’s sub-optimal exercise decisions.

<table>
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<th>$\beta$</th>
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<th>RN values ($</th>
<th>Executive values ($</th>
<th>Company cost ($</th>
<th>Difference (cost-value)/cost</th>
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Figure 1. Rainbow grid by Rubinstein (1994). This figure illustrates the first two moves of the rainbow grid. Panel A plots the first move. The initial node is (1,1). In the first move, the first asset can move up by a factor of $u$ or move down by a factor of $d$ with equal probability. Given the first asset moves up, the second asset can move up by a factor of $A$ or move down by a factor of $B$ with equal probability. Given the first asset moves down, the second can move up by a factor of $C$ or move down by a factor of $D$ with equal probability. The four nodes after the first move are ($u$, A), ($u$, B), ($d$, C), and ($d$, D). Panel B plots the second move. From each of the four nodes ($u$, A), ($u$, B), ($d$, C), and ($d$, D), there are four possible moves with equal probability. To make the grid recombine, it is necessary that $AD=BC$. Thus, after the second move, there are nine nodes: ($u^2$, A²), ($u^2$, AB), ($u^2$, B²), ($ud$, AC), ($ud$, AD), ($du$, BD), ($d²$, C²), ($d²$, CD), and ($d²$, D²). In general, after $t$ moves, there will be $(t+1)^2$ nodes. The grid evolves as a square pyramid.
Figure 2. Percent overvaluation of the S1 executive stock option value relative to the S2 executive stock option value. The S1 model is the one state-variable model, and the S2 model is the two state-variable model. The S1 model assumes that the executive invests his entire outside wealth in the risk-free asset, and the S2 model assumes that he invests his outside wealth in an optimal combination of the risk-free asset and the market portfolio. The bar charts show the values of $100 \times (S1-S2)/S2$. The proportion of stock wealth, $\alpha$, refers to how much of his non-option wealth is invested in company stock. The log return of the underlying stock is normally distributed with mean $\ln(R_f + \beta \cdot rmrf) - \sigma_s^2/2$, and standard deviation $\sigma_s=40\%$. Here $\beta=1$ is the stock’s beta, $R_f - 1=5\%$ is the annual riskfree rate, and $rmrf=6\%$ or $8\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f + rmrf) - \sigma_m^2/2$, and standard deviation $\sigma_m=20\%$. The stock has an annual dividend yield of 1%, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5$ million of non-option wealth split between company stock and outside investment. The executive’s relative risk aversion is given by $\gamma=2$ as in Panels A and C of Table 1.
Figure 3. Ranges of $\alpha$, the proportion of non-option wealth invested in company stock, for which the S1 executive stock option value is higher than the RN value. The S1 model is one state-variable mode, and it computes the executive value of option assuming that he invests his entire outside wealth in the risk-free asset. The ranges are reported as a function of the stock price, $S_0$, and the market risk premium, $rmrf$. The RN value is the risk-neutral value of stock option estimated using the standard binomial model. The log return of the underlying stock is normally distributed with mean $ln(R_f + \beta \cdot rmrf) - \frac{\sigma_s^2}{2}$, and standard deviation $\sigma_s=40\%$. Here $\beta=1$ is the stock’s beta, $R_f-1=5\%$ is the annual risk free rate, and $rmrf=6\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $ln(R_f + rmrf) - \frac{\sigma_m^2}{2}$, and standard deviation $\sigma_m=20\%$. The stock has an annual dividend yield of $1\%$, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5$ million of non-option wealth split between company stock and outside investment. The executive’s relative risk aversion is given by $\gamma=2.5$. 

Figure 3.
Figure 4. The sensitivity of executive stock option values to expected market risk premium in the S2 and S1 models. The S2 model is the two state-variable model, and the S1 model is the one state-variable model. The first three figures plot the ratios of the S2 and S1 executive stock option values to the RN value of an option for three different levels of stock price $S$. In each case the option has an exercise price of $30, 10$ years to maturity, and $3$ years to vesting. The fourth figure plots the executive values of restricted stock with a restriction period of $10$ years. The log return of the underlying stock is normally distributed with mean $\ln(R_f + \beta \cdot rmrf) - \sigma_s^2/2$, and standard deviation $\sigma_s=40\%$. Here $\beta=1$ is the stock’s beta, $R_f-1=5\%$ is the annual riskfree rate, and $rmrf=6\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f+rmrf) - \sigma_m^2/2$, and standard deviation $\sigma_m=20\%$. The stock has an annual dividend yield of $1\%$, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has $25,000$ options or shares of restricted stock and $5$ million of non-option wealth evenly split between company stock and outside investment. The executive’s relative risk aversion is given by $\gamma=2.5$. The RN value is the risk-neutral value of stock option estimated using the standard binomial model, the S2 value is the executive value of option assuming that he optimally allocates his outside wealth between the riskfree asset and the market portfolio, and the S1 value is the executive value of option assuming that he invests his entire outside wealth in the riskfree asset. The solid line represents the ratio of the S2 value to RN value for various values of market risk premium, and the dashed line represents the ratio of the S1 value to RN value.
Figure 5. The sensitivity of executive stock option value to executive’s relative risk aversion, $\gamma$, in the S2 and S1 models. The S2 model is the two state-variable model, and the S1 model is the one state-variable model. The stock price is $30. The option has an exercise price of $30, 10 years to maturity, and 3 years to vesting. The log return of the underlying stock is normally distributed with mean $\ln(R_f + \beta \cdot \text{rmrf}) - \sigma_s^2/2$, and standard deviation $\sigma_s = 40\%$. Here $\beta = 1$ is the stock’s beta, $R_f - 1 = 5\%$ is the annual riskfree rate, and $\text{rmrf} = 6\%$ is the expected annual market risk premium. The log return of market is normally distributed with mean $\ln(R_f + \text{rmrf}) - \sigma_m^2/2$, and standard deviation $\sigma_m = 20\%$. The stock has an annual dividend yield of 1%, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5$ million of non-option wealth evenly split between company stock and outside investment. The solid line represents the S2 value, which is the executive value of option assuming that he optimally allocates his outside wealth between the riskfree asset and the market portfolio. The dashed line represents the S1 value, which is the executive value of option assuming that he invests his entire outside wealth in the riskfree asset.
Figure 6. The sensitivity of executive stock option value to diversification level in the S2 and S1 models. The S2 model is the two state-variable model, and the S1 model is the one state-variable model. The proportion of executive’s non-option wealth invested in the company stock, \( \alpha \), is used as an inverse measure of the diversification level. The stock price is $30. The option has an exercise price of $30, 10 years to maturity, and 3 years to vesting. The log return of the underlying stock is normally distributed with mean \( \ln(R_f + \beta \cdot rmrf) - \frac{\sigma_s^2}{2} \), and standard deviation \( \sigma_s = 40\% \). Here \( \beta = 1 \) is the stock’s beta, \( R_f - 1 = 5\% \) is the annual riskfree rate, and \( rmrf = 6\% \) is the expected annual market risk premium. The log return of market is normally distributed with mean \( \ln(R_f + rmrf) - \frac{\sigma_m^2}{2} \), and standard deviation \( \sigma_m = 20\% \). The stock has an annual dividend yield of 1\%, but the market has no dividend yield, as it refers to an index fund that reinvests all its dividends. The executive has 25,000 options or shares of restricted stock and $5 million of non-option wealth evenly split between company stock and outside investment. The executive’s relative risk aversion is given by \( \gamma = 2.5 \). The solid line represents the S2 value, which is the executive value of option assuming that he optimally allocates his outside wealth between the riskfree asset and the market portfolio. The dashed line represents the S1 value, which is the executive value of option assuming that he invests his entire outside wealth in the riskfree asset. The dotted line represents the RN value, which is the risk-neutral value of stock option estimated using the standard binomial model.