Incentive Contracts in Delegated Portfolio Management

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May 2008

We are grateful to two anonymous referees and Raman Uppal (the editor) for their helpful comments.

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Abstract

This paper analyzes optimal non-linear portfolio management contracts. We consider a setting where the investor faces moral hazard with respect to the effort and risk choices of the portfolio manager. The manager's employment contract promises her: (a) a fixed payment, (b) a proportional asset-based fee, (c) a benchmark-linked fulcrum fee, and (d) a benchmark-linked option-type "bonus" incentive fee. We show that the option-type incentive helps overcome the effort-underinvestment problem that undermines linear contracts. More generally, we find that for the set of contracts we consider, with the appropriate choice of benchmark it is always optimal to include a bonus incentive fee in the contract. We derive the conditions that such a benchmark must satisfy. Our results suggest that current regulatory restrictions on asymmetric performance-based fees in mutual fund advisory contracts may be costly.

When investors delegate the portfolio management decision to a professional manager, they are faced with the classic problem of designing management-fee contracts. Neither the manager's information nor the effort expended by him is directly observable by investors. Consequently, the appropriate contract should motivate manager to exert costly effort to gather information relevant to security payoffs. The contract should also induce the manager to subsequently use this information in choosing a portfolio with the desirable risk characteristics. Importantly, as recognized by Stoughton (1993), the effort incentives and the risk-taking incentives interact in a particular way in the delegated portfolio management setting, and therefore, the solutions offered by the generic principal-agent literature are not readily applicable. For example, Stoughton shows that, due to the feedback effect of risk incentives on the effort incentive, contracts in which the management fee is *linearly* related to portfolio returns lead to an underinvestment in effort expended by the manager and such an underinvestment problem cannot be addressed by increasing the manager's stake.[1] Further, as shown by Admati and Pfleiderer (1997), this underinvestment problem cannot be resolved by the use of benchmark-adjusted linear compensation contracts.

A natural question then is whether *non-linear* portfolio management contracts can help overcome the effort-underinvestment problem faced by investors while at the same time providing the appropriate risk incentives. Furthermore, if a non-linear contract is to be employed, what form should it take? Stoughton (1993) shows that within a security-analyst context in which the portfolio manager simply reveals his information to the investor, quadratic contracts proposed by Bhattacharya and Pfleiderer (1985) help overcome the effort underinvestment problem. However, as Stoughton points out, such contracts cannot be implemented in the more realistic portfolio management environment where investors delegate the portfolio choice decision to the manager. Hence, the question of the optimal management fee contract in this environment remains largely unanswered.

[1] These results contrast with those from the standard agency literature that suggest that linear contracts are optimal in a second best sense under fairly general conditions (see, for example, Holmstrom and Milgrom (1987)). The difference stems from the structure and the timing of the delegated portfolio management problem under which the manager first expends costly effort to acquire an information signal, and then chooses the portfolio allocation based on the signal. Importantly, the manager can alter the scale of his response to the signal, thereby controlling both the level and the volatility of the portfolio return. This structure limits the effectiveness of linear contracts.

In this paper we adopt Stoughton's framework in order to analyze the optimal contracting problem. We examine a model in which a risk-averse portfolio manager proposes an employment contract while soliciting funds from a representative potential investor. The contract specifies the manager's compensation, which consists of (i) a fixed payment, (ii) a proportional management fee equal to a portion of the assets under management, (iii) a symmetric or "fulcrum" performance incentive fee that yields a bonus or a penalty as a function of the portfolio performance relative to a benchmark, and (iv) an option-type performance fee that yields a bonus depending on the portfolio performance relative to a benchmark, without a corresponding penalty in the event of underperformance. Once the contract terms are agreed upon by the investor, the manager invests the funds on the investor's behalf. The manager's investment opportunity set consists of (a) the market index portfolio representing the passive component of the manager's portfolio, (b) a risky asset that represents the actively managed component of the portfolio, and (c) the riskfree asset. The manager first undertakes the level of costly effort in order to collect information about the actively managed risky asset's payoff. His effort level determines the precision of the information. Conditional on this information, he then chooses the portfolio allocation.

Our analysis proceeds in two steps. In the first step, we take the contract as being exogenous, and study how the managerial effort is influenced by the contract composition. In the second step of the analysis, we address the optimal contract choice problem. Our analysis leads to three main conclusions. One, the managerial effort incentive is critically dependent on the choice of an appropriate benchmark in the design of the option-type performance fee. Two, when the benchmark is appropriately chosen the use of an option-type "bonus" incentive fee can motivate the portfolio manager to expend increased effort. Three, for the set of contracts considered here, it is always optimal to include a "bonus" incentive fee in the managerial contract. We assess the economic magnitude of the potential efficiency gains by calibrating the model to empirically observed values for the key parameters based on a sample of U.S. equity mutual funds. We find that the inclusion of the option-type incentive fee in the managerial contract leads to an improvement in information production from the manager of over 90 percent relative to the optimal linear contract.

To highlight the relevance of the benchmark in the bonus incentive fee for managerial effort, we demonstrate the existence of two main scenarios characterized by the choice of the benchmark. In the first case, the benchmark is specified such that it closely tracks the investment style of the manager, and thereby, successfully purges the factors that are beyond the manager's control, but that have an impact on the portfolio outcome. In this case, the option-type performance fee can enhance the manager's effort incentive, and therefore, help mitigate the effort underinvestment problem. In the second case, the benchmark fails to purge the factors that are beyond the manager's control. In this scenario the use of the bonus fee is counterproductive and in effect encourages the manager to shirk effort and rely solely on luck in order to beat the benchmark. Therefore, if the benchmark is not chosen with care, the option-type "bonus" incentive fee is actually detrimental to the manager's effort incentive.

While the use of a well designed option-type bonus performance fee can improve the effort incentives of the portfolio manager, it distorts the risk incentives and leads to sub-optimal risk sharing between the manager and the investor. Such concerns lead to a natural question: "Are the resultant gains from the improved effort choices of portfolio managers sufficiently large to offset the cost of such an incentive scheme?" We show analytically that for the set of contracts considered by us, it is *always* optimal to include a bonus performance incentive fee in the managerial contract.

In contrast to the extensive theoretical literature on the principal-agent contracting problem within the conventional corporate setting, there are relatively few studies that are specific to the delegated portfolio management environment. Our analysis makes a number of contributions to this particular strand of the theoretical literature. First, we obtain analytical results that provide an explicit link between the managerial effort choices and option-type performance incentive fees. In contrast to previous studies such as Starks (1987), we show that in a setting where the portfolio manager optimally determines both the effort allocation and the portfolio allocation, the use of the option-type performance incentive fees can in fact improve outcomes relative to the case with pure fulcrum performance fees. Furthermore, our study differs in important ways from the general literature in the principal-agent setting, where the focus has been mainly on the risk incentives created by the option-type incentive contract.[2]

Second, in contrast to extant results that suggest that benchmark-based compensation schemes are at best irrelevant (e.g., Admati and Pfleiderer (1997)), our analysis highlights the critical role played by the choice of benchmark in motivating managerial effort. The key difference is that Admati and Pfleiderer analyze a managerial compensation function that is linear in the portfolio returns. In contrast, we examine a more general set of compensation contracts that allows for a component that is a convex function of the portfolio returns, in addition to a component that is linear in the portfolio returns. Our results with respect to the significance of the benchmark choice are consistent with the widespread use of benchmark-based performance evaluation, either explicitly or implicitly, by investors.

Finally, our analysis provides an answer to the long-standing question of whether regulatory restrictions on the form of performance-based fees in, say, mutual funds are theoretically justified.[3] Our analytical results suggest that a universal prohibition of asymmetric performance-based fees is costly from a social welfare perspective.[4] In this respect our results provide a justification for the existence of such "bonus" performance-based fees in mutual funds prior to the 1970 amendment that banned them. Interestingly, our analysis also provides theoretical justification for the empirical findings of Golec and Starks (2004). They document that mutual funds that were forced to alter their managerial contracts in

[2] Examples of the extended literature include Ross (2004), Chen and Pennacchi (2005), and Hemmer, Kim, and Verrecchia (2000). Studies which primarily focus on the manager's risk incentives in the portfolio management setting include Carpenter (2000) and Grinblatt and Titman (1989).

[3] In 1970 Congress brought investment company advisers under the purview of the *Investment Advisers Act* of 1940. At the same time it amended Section 205 of the Act to exempt these advisers from the general prohibition against performance-based fees. However, it required that performance-based fees should be symmetrical in terms of rewards or penalties around the chosen benchmark, and therefore ruled out the use of "bonus" or option-type performance incentive fees.

[4] While the analysis in this paper is based on the case where the manager has absolute market power, our results on the efficiency gains also hold in the less extreme cases where the investor is able to share a portion of the gain. Of course, a gain in the efficiency of information production is socially desirable, regardless of how it is divided up.

order to comply with the new restrictions that went into effect in 1971, lost both assets and shareholders in the aftermath.[5]

The remainder of the paper is organized as follows. Section 1 presents the modeling framework. In Section 2, we analyze the manager's optimization problem under a contract which includes the option-type performance fee. A key result of this section shows that the bonus incentive fee helps mitigate the effort-underinvestment problem that plagues the linear contract. Section 3 analyzes the optimal contract choice problem by simultaneously taking into account the impact of the three key aspects of the decision problem, namely risk sharing, the manager's effort incentive, and the manager's portfolio choice. This section also presents numerical results based on a calibration of the model using data on a sample of U.S. mutual funds. Our conclusions are presented in Section 4.

1. Model Specification

We consider a one-period stationary model in which a professional portfolio manager who can exert costly effort to generate private information meets a representative investor who has investment capital W_0. At the start of the period, the manager proposes a contract to the investor under which the manager invests W_0 on the investor's behalf. Upon the investor's acceptance of the terms of the contract, the manager voluntarily undertakes a level of costly effort to collect information about assets payoffs and proceeds to make the portfolio allocation decision. At the end of the period, the asset returns are realized and the total wealth of the managed portfolio is returned to the investor, while the manager is compensated according to the terms of his contract. We assume that the investor's utility is driven to her reservation level due to competition for the manager's services.[6] We further assume that the investor can alternatively invest through other channels and get a certainty-equivalent return of R_0.

[5] Our results complement those of de Meza and Webb (2007), and Dittman, Maug, and Spault (2007). Other related studies include Das and Sundaram (2002), Palomino and Prat (2003), and Ou-Yang (2003).

[6] This is, for example, consistent with the argument made by Berk and Green (2004). Nevertheless, all our key results will remain unchanged if the surplus is divided between the investor and the manager in some other pre-specified fashion, including, for example, the case where the investor retains the entire surplus.

1.1 Investment Opportunity Set and Information

The investment opportunity set for the manager consists of three assets: a riskfree asset with constant return expressed in gross terms, $R_f > 1$; a market index portfolio (the passively managed asset) with a return denoted by R_m, and the excess return $R_m^e \equiv R_m - R_f$; and a representative stock (stock *a*, the actively managed asset) with a return denoted by R_a, which can be decomposed as $R_a = R_f + \beta_a R_m^e + R_a^e$ with $E(R_a^e) = 0$ and $corr(R_m^e, R_a^e) = 0$. Upon expending a level of effort, ε, the manager learns the value of a "firm-specific" signal, s, that is correlated with the stock's excess return, R_a^e. The manager's effort level determines the informativeness of the signal, captured by $\rho = corr(R_a^e, s)$. The signal s is uninformative about R_m^e (i.e., $corr(R_m^e, s) = 0$). After collecting information, the manager makes his portfolio allocation decision. For a portfolio allocation of (w_1, w_2) to the market index portfolio and stock *a*, the portfolio return realized at the end of the period is

$$R_w = R_f + (w_1 + w_2\beta_a)R_m^e + w_2 R_a^e. \tag{1}$$

For convenience, we will henceforth denote a vector $w = (w_m, w_a)$, with $w_m = w_1 + w_2\beta_a$, and $w_a = w_2$, and refer to this vector as the manager's portfolio allocation. We note that stock *a* in our model represents any generic stock that the manager researches. As his signal is firm specific, we anticipate that the manager will be long (or increase the holdings of) the stocks for which he receives a positive signal, and short (or reduce the holdings of) the stocks for which he receives a negative signal. We denote the portfolio's excess return $R_w^e \equiv R_w - R_f$, and the terminal value of the portfolio, $W \equiv W_0 R_w$.[7]

We make the following assumptions about the marginal distributions for the asset returns and the information signal. The market excess return is assumed to follow a discrete distribution with the following values: $R_m^e = \delta_m$ with probability p, and $R_m^e = -\delta_m$ with probability $1-p$. The excess return

[7] Consistent with the typical practice in the case of say, mutual funds, we restrict the portfolio manager's opportunity set to rule out investments in derivatives contracts.

of the actively managed asset, R_a^e, can be either δ_a or $-\delta_a$, with equal probability. The manager's signal, s, with equal probabilities, takes a value of either 1 (good news about R_a^e) or 0 (bad news).

The joint distribution of the three variables (R_m^e, R_a^e, and s) is now uniquely determined. Conditional on the realization of s, we characterize the four states in the asset-return space by whether the market portfolio and/or the actively managed asset are up (U) or down (D) in value. Thus, there are eight states in the model, four states for the case when $s = 1$, and symmetrically another four states corresponding to $s = 0$. Conditional on $s = 1$, the four states in the asset-return space have the following distribution:

State (UU): $R_m^e = \delta_m$ and $R_a^e = \delta_a$ with probability $p(1+\rho)/2$;

State (UD): $R_m^e = \delta_m$ and $R_a^e = -\delta_a$ with probability $p(1-\rho)/2$;

State (DU): $R_m^e = -\delta_m$ and $R_a^e = \delta_a$ with probability $(1-p)(1+\rho)/2$;

State (DD): $R_m^e = -\delta_m$ and $R_a^e = -\delta_a$ with probability $(1-p)(1-\rho)/2$.

Clearly, $s = 1$ represents good news about R_a^e, indicating a conditional marginal probability of $\frac{1+\rho}{2}$ for $R_a^e > 0$. The probabilities for the above four states conditional on $s = 0$ are: $p(1-\rho)/2$, $p(1+\rho)/2$, $(1-p)(1-\rho)/2$, and $(1-p)(1+\rho)/2$, respectively. Note that the realization of bad news ($s = 0$) implies a conditional marginal probability of $\frac{1+\rho}{2}$ for $R_a^e < 0$.[8]

1.2 Utility Specification and the Cost of Effort

Henceforth, we assume CARA preferences for both the manager and the investor. The manager's utility is given by $U(C,\varepsilon) = -\exp[-a(C - V(\varepsilon))]$, where C denotes consumption and $V(\varepsilon)$ represents the cost of

[8] It is usual practice to think of a less accurate information signal as being equivalent to the sum of a more accurate signal plus a random noise component. In our discrete-distribution setup, a similar intuition holds. A manager with a signal of lower accuracy is practically equivalent to a manager with a signal of higher accuracy but who has the tendency to misinterpret her signal with a certain probability.

managerial effort, ε. The investor's utility is given by $U_p(C) = -\exp[-bC]$. The parameters a and b are the risk-aversion coefficients for the manager and the investor respectively. In the above specification of the manager's utility, the cost function of effort is assumed to be a generic function. We make the same minimal assumption about $V(\varepsilon)$ as Stoughton (1993):

$$V'(\varepsilon) > 0, \text{and } V''(\varepsilon) \geq 0. \tag{2}$$

Note that from the manager's perspective, the choice of the effort level boils down to the perceived tradeoff between the cost and the benefits of effort. Hence, a value-based definition of managerial effort can allow for an economically meaningful comparison between the costs and the benefits of acquiring information. To that end, we define the effort level (ε), as a linear transformation of the gain in the certainty-equivalent utility for the manager from the acquisition of information with accuracy ρ, if the manager were to invest for himself.[9] Such a definition of ε takes the following simple form:

$$\varepsilon = -\frac{1}{2}\ln\left(1-\rho^2\right). \tag{3}$$

Clearly, $\dfrac{d\varepsilon}{d\rho} = \dfrac{\rho}{(1-\rho^2)} > 0$. In other words, observing a signal with higher accuracy requires higher effort from the manager. It is worth emphasizing that we do not lose generality from the specification in (3), because the cost function of effort, $V(\varepsilon)$, is still in a generic form.[10] Given our specification of the managerial effort, the second condition in (2) has *precisely* the following economic interpretation: it is increasingly costly for the manager to improve his *certainty-equivalent utility* by acquiring increasingly accurate information if he were to invest for himself.[11]

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[9] If the manager's expected utility is Y, then the certainty-equivalent utility, by definition, is $-\dfrac{1}{a}\ln(-Y)$.

[10] Not surprisingly, the analytical results do not rely on this effort specification. By using this specific setup, we are saved from the trouble of repetitively citing some form of "single crossing" condition so that the voluntary effort choice problem has a definite solution.

[11] We note here that, for this interpretation to be equivalent to the second inequality in (2), ε has to be defined as in (3), or as its positive linear transformation.

1.3 Managerial Contract

The managerial contract is allowed to consist of four components: (a) the fixed payment with value F; (b) the proportional management fee that pays a pre-specified portion of the assets under management with contingent payoff $c_1 W$; (c) the "fulcrum" incentive fee with payoff $c_2(W - W_{BL})$, where the performance is measured against a pre-specified benchmark, W_{BL}; and (d) the option-type "bonus" incentive fee with payoff $\lambda \max(W - W_{BO}, 0)$, where W_{BO} is another pre-specified benchmark. The subscript "*BL*" ("*BO*") denotes the benchmark for the *linear* (*option-type*) fee component. We use the functional notation $W_a(W)$ to denote the manager's final reward contingent on his performance represented by the terminal portfolio value, W. Based on the contract, we have

$$W_a(W) = F + c_1 W + c_2(W - W_{BL}) + \lambda \max(W - W_{BO}, 0). \tag{4}$$

We assume that the benchmarks, W_{BL} and W_{BO}, are themselves the terminal values of portfolios formed by allocating the initial wealth (W_0) in different proportions to the market index portfolio and the riskfree asset. In particular, we assume that W_{BL} represents the terminal value of a portfolio that allocates a proportion w_{BL} to the market index and $(1 - w_{BL})$ to the riskfree asset. Similarly, W_{BO} represents the terminal value of a portfolio with an allocation of w_{BO} to the market index and $(1 - w_{BO})$ to the riskfree asset. The corresponding excess returns to these benchmark portfolios are:

$$R^e_{BL} \equiv W_{BL}/W_0 - R_f = w_{BL} R^e_m, \tag{5}$$

and[12]

$$R^e_{BO} \equiv W_{BO}/W_0 - R_f = w_{BO} R^e_m. \tag{6}$$

In summary, the dimensions of the managerial contract are captured by a set of six parameters: F, c_1, c_2, λ, w_{BL}, and w_{BO}. Three additional parameters (c, m, and θ) are used to simplify the

[12] The often-observed practice, where the same benchmark is used for both kinds of fees, can be captured by imposing the constraint: $w_{BL} = w_{BO}$. Such a constraint has no effect on our analysis of the manager's incentives, and therefore, can be assumed as being present wherever the reader finds it desirable.

notation throughout the rest of this paper: $c = c_1 + c_2$, $m = c + \lambda / 2$, and $\theta = \frac{\lambda / 2}{m}$. Note first that c represents the pay-performance sensitivity of the contract due to the proportional fee and the fulcrum fee. The term $\lambda / 2$ represents the "average" pay-performance sensitivity from the option like "bonus" fee. In this context the word "average" is to be understood purely as a mathematical formality. Similarly, m represents the "average" pay-performance sensitivity of the entire contract. Also note that the term θ captures the relative weight of the pay-performance sensitivity contributed by the option-type "bonus" fee in the contract. When $\theta = 0$, which implies $\lambda = 0$, the contract consists exclusively of the linear components with no "bonus" fee. As θ become larger, which can be achieved by decreasing c, increasing λ, or both, the "bonus" fee becomes an increasingly important component of the overall contract. We assume that $m > 0$, and $\theta \in (-1,1)$. We allow λ to be negative, in order to examine the possibility that the optimal contract is concave instead of convex. Nevertheless, our primary focus is on the case where λ is non-negative.

2. Managerial Effort Choice under Alternative Contract Forms

In this section, we study the manager's portfolio allocation and effort choice problem under different contract forms. In order to examine how the contract form influences the manager's decision, for this part of the analysis we take the contract as being exogenously determined. Our primary objective at this stage is to investigate the ability of the option-type "bonus" incentive fee to resolve the effort-underinvestment problem that exists under the linear contract. One of the key results of the paper is contained in Theorem 1 in Section 2.4, which shows that an appropriately designed "bonus" fee can in fact be used to improve the manager's effort incentive. The importance of the benchmark choice in the design of the "bonus" fee is highlighted in the discussion in Section 2.5.

2.1 First Best Solution and Managerial Effort-Underinvestment under the Linear Contract

We begin our discussion of the managerial effort choice by presenting the classical effort-underinvestment problem within our setting by comparing the linear contract solution with the first best solution. A linear contract corresponds to the case with $\lambda = 0$ in expression (4). The solution to the manager's optimization problem is straightforward. We report the result below.

Proposition 1: Under the linear contract, the manager's optimal portfolio allocations are

$$w_{m,l} = \frac{c_2}{m} w_{BL} + \frac{\ln\left[\frac{p}{1-p}\right]}{2amW_0\delta_m}; \tag{7a}$$

$$w_{a,l}\Big|_{s=1} = \frac{\ln\left[\frac{1+\rho}{1-\rho}\right]}{2amW_0\delta_a}; \text{ and} \tag{7b}$$

$$w_{a,l}\Big|_{s=0} = -w_{a,l}\Big|_{s=1}, \tag{7c}$$

where the subscript l denotes the "linear contract". The resulting manager's certainty-equivalent utility, denoted by Π_l, is[13]

$$\Pi_l = F + c_1 W_0 R_f - \frac{1}{2a}\ln\bigl(4p(1-p)\bigr) + \frac{1}{a}\varepsilon_l - V(\varepsilon_l), \tag{8}$$

where ε_l, the optimal managerial effort choice, is determined by the first order condition:

$$V'(\varepsilon_l) = \frac{1}{a}. \tag{9}$$

As a benchmark case, we next examine the contracting problem when the manager's effort choice, the signal observed by him, and his portfolio allocations are verifiable and therefore can be contracted on. This is a setting devoid of moral hazard. The optimal solution of this problem is commonly referred to as the "first best solution" in the principal-agency literature. The manager's optimization problem is the following:

$$\max_{\varepsilon, w_m, w_a, W_a(W)} E\bigl[U_a\bigl(W_a(W) - V(\varepsilon)\bigr)\bigr], \tag{10}$$

$$\text{s.t. } W = W_0\left(w_m R_m^e + w_a R_a^e + R_f\right), \tag{11}$$

[13] Unlike expected utility, the certainty-equivalent utility represents the economic agents' well-being in a scale that is comparable with the initial wealth. We find it more accessible to interpretation, and therefore use it throughout the paper in place of expected utility when presenting results.

$$E\left[U_p\left(W - W_a(W)\right)\right] \geq U_p\left(W_0 R_0\right). \tag{12}$$

Inequality (12) represents the investor's participation constraint where the constant R_0 represents the opportunity cost of capital to the investor. The manager's terminal wealth represented by $W_a(W)$ can be any measurable function of the terminal portfolio value, W. This function in effect specifies a sharing rule between the investor and the manager. The functional form of $W_a(W)$ is at the discretion of the manager. The solution to the optimization problem in (10)-(12), noted below, is the familiar result that the linear sharing rule is optimal. The derivation is omitted.

Proposition 2: The first best solution of the principal-agent problem is:

$$w_{m,fb} = \frac{(a+b)\ln\left[\frac{p}{1-p}\right]}{2abW_0\delta_m}; \tag{13a}$$

$$w_{a,fb}\Big|_{s=1} = \frac{(a+b)\ln\left[\frac{1+\rho}{1-\rho}\right]}{2abW_0\delta_a}; \tag{13b}$$

$$w_{a,fb}\Big|_{s=0} = -w_{a,fb}\Big|_{s=1}\text{; and} \tag{13c}$$

$$W_{a,fb}(W) = \frac{b}{a+b}W + V(\varepsilon) + W_0 - W_C + \frac{1}{a}\ln\left\{E\left[\exp\left(-\frac{ab}{a+b}W\right)\right]\right\}. \tag{14}$$

In the expressions above, the subscript fb denotes "first best" while $W_{a,fb}$ represents the first best sharing rule between the two parties. The certainty-equivalent utility for the manager, denoted by Π_{fb}, is:

$$\Pi_{fb} = W_0\left(R_f - R_0\right) - \frac{a+b}{2ab}\ln\left(4p(1-p)\right) + \frac{a+b}{ab}\varepsilon_B - V(\varepsilon_B). \tag{15}$$

The first-best effort level, ε_{fb}, is determined by the first order condition:

$$V'(\varepsilon_{fb}) = \frac{1}{a} + \frac{1}{b}. \tag{16}$$

Comparing Equation (9) with Equation (16), we have that $V'(\varepsilon_l) < V'(\varepsilon_{fb})$ and therefore the *effort-underinvestment problem* under the linear contract: $\varepsilon_l < \varepsilon_{fb}$.[14] Notice that the condition in (9) is independent of the slope coefficients of the linear contract (both c_1 and c_2) and the specification of the benchmark (w_{BL}). We thus conclude that, with a linear contract, neither changing the pay-performance sensitivity of the contract, nor using a benchmark in the fulcrum incentive fee, helps in resolving the effort-underinvestment problem, a conclusion known as the *irrelevance result.* In other words, Stoughton's result holds in essentially the same fashion in the current model.

2.2 Characterization of the Manager's Portfolio Choice Problem under the Incentive Contract

We now analyze the manager's problem when facing a contract in the form of Equation (4). We solve the manager's overall optimization problem in two steps. In the first step, we take the manager's effort level as given, and solve for the manager's optimal portfolio choices. In the second step, we plug in the optimal portfolio choices and solve for the manager's effort choice.

We start by examining the contribution of the option-type bonus incentive fee to the manager's payoff. The manager ends up earning the bonus incentive fee ex post (i.e., $\lambda \max(W - W_{BO}, 0) > 0$) if and only if he outperforms the benchmark, i.e., $R_w > R_{BO}$, or equivalently,

$$-\left(w_m - w_{BO}\right) R_m^e < w_a R_a^e . \tag{17}$$

It is clear from this inequality that earning the incentive fee is contingent on the state of nature characterized by the signs of the excess returns on the two assets. Intuitively, in order to motivate the manager to exert effort to improve his signal precision, we would want to reward him when the state of nature ex post confirms his ex ante signal. In other words, we want Inequality (17) to be satisfied in the states where $R_a^e = \delta_a$ when $s = 1$, and in the states where $R_a^e = -\delta_a$ when $s = 0$. We note that it is only

[14] If we adopt the perspective that coefficient b, representing the aggregate risk aversion of all shareholders of a fund, is much smaller than the manager's risk aversion coefficient, a, then the difference between $V'(\varepsilon_l)$ and $V'(\varepsilon_{fb})$ will be substantial, highlighting the severity of the underinvestment problem with a linear contract.

natural for the manager to choose $w_2 \geq 0$ when $s=1$ and $w_2 \leq 0$ when $s=0$. Substituting these values for R_a^e and w_2 in (17) and using the fact that $R_m^e = \pm\delta_m$, we arrive at the following inequality:

$$|w_m - w_{BO}| < |w_a| \cdot \delta_a / \delta_m . \tag{18}$$

Consider the case when $s=1$, for example. Inequality (18) implies that the bonus fee option is 'in the money' in states (UU) and (DU), i.e., when the actively managed asset is up in value while the market index portfolio can be either up or down. The bonus fee payoff is zero in the other two states (i.e., states (UD) and (DD)). Intuitively, the above inequality is satisfied when the benchmark allocation to the market index, w_{BO}, is 'sufficiently' close to the manager's allocation to the index, w_m at the equilibrium. Whether the respective portfolio weights are 'close enough' depends on the manager's portfolio choices, namely, w_m and w_a, which we need to solve for. Therefore, we need to exhaust all possible cases including those cases where Inequality (18) does not hold. Also, we need to confirm our intuition that it is optimal to reward the manager through a bonus fee in states (UU) and (DU) in the case where $s=1$. Mathematically, the following three scenarios exhaust all the possibilities embedded in Inequality (18): [15]

$$\text{Scenario (a)} \qquad |w_m - w_{BO}| \leq |w_a| \cdot \frac{\delta_a}{\delta_m},$$

$$\text{Scenario (b)} \qquad w_m - w_{BO} > |w_a| \cdot \frac{\delta_a}{\delta_m},$$

$$\text{Scenario (c)} \qquad w_m - w_{BO} < -|w_a| \cdot \frac{\delta_a}{\delta_m}.$$

Under Scenario (b), the manager's allocation to the market index portfolio is much higher than the benchmark's allocation to the index portfolio. For the case $s=1$ and therefore $w_a \geq 0$, in Scenario (b) the bonus is 'in the money' in states (UU) and (UD), i.e., when $R_m^e > 0$, regardless of the sign of R_a^e. Under Scenario (c), the manager's portfolio allocation to the market index portfolio is much lower than the benchmark's allocation to the index portfolio. Consequently, the bonus fee is 'in the money' in states (DU) and (DD), i.e., when $R_m^e < 0$, regardless of the sign of R_a^e. We solve for the manager's optimal

[15] Including the equality in Scenario (a) is a matter of convention for the consideration of completeness.

portfolio choices within each of these three scenarios, starting with Scenario (a). We then arrive at the complete solution of the manager's optimal portfolio allocation problem by checking for consistency within each scenario and his optimal choice across the scenarios.

2.3 Manager's Portfolio and Effort Choices under Option-type Bonus Fee Contract

We employ the following notation to highlight the recurring terms in the manager's objective function and therefore help simplify our exposition:

$$u \equiv w_m\delta_m + w_a\delta_a \text{, and} \tag{19}$$

$$v \equiv w_m\delta_m - w_a\delta_a \,. \tag{20}$$

We further define $N_{BL} \equiv w_{BL}\delta_m$, and $N_{BO} \equiv w_{BO}\delta_m$. The terms u and v represent the excess return of the overall portfolio R_w^e, in the two states (UU) and (UD) respectively. The terms N_{BL} and N_{BO} represent the excess returns of the benchmark portfolios, R_{BL}^e and R_{BO}^e respectively, in these two states. In Scenario (a), the manager's objective function, conditional on receiving a positive signal (i.e., $s=1$), is the product of the constant, $-\exp\left[-a\left(F + c_a W_0 R_f\right)\right]$, and the following minimization problem:

$$\begin{aligned}
&\min_u \; p\frac{1+\rho}{2}\left\{\exp\left[-aW_0\left(c_1 u + c_2(u - N_{BL}) + \lambda(u - N_{BO})\right)\right]\right\} \\
&\qquad + (1-p)\frac{1-\rho}{2}\left\{\exp\left[-aW_0\left(-c_1 u - c_2(u - N_{BL})\right)\right]\right\} \\
&+ \min_v \; p\frac{1-\rho}{2}\left\{\exp\left[-aW_0\left(c_1 v + c_2(v - N_{BL})\right)\right]\right\} \\
&\qquad + (1-p)\frac{1+\rho}{2}\left\{\exp\left[-aW_0\left(-c_1 v - c_2(v - N_{BL}) + \lambda(N_{BO} - v)\right)\right]\right\}.
\end{aligned} \tag{21}$$

After solving (21) for u and v, the manager's optimal portfolio choice is determined jointly by Equations (19) and (20). It is equally likely that the manager receives a negative signal ($s=0$), and his portfolio choice problem can be solved the same way. In fact, due to the exact symmetry, all the manager does in this case is to reverse the sign of w_a. We then combine the manager's utility in both cases, include the cost of effort into his objective function, and then solve for his effort choice. The results are summarized in the following lemma.

Lemma 1: In the case of Scenario (a), the manager's optimal portfolio weights are

$$w_{m,ic} = \frac{c_2}{m} w_{BL} + \theta w_{BO} + \frac{\ln\left[\frac{p}{1-p}\right]}{2amW_0\delta_m}; \tag{22a}$$

$$w_{a,ic}\Big|_{s=1} = \frac{\ln\left[\frac{1+\rho}{1-\rho}\right]}{2amW_0\delta_a} + \frac{\ln\left[\frac{1+\theta}{1-\theta}\right]}{2amW_0\delta_a}; \text{ and} \tag{22b}$$

$$w_{a,ic}\Big|_{s=0} = -w_{a,ic}\Big|_{s=1}. \tag{22c}$$

We use the subscript "ic" to denote that these portfolio weights are the solutions to the optimization problem under the "incentive contract". The manager's optimal effort, ε_{ic} is determined implicitly by

$$V'(\varepsilon_{ic}) = \frac{1}{a}\left(1 + \frac{1}{\rho_{ic}} \cdot \theta\right), \tag{23}$$

where ρ_{ic} is a function of ε_{ic} and is implicitly defined by Equation (3) (i.e., $\rho_{ic} = \sqrt{1-e^{-2\varepsilon_{ic}}}$). An explicit form of the manager's utility and a necessary condition for the manager's optimization problem to fit Scenario (a) are given in the Appendix as expressions (A1) and (A2), respectively.

We can see from Equation (22b) that the manager's allocation to the actively managed asset is determined by two factors represented by the two terms in the expression for $w_{a,ic}$. The first term in (22b) takes the exact same form as that in the linear contract (Equation (7b)). Conditional on receiving a positive signal about R_a^e (i.e., conditional on $s=1$), the manager invests more in the actively managed asset when the signal is more informative (i.e., if ρ is higher). The second term in (22b) is independent of information precision and depends solely on the convexity of the contract, captured by θ. We can also see from Equation (22a) that the manager's allocation to the market index $w_{m,ic}$, is sensitive to the relative weights for the two fee components (i.e., c_2/m and θ) in the contract and to the corresponding benchmark weights (i.e., w_{BL} and w_{BO}). Following a similar solution technique to that used for Scenario (a), we solve the manager's optimization problem under Scenarios (b) and (c). We record the results in the following lemma.

Lemma 2: In the case of Scenarios (b) and (c), the manager's optimal portfolio weights are

$$w_{m,ic} = \frac{c_2}{m} w_{BL} + \theta w_{BO} + \frac{\ln\left[\frac{p}{(1-p)}\right]}{2amW_0\delta_m} \pm \frac{\ln\left[\frac{1+\theta}{1-\theta}\right]}{2amW_0\delta_m}; \tag{24a}$$

$$w_{a,ic}\Big|_{s=1} = \frac{\ln\left[\frac{1+\rho}{1-\rho}\right]}{2amW_0\delta_a}; \text{ and} \tag{24b}$$

$$w_{a,ic}\Big|_{s=0} = -w_{a,ic}\Big|_{s=1}. \tag{24c}$$

where the last term in the expression for $w_{m,ic}$ above takes the positive sign in Scenario (b), and it takes the negative sign in Scenario (c). The managerial effort is implicitly determined by the condition:

$$\frac{\partial}{\partial\zeta} g(\zeta,\theta) / g(\zeta,\theta) + aV'(\varepsilon_{ic}) \frac{\rho_{ic}}{2\zeta} = 0 \tag{25}$$

where $\zeta = \frac{1+\rho_{ic}}{1-\rho_{ic}}$, $g(\zeta,\theta) \equiv \frac{2\zeta}{\zeta+1} \zeta^{-(1+\theta)/2}\left(1+\zeta^{\theta}\right)$, and as in (15), $\rho_{ic} = \sqrt{1-e^{-2\varepsilon_{ic}}}$. An explicit form of the manager's utility and a necessary condition for the manager's optimization problem to fit Scenario (b) are given in the Appendix as (A3) and (A4), and for Scenario (c) as (A5) and (A6).

The final solution for the manager's portfolio decision problem involves comparing the desirability of the outcomes across the three scenarios. The above lemmas provide the necessary conditions on the control variables for the manager's problem to fit into one of the three scenarios. We now derive the corresponding necessary and sufficient conditions, and therefore have a complete description of the manager's optimal strategy. In the following proposition, we present the solution to the manager's optimization problem as a set of rules that determines the manager's portfolio choices and ultimately determines the manager's optimal effort level.

Proposition 1: Conditional on the precision (ρ) of the signal observed by the manager, his optimal portfolio decision is as follows:

(a) In the case when the information precision is sufficiently high such that

$$\ln\frac{1+\rho}{1-\rho} \geq \left|\ln\frac{p}{1-p} - 2caW_0\delta_m(w_{BO} - \frac{c_2}{c} w_{BL})\right| \tag{26}$$

the manager's optimal portfolio decision $(w_{m,ic}, w_{a,ic})$ is specified by Lemma 1.

(b) In the case that Inequality (26) does not hold, then either

$$\ln\frac{1+\rho}{1-\rho} < \ln\frac{p}{1-p} - 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) \tag{27}$$

and the manager's optimal portfolio decision is specified by Lemma 2 for Scenario (b); or

$$\ln\frac{1+\rho}{1-\rho} < -\ln\frac{p}{1-p} + 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) \tag{28}$$

and the manager's optimal portfolio decision is specified by Lemma 2 for Scenario (c).

The optimal effort level of the manager is subsequently determined by either Equation (23) in Scenario (a), or Equation (25) in Scenarios (b) and (c).

Proof. See Appendix.

Proposition 1 specifies explicitly and completely the manager's decision process in making his portfolio choice after observing a signal with precision ρ. The manager's optimal effort choice is then the outcome of maximizing his utility using either Equation (23) or Equation (25) depending on which one is consistent.[16] At equilibrium with $\rho = \rho_{ic}$, the conditions in the proposition shall ultimately be viewed as restrictions on the contract space. With this in mind, it may be helpful to rearrange (26) and express it as a constraint on the choice of benchmarks:

$$2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) - \ln\frac{p}{1-p} \quad \in \quad \left[-\ln\frac{1+\rho_{ic}}{1-\rho_{ic}} \quad , \quad \ln\frac{1+\rho_{ic}}{1-\rho_{ic}}\right] \tag{29}$$

From Equation (23) or (25), the only contract parameter that explicitly affects ρ_{ic} is θ. Hence, ρ_{ic} in (29) is a function of θ, w_{BO}, and w_{BL}. Therefore, (29) can be viewed as a joint restriction on the choice of contract parameters w_{BO}, w_{BL}, and θ. With this understanding in mind, we are in a position to analyze the implication of the contract form for the managerial effort choice.

2.4 Motivating the Manager Using the Bonus Fee

In the following theorem, we present the effort implications of the option-type "bonus" fee.

Theorem 1: Corresponding to the three cases in Proposition 1, we have the following:

[16] Depending on the specification of the contract parameters, Equation (23) is said to be consistent if it yields a solution for ρ_{ic} that satisfies Inequality (26). Similarly Equation (25) is said to be consistent if it yields a solution for ρ_{ic} that satisfies either (27) or (28). By making use of the results in Theorem 1, one can show that at least one of the two equations will be consistent. In the case that both are consistent, a direct comparison between the two levels of resulting managerial utility will resolve any remaining ambiguity.

(a) Among contracts that satisfy the relation in (29), the manager's optimal effort level, ε_{ic}, is an increasing function of θ for $\theta \geq 0$. In particular, as $\theta = 0$ corresponds to the case of a linear contract, we have $\varepsilon_{ic} > \varepsilon_l$ for $\theta > 0$. On the other hand, in the case that $\theta < 0$, the manager's optimal effort level is lower than that in the linear contract. That is, $\varepsilon_{ic} < \varepsilon_l$ for $\theta < 0$.

(b) Among contracts that violate the relation in (29), and if $\rho_{ic} < \frac{\exp(2)-1}{\exp(2)+1}$ (i.e., if $\rho_{ic} < 0.7616$), the manager's effort level, ε_{ic}, is a decreasing function of θ for $\theta \geq 0$. We assume here that ε_{ic} is an interior solution, and the second order condition of the effort choice problem is satisfied.

Proof. See Appendix.

Theorem 1 contains the main result of this paper, namely the relevance of the option-type bonus fee in motivating the manager, and the importance of the benchmark choice. According to the theorem, changing the relative weight of the manager's bonus fee in the managerial contract (θ) changes the manager's optimal effort choice (ε_{ic}). The precise relation between ε_{ic} and θ depends critically on the specification of the benchmarks (i.e., on the choice of w_{BL} and w_{BO}). In the case that w_{BL} and w_{BO} are so chosen that the relation in (29) is satisfied, increasing the relative weight of the manager's bonus fee in the contract (i.e., increasing θ) leads to higher effort (i.e., an increase in ε_{ic}). Under this scenario, the additional option-type bonus fee motivates the manager to work harder and potentially reduces the cost of effort-related moral hazard.[17] On the other hand, increasing the relative weight of the bonus fee in the manager's compensation when the relation in (29) is violated, will in fact lead to a reduction in the manager's effort. In this case, the use of the bonus fee is counterproductive and in effect encourages the manager to shirk effort and rely solely on luck in order to beat the benchmark. It is therefore important to choose the benchmarks carefully so that the relation in (29) is satisfied. We will return to a discussion of these conditions in the next subsection. With the relation in (29) being satisfied, we can either increase λ

[17] It is worth emphasizing that the optimal contract composition (including the composition of the benchmark) that would induce effort at minimal cost depends, among other things, on the effort cost. We address this issue in the next section when we formally analyze the optimal contract choice problem.

or decrease c in order to increase the relative weight of the bonus fee, θ, and therefore induce higher managerial effort. In fact, we have the following corollary of the theorem.

Corollary. Given a contract of the form in (4) and under any of the three scenarios, the manager's optimal effort level, ε_{ic}, is completely determined by the manager's risk aversion (a), the relative weight of the option-type bonus fee (θ) in the manager's compensation, and the functional form of the effort cost, $V(\cdot)$.

The proof of this corollary is immediate. It follows by noting that the first order conditions of effort choice (Equations (23) and (25)) can ultimately be expressed in terms of only the above parameters (i.e., a and θ) and, of course, the variable ε_{ic} itself. An implication of the above corollary is that increasing the overall pay-performance sensitivity of the contract, i.e., increasing m (the sum of c_1, c_2, and $\lambda/2$), while keeping the relative weight of the bonus fee constant, i.e., while keeping the ratio θ constant, will not affect the manager's optimal effort choice. This strengthens Stoughton's result under our setting, namely, that the average slope of the contract is irrelevant for the manager's effort.

2.5 The Relevance of the Benchmark

We now interpret condition (29) in order to provide more meaningful guidelines with respect to the benchmark choice. In this context we note that, (29) can be transformed to the condition: $\left|w_{m,ic} - w_{BO}\right| < w_{a,l}\ \delta_a / \delta_m$. Intuitively, the appropriate benchmark weight on the passive asset (w_{BO}) should track the manager's allocation to the passive asset ($w_{m,ic}$) closely enough such that the bonus fee will be in the money only if the manager's ex ante signal is confirmed ex post by the realized state of nature. This, in turn, motivates the manager to work hard to enhance the precision of his signal, and to appropriately invest in the actively managed asset, in order to improve the chances of earning a "bonus" fee as well its magnitude. Therefore, when the benchmark is chosen to accurately reflect the investment style of the manager, the use of the benchmark-linked incentive improves managerial effort. We therefore have confirmed the intuition that motivates our definition of Scenario (a). In essence, this result represents an application of the Holmström (1979) "informativeness principle" to the delegated portfolio management problem. In this sense the use of style benchmarks, as advocated by Sharpe (1992), can be quite beneficial, even from a managerial incentive standpoint.

On the other hand, as discussed previously under Scenarios (b) and (c), a benchmark may involve a weight on the passive index portfolio that is too little (too much), relative to the manager's actual passive asset allocation. The option component of the manager's "bonus" fee will now be in the money if the passive index portfolio happens to have a favorable (unfavorable) outcome, regardless of the performance of the actively managed asset. In this case, there is little incentive for the manager to improve his information precision. As a result, the manager will actually be more reluctant to exert effort, and consequently the incentive fee in this form encourages the manager to shirk.

2.6 Understanding the Incentive Effects of the "Bonus" Incentive Fee

From Theorem 1, we conclude that the inclusion of an appropriately specified "bonus" incentive fee in the contract provides the manager incentives to collect costly information. To get a sense of the potential magnitude of this incentive effect, let us take a concrete example. Suppose the manager faces an environment where the typical information signal has a correlation coefficient of 16.8% with the true state of nature, i.e. $\rho = 16.8\%$.[18] Compared to a linear contract, how much harder will the manager work if he is given the "Two and Twenty" incentive contract often observed in hedge funds? That is, a contract where $c = 0.02$, $\lambda = 0.20$, and thus, $\theta = 0.83$. According to (23), the marginal cost of effort increases multifold from $V' = \frac{1}{a}$ in the case of a linear contract to $V' = \frac{5.96}{a}$ in the case of the "bonus" incentive contract. Thus, the option-type "bonus" fee not only monotonically increases managerial effort in a qualitative sense, but also does so in a very economically meaningful way.

[18] This value is calibrated from U.S. stock mutual fund data and is used in the numerical analysis in Section 3.5.

3. Optimal Contract Choice

3.1 The Manager's Optimization Problem with Endogenous Contract Choice

After studying the manager's incentive problem while taking the contract as given, we conclude that, by including the option-type "bonus" fee with an appropriately specified benchmark, the manager will be motivated to work harder to collect valuable information. However, as we have not considered the cost of including the "bonus" fee, it still leaves unanswered the question: Are incentive contracts desirable? To answer this question we need to take into account the investor's participation constraint, which dictates a tradeoff among the components of the contract. We focus on a contract featuring option-type "bonus" incentive fees with the appropriately chosen benchmark that is consistent with the manager's allocation to the passive asset.

To directly capture the tradeoff between the "bonus" fee and the fixed salary, we can solve the binding participation constraint of the investor (as in Inequality (12)) for F, and plug it, together with the manager's decisions solved in Lemma 1, into the manager's certainty-equivalent utility function. As a result, we are able to formulate the optimal contract problem for the manager as the following unconstrained maximization problem, with an explicit objective function, and five control variables, namely, c_1, c_2, w_{BL}, λ, and w_{BO}. The manager's objective function representing his certainty-equivalent utility is:

$$\begin{aligned} \max_{c_1,c_2,w_{BL},\lambda,w_{BO}} \; & W_0\left(R_f - R_0\right) - V(\varepsilon_{ic}) - \frac{1}{a}\ln\left[\begin{array}{l} p^{(1-\theta)/2}(1-p)^{(1+\theta)/2}\exp\left(aW_0\delta_m\theta\left(cw_{BO}-c_2w_{BL}\right)\right) \\ + p^{(1+\theta)/2}(1-p)^{(1-\theta)/2}\exp\left(-aW_0\delta_m\theta\left(cw_{BO}-c_2w_{BL}\right)\right) \end{array}\right] \\ & - \frac{1}{2a}\left[(1-\theta)\ln\left(\frac{1+\rho_{ic}}{1-\theta}\right) + (1+\theta)\ln\left(\frac{1-\rho_{ic}}{1+\theta}\right)\right] - \frac{1}{b}\ln\left[D_1 + D_2 + D_3 + D_4\right], \end{aligned} \tag{30}$$

where ε_{ic} is determined by Equation (23) and $\rho_{ic} = \sqrt{1-e^{-2\varepsilon_{ic}}}$. The derivation of the above expression is provided in the Appendix. The expressions for D_1, D_2, D_3, and D_4 are given by (A11)-(A14) in the appendix. Below, we characterize the solution of this optimization problem.

3.2 The Optimal Linear Contract

We begin by searching for the optimal contract within the set of all *linear* contracts, a case of interest by itself. That is, we now impose the constraint, $\lambda = 0$ (i.e., $\theta = 0$). The resulting simplified expression for the manager's objective is given by:

$$\max_{c_1, c_2, w_{BL}} W_0\left(R_f - R_0\right) - V(\varepsilon_l) - \frac{1}{2a}\left(\ln p + \ln(1-p)\right) - \frac{1}{2a}\left(\ln(1+\rho_l) + \ln(1-\rho_l)\right) - \frac{1}{b}\ln\left[D_1 + D_2 + D_3 + D_4\right], \tag{31}$$

where ε_l is determined by Equation (9). Only the last term in the above expression depends on the contract coefficients. Therefore, the search for optimal contract parameters c_1, c_2, and w_{BL} boils down to minimizing the arithmetic average of four positive terms $\left(D_1, D_2, D_3, \text{and } D_4\right)$[19] whose geometric average, through direct computation, is $\sqrt{p(1-p)(1+\rho_l)(1-\rho_l)}$. As the arithmetic average cannot be less than the geometric average and the latter is independent of all control variables of the optimization problem in (31), the geometric average provides a natural lower bound for the minimization, which can be attained when $D_1 = D_2 = D_3 = D_4$. The necessary and sufficient condition is: $c = \frac{b}{a+b}$, and $c_2 w_{BL} = 0$. We have arrived at the following proposition.

Proposition 3: In the optimal *linear* portfolio management contract, $c_1 = \frac{b}{a+b}$ and $c_2 = 0$.[20]

According to Proposition 3, the benchmark-linked fulcrum fee should not be used in the optimal linear contract. It therefore confirms the benchmark irrelevance result of Admati and Pfleiderer (1997). The merit here is that the result is arrived at directly from the optimization of the manager's objective function and therefore it takes into account all of the aforementioned contracting issues simultaneously.[21]

[19] The expressions for these terms in the case of a linear contract are given by (A21)-(A24) in Appendix C.
[20] The other possibility is to have c_2 be nonzero but $w_{BL} = 0$. However, in our model, setting $w_{BL} = 0$ is equivalent to selecting an appropriate combination of the fixed fee and the proportional asset-based fee. Therefore, by skipping the case with $w_{BL} = 0$, we do not miss out on any contracts in our optimal set.
[21] The result in Proposition 3 is anticipated by combining and comparing results in Propositions 1 and 2.

3.3 The Optimal Incentive Contract

We have the form of the optimal linear contract according to Proposition 3. Are there circumstances under which this contract is the optimal contract within the broader contract space examined in this paper? We answer this question in the following theorem.

Theorem 2: In the optimal managerial contract, $\lambda > 0$. More specifically, the optimal linear contract in Proposition 3 dominates all contracts with $\lambda < 0$, and there are contracts with $\lambda > 0$ that dominate the optimal linear contract.

Proof. See Appendix.

According to Theorem 2, it is always optimal for the investor to specify a contract that includes the option-type "bonus" incentive fee component. We prove the theorem by studying the marginal effect on the manager's objective function of introducing a small amount of "bonus" incentive fee in the optimal linear contract. We show that at the margin, the benefits of including the "bonus" incentive fee, in terms of the improved effort incentives, outweigh the potential costs in terms of suboptimal risk sharing and distortions in the managerial risk incentives. Clearly, due to the nature of this argument, Theorem 2 is a qualitative result. Although in theory it is always desirable to include the option-type "bonus" fee in the contract, the economic significance of such a fee is not directly addressed in the theorem. However, based on the discussion in Subsection 2.6., we know that the managerial effort incentives induced by the option-type bonus fee contract are indeed economically meaningful. We provide further evidence on this issue based on a calibration of the model in Subsection 3.5.

3.4 Alternative Forms of the Portfolio Management Contract

Even though the different elements of the generic contract that we examine are quite similar to what is observed in practice, it is reasonable to ask whether our results continue to hold when other forms of fulcrum fee or incentive fees are allowed. For example, is the option-type incentive fee still optimal when the fulcrum fee is say, convex (concave) in gains and concave (convex) in losses? Similarly, can the non-linear component of the contract take the simpler form of a lump-sum bonus payment for exceeding a performance threshold (and zero otherwise) rather than the option-type fee considered here? Our results suggest that our conclusion regarding the optimality of the option-type fee is robust to these extensions. We begin by stating the following proposition that shows that all forms of the fulcrum fee contracts, not just the linear fulcrum fee considered here, lead to the identical level of managerial effort.

Proposition 4: Consider the general fulcrum fee contract form:

$$W_a(W) = F + c_1 W + g(W - W_{BL}) \tag{32}$$

where $g(x)$ is an odd function (i.e., $\forall x \in R$, $g(-x) = -g(x)$). The manager's optimal effort choice is determined by Equation (9), and is therefore independent of the form of the fulcrum fee.

Proof. See Appendix.

Proposition 4 implies that all forms of fulcrum fee contracts will result in the same managerial effort level under our setting. It is clear then that any non-linear fulcrum fee contract is dominated by the linear fulcrum fee due to the former's (a) suboptimal risk sharing features, and (b) suboptimal managerial risk incentive (i.e., portfolio allocation incentive). However, as we have shown earlier, the option-type bonus fee when used in addition to the linear fulcrum fee, can in fact improve the managerial effort and utility. Therefore, the optimality of the option-type incentive fee is robust to more general forms of fulcrum fee arrangements. We next examine whether a non-linear component in the form of a simple lump-sum bonus would suffice in lieu of the option-type bonus incentive considered by us. We state the answer to this question in the form of the following proposition.

Proposition 5: Consider a contract that adds to our general contract an additional item that corresponds to a lump-sum bonus fee if the manager outperforms a benchmark, and is zero otherwise. That is, consider a contract with the following compensation structure:[22]

$$W_a(W) = F + c_1 W + c_2(W - W_{BL}) + \lambda \max(W - W_{BO}, 0) + \xi \cdot \{W > W_{BP}\} \tag{33}$$

[22] We adopt de Finetti's notation and use the same symbol for a set and its indicator function.

Here W_{BP} is the benchmark for the lump-sum bonus fee. The manager's optimal effort choice under this contract is the same as that under contract (4). Therefore, adding a lump-sum bonus component to the contract does not alter the manager's effort incentive.

Proof. See Appendix.

From the above proposition, when restricting λ to be zero, it follows that adding a lump-sum bonus fee to a linear contract does not improve the manager's effort level. For that given level of effort, the linear contract in Proposition 3 will achieve the optimal risk sharing and lead to the appropriate managerial risk incentive. Consequently, unlike the option-type incentive fee, the inclusion of a simple lump sum bonus fee to a linear contract will in fact be detrimental to the manager's utility. Therefore, the conclusion in Theorem 2 that $\lambda > 0$ in the optimal contract, still holds.

3.5 Numerical Results

In this section we provide an assessment of the potential efficiency gains from the adoption of the option-type bonus incentive fee. For the sake of simplicity, we will assume in this subsection that the fund management contract does not include a benchmark-linked fulcrum fee. To calibrate the model, we assume that the typically observed linear contract in the mutual fund industry is the optimal linear contract, and the typically observed portfolio allocation to stocks by equity funds is the manager's optimal response to the linear contract. Specifically, we assume that the optimal linear contract has a slope of 1 percent. The manager's allocation to the market index portfolio is assumed to be 90 percent.

In order to get a reasonable estimate of the equilibrium value of the information precision parameter, ρ, we use data on a sample of stock mutual funds from the CRSP Survivor-Bias Free US Mutual Fund Database for the period 1962-2004. The sample includes 2,839 actively managed funds belonging to the "Aggressive Growth", "Growth and Income", and "Long Term Growth" categories. In the context of the model, with probability $(1+\rho)/2$, the manager achieves a positive Jensen's alpha, ex post. For our sample of funds we find that the Jensen's alpha based on gross returns is positive for 58.44% of the funds. Setting this figure equal to $(1+\rho)/2$ yields a value of 16.88 percent for ρ. We assume that this is the optimal choice of information precision for the manager under a linear contract. The market excess return was positive in 309 out of 516 months during the above period, i.e., 59.84 percent of the time. Accordingly, we set $p = 0.5984$. The monthly market risk premium was 61 basis points and the monthly market volatility was 4.21 percent during this period. This corresponds to a value of 16.30 percent for the annualized square root of the second moment for the market excess return, which we use as the value for parameter δ_m.

When adding a bonus fee to a linear contract, we can utilize expressions (22a-22c) and rewrite Condition (18) as $\left|w_{BO} - w_{m,l}\right| < \frac{1}{2caW_0\delta_m}\ln\left(\frac{1+\rho}{1-\rho}\right)$. Given the above calibration, this condition is satisfied if $0.13 < w_{BO} < 1.67$. In other words, as long as the benchmark weight on the market portfolio falls in the interval between 0.13 and 1.67, the corresponding bonus fee improves the managerial effort. While not very restrictive, this condition does imply that, for a typical mutual fund, the use of a bonus fee with the risk-free rate as the benchmark will hurt the manager's effort incentive.

To further characterize the optimal contract, we specify the following information cost function:

$$V(\rho) = \gamma\left\{\frac{1}{1+\hat{\rho}}\ln[1+\rho] + \frac{1}{1-\hat{\rho}}\ln[1-\rho] - \frac{2\hat{\rho}^2}{1-\hat{\rho}^2}\ln\left[1-\frac{\rho}{\hat{\rho}}\right]\right\} \tag{34}$$

where γ is a positive parameter, and $\hat{\rho}$ is a parameter that falls in the interval [0, 1]. We use the parameter γ to capture different levels of the information related costs. The parameter $\hat{\rho}$ represents the highest possible information precision and we set it equal to 0.50. We note that the above cost function satisfies all theoretical requirements in the context of our model and allows for an explicit solution for the optimal managerial effort. We initially choose a value of γ to fit the empirically observed value for ρ, i.e., 16.88 percent for a linear contract. We find that the optimal contract under this setting consists of a proportional fee of 0.74 percent of the assets under management, and a bonus fee equal to 7.35 percent of the outperformance against the benchmark. The optimal benchmark involves an 84 percent allocation to the market (portfolio) and 16 percent to T-bills. This contrasts with the optimal linear contract with a proportional asset-based fee of 1 percent.

Based upon the above parameters we find that the information precision under the optimal bonus fee contract increases to 32.29 percent from the linear contract level of 16.88 percent, an economically significant improvement. We next calculate the certainty-equivalent utility for the manager under three scenarios: the optimal linear contract, the optimal contract with the option-type bonus component, and the first best solution. We find that under the optimal bonus contract, the manager's certainty-equivalent utility is 4.6 percent of the initial assets under management (W_0). This represents an improvement of over 100 percent of the manager's certainty-equivalent utility under the linear contract (2.1 percent of W_0).[23]

We next experiment with different choices for the information cost coefficient γ while keeping all other model parameters unchanged. Panel A of Figure 1 depicts the manager's certainty-equivalent utility, expressed as a proportion of the initial assets (W_0), under the optimal incentive contract, the optimal linear contract, and in the first best solution respectively, for varying levels of γ. As can be seen, the manager's utility is consistently higher under the incentive contract compared to the linear contract, thereby reducing the gap between the contract-induced manager utility and the first-best level of utility.

Panel B of Figure 1 shows the composition of the optimal bonus contract as a function of the cost of information. The composition of a contract is characterized by the "average" pay-performance sensitivity (*m*) and the relative weight on the "bonus" fee (θ). Initially, as information become costly, it becomes increasingly important to motivate the manager to exert effort leading to a larger relative weight of the option-type bonus fee in the contract. This is accompanied by a correspondingly higher average pay-performance sensitivity to offset the shifted risk incentive of the manager. However, as the information cost increases further, the equilibrium level of information precision becomes small. At this point, aligning the manager's risk incentive becomes the dominant concern. Therefore, we see a decrease in the relative weight assigned to the "bonus" fee along with an increase in the average pay-performance sensitivity of the contract.

[23] Recall that in our analysis we have assumed that the manager has market power and thus captures all of the efficiency gains. Under the alternative assumption that the gains are captured by the investor, the option type bonus fee contract leads to an improvement in the investor's certainty-equivalent return of over 50 percent.

We next explore the manager's certainty-equivalent utility and the optimal contract composition under varying levels of investor risk aversion, while keeping the manager's risk aversion and all other parameters fixed. In particular, we allow the ratio of the investor's risk aversion coefficient (b) to the manager's risk aversion coefficient (a) to vary from a low of 0.001 to a high of 0.01. The results are presented in Panels C and D of Figure 1. Panel C of the figure depicts the manager's certainty-equivalent utility, expressed as a proportion of the initial assets (W_0). As the investor becomes less risk averse, the overall investment opportunity set looks more appealing to her. Therefore, we see a higher surplus left over to the manager when the investor is less risk averse. In Panel D of the figure we see that the relative weight on the "bonus" fee becomes higher as the investor becomes less risk averse. This is due to two reasons. First, as can be seen from Equation (16), as the investor becomes less risk averse, i.e., as b becomes smaller, the optimal managerial effort in the first best solution becomes higher. At the same time, the optimal managerial effort from a linear contract stays unchanged, as shown in Equation (9). Therefore, the underinvestment problem in managerial effort under a linear contract becomes more severe and hence, the need to use the "bonus" fee to motivate the manager becomes more urgent. Furthermore, with declining investor risk aversion, concerns over excessive managerial risk-taking as a consequence of the inclusion of a "bonus" fee in the contract, become less important. In Panel D we also see that the overall pay-performance sensitivity of the contract decreases as the investor becomes less risk averse. This is because the less risk-averse investor optimally bears more risk.

Stoughton (1993) considers a truth revelation mechanism to overcome the moral hazard problem in the context of delegated portfolio management. While the mechanism cannot be implemented in the realistic portfolio management environment, we nevertheless adapt it to our setting to serve as a benchmark for evaluating the efficiency of the incentive contract. Under this mechanism the manager simply reports his forecast about the excess return on the actively managed asset to the investor who then implements the portfolio allocation based on the reported forecast. The manager receives a payoff equal to $\phi + d$ if he is ex post correct, and $\phi - d$ otherwise. The compensation can thus be specified by two parameters: the arithmetic average of the two levels, denoted by ϕ, and the difference of the two levels, denoted by $2d$. The appendix provides a brief analysis of this mechanism. The reader is referred to Stoughton (1993) for additional details.

We find that, within the range of parameter variation considered in Figure 1, while the manager's effort under the information revelation mechanism (denoted by the subscript "*ir*") is often higher compared to that under the optimal incentive contract, the difference is very small. In fact, the ratios ρ_{ir}/ρ_{ic} all fall into the interval $[0.99,1.01]$. This difference becomes noticeable only when the aggregate risk aversion of investors becomes comparable to that of the manager. If for instance we take the ratio b/a as 0.05, a value higher than what we considered in Figure 1 (Panels C and D), the ratio ρ_{ir}/ρ_{ic} is 1.08, which is noticeably higher than those ratios considered earlier but still very close to 1. Interestingly, in this case, the ratio of the manager's certainty-equivalent utility under the information revelation mechanism to that under the incentive contract $\left(\Pi_{ir}/\Pi_{ic}\right)$ is only 0.66. That is, the manager is much better off under the optimal incentive contract than under the information revelation mechanism despite the fact that the latter generates higher managerial effort. On the other hand, when the investor approaches risk neutrality, the ratio Π_{ir}/Π_{ic} increases. For example, it reaches 1.16 for the case where $b/a = 0.001$. This highlights the intuition that by severing the feedback relation between portfolio allocation and information collection, the information revelation mechanism is quite effective in motivating the manager. However, when the investor becomes more risk averse, such a mechanism proves inferior to the incentive contract as it does not allow the investor to effectively share the portfolio risk with the manager.

4. Conclusion

The question of how to best design contracts in the delegated portfolio management setting has long engaged academics, regulators, and the investment community. Three considerations play important roles in determining the desirability of a contract in this setting. We want to find contracts that (a) provide the portfolio manager with the right incentives to exert costly effort to collect information, (b) lead the manager to choose a portfolio with the desirable risk characteristics, and (c) provide effective risk-sharing between the manager and the investors. In this study, we address the optimal contract design problem by effectively considering all three of the above issues. We start with a simplifying assumption on the joint distribution of asset returns and information that introduces substantial symmetry into the structure of the model while preserving the richness of the contracting problem's intrinsic decision structure.

Our key results can be summarized as follows. One, when the benchmark is appropriately chosen to purge the impact of factors beyond the manager's control, the use of the option-type "bonus" performance fee in the contract leads to increased effort from the portfolio manager. Two, with a poorly chosen benchmark, the option-type incentive fee actually hurts the manager's effort incentive. Three, for the general set of contracts considered here, with the appropriate choice of benchmark, it is always optimal to include an option-type incentive fee in the contract. We derive the conditions that the benchmark must satisfy. Our results highlight the importance of benchmark choice in the design of performance-based fees in portfolio management contracts. Further, they suggest that a universal prohibition on the use of asymmetric performance-based fees in mutual fund advisory contracts may in fact be harmful to shareholder interests.

Appendix

Proof of Lemma 1. The first order condition, which is also the sufficient condition of optimization for u in Expression (21) is

$$\left(\frac{p}{1-p}\right)\left(\frac{1+\rho}{1-\rho}\right)\left(\frac{c+\lambda}{c}\right)=\left\{\exp\left[aW_0\left(2c_1u+2c_2(u-N_{BL})+\lambda(u-N_{BO})\right)\right]\right\}.$$

Solving for u we get

$$u=\frac{\ln\left(\frac{p}{1-p}\right)+\ln\left(\frac{1+\rho}{1-\rho}\right)+\ln\left(\frac{c+\lambda}{c}\right)}{2amW_0}+\frac{c_2}{m}N_{BL}+\theta N_{BO}.$$

Similarly, we can solve the optimization problem for v. We get

$$v=\frac{\ln\left(\frac{p}{1-p}\right)-\ln\left(\frac{1+\rho}{1-\rho}\right)-\ln\left(\frac{c+\lambda}{c}\right)}{2amW_0}+\frac{c_2}{m}N_{BL}+\theta N_{BO}.$$

From Equations (19) and (20), we have $w_m=\frac{u+v}{2\delta_m}$ and $w_a=\frac{u-v}{2\delta_a}$. Note that $\frac{c+\lambda}{c}=\frac{1+\theta}{1-\theta}$. Direct computation then leads to Equations (22a) and (22b) in the lemma. Equation (22c) is due to symmetry. It is then straightforward to compute the manager's certainty-equivalent utility, which we report below:

$$\begin{aligned}\Pi_{ic,a}=&\;F+c_1W_0R_f+\frac{1}{2a}\left((1-\theta)\ln(1-\theta)+(1+\theta)\ln(1+\theta)\right)\\&-\frac{1}{a}\ln\left[\begin{array}{l}p^{(1-\theta)/2}(1-p)^{(1+\theta)/2}\exp\left(aW_0\delta_m\theta\left(cw_{BO}-c_2w_{BL}\right)\right)\\+p^{(1+\theta)/2}(1-p)^{(1-\theta)/2}\exp\left(-aW_0\delta_m\theta\left(cw_{BO}-c_2w_{BL}\right)\right)\end{array}\right]\\&+\max_{\varepsilon}\left[-\frac{1}{2a}\left((1-\theta)\ln(1+\rho)+(1+\theta)\ln(1-\rho)\right)-V(\varepsilon)\right].\end{aligned}\tag{A1}$$

The first order condition of the above maximization problem of effort choice ε is:

$$2aV'(\varepsilon)+\left(\frac{1-\theta}{1+\rho}-\frac{1+\theta}{1-\rho}\right)\cdot\frac{d\rho}{d\varepsilon}=0.$$

Total differentiation of Equation (3) yields $\frac{d\rho}{d\varepsilon}$. After substituting the result in the above expression and upon further simplification, we arrive at Equation (23). To be consistent, we need $w_{m,ic}$ and $w_{a,ic}$ to fit in Scenario (a). Inequality (18), after plugging in the solutions for $w_{m,ic}$ and $w_{a,ic}$, turns into the following condition:

$$\ln\frac{1+\rho}{1-\rho}\geq\left|\ln\frac{p}{1-p}-2caW_0\delta_m(w_{BO}-\frac{c_2}{c}w_{BL})\right|-\ln\frac{c+\lambda}{c}.\tag{A2}$$

Appendix for Lemma 2. The manager's optimal certainty-equivalent utility for Scenario (b) is

$$\begin{aligned}\Pi_{ic,b}=&\;F+c_1W_0R_f+\frac{1}{2a}\left[(1-\theta)\left(\ln(1-\theta)-\ln p\right)+(1+\theta)\left(\ln(1+\theta)-\ln(1-p)\right)\right]\\&+W_0\delta_m\theta\left(c_2w_{BL}-cw_{BO}\right)\\&+\max_{\varepsilon}\left[-\frac{1}{a}\ln\left((1+\rho)^{(1-\theta)/2}(1-\rho)^{(1+\theta)/2}+(1+\rho)^{(1+\theta)/2}(1-\rho)^{(1-\theta)/2}\right)-V(\varepsilon)\right].\end{aligned}\tag{A3}$$

A necessary condition for the manager's optimization problem to fit Scenario (b) is:

$$\ln\frac{1+\rho}{1-\rho} < \ln\frac{p}{1-p} - 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) + \ln\frac{c+\lambda}{c}. \quad \text{(A4)}$$

The manager's optimal certainty-equivalent utility for Scenario (c) is:

$$\begin{aligned}\Pi_{ic,c} = {} & F + c_1W_0R_f + \frac{1}{2a}\left[(1-\theta)\left(\ln(1-\theta) - \ln(1-p)\right) + (1+\theta)\left(\ln(1+\theta) - \ln p\right)\right] \\ & + W_0\delta_m\theta\left(c_2w_{BL} - cw_{BO}\right) \\ & + \max_{\varepsilon}\left[-\frac{1}{a}\ln\left((1+\rho)^{(1-\theta)/2}(1-\rho)^{(1+\theta)/2} + (1+\rho)^{(1+\theta)/2}(1-\rho)^{(1-\theta)/2}\right) - V(\varepsilon)\right].\end{aligned} \quad \text{(A5)}$$

A necessary condition for the manager's optimization problem to fit Scenario (c) is

$$\ln\frac{1+\rho}{1-\rho} < 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) - \ln\frac{p}{1-p} + \ln\frac{c+\lambda}{c}. \quad \text{(A6)}$$

Notice that the optimization problem of ε is identical in (A3) and (A5). The first order condition of the optimization problem, after simplification, is given by Equation (25).

Proof of Proposition 1: After Lemmas 1 and 2, the manager's choice of portfolio weights is still not yet completely solved for, because conditions (A2), (A4) and (A6) are not mutually exclusive. First, when $\ln\frac{1+\rho}{1-\rho}$ is in the interval

$$\left[\ln\frac{p}{1-p} - 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) - \ln\frac{c+\lambda}{c}\ ,\ \ \ln\frac{p}{1-p} - 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}) + \ln\frac{c+\lambda}{c}\right) \quad \text{(A7)}$$

the manager needs to compare the portfolio solution according to Scenario (a) described by Lemma 1 with that according to Scenario (b) described by Lemma 2. For any given level of ρ that satisfies (A9), we compare the manager's utility under the optimal portfolio choice for Scenario (a) (Equation (A1) without taking the maximization on ε) with that for Scenario (b) (Equation (A3)). We find that the manager will choose to go with Scenario (a) rather than Scenario (b) if and only if

$$\ln\frac{1+\rho}{1-\rho} \geq \ln\frac{p}{1-p} - 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}), \quad \text{(A8)}$$

which is exactly at the middle point of the interval in (A7). Similarly, we compare the manager's interior optimal utility for Scenario (a) with that for the Scenario (c) (Equation (A5)). We arrive at the condition that the manager will choose to go with Scenario (a) rather than Scenario (c) if and only if,

$$\ln\frac{1+\rho}{1-\rho} \geq -\ln\frac{p}{1-p} + 2caW_0\delta_m(w_{BO} - \frac{c_2}{c}w_{BL}). \quad \text{(A9)}$$

After this, there is no more ambiguity in the manager's portfolio decision. Combining (A8) and (A9), we arrive at (26) in the proposition. The rest follows.

Proof of Theorem 1: Case (a): Through differentiation of Equation (23) in Section 2.3, we have

$$\frac{\partial\varepsilon_{ic}}{\partial\theta} = \frac{\rho^2}{\theta(1-\rho)^2 + a\rho^3V''(\varepsilon_{ic})} > 0,$$

for $\theta > 0$, as V'' is positive. As for the case where $\lambda < 0$ (i.e. $\theta < 0$), notice that from Equation (23) we have $V'(\varepsilon_{ic}) < 1/a$. By comparing this inequality with Equation (9), we conclude that $\varepsilon_{ic} < \varepsilon_l$.

Case (b): The proof of the corollary in this case makes use of the following claim.

Claim**:** for $1 < \zeta < \exp(2)$ and $0<\theta<1$, $(1+\theta)\left(\zeta^{(1-\theta)/2} + \zeta^{-(1-\theta)/2}\right) - (1-\theta)\left(\zeta^{(1+\theta)/2} + \zeta^{-(1+\theta)/2}\right) > 0$.

Proof of the claim**:** To prove the claim is to prove $(1+\theta)\zeta^{(1-\theta)/2}\left(1+\zeta^{-(1-\theta)}\right) > (1-\theta)\zeta^{(1+\theta)/2}\left(1+\zeta^{-(1+\theta)}\right)$. Notice that $\left(1+\zeta^{-(1-\theta)}\right) > \left(1+\zeta^{-(1+\theta)}\right)$ because $\zeta > 1$, and $\theta > 0$. Therefore, a stronger statement than our claim is: $\zeta < \left(\frac{1+\theta}{1-\theta}\right)^{\frac{1}{\theta}}$. Notice first, that $\lim_{\theta \to 0^+}\left(\frac{1+\theta}{1-\theta}\right)^{\frac{1}{\theta}} = \exp(2) > \zeta$. We now only need to prove $\frac{d}{d\theta}\left(\frac{1+\theta}{1-\theta}\right)^{\frac{1}{\theta}} > 0$ for $\theta > 0$. It suffices to prove $\frac{d}{d\theta}\left[\frac{1}{\theta}\ln\left(\frac{1+\theta}{1-\theta}\right)\right] < 0$, or equivalently, $\ln\left(\frac{1+\theta}{1-\theta}\right) - \frac{2\theta}{1-\theta^2} < 0$. Notice that $\ln\left(\frac{1+\theta}{1-\theta}\right) - \frac{2\theta}{1-\theta^2}\Big|_{\theta=0} = 0$. Therefore, we only need to prove that $\frac{d}{d\theta}\left(\ln\left(\frac{1+\theta}{1-\theta}\right) - \frac{2\theta}{1-\theta^2}\right) < 0$ for $\theta > 0$. That is, to prove $\frac{-4\theta^2}{1-\theta^2} < 0$, which holds of course. End of the proof of the claim.

We now get back to the proof of the theorem. Through direct computation, we have:

$$\frac{\partial}{\partial\zeta}g(\zeta,\theta) = -\frac{1}{(\zeta+1)^2}\left[(1+\theta)\left(\zeta^{(1-\theta)/2} - \zeta^{-(1-\theta)/2}\right) + (1-\theta)\left(\zeta^{(1+\theta)/2} - \zeta^{-(1+\theta)/2}\right)\right] < 0,$$

and $\frac{\partial}{\partial\theta}g(\zeta,\theta) = \frac{\zeta^{(1-\theta)/2}}{\zeta+1}\left(\zeta^{\theta} - 1\right)\ln\zeta > 0$. Furthermore,

$$\frac{\partial^2}{\partial\theta\partial\zeta}g(\zeta,\theta) = \frac{1}{(\zeta+1)^2}\left\{\begin{array}{l} -\left(\zeta^{(1-\theta)/2} - \zeta^{-(1-\theta)/2}\right) + \left(\zeta^{(1+\theta)/2} - \zeta^{-(1+\theta)/2}\right) \\ +\frac{1}{2}\left[(1+\theta)\left(\zeta^{(1-\theta)/2} + \zeta^{-(1-\theta)/2}\right) - (1-\theta)\left(\zeta^{(1+\theta)/2} + \zeta^{-(1+\theta)/2}\right)\right]\ln\zeta \end{array}\right\}.$$

We note that

$$\frac{\partial}{\partial\theta}\left\{-\left(\zeta^{(1-\theta)/2} - \zeta^{-(1-\theta)/2}\right) + \left(\zeta^{(1+\theta)/2} - \zeta^{-(1+\theta)/2}\right)\right\} = \frac{1}{2}\left[\left(\zeta^{(1-\theta)/2} + \zeta^{-(1-\theta)/2}\right) + \left(\zeta^{(1+\theta)/2} + \zeta^{-(1+\theta)/2}\right)\right]\ln\zeta > 0,$$

and $-\left(\zeta^{(1-\theta)/2} - \zeta^{-(1-\theta)/2}\right) + \left(\zeta^{(1+\theta)/2} - \zeta^{-(1+\theta)/2}\right)\Big|_{\theta=0} = 0$. Therefore,

$$\left(\zeta^{(1-\theta)/2} - \zeta^{-(1-\theta)/2}\right) + \left(\zeta^{(1+\theta)/2} - \zeta^{-(1+\theta)/2}\right) > 0 \text{ for } \theta > 0.$$

This inequality together with the inequality in the claim proved above, implies that $\frac{\partial^2}{\partial\theta\partial\zeta}g(\zeta,\theta) > 0$. Therefore, we have:

$$\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial\zeta}g(\zeta,\theta)/g(\zeta,\theta)\right) = \frac{g(\zeta,\theta)\frac{\partial^2}{\partial\theta\partial\zeta}g(\zeta,\theta) - \frac{\partial}{\partial\zeta}g(\zeta,\theta)\frac{\partial}{\partial\theta}g(\zeta,\theta)}{g^2(\zeta,\theta)} > 0.$$

As per assumption, the second order condition of the manager's optimal effort choice problem is satisfied:

$$\frac{\partial}{\partial\varepsilon}\left(\frac{\partial}{\partial\zeta}g(\zeta,\theta)/g(\zeta,\theta) + aV'(\varepsilon)\frac{\rho}{2\zeta}\right)\Bigg|_{\varepsilon=\varepsilon_{ic}} > 0.$$

Therefore, from Equation (17), we have $\frac{\partial\varepsilon_{ic}}{\partial\theta} = \left.\frac{-\frac{\partial}{\partial\theta}\left(\frac{\partial}{\partial\zeta}g(\zeta,\theta)/g(\zeta,\theta)\right)}{\frac{\partial}{\partial\varepsilon}\left(\frac{\partial}{\partial\zeta}g(\zeta,\theta)/g(\zeta,\theta) + aV'(\varepsilon)\frac{\rho}{2\zeta}\right)}\right|_{\varepsilon=\varepsilon_{ic}} < 0.$

Deriving the reduced-form objective function for the manager (Expression (30)): Due to symmetry, we only need to look at the case with $s = 1$. By plugging in the manager's optimal portfolio strategy, specified by Equations (22a-22c) in Lemma 1 into Equation (1), subsequently applying Equation (4), and finally combining the results, we can get the distribution for $W - W_a(W)$, the investor's payoff. The investor's certainty-equivalent utility can be readily computed, which leads to an explicit form of the investor's participation constraint:

$$W_0\left(R_f - R_0\right) - \frac{1}{b}\ln\left[D_1 + D_2 + D_3 + D_4\right] = F + c_1 W_0 R_F, \tag{A10}$$

where

$$D_1 = p \times \frac{1+\rho}{2} \times G_1^{-(2-\lambda)} \times G_2^{-(1+c)} \times \left[\frac{p}{1-p}\frac{1+\rho}{1-\rho}\frac{1-\theta}{\theta}\right]^{F_1}, \tag{A11}$$

$$D_2 = p \times \frac{(1-\rho)}{2} \times G_1^{-(2+\lambda)} \times G_2^{-(1-c)} \times \left[\frac{p}{1-p}\frac{1-\rho}{1+\rho}\frac{\theta}{1-\theta}\right]^{F_2}, \tag{A12}$$

$$D_3 = (1-p) \times \frac{(1+\rho)}{2} \times G_1^{2-\lambda} \times G_2^{1+c} \times \left[\frac{1-p}{p}\frac{1+\rho}{1-\rho}\frac{1-\theta}{\theta}\right]^{F_1}, \tag{A13}$$

$$D_4 = (1-p) \times \frac{1-\rho}{2} \times G_1^{2+\lambda} \times G_2^{1-c} \times \left[\frac{1-p}{p}\frac{1-\rho}{1+\rho}\frac{\theta}{1-\theta}\right]^{F_2}. \tag{A14}$$

In the above expressions, $G_1 = \exp\left[\frac{c_2}{2m} b W_0 N_{BL}\right]$, $G_2 = \exp\left[\theta b W_0 N_{BO}\right]$, $F_1 = -\frac{b}{2a}\left(\frac{1}{m} - 1 - \theta\right)$, and $F_2 = -\frac{b}{2a}\left(\frac{1}{m} - 1 + \theta\right)$. Substituting the left hand side of (A10) for $F + c_1 W_0 R_F$ in the manager's certainty-equivalent utility (i.e., A1), we arrive at (32) in Section 3.1. In the special case when $\lambda = 0$, and therefore $\theta = 0$, we have

$$D_1 = \frac{1}{2} G_3^{-1} p^{1-\frac{b(1-c)}{2ac}} (1-p)^{\frac{b(1-c)}{2ac}} (1+\rho)^{1-\frac{b(1-c)}{2ac}} (1-\rho)^{\frac{b(1-c)}{2ac}}, \tag{A15}$$

$$D_2 = \frac{1}{2} G_3^{-1} p^{1-\frac{b(1-c)}{2ac}} (1-p)^{\frac{b(1-c)}{2ac}} (1+\rho)^{\frac{b(1-c)}{2ac}} (1-\rho)^{1-\frac{b(1-c)}{2ac}}, \tag{A16}$$

$$D_3 = \frac{1}{2} G_3 p^{\frac{b(1-c)}{2ac}} (1-p)^{1-\frac{b(1-c)}{2ac}} (1+\rho)^{1-\frac{b(1-c)}{2ac}} (1-\rho)^{\frac{b(1-c)}{2ac}}, \text{ and} \tag{A17}$$

$$D_4 = \frac{1}{2} G_3 p^{\frac{b(1-c)}{2ac}} (1-p)^{1-\frac{b(1-c)}{2ac}} (1+\rho)^{\frac{b(1-c)}{2ac}} (1-\rho)^{1-\frac{b(1-c)}{2ac}}, \tag{A18}$$

where $G_3 = \exp\left[\frac{c_2}{c} b W_0 N_{BL}\right]$.

Proof of Theorem 2: We first prove that the optimal linear contract is dominated by a contract with $\lambda > 0$. Let us denote the manager's objective function in expression (30) in Section 3.1 by $\Pi(c, c_2, w_{BL}, \lambda, w_B)$. We change to the equivalent set of control variables: { m, c_2, w_{BL}, θ, w_B }. To prove the theorem, we need to first prove that

$$\left.\frac{\partial \Pi}{\partial \theta}\right|_{m=\frac{b}{a+b}, c_2=0, \theta=0} > 0. \tag{A19}$$

Applying the expression for Π in Expression (30) in Section 3.1, we get

$$\frac{\partial \Pi}{\partial \theta} = -\frac{1}{2a}\left(2\frac{\partial \ln \Pi_1}{\partial \theta} + \frac{\partial \Pi_2}{\partial \theta}\right) - \frac{1}{b}\sum_{i=1}^{4}\frac{\partial D_i}{\partial \theta} \Big/ \sum_{i=1}^{4} D_i, \tag{A20}$$

where

$$\begin{aligned}\Pi_1 &= p^{(1-\theta)/2}(1-p)^{(1+\theta)/2}\exp\left(aW_0\delta_m\theta\left(cw_{BO} - c_2 w_{BL}\right)\right) \\ &+ p^{(1+\theta)/2}(1-p)^{(1-\theta)/2}\exp\left(-aW_0\delta_m\theta\left(cw_{BO} - c_2 w_{BL}\right)\right),\end{aligned}$$

and

$$\Pi_2 = (1-\theta)\ln\left(\frac{1+\rho(\varepsilon)}{1-\theta}\right) + (1+\theta)\ln\left(\frac{1-\rho(\varepsilon)}{1+\theta}\right) - 2aV(\varepsilon)\bigg|_{\varepsilon=\varepsilon_{ic}} .$$

Here ε_{ic} is a function of θ implicitly defined by Equation (25). One can directly verify that

$$\frac{\partial \ln \Pi_1}{\partial \theta}\bigg|_{m=\frac{b}{a+b},c_2=0,\theta=0} = \frac{1}{\Pi_1}\frac{\partial \Pi_1}{\partial \theta}\bigg|_{m=\frac{b}{a+b},c_2=0,\theta=0} = 0 . \tag{A21}$$

Because ε_{ic} solves the maximization problem in (A1), we have $\frac{\partial \Pi_2}{\partial \varepsilon}\Big|_{\varepsilon=\varepsilon_{ic}} = 0$. This serves as the envelope condition. Because of the envelope condition, we can take the derivative of Π_2 with respective to θ without the concern that ε_{ic} depends on θ. We have the following partial differentiation:

$$\frac{\partial \Pi_2}{\partial \theta}\bigg|_{m=\frac{b}{a+b},c_2=0,\theta=0} = \ln(1-\rho(\varepsilon_l)) - \ln(1+\rho(\varepsilon_l)) . \tag{A22}$$

In the derivation of the above equation, we used the fact that $\varepsilon_{ic} = \varepsilon_l$, when $\theta = 0$. Taking partial derivative for each D_i, summing up the four items, and then dividing by $\sum D_i$ we can show that:

$$\sum \frac{\partial D_i}{\partial \theta} \bigg/ \sum D_i \bigg|_{m=\frac{b}{a+b},c_2=0,\theta=0} = 0 . \tag{A23}$$

Plugging (A21), (A22), and (A23) into (A20), we have

$$\frac{\partial \Pi}{\partial \theta}\bigg|_{m=\frac{b}{a+b},c_2=0,\theta=0} = \frac{1}{2a}\big(\ln(1+\rho(\varepsilon_l)) - \ln(1-\rho(\varepsilon_l))\big) > 0 .$$

We have thus proved inequality (A19). What is left is to prove that the optimal linear contract dominates all contracts with $\lambda < 0$. Although intuitive, the rigorous proof is long and tedious. We only sketch out the main steps here. Consider a contract with $\lambda < 0$. The manager will voluntarily choose the effort level, denoted by ε_{ic}. For this fixed effort level (ε_{ic}), changing the contract parameters in (30) to the set that is in accordance with the optimal linear contract will lead to an improvement in the objective function due to the optimal contract's more appropriate risk incentive and better risk sharing properties. Then one can prove that, with the optimal linear contract parameters, the objective function in (30) is monotonic in ρ for all cases with $\varepsilon_{ic} < \varepsilon_l$. From Corollary 1, we know that $\varepsilon_{ic} < \varepsilon_l$. Therefore, increasing the effort level to ε_l leads to a further increase in the value of the objective function in (30). The above chain of comparisons leads to the conclusion of the theorem.

Proof of Proposition 4: The manager's objective function is:

$$\begin{aligned}&\min_u \; p\frac{1+\rho}{2}\left\{\exp\left[-a\left(c_1W_0u + g(W_0(u-N_{BL}))\right)\right]\right\} + (1-p)\frac{1-\rho}{2}\left\{\exp\left[-a\left(-c_1W_0u - g(W_0(u-N_{BL}))\right)\right]\right\}\\ &+\min_v \; p\frac{1-\rho}{2}\left\{\exp\left[-a\left(c_1W_0v + g(W_0(v-N_{BL}))\right)\right]\right\} + (1-p)\frac{1+\rho}{2}\left\{\exp\left[-a\left(-c_1W_0v - g(W_0(v-N_{BL}))\right)\right]\right\}\end{aligned}$$

For the moment, we denote that $D = \exp\left(-a\left(c_1u + g(u-N_{BL})\right)\right)$, and look at the optimization problem that involves *u*. The problem transforms to: $\min_{D>0}\left(p(1+\rho)D + (1-p)(1-\rho)/D\right)/2$. We are in effect minimizing the arithmetic average of two items whose geometric average is $\sqrt{p(1-p)(1+\rho)(1-\rho)}$, which no longer depends on the control variable and is therefore the minimum.[24] The minimization problem for v can be solved in the same fashion. It is then straightforward to check that the resulting managerial utility is the same as in Equation (8), which then leads to the effort choice condition (9).

Proof of Proposition 5: In the following proof, we will assume that condition (a) under Theorem 1 is satisfied. The other two cases follow similarly. We denote that:

$$\rho' = \frac{\frac{1+\rho}{2}\exp(-a\xi)-\frac{1-\rho}{2}}{\frac{1+\rho}{2}\exp(-a\xi)+\frac{1-\rho}{2}} \ . \tag{A24}$$

After some manipulation, the manager's portfolio choice problem can be expressed in exactly the same form as that in Expression (21) in Section 2.3, except the term ρ is replaced by ρ' here. We can therefore invoke Lemma 1 to solve the portfolio allocation problem. The proof of the proposition then follows.

Information Revelation Mechanism: The manager reports the signal to the investor and gets $\phi+d$ if he is ex post correct, and $\phi-d$ otherwise. The manager's incentive compatibility condition to report the signal truthfully is simply: $d \geq 0$. The manager's incentive compatibility condition for information collection is straightforward:

$$\rho_{ir} = \arg\max_{\rho} \frac{1}{a}\ln 2 + \phi - \frac{1}{a}\ln\left[(1+\rho)e^{-ad} + (1-\rho)e^{ad}\right] - V(\rho) \tag{A25}$$

Here the subscript ir denotes the information revelation mechanism. It is clear from (A25) that only the term d , and not ϕ , motivates the manager. The investor anticipates, correctly in equilibrium, the signal precision ρ_{ir} . Start with the investor's payoff distribution, which leads to the derivation of her expected utility. After some simplification, the investor's expected utility is the product of $-\exp\left(-b(W_0R_f-\phi)\right)$ and the following minimization problem:

$$\min_{u,v}\left[p\frac{1+\rho}{2}e^{bd}e^{-bW_0u} + (1-p)\frac{1-\rho}{2}e^{-bd}e^{bW_0u} + p\frac{1-\rho}{2}e^{-bd}e^{-bW_0v} + (1-p)\frac{1+\rho}{2}e^{bd}e^{bW_0v}\right].$$

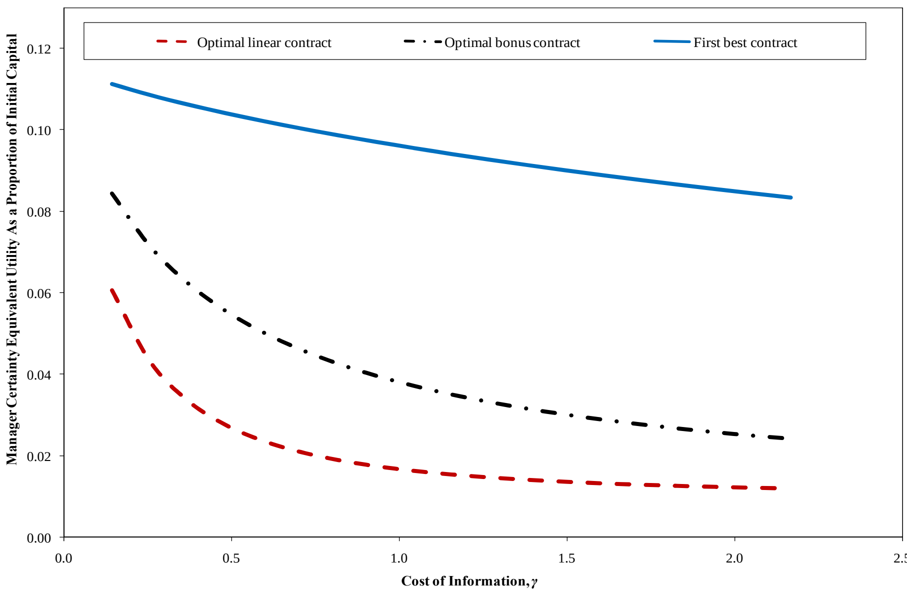
Again, we can view the above as minimizing an arithmetic average of four terms whose geometric average is $2\sqrt{p(1-p)(1-\rho^2)}$, which does not depend on the control variables and hence, is the minimum. From this, the investor's participation constraint implies: $\phi = -\frac{1}{b}\ln 2\sqrt{p(1-p)(1-{\rho_{ir}}^2)} - W_0(R_0-R_f)$. The manager has the freedom to choose the compensation parameter d . By applying the investor's participation constraint, the manager's certainty-equivalent utility is obtained by the following maximization:

$$\max_{b}\frac{b-a}{ab}\ln 2 - W_0(R_0-R_f) - \frac{1}{2b}\ln p(1-p)(1-{\rho_{ir}}^2) - \frac{1}{a}\ln\left[(1+\rho_{ir})e^{-ad} + (1-\rho_{ir})e^{ad}\right] - V(\rho_{ir})\,.$$

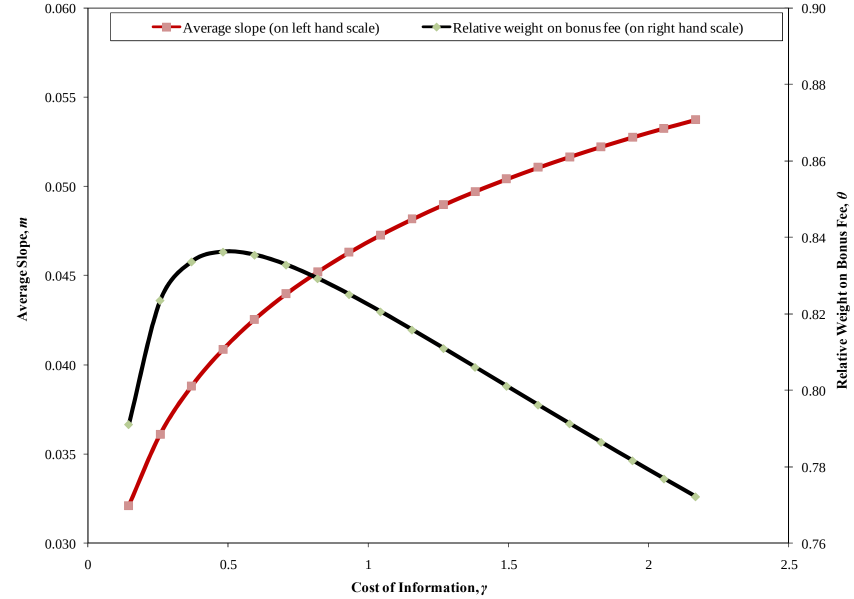
In the above optimization, ρ_{ir} is viewed as a function of the control variable d implicitly defined by (A25).

[24] To be exact, we need to ensure that after solving for D, we are able to obtain a solution for u. We can, for example, assume that $g(x)$ is continuous.

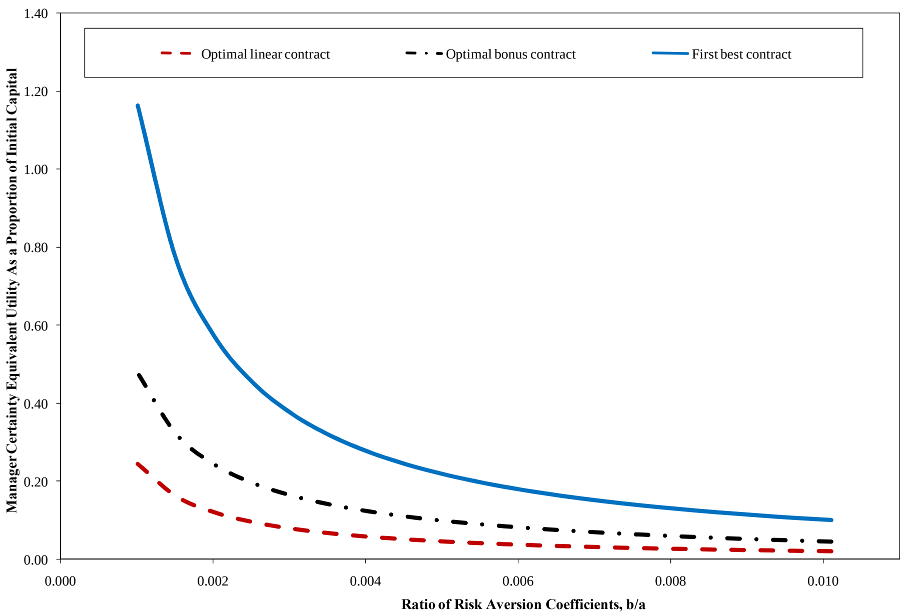
Figure 1



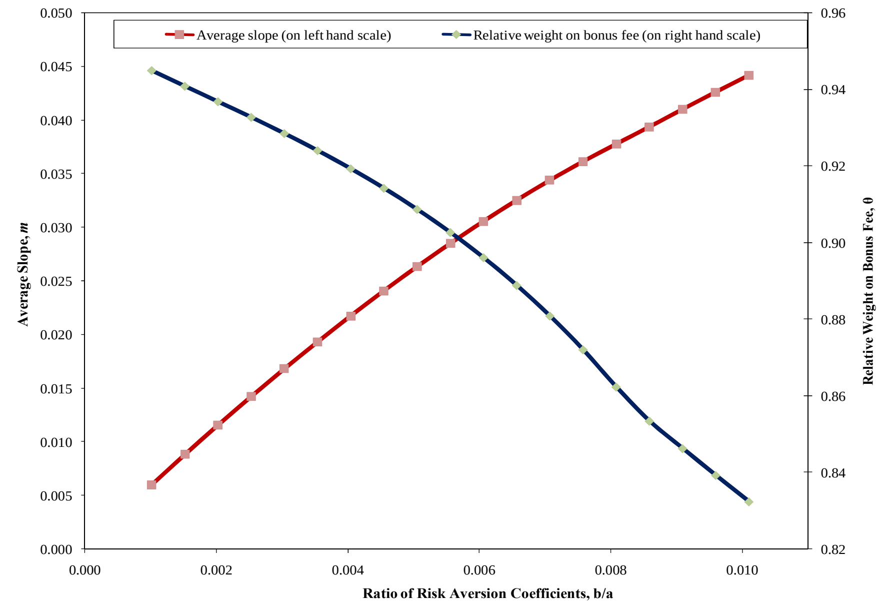
A. Manager's Certainty-Equivalent Utility Under Alternative Contracts



B. Composition of Optimal Bonus Contract



C. Manager's Certainty-equivalent Utility As a Function of the Ratio of the Investor's Risk Aversion to the Manager's Risk Aversion, b/a



D. Optimal Contract Composition As a Function of the Ratio of the Investor's Risk Aversion to the Manager's Risk Aversion, b/a

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