On the Consequences of Mutual Fund Tournaments

WEI LI* and ASHISH TIWARI**

This Version: April 2008

Keywords: Mutual Fund Tournaments, Implicit Incentives, Performance Chasing

* Department of Finance, Ourso College of Business Administration, Louisiana State University, Baton Rouge, LA 70803-1000. Phone: (225) 578-6335; Fax: (225) 578-6336; Email: wli@lsu.edu.
** Department of Finance, Henry B. Tippie College of Business, The University of Iowa, Iowa City, IA 52242-1000. Phone: (319) 353-2185; Fax: (319) 335-3690; Email: ashish-tiwari@uiowa.edu.

We are grateful to David Bates, Jeff Busse (WFA discussant), Bob Chirinko, B. Ravikumar, Tom George, Tom Rietz, J. Sa-Aadu, Laura Starks, Avanidhar Subrahmanyam, Laura Veldkamp (FMA discussant), and Paul Weller for their helpful comments and suggestions.
Abstract

We analyze the implications of the tournament-like competition in the mutual fund industry for shareholder welfare using a framework that addresses both (a) the risk taking incentives, and (b) the effort incentives facing fund managers. We compare the ex ante shareholder utility under two competing regimes: (a) a tournament regime characterized by future fund flows accruing to the winner fund, and (b) a no-tournament regime in which future flows are not sensitive to past performance. Our analysis suggests that mutual fund tournaments can be beneficial when the informational costs and the cash flow sensitivity to past performance are moderate. Using monthly as well as daily data for a sample of domestic stock funds, we present empirical evidence consistent with the key predictions of our theoretical model.
On the Consequences of Mutual Fund Tournaments

The tendency to chase past performance is one of the best known facts regarding the behavior of mutual fund investors.¹ Funds with superior relative performance within a category attract the lion’s share of the cash inflows in the future. Consequently, economists have often viewed the competition in the fund industry as a tournament in which players are competing with each other for investor cash flows. As shown by Brown, Harlow, and Starks (1996), the tournament nature of the mutual fund industry provides adverse incentives to losing fund managers. Specifically, they find that funds that trail at the half-year mark tend to subsequently increase the volatility of their portfolios. Similarly, Chevalier and Ellison (1997) study the relationship between fund flows and past performance of mutual funds and conclude that the flow-performance relationship provides incentives for the funds to alter the risk of their portfolios at the interim stage.²

While the extant literature has naturally focused on the perverse incentives created by the flow-performance relationship in mutual funds, it is unclear how such incentives impact the information gathering efforts of fund managers. More specifically, what are the shareholder welfare consequences of the tournament-like competition among funds? In this study we adopt a tournament framework to analyze the incentives of fund managers with a view to answering this question. The key feature of the analysis is its focus on both the risk taking incentives and the effort incentives generated by fund flows.

Recent rule changes enacted by the SEC require mutual funds to disclose, among other things, details of the fund portfolio manager compensation structure. The increased disclosure, it is hoped, would

¹ A number of studies have documented a strong positive relation between a fund’s past performance and its future fund inflows. See, for example, Ippolito (1992), Chevalier and Ellison (1997), Sirri and Tufano (1998), and Sapp and Tiwari (2004).

² See, also, Koski and Pontiff (1999). In related work, Busse (2001) analyzes daily returns for a sample of 230 equity funds over the period 1985–1995, but fails to find support for the hypothesis that fund managers actively alter the risk of their portfolios in response to past performance. However, he recognizes that “uncovering a more complex behavior pattern should be a fruitful area for future research” (p. 73). As we show later, both our theoretical model and the empirical results confirm that the risk taking incentives of fund managers are a function of (a) the fund cash flow sensitivity to past performance, i.e., the strength of the tournament effect, and (b) the magnitude of the interim performance gap separating the funds.
lead to better decision making by investors.\(^3\) For example, in discussing the benefits of the increased disclosure regarding the fund manager compensation the SEC stated in its release “… requiring a fund to provide disclosure regarding the structure of, and method used to determine, the compensation of its portfolio managers will help investors better understand a portfolio manager's incentives in running a fund…” In this context it is evident that an evaluation of the managerial incentives should take into consideration not only the explicit employment contract but also the incentive effects of the implicit “contract” created by fund flows. The goal of the present study is to examine the incentive effects of the fund flows with a view to better understand the link between mutual fund tournaments and shareholder welfare. Our key result, discussed below, is that under certain conditions mutual fund tournaments can be beneficial for shareholders.

As a first step towards the above objective, we develop a framework to analyze the risk-taking incentives of fund managers in a setting that abstracts from the issue of superior managerial information or skill. We analyze a model in which two risk-neutral fund managers with exogenously determined unequal performances at an interim stage, compete for investor cash flows. Given the unequal performances of the managers at the interim stage, we may think of one manager as the interim winner, and the other as the interim loser. Managers’ compensation is assumed to be a fixed proportion of the assets under management. Each manager has the choice of investing in a market index in addition to a security that represents pure idiosyncratic risk. Our focus is on the portfolio choices made by the two managers after they observe their interim performances.

We show that there exists an equilibrium in which it is optimal for the fund manager who is trailing behind at the interim stage (i.e., the interim loser) to increase the *idiosyncratic* risk of her portfolio. The intuition for the portfolio choices made by the two managers after their interim performance records become known is straightforward. The interim winner fund manager has an

\(^3\) The SEC adopted the new disclosure requirements in Investment Company Release No. 26533 (August 23, 2004). The stated objective was to “…provide greater transparency regarding portfolio managers, their incentives in managing a fund, and potential conflicts of interest. These proposals were designed to assist investors in evaluating fund management and making investment decisions.”
incentive to try to “lock in” her gains by maintaining an indexed position after it becomes known that she is ahead of her competitor. On the other hand, the optimal policy for the interim loser manager is to hold a portfolio that differs from that of the interim winner, as that is the only way she can hope to make up the initial performance shortfall. In light of the winner manager’s incentives to adopt an indexed position, the optimal policy for the interim loser is to distinguish her portfolio by taking on idiosyncratic risk. Importantly, our result regarding the interim loser manager’s incentive to increase her portfolio’s idiosyncratic risk exposure differs from the previous empirical studies (e.g., Brown, Harlow, and Starks (1996)) whose primary focus is on the interim loser’s incentives to increase the total volatility of her portfolio.

We extend our basic model by allowing for the possibility of costly information acquisition by risk averse fund managers. This framework allows us to analyze the tradeoff between the risk taking incentives created by the previously documented cash flow-performance relationship on the one hand, and the effort incentives created by this implicit reward mechanism, on the other. To assess the impact of the tournament nature of the competition in the fund industry on shareholder welfare, we compare the ex ante shareholder utility under two hypothetical regimes describing the industry: (a) a tournament regime characterized by future fund flows accruing to the winner fund, and (b) a no-tournament regime in which future flows are not sensitive to past performance. We numerically analyze the equilibrium outcomes under the two regimes and focus in particular on the effort choices and the portfolio choices of fund managers, and the resulting ex-ante welfare gains, if any, under the tournament regime.

Our analysis suggests that mutual fund tournaments can be welfare enhancing when the informational costs are moderate and when the cash flow sensitivity to past performance is not too high. Intuitively, when informational costs are low, the positive gains from the increased managerial effort under the tournament setting outweigh the adverse effects of increased risk taking by the losing (trailing) fund manager. In contrast, when the information costs are relatively high, shareholder utility is lower in the tournament regime relative to the no-tournament regime, since under the tournament regime the trailing fund manager has an incentive to take on excessive idiosyncratic risk. Similarly, the risk taking
incentive dominates the effort incentive when the cash flow sensitivity to past performance is very high, leading to reduced shareholder welfare in the tournament regime. These results have implications for optimal fund design with respect to features such as the choice of open vs. closed-end format, among others. For example, they suggest why the closed-end fund format which by design minimizes cash flow sensitivity, may be particularly suited to funds that operate in asset markets with potentially high costs of information, e.g., emerging equity markets.

Our theoretical analysis yields a number of testable predictions. In particular, it suggests that the increase in the idiosyncratic risk of the interim loser manager’s portfolio is directly related to the magnitude of the performance gap at the interim stage, and to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect). The analysis also suggests that the incentive to increase risk following poor performance is meaningful for only those funds that are within striking distance of the winning fund(s) at the interim evaluation stage, i.e., for funds for whom the performance deficit is not excessively large.

We use monthly data for a sample of domestic stock funds for the period 1962-2004 obtained from the CRSP Survivor-Bias Free US Mutual Fund Database to test the key implications of our basic model. The funds in the sample are grouped into three categories, namely, Aggressive Growth, Growth and Income, and Long Term Growth. The empirical evidence suggests that, consistent with the model, changes in the idiosyncratic risk of the funds are negatively related to their relative return performance at the interim evaluation stage. The relationship is statistically significant for funds with the greatest cash flow sensitivity to performance, namely, Aggressive Growth funds, as well as for the Growth and Income funds. Our findings confirm that the incentives of fund managers to take on idiosyncratic risk are related to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect).

We replicate our tests using daily data for a subset of our mutual fund sample that includes funds existing during the period 2001-2004. We find that when we do not control for the performance gap among funds, there does not appear to be a negative relation between past fund performance and the
subsequent change in their idiosyncratic risk exposure. This result is qualitatively similar to the conclusion reached by earlier studies that utilize daily fund returns (see, for example, Busse (2001)). However, we show that for funds with above the median level of performance during the January-September period, there is a significantly negative relation between past performance and change in idiosyncratic risk during the last quarter of the year. These results are broadly consistent with the implications of our theoretical analysis. They also suggest that the failure to control for the performance gap at the interim evaluation stage may help explain the differences in conclusions reached by earlier studies that relied on daily versus monthly data.

Our study contributes to a growing literature that examines managerial incentives and investor behavior in the mutual fund industry. On the theoretical front, a number of recent studies analyze the incentives facing mutual fund managers in a “tournament” framework (see, e.g., Bagnoli and Watts (2000), Goriaev, Palomino, and Prat (2003), and Taylor (2003)). However, the focus of these papers is on the risk taking incentives of portfolio managers. In contrast, we analyze both the risk taking incentives and the effort incentives of fund managers in a tournament framework with a view to assessing their impact on shareholder welfare. Our basic model is closest in spirit to that of Taylor (2003) who develops a model of a mutual fund tournament in which two fund managers with unequal midyear performances compete for new cash inflows. Taylor derives a mixed-strategy equilibrium for the case when both funds are actively managed. The pure strategies that are the basis for the mixed-strategies are those of the extreme form, i.e., they involve investing fully in risky assets or investing fully in the risk-free asset. In contrast, we allow the fund managers the option to take on idiosyncratic risk in addition to investing in the market portfolio and the risk free asset, and focus on the pure strategies equilibrium.

Our extended model which incorporates costly information acquisition by fund managers, allows us to analyze the tradeoff between the effort incentives and the risk taking incentives of the managers in a tournament setting. To our knowledge, ours is the first study of the impact of the tournament-like competition among fund managers on shareholder welfare that accounts for both the effort incentives and the risk taking incentives created by this competitive environment. The exclusive focus in the extant
literature on the risk taking incentives created by mutual fund tournaments has naturally led to a perception that the tournament-like competition may be unambiguously harmful to fund shareholders. By analyzing both the risk taking incentives and the effort incentives of fund managers, we are able to provide a more accurate view of the impact of such competition on fund shareholders’ welfare. In particular, we are able to characterize the conditions under which mutual fund tournaments may be beneficial to shareholders.

The use of tournaments involving relative performance evaluation as incentive devices has been studied by Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Meyer and Vickers (1997), among others. Our study complements this literature by focusing on a setting that involves explicit incentives for fund managers as well as the implicit incentives generated by the cash flow-performance relationship.

Other relevant papers in this area include Chen and Pennacchi (2005), Hu, Kale, and Subramanian (2002), and Basak, Pavlova, and Shapiro (2004). In the analytical framework of these papers, managers’ performance is measured against a benchmark. In this sense, the modeling framework of these papers does not involve a tournament structure within which competition among the players is a key feature and relative performance among peers is used in determining the reward. The model presented in this paper on the other hand, is based on a tournament structure of competition among fund managers. Further, the above papers focus solely on the fund managers’ risk taking incentives and do not address the information/effort incentives. In contrast, the model we present here allows us to address the effect that the tournament nature of the competition in the mutual fund industry has on investors’ welfare.

The rest of the paper proceeds as follows. Section 1 presents our basic model. Section 2 extends the basic model to a framework that incorporates costly information. Section 3 presents empirical

---

4 Lazear and Rosen (1981) examine the efficiency of three compensation schemes, namely, a linear piece rate, comparison against a fixed benchmark, and a tournament, in the context of labor contracts. In their modeling framework the output of each agent consists of the agent specific effort and an additive common shock. They show that the tournament scheme is the most efficient contracting arrangement when the variance of the common shock is relatively high.
evidence on the key implications of our model while Section 4 offers concluding remarks. All technical proofs are relegated to the appendix.

1. The Model

Consistent with recent theoretical studies in the area, we begin our analysis by considering a one-period model in which we explore the nature of managerial risk taking incentives and the implications for portfolio performance when portfolio managers are faced with investors who chase recent performance. We further assume that there is no asymmetric information. Subsequently, in Section 2 we extend our basic model to allow the manager to spend effort to potentially become informed. The latter extension allows us to analyze the welfare implications of the tournament-like competition among fund managers.

1.1 Model Structure

Consider two risk neutral managers who manage a pool of assets for a single period. Let the single period be divided into two (not necessarily equal) sub-periods (indexed by dates \( t = 0 \) to \( t = 1 \), and \( t = 1 \) to \( t = 2 \)), with the end of the first sub-period marking an interim evaluation stage at which managers observe their performances to date, and make portfolio choices for the second sub-period. Figure 1 contains a timeline representing the sequence of events. Let \( m_g \) and \( m_b \) be the year-to-date cumulative return performances of the two managers at the end of the first sub-period (i.e., at the interim evaluation stage), with the assumption that \( m_g > m_b \). Based on their relative performances in the first sub-period, we denote manager \( g \) (\( b \)) a “winning” (“losing”) manager. Let the performance gap between manager \( g \) and manager \( b \) at the interim stage (date \( t = 1 \)) be denoted by \( m_\beta \), where \( m_\beta = m_g - m_b \).

1.1.1 Managers’ compensation. We assume that both managers begin with an asset base of size \( s \) at date \( t = 0 \). At date \( t = 2 \), the manager with the higher average return over the two sub-periods realizes additional fund inflows. We use notation \( \lambda \), to denote the relative size of this additional fund inflow with respect to the original asset base, \( s \). In other words, we assume that there is a group of investors, with aggregate wealth \( \lambda s \), who chase the winner fund. We further assume that managers are paid a fixed percentage \( k \) of
the assets under their management at date $t = 2$.\footnote{This assumption appears reasonable even though until the recent SEC rule changes, mentioned earlier, the terms of the employment contract of the fund portfolio managers were not required to be publicly disclosed. However, it is well known that the vast majority of Investment Advisory firm fee contracts are based solely on assets under management (see, for example, Deli (2002)). It is reasonable to expect that a fund manager’s compensation will have a positive correlation with the profits of the Investment Advisory firm that employs her.} Hence, each manager’s objective is to enlarge the size of their asset base which can be achieved by earning a higher return over the single period and by attracting the performance chasing investors at the end of the period.

1.1.2 Investment opportunity set and asset returns. Each manager faces an identical investment opportunity that consists of a riskfree asset with return $R_f > 0$, and a market index portfolio with return $R_m$. In addition to the market index portfolio, each manager has the option to invest in a portfolio that offers pure idiosyncratic risk (i.e., it offers zero expected risk premium).\footnote{In a recent paper Berk and Green (2004) develop a model of competitive allocation of capital to mutual funds that rationalizes the response of fund flows to past performance. In their model also fund managers have the option of allocating a portion of the fund’s assets to an actively managed account and the balance to an indexed account.} Let $R_g$ denote the random return realized by the idiosyncratic portfolio chosen by the winning manager and let $R_b$ be the random return realized by the idiosyncratic portfolio chosen by the losing manager. We assume $R_m$, $R_g$, and $R_b$ are mutually independent normal random variables, where $R_m \sim N(\mu, \sigma_m)$ with $\mu > R_f$, and $R_g, R_b \sim N(R_f, \sigma_f)$. The properties of the asset returns are public knowledge, so neither manager possesses superior knowledge.\footnote{While the assumption that managers do not possess superior information, is certainly consistent with empirical evidence on the performance of actively managed funds (see, for example Jensen (1968), Gruber (1996), Grinblatt and Titman (1992), Carhart (1997), among others), it is not critical for our analysis. Our results hold as long as the two managers do not differ in their abilities.}

1.2 Analysis of the Model

We now focus on the portfolio choices made by each manager at the start of the second sub-period. Manager $g$’s portfolio allocation decision is characterized by $\alpha_g = \begin{bmatrix} \alpha_g^1 \\ \alpha_g^2 \end{bmatrix} \geq 0$, with $\alpha_g^1 + \alpha_g^2 \leq 1$, where $\alpha_g^1$ is the proportion of the total assets she decides to invest in the market index portfolio, $\alpha_g^2$ is the
proportion invested in the idiosyncratic portfolio, and $1 - \left(\alpha_i^g + \alpha_i^b\right)$ is the proportion invested in the riskfree asset. Similarly manager $b$’s decision is characterized by $\alpha_b^b = \begin{bmatrix} \alpha_{i_b}^b \\ \alpha_{s}^b \end{bmatrix} \geq 0$, with $\alpha_{i_b}^b + \alpha_{s}^b \leq 1$.

At the end of the single period, one of the two funds will end up with a higher average return measured over the two sub-periods, and become the ultimate “winner” of the tournament. For the “winner” of the tournament, the end-of-period asset base under management will grow not only by the rate of the portfolio return, but also by an additional amount of $\lambda s$. The “loser” fund will not get any additional cash inflow. We use an indicator variable, $\Pi[\alpha^g, \alpha^b]$, to indicate the event that manager $g$ wins the tournament. That is, the indicator variable equals 1 when manager $g$ ends up as the ultimate winner and it equals 0 when manager $b$ ends up as the ultimate winner. Formally, we have

$$
\Pi[\alpha^g, \alpha^b] = 1 \text{ iff } \frac{(m_g + \alpha_{i_g}^g (R_m - R_f) + \alpha_{s}^g (R_g - R_f))}{2} > \frac{(m_b + \alpha_{i_b}^b (R_m - R_f) + \alpha_{s}^b (R_b - R_f))}{2},
$$

$$
\Pi[\alpha^g, \alpha^b] = 0 \text{ otherwise.}
$$

The managers’ end-of-period compensation is $k$ times the end-of-period asset base:

$$
C_g = k s \left(1 + m_g \right) \left(1 + R_f + \alpha_{i_g}^g (R_m - R_f) + \alpha_{s}^g (R_g - R_f)\right) + \lambda \Pi[\alpha^g, \alpha^b],
$$

and

$$
C_b = k s \left(1 + m_b \right) \left(1 + R_f + \alpha_{i_b}^b (R_m - R_f) + \alpha_{s}^b (R_b - R_f)\right) + \lambda (1 - \Pi[\alpha^g, \alpha^b]).
$$

It is straightforward to note that

$$
E[\Pi] = \Phi \left( \frac{m_g + (\alpha_{i_g}^g - \alpha_{i_b}^b)(\mu - R_f)}{\sqrt{(\alpha_{s}^g)^2 + \alpha_{s}^b)^2} \sigma_i^2 + (\alpha_{i_g}^g - \alpha_{i_b}^b)^2 \sigma_m^2} \right),
$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal variable. We note that the above expression represents the probability of manager $g$ finishing the game as the ultimate winner. In the

---

8 Consistent with the restrictions facing the typical mutual fund, we disallow short sales.
subsequent discussion we use the notation \( \phi(\cdot) \) to denote the standard normal probability density function.

Since the constant \( k \) (proportional management fee charged by the managers) and \( s \) (the initial size of each fund) have no bearing on a manager’s strategy, we will assume in this section, for brevity, that \( ks = 1 \). We now consider the expected utility of the two managers for a strategies pair \( \{ \alpha^g, \alpha^b \} \). The winning manager’s expected utility is given by

\[
U_g(\alpha^g, \alpha^b) = (1 + m^g)(1 + R^g + \alpha^g(\mu - R^g)) + \lambda \Phi\left( \frac{m^g + (\alpha^g - \alpha^b)(\mu - R^g)}{\sqrt{(\alpha^g + \alpha^b)^2 + (\alpha^g - \alpha^b)^2 \sigma^2}} \right),
\]

and the losing manager’s expected utility is

\[
U_b(\alpha^g, \alpha^b) = (1 + m^b)(1 + R^b + \alpha^b(\mu - R^b)) + \lambda - \lambda \Phi\left( \frac{m^b + (\alpha^g - \alpha^b)(\mu - R^b)}{\sqrt{(\alpha^g + \alpha^b)^2 + (\alpha^g - \alpha^b)^2 \sigma^2}} \right)
\]

Manager \( b \) chooses \( \alpha^b \) to maximize her expected utility given by Equation (5). The following result characterizes the portfolio choices made by the two managers at date \( t = 1 \), in equilibrium.

**PROPOSITION 1.** In equilibrium, \( \alpha^g_1 \geq \alpha^b_1 \), where the equality holds only when \( \alpha^g_1 = \alpha^b_1 = 1 \). Moreover, \( \alpha^g_2 = 0 \) and \( \alpha^b_1 + \alpha^b_2 = 1 \).

**Proof.** See Appendix A.

Proposition 1 establishes that, in equilibrium, (a) manager \( g \) (i.e., the winning manager at the interim stage) will invest a proportion of her portfolio in the market index that is at least as large as the corresponding proportion chosen by manager \( b \), (b) manager \( g \)’s portfolio will not involve any exposure to idiosyncratic risk, and (c) manager \( b \) will choose to not invest in the risk free asset. We also note that the manager \( g \) will always desire a portfolio allocation to the market index that is strictly greater than the manager \( b \)’s allocation to the market index when the short sale constraint is not binding. Intuitively, the manager who is ahead at the interim stage (date \( t = 1 \)) would like to “lock in” her gains while the interim
loser attempts to increase her portfolio’s idiosyncratic risk in an attempt to catch up. Note that in our setting the interim loser fund has an incentive to increase her portfolio’s idiosyncratic risk, rather than the systematic risk of the portfolio. The intuition for this result is that the only way for the loser fund to catch up to (and possibly surpass) the winner fund is to hold a portfolio that is different from that held by the winner fund. Given the winner fund manager’s incentive to hold an indexed position, the loser fund manager attempts to take on additional idiosyncratic risk. Our result that the interim loser fund manager has an incentive to increase idiosyncratic risk is different from earlier studies (e.g., Taylor (2003)) that focus primarily on the losing fund manager’s incentive to affect the total volatility of her portfolio.

The following lemma further describes the equilibrium in terms of the optimal portfolio allocations, $\alpha^x\star$, and $\alpha^b\star$.

**Lemma 1.** In equilibrium either $\alpha^x\star = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, or $\alpha^b\star = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, or both.

**Proof.** See Appendix A.

We focus on the equilibrium with $\alpha^x\star = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Later, we provide a sufficient condition for this to be the case. We make the following assumption that

$$
(1 + m_b)(\mu - R_j) \geq \lambda \phi \left( \frac{m_b}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \frac{m_b}{\sqrt{\sigma_i^2 + \sigma_m^2}}.
$$

(6)

The left hand side of the above inequality is the expected rate of increase of manager $b$’s assets if the manager chooses to increase her allocation to the market index portfolio, and the right hand side represents the additional reward the manager expects to get for having a larger chance of winning the tournament by lowering the allocation to the market index. Intuitively, the above assumption states that the tournament effect is not so overwhelming that manager $b$ would want to go to the extreme of allocating 100% of her portfolio to the idiosyncratic risky asset in order to win the tournament. Inequality (6) is equivalent to
Thus, the assumption represented by inequality (6) ensures that the interim loser fund has a non-zero allocation to the market index (i.e., $\alpha_i^b > 0$) in the equilibrium.

We next have the following statement regarding the existence of an equilibrium.

**PROPOSITION 2.** In the case that 
\[
\frac{m_s}{\sqrt{\sigma_i^2 + \sigma_m^2}} \geq C_1,
\]
the pair 
\[
\begin{bmatrix}
\alpha^g \\
\alpha^b
\end{bmatrix}
= \begin{bmatrix}
1 \\
0
\end{bmatrix},
\]
\[
\begin{bmatrix}
\alpha^{g*} \\
\alpha^{b*}
\end{bmatrix}
= \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]
constitutes a Nash equilibrium where $C_1 \propto \lambda$ (see Appendix A for the definition of $C_1$).

**Proof.** See Appendix A.

Proposition 2 states that if the interim performance gap, $m_s$, is larger than $C_1\sqrt{\sigma_i^2 + \sigma_m^2}$, it will be optimal for manager $b$ to choose strategy $\alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. As a result, her chance of winning the tournament is exactly zero. Essentially this means that manager $b$ withdraws from the tournament. The intuition for this outcome is as follows. When the performance gap is large, the chance for manager $b$ to make up such a large gap before the end of the game is so slim that she simply gives up and decides instead to focus on the reward from the linear employment contract. Moreover, the bound on the size of the performance gap is proportional to $\lambda$, the reward from winning the tournament in the form of the end-of-period cash inflow. Intuitively, the higher the potential reward for winning the tournament, the stronger the desire for manager $b$ to stay on and compete in the tournament. Given a large enough value of $\lambda$, she might still consider participating in the tournament even when she is lagging behind significantly at the interim stage.

The following proposition establishes a link between manager $b$’s allocation to the idiosyncratic risky asset and the strength of the tournament effect captured by $\lambda$, and the interim performance gap, $m_s$. 

13
PROPOSITION 3. In case that \( \alpha^{*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), and \( \alpha^{*}_{i} < 1 \), we have \( \frac{\partial}{\partial \lambda}(1 - \alpha^{*}_{i}) > 0 \), and

\[
\frac{\partial}{\partial m_{\delta}} \Phi \left( \frac{m_{\delta}}{\sqrt{\sigma_{i}^{2} + \sigma_{m}^{2}}} \right) > 0.
\]

Proof. See appendix A.

Proposition 3 implies that when \( \alpha^{*} = \begin{bmatrix} \alpha^{*}_{i} \\ 1 - \alpha^{*}_{i} \end{bmatrix} \) is manager b’s optimal response with \( \alpha^{*}_{i} < 1 \), the higher the stakes in the tournament (i.e. the larger value of \( \lambda \)), the greater the weight on the idiosyncratic risky asset in manager b’s portfolio. Hence, the interim loser manager’s propensity to take on idiosyncratic risk is directly related to the strength of the tournament effect. The above proposition also points out that in the equilibrium, everything else equal, the larger the lead enjoyed by manager g at the interim stage of the game (date \( t = 1 \)), the greater the probability of manager g finishing the game as the ultimate winner, after taking into consideration the losing manager’s incentive of taking on more idiosyncratic risk to reduce this probability.

Thus far, our discussion of the equilibrium is based on the assumption that it exists. The following theorem establishes the existence of the equilibrium under the condition that the interim performance gap (i.e. \( m_{\delta} \)) between the two managers is not too large.

THEOREM 1. There is a positive value \( C \) that depends on other parameters except \( m_{\delta} \), such that if

\[
m_{\delta} < C \sqrt{\sigma_{i}^{2} + \sigma_{m}^{2}},
\]

the pair \( \alpha^{*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha^{*} = \begin{bmatrix} \alpha^{*}_{i} \\ 1 - \alpha^{*}_{i} \end{bmatrix} \) will be an equilibrium, where \( \alpha^{*}_{i} = 1 - \frac{1}{x_{i}} \).

Furthermore, \( \alpha_{i}^{*} < 1 \), \( \frac{\partial(1 - \alpha^{*}_{i})}{\partial m_{\delta}} > 0 \) and \( \lim_{m_{\delta} \to 0} (1 - \alpha^{*}_{i}) = 0 \). To be exact, we have that

\[
1 - \alpha^{*}_{i} = O(\sqrt{m_{\delta}}).
\]

Proof. See Appendix A for a more precise version of the theorem and the proof. Also notice that under the condition of the theorem, the claims made in Proposition 1 and 3 are true.
**Corollary 1.** The Sharpe Ratio of fund $b$ is lower than that of fund $g$, and it is decreasing with respect to $m_s$ and $\lambda$.

**Proof.** See Appendix A.

In addition to assuring the existence of the equilibrium, this theorem also points out that manager $b$’s portfolio holding of the idiosyncratic risky asset is an increasing function of the interim performance gap, $m_s$. That is, the larger the gap, the more the aggressive is manager $b$’s portfolio allocation with respect to the magnitude of idiosyncratic risk in her portfolio. As the performance gap approaches zero, manager $b$’s portfolio holding of idiosyncratic risk also approaches zero. Furthermore, the corollary to the theorem shows that the interim loser fund has a lower Sharpe ratio compared to the interim winner fund.

The above discussion formally analyzes the impact of the explicit and implicit incentives of the fund managers on their portfolio choices, in particular their choice of idiosyncratic risk exposure. A key question is: “How do these incentives impact shareholder welfare in light of their potential influence on the information gathering efforts of portfolio managers?” We address this issue in the next section by extending our basic model to a setting that allows for costly information and managerial risk aversion.

2. **The Model with Costly Information**

We now allow the two managers to potentially become informed by expending resources, after observing their relative performances at the interim stage. Further, we relax the previous assumption of risk neutrality and allow managers to be risk averse. As before, we focus on their portfolio strategies once the interim performances are observed.

2.1 **Model Structure**

Each manager has access to a market portfolio and the risk free asset. In addition, each manager can choose to have an actively managed component within her overall portfolio. As before, we assume that the actively managed component is uncorrelated with the market portfolio. The sequence of events, depicted in Figure 2, is similar to our base case model with one difference. Specifically, after observing their relative performances at date $t = 1$, both managers can choose to spend costly effort in order to
potentially acquire information about the risky assets’ payoffs. The probability of becoming informed is directly related to the effort expended by the manager. Specifically, a manager can expend effort, \( e \in [0,1] \) with cost \( f(e) \), which leads to a probability \( e \) of the manager becoming informed. The cost function \( f(e) \) is assumed to be convex. The actively managed component of the informed manager’s portfolio has a return, \( R_{a,s} \), where \( R_{a,s} \sim N(R_f + \rho, \sigma_i) \), with \( \rho < \mu - R_f \). The uninformed manager’s active portfolio has a return, \( R_{a,u} \), where \( R_{a,u} \sim N(R_f, \sigma_i) \). Note that the term \( \rho \) represents the incremental return earned by the active component of the informed manager’s portfolio and is analogous to the standard Jensen’s alpha measure. Below we analyze the fund manager’s incentives and the resulting utility of the fund shareholders in two polar cases. We begin by considering a no-tournament regime in which investor cash flows are not sensitive to past performances of the two funds. We then explore the contrasting tournament regime in which managers compete for future cash inflows which in turn are sensitive to the past relative performances of the managers. We assume that in the no-tournament regime the managers (the agents) have the reduced form mean-variance utility with risk aversion coefficient \( \gamma_a \), i.e., \( U_a(W) = E(W) - \frac{1}{2} \gamma_a \text{Var}(W) \).

2.2 The No-Tournament Case

At date \( t = 0 \), a fund manager’s optimization problem can be expressed as

\[
\begin{align*}
\text{Max} \quad & e \left( 1 + R_f + \alpha_1^s(\mu - R_f) + \alpha_2^s\rho - \frac{1}{2} \gamma_a \left( \alpha_1^s \sigma_m^2 + \alpha_2^s \sigma_i^2 \right) \right) \\
& + (1-e) \left( 1 + R_f + \alpha_1^u(\mu - R_f) - \frac{1}{2} \gamma_a \left( \alpha_1^u \sigma_m^2 + \alpha_2^u \sigma_i^2 \right) \right) - f(e)
\end{align*}
\]

\( \quad (11) \)

\[9\] We assume the manager can be either informed or uninformed, and the effort expended by the manager increases her chance of becoming informed. Several points may be noted in respect of our assumption: (1) it allows for tractability, (2) it allows us to analyze the manager’s behavior in equilibrium when she is informed as opposed to when she is not informed - we find that such a comparison is informative, and (3) such an assumption is often used in the standard principal-agent literature. An alternative may be to assume that the manager’s effort continuously increases her portfolio’s Jensen’s alpha, for example.
In the above expression, $\alpha_1^s$, and $\alpha_2^s$ represent the portfolio weights for the market index and the actively managed component chosen by the manager if she becomes informed. Similarly, $\alpha_1^u$, and $\alpha_2^u$ represent the corresponding portfolio weights chosen by the manager if she stays uninformed. The solution to this optimization problem is straightforward. We first solve for the manager’s optimal portfolio choice and then solve for her optimal effort choice. The resulting solution is used as a benchmark for the analysis of investors’ welfare in the subsequent numerical analysis.

2.3 The Tournament Case

Similar to our basic model in Section 1, we now consider the case where there is an exogenously determined performance gap, $m_g$, at the interim evaluation stage. We refer to the manager who has a performance advantage as the winning manager (indexed by $g$) and the other manager as the losing manager (indexed by $b$). Similar to expression (3) in Section 1.2, the uninformed manager $g$’s probability of winning the tournament at date $t = 2$, when competing against the uninformed manager $b$, is given by

\[
\Phi\left(\frac{m_g + (\alpha_{1u}^g - \alpha_{1u}^b)(\mu - R_f)}{\sqrt{(\alpha_{2u}^g + \alpha_{2u}^b)^2 + (\alpha_{1u}^g - \alpha_{1u}^b)^2 \sigma_m^2}}\right).
\]

We denote this probability as $P_{uu}$, where the subscript $uu$ denotes that both managers are uninformed. In the same fashion, we can express the probability of manager $g$ winning the tournament in the case that manager $g$ is uninformed, and manager $b$ is informed as $P_{us}$, where the subscript $us$ denotes that manager $g$ is uninformed and manager $b$ receives the information signal and becomes informed. Similarly, the probability of manager $g$ winning the tournament in the case when she is informed but manager $b$ is uninformed is denoted by $P_{su}$, and the probability of manager $g$ winning the tournament when both she and manager $b$ are informed is denoted by $P_{ss}$.

Given the losing manager’ (manager $b$’s) strategy, the objective function for the winning manager (manager $g$), after taking into consideration the reward from winning the tournament, is
A similar expression defines the losing manager’s utility.

The manager’s objective function can be viewed as consisting of two components, namely, the utility in the absence of a tournament effect and the expected gain from winning the tournament. While it is not a standard type of objective function, a similar objective function has been used previously in the mutual fund tournament literature (see, for example, Goriaev et al. (2003)). Our assumption of the form of the manager’s objective function is for reasons of tractability. There are a number of factors that argue in favor of such an assumption. First, it seems reasonable to assume that managers mentally separate the two objectives of (a) earning the rewards from the growth of fund assets due to strong portfolio return performance, and (b) winning the tournament, which in turn leads to incremental growth in fund assets due to cash inflows accruing to the winner. Second, introducing a correlation between the asset returns and the probability of winning the tournament will make the objective function a lot more complicated, with little corresponding gain in intuition.

2.4 Numerical Results

In this section we numerically explore the equilibrium outcomes under the tournament regime and the no-tournament regime. In doing so, we implicitly abstract from the motivations of the fund investors. In other words, we take the existence of a tournament regime or the no-tournament regime as given. Our ultimate objective is to assess the relative welfare of the fund shareholders in the two contrasting regimes. In the process, we hope to shed light on a number of other related issues. For example, how does the existence or the absence of a tournament setting affect the effort choices and the portfolio choices of fund managers? Similarly, how are the effort and portfolio choices of fund managers affected by the cost of effort? The results are based on numerical optimization of the utility objective.
functions of the managers in the tournament regime and the no-tournament regime identified above. The optimized portfolio and effort choices of the managers, in turn determine the shareholders’ utility which we analyze below.

We assume that the investors have mean-variance utility preferences with risk aversion coefficient $\gamma_p$, i.e., $U_p(W) = E(W) - \frac{1}{2} \gamma_p Var(W)$. Further, we specify the cost function $f(e)$, as the convex function $f(e) = c \cdot \exp(e)$. We allow for a range of values of the cost coefficient parameter ‘c’ to assess how investor utility is affected when information costs are high relative to settings where information costs are relatively low. The key choice parameters for the numerical analysis include the value of information ($\rho$), the cost parameter (c), the interim performance gap $\delta_m$, the cash flow sensitivity to past performance, $\lambda$, as well as the asset return moments. Below we describe our choice of the key parameter values used in obtaining our base set of results.

Although not critical for the analysis, for the sake of consistency we adopt the commonly employed convention in the literature that the mutual fund tournament occurs on an annual basis. We adopt the following values for the key parameters: the risk free rate is 0.50% per month, the monthly market risk premium is 0.54%, the market index’s standard deviation is 4.18% per month, while that for the actively managed component is 6% per month, and the actively managed component of the fund has a monthly Jensen’s alpha of 0.45% should the manager become informed. Both managers and the investors are assumed to have a risk aversion coefficient of 2, i.e., $\gamma_a = \gamma_p = 2$. In the base case scenario, the values of the variables $\lambda$, $c$, and the interim performance gap $m_\delta$, are: $\lambda = 5\%$, $c = 1\%$, and $m_\delta = 0.50\%$ per month. In the subsequent analysis, we allow for a range of values for the variables $\lambda$, $c$, and the interim performance gap $m_\delta$ to assess how these changes impact investor welfare under the tournament regime. In choosing these values we are guided, where possible, by the empirically observed values for these variables (or their suitable proxies).
Under the tournament regime, the utility of the fund shareholders is influenced by the portfolio and effort choices made by the managers after observing their relative performances at the interim stage. Since each manager is equally likely to end up as the interim winner (or loser), we compute the ex ante utility of fund shareholders as the average of the utility obtained from investing with the interim winner fund and the utility obtained from investing with the interim loser fund.

Figure 3A depicts the difference in ex ante shareholder utility under the tournament regime and the corresponding ex ante shareholder utility under the no-tournament regime, as a function of $\lambda$, i.e., the strength of the tournament effect. The utility difference is the potential welfare gain arising from the tournament-like competition among fund managers for different values of $\lambda$, keeping other factors constant. Figures 3B and 3C depict how the portfolio choices (in terms of the weight on the actively managed component) and the effort choices of the different types of managers vary with $\lambda$ in the tournament regime. In these and subsequent figures the effort choice of the winning manager is labeled as ‘effort_g’ while the effort choice of the losing (trailing) manager is labeled ‘effort_b’. The active component portfolio allocation of the winning manager when she is informed is labeled ‘act_g_info’. Similarly, the active component portfolio allocation of the losing manager when she is informed is labeled ‘act_b_info’ and her active component allocation when she is uninformed is labeled ‘act_b_uninfo’. For comparison, Figure 3B depicts the optimal managerial effort choice (labeled ‘effort_NT’) under the no-tournament regime. Similarly, Figure 3C depicts the optimal portfolio allocation to the active component when the manager is informed (labeled ‘act_NT_info’) under the no-tournament regime. The figure also depicts the active component allocations of the losing manager when she is informed (labeled ‘act_b_info’) and when she is uninformed (labeled ‘act_b_uninfo’).

As can be seen from Figure 3A, for low values of $\lambda$, the tournament nature of competition among fund managers, leads to ex-ante investor welfare gains. Further insight into this result can be gained from examining the effort choices and the portfolio choices of the fund managers. As can be seen
from Figure 3B, an increase in $\lambda$ leads to increasing effort on the part of the losing manager in the tournament setting. There is also a corresponding increase in the active component allocation of the losing manager (Figure 3C). At relatively low values of $\lambda$, the increased effort on the part of the losing manager is enough to offset the potential ex ante shareholder utility loss from the increasing active allocation made by the losing manager. On the other hand, too high a value of $\lambda$ will lead to an excessive amount of risk taking (in terms of her active bets) by manager $b$ leading to an overall welfare loss.

When the value of $\lambda$ is about 6%, we can see from Figure 3C that manager $b$ starts to dramatically increase her allocation to the active component of her portfolio (i.e., reduce her exposure to the market index) in the case when she does not have information to warrant such a portfolio allocation. A curious effect of such behavior is that, in equilibrium, the investor’s ex ante utility increases in this range under the tournament regime. The gain in investor’s welfare in this range of $\lambda$ (i.e., when $\lambda$ is in the vicinity of 6%) is primarily on account of manager $g$’s actions. The risk taking behavior of manager $b$ reduces manager $g$’s incentive to index, and further leads to a stronger incentive for her to spend effort in order to become informed. As seen in Figure 3A, at increasingly higher values of $\lambda$, the welfare gains become negative.

Figure 4A depicts the ex ante shareholder welfare gains under the tournament regime as a function of the cost of effort coefficient ‘$c$’. As may be seen from Figure 4A, it is in the region of moderate values of the cost coefficient, that positive welfare gains are realized. In order to provide some intuition for this result, we first examine the managers’ effort choices depicted in Figure 4B. As expected, both managers’ effort levels are decreasing with respect to the cost coefficient. Although difficult to see from Figure 4B, manager $g$’s effort level would be higher in the tournament regime relative to the no-tournament regime when the cost coefficient is low. The reason for this is that manager $g$ is motivated by the likely competition from manager $b$ who may potentially become informed. When the information cost

---

Note that under the tournament regime, the optimal active component portfolio allocation for the winning manager (manager $g$) is zero when she is uninformed. Similarly, under the no-tournament regime the optimal active allocation for both managers is also zero when they are uninformed.
is higher, manager \( b \) will accordingly choose a lower level of effort, resulting in a lower probability of her becoming informed. Manager \( g \) correctly anticipates this, and thus, hoping to beat an uninformed and trailing opponent, has a stronger incentive to index. This can be seen from Figure 4C where manager \( g \)’s active allocation starts to decline when the cost coefficient increases beyond about 0.008. Such indexing incentive further leads to lowered effort level from manager \( g \). Thus, at high levels of informational costs, manager \( g \)’s effort declines below the level she would have chosen in the no-tournament setting. This, in turn, leads to shareholder welfare loss under the tournament regime in the high cost region. It is when the cost is in the intermediate region, that the benefit from the tournament-like competition resulting from the stronger incentive to work hard by manager \( b \), outweighs the costs of tournament. The potential costs of the tournament-like competition among the funds result from (a) the relatively weaker incentive for manager \( g \) to acquire information, and (b) the deviation of both managers portfolios from their optimal portfolio allocations, i.e. the incentive of manager \( g \) to index her portfolio and the incentive of manager \( b \) to take on excessive risk.

We next examine the impact of mutual fund tournaments on shareholder utility as the value of information for the fund managers, \( \rho \), is altered. Figures 5A, 5B and 5C depict the results for this case. As can be seen from Figure 5A, when the value of managerial information (\( \rho \)) is high, the welfare gains from the tournament-like competition among fund managers are positive. For low values of \( \rho \), information is not sufficiently valuable to enable welfare gains from the tournament setting, even though the effort of the losing manager (manager \( b \)) is increasing in \( \rho \) (see Figure 5B). Hence, for low values of \( \rho \), the welfare gains are negative. Further insight can be gained by examining the portfolio weights on the actively managed component for both managers, as well as the optimal allocation in the absence of a tournament, depicted in Figure 5C. As may be seen, while the active allocation is increasing with \( \rho \) for both managers, manager \( b \)’s allocation is more aggressive throughout. This is consistent with our earlier results.
Finally, we examine how the managers’ choices are affected by the size of the interim performance gap, $m_δ$. These results are depicted in Figure 6. As can be seen, as the performance gap increases, initially manager $b$ will take more risk whether she is informed or is not informed. Consistent with our base model in Section 1, when the gap is sufficiently large, manager $b$ will reduce her active component allocation when she is uninformed. In fact her active allocation eventually converges (when the performance gap equals about 0.018) to the optimal active portfolio allocation under the no-tournament regime.¹¹ Not surprisingly, it takes a much larger performance gap to discourage manager $b$ from taking excessively large active positions when she is informed. In the latter case, her active component allocation converges to the optimal active portfolio allocation under the no-tournament regime, only when the performance gap approaches 0.12.

To summarize, the above results establish that the tournament nature of the competition among fund managers leads to improved shareholder welfare for (a) low to moderate levels of sensitivity of cash flow to performance ($λ$), and (b) intermediate levels of information cost. Similarly, the tournament regime can be welfare enhancing when the value of information is high. In the next section we provide evidence on the performance chasing behavior of fund investors and on the issue of whether funds appear to alter the idiosyncratic (non-market) risk of their portfolios in response to these incentives.

3. Empirical Evidence

Our analysis provides a number of testable predictions relating to the risk taking incentives of fund managers. In particular, our results suggest that the increase in the idiosyncratic risk of the interim loser manager’s portfolio is directly related to the magnitude of the performance gap at the interim stage, and to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect). We now examine the empirical evidence on these issues.

¹¹ Recall that when the manager is uninformed, her optimal active component allocation under the no-tournament regime is in fact, zero.
3.1 Sample Description

We use data from the CRSP Survivor-Bias Free US Mutual Fund Database. Our sample includes all domestic common stock funds in the categories “Aggressive Growth (AG)”, “Growth and Income (GI)”, and “Long Term Growth (LG)” that exist at any time during the period January 1962 to December 2004. From this group of funds we exclude all stock index funds as the incentives of their fund managers and investors are quite different from that of the participants in active funds. Our final sample contains 2,839 fund-entities comprising 31,382 fund-years, and is described in Table 1.

Panel A of Table 1 presents the characteristics of the funds in the sample. There are 1,313 Long Term Growth funds accounting for the majority of the sample. The Aggressive Growth and the Growth and Income categories account for 772 and 754 funds, respectively. As expected, Aggressive Growth funds have the highest average annual portfolio turnover rate at 85.14%, compared to 47.67% for Growth and Income funds at the other extreme. Aggressive Growth funds also have the highest average expense ratio at 0.95%. Panel B of Table 1 depicts the growth in the number of funds and the aggregate total net assets (TNA) in each category over the period 1962-2004. As can be seen, each category experienced steady growth during the early part of the sample period until 1990. During the nineties, both the number of funds and the assets under management exploded in each category. Interestingly, the period 2000-2004 witnessed a significant reduction in the number of funds in each category as well as in the asset base for the Aggressive Growth and Long Term Growth categories. These trends broadly reflect the relatively poor performance of the stock market during this period.

3.2 Evidence of Performance Chasing Behavior by Investors

A number of previous studies have documented evidence of performance chasing behavior by fund investors. We begin our analysis by confirming this behavior in our sample using a cross-sectional regression framework. We examine quarterly cross-sectional regressions of the normalized cash flow experienced by a fund on several factors known to influence fund flows. The net cash flow to fund $i$ during quarter $t$ is measured as follows:
Here $TNA_{i,t}$ refers to the total net assets at the end of quarter $t$, $r_{i,t}$ is the fund’s return for quarter $t$, and $MGTNA_{i,t}$ is the increase in the total net assets due to mergers during quarter $t$. The normalized cash flow is defined as the quarterly net cash flow divided by the TNA at the beginning of the quarter. The independent variables used in the regressions include the one quarter lagged normalized cash flow for the fund, the fund’s return during the previous quarter (net of the average return for the fund’s category), the logarithm of total net assets at the end of the previous quarter, fund asset turnover, fund expenses, and the maximum load fees charged by the fund. These variables are suggested by previous studies of the determinants of fund flows (see, for example, Ippolito (1992), Chevalier and Ellison (1997), Sirri and Tufano (1998), and Sapp and Tiwari (2004)). Table 2 presents the estimated coefficients from these regressions. The reported coefficient estimates are time-series averages of 156 quarterly cross-sectional regression estimates over the period from January 1966 to December 2004. We present results for the entire sample as well as for funds within each of the three categories examined by us.

It can be seen from the overall results that fund net cash flows are significantly positively related to the previous quarter performance of the fund, confirming the results of previous studies. Additionally, cash flows are significantly negatively related to fund size reflecting the fact that smaller funds attract proportionately larger percentage net cash flows compared to bigger funds. We also note that consistent with intuition, cash flows are negatively related to expense ratios and the maximum front-end load fees charged by the fund, although the individual coefficients are not significant for some categories. Comparing the results across the three categories of funds, it is interesting to note that the cash flows experienced by the Aggressive Growth funds have the greatest sensitivity to past relative performance, which presumably reflects the active nature of the investor clientele that is drawn to such funds. For example, the results suggest that in the case of Aggressive Growth funds, outperforming the category average by 1 percent translates into an incremental quarterly cash flow equal to 0.47 percent of a fund’s assets. The corresponding figures representing the incremental quarterly cash flow from beating the
category average by 1 percent are 0.43 percent and 0.39 percent in the case of Growth and Income funds and Long Term Growth funds, respectively. In this sense the implicit incentives created by the existence of the positive cash flow-performance relationship appear to be quite meaningful for the funds.

In summary, the results presented in Table 2 confirm that the recent relative performance of a fund has significant explanatory power for its future cash flows, even after controlling for a number of other fund characteristics. In particular, the “winner” funds attract significant future cash inflows. In the next sub-section we explore the risk taking behavior of the funds in the context of the above incentives.

3.3 Do Funds Alter their Idiosyncratic Risk in Response to Above Incentives?

We now examine the key question, namely, whether funds increase their idiosyncratic risk exposure following poor performance at an interim evaluation stage. To assess this issue, we divide each calendar year in the sample period into two equal sub-periods of six months each. We compute the idiosyncratic risk of the fund for each sub-period of the year as the standard deviation of the estimated monthly fund residual using the market model for a particular sub-period. Specifically, the idiosyncratic risk for fund $i$ for sub-period $s$ ($s = 1, 2$) of year $T$, denoted by $\sigma(e_{i,s,T})$, is given by the standard deviation of the monthly residual $e_{i,s,t}$. The fund residual is estimated as:

$$e_{i,s,T} = r_{i,t} - (\hat{\alpha}_{i,T} + \hat{\beta}_{i,T} r_{m,t})$$

where $\Gamma(s) = \{1, \ldots, 6\}$ for $s = 1$; and $\Gamma(s) = \{7, \ldots, 12\}$ for $s = 2$

In the above equation $r_{i,t}$ denotes the return in excess of the one month US T-bill return for fund $i$ during month $t$, and $r_{m,t}$ denotes the excess return on the CRSP value-weighted market portfolio for month
We obtain the parameters estimates \( \hat{\alpha}_{i,T} \) and \( \hat{\beta}_{i,T} \) using monthly fund returns for the 24 months immediately preceding year \( T \). We estimate the change in idiosyncratic risk of fund \( i \) for year \( T \) as

\[
\Delta \sigma_{i,T} = \sigma(\epsilon_{i,2,T}) - \sigma(\epsilon_{i,1,T})
\]

Thus, for each fund we have a time series of the yearly changes in the idiosyncratic risk from sub-period 1 to sub-period 2. For each category \( k \), we conduct annual cross-sectional regressions of the change in idiosyncratic risk of a fund on the return performance of the fund during the first sub-period of the year:

\[
\Delta \sigma_{i,T} = a_{k,T} + b_{k,T} R_{i,1,T} + u_{k,T}
\]

In the above equation \( R_{i,1,T} \) denotes the total return for fund \( i \) during the first sub-period of calendar year \( T \), net of the mean return earned by all funds in its peer group (category) for the sub-period. We next compute the time series averages of the yearly cross-sectional regression coefficients \( a_{k,T} \) and \( b_{k,T} \).

Panel A of Table 3 reports the estimated intercept and slope coefficients from Equation (18) for the three categories of funds. The estimates reported in the first vertical panel (Model I) are based on the regression involving the change in idiosyncratic risk given by Equation (17), as the dependent variable. We also report in the second vertical panel (Model II), estimates for regressions that use the proportional change in the idiosyncratic risk as the dependent variable. We note that in each category, there is a negative relation between fund relative performance during the first half of the year and the subsequent change in the fund’s idiosyncratic risk. Funds appear to increase their idiosyncratic risk exposure following poor performance at the interim stage. Equally striking is the fact that the relationship appears to be the strongest, and statistically significant, for funds in the Aggressive Growth and Growth and Income categories. Interestingly, it was these two categories for which the tournament effect (i.e., the sensitivity of cash flow to past relative performance) was found to be the most intense based on our

\[12\] As a robustness check we repeated our tests by calculating the fund residuals using a four factor model that incorporates the three Fama-French factors along with a momentum factor. Though not reported here for the sake of brevity, our results are qualitatively unchanged under this alternative specification.

\[13\] Estimating parameters over a (lagged) 24-month period allows for more precise estimates. The alternative would be to estimate the alpha and beta parameters using just six months of data for each period, which would lead to highly imprecise estimates.
earlier results in Table 2. These results are broadly consistent with our theoretical framework that suggests that the changes in idiosyncratic risk are related to the performance gap at the interim stage, and to the strength of the tournament effect.

Recall that in our theoretical analysis, Theorem 1 places a restriction on the magnitude of the performance gap, $m_\delta$, between the winning and losing managers (see also the discussion following Proposition 2). Intuitively, managers who trail too far behind at the interim stage and who do not have a realistic chance of making up the shortfall may exit the “tournament”, i.e., they may choose to not increase their idiosyncratic risk in the second half of the year. This suggests that the risk taking incentives would be stronger for those fund managers who feel they are still “in the game” and have a chance of making up the performance deficit in the second half of the evaluation period. To test this we repeat the analysis in Panel A of Table 3 by considering two groups of funds within each category. Specifically, in each year we rank funds into two groups within each category, based on their return performance during the first sub-period (i.e., the first six months of the year). Based on our analytical framework, we would expect the negative relation between idiosyncratic risk change and the first sub-period return to be stronger for funds in the high performance group in each category.

Panel B of Table 3 presents evidence on the above issue. The panel shows the estimated intercept and the slope coefficients for the low and the high performance ranks in each category. Also shown is the difference in the slope coefficients for the two groups. As can be seen, funds that rank above the median in each category (i.e, funds having a “high” performance rank) consistently have a negative slope coefficient signifying an inverse relation between past relative performance and subsequent change in idiosyncratic risk. The slope coefficient is significantly negative for the “high” performance category of Aggressive Growth funds and the Growth and Income funds. Furthermore, the difference between the slope coefficients of the low and the high performance ranks is positive for all categories of funds and it is significant for the Aggressive Growth funds. These findings are consistent across the two models utilized.
3.4 Evidence Using Daily Fund Returns

Busse (2001) notes that daily fund returns produce more efficient estimates of fund volatility measures compared to monthly returns, leading to different inferences about the risk taking behavior of mutual funds. Accordingly, we repeat the analysis presented in Table 3 using a sample of daily fund returns. The CRSP Survivor-Bias Free US Mutual Fund Database includes daily fund returns beginning in January 2001. This allows us to obtain daily fund returns for a subset of our original sample that includes funds that exist during the period 2001 to 2004. The reduced sample consists of 1,655 funds representing 5,313 fund-years and includes 418 Aggressive Growth funds, 439 Growth and Income funds, and 798 Long Term Growth funds.

Similar to the analysis using monthly returns, we examine changes in fund idiosyncratic risk across two sub-periods within each year. Since it is plausible that the potential changes in fund idiosyncratic risk do not occur until the last quarter of the year, we consider two distinct assessment periods within each year for the purpose of this analysis. The first assessment period (labeled ‘6,6’) includes an interim evaluation period January-June, followed by a post-evaluation period July-December. This corresponds to the assessment period used for the monthly returns analysis. The second assessment period (labeled ‘9,3’) includes an interim evaluation period January-September, followed by a post-evaluation period October-December.14 In contrast to the monthly data, the use of daily data makes it feasible for us to carry out the analysis for this latter assessment period.

Using the daily excess fund returns and the daily excess return on the market portfolio, we estimate the fund-specific residuals, as described in Equation (16), for each fund within each sub-period (i.e., the interim evaluation period and the post-evaluation period) of a given year. Based on the estimated standard deviations of the residuals in each sub-period of a given year, we estimate the change in the residual risk of a fund from the first period to the second period of a year, denoted by $\Delta \sigma_{i,T}$. Finally,

---

14 This period corresponds to the assessment period used by Chevalier and Ellison (1997).
similar to the analysis in the previous section, we estimate the following pooled cross-sectional regression (labeled ‘Model I’) for each category of fund, $k$:

$$\Delta \sigma_{i,T} = a_k + b_k R_{i,1,T} + u_k$$  \hspace{1cm} (19)$$

In the above equation $R_{i,1,T}$ denotes the total return for fund $i$ during the first sub-period of calendar year $T$, net of the mean return earned by all funds in its peer group (category) for the sub-period. Similar to the analysis with monthly returns presented in the last sub-section, we also estimate a second pooled cross-sectional regression (labeled ‘Model II’) in which the dependent variable is the proportional change in the fund idiosyncratic risk.

It is well known that daily fund returns are likely to be auto-correlated due to non-synchronous trading of the underlying securities, for example.\textsuperscript{15} This can potentially lead to incorrect inference in finite samples. To address this concern we simulate fund returns every year under the null hypothesis that managers do not strategically alter fund risk in response to past performance. The simulation design is similar to that employed by Busse (2001). Specifically, we obtain the time-series of daily fund specific residuals, $\varepsilon_{i,d}$, for day $d$ within each sub-period $s$ ($s = 1, 2$) in each year $T$ of the sample period:

$$\varepsilon_{i,d} = \left( y_{i,d} - \left( \hat{\alpha}_{i,s,T} + \hat{\beta}_{i,s,T} r_{m,d} \right) \right)$$

We sample randomly with replacement from the time series of the above residuals while preserving the cross-correlation structure of the residuals across the funds within a category. Using the re-sampled residuals and the estimated coefficients $\hat{\alpha}_{i,s,T}$ and $\hat{\beta}_{i,s,T}$, we create a set of simulated fund returns. This re-sampling scheme in which the fund-specific residuals for the first sub-period of a year are re-sampled independently of the residuals for the second sub-period, ensures that there is no relation between past fund performance and its subsequent risk. In other words, the simulated fund returns are devoid of any tournament effects. Using the simulated returns we estimate the pooled cross-sectional regression specified in Equation (19). For each category of funds $k$ we record the estimated coefficients.
and the associated t-statistic based on the Newey-West covariance matrix. We repeat this process 1000 times. In each case we record the proportion of times in 1000 replications that the simulated t-statistic exceeds the value of the corresponding t-statistic using the original data. We assess the significance of the reported t-statistics using these empirical p-values.

Table 4 presents the results from the daily returns analysis. Panel A presents the slope coefficients, $b_k$, from each of the two estimated models for the two assessment periods. Based on the results in Panel A there is no evidence of a reliably negative relation between the change in idiosyncratic risk and the past relative performance of funds. Regardless of the assessment period it appears that funds do not increase their idiosyncratic risk exposure following poor performance at the interim evaluation stage. This finding appears to be qualitatively consistent with the regression-based results in previous studies that employ daily returns such as Busse (2001). Note however, that our earlier theoretical analysis implies that the tournament incentives are related to the size of the performance gap facing trailing funds. Based on the model’s prediction (see, for example, the discussion following Proposition 2 in the text), risk taking incentives are expected to be meaningful only for funds that are within striking distance of the performance leaders, i.e., funds that do not trail by an excessive amount at the interim evaluation stage. Consequently, it is important to account for the performance gap when assessing the relation between past fund performance and changes in risk.

Panel B of Table 4 provides evidence on the above issue. The panel shows the slope coefficients and the associated t-statistics from the pooled cross-sectional regressions for two groups of funds within each category. Funds with first-period return below the median return for the category in a given year are classified in the ‘low’ performance rank and funds with first-period return above the category median return are classified in the ‘high’ performance rank. Based on the results for the (6,6) assessment period, we do not find a reliably negative relation between past performance and changes in idiosyncratic risk. Interestingly, however, the results for the (9,3) assessment period show that for funds in the high

---

15 Similarly, to the extent that fund portfolio holdings are similar for funds within a category, daily fund returns are likely to be cross-correlated.
performance rank, there is a significantly negative relation between the fund performance during the first nine months and the change in idiosyncratic risk during the last quarter of the year. In other words, funds that do not suffer from too large of a performance deficit appear to increase their idiosyncratic risk following relatively poor performance in the first nine months of the year. Furthermore, the difference between the slope coefficients of the low and the high performance ranks is positive and significant for all categories of funds. This conclusion is robust across the two models employed by us as well as being consistent across the three fund categories.\textsuperscript{16} Hence, the results for the (9,3) assessment period in Panel B of Table 4 using daily returns are qualitatively similar to our earlier results based on monthly returns. Interestingly, these results are in stark contrast to those reported in Panel A where we do not control for the interim stage performance ranking of the funds. Collectively, these results suggest that the failure to control for the performance gap at the interim evaluation stage may partially explain the discrepancy between the results of previous studies that utilize monthly versus daily data.

In summary, our empirical results provide support for some key implications of the theoretical model analyzed by us. They suggest that the changes in fund idiosyncratic risk are negatively related to the relative performance of the funds at an interim evaluation stage. The relationship appears to be the strongest for funds with the greatest cash flow sensitivity to relative performance, namely, Aggressive Growth funds. Hence, consistent with the theoretical model, the incentives of fund managers to take on idiosyncratic risk are related to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect). Furthermore, as suggested by the theoretical analysis, a fund’s incentive to increase its idiosyncratic risk exposure following poor performance is related to the performance gap facing the fund at the interim evaluation stage.

\textsuperscript{16} We replicated the analysis in Table IV by calculating fund residuals using a four factor model that includes the momentum factor in addition to the three Fama-French factors. For funds in the high performance group in each category we again find a negative relation between the past performance and changes in idiosyncratic risk, although the relationship is not statistically significant in all cases.
4. Discussion and Conclusion

This paper provides a framework for assessing the impact of fund manager incentives, in particular the implicit incentives, on shareholder welfare. The analysis accounts for both the risk taking incentives and the effort incentives that are generated by the flow-performance relationship in mutual funds. This allows us to explore the link between shareholder welfare and the strength of the tournament effect characterizing the competitive environment of the fund industry.

As a first step we analyze a model in which two risk-neutral fund managers with unequal performances at an interim stage, compete for investor cash flows. Neither manager possesses superior information. Our focus is on the portfolio choices made by the two managers after they observe their interim performances. We show that there exists an equilibrium in which it is optimal for the fund manager whose performance lags behind at the interim stage (i.e., the interim loser) to increase the idiosyncratic risk of her portfolio.

We extend our basic model by allowing for the possibility of costly information acquisition by risk averse fund managers. We analyze the tradeoff between the risk taking incentives created by the cash flow-performance relationship in mutual funds on the one hand, and the effort incentives created by the implicit reward mechanism, on the other. We compare the ex ante shareholder utility under two competing regimes: (a) a tournament regime characterized by future fund flows accruing to the winner fund, and (b) a no-tournament regime in which future flows are not sensitive to past performance. We numerically analyze the equilibrium outcomes under the two regimes with a view to assessing the shareholder welfare implications of the tournament regime.

Our main conclusion is that mutual fund tournaments can be welfare enhancing when the informational costs are moderate and when the cash flow sensitivity to past performance is not too high. Similarly, the tournament-like competition among funds can be beneficial to shareholders when the value of potential managerial information is high. In contrast, when the information costs are relatively high, shareholder utility is lower in the tournament regime relative to the no-tournament regime. Our results have implications for optimal fund design with respect to features such as the choice of open vs. closed-
end format, among others. For example, they suggest why the closed-end fund format which by design minimizes cash flow sensitivity, may be particularly suited to funds that operate in asset markets with potentially high costs of information, e.g., emerging equity markets.17

The extant literature has focused exclusively on the risk taking incentives created by mutual fund tournaments. This focus has naturally led to a perception that the tournament-like competition may be harmful to fund shareholders. By analyzing both the risk taking incentives and the effort incentives of fund managers, we are able to provide a more accurate view of the impact of such competition on fund shareholders’ welfare. In particular, we are able to characterize the conditions under which mutual fund tournaments may be beneficial to shareholders.

Our theoretical analysis affords a number of testable predictions. Specifically, our results suggest that the increase in the idiosyncratic risk of the interim loser manager’s portfolio is directly related to the magnitude of the performance gap at the interim stage, and to the strength of the investor (cash flow) response to the relative performance rankings of the funds (i.e., the strength of the tournament effect). Our empirical analysis of a sample of stock funds using both monthly as well as daily data provides support for these implications.

Recently the SEC has made a number of rule changes that, among other things, require mutual funds to disclose the details of the compensation structures for their portfolio managers. The stated intent of the enhanced disclosure requirements is to allow investors to have a better understanding of managerial incentives which presumably will lead to better decision making. In this context it is important to recognize that an assessment of managerial incentives should, in addition to the explicit employment contract, also consider the incentive effects of the implicit “contract” created by mutual fund flows. Our study represents a first step targeted at the latter issue. An interesting extension would be to explore the design of an optimal contract featuring both explicit and implicit incentives in the delegated portfolio management context. We leave this task for future research.

17 For an alternative liquidity based explanation for the choice of the closed-end organizational form by funds specializing in foreign securities, see the discussion in Nanda, Narayanan, and Warther (2000, p. 440).
References


Appendix

**Proof of Proposition 1**: We prove the proposition in 4 steps.

1. For any losing manager’s feasible strategy, \( \alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \), the winning manager’s strategy \( \begin{bmatrix} \theta_1 \\ 0 \end{bmatrix} \) strictly dominates any one of her feasible strategies \( \begin{bmatrix} \alpha^g_1 \\ \alpha^g_2 \end{bmatrix} \) with \( \alpha^g_1 < \theta_1 \), and any one of her feasible strategies \( \begin{bmatrix} \theta_1 \\ \alpha^g_2 \end{bmatrix} \) with \( \alpha^g_2 > 0 \).

   For \( \alpha^g_1 \leq \theta_1 \), we have:
   \[
   \frac{m_g + (\alpha^g_1 - \theta_1)(\mu - R_f)}{\sqrt{m_g + (\alpha^g_2 + \theta_2^2)^2 + (\alpha^g_3 - \theta_3)^2 \sigma_m^2}} \leq \frac{m_s}{\theta_2^2},
   \]
   and therefore,
   \[
   \Phi \left( \frac{m_g + (\alpha^g_1 - \theta_1)(\mu - R_f)}{\sqrt{m_g + (\alpha^g_2 + \theta_2^2)^2 + (\alpha^g_3 - \theta_3)^2 \sigma_m^2}} \right) \leq \Phi \left( \frac{m_s}{\theta_2}, \sigma_i \right),
   \]
   where the equality holds only if \( \alpha^g_1 = \theta_1 \) and \( \alpha^g_2 = 0 \). We thus have, for \( \alpha^g_1 \leq \theta_1 \),
   \[
   U_g \left( \alpha^g = \begin{bmatrix} \alpha^g_1 \\ \alpha^g_2 \end{bmatrix}; \alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) = (1 + m_g)(1 + R_f + \alpha^g_1(\mu - R_f)) + \lambda \Phi \left( \frac{m_g + (\alpha^g_1 - \theta_1)(\mu - R_f)}{\sqrt{m_g + (\alpha^g_2 + \theta_2^2)^2 + (\alpha^g_3 - \theta_3)^2 \sigma_m^2}} \right)
   \leq (1 + m_g)(1 + R_f + \alpha^g_1(\mu - R_f)) + \lambda \Phi \left( \frac{m_g}{\theta_2}, \sigma_i \right)
   \leq (1 + m_g)(1 + R_f + \theta_1(\mu - R_f)) + \lambda \Phi \left( \frac{m_g}{\theta_2}, \sigma_i \right)
   = U_g \left( \alpha^g = \begin{bmatrix} \theta_1 \\ 0 \end{bmatrix}; \alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right),
   \]
   where the equality holds only if \( \alpha^g_1 = \theta_1 \) and \( \alpha^g_2 = 0 \).

2. For any losing manager’s feasible strategy, \( \alpha^b = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \), the winning manager’s strategy \( \begin{bmatrix} \alpha^g_1 \\ 0 \end{bmatrix} \), if \( \alpha^g_1 \geq \theta_1 \), strictly dominates any one of her feasible strategies \( \begin{bmatrix} \alpha^g_1 \\ \alpha^g_2 \end{bmatrix} \) with \( \alpha^g_2 > 0 \).

   For \( \alpha^g_1 \geq \theta_1 \), we have:
   \[
   \frac{m_s + (\alpha^g_1 - \theta_1)(\mu - R_f)}{\sqrt{m_s + (\alpha^g_2 + \theta_2^2)^2 + (\alpha^g_3 - \theta_3)^2 \sigma_m^2}} \leq \frac{m_s + (\alpha^g_1 - \theta_1)(\mu - R_f)}{\sqrt{\theta_2^2 \sigma_i^2 + (\alpha^g_3 - \theta_3)^2 \sigma_m^2}},
   \]
   and therefore,
\[
\Phi \left( \frac{m_\beta + (\alpha^e_1 - \theta_1)(\mu - R_f)}{\sqrt{\left(\alpha^e_2 + \theta_2^2\right)\sigma_i^2 + (\alpha^e_i - \theta_1)^2\sigma_m^2}} \right) \leq \Phi \left( \frac{m_\beta + (\alpha^e_1 - \theta_2)(\mu - R_f)}{\sqrt{\theta_2^2\sigma_i^2 + (\alpha^e_i - \theta_2)^2\sigma_m^2}} \right),
\]
where the equality holds only if \( \alpha^e_i = \theta_2 \). We thus have, for \( \alpha^e_i \geq \theta_2 \),
\[
U^e_s \left( \alpha^e = \begin{bmatrix} \alpha^e_i \\ \alpha^e_2 \end{bmatrix}; \alpha^i = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) \leq U^e_s \left( \alpha^e = \begin{bmatrix} \alpha^e_i \\ 0 \end{bmatrix}; \alpha^i = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right),
\]
where the equality holds only if \( \alpha^e_2 = 0 \).

3. We have
\[
\frac{\partial}{\partial \alpha^e_i} U^e_s \left( \alpha^e = \begin{bmatrix} \alpha^e_i \\ \alpha^e_2 \end{bmatrix}; \alpha^i = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) \bigg|_{\alpha^i = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}} > 0.
\]
We then have
\[
U^w \left( \alpha^w = \begin{bmatrix} \alpha^w_1 \\ 0 \end{bmatrix}; \alpha^i = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \right) = (1 + m_w)(1 + R_f + \alpha^w_1(\mu - R_f)) + \lambda \Phi \left( \frac{m_\beta + (\alpha^w_1 - \theta_1)(\mu - R_f)}{\sqrt{\theta_2^2\sigma_i^2 + (\alpha^w_1 - \theta_2)^2\sigma_m^2}} \right),
\]
where \( \phi(\cdot) \) denotes the standard normal density function.

4. For any winning manager’s feasible strategy, \( \begin{bmatrix} \alpha^b_1 \\ \alpha^b_2 \end{bmatrix} \), if \( \alpha^e_i \geq \alpha^b_i - \frac{m_\beta}{(\mu - R_f)} \) (especially if \( \alpha^e_i \geq \alpha^b_i \)), then the losing manager’s strategy \( \begin{bmatrix} \alpha^b_1 \\ 1 - \alpha^b_2 \end{bmatrix} \) strictly dominates any one of her feasible strategies \( \begin{bmatrix} \alpha^b_1 \\ \alpha^b_2 \end{bmatrix} \) with \( \alpha^b_1 + \alpha^b_2 < 1 \).

If \( \alpha^b_1 + \alpha^b_2 < 1 \), in the presence of the short sale constraint, we have \( \alpha^e_2 < (1 - \alpha^b_2)^2 \), and therefore
\[
\sqrt{(\alpha^e_2 + \alpha^b_2)}\sigma_i^2 + (\alpha^e_i - \alpha^b_i)^2\sigma_m^2 < \sqrt{(\alpha^e_2 + (1 - \alpha^b_2)^2)}\sigma_i^2 + (\alpha^e_i - \alpha^b_i)^2\sigma_m^2.
\]
In the case that \( \alpha^e_i \geq \alpha^b_i - \frac{m_\beta}{(\mu - R_f)} \), we have \( \Delta + (\alpha^e_i - \alpha^b_i)(\mu - R_f) > 0 \), and therefore
\[
\frac{m_\beta + (\alpha^e_i - \alpha^b_i)(\mu - R_f)}{\sqrt{(\alpha^e_2 + \alpha^b_2)^2\sigma_i^2 + (\alpha^e_i - \alpha^b_i)^2\sigma_m^2}} > \frac{m_\beta + (\alpha^e_i - \alpha^b_i)(\mu - R_f)}{\sqrt{(\alpha^e_2 + (1 - \alpha^b_2)^2)^2\sigma_i^2 + (\alpha^e_i - \alpha^b_i)^2\sigma_m^2}}.
\]
This shows that the first order optimality condition for the portfolio allocation of manager equilibrium, the short sale constraint is binding for at least one of the two managers. If the short sale constraint is binding for manager $a$ from Statement 1, we have that in the equilibrium to be 

$$
(1 + m_b)(1 + R_f + \alpha^*_a(\mu - R_f)) + \lambda - \lambda \Phi \left( \frac{m_s + (\alpha^*_a - \alpha^*_b)(\mu - R_f)}{\sqrt{\alpha^2_2 + \alpha^2_2 \sigma_i^2 + (\alpha^*_a - \alpha^*_b)^2 \sigma_m^2}} \right)
$$

< (1 + m_b)(1 + R_f + \alpha^*_a(\mu - R_f)) + \lambda - \lambda \Phi \left( \frac{m_s + (\alpha^*_a - \alpha^*_b)(\mu - R_f)}{\sqrt{\alpha^2_2 + (1 - \alpha^*_b)^2 \sigma_i^2 + (\alpha^*_a - \alpha^*_b)^2 \sigma_m^2}} \right)

$$
= U_b \left( \begin{bmatrix} \alpha^*_a \\ \alpha^*_b \end{bmatrix} ; \alpha^b = \begin{bmatrix} \alpha^*_a \\ 1 - \alpha^*_b \end{bmatrix} \right)
$$

To finish the proof, we derive the claim of the proposition from the above four proved statements. From Statement 1, we have that in the equilibrium $\alpha^*_a \geq \alpha^*_i$. From Statement 2, we have $\alpha^*_a = 0$. Then from Statement 3, we have that $\alpha^*_a \geq \alpha^*_b$ with the equation holding only if $\alpha^*_a = \alpha^*_b = 1$. From Statement 4, we have that $\alpha^*_a + \alpha^*_b = 1$. Thus, the proposition is proved.

**Proof of Lemma 1:** Given the assumption that the equilibrium exists, we can denote the equilibrium to be $\begin{bmatrix} \alpha^*_a \\ \alpha^*_b \end{bmatrix}$ because of the result in Proposition 1. We have,

$$
U_g \left( \begin{bmatrix} \alpha^*_a \\ 0 \end{bmatrix} ; \alpha^b = \begin{bmatrix} \alpha^*_b \\ 1 - \alpha^*_b \end{bmatrix} \right) = (1 + m_g)(1 + R_f + \alpha^*_a(\mu - R_f)) + \lambda - \lambda \Phi \left( \frac{m_s + (\alpha^*_a - \alpha^*_b)(\mu - R_f)}{\sqrt{1 - \alpha^*_b)^2 \sigma_i^2 + (\alpha^*_a - \alpha^*_b)^2 \sigma_m^2}} \right)
$$

$$
U_h \left( \begin{bmatrix} \alpha^*_a \\ 0 \end{bmatrix} ; \alpha^b = \begin{bmatrix} \alpha^*_b \\ 1 - \alpha^*_b \end{bmatrix} \right) = (1 + m_b)(1 + R_f + \alpha^*_a(\mu - R_f)) + \lambda - \lambda \Phi \left( \frac{m_s + (\alpha^*_a - \alpha^*_b)(\mu - R_f)}{\sqrt{(1 - \alpha^*_b)^2 \sigma_i^2 + (\alpha^*_a - \alpha^*_b)^2 \sigma_m^2}} \right)
$$

Combining the derivatives of the above two functions, we get

$$
\frac{\partial}{\partial \alpha^*_a} U_g \left( \begin{bmatrix} \alpha^*_a \\ 0 \end{bmatrix} ; \alpha^b = \begin{bmatrix} \alpha^*_b \\ 1 - \alpha^*_b \end{bmatrix} \right) - \frac{\partial}{\partial \alpha^*_b} U_h \left( \begin{bmatrix} \alpha^*_a \\ 0 \end{bmatrix} ; \alpha^b = \begin{bmatrix} \alpha^*_b \\ 1 - \alpha^*_b \end{bmatrix} \right)
$$

$$
= m_s(\mu - R_f) + \lambda \Phi \left( \frac{m_s + (\alpha^*_a - \alpha^*_b)(\mu - R_f)}{\sqrt{(1 - \alpha^*_b)^2 \sigma_i^2 + (\alpha^*_a - \alpha^*_b)^2 \sigma_m^2}} \right) \frac{m_s + (\alpha^*_a - \alpha^*_b)(\mu - R_f)(1 - \alpha^*_b) \sigma_i^2}{(1 - \alpha^*_b)^2 \sigma_i^2 + (\alpha^*_a - \alpha^*_b)^2 \sigma_m^2}
$$

$$
> 0
$$

This shows that the first order optimality condition for the portfolio allocation of manager $g$ and the first order condition for manager $b$ will not be satisfied at the same time. As a consequence, in the equilibrium, the short sale constraint is binding for at least one of the two managers. If the short sale constraint is binding for manager $g$, we have, by Proposition 1, $\alpha^*_g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. If the short sale constraint is
binding for manager \( b \), we have either \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) or \( \alpha^b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \). In the case that \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), we have, again by Proposition 1, \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \). Therefore, in equilibrium, either \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), or \( \alpha^b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \), or both.

**Discussion of the first order condition for the portfolio allocation of manager \( b \):** In general, the first order optimality condition for the portfolio allocation of manager \( b \) is

\[
0 = \frac{\partial}{\partial \alpha^b_i} U_b \left( \alpha^b = \begin{bmatrix} \alpha^b_i \\ 1 - \alpha^b_i \end{bmatrix} \right)
\]

\[
= (1 + m_s)(\mu - R_f) - \lambda \phi \left( \frac{m_s}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{1 - \alpha^b_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) - \frac{1}{\sqrt{\sigma_i^2 + \sigma_m^2}} (1 - \alpha^b_i)^2.
\]

In the equilibrium, either the above first order condition is satisfied, or, \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \). The first order condition is equivalent to

\[
(1 + m_s)(\mu - R_f) = \lambda \phi \left( \frac{m_s}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{1 - \alpha^b_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) - \frac{1}{\sqrt{\sigma_i^2 + \sigma_m^2}} x^2, \text{ where } x = \frac{1}{1 - \alpha^b_i}.
\]

Notice that both sides are positive. We can therefore take natural log of both sides, which yields the equivalent condition:

\[
h(x) = \ln \left( \frac{(1 + m_s)(\mu - R_f)}{\lambda \phi} \right) + \frac{1}{2} \left( \frac{m_s}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 - 2 \ln x = 0. \tag{A1}
\]

It is straightforward to see that \( h(x) \) and \( \frac{\partial}{\partial \alpha^b_i} U_b \left( \alpha^b = \begin{bmatrix} \alpha^b_i \\ 1 - \alpha^b_i \end{bmatrix} \right) \) have the same sign. Taking derivatives of \( h(x) \), we have:

\[
h'(x) = \frac{m_s}{\sqrt{\sigma_i^2 + \sigma_m^2}} x + \frac{(\mu - R_f)}{\sqrt{\sigma_i^2 + \sigma_m^2}} - \frac{2}{x}, \text{ and } h''(x) = \left( \frac{m_s}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 + \frac{2}{x^3} > 0.
\]

Therefore, \( h(x) \) is a strictly convex function and thus has at most two roots. It is clear that \( \lim_{x \to 0^+} h(x) = +\infty \). In the case that \( h(x) \) has only one root or no root, by the middle value theorem of continuous function, we have \( h(x) \geq 0 \) for \( x \in [1, \infty) \). Therefore \( U_b \) will be a strictly increasing function with respect to the variable \( \alpha^b_i \). We can then conclude that, in this case, \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) will be manager \( b \)'s optimal response.

In the case that function \( h(x) \) has two roots, we denote the roots by \( x_1 \) and \( x_2 \) with \( 1 < x_1 < x_2 \). Because \( h(x) \) is a convex function, we have \( h(x) > 0 \) for \( x \in [1, x_1) \cup (x_2, \infty) \), and \( h(x) < 0 \) for \( x \in (x_1, x_2) \). Thus \( U_b \) is increasing for \( \alpha^b_i \in [0, 1 - 1/x_1) \cup (1 - 1/x_2, \infty) \), and decreasing for \( \alpha^b_i \in [1 - 1/x_1, 1 - 1/x_2] \). Therefore, \( \arg \max_{\alpha^b_i} U_b \left( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = 1 \) or \( 1 - \frac{1}{x_1} \).
We derive some properties regarding the equation \( h(x) = 0 \), especially regarding \( x_* \), and record the results in the following several lemmas. The existence of roots for \( h(x) \) is given by the following lemma:

**LEMMA 2**: \( h(x) \) has two roots in \((0, +\infty)\) if and only if the following condition holds:

\[
\frac{m_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} < C_i
\]

where

\[
C_i = \frac{\lambda}{4(1 + m_j)(\mu - R_j)} - \phi \left[ 1 - \left( \frac{2(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 \right] \left( \frac{2(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 + 8 - \left( \frac{2(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2
\]

In addition, \( h(x) \) has one root if and only if \( \frac{m_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} = C_i \).

**Proof**: The only critical point of \( h(x) \), located by the first order condition \( h'(x) = 0 \), is at

\[
x_* = \frac{1}{2} \left( \frac{m_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^{-1} \left( \frac{2(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2 + 8 - \left( \frac{2(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right)^2.
\]

Therefore, \( h(x^*) = \min h(x) \). The ending behavior of \( h(x) \) is given by:

\[
\lim_{x \to +\infty} h(x) \geq \lim_{x \to +\infty} \frac{m_i^2}{2(\sigma_i^2 + \sigma_m^2)} x^2 - 2x = +\infty,
\]

\[
\lim_{x \to +\infty} h(x) \geq -2 \lim_{x \to +\infty} \ln x = +\infty.
\]

Since \( h(x) \) is convex, we then have that \( h(x) \) has one root in \((0, +\infty)\) if and only if \( h(x^*) = 0 \), and it has two distinct roots in \((0, +\infty)\) if and only if \( h(x^*) < 0 \). By substituting \( x^* \) into \( h(x) \), we get the proof the lemma.

The above Lemma 2 imposes a restriction on the performance gap between the winning manager and the losing manager. The gap, \( m_j \), can not be too large in order for \( h(x) \) to have roots. If the condition of the lemma does not hold, \( h(x) \) will always be positive. It will then lead to the conclusion that it is optimal for the losing manager to choose \( \alpha' = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) as her optimal strategy. Notice that the bound for the gap in the above condition is proportional to \( \lambda \), the reward from winning the tournament.

Another way to express the bound in the above lemma is to say that there is at least one feasible portfolio holding for the losing manager at which this manager would prefer to increase her exposure to idiosyncratic risk. Increasing the exposure to idiosyncratic risk affects the losing manager’s utility in two ways. First, increasing the portfolio allocation to idiosyncratic risk will increase the variance of the difference between her portfolio and the winning manager’s portfolio. This, in turn, will introduce more noise in the outcome of the tournament, and therefore has the potential of increasing the chance for the losing manager to make up the gap and win the tournament. Second, an increase in the idiosyncratic risk exposure implies a reduction in the allocation to the market index and hence, a reduction in the expected return. The lower expected return has two consequences for the losing manager’s utility. It will lower the losing manager’s expected reward from the explicit linear contract, and it will lower her chance of
winning the tournament. The losing manager has to take all these three factors into consideration when making the portfolio decision.

For the rest of the section, we will maintain the assumption that the condition of Lemma 2 holds. That is, we assume that \( h(x) \) has two roots. It will be sufficient for that condition to hold if we assume that the tournament has some effect on at least one of the managers’ portfolio decision. We will maintain the notation \( x_1 \) to denote the smaller one of the two roots of \( h(x) \).

**Proof of Proposition 2:** It follows immediately from Lemma 2 that, under the conditions of the proposition, \( \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) is optimal for manager \( b \) given \( \alpha^g = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \). The optimality of \( \alpha^g = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) for manager \( g \) is assured by Proposition 1. Thus \( \left\{ \alpha^w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \) is a Nash equilibrium.

**LEMMA 3:** \( \frac{\partial x_1}{\partial \lambda} = \frac{1}{h'(x_1) \lambda} < 0 \)

*Proof:* Since \( x_1 \) is the smaller of the two roots of the convex function \( h(x) \), therefore, \( h(x_1) = 0 \) and \( h'(x_1) < 0 \). From \( h(x_1) = 0 \), we have

\[
\ln\left(1 + m_i (\mu - R_i) \sqrt{\sigma_i^2 + \sigma_i^2 \sqrt{2\pi}}\right) - \ln \lambda - \ln m_i + \frac{1}{2} \left( \frac{m_{g} \sqrt{\sigma_i^2 + \sigma_i^2}}{\sqrt{\sigma_i^2 + \sigma_i^2}} x_i + \frac{(\mu - R_i)}{\sqrt{\sigma_i^2 + \sigma_i^2}} \right)^2 - 2 \ln x_i = 0
\]

Differentiating the above equation, we get \( \frac{\partial x_1}{\partial \lambda} = \frac{1}{h'(x_1) \lambda} < 0 \). Thus the lemma is proved.

**LEMMA 4:** \( \frac{\partial [m_i x_i]}{\partial m_{g}} > 0 \), and \( \lim_{m_{g} \to 0} m_{g} x_i = 0 \). Moreover, we have \( \lim_{m_{g} \to 0} x_i = +\infty \), and to be more precise, we have \( x_i = O(m_{g}^{-1/2}) \).

*Proof:* By \( h'(x_i) < 0 \), we have \( \frac{m_{g} x_i + (\mu - R_i)}{\sqrt{\sigma_i^2 + \sigma_i^2}} \frac{m_{g}}{\sqrt{\sigma_i^2 + \sigma_i^2}} - 2 < 0 \). Differentiate equation \( h(x_i) = 0 \), we get that

\[
\frac{\partial x_i}{\partial m_{g}} = \frac{1}{m_{g}} \left[ \frac{m_{g} x_i + (\mu - R_i)}{\sqrt{\sigma_i^2 + \sigma_i^2}} \right] \frac{x_i}{\sqrt{\sigma_i^2 + \sigma_i^2}} - \frac{2}{x_i} \]

Therefore,

---

18 The notation \( O \) means that both \( x_i \) and \( m_{g}^{-1/2} \) approach infinity with the same order of speed when \( m_{g} \to 0' \).
\[
\frac{\partial [m_{x_1}]}{\partial m_{x_1}} = x_1 + m_{x_1} \frac{\partial x_1}{\partial m_{x_1}}
\]

\[
= - \left[ \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} x_1 + \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right] \left[ \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} x_1 - 2 \right]^{-1} = -1/h'(x_1) > 0
\]

We thus have that \( m_{x_1} \) is an increasing function of \( m_{x_1} \). Moreover, \( m_{x_1} > 0 \). Therefore, \( (1 + m_{x_1})(\mu - R_f) = \lambda \phi \left( \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right) \left( \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} x_1 \right)^2 \), \( \lim_{m_{x_1} \to 0^+} m_{x_1} \) exists, is bounded and non-negative.

An equation equivalent to \( h(x_1) = 0 \) is:

\[
(1 + m_{x_1})(\mu - R_f) = \lambda \phi \left( \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right) \left( \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} x_1 \right)^2 \tag{A4}
\]

Taking the limit of both sides of the above equality by letting \( m_{x_1} \to 0^+ \), we have:

\[
0 = \lambda \phi \left( \lim_{m_{x_1} \to 0^+} m_{x_1} x_1 \right) \left( \lim_{m_{x_1} \to 0^+} m_{x_1} x_1 \right)^2 \left( \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right) \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} x_1 \right)^2 ,
\]

which implies that \( \lim_{m_{x_1} \to 0^+} m_{x_1} x_1 = 0 \). From equation (A2) we have

\[
\lim_{m_{x_1} \to 0^+} m_{x_1} x_1^2 = \frac{1}{\lambda} (1 + m_{x_1})(\mu - R_f) \sqrt{\sigma^2 + \sigma_m^2} \left[ \phi \left( \lim_{m_{x_1} \to 0^+} m_{x_1} x_1 \right) \right. \\
\left. \phi \left( \frac{m_{x_1}}{\sqrt{\sigma^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right) \right]^{-1}
\]

\[
= \frac{1}{\lambda} (1 + m_{x_1})(\mu - R_f) \sqrt{\sigma^2 + \sigma_m^2} \left[ \phi \left( \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right) \right]^{-1}
\]

The right hand side of the above identity is positive. Therefore, we have that \( \lim_{m_{x_1} \to 0^+} x_1 = +\infty \), and to be more precise, \( x_1 = O(m_{x_1}^{-1/2}) \). Thus the lemma is proved.

**Lemma 5**: \( \frac{\partial x_i}{\partial m_{x_1} < 0 \) for sufficiently small \( m_{x_1} \). In fact, it is true for any

\[
m_{x_1} \in \left( 0, \sqrt{\sigma^2 + \sigma_m^2} \cdot \min(C_1, C_2) \right),
\]

where \( C_1 \) is given as in Lemma 2, and

\[
C_2 = \frac{\lambda}{4(1 + m_{x_1})(\mu - R_f)} \phi \left[ \frac{1}{2} \left( \frac{(\mu - R_f)^2}{\sqrt{\sigma^2 + \sigma_m^2}} + 4 \right) + \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}} \right]^n
\]

\[
\left( \frac{(\mu - R_f)^2}{\sqrt{\sigma^2 + \sigma_m^2}} + 4 \right) - \frac{(\mu - R_f)}{\sqrt{\sigma^2 + \sigma_m^2}}^2 \]

**Proof**: In the derivation of Lemma 3, we see
\[
\frac{\partial x_i}{\partial m_\delta} = \frac{1}{m_\delta} - \left( \frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} x_i + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \frac{x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}}.
\]

Since \( h'(x_i) < 0 \), we have \( \frac{\partial x_i}{\partial m_\delta} < 0 \) if and only if,

\[
1 > \left( \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}}.
\]

The right hand side is a quadratic form of \( \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} \). It is straightforward to solve the above inequality.

Given \( \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} > 0 \), we have \( \frac{\partial x_i}{\partial m_\delta} < 0 \) if and only if

\[
0 < \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} < \frac{1}{2} \left\{ \frac{(\mu - R_j)^2}{\sigma_i^2 + \sigma_m^2} + 4 - \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right\}.
\]

(A6)

By Lemma 3, we have \( \lim_{m_\delta \to 0^+} m_\delta x_i = 0 \). Therefore, the above inequalities will be satisfied for sufficiently small \( m_\delta \). To answer the question of how small is sufficiently small, we evoke the equality \( h(x_i) = 0 \).

With this equation, we can see that the necessary condition for

\[
\frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} = C_2.
\]

is \( \frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} = C_2 \). Therefore, for any \( m_\delta \in (0, C_2 \sqrt{\sigma_i^2 + \sigma_m^2}) \), we always have

\[
\frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} \neq \frac{1}{2} \left\{ \frac{(\mu - R_j)^2}{\sigma_i^2 + \sigma_m^2} + 4 - \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right\}.
\]

This together with the fact that inequality (A6) is satisfied by sufficiently small \( m_\delta \), implies that inequality (A6) is satisfied by any \( m_\delta \in (0, C_2 \sqrt{\sigma_i^2 + \sigma_m^2}) \), where we use the fact that \( \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} \) is a continuous function of \( m_\delta \). Thus the lemma is proved.

**Proof of Proposition 3:** In case that \( \alpha^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), and \( \alpha_i^* < 1 \), the condition in Lemma 2 is satisfied. Therefore, Lemma 3 and Lemma 4 imply that \( \alpha_i^* = 1 - 1/x_i \), and therefore,

\[
\frac{\partial (1 - \alpha_i^*)}{\partial \lambda} = \frac{\partial}{\partial \lambda} (1/x_i) = -\frac{1}{x_i^2} \frac{\partial x_i}{\partial \lambda} > 0,
\]

because of Lemma 3. This proves the first part of the proposition.

Taking the derivative and using Lemma 4, we have

\[
\frac{\partial}{\partial m_\delta} \Phi \left( \frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} 1 + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) = \Phi \left( \frac{m_\delta}{\sqrt{\sigma_i^2 + \sigma_m^2}} 1 + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right) \frac{x_i}{m_\delta} > 0.
\]

Thus the proposition is proved.
**Lemma 6**: If \( \alpha^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), then for sufficiently small \( m_\delta \) we have \( \alpha^*_i < 1 \). In fact, it is true for any \( m_\delta \in \left(0, \sqrt{\sigma_i^2 + \sigma_m^2 \cdot \min(C_i, C_j)}\right) \), where \( C_i = \bar{m}_\delta / \sqrt{\sigma_i^2 + \sigma_m^2} \) with \( \bar{m}_\delta \) being a root in \( \left(0, \sqrt{\sigma_i^2 + \sigma_m^2 \cdot \min(C_i, C_j)}\right) \). Further,

\[
(1 + m_\delta)(\mu - R_j) = \lambda \cdot \left(1 - \Phi \left[ \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right] \right) x_i \tag{A7}
\]

if there such a root, and \( C_i = +\infty \) otherwise.

**Proof**: Notice that

\[
\frac{\partial}{\partial m_\delta} \left(1 - \Phi \left[ \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right] \right) x_i = -\phi \left[ \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right] \frac{x_i^2}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \left(1 - \Phi \left[ \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right] \right) \frac{\partial x_i}{\partial m_\delta} < 0
\]

for \( m_\delta \) in \( \left(0, \sqrt{\sigma_i^2 + \sigma_m^2 \cdot \min(C_i, C_j)}\right) \), because the first term is clearly less than zero, and the second term is less than zero because of Lemma 4. Therefore, the right hand side expression of Equation (A7) as a function of \( m_\delta \) is decreasing in \( \left(0, \sqrt{\sigma_i^2 + \sigma_m^2 \cdot \min(C_i, C_j)}\right) \). Thus equation (A7) has at most one root in the interval. And if there is a root, any \( m_\delta \) less than the root will satisfy the inequality with left hand side of (A7) strictly less than the right hand side. Furthermore, we note the fact that

\[
\lim_{m_\delta \to 0^+} \lambda \cdot \left(1 - \Phi \left[ \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right] \right) x_i = \lambda \cdot \left(1 - \Phi \left[ \lim_{m_\delta \to 0^+} \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_j)}{\sqrt{\sigma_i^2 + \sigma_m^2}} \right] \right) \lim_{m_\delta \to 0^+} x_i = +\infty
\]

Therefore, in the case that Equation (A7) has no root in the interval, we will always have the left hand side of (A7) being strictly less than the right hand side. Here the continuity of the right hand side of (4) as a function of \( m_\delta \) is used.

In summary, we have that the left hand side of Equation (A7) is strictly less than the right hand side if \( m_\delta \in \left(0, \sqrt{\sigma_i^2 + \sigma_m^2 \cdot \min(C_i, C_j)}\right) \). The earlier discussion about the first order condition of the losing manager’s optimization problem leads us to conclude that

\[
\arg \max_{\alpha_i} U_b \left[ \alpha_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha_i^b = \begin{bmatrix} \alpha^b_i \\ 1 - \alpha^b_i \end{bmatrix} \right] = 1 \text{ or } 1 - \frac{1}{x_i}. By comparing the value of } U_b \text{ at } \alpha^b_i = 1 \text{ with the value of } U_b \text{ at } \alpha^b_i = 1 - 1/x_i, \text{ we have the necessary and sufficient condition for } \alpha^*_i < 1 \text{ to be that the condition (1) in Lemma 2 holds and that at the same time the left hand side of Equation (A7) is strictly less than the right hand side. For each } m_\delta \in \left(0, \sqrt{\sigma_i^2 + \sigma_m^2 \cdot \min(C_i, C_j)}\right), \text{ both the above conditions are satisfied. Thus the lemma is proved.}
\]

Based on the earlier lemmas, we derive the following sufficient condition for the existence of the equilibrium.
THEOREM 1: Let $\alpha_i^* = 1 - 1/x_i$. The pair $\left\{ \begin{array}{l} \alpha^* = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \alpha^b = \begin{bmatrix} \alpha_1^* \\ 1 - \alpha_1^* \end{bmatrix} \end{array} \right.$ will be an equilibrium for any $m_\delta \in \left[0, \sqrt{\sigma_i^2 + \sigma_m^2} \cdot \min(C_1, C_2, C_3, C_4)\right]$, (A8)

where $C_4 = \lambda (\mu - R_f) \sigma_i^2 (1 + m_i) \sigma_m^2 \sqrt{\frac{(\mu - R_f) \sqrt{\sigma_i^2 + \sigma_m^2}}{\sigma_m^2}}$. Furthermore, $\frac{\partial (1 - \alpha_1^*)}{\partial m_\delta} > 0$ and $\lim_{m_\delta \to 0} (1 - \alpha_1^*) = 0$. To be more precise, we have that $1 - \alpha_1^* = O(\sqrt{m_\delta})$.

Proof: The optimality of the losing manager’s strategy $\alpha^* = \begin{bmatrix} 1 - 1/x_i \\ 1/x_i \end{bmatrix}$ has already been established in Lemma 6. We now prove the optimality of the winning manager’s strategy $\alpha^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ under the condition of the theorem. We evoke the equation $h(x_i) = 0$, which is equivalent to

$$m_\delta = (1 + m_i)(\mu - R_f) \lambda \phi \left( \frac{m_\delta x_i}{\sqrt{\sigma_i^2 + \sigma_m^2}} + \frac{(\mu - R_f)}{m_\delta x_i} \lambda \phi \right)^2.$$  

With this equation, we can see that the necessary condition for $(\mu - R_f) \sigma_i^2 - m_\delta x_i \sigma_m^2 = 0$ is $m_\delta = C_4 \sqrt{\sigma_i^2 + \sigma_m^2}$. Therefore, there is no root for $(\mu - R_f) \sigma_i^2 - m_\delta x_i \sigma_m^2 = 0$ with $m_\delta \in (0, C_4 \sqrt{\sigma_i^2 + \sigma_m^2})$. This coupled with the fact that $\lim_{m_\delta \to 0} m_\delta x_i = 0$, implies that $(\mu - R_f) \sigma_i^2 - m_\delta x_i \sigma_m^2 > 0$ for $m_\delta \in (0, C_4 \sqrt{\sigma_i^2 + \sigma_m^2})$. We next compute the derivative of $U_\delta$ with respect to $\alpha_i^b$.

$$\frac{\partial}{\partial \alpha_i^b} U_\delta \left( \begin{array}{l} \alpha^* = \begin{bmatrix} \alpha_1^* \\ 0 \end{bmatrix} \\ \alpha^b = \begin{bmatrix} 1 - 1/x_i \\ 1/x_i \end{bmatrix} \end{array} \right)$$

$$= (1 + m_i)(\mu - R_f) + \lambda \phi \left( \frac{m_\delta x_i + (x_i \alpha_1^* - x_i + 1)(\mu - R_f)}{\sqrt{\sigma_i^2 + (x_i \alpha_1^* - x_i + 1)^2 \sigma_m^2}} \right)$$

$$= (1 + m_i)(\mu - R_f) + \lambda \phi \left( \frac{m_\delta x_i + (x_i \alpha_1^* - x_i + 1)(\mu - R_f)}{\sqrt{\sigma_i^2 + (x_i \alpha_1^* - x_i + 1)^2 \sigma_m^2}} \right)$$

$$> 0$$
The last inequality is true for all \( \alpha_i^* \in [\alpha_i^1, 1] \), and for \( m_s \in \left( 0, C_i \sqrt{\sigma_i^2 + \sigma_m^2} \right) \). Therefore, \( \alpha^{x*} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) is optimal for the winning manager. Furthermore, \( \frac{\partial (1 - \alpha_i^*)}{\partial m_s} = -\frac{1}{x_i^2} \frac{\partial x_i}{\partial m_s} < 0 \) by lemma 5, and \( \lim_{m_s \to 0} (1 - \alpha_i^*) = \lim_{m_s \to 0} \frac{1}{x_i} = 0 \) by lemma 4. Also by lemma 4, \( 1 - \alpha_i^* = O(\sqrt{m_s}) \).

Thus we have proved the theorem.

**Proof of Corollary 1:** The Sharpe Ratio of fund \( i \), denoted by \( SR^i \) is:

\[
SR^i = \frac{\alpha_i^1 (\mu - R_f)}{\sqrt{\left(\alpha_i^1 \sigma_m^2\right)^2 + \left((1 - \alpha_i^1) \sigma_f^2\right)^2}}.
\]

Taking the derivative of \( SR^i \) with respect to variable \( \alpha_i^1 \) and after simplification, we have

\[
\frac{\partial}{\partial \alpha_i^1} (SR^i) = \frac{(\mu - R_f) \sigma_i^2}{\sqrt{\left(\alpha_i^1 \sigma_m^2\right)^2 + \left((1 - \alpha_i^1) \sigma_f^2\right)^2}} (1 - \alpha_i^1).
\]

Therefore, \( \frac{\partial}{\partial \alpha_i^1} (SR^i) > 0 \) for \( \alpha_i^1 < 1 \). From Theorem 1, in the equilibrium, we have \( \alpha_i^{x*} = 1 \) and \( \alpha_i^{b*} < 1 \), and thus \( SR^b < SR^x \). Moreover, we have that \( \frac{\partial}{\partial m_s} (SR^b) = \frac{\partial}{\partial \alpha_i^b} (SR^b) \cdot \frac{\partial \alpha_i^{b*}}{\partial m_s} < 0 \) from Theorem 1, and \( \frac{\partial}{\partial \lambda} (SR^b) = \frac{\partial}{\partial \alpha_i^b} (SR^b) \cdot \frac{\partial \alpha_i^{b*}}{\partial \lambda} < 0 \) from Proposition 3.
Figure 1: One Period Model

$t = 0$
1. Initial asset base for both managers = $s$

$t = 1$
1. Interim performance observed
2. Both managers choose portfolios for second sub-period

$t = 2$
1. Winning manager receives addition inflow of funds equal to $\lambda$
2. Managers compensated based on total asset base
**Figure 1A: Payoff Table**

**A. Payoffs to managers at the end of each period:**

<table>
<thead>
<tr>
<th>Manager</th>
<th>Period 1</th>
<th>Period 2</th>
<th>period 3</th>
</tr>
</thead>
</table>
| $g$     | $k \exp(F^g)$                                | $k((1-k)\exp(F^g) + \lambda) \exp(S^g)$      | If $S^b - S^g < F^g$  
$                |                                               |                                               | $k((1-k)((1-k)\exp(F^g) + \lambda)\exp(S^g) + \lambda) \exp(T^g)$ |
|         |                                               |                                               | Otherwise  
$                |                                               |                                               | $k(1-k)(1-k)\exp(F^g) + \lambda)\exp(S^g + T^g)$ |
| $b$     | $k \exp(F^b)$                                | $k(1-k)\exp(F^b + S^b)$                     | If $S^b - S^g < F^g$  
$                |                                               |                                               | $k(1-k)^2 \exp(F^b + S^b + T^b)$ |
|         |                                               |                                               | Otherwise  
$                |                                               |                                               | $k[(1-k)^2 \exp(F^g + S^g) + \lambda]\exp(T^g)$ |

**B. Returns to the investors in each period:**

<table>
<thead>
<tr>
<th>Invest with manager</th>
<th>Return in period 1</th>
<th>Return in period 2</th>
<th>Return in period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>$(1-k)\exp(F^g)$</td>
<td>$(1-k)\exp(S^g)$</td>
<td>$(1-k)\exp(T^g)$</td>
</tr>
<tr>
<td>$b$</td>
<td>$(1-k)\exp(F^b)$</td>
<td>$(1-k)\exp(S^b)$</td>
<td>$(1-k)\exp(T^b)$</td>
</tr>
</tbody>
</table>
Figure 2: Sequence of Events for Model with Costly Information Acquisition

- **t = 0**
  1. Initial asset base for both managers is normalized to be 1

- **t = 1**
  1. Interim performance observed
  2. Both managers decide the effort level
  3. Each manager becomes either informed or uninformed
  4. Both managers choose portfolios for second sub-period

- **t = 2**
  1. Winning manager receives additional inflow of funds equal to $\lambda$
  2. Managers compensated based on total asset base
Note: The series labeled ‘effort_g’, and ‘effort_b’, represent the optimal effort choices of manager \( g \), and manager \( b \) in the tournament regime, while ‘effort_NT’ represents the optimal managerial effort in the no-tournament regime. The series labeled ‘act_g_info’, and ‘act_b_info’, represent the optimal active allocations of manager \( g \) and manager \( b \) when informed, in the tournament regime, while ‘act_NT_info’ represents the optimal active allocation in the no-tournament regime when the manager is informed. The series labeled ‘act_b_uninfo’ represents the optimal active allocations of manager \( b \) when she is uninformed, in the tournament regime.
Note: The series labeled ‘effort_g’, and ‘effort_b’, represent the optimal effort choices of manager \( g \), and manager \( b \) in the tournament regime, while ‘effort_NT’ represents the optimal managerial effort in the no-tournament regime. The series labeled ‘act_g_info’, and ‘act_b_info’, represent the optimal active allocations of manager \( g \) and manager \( b \) when informed, in the tournament regime, while ‘act_NT_info’ represents the optimal active allocation in the no-tournament regime when the manager is informed.
Figure 5.A

Investor Welfare Gain V/s Value of Information ($\rho$)

Figure 5.B

Managerial Effort V/s Value of Information ($\rho$)

Figure 5.C

Active Component Allocation V/s Value of Information ($\rho$)

Note: The series labeled ‘effort_g’, and ‘effort_b’, represent the optimal effort choices of manager $g$, and manager $b$ in the tournament regime, while ‘effort_NT’ represents the optimal managerial effort in the no-tournament regime. The series labeled ‘act_g_info’, and ‘act_b_info’, represent the optimal active allocations of manager $g$ and manager $b$ when informed, in the tournament regime, while ‘act_NT_info’ represents the optimal active allocation in the no-tournament regime when the manager is informed.
Figure 6

Active Component Allocation V/s Intermediate Performance Gap $m_\delta$

Note: The series labeled ‘act_g_info’, and ‘act_b_info’, represent the optimal active allocations of manager g and manager b when informed, in the tournament regime, while ‘act_NT_info’ represents the optimal active allocation in the no-tournament regime when the manager is informed. The series labeled ‘act_b_uninfo’ represents the optimal active allocations of manager b when she is uninformed, in the tournament regime.
Table 1
Sample Description
The table presents summary statistics on the mutual fund sample obtained from the CRSP Survivor-Bias Free US Mutual Fund Database. The sample includes U.S. equity mutual funds classified as Aggressive Growth (AG), Growth and Income (GI), and Long Term Growth (LG) that existed at any time during January 1962 to December 2004. We exclude index funds from the sample. The final sample consists of 2,839 fund-entities representing 31,382 fund-years. TNA (total net assets) refers to the year-end market value of fund assets and is shown in billions of dollars. Turnover is defined as the minimum of aggregate purchases or sales of securities during the year, divided by the average TNA, front-end load is the maximum percent charges applied at the time of purchase, total load is the sum of maximum front-end load fees and maximum sales charges paid when withdrawing money from the fund, and expense ratio is the percentage of total investment that shareholders pay for the fund’s operating expenses. For each item (other than number of funds, and TNA) we first compute the TNA-weighted cross-sectional averages in each year from 1962 to 2004. The reported statistics in Panel A are computed as the time series averages of the annual cross-sectional average figures for each item.

Panel A: Characteristics of stock funds in the sample

<table>
<thead>
<tr>
<th>Item</th>
<th>AG</th>
<th>GI</th>
<th>LG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of funds</td>
<td>772</td>
<td>754</td>
<td>1313</td>
</tr>
<tr>
<td>Percent stock holding</td>
<td>89.81</td>
<td>84.23</td>
<td>89.33</td>
</tr>
<tr>
<td>Percent cash holding</td>
<td>7.71</td>
<td>7.31</td>
<td>7.73</td>
</tr>
<tr>
<td>Turnover (%)</td>
<td>85.14</td>
<td>47.67</td>
<td>58.37</td>
</tr>
<tr>
<td>Front-end load (%)</td>
<td>4.30</td>
<td>5.24</td>
<td>4.59</td>
</tr>
<tr>
<td>Total load (%)</td>
<td>4.44</td>
<td>5.39</td>
<td>4.76</td>
</tr>
<tr>
<td>Expense ratio (%)</td>
<td>0.95</td>
<td>0.66</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Panel B: Time trend in number of funds and aggregate TNA

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AG</td>
<td>Funds</td>
<td>39</td>
<td>165</td>
<td>104</td>
<td>150</td>
<td>210</td>
<td>369</td>
<td>407</td>
<td>265</td>
</tr>
<tr>
<td></td>
<td>TNA</td>
<td>0.5</td>
<td>6.2</td>
<td>7.9</td>
<td>26.4</td>
<td>47.8</td>
<td>245.4</td>
<td>577.2</td>
<td>486.8</td>
</tr>
<tr>
<td>GI</td>
<td>Funds</td>
<td>95</td>
<td>131</td>
<td>126</td>
<td>179</td>
<td>318</td>
<td>423</td>
<td>443</td>
<td>273</td>
</tr>
<tr>
<td></td>
<td>TNA</td>
<td>10.8</td>
<td>21.6</td>
<td>22.0</td>
<td>49.5</td>
<td>91.7</td>
<td>330.1</td>
<td>662.4</td>
<td>757.0</td>
</tr>
<tr>
<td>LG</td>
<td>Funds</td>
<td>64</td>
<td>144</td>
<td>124</td>
<td>231</td>
<td>382</td>
<td>571</td>
<td>737</td>
<td>393</td>
</tr>
<tr>
<td></td>
<td>TNA</td>
<td>3.4</td>
<td>13.0</td>
<td>15.4</td>
<td>35.0</td>
<td>63.1</td>
<td>228.8</td>
<td>603.5</td>
<td>543.8</td>
</tr>
</tbody>
</table>
**Table 2**

**Determinants of Quarterly Fund Cash Flows**

This table presents the coefficients from regressions of realized quarterly normalized cash flow for a fund against the fund’s lagged quarterly normalized cash flow, the fund’s total return in the previous quarter (net of the average return for the fund’s category), the logarithm of total net assets, turnover, expense ratio, and the maximum front-end load fees. The normalized quarterly cash flow for a fund during a quarter is computed as the quarterly cash flow for the fund divided by the total net assets (TNA) at the beginning of the quarter. Turnover is defined as the minimum of aggregate purchases or sales of securities during the year, divided by the average TNA, expense ratio is the proportion of total investment that shareholders pay for the fund’s operating expenses, and the maximum front-end load fees are the maximum percent charges applied at the time of purchase. The complete sample of funds covers the period January 1962 to December 2004 and is described in Table 1. Due to some missing data items in the initial years, the coefficients reported here are based on averages of 156 quarterly cross-sectional regressions from 1966 to 2004. The t-statistics based on the Newey-West covariance matrix are reported in parenthesis. The notations *, **, and *** represent statistical significance levels of 10 percent, 5 percent, and 1 percent in two-tailed tests. To minimize clutter significance is denoted only for the slope coefficients.

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Category</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All funds</td>
<td>AG</td>
<td>GI</td>
<td>LG</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.081</td>
<td>0.149</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(4.67)</td>
<td>(4.13)</td>
<td>(4.07)</td>
<td>(4.61)</td>
</tr>
<tr>
<td>Previous quarter’s cash flow</td>
<td>0.130</td>
<td>0.097</td>
<td>0.147</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.93)</td>
<td>(2.13)</td>
<td>(1.19)</td>
</tr>
<tr>
<td>Previous quarter’s return</td>
<td>0.435</td>
<td>0.469</td>
<td>0.427</td>
<td>0.392</td>
</tr>
<tr>
<td></td>
<td>(2.66) **</td>
<td>(6.39) ***</td>
<td>(4.63) ***</td>
<td>(5.80) ***</td>
</tr>
<tr>
<td>Logarithm of total net assets</td>
<td>-0.017</td>
<td>-0.020</td>
<td>-0.014</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(-6.66) ***</td>
<td>(-4.59) ***</td>
<td>(-4.93) ***</td>
<td>(-4.99) ***</td>
</tr>
<tr>
<td>Expense ratio</td>
<td>-2.123</td>
<td>-4.188</td>
<td>-1.939</td>
<td>-1.332</td>
</tr>
<tr>
<td></td>
<td>(-3.32) ***</td>
<td>(-2.22) **</td>
<td>(-2.21) **</td>
<td>(-1.09)</td>
</tr>
<tr>
<td>Turnover</td>
<td>0.003</td>
<td>0.001</td>
<td>0.012</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(1.23)</td>
<td>(0.06)</td>
<td>(2.16) **</td>
<td>(0.42)</td>
</tr>
<tr>
<td>Maximum front-end load</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(-1.73) *</td>
<td>(-0.49)</td>
<td>(-1.78) *</td>
<td>(-1.28)</td>
</tr>
</tbody>
</table>
Table 3
Changes in Fund Idiosyncratic Risk and Past Performance: Monthly Returns

We divide each year in the sample period into two six monthly sub-periods. We estimate the change in the idiosyncratic risk of a fund from the first half of the year (January to June) to the second half of the year (July to December). The idiosyncratic risk for fund $i$ for sub-period $s$ ($s = 1, 2$) of year $T$, denoted by $\sigma(\varepsilon_{i,s,T})$, is given by the standard deviation of the monthly residual return $\varepsilon_{i,s,d}$. The fund residual return is estimated as:

$$\varepsilon_{i,s,d} = \{r_{i,d} - (\hat{a}_{i,T} + \hat{\beta}_{i,T} r_{m,t})\}_{t \in \Gamma(s)},$$

where $\Gamma(s) = \{1,...,6\}$ for $s = 1$; and $\Gamma(s) = \{7,...,12\}$ for $s = 2$.

In the above equation $r_{i,d}$ denotes the return in excess of the one month US T-bill return for fund $i$ during month $t$, and $r_{m,t}$ denotes the excess return on the CRSP value-weighted market portfolio for month $t$. We obtain the parameters estimates $\hat{a}_{i,T}$ and $\hat{\beta}_{i,T}$ using monthly fund returns for the 24 months immediately preceding year $T$. The change in idiosyncratic risk of fund $i$ for year $T$ is measured as $\Delta \sigma_{i,T} = \sigma(\varepsilon_{i,2,T}) - \sigma(\varepsilon_{i,1,T})$. For each fund within category $k$, the estimated change in idiosyncratic risk from the first half of the year to the second half is regressed on the total return of the fund during the first six months adjusted for the mean return of all funds within the category $\{R_{i,1,T}\}$ during the six month period:

$$\Delta \sigma_{i,T} = a_{k,T} + b_{k,T} R_{i,1,T} + u_{k,T}$$

The coefficients reported below in Panel A are time-series averages of the 41 yearly cross-sectional regression coefficients from 1964 to 2004. Panel B reports coefficients for two groups of funds within each category: funds with return (in the first half of the year) above the median return and funds with return below the median return. It also reports the difference in slope coefficients for each of the two groups within a category. Model I estimates are based on the above regression where the dependent variable is the change in idiosyncratic risk. Model II estimates are based on a regression where the dependent variable is the proportional change in idiosyncratic risk. Also shown are the t-statistics based on the Newey-West covariance matrix. The notations *, **, and *** represent statistical significance levels of 10 percent, 5 percent, and 1 percent in two-tailed tests. To avoid clutter significance is denoted only for the slope coefficients. The initial sample is described in Table 1. The analysis here is based on a reduced sample of 2,682 funds due to the requirement that each fund should have twelve months of returns available in a given calendar year.

<table>
<thead>
<tr>
<th>Category</th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A t-stat</td>
<td>b t-stat</td>
</tr>
<tr>
<td>AG</td>
<td>0.001</td>
<td>0.34</td>
</tr>
<tr>
<td>GI</td>
<td>0.001</td>
<td>0.09</td>
</tr>
<tr>
<td>LG</td>
<td>0.001</td>
<td>0.38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel A: Regression of change in idiosyncratic risk on past return</th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>A t-stat</td>
<td>b t-stat</td>
</tr>
<tr>
<td>AG</td>
<td>0.001</td>
<td>0.34</td>
</tr>
<tr>
<td>GI</td>
<td>0.001</td>
<td>0.09</td>
</tr>
<tr>
<td>LG</td>
<td>0.001</td>
<td>0.38</td>
</tr>
<tr>
<td>Category</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.004</td>
<td>1.25</td>
</tr>
<tr>
<td>High</td>
<td>0.003</td>
<td>1.38</td>
</tr>
<tr>
<td>Difference</td>
<td>0.075</td>
<td>3.10 ***</td>
</tr>
<tr>
<td>GI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.000</td>
<td>0.11</td>
</tr>
<tr>
<td>High</td>
<td>0.001</td>
<td>1.37</td>
</tr>
<tr>
<td>Difference</td>
<td>0.021</td>
<td>1.09</td>
</tr>
<tr>
<td>LG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>0.001</td>
<td>0.99</td>
</tr>
<tr>
<td>High</td>
<td>0.002</td>
<td>1.24</td>
</tr>
<tr>
<td>Difference</td>
<td>0.053</td>
<td>1.56</td>
</tr>
</tbody>
</table>

**Panel B: Regression of change in idiosyncratic risk on past return by performance rank**

<table>
<thead>
<tr>
<th>Category</th>
<th>Performance Rank</th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>a  t-stat</td>
<td>b  t-stat</td>
</tr>
<tr>
<td>AG</td>
<td>Low</td>
<td>0.290</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.285</td>
<td>3.55</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>1.594</td>
<td>2.54 **</td>
</tr>
<tr>
<td>GI</td>
<td>Low</td>
<td>0.209</td>
<td>3.89</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.251</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>0.333</td>
<td>0.32</td>
</tr>
<tr>
<td>LG</td>
<td>Low</td>
<td>0.227</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.248</td>
<td>3.86</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>0.015</td>
<td>0.02</td>
</tr>
</tbody>
</table>
### Table 4
**Changes in Fund Idiosyncratic Risk and Past Performance: Daily Returns**

We divide each year in the sample period into two periods. In the first case (labeled ‘Assessment Period (6,6)’) the two periods are each of six months duration with the first period extending from January to June and the second period extending from July to December. In the second case (labeled ‘Assessment Period (9,3)’), the first period extends from January to September while the second period extends from October to December. We estimate the change in the idiosyncratic risk of a fund from the first period of the year to the second period of the year. The idiosyncratic risk for fund $i$ for sub-period $s$ ($s = 1, 2$) of year $T$, denoted by $\sigma(e_{i,s,T})$, is given by the standard deviation of the daily residual return $e_{i,d}$. The fund residual return for day $d$ is estimated as: $e_{i,d} = \left[ r_{i,d} - (\hat{a}_{i,s,T} + \hat{\beta}_{i,s,T} r_{m,d}) \right]$. Here $r_{i,d}$ denotes the excess return for fund $i$ during day $d$, and $r_{m,d}$ denotes the corresponding excess return on the CRSP value-weighted market portfolio. The change in idiosyncratic risk of fund $i$ for year $T$ is measured as $\Delta \sigma_{i,T} = \sigma(e_{i,2,T}) - \sigma(e_{i,1,T})$. We estimate the following pooled cross-sectional regression for each fund category $k$:

$$\Delta \sigma_{i,T} = \alpha_k + \beta_k R_{i,1,T} + u_k$$

In the above regression, the variable $R_{i,1,T}$ denotes the total return of the fund during the first period of the year adjusted for the mean return of all funds within its category. Panel A reports estimates of the pooled cross-sectional regression slope coefficients for the period 2001 to 2004. Panel B reports the slope coefficients from pooled cross-sectional regressions for two groups of funds within each category: funds with return (in the first period of the year) above the median return and funds with return below the median return. It also reports the difference in slope coefficients for each of the two groups within a category. Model I estimates are based on the above regression where the dependent variable is the change in idiosyncratic risk. Model II estimates are based on a regression where the dependent variable is the proportional change in idiosyncratic risk. Also shown are the t-statistics based on the Newey-West covariance matrix and the corresponding empirical p-values (in parenthesis) that are based on a simulation procedure described in the text. The initial sample is described in Table 1. The analysis in this table is based on a smaller sample of 1,655 funds (representing 5,313 fund years) with daily returns that exist at any time during 2001 to 2004.

#### Panel A: Regression of change in idiosyncratic risk on past return

<table>
<thead>
<tr>
<th>Category</th>
<th>Assessment Period (6,6)</th>
<th></th>
<th></th>
<th>Assessment Period (9,3)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model I</td>
<td>Model II</td>
<td>Model I</td>
<td>Model II</td>
<td>Model I</td>
<td>Model II</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>t-stat</td>
<td>b</td>
<td>t-stat</td>
<td>b</td>
<td>t-stat</td>
</tr>
<tr>
<td>AG</td>
<td>0.002</td>
<td>2.15</td>
<td>0.321</td>
<td>3.27</td>
<td>0.003</td>
<td>2.97</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td></td>
<td>(0.07)</td>
<td></td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>GI</td>
<td>0.003</td>
<td>1.96</td>
<td>0.243</td>
<td>1.28</td>
<td>0.002</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td></td>
<td>(0.47)</td>
<td></td>
<td>(0.29)</td>
<td></td>
</tr>
<tr>
<td>LG</td>
<td>0.004</td>
<td>3.35</td>
<td>0.347</td>
<td>3.14</td>
<td>0.003</td>
<td>2.95</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td></td>
<td>(0.15)</td>
<td></td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td>Category</td>
<td>Performance Rank</td>
<td>Assessment Period (6,6)</td>
<td></td>
<td></td>
<td>Assessment Period (9,3)</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>-------------------------</td>
<td>---</td>
<td>---</td>
<td>-------------------------</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Model I</td>
<td>Model II</td>
<td>b</td>
<td>t-stat</td>
<td>b</td>
<td>t-stat</td>
</tr>
<tr>
<td>AG</td>
<td>Low</td>
<td>0.006</td>
<td>2.01</td>
<td>(0.23)</td>
<td>0.274</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.000</td>
<td>0.30</td>
<td>(0.87)</td>
<td>0.308</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>0.006</td>
<td>1.67</td>
<td>(0.33)</td>
<td>-0.033</td>
<td>-0.14</td>
</tr>
<tr>
<td>GI</td>
<td>Low</td>
<td>0.011</td>
<td>2.63</td>
<td>(0.16)</td>
<td>0.278</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.002</td>
<td>2.11</td>
<td>(0.21)</td>
<td>0.809</td>
<td>3.30</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>0.009</td>
<td>2.09</td>
<td>(0.02)</td>
<td>-0.531</td>
<td>-1.26</td>
</tr>
<tr>
<td>LG</td>
<td>Low</td>
<td>0.014</td>
<td>4.74</td>
<td>(0.08)</td>
<td>0.903</td>
<td>4.41</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.000</td>
<td>-0.55</td>
<td>(0.78)</td>
<td>0.101</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Difference</td>
<td>0.015</td>
<td>4.71</td>
<td>(0.00)</td>
<td>0.802</td>
<td>3.28</td>
</tr>
</tbody>
</table>