# Executive Stock and Option Valuation in a Two State-Variable Framework

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*We provide a discrete-time executive option valuation model that allows optimal investment of the executive's outside wealth in the risk-free asset and the market portfolio. The model represents an improvement over one state-variable models that allow outside investment only in the risk-free asset.*

*First, the model is consistent with portfolio theory and the CAPM. Second, it produces executive option values that are always lower than risk-neutral values. Third, values are less subjective, as the model gives significantly lower sensitivities of option values to the unobservable expected market risk premium, executives' risk aversion, and executives' diversification level. Fourth, for common parameter values the executive option values are significantly lower than in the one state-variable model, but the company costs are not much lower. This suggests more difference between executive value and company cost than previously documented.*

*The methodology can be easily applied to the valuation of American-style indexed strike price executive options.*

Stock options have become the most significant part of compensation for senior executives and a significant part of compensation for other employees in U.S. firms. According to Compustat's ExecuComp data, new stock options granted by all listed firms during 2002 had a Black-Scholes value of $98 billion, or 1.2% of the year-end market value of these firms.

While restricted stock has so far represented a smaller part of compensation than stock options, it is gaining in popularity, and together these two forms of compensation account for a significant part of the new wealth created by U.S. firms. It should not be surprising that determining the executive value and the company cost of restricted stock and option grants has become a topic of considerable interest.

We illustrate some important limitations of the current models, and present a new model that overcomes these limitations while satisfying traditional portfolio theory.

The value to an executive of stock options and restricted stock is generally believed to be lower than their value to a diversified investor or shareholder. Diversified investors can hedge the option risk, which makes their option values equal the American option value derived in the risk-neutral Black-Scholes framework. Diversified investors can also buy or sell stock openly in the market, which makes their stock values equal to the market price. Executives can neither hedge nor sell, which makes them undiversified, especially if they hold additional amounts of company stock. The value of their options is therefore lower, and it is usually determined by utility-based models that ask what minimum dollar amount would make executives forgo the stock options or the restricted stock.

Lambert, Larcker, and Verrecchia [1991], Huddart [1994], Kulatilaka and Marcus [1994], Carpenter [1998], and Hall and Murphy

[2002] describe utility-based models that share many common characteristics. We call these models one state-variable models (henceforth *S1 models*), as they consider the stock price the only state-variable in determining executive values.

Hall and Murphy [2002] assume the executive has a power utility function and invests outside wealth in the risk-free asset. Assuming further that the stock returns are lognormally distributed, they determine the certainty-equivalent amount that would make the executive give up the options.

Hall and Murphy show why executives often argue that Black-Scholes values are too high; how this divergence is related to risk aversion and lack of diversification; why executives exercise options earlier than maturity; why virtually all options are granted at the money; why companies often reset exercise prices of underwater or out-of-the-money options; and why in the absence of strong incentive effects stock options (and to a lesser extent restricted stock) are an inefficient form of executive compensation.

While S1 models do an excellent job of explaining the direction of such results, the executive option values they derive suffer from important limitations. First, we find the option values to executives are often too high, exceeding the option values derived using the traditional risk-neutral model (henceforth the *RN model*). For example, an executive holding \$3.75 million of outside wealth, \$1.25 million of company stock, and 10,000 at-the-money stock options assigns an S1 value of \$17.05 to each option, which exceeds the RN value of \$16.55 (other parameters: risk aversion coefficient 2.0, stock price \$30.00, stock volatility 30%, dividend yield 0%, time to maturity ten years, European-style exercise, risk-free rate 6%, stock beta 1.00, and market risk premium 6.5%).[1]

This finding is inconsistent with traditional portfolio theory in that an individual without superior information assigns a lower than market value to any non-negligible stock holding in excess of its market weight. This overvaluation in the S1 model arises because an investor constrained to hold all outside wealth only in the risk-free asset overvalues any marginal holding of a risky asset with an expected return higher than the risk-free return.

A second limitation of S1 models is that the derived executive option values are highly sensitive to assumptions of *rmrf*, or the expected market risk premium, that determines the expected stock returns. Using the same parameters as above, except with only \$1.70 million of outside wealth and \$3.30 million of company stock, an *rmrf* value of 3% gives an option value of \$4.57, while an *rmrf* value of 8% gives an option value of \$8.83. This result is in sharp contrast with the RN model, which gives an option value of \$16.55 whatever the *rmrf* value. This also makes the empirical implementation of S1 models difficult when the researcher must estimate the option values over several years for hundreds of executives with possible heterogeneous and time-varying beliefs concerning the unobservable *rmrf* value.

We argue that both the overvaluation of stock options (at least for not so undiversified executives and employees) and the high *rmrf* sensitivity of option prices reflect a common cause, which constitutes a third limitation of all S1 models. These models assume that executives invest all their outside wealth in the risk-free asset. This assumption is inconsistent with portfolio theory, which implies that investors with finite risk aversion should hold combinations of the risk-free asset and the market portfolio.

Allowing optimal investments of outside wealth in the market portfolio helps the model in several ways. First, starting from the optimal combination of only the risk-free asset and the market portfolio, an executive values a small incremental investment in any risky asset at its market value, and increasing investments in the same asset at less than market value. In other words, a nearly fully diversified executive values a stock option at nearly its risk-neutral value, and restricted stock at nearly its market value, while an undiversified executive attaches strictly lower values. In either case, there is no violation of what clearly must be the upper bound on executive value.

Second, the high *rmrf* sensitivity of S1 option values arises because an increase in *rmrf* increases the attractiveness of stock and options in this model, but has no effect on the attractiveness of outside wealth invested only in the risk-free asset. Suppose the executive invests a proportion of outside wealth in the market portfolio and is also free to choose the optimal proportion. An increase in *rmrf* in this case simultaneously increases the attractiveness of outside investments, first because the expected market return increases, and second because the proportion of outside wealth invested in the market portfolio increases. The combined result is a sharp reduction in the *rmrf* sensitivity of executive stock and option values.

Allowing market investment with outside wealth poses some challenge, as it introduces two state variables in the valuation problem. Fortunately, Rubinstein [1994] provides a rainbow option pricing model that can be used to value contingent claims on two jointly lognormal state variables. We modify Rubinstein's model to build a three-dimensional pyramidal grid, and show that it can be used to value all European- and American-style options in portfolios that are invested in the risk-free asset, the market portfolio, the company stock, and the stock options.

Our model accommodates all complications related to the executive's risk aversion, vesting period, and early exercise. We choose optimal proportions of the risk-free asset and the market portfolio with outside wealth by using a numerical search process. The resulting two state-variable executive option pricing model (henceforth the *S2 model*) corrects many limitations of the typical executive option pricing models.

The major contributions may be summarized as follows:

1. The executive option pricing model is consistent with portfolio theory and the capital asset pricing model (CAPM).[2] It allows optimal investments in the risk-free asset and the market portfolio, and values forced investments in company stock and options at less than the market value determined by diversified and therefore risk-neutral investors. The one state-variable S1 model is a special case of the two state-variable S2 model when the proportion of market investment with outside wealth is exogenously set to zero. Restricted stock is also a special case of the S2 model when the option strike price is set to zero or an arbitrarily low value.
2. The S2 values of stock options and restricted stock are uniformly and significantly lower than the S1 values for the same set of parameters (the executive's risk aversion, the proportion of non-option wealth invested in company stock, and the stock and market return distributions).
3. Although the S2 option values are significantly lower than the S1 option values, the average time to exercise is not much lower. Early exercise reduces the company cost, which is computed as what a diversified and risk-neutral investor would pay for the same option. Thus, for the same parameters, the S2 model shows more divergence between the company cost and the executive value of both stock options and restricted stock. This greater divergence requires that stock options and restricted stock must provide stronger incentive effects to executives to increase the firm value.
4. In most cases, the *rmrf* sensitivity of the S2 option value is a small fraction of the *rmrf* sensitivity of the S1 option value. The S2 option values are also less sensitive to the two remaining unobservable parameters, which are the executive's risk aversion and the proportion of non-option wealth invested in the company stock. The lower sensitivities lead to less subjective estimates of executive value. This helps company accountants, regulators, and empirical researchers.
5. The S2 model can be easily extended to compute the executive value and the company cost of American-style options whose strike price is adjusted for changes in the market level according to any pre-set formula. Such indexed strike price options are currently not very common, but their popularity may increase if there is shareholder activism to reward executives for their performance net of market performance.[3]

## I. METHODOLOGY

The traditional risk-neutral option pricing theory assumes that an investor can hedge the risk of a call option by short-selling a certain amount of the underlying stock. According to this assumption, the RN value of a call option equals the present value of its expected cash flow under the risk-neutral distribution. To an outside investor who faces no constraints in short-selling the stock, the option value equals the RN value. The RN value, however, overstates the option value to an undiversified risk-averse executive who cannot sell his options or short-sell the stock of his company. The executive value must take into consideration his unique portfolio of option and non-option wealth, his utility function, and the actual stock return distribution.[4]

### Two State-Variable Model (S2 Model)

We estimate the executive value of options as the amount of outside wealth that gives the executive the same expected utility as the options. The executive has non-option wealth of $W_0$, of which a proportion $\alpha$ is invested in the company stock, and the rest is invested in outside wealth. He further optimally invests a proportion $p$ of his outside wealth in the market portfolio and the rest in the risk-free asset.[5]

When the options mature in $T$ years, assuming no early exercise, the executive's total wealth will equal

$$W_T = W_0[\alpha\, e^{\delta T}\frac{S_T}{S_0} + (1-\alpha)(p\frac{M_T}{M_0} + (1-p)R_f^{\,T})] + \\ N_{opt}Max(S_T - X, 0) \qquad (1)$$

where $\delta$ is the continuous dividend yield on the stock, $S_T$ is the stock price in year $T$, $S_0$ is the current stock price, $M_T$ is the market level in year $T$, $M_0$ is the current market level, $R_f$ is the annual risk-free rate, $N_{opt}$ is the number of options the executive holds, and $X$ is the exercise price of the options.

Note that $S_T$ excludes dividends paid between now and year $T$. We assume the executive reinvests all dividends in the stock, thus ending up with stock wealth of $e^{\delta T} W_0 \alpha S_T/S_0$ in year $T$. The executive invests in the market portfolio through an index fund that reinvests all dividend proceeds. Therefore, $M_T$ already includes all dividends paid between today and year $T$ (which is just a matter of notational convenience).

Executive options are generally exercisable after an initial vesting period. If the executive exercises the options at any time $t < T$, we assume he invests the exercise proceeds in the risk-free asset and the market portfolio according to his optimal portfolio choice $p$.[6]

In this case, the executive's total wealth in year $T$ is given by

$$W_T = W_0\alpha\frac{S_T}{S_0}e^{\delta T} + p[W_0(1-\alpha)\frac{M_t}{M_0} + N_{opt}Max(S_t - X, 0)]\frac{M_T}{M_t} + (1-p)[W_0(1-\alpha)R_f^{\,t} + N_{opt}Max(S_t - X,0)]R_f^{\,T-t} \quad (2)$$

We define the executive's utility function with regard to terminal wealth at year $T$ as

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma} \quad (3)$$

The executive chooses $p$ to maximize expected utility:

$$p^* = \underset{p}{\operatorname{argmax}}\left[E\left(U(W_T)\right)\right] \quad (4)$$

In the one state-variable S1 model, the executive can invest outside wealth only in the risk-free asset. It becomes a special case of our S2 model where $p$ is exogenously specified as 0.

Utility maximization also determines the executive's option exercise decision. Once the options are vested, the executive exercises whenever the expected utility of exercising the options exceeds the expected utility from holding them.

We estimate the executive value of an option as the amount of outside wealth equivalent, *OWE*, that solves

$$E\left(U(W_T^{OWE})\right) = E\left(U(W_T^{OPT})\right) \quad (5)$$

where $W_T^{OWE}$ is given by

$$W_T^{OWE} = W_0\alpha\, e^{\delta T}\frac{S_T}{S_0} + [W_0(1-\alpha) + N_{opt}OWE] \times (p^*\frac{M_T}{M_0} + (1-p^*)R_f^{\,T}) \quad (6)$$

$W_T^{OPT}$ is given by Equation (1) if the options are not exercised prior to maturity, and by Equation (2) if the options are exercised at any time before maturity.[7]

Solving Equation (5) numerically for *OWE* gives the S2 model executive value of a stock option. We can easily extend this model to estimate the executive value of restricted stock by setting the exercise price arbitrarily close to zero.

In concept, our *OWE* is similar to the certainty-equivalent amount used by Hall and Murphy and others. We prefer the term, *OWE*, as in our case the outside investment opportunity set includes the uncertain market portfolio in addition to the certain risk-free asset.

## Rainbow Grid

The S2 model has two state-variables: the stock price $S_t$, and the market portfolio $M_t$. Following common practice, we adopt the CAPM to calculate expected stock returns, and also assume that in any year $t$ the log stock returns and the log market returns follow a joint-normal distribution:[8]

$$\begin{pmatrix}\ln(M_t) - \ln(M_{t-1}) \\ \ln(S_t) - \ln(S_{t-1})\end{pmatrix} = \begin{pmatrix}\ln(R_{M,t}) \\ \ln(R_{S,t})\end{pmatrix} \sim N\left(\begin{pmatrix}\mu_M \\ \mu_S\end{pmatrix}, \begin{pmatrix}\sigma_M^2 & \sigma_{S,M} \\ \sigma_{S,M} & \sigma_S^2\end{pmatrix}\right)$$

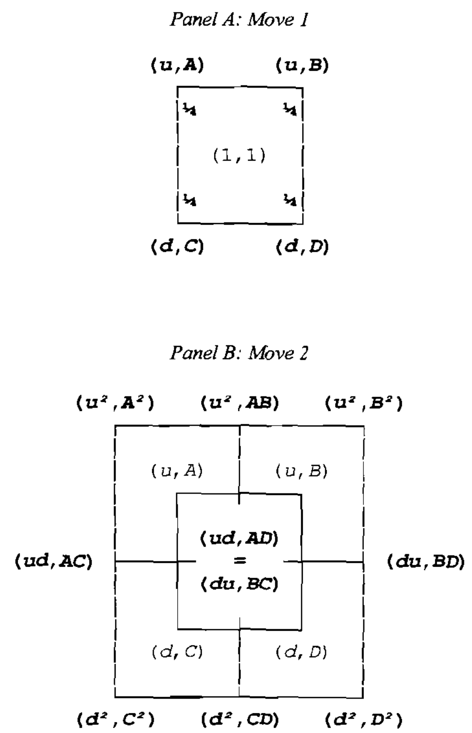
$$\mu_M = \ln(R_f + rmrf) - \frac{1}{2}\sigma_M^2$$

$$\mu_S = \ln(R_f + \beta \cdot rmrf) - \delta - \frac{1}{2}\sigma_S^2 \quad (7)$$

where $R_{M,t}$ is the annual market return in year $t$, $R_{S,t}$ is the annual stock return, $\mu_M$ is the expected log market return, $\mu_S$ is the expected log stock return, $\sigma_M^2$ is the variance of the log stock return, $\sigma_{S,M}$ is the covariance between the log stock return and the log market return, $\sigma_S^2$ is the variance of the log stock return, *rmrf* is the executive's expected annual market risk premium, and $\beta$ is the systematic risk of the stock. Note that $\mu_M$ includes dividends since we assume the executive invests in the market

## Exhibit 1

**Rubinstein [1994] Rainbow Grid**




portfolio through an index fund that reinvests all dividend proceeds.

To accommodate the two state-variables and the early exercise feature of American-style options, we modify the rainbow option pricing model of Rubinstein [1994]. His model is designed to value options whose payoffs depend on the realization of two less-than-perfectly correlated assets. Rubinstein constructs a three-dimensional binomial grid to approximate the movements of the two assets.

The mechanics are best explained in Exhibit 1. Panel A shows the first-period moves. The initial node is (1, 1). The first state-variable, the market level in our case, moves up by a factor of $u$ or down by a factor of $d$ with equal probability. Given that the market moves up, the second state variable, the stock price in our case, can move up by a factor of $A$ or down by a factor of $B$ with equal probability. Similarly, given that the market moves down, the stock can move up by a factor of $C$ or down by a factor of $D$ with equal probability. Thus, at the end of the first period, there are four nodes, and the probability of reaching any of these four nodes from the previous node is one-fourth. The four nodes after the first move are $(u, A)$, $(u, B)$, $(d, C)$, and $(d, D)$.

Panel B shows the second-period moves. There are four possible moves from each of the four nodes. To make the grid recombine and evolve in a compact fashion, it is necessary to assume that $AD = BC$. This leads to $3^2 = 9$ nodes at the end of the second period: $(u^2, A^2)$, $(u^2, AB)$, $(u^2, B^2)$, $(ud, AC)$, $(ud, AD)$, $(du, BD)$, $(d^2, C^2)$, $(d^2, CD)$, and $(d^2, D^2)$. In general, there are $(t + 1)2$ nodes after $t$ periods. The probability of reaching a node with $i$ up moves in the market portfolio and $j$ up moves in the stock after $t$ periods is given by

$$\Pr(t, i, j) = {}^tC_i\,{}^tC_j(1/4)^t \tag{8}$$

where ${}^tC_i = \dfrac{t!}{i!(t-i)!}$

In Rubinstein's original model, the six parameters $\{u, d, A, B, C, D\}$ are determined so that the expected returns of the two assets equal the risk-free return net of their individual dividend yield. We modify this condition by setting the expected log annual market return to $\mu_M$, and the expected log annual stock return to $\mu_S$. Assuming $h$ periods per year, and therefore $h \times T$ periods in total, the expected market return per period is $e^{\mu_M \Delta t}$, and the expected stock return per period is $e^{\mu_S \Delta t}$, where $\Delta t = 1/h$ is the time interval of one period.

The six parameters are now determined by using the two expected return conditions, two variance conditions, one covariance condition, and one grid recombining condition as follows:

$$\begin{aligned}
u &= \exp(\mu_M \Delta t + \sigma_M \sqrt{\Delta t}) \\
d &= \exp(\mu_M \Delta t - \sigma_M \sqrt{\Delta t}) \\
A &= \exp[\mu_S \Delta t + \sigma_S \sqrt{\Delta t}(\rho + \sqrt{1-\rho^2})] \\
B &= \exp[\mu_S \Delta t + \sigma_S \sqrt{\Delta t}(\rho - \sqrt{1-\rho^2})] \\
C &= \exp[\mu_S \Delta t - \sigma_S \sqrt{\Delta t}(\rho - \sqrt{1-\rho^2})] \\
D &= \exp[\mu_S \Delta t - \sigma_S \sqrt{\Delta t}(\rho + \sqrt{1-\rho^2})]
\end{aligned} \tag{9}$$

where $\rho$ is the correlation between the log stock return and the log market return.

### Implementation

The first step in implementation is to find $E[U(W_T^{OPT})]$ in Equation (5). This step is straightforward for European-style options. We start by constructing a rainbow grid with $h \times T$ periods. Next, for each of the $(h \times T + 1)^2$ terminal nodes, we calculate the terminal wealth $W_T$

by Equation (1), the corresponding utility by Equation (3), and the corresponding probability by Equation (8). Finally, we calculate $E[U(W_T^{OPT})]$ as the probability-weighted average utility of all terminal nodes.

For American-style options, finding $E[U(W_T^{OPT})]$ requires much more computation. We first calculate the utility at each of the $(hT + 1)^2$ terminal nodes assuming no early exercise. Then, starting backward from one period before the last, we calculate at each node 1) the expected utility of holding the option for one more period, and 2) the expected utility of early exercising.

The expected utility of holding the option for one more period is the simple average of the expected utilities at each of the four nodes following, since the probability of reaching any node following from the current node is always $1/4$. The expected utility of early exercising is calculated in two steps.

First, at all terminal nodes attainable from the current node, we calculate the terminal wealth $W_T$ by Equation (2), the corresponding utility, and the corresponding probability. Note that the corresponding probability is conditional on the probability of reaching the current node. Second, we calculate the expected utility from early exercising as the probability-weighted average utility of all attainable terminal nodes.

We then compare the expected utility of holding the option for one more period to the expected utility of early exercising, and choose the higher one as the expected utility for this node. We repeat this process at each node until the initial node is reached, and the expected utility at the initial node is $E[U(W_T^{OPT})]$ in Equation (5).

The overall procedure is similar to evaluating American-style options in other contexts, but with one difference. For options with an initial vesting period, we compare the expected utility of holding the option for one more period to the expected utility of early exercising only at any node on or after the vesting date. The expected utility at any node before the vesting date is simply the expected utility of holding the option.

The next step in implementation is to find the optimal $p$ that maximizes the expected utility. We use a grid search method by calculating the expected utility for 101 values of $p$ from 0 to 1 in increments of 0.01, and choose the $p$ that gives the highest expected utility.

The final step in implementation is to solve Equation (5) for $OWE$. We solve this equation numerically.

One point needs special mention. If option wealth is low relative to total wealth, the executive's expected utility changes only slightly when the options are excluded. Therefore, to obtain accurate option values, it is necessary to specify very tight convergence criteria.[9]

### Computation Issues

For the same number of periods, the rainbow grid in the S2 model requires far more computation than the traditional one-dimensional grid. For $N$ periods, the rainbow grid has $(N + 1)(N + 2)(2N + 3)/6$ nodes. In addition, evaluating the early exercise decision for a node at period $n$ requires calculating the expected utility across $(N - n + 1)^2$ terminal nodes. As a result, it is computationally infeasible for us to have more than four periods per year for a ten-year option. Fortunately, given the high node-density of the rainbow grid, that is enough periods for good convergence. In fact, results obtained using a grid with one period per year are not much different from the results obtained using a grid with four periods per year.[10]

We allow two imperfections in the portfolio optimization step for computational reasons. First, the executive is not allowed to rebalance his outside portfolio. In an ideal world, the executive would choose the optimal $p$ at every node of the grid, but this is computationally infeasible, given the large number of nodes in the rainbow grid. This imperfection makes the portfolio choice with outside wealth suboptimal, which diminishes the attractiveness of outside wealth.[11] A related imperfection is to assume that the same optimal $p$ describes the outside portfolio choice when the portfolio includes the options and when it includes the $N_{opt}OWE$, which also diminishes the attractiveness of outside wealth.

Both these imperfections work against us in finding the difference between the S2 executive option value and the S1 executive option value. This difference would be greater if the executive were allowed to rebalance his outside portfolio.

Second, the optimal $p$ value is restricted to lie between 0 and 1, implying that the executive cannot short-sell the risk-free asset or the market portfolio. This restriction is necessary because of the first imperfection. When $p$ is fixed over time, short-selling the risk-free asset or the market portfolio leads to a non-zero probability of negative terminal wealth, which is not defined under the power utility function.

As a practical matter, this restriction does not seem to affect our results for reasonable parameter values. We later show that most optimal $p$ values fall between 0 and 1 for reasonable parameters. Besides, a $p$ value outside the range of 0 to 1 may not represent the portfolio choice of a typical executive. Certainly, with positive net supply of both the risky and risk-free assets in the economy, this cannot be the portfolio choice of an average investor.

## II. VALUATION RESULTS

We describe our results in terms of differences between S1 and S2 values and sensitivities.

### Executive Option Values

Both the two state-variable S2 model and the one state-variable S1 model measure executive option value by the amount of outside wealth that gives the executive the same expected utility as options. The S2 model allows the executive to invest outside wealth optimally in a combination of the risk-free asset and the market portfolio; the S1 model allows investment of outside wealth only in the risk-free asset. The investment opportunity set for outside wealth is therefore more attractive in the S2 model than in the S1 model. For any given option holding, less outside wealth is required to generate the same expected utility as options in the S2 model than in the S1 model. As a result, the S2 executive option value is lower than the S1 executive option value.

Exhibit 2 reports the simulation results for an executive who has 25,000 options and \$5 million in non-option wealth. The options have an exercise price of \$30, ten years to maturity, and three years to vesting. The restricted stock has a restriction period of ten years, and is treated as an option with zero exercise price. The annual risk-free rate equals 5%; the annual standard deviations of stock and market returns equal 40% and 20%; the stock beta equals 1.0; and the annual dividend yield equals 1% (base measures we generally use throughout the analysis).

The four panels in Exhibit 2 consider different combinations of the expected market risk premium, *rmrf*, and the executive's risk aversion, $\gamma$. Within panels, we report results for different values of the proportion of non-option wealth invested in company stock, $\alpha$, and the current stock price, $S_0$. Each cell reports the ratio of the S2 value to RN value, the optimal $p$ in the S2 model, and the ratio of the $S_1$ value to RN value. The RN value itself is reported at the head of the exhibit.

Panel A reports the results for *rmrf* = 6% and $\gamma = 2$. It shows that the ratio of S2 to the RN value is always less than or equal to the ratio of S1 to the RN value. The two ratios are equal only when the optimal $p$ in the S2 model is zero; i.e., when the two models are identical. This happens when nearly all the executive's wealth is tied up in the company stock ($\alpha = 0.99$).

An extremely undiversified executive is very averse to any additional risk, and invests outside wealth completely in the risk-free asset. A more diversified executive in the S2 model invests a positive proportion of outside wealth in the market portfolio, making the S2 value lower than the S1 value. In other words, the S1 value overstates the executive option value for all but extremely undiversified executives. This result holds for out-of-the-money options, at-the-money options, in-the-money options, and restricted stock.

The difference between the S2 and the S1 executive option values is economically significant. For at-the-money (ATM) options (corresponding to a stock price of \$30) held by an executive with 50% of non-option wealth invested in the company stock, the RN value of the option equals \$15.70. Given *rmrf* and $\gamma$ values of 6% and 2, Panel A further shows that the S2 value equals \$0.457 × 15.70 = \$7.17, while the S1 value equals \$0.550 × 15.70 = \$8.64. The difference between these two values equals \$1.47, or 21% of the S2 value. This difference increases as $\alpha$ declines; it seems to be the highest for options that are not too far from the money.

Panel B reports the results for *rmrf* = 6% and $\gamma = 3$. The intuition gained in Panel A remains unchanged, but the differences narrow between the S2 and the S1 values. This occurs because a more risk-averse executive invests a lower proportion of outside wealth in the market portfolio. For the same benchmark ATM option, the S2 and the S1 values equal \$0.356 × 15.70 = \$5.59 and \$0.403 × 15.70 = \$6.33; the difference equals 13% of the S2 value.

Panel C reports the results for *rmrf* = 8% and $\gamma = 2$. A higher market risk premium combined with moderate risk aversion now makes the S2 value lower than the S1 value in every single case because the executive always chooses to invest some proportion (in fact, the majority) of outside wealth in the market portfolio. For the benchmark ATM option, the S2 and the S1 values equal \$0.459 × 15.70 = \$7.21 and \$0.611 × 15.70 = \$9.59, and the difference equals 33% of the S2 value.

Panel D reports the results for *rmrf* = 8% and $\gamma = 3$. As before, the differences between the S2 and the S1 values narrow again. For the benchmark option, the S2 and the S1 values now equal \$0.356 × 15.70 = \$5.59 and \$0.441 × 15.7 = \$6.92, and the difference equals 24% of the S2 value.

Exhibit 3 uses a bar chart to show the percentage difference between the S2 and S1 values (calculated as 100 × (S1 − S2)/S2) for selected combinations of the

# Exhibit 2

**Ratios of Executive Value to Risk-Neutral Value for Stock Options and Restricted Stock in Two Models**

| | Stock options | | | | | Restricted stock | |
|---|---|---|---|---|---|---|---|
| Stock price ($) | 5.00 | 15.00 | 30.00 | 45.00 | 60.00 | 30.00 | |
| RN value ($) | 0.68 | 5.34 | 15.70 | 27.70 | 40.44 | 30.00 | |
| $\alpha$ | | | | | | | |
| *Panel A: rmrf = 6%, $\gamma = 2$* | | | | | | | |
| 0.99 | **0.005** | **0.039** | **0.100** | **0.155** | **0.205** | **0.122** | *S2/RN* |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | *p* |
| | 0.005 | 0.039 | 0.100 | 0.155 | 0.205 | 0.122 | *S1/RN* |
| 0.75 | **0.044** | **0.180** | **0.323** | **0.407** | **0.460** | **0.308** | |
| | 0.50 | 0.49 | 0.46 | 0.45 | 0.45 | 0.48 | |
| | 0.052 | 0.211 | 0.363 | 0.447 | 0.499 | 0.344 | |
| 0.50 | **0.106** | **0.303** | **0.457** | **0.535** | **0.581** | **0.434** | |
| | 0.60 | 0.59 | 0.57 | 0.56 | 0.55 | 0.59 | |
| | 0.142 | 0.388 | 0.550 | 0.620 | 0.658 | 0.517 | |
| 0.25 | **0.237** | **0.468** | **0.601** | **0.658** | **0.689** | **0.579** | |
| | 0.65 | 0.64 | 0.62 | 0.61 | 0.61 | 0.64 | |
| | 0.368 | 0.664 | 0.777 | 0.810 | 0.822 | 0.739 | |
| 0.01 | **0.767** | **0.821** | **0.832** | **0.832** | **0.833** | **0.825** | |
| | 0.70 | 0.69 | 0.67 | 0.66 | 0.65 | 0.68 | |
| | 1.922 | 1.542 | 1.313 | 1.199 | 1.131 | 1.220 | |
| *Panel B: rmrf = 6%, $\gamma = 3$* | | | | | | | |
| 0.99 | **0.000** | **0.007** | **0.025** | **0.045** | **0.063** | **0.048** | |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | |
| | 0.000 | 0.007 | 0.025 | 0.045 | 0.063 | 0.048 | |
| 0.75 | **0.012** | **0.100** | **0.220** | **0.294** | **0.342** | **0.215** | |
| | 0.17 | 0.17 | 0.15 | 0.14 | 0.13 | 0.15 | |
| | 0.014 | 0.109 | 0.233 | 0.307 | 0.354 | 0.226 | |
| 0.50 | **0.042** | **0.203** | **0.356** | **0.434** | **0.478** | **0.334** | |
| | 0.30 | 0.29 | 0.27 | 0.26 | 0.26 | 0.28 | |
| | 0.054 | 0.243 | 0.403 | 0.478 | 0.519 | 0.375 | |
| 0.25 | **0.128** | **0.360** | **0.506** | **0.570** | **0.603** | **0.482** | |
| | 0.37 | 0.36 | 0.35 | 0.34 | 0.33 | 0.35 | |
| | 0.190 | 0.478 | 0.619 | 0.667 | 0.687 | 0.582 | |
| 0.01 | **0.678** | **0.766** | **0.782** | **0.781** | **0.778** | **0.768** | |
| | 0.45 | 0.43 | 0.41 | 0.40 | 0.39 | 0.41 | |
| | 1.652 | 1.353 | 1.157 | 1.058 | 1.000 | 1.087 | |

market risk premium, the proportion of non-option wealth invested in company stock, and the current stock price (relative to the fixed exercise price of $30) as in Panels A and C of Exhibit 2. The difference between the two values is negligible when the proportion equals 0.99, but quite significant when the proportion of wealth in company stock equals 0.50 or 0.01.

The combined results of Exhibit 2 and Exhibit 3 illustrate that the one state-variable S1 model significantly overvalues executive stock options and restricted stock.

## Violation of RN Value Upper Bound

As company executives are barred from selling or hedging their options, they attach less value to the options than an unrestricted outside investor. The option value to an outside investor, or the RN value, imposes an upper bound that any estimated executive option value should not exceed. Hall and Murphy [2002] note that one limitation of the S1 model is that this upper bound is sometimes violated.

Exhibit 2 shows that the S1 value exceeds the RN values when the company stock constitutes only a small proportion of an executive's total non-option wealth (e.g., $\alpha = 0.01$), but the S2 value is always lower than the RN value—because an investor who holds only the risk-free asset is locally risk-neutral. If the expected return of a risky asset is higher than the risk-free rate, the investor will always prefer $1 in the risky asset to $1 in the risk-free asset.

In the S1 model, an executive's outside wealth is invested in the risk-free asset only. The executive also will be locally risk-neutral, with little wealth in the company stock. If the expected return (based on RN value) of an option is higher than the risk-free rate, the executive derives higher utility from $1 in the option (of RN value) than from $1 in the risk-free asset, and will thus be willing to trade more than $1 of outside wealth in the risk-free asset for $1 in options (of RN value).

**EXHIBIT 2** (continued)

**Ratios of Executive Value to Risk-Neutral Value for Stock Options and Restricted Stock in Two Models**

| | Stock options | | | | | Restricted stock | |
|---|---|---|---|---|---|---|---|
| Stock price ($) | 5.00 | 15.00 | 30.00 | 45.00 | 60.00 | 30.00 | |
| RN value ($) | 0.68 | 5.34 | 15.70 | 27.70 | 40.44 | 30.00 | |
| $\alpha$ | | | | | | | |
| *Panel C: rmrf = 8%, $\gamma$ = 2* | | | | | | | |
| 0.99 | **0.007** | **0.046** | **0.113** | **0.171** | **0.223** | **0.133** | **S2/RN** |
| | 0.55 | 0.53 | 0.55 | 0.62 | 0.69 | 0.51 | *p* |
| | 0.008 | 0.048 | 0.118 | 0.179 | 0.234 | 0.142 | *S1/RN* |
| 0.75 | **0.053** | **0.187** | **0.327** | **0.409** | **0.462** | **0.310** | |
| | 0.90 | 0.89 | 0.86 | 0.84 | 0.84 | 0.90 | |
| | 0.069 | 0.245 | 0.403 | 0.486 | 0.538 | 0.384 | |
| 0.50 | **0.122** | **0.311** | **0.459** | **0.535** | **0.580** | **0.436** | |
| | 0.92 | 0.91 | 0.88 | 0.89 | 0.88 | 0.92 | |
| | 0.183 | 0.451 | 0.611 | 0.676 | 0.709 | 0.578 | |
| 0.25 | **0.260** | **0.475** | **0.599** | **0.656** | **0.688** | **0.581** | |
| | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.93 | |
| | 0.473 | 0.780 | 0.874 | 0.894 | 0.898 | 0.833 | |
| 0.01 | **0.787** | **0.825** | **0.831** | **0.829** | **0.830** | **0.825** | |
| | 0.94 | 0.93 | 0.91 | 0.91 | 0.90 | 0.94 | |
| | 2.587 | 1.921 | 1.569 | 1.400 | 1.300 | 1.409 | |
| *Panel D: rmrf = 8%, $\gamma$ = 3* | | | | | | | |
| 0.99 | **0.000** | **0.008** | **0.030** | **0.052** | **0.072** | **0.054** | |
| | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | |
| | 0.000 | 0.008 | 0.030 | 0.052 | 0.072 | 0.054 | |
| 0.75 | **0.014** | **0.102** | **0.221** | **0.296** | **0.345** | **0.218** | |
| | 0.42 | 0.41 | 0.39 | 0.37 | 0.36 | 0.40 | |
| | 0.017 | 0.124 | 0.255 | 0.331 | 0.379 | 0.248 | |
| 0.50 | **0.046** | **0.205** | **0.356** | **0.434** | **0.480** | **0.338** | |
| | 0.51 | 0.50 | 0.48 | 0.47 | 0.46 | 0.49 | |
| | 0.067 | 0.278 | 0.441 | 0.515 | 0.554 | 0.414 | |
| 0.25 | **0.136** | **0.361** | **0.505** | **0.569** | **0.603** | **0.484** | |
| | 0.56 | 0.55 | 0.54 | 0.53 | 0.52 | 0.55 | |
| | 0.240 | 0.552 | 0.687 | 0.726 | 0.741 | 0.647 | |
| 0.01 | **0.679** | **0.762** | **0.779** | **0.778** | **0.775** | **0.768** | |
| | 0.62 | 0.60 | 0.58 | 0.57 | 0.57 | 0.59 | |
| | 2.191 | 1.656 | 1.354 | 1.209 | 1.124 | 1.244 | |

S2 or S1 value exceeds RN value.

In the S2 model, the executive's outside wealth is optimally invested in the risk-free asset and the market portfolio. According to the CAPM, any investor's optimal portfolio consists of only the risk-free asset and the market portfolio. Forcing the holding of options pushes the executive away from the optimal portfolio. Therefore, the executive always derives higher utility from $1 in outside wealth than from $1 in options (of RN value). Thus, the executive is willing to trade $1 in options (of RN value) for less than $1 in outside wealth.

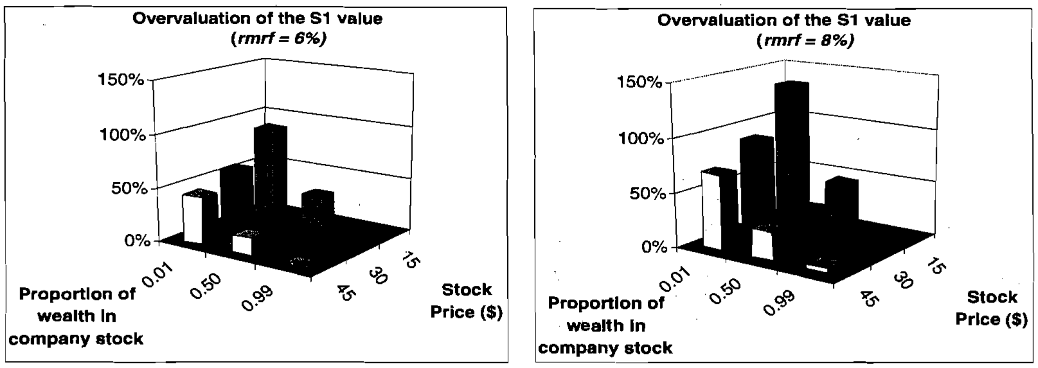
We prove the intuition rigorously in the appendix in a simple one-period model for restricted stock. We show that in the one state-variable S1 model, as long as the company has positive $\beta$ and *rmrf* is positive, a sufficiently diversified executive will value restricted stock at more than its market price. In the two state-variable S2 model, a less-than-fully diversified executive always values a restricted stock at less than its market price, and a fully diversified executive, like an outside investor, values the restricted stock at its market price. Numerical simulations in the one-period model show that this result also holds for options, although it is difficult to prove it mathematically because of the non-linear payoff structure of options.

In multiperiod simulations, it is possible to have an S2 value higher than the RN value if *rmrf* is very high, say, 20%.[12] When *rmrf* is very high, the executive's optimal portfolio involves short-selling the risk-free asset and investing the proceeds in the market. Our algorithm does not allow short sales, however, which makes the executive's outside portfolio suboptimal and less attractive than the stocks and options. As a result, the S2 value can be higher than the RN value in this constrained setting when *rmrf* is very high. For more reasonable parameter values, the S2 value is always lower than the RN value.

Exhibit 4 reports the range of the proportion ($\alpha$) of non-option wealth invested in company stock over which the S1 value of an option exceeds the RN value for given stock prices and the market risk premium *rmrf*. Given a

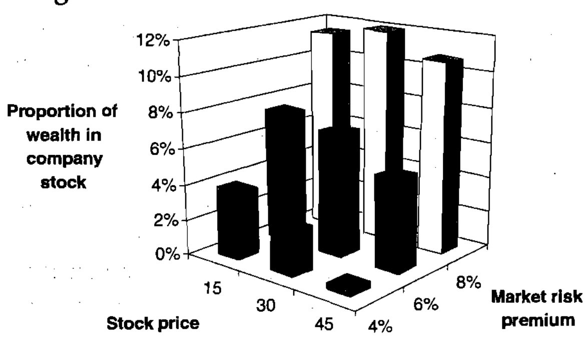
## Exhibit 3

**Percent Overvaluations of S1 Executive Stock Option Values**




## Exhibit 4

**When S1 Executive Stock Option Value is Higher than RN Value**




relative risk aversion coefficient of 2.5 and a 6% market risk premium, the S1 executive option value exceeds the RN value when $\alpha$ is lower than 7.4%, 6.9%, and 5.2% for stock prices of $15, $30, and $45.

The threshold levels are higher when the executive holds fewer options or the expected stock return is higher. Given the same market risk premium and a holding of 5,000 options, the corresponding $\alpha$ thresholds are 9.6%, 11.8%, and 12.6%. If we increase the market risk premium to 8% and the stock beta to 2.0, the $\alpha$ thresholds increase to 29.5%, 33.5%, and 34.1% with a holding of 25,000 options.

The combined analysis of Exhibits 2, 3, and 4 shows that the S1 values are generally too high and may even exceed the RN values for not so undiversified executives. This can be especially problematic in the case of options held by junior executives and non-executive employees, who face fewer constraints and probably invest only a low proportion of their total wealth in their company's stock. For the same parameter values, the S2 value never exceeds the RN value.

### Sensitivity to Unobservable Parameters

One drawback of all utility-based option valuation models is that solution depends on several parameters that are observable only to executives themselves, and not to company accountants, regulators, researchers, or the general public. In our case, these parameters are the executive's expected market risk premium *rmrf*, risk aversion $\gamma$, and proportion $\alpha$ of non-option wealth invested in the company stock.

Rubinstein [1995, p. 8] points out in a similar context:

> The model uses sixteen input variables, many of them difficult to estimate. A firm seeking to overvalue its options could actually report values almost double those reported by an otherwise similar firm seeking to undervalue its options.

The unobservable parameters also make empirical testing with utility-based models difficult, because the result depends on the choice of parameter values. For these reasons, we should prefer a model less sensitive to the unobservable parameters. We show that the two state-variable S2 executive option value is less sensitive to these unobservable parameters than the one state-variable S1 executive option value.

***Sensitivity to expected market risk premium.*** Exhibit 5 shows that the S2 value is much less sensitive to changes in *rmrf* than the S1 value. We assume the executive has 25,000 options, $5 million of non-option wealth divided evenly between the company stock and the outside portfolio, and a risk aversion coefficient of 2.5. When the *rmrf* increases from 2% to 10%, the ratio of the S2 value to the RN value of an at-the-money option rises from 0.380 to 0.402, which represents a 5.8% increase. The ratio of the S1 value to the RN value rises from 0.380 to 0.565, which represents a 48.7%

## EXHIBIT 5
**Sensitivity to Expected Market Risk Premium**

| Stock price ($) | RN value ($) | Expected market risk premium *rmrf* 2% | 4% | 6% | 8% | 10% | |
|---|---|---|---|---|---|---|---|
| *Panel A: Option* | | | | | | | |
| 15.00 | 5.34 | **0.225** | **0.240** | **0.244** | **0.249** | **0.255** | ***S2/RN*** |
| (OTM) | | 0.00 | 0.16 | 0.41 | 0.66 | 0.92 | *p* |
| | | 0.225 | 0.262 | 0.303 | 0.348 | 0.399 | *S1/RN* |
| 30.00 | 15.70 | **0.380** | **0.399** | **0.400** | **0.402** | **0.402** | |
| (ATM) | | 0.00 | 0.14 | 0.39 | 0.64 | 0.90 | |
| | | 0.380 | 0.422 | 0.466 | 0.514 | 0.565 | |
| 45.00 | 27.70 | **0.458** | **0.478** | **0.481** | **0.480** | **0.479** | |
| (ITM) | | 0.00 | 0.13 | 0.37 | 0.63 | 0.89 | |
| | | 0.458 | 0.498 | 0.540 | 0.585 | 0.633 | |
| *Panel B: Restricted stock* | | | | | | | |
| 30.00 | 30.00 | **0.350** | **0.372** | **0.378** | **0.380** | **0.382** | |
| | | 0.00 | 0.13 | 0.39 | 0.66 | 0.93 | |
| | | 0.350 | 0.390 | 0.434 | 0.482 | 0.534 | |

increase. The out-of-the-money option, the in-the-money option, and the restricted stock show similar patterns.

Exhibit 6 plots the results of Exhibit 5 to provide a visual comparison of the two models.[13]

The reduced *rmrf* sensitivity of the S2 values is reassuring because it is closer to the *rmrf* sensitivity of the RN model than the S1 model. (Recall that the RN values are totally insensitive to changes in *rmrf*.) The reasons behind this result are as follows.

In the S1 model, the executive can invest outside wealth only in the risk-free asset, so the attractiveness of outside wealth remains unchanged, no matter how much *rmrf* increases. As *rmrf* increases, however, the CAPM implies that the expected stock return increases by a factor of $\beta$. As long as the company has a positive beta, the expected return increases for both the stock and the option. Therefore, an increase in *rmrf* increases the attractiveness of the option portfolio compared to outside wealth. The executive now requires a greater amount of outside wealth to give up the options. This explains why the S1 value is very sensitive to changes in *rmrf*.

In the S2 model, the executive optimally allocates outside wealth between the risk-free asset and the market portfolio. An increase in *rmrf* now increases the expected return of outside wealth in two ways. First, the expected market return increases, which increases the overall expected return of the outside wealth. Second, as the market portfolio becomes more attractive than the risk-free asset, the executive also invests a higher proportion of outside wealth $p$ in the market, which further increases the overall expected return of the outside wealth. This is apparent from Exhibit 5, which shows that the optimal $p$ increases with *rmrf*.

The net result is that an increase in *rmrf* makes both the option portfolio and the outside portfolio more attractive, but their relative attractiveness does not change by much. This explains why the S2 model value is much less sensitive to changes in *rmrf*.

***Sensitivity to executive's risk aversion $\gamma$.*** Exhibit 7 shows that the difference between the S2 value and the S1 value increases as the executive's risk aversion $\gamma$ declines, which makes the S2 value less sensitive to changes in $\gamma$. (The stock price in Exhibit 7 equals $30; *rmrf* equals 6%; and the remaining parameters are the same as in Exhibit 5.)

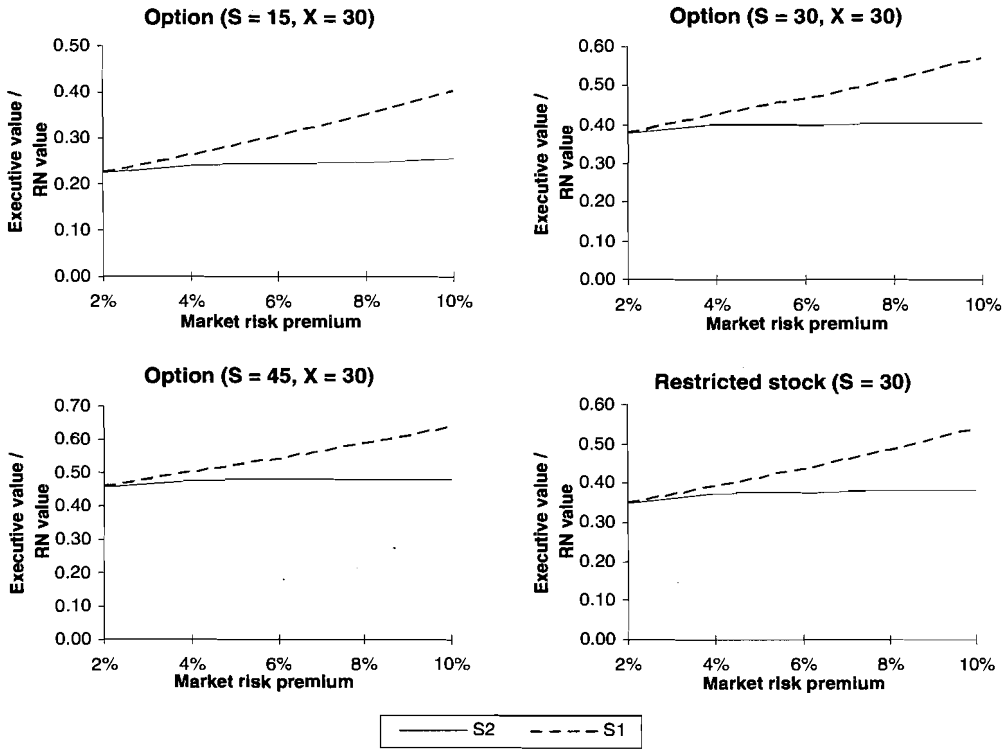
This is a direct result of the overvaluation of option values implied by the S1 model for low $\gamma$, but not for high $\gamma$. When $\gamma$ is low, an executive in the S2 model optimally invests a large proportion of outside wealth in the market portfolio, while in the S1 model outside wealth can be invested only in the risk-free asset. This makes the options too attractive compared to outside wealth for the executive in the S1 model, leading the model to overestimate the option value.

When $\gamma$ is high, even an executive in the S2 model invests almost all outside wealth in the risk-free asset, much as in the S1 model. This is why the two values converge as $\gamma$ increases.

***Sensitivity to proportion of non-option wealth invested in company stock.*** Exhibit 8 shows that the difference between the S2 value and the S1 value increases as the proportion $\alpha$ of non-option wealth invested in company stock declines, which makes the S2 value less sensitive to changes in $\alpha$. This is also a direct result of the overvaluation of option values implied by the S1 model for low $\alpha$, but not for high $\alpha$.

When $\alpha$ is low, an executive in the S2 model optimally invests a large proportion of outside wealth in the market portfolio, while in the S1 model outside wealth can

## EXHIBIT 6
**Sensitivity to Expected Market Risk Premium Graphed**




be invested only in the risk-free asset. This again makes the options too attractive compared to outside wealth for the executive in the S1 model, leading the model to overestimate the option value. When $\alpha$ is very low, we have shown that the S1 value even crosses the RN value upper bound.

When $\alpha$ is high, an executive in the S2 model invests almost all outside wealth in the risk-free asset, much as in the S1 model. This is why the two values converge as $\alpha$ increases.

## III. EARLY EXERCISE, COMPANY COST, AND REQUIRED INCENTIVE EFFECTS

The company cost of stock option grants is their opportunity cost to diversified and risk-neutral shareholder investors. This may be quite different from their executive value. The company cost also differs from the RN value we have discussed. The RN value represents the option value to risk-neutral investors assuming that these investors optimally exercise the option, but the company cost represents the option value to the same investors assuming that the executives optimally exercise the option.

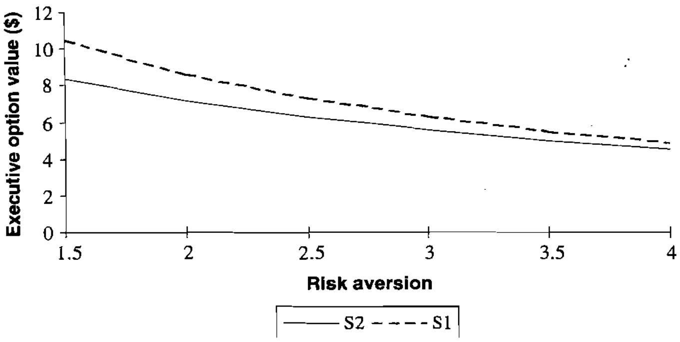
Since the executive exercise policy is suboptimal from the perspective of risk-neutral investors, the company cost must be lower than the RN value. The company cost, however, must be higher than the executive value, because the shareholder investors are well diversified and require a lower return for bearing the unsystematic risk of stock options.

Kulatilaka and Marcus [1994], Huddart and Lang [1996], Carpenter [1998], and Hall and Murphy [2002] discuss computation of the company cost versus the executive value of stock options. The difference between these two quantities represents the required incentive effects of option compensation.[14]

If option compensation does not motivate executives to increase company value by at least the difference amount, it may be more desirable to offer cash compensation unrelated to stock price performance (putting aside any other benefits of option compensation such as tax savings and accounting considerations). A computation of the expected life and the company cost of executive options using the two state-variable S2 model shows that the one state-variable S1 model understates the difference between executive value and company cost.

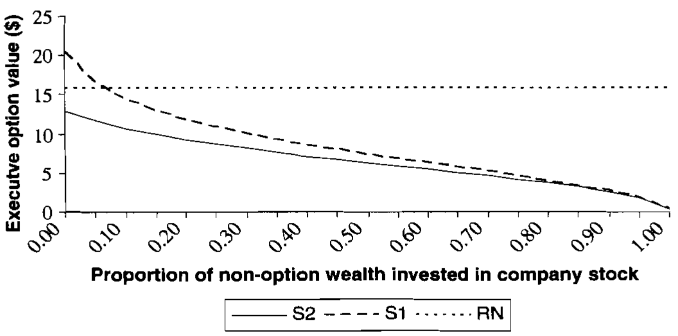
## EXHIBIT 7

**Sensitivity to Executive's Relative Risk Aversion**




## EXHIBIT 8

**Sensitivity to Proportion of Non-Option Wealth Invested in Company Stock**




### Expected Life of Options

We use the expected life of options to summarize the early exercise decisions. The expected life is calculated as the probability-weighted average time between the grant date and the exercise date (or the maturity date in the case of no exercise).[15]

Exhibit 9 shows that the expected life in the S2 model is shorter than in the S1 model. This is because the executive invests the exercise proceeds in the outside portfolio, and a more attractive outside portfolio in the S2 model induces the executive to exercise options earlier than in the S1 model. The difference increases with the degree the options are in the money (which causes early exercise to begin with), with lower $\alpha$ (which causes the S1 model to overvalue the option and makes early exercise less attractive), and with higher *rmrf* (which results in a higher expected stock and option return and makes early exercise followed by investment in the risk-free asset less attractive in the S1 model).

### Company Cost and Incentive Effects

Hall and Murphy [2002] estimate the company cost of stock options using a barrier option methodology. We modify this methodology for our purposes. We start by constructing the stock price thresholds that trigger early exercise. In each period, and for each value of cumulative market return, we find the stock price above which an executive always exercises the option. We next estimate a barrier function at each period by regressing the stock price thresholds on the cumulative market returns.[16]

We expect a positive regression coefficient for reasons as follows. If market returns have been high, the executive's outside wealth is likely to account for a greater proportion of total wealth. As a result, the executive is likely to be more diversified and more willing to hold options, leading to a higher exercise threshold. The regression coefficients are consistent with this intuition.

We build a parallel risk-neutral grid and calculate the implied risk-neutral exercise barrier at each period and for each value of cumulative market return using the barrier function we have estimated. We finally estimate the company cost of options as their risk-neutral value, given the exogenous exercise barriers.

Exhibit 9 also shows the executive value, the company cost, and the difference between the two as computed using the two state-variable S2 model and the one state-variable S1 model for stock options. We present the results for different values of $\alpha$, depth in the money, and *rmrf*.

In all cases, the company cost in the S2 model is slightly lower than in the S1 model, although the executive value in the S2 model is substantially lower than in the S1 model. As a result, the two quantities differ more in the S2 model than in the S1 model. For example, the executive value is 54% lower than the company cost in the S2 model for at-the-money options when $\alpha = 0.50$ and *rmrf* = 6%, compared to 47% lower in the S1 model.

The divergence between executive value and company cost is particularly acute when $\alpha = 0.10$ and *rmrf* = 8%. For this choice of parameters, the executive value is 29% lower than the company cost in the S2 model, but 6% higher in the S1 model (which is counter-intuitive, and results from violation of the RN value upper bounds).

Overall, the S2 model results show that stock options must provide greater benefits in the form of incentive effects to executives to increase the firm value in order to overcome the greater difference between executive value and company cost.

## EXHIBIT 9

### Executive Value, Expected Life, and Company Cost of Stock Options

| α | Stock price ($) | RN value ($) | Executive value ($) | | Expected life (years) | | Company cost ($) | | Difference (cost – value)/cost | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | S2 | S1 | S2 | S1 | S2 | S1 | S2 | S1 |
| ***Panel A: rmrf = 6%*** | | | | | | | | | | |
| 0.90 | 15.00 | 5.34 | **0.36** | 0.37 | **7.72** | 7.73 | **3.91** | 3.90 | **0.91** | 0.91 |
| 0.90 | 30.00 | 15.70 | **2.54** | 2.57 | **5.59** | 5.59 | **12.46** | 12.44 | **0.80** | 0.79 |
| 0.90 | 45.00 | 27.70 | **6.34** | 6.40 | **4.43** | 4.49 | **23.56** | 23.56 | **0.73** | 0.73 |
| 0.50 | 15.00 | 5.34 | **1.30** | 1.62 | **8.16** | 8.21 | **4.45** | 4.46 | **0.71** | 0.64 |
| 0.50 | 30.00 | 15.70 | **6.29** | 7.32 | **6.33** | 6.44 | **13.69** | 13.85 | **0.54** | 0.47 |
| 0.50 | 45.00 | 27.70 | **13.31** | 14.97 | **5.20** | 5.35 | **25.05** | 25.27 | **0.47** | 0.41 |
| 0.10 | 15.00 | 5.34 | **3.09** | 4.74 | **8.84** | 9.06 | **4.98** | 5.10 | **0.38** | 0.07 |
| 0.10 | 30.00 | 15.70 | **10.64** | 14.41 | **7.40** | 8.01 | **14.83** | 15.25 | **0.28** | 0.05 |
| 0.10 | 45.00 | 27.70 | **19.76** | 25.14 | **6.28** | 7.16 | **26.40** | 27.09 | **0.25** | 0.07 |
| ***Panel B: rmrf = 8%*** | | | | | | | | | | |
| 0.90 | 15.00 | 5.34 | **0.37** | 0.43 | **7.50** | 7.50 | **3.98** | 3.95 | **0.91** | 0.89 |
| 0.90 | 30.00 | 15.70 | **2.58** | 2.86 | **5.35** | 5.41 | **12.53** | 12.53 | **0.79** | 0.77 |
| 0.90 | 45.00 | 27.70 | **6.44** | 7.00 | **4.29** | 4.32 | **23.60** | 23.71 | **0.73** | 0.70 |
| 0.50 | 15.00 | 5.34 | **1.33** | 1.86 | **8.01** | 8.05 | **4.51** | 4.53 | **0.70** | 0.59 |
| 0.50 | 30.00 | 15.70 | **6.30** | 8.07 | **6.15** | 6.32 | **13.75** | 13.99 | **0.54** | 0.42 |
| 0.50 | 45.00 | 27.70 | **13.30** | 16.22 | **5.04** | 5.26 | **25.05** | 25.50 | **0.47** | 0.36 |
| 0.10 | 15.00 | 5.34 | **3.11** | 5.61 | **8.73** | 9.04 | **5.02** | 5.17 | **0.38** | -0.09 |
| 0.10 | 30.00 | 15.70 | **10.58** | 16.40 | **7.23** | 8.11 | **14.85** | 15.41 | **0.29** | -0.06 |
| 0.10 | 45.00 | 27.70 | **19.66** | 28.06 | **6.11** | 7.38 | **26.43** | 27.32 | **0.26** | -0.03 |

## IV. VALUATION OF INDEXED STRIKE PRICE OPTIONS

Indexed options, whose strike (or exercise) price is tied to a market or industry benchmark, have attracted some attention in recent years. These options can filter out the effect of marketwide or industrywide trends in order to reward executives on stock price performance.

Johnson and Tian [2000] describe a risk-neutral methodology for the valuation of indexed options. They derive closed-form solutions and describe the very different incentive effects of indexed strike price options. The methodology does not adjust for the executive's subjective valuation arising from inability to hedge the options, nor can it value the general American-style option with an initial vesting period.

Our S2 methodology based on a two state-variable grid can easily be extended to estimate the risk-neutral value, the executive value, and the company cost of an American-style option whose exercise price is any arbitrary function of the current market level, but we cannot accommodate indexing to a benchmark other than this second state-variable.

Exhibit 10 shows the results for indexed strike options whose strike price equals $30 multiplied by the ratio of the current market level at the time of exercise to the starting market level at the time of grant. It also shows the comparative results for the traditional fixed strike options whose strike price always equals $30.[17]

The rows in Exhibit 10 show the results for a range of values of the correlation between log stock returns and log market returns, stock volatility, beta, and depth in the money. The RN value, the executive value, and the company cost are always lower for the indexed strike price options than for the fixed strike price options. The difference between RN and executive values widens as the correlation increases.

When the correlation equals 0.50 and the stock volatility equals 0.30, the RN value, the executive value, and the company cost for at-the-money options equal $9.04, $3.79, and $8.34 for indexed strike options, and $13.34, $6.11, and $11.68 for fixed strike options. The last column shows the difference between executive value and company cost. The executive value is 55% lower than the company cost for indexed strike options, and 48% lower for fixed strike options.

In general, there is a greater percentage difference between executive value and company cost for the indexed strike price options than for the fixed strike price options. The comparison becomes worse as the correlation increases.

## V. IMPLICATIONS FOR FASB PROPOSAL ON ACCOUNTING TREATMENT OF STOCK OPTIONS

Accounting for stock option compensation has been a controversial issue in recent years. Currently, companies may choose to record their option grants at either intrinsic value or fair value. The intrinsic value, or the difference between the stock price and the exercise price

## EXHIBIT 10

**Value of Indexed Strike Price Options with S2 Model (rmrf = 6%)**

| | | | Stock price ($) | RN values ($) | | Executive values ($) | | Company cost ($) | | Difference (cost – value)/cost | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_{ms}$ | $\sigma_s$ | $\beta$ | | Indexed strike | Fixed strike | Indexed strike | Fixed strike | Indexed strike | Fixed strike | Indexed strike | Fixed strike |
| 0.25 | 0.20 | 0.25 | 15.00 | **1.46** | 1.97 | **0.41** | 0.53 | **1.38** | 1.82 | **0.70** | 0.71 |
| | | | 30.00 | **8.41** | 11.00 | **4.57** | 5.43 | **7.91** | 9.73 | **0.42** | 0.44 |
| | | | 45.00 | **19.02** | 23.20 | **12.88** | 14.65 | **18.32** | 20.76 | **0.30** | 0.29 |
| 0.50 | 0.30 | 0.75 | 15.00 | **1.76** | 3.68 | **0.33** | 0.97 | **1.61** | 3.17 | **0.80** | 0.69 |
| | | | 30.00 | **9.04** | 13.34 | **3.79** | 6.11 | **8.34** | 11.68 | **0.55** | 0.48 |
| | | | 45.00 | **19.70** | 25.23 | **10.91** | 14.23 | **18.73** | 22.83 | **0.42** | 0.38 |
| 0.75 | 0.40 | 1.50 | 15.00 | **2.06** | 5.34 | **0.35** | 1.79 | **1.80** | 4.34 | **0.81** | 0.59 |
| | | | 30.00 | **9.62** | 15.70 | **3.91** | 7.99 | **8.76** | 13.76 | **0.55** | 0.42 |
| | | | 45.00 | **20.32** | 27.70 | **10.91** | 16.24 | **19.34** | 25.15 | **0.44** | 0.35 |

on the grant date, typically equals zero, as most options are granted at the money. The fair value, however, is typically calculated using established option pricing models such as the Black-Scholes model.

In a recent exposure draft, the Financial Accounting Standards Board (FASB) proposes to require U.S. public companies to recognize stock option compensation at fair value in their income statements. The FASB recommends calculation of the fair value of option grants as the risk-neutral value of the options using a lattice model, with adjustment to reflect the executive's exercise behavior.

From the company's point of view, it is true that the cost of granting options should be calculated in the risk-neutral setting as this is the amount a diversified outside investor is willing to pay for the options. But because executives are risk-averse and undiversified, the value of options to them personally can be substantially lower than the company cost. Thus the executive value estimated by our model is the appropriate measure in empirical work concerning the incentive effects of options.

Our model is also useful in estimating the company cost of option grants, because it helps us understand option exercise patterns. Undiversified and risk-averse executives tend to exercise options sooner than a diversified investor optimally would. This makes option grants cost less to the company than the risk-neutral value estimated by standard option pricing models.

The utility-based model that we build to determine the optimal exercise behavior of the undiversified executives is the theoretically correct approach; it helps us understand how various parameters affect the option exercise pattern. The disadvantage of this approach is that it requires assumptions about the executive's portfolio of option holdings, risk aversion, diversification level, and expected market risk premium.

In its exposure draft, the FASB recommends companies use historical exercise data to calculate the company cost of the options. While this alternative approach requires fewer assumptions about the unobservable parameters, it also assumes the option exercise pattern is exogenously given, with no relation to many parameters such as time to maturity and market conditions. A study that combines our model with historical option exercise data may improve the estimation of company cost of option grants—a worthwhile research project.

## VI. CONCLUSIONS

Recent annual grants of executive and employee stock options have accounted for around 1% of the market value of the firms included in the ExecuComp database (estimated at their Black-Scholes values). Large and recurrent option grants have attracted considerable attention from shareholders, accountants, regulators, and researchers. While the value of stock options to executives is much lower than the cost to the company incurred by diversified shareholders, how much lower has remained a question.

We identify several shortcomings of current one state-variable models (S1 models) that allow investment of an executive's outside wealth only in the risk-free asset. We show that the one state-variable models produce counter-intuitive valuation results and that there may be limits on their use because their results are sensitive to certain unobservable model parameters.

Our new two state-variable model (the S2 model) allows investment of the executive's outside wealth in combinations of the risk-free asset and the market portfolio. We show that the two state-variable model: 1) is consistent with traditional portfolio theory and the capital asset pricing model; 2) accommodates all option features; 3) is computationally tractable; and 4) overcomes many of the shortcomings of the one state-variable models.

We use a three-dimensional binomial grid proposed by Rubinstein [1994] to model the movement of two less-than-perfectly correlated state variables, which in our case are the stock price and the market level. This grid can be used to value any dynamic portfolio strategy whose outcome depends on these two state variables in addition to the risk-free return. That the executive can optimally allocate non-company outside wealth between the risk-free asset and the market portfolio is a critical distinction. We use theoretical arguments as well as simulation results to show that our model has several desirable features that are lacking in previous models for determining the executive value and the company cost of stock options and restricted stock.

Our first contribution is that the model allows an outside investment opportunity set that is suggested by traditional portfolio theory. The model produces executive option values that are always lower than risk-neutral values. Intuitively, the risk-neutral values must serve as an upper bound on executive values.

Our model shows option values are significantly less sensitive to the unobservable expected market risk premium (which determines the expected stock returns), executive risk aversion, and the proportion of executives' non-option wealth invested in company stock. This is an advantage for company accountants and regulators who want less subjective values. It also helps empirical researchers who typically examine over a long period a large cross-section of executives with possibly heterogeneous and time-varying beliefs about the expected market risk premium.

For most common parameters, the values of options to executives according to our two state-variable model are significantly lower than in the one state-variable models, but the company costs are not that much lower. This suggests there is more of a difference than previously documented between executives' option values and company costs that must be offset by incentives for executives to increase firm value.

Finally, our methodology can be easily applied to the valuation of American-style indexed strike price options with vesting restrictions. Such options may become more popular if there is shareholder activism to reward executives for performing better than the market.

## APPENDIX

### Violation of RN Value Bounds in S1 Model

We consider a simplified version of the model in the text to prove that the S1 value can exceed the RN value but the S2 value cannot. We illustrate this point for the simpler case of restricted stock and an executive whose portfolio consists of restricted company stock and unrestricted outside wealth. In the S1 model, the executive can invest outside wealth only in the risk-free asset. In the S2 model, the executive optimally allocates his outside wealth between the risk-free asset and the market portfolio.

In this one-period model, the executive's initial wealth $W_0$ consists of two components: outside wealth $W_{out}$ and restricted stock wealth $W_{stk}$ (computed as the number of shares of restricted stock multiplied by the market value):

$$W_0 = W_{out} + W_{stk} \tag{A-1}$$

The executive's end-of-period wealth, $W_T$, is given by

$$W_T = W_{out}[R_f + p(R_m - R_f)] + W_{stk}R_s \tag{A-2}$$

where $R_f$ is the risk-free rate, $p$ is the proportion of outside wealth invested in the market portfolio, $R_m$ is the market return, and $R_s$ is the stock return. Notice the restricted stock becomes unrestricted at the end of the holding period. The market return dynamics are given by:

$$\begin{aligned} R_m &= R_f + rmrf + \varepsilon_m \\ E(\varepsilon_m) &= 0 \\ Var(R_m) &= E(\varepsilon_m{}^2) = \sigma_m{}^2 \end{aligned} \tag{A-3}$$

where $rmrf$ is the market risk premium, and $\varepsilon_m$ is the unexpected market return with mean zero and standard deviation $\sigma_m$.

The stock return dynamics are given by

$$\begin{aligned} R_s &= R_f + \beta(R_m - R_f) + \varepsilon_s \\ &= R_f + \beta rmrf + \beta\varepsilon_m + \varepsilon_s \\ E(\varepsilon_s) &= 0 \\ E(\varepsilon_s{}^2) &= (1-\rho^2)\sigma_s{}^2 \quad \mathrm{cov}(\varepsilon_s, \varepsilon_m) = 0 \\ \mathrm{Var}(R_s) &= \beta^2 E(\varepsilon_m{}^2) + E(\varepsilon_s{}^2) = \sigma_s{}^2 \quad \beta = \frac{\rho\sigma_s}{\sigma_m} \end{aligned} \tag{A-4}$$

Here $\varepsilon_s$ is the unique risk of stock, $\beta$ is the stock beta, $\rho$ is the correlation between $R_m$ and $R_s$, and $\sigma_s$ is the standard deviation of stock returns. Notice these return dynamics are slightly different from our model in the text, which assumes that stock returns and market returns are joint-lognormally distributed.

The executive's utility function is defined by end-of-period wealth $W_T$ as follows

$$E[U(W_T)] = E\left(\frac{W_T^{1-\gamma}}{1-\gamma}\right) \quad \text{(A-5)}$$

We value the executive's restricted stock using the outside wealth equivalent (*OWE*) method. *OWE* measures how much outside wealth the executive is willing to give up for an increase in restricted stock grant with the market value of \$1. Notice this definition is also a little different from our model in the text, where *OWE* measures how much outside wealth the executive is willing to give up for one share of stock or one option. Using standard microeconomic principles:

$$OWE = \frac{\partial E(U)/\partial W_{stk}}{\partial E(U)/\partial W_{out}} \quad \text{(A-6)}$$

If *OWE* is greater than one, the executive is willing to give up outside wealth with the market value of more than \$1 for additional restricted stock with the market value of \$1. This means the executive will actually buy restricted company stock at market price with outside wealth. Intuitively, this should not happen.

We show that *OWE* can be greater than one in the S1 model, but *OWE* is always less than or equal to one in the S2 model. The two marginal utilities are given by:

$$\frac{\partial E(U)}{\partial W_{stk}} = E\left(U'(W_T)\frac{\partial W_T}{\partial W_{stk}}\right)$$
$$= E[U'(W_T)](R_f + \beta rmrf) + \text{cov}(U'(W_T), R_s) \quad \text{(A-7)}$$

$$\frac{\partial E(U)}{\partial W_{out}} = E\left(U'(W_T)\frac{\partial W_T}{\partial W_{out}}\right)$$
$$= E[U'(W_T)](R_f + prmrf) + \text{cov}(U'(W_T), pR_m) \quad \text{(A-8)}$$

which use the simple relationship $E(xy) = E(x)E(y) + \text{cov}(x, y)$. In the S1 model, the executive can invest unrestricted wealth only in the risk-free asset, so $p$ is set to zero. We can calculate the difference between marginal utility of restricted stock and marginal utility of outside wealth in this case as

$$\frac{\partial E(U)}{\partial W_{stk}} - \frac{\partial E(U)}{\partial W_{out}} = E[U'(W_T)]\beta rmrf + \text{cov}(U'(W_T),R_s) \quad \text{(A-9)}$$

If the Equation (A-9) result is greater than zero, *OWE* is greater than one, implying that the executive values restricted stock at more than its market value. Setting $p$ to zero also gives us

$$W_T = W_{out}R_f + W_{stk}R_s \quad \text{(A-10)}$$

For an executive with no restricted company stock, $W_{stk} = 0$. Terminal wealth $W_T = W_{out}R_f$ is a constant, which implies that $\text{cov}(U'(W_T),R_s) = 0$. Thus, Equation (A-9) becomes

$$\frac{\partial E(U)}{\partial W_{stk}} - \frac{\partial E(U)}{\partial W_{out}} = E[U'(W_T)]\beta rmrf \quad \text{(A-11)}$$

Since the utility function is concave, its first-order derivative is positive. The Equation (A-11) result is greater than zero if the company has positive $\beta$ and *rmrf* is positive. This implies that the executive value of restricted stock exceeds its market value. As the executive's restricted stock wealth $W_{stk}$ increases, however, cov $(U'(W_T),R_S)$ becomes more negative. For a sufficiently high $W_{stk}$, Equation (A-9) becomes negative. For any $W_{stk}$ lower than this critical point, the executive in the S1 model values restricted stock at more than its market value.

Now we will show that the executive value of restricted stock is never higher than its market value in the S2 model. In this model, the executive optimally allocates $p$ proportion of outside wealth to the market portfolio and the remaining $(1 - p)$ proportion to the risk-free asset. The first-order condition gives:

$$\frac{\partial E(U)}{\partial p} = E\left(U'(W_T)\frac{\partial W_T}{\partial p}\right)$$
$$= -W_{out}\left\{\begin{matrix} E[U'(W_T)]rmrf + \\ \text{cov}(U'(W_T), R_m) \end{matrix}\right\} = 0 \quad \text{(A-12)}$$

If the executive has non-zero outside wealth, we have:

$$E[U'(W_T)]rmrf + \text{cov}(U'(W_T),R_m) = 0 \quad \text{(A-13)}$$

By substituting Equation (A-13) into Equations (A-7) and (A-8), we have:

$$\frac{\partial E(U)}{\partial W_{stk}} = E\left(U'(W_T)\frac{\partial W_T}{\partial W_{stk}}\right)$$
$$= E[U'(W_T)]R_f + \text{cov}(U'(W_T), R_s - \beta R_m) \quad \text{(A-14)}$$

$$\frac{\partial E(U)}{\partial W_{out}} = E\left(U'(W_T)\frac{\partial W_T}{\partial W_{out}}\right) = E[U'(W_T)]R_f \quad \text{(A-15)}$$

The difference between the two marginal utilities therefore equals:

$$
\begin{aligned}
\frac{\partial E(U)}{\partial W_{stk}} - \frac{\partial E(U)}{\partial W_{out}} &= \text{cov}(U'(W_T), R_s - \beta R_m) \\
&= \text{cov}(U'(W_T), (1-\beta)R_f + \varepsilon_s) \\
&= \text{cov}(U'(W_T), \varepsilon_s) \qquad \text{(A-16)}
\end{aligned}
$$

By Equations (A-2) and (A-4), $\text{cov}(\varepsilon_S, \varepsilon_m) = 0$, $W_T$ increases monotonically with $\varepsilon_S$ if $W_{stk} > 0$. Since $U'$ declines monotonically with $W_T$, $U'$ becomes negatively correlated with $\varepsilon_S$. Thus, the right-hand side of Equation (A-16) is always less than or equal to zero. The equality holds only when $W_{stk}$ is zero or $\rho^2$ is one. When $W_{stk}$ is zero, $W_T$ is not correlated with $\varepsilon_S$. When $\rho^2$ is one, the market and the stock are perfectly correlated, implying $\varepsilon_S = 0$.

Overall, in the S2 model the marginal utility of outside wealth with a market value of $1 is always greater than or equal to the marginal utility of the restricted wealth with a market value of $1. The executive value of restricted stock is therefore always less than or equal to its market value. The equality holds only if the executive holds no restricted stock or the restricted stock is perfectly correlated with the market portfolio.

## ENDNOTES

The authors have benefited from the comments of seminar participants at the University of Iowa. They are also obliged to Mark Rubinstein and Raghu Sundaram for many comments that improved this article substantially.

[1]The calculations in these examples are based on a spreadsheet provided on Professor Kevin Murphy's website. We set his step size to 0.10.

[2]Several other authors consider outside investments in the market portfolio, but their results do not apply to the valuation of a substantial block of American-style executive stock options with vesting restrictions. First, Kahl, Liu, and Longstaff [2003] study the optimal investment and consumption choices of executives who own illiquid or restricted stock besides the risk-free asset and the market portfolio in a continuous-time framework, but they do not consider options. Second, Hall and Knox [2002] analyze the incentive effects of underwater options, and Holland and Elder [2003] analyze a financing explanation for options. Both consider European-style options and allow the same outside investments.

Finally, Ingersoll [2002] provides closed-form solutions for the general executive stock option values by using a pricing kernel approach. In his a model, the executive holds a combination of the company stock and optimal proportions of the risk-free asset and the market portfolio. By solving the optimization problem, Ingersoll obtains the undiversified executive's subjective values of the risk-free rate, the stock price, and the stock return. These subjective parameter values can be used to price options using standard Black-Scholes and binomial model techniques, but the resulting option values are valid only for a marginal (or small) option holding. Unlike our model, the Ingersoll methodology does not apply to executives who hold a substantial amount of options.

[3]Johnson and Tian [2000] also provide a model of pricing indexed strike price options, but they use risk-neutral parameters and allow only European-style exercise.

[4]Bettis, Bizjak, and Lemmon [2001] suggest that executives sometimes hedge their option risk by entering into zero-cost collars and equity swaps in the over-the-counter market. Hall and Murphy [2002] suggest that such transactions are quite rare. Such transactions would undoubtedly kill the incentive effect of stock options.

[5]We have assumed in traditional portfolio theory that the market portfolio is efficient and all investors hold optimal (i.e., utility-maximizing) combinations of the risk-free asset and the market portfolio. In this framework the executive also invests his outside wealth in an optimal combination of the risk-free asset and the market portfolio. In a more realistic setting with heterogeneous beliefs and information, it is possible that the executive holds other investments (such as real estate, hedge funds, and commodities) that are excluded from the typical proxies for the market portfolio. Intuitively, a wider investment opportunity set may enhance the attractiveness of a dollar of outside wealth relative to a dollar (in market value) of the firm's stock and options, which in turn may further reduce their executive value relative to the risk-neutral price. The modeling of option values with such an expanded investment opportunity set is beyond the scope of this article, however.

[6]In general, the executive's portfolio choice $p$ at any time $t$ may be different from his portfolio choice today. As we explain later, we do not allow the executive to update $p$ over time for computational reasons.

We assume implicitly that the executive maintains a stable ownership of company stock. This is justified on two grounds. First, Ofek and Yermack [2000] show that executives with large amounts of company stock respond to additional stock received by new stock grants or options exercise by selling equivalent amounts of previously owned stock. Second, many companies offer cashless exercise programs, where the executive pays nothing and receives the spread between the stock price and the exercise price.

[7]Once again, a complete optimization would require that the optimal portfolio choice $p^*$ in Equation (6) be different from the optimal portfolio choice in Equation (4). We make them equal for computational reasons. Test results show that the two can differ significantly for very large option holdings (as expected), but the resulting option values are not very different (perhaps due to flatness of the expected utility function near $p^*$).

[8]As in other executive option pricing models, we also assume that the stock return distribution is independent of whether the executive receives cash or stock options. This assumption can be justified on grounds that most companies grant options pursuant to some established compensation policy, so the incentive effects of options may be largely included in the stock prices.

[9]We use the *nlp* procedure in SAS to solve Equation (5). The convergence criteria are set to $10^{-25}$.

[10]We use a grid with one period per year in determining the optimal $p$. The optimal $p$ is then used to calculate $E[U(W_T^{OPT})]$ in Equation (5) in a grid with four periods per year.

[11]For an option with ten years to maturity, a rainbow grid with four periods per year has 23,821 nodes. Allowing the executive to rebalance at every individual node would lead to $101^{23,821}$ possible portfolio allocation strategies.

[12]In single-period simulations, we allow the executive to short-sell the risk-free asset or the market portfolio to the extent that terminal wealth $W_T$ is positive at all four nodes.

[13]Both Exhibit 4 and Exhibit 5 show that the S2 values are even less sensitive to *rmrf* when it lies between 4% and 10%. For example, the S2 value of the at-the-money option changes by less than 1.0% over this range.

The implications of this observation are, first, that this may be the more likely range of *rmrf* values; *rmrf* is the arithmetic market risk premium, and the geometric market risk premium is lower by $0.5\sigma_M^2 = 0.02$. Thus, an *rmrf* value of 2% implies a geometric market risk premium of 0%.

Second, note that the $p$ proportion of outside wealth invested in the market portfolio takes the value 0.00 in the S2 model for an *rmrf* value of 2%. This is the likely result of suboptimally constraining the $p$ values to lie between 0 and 1. As an experiment, we relax this constraint in a one-period model, which still does not result in negative wealth in any state. We find that the optimal $p$ now becomes negative for an *rmrf* value of 2%, and the overall *rmrf* sensitivity is even lower.

[14]The difference between the executive value and the company cost also follows from the principal-agent literature. Holmstrom [1979] shows that when a risk-neutral principal deals with a risk-averse agent in the presence of moral hazard, first-best outcomes are impossible, since they would require the principal to bear all the risk, leaving the agent unmotivated. Thus, in equilibrium the compensation will necessarily be less valuable to the agent (the executive) than to the principal (the shareholders).

[15]We prefer to report the expected life instead of the expected time to exercise as some options may never be exercised.

[16]In general, the barrier function can be a polynomial expansion of the cumulative market return, but the second moment tends to capture the threshold oscillation caused by a coarse grid, which impedes the convergence of the company cost calculations. We therefore use a linear barrier function that results in smooth convergence.

[17]Note the market's dividend yield is excluded in determining the exercise price but the executive's investment in the market portfolio through an index fund does earn the dividend-included return.

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