Does Stock Return Predictability Imply Improved Asset Allocation and Performance?
Evidence from the U.S. Stock Market (1954-2002)

Puneet Handa *
Ashish Tiwari **

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* John Hawkinson Research Fellow in Finance and Associate Professor, Henry B. Tippie College of Business, University of Iowa, Iowa City 52242, Ph.: 319-335-2731, E-mail: puneet-handa@uiowa.edu.

** Matthew Bucksbaum Research Fellow in Finance and Assistant Professor, Henry B. Tippie College of Business, University of Iowa, Iowa City 52242, Ph.: 319-353-2185, E-mail: ashish-tiwari@uiowa.edu.

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Abstract

This paper provides evidence on the economic significance of U.S. stock return predictability within an asset allocation framework in a real-time context. We examine the performance of a Bayesian, risk averse investor (the mutual fund investor) who relies on conditioning information (e.g., dividend yield, T-bill yield, default spread and term spread) to forecast future returns, and contrast it with that of an otherwise identical investor who believes that the returns are \textit{i.i.d.} (the \textit{i.i.d.} investor). Our major finding is that the relative performance of the mutual fund strategy is unstable over time, being noticeably poor during the most recent sub-period (1989-2002). In marked contrast, the performance of the mutual fund strategy is significantly better when it relies on a model-based approach, characterized by varying degrees of prior confidence in the CAPM.
Does Stock Return Predictability Imply Improved Asset Allocation and Performance?

I. Introduction

The predictability of excess stock returns is by now a well-established phenomenon in the asset pricing literature. As Cochrane (1999) notes in his review article, during the past two decades several studies have demonstrated that a substantial amount of stock return variation can be explained by variables such as the dividend yield and T-bill yield, among others. There are legitimate concerns expressed in the literature about the statistical significance of the evidence and its out-of-sample validity. However, as Cochrane notes, the consensus view now is that stock returns are predictable.

In contrast, there has been less discussion and agreement on the economic significance of stock return predictability, i.e., whether the documented predictability leads to improved performance. Breen, Glosten and Jagannathan (1989) find that the predictive ability of the one-month Treasury bill rates is economically significant. In a similar vein, Solnik (1993) shows that the use of conditioning information leads to economically significant improvement in the performance of international asset allocation strategies. On the other hand, Pesaran and Timmermann (1995) show that the predictive ability of a number of economic factors varies over time although it is economically significant during the volatile markets of the 1970s. More recently, Cooper, Gutierrez, and Marcum (2001), and Cooper and Gulen (2002) find that predictability is not evident in a real-time context.

Except for Solnik (1993), the above studies adopt a framework that involves shifting one’s portfolio entirely to stocks or to T-bills, instead of a utility-based asset allocation framework which would be more reflective of the impact of predictability on investor decisions. Most of these studies also ignore the uncertainty inherent in the parameters of the forecasting

1 For a critique of the evidence on grounds of mis-specified test statistics and/or finite sample biases, see Richardson and Stock (1989), Richardson and Smith (1991), Kim, Nelson and Startz (1991), Nelson and Kim (1993), and Goetzmann and Jorion (1993), among others. More recently, Bossaerts and Hillion (1999) and Goyal and Welch (1999) have used statistical measures of model performance to question the out-of-sample predictive ability of the return forecasting models. The impact of data snooping biases has been examined by Lo and Mackinlay (1990), and Foster, Smith and Whaley (1997).
model used to form expectations of future asset returns. From the standpoint of practical advice to portfolio investors, a natural question to ask is whether the documented stock return predictability translates into improved out-of-sample asset allocation and performance when investors account for parameter uncertainty. This question is important because as Kandel and Stambaugh (1996) argue, the issue of statistical significance of a predictive model is quite distinct from its impact on portfolio allocation. In particular, a predictive variable may significantly impact portfolio allocation despite having low statistical significance. Hence, an asset allocation framework provides us with an intuitively appealing metric by which to judge the value of information contained in a predictor variable, namely the quality of portfolio allocation resulting from its use.

There is relatively little evidence in the literature on the above issue, i.e., whether predictability of stock returns leads to improved asset allocation and performance, in a real-time context. In this paper we provide evidence on this issue in the context of a Bayesian investor who accounts for parameter uncertainty and faces constraints such as the presence of transaction costs, limited short selling, and real-time decision making. Previous studies such as Kandel and Stambaugh (1996) and Barberis (2000) have provided valuable insights in regard to the impact of return predictability on portfolio allocation in a Bayesian setting. However, these studies make use of the full historical sample information and the end-of-sample values of the predictor variables. Our analysis complements and adds to the previous literature by shedding light on the portfolio allocation implications of return predictability when investors are faced with parameter uncertainty and market frictions in a real-time context.

At the start of each month, our Bayesian investor makes portfolio allocations based on the belief that stock returns are predictable. The portfolios allocations are updated every month.

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2 We recently became aware of a study by Wei and Zhang (2000) that examines return predictability in an asset allocation framework. This study differs from our work as it adopts a non-Bayesian framework. The benefits of international diversification with regime shifts have been examined by Ang and Bekaert (2002) who find that such benefits persist despite the presence of bear market regimes. They do not consider transaction costs or parameter uncertainty in their analysis. In a different context, Fleming, Kirby, and Ostdiek (2001) study volatility timing in an asset allocation framework.

3 In our context, the real-time environment allows the investor access to just the historical data while making portfolio allocation decisions.

4 As Kandel and Stambaugh (1996) note, their analysis intentionally omits any consideration of the typical performance of the optimal conditional asset allocations. Our analysis may be viewed as providing some insights into the “typical” performance of the strategy that relies on predictability.
We contrast his performance to that of a Bayesian investor who believes that stock returns are independent and identically distributed (i.i.d.) and ignores any evidence of stock return predictability in making his monthly portfolios allocation choices. Both investors are assumed to be mean-variance optimizers with a single-period horizon. We refer to the former strategy as the *mutual fund* strategy and the latter as the *i.i.d.* strategy. Our study spans the period 1954-2002.

Our research design allows for both a data-based approach as well as an alternative model-based approach to portfolio allocation. Under the data-based approach the *mutual fund* investor uses conditioning variables such as lagged values of the dividend yield, one month T-bill yield, default spread, and term spread, to form expectations of next month’s stock returns. A key feature of our research design is that at the start of each month the *mutual fund* strategy is allowed to select an optimal forecasting model based on a recursive utility maximization criterion. At the start of each month, the *mutual fund* strategy selects an optimal model from the full set of 16 (= 2^4) possible models, including the *i.i.d.* model. The model chosen by the *mutual fund* strategy is the one that performs the best in terms of the realized utility, to date. The *mutual fund* strategy and the *i.i.d.* strategy are allowed to form portfolios by investing in a risk-free asset and (a) a single stock portfolio, or (b) three stock portfolios.

We also adopt the alternative model-based approach of Pástor and Stambaugh (1999, 2000) and Pástor (2000) under which the *mutual fund* strategy is allowed to form expectations of next month’s stock returns based on varying degrees of prior confidence in the Capital Asset Pricing Model. Under this framework we allow both strategies to invest in ten industry-based portfolios in addition to the risk free asset. As an extension of this framework we allow the *mutual fund* strategy to account for time-variation in alphas, betas, and the market risk premium. Similar to Shanken (1990), Ferson and Harvey (1999), Avramov (2004), and Avramov and Chordia (2004), we model the alphas and betas as well as the market risk premium as linear functions of the conditioning variables.

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5 Two recent studies, by Avramov (2002) and Cremers (2002), use Bayesian model averaging techniques to examine predictability in the presence of model uncertainty. They use statistical measures to conclude that the out-of-sample performance of the Bayesian approach is superior to that of the classical statistical model selection methods. However, neither study addresses the question of economic significance of return predictability in a real-time context.
We calculate the certainty equivalent rates of return (CER) for each strategy to judge its relative performance. The CER for each strategy represents the rate of return earned with certainty that would provide the investor with the same utility as the utility realized from the portfolio allocations resulting from the strategy. We also report the adjusted Sharpe ratio of Graham and Harvey (1997) for the *mutual fund* strategy and assess its market timing performance. In our analysis we incorporate several features of the economic environment including out-of-sample forecasting, parameter uncertainty, portfolio allocation across both single and multiple risky assets, transactions costs and limited short selling and margin purchases. Our study’s design allows investors to continuously update the parameters of the predictive model utilized by them. This is potentially important if the dynamics of the stock return process or the predictive ability of conditioning variables changes over time.

Our construction of the two hypothetical strategies can be thought of as a device to explore the value of information contained in (a) the predictor variables in the data-based approach, and (b) the asset pricing model utilized in the model-based approach. If, for example, the predictor variables contain useful information about future returns, in the context of our repeated experiments, we expect the *mutual fund* strategy to outperform the *i.i.d.* strategy. Our approach is similar in spirit to the utility-based metric proposed by McCulloch and Rossi (1990) to judge the economic significance of departures from the arbitrage pricing theory. As McCulloch and Rossi note, such an approach may result in conclusions that differ markedly from traditional significance testing.

We examine the relative performance of the *mutual fund* and *i.i.d.* strategies in real-time over the period 1954-2002, with the first five years used as an initial “learning” period. Specifically, we allow each strategy to use only historical data from 1954 to date, to make portfolio allocations for the next period. The *mutual fund* strategy that relies on a data-based approach has unstable out-of-sample performance that is period specific. It is unable to outperform the *i.i.d.* strategy on a consistent basis. The *mutual fund* strategy’s performance is generally favorable during the sub-periods 1959-73 and 1974-1988. For example, in the case where portfolio allocations involve the risk free asset and a single stock portfolio, the *mutual

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6 In particular, the multiple risky asset scenario allows us to capture sectoral differences that may exist over economic cycles (see for example, Perez-Quiros and Timmermann (2000)).
fund strategy outperforms the i.i.d. strategy by 37 basis points per month during the two sub-periods in terms of CER, when averaged across the three levels of risk aversion considered by us. However, the strategy’s performance is noticeably poor during the most recent sub-period, 1989-2002, where it underperforms the i.i.d. strategy with an average CER difference of –38 basis points per month. Moreover, the portfolio allocations of the mutual fund strategy are considerably more volatile than the allocations of the i.i.d. strategy. We also find that the mutual fund strategy performs only marginally better than a random coin toss in terms of its market timing ability. When judged in terms of statistical significance, for the full sample period, the performance of the mutual fund strategy is not significantly different than that of the i.i.d. strategy.

To get a better understanding of the performance of the mutual fund strategy, we focus on the model choices made by the strategy over time. We find that the mutual fund strategy makes use of the i.i.d. model infrequently, suggesting that there is support for predictability in the data. However, we find that the shifts in the economic environment over time make it difficult for the mutual fund strategy to uncover the appropriate model for a particular time period leading to unstable performance across sub-periods. In our base set of results, the mutual fund strategy chooses the optimal forecasting model each month based on the candidate models’ performance to date. Shortening the length of the historical window used for model evaluation and selection does not qualitatively alter our results.

In contrast to the data-based approach, the results from the model-based approach, under which the mutual fund strategy relies on varying degrees of prior belief in the CAPM to form expectations of future stock returns, are much more favorable for the strategy. We find that under the standard CAPM approach, the mutual fund strategy significantly outperforms the i.i.d. strategy for moderate to high levels of risk aversion. The performance advantage is significant in both statistical and economic terms, especially when the investor has a high degree of prior confidence in the CAPM. Allowing for time variation in the alphas, betas, and the market risk premium does not lead to significant improvement relative to the standard CAPM framework.

Our results suggest that variables such as the T-bill yield, dividend yield, default spread, and term spread are valuable for asset allocation. This is borne out by the fact that the Bayesian mutual fund investor continues to use these variables instead of opting for the i.i.d. model in most cases. Also, in economic terms, there are impressive gains from using these variables in
some sub-periods. However, the gains are not consistent over time. Shifts in the economic environment often make it difficult to identify the model that is most appropriate in a particular sub-period. Thus, our results provide support for predictability in the data. At the same time they suggest that translating the predictability into consistently superior performance is a challenging task and therefore allocation strategies relying on data-based predictor variables should be tempered with caution. Our analysis helps explain the gap between the findings of stock return predictability in the academic literature on the one hand, and the documented lack of superior performance of the average professional money manager, on the other. Interestingly, our results also serve to highlight the value of a model-based approach motivated by theoretical considerations, for achieving optimal asset allocations.

Recently, studies such as Balduzzi and Lynch (1999) and Campbell and Viciera (1999) have argued that the utility costs of ignoring the sample evidence of stock return predictability are substantial. These studies typically involve a calibration of a data generating process for excess stock returns based upon the U.S. sample history. In this context, our analysis can be viewed as a real-time experiment designed to capture the actual experience of a strategy that relies on predictability and is subject to realistic constraints such as transactions costs, and limited short selling and margin purchases. As discussed below, our results suggest that the utility gains from relying on predictability that have been anticipated by previous studies, may represent an ideal that is difficult to attain for investors making decisions in real-time. In this respect our results complement those of Carlson, Chapman, Kaniel, and Yan (2001) who examine the impact of return predictability on the optimal portfolio and consumption choices of an investor in a general equilibrium setting. They find that allocations based on the observed sample estimates can be severely biased and have large variance in repeated simulations of the model economy. They conclude that calibration exercises based on point estimates from a simple sample path can be quite misleading. A similar note of caution is sounded by Aït-Sahalia and

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7 For example, beginning with Jensen (1968), a large number of studies have shown that the average actively managed mutual fund fails to outperform the relevant benchmark(s), net of the management fees (see, also, Gruber (1996), and Carhart (1997), among others).

8 In related work Johannes, Polson, and Stroud (2002) examine the value of market timing and volatility timing while allowing for estimation risk. They find that market timing strategies perform poorly in contrast to the performance of volatility timing strategies. Similarly, Gomes (2003) finds that utility gains from strategies based on exploiting short-run stock return autocorrelation are quite small and not robust to parameter uncertainty, in contrast to the performance of strategies based on volatility timing that yield significant gains. Both studies analyze strategies
Brandt (2001) who solve the sample analogues of the conditional Euler equations that characterize investors’ optimal portfolio choice, to determine which economic variables are important for such a choice. They conclude that “…the magnitude of predictability in returns is small and subject to a tremendous amount of noise, especially at short horizon.” (p. 1349).

The paper is organized as follows. In Section II, we outline how we implement the portfolio allocation problem in a Bayesian context. Section III describes our research design while Section IV presents descriptive statistics and the results of preliminary regression analysis. In Section V we present the results of the data-based approach where the mutual fund investor relies on the predictive ability of the conditioning variables to make asset allocations. Section VI presents results for the alternative model-based approach while Section VII concludes.

II. Asset allocation

A. Investor preferences

We assume that investors’ expected utility is defined over the expected return, $E(r_p)$, and variance, $\sigma^2_p$, of their portfolio:

$$E(U) = E(r_p) - \frac{1}{2} \lambda \sigma^2_p$$  \hspace{1cm} (1)

where $\lambda$ represents the investors’ degree of relative risk aversion.\(^9\) We consider two investors. In our framework the mutual fund investor, believes that excess stock returns are predictable. Under the data-based approach to asset allocation, each month the investor estimates the parameters of the following model using historical data, and uses the estimates as input in forecasting the next month’s return:

$$\tilde{r}_{i,t} = X_{t-1}B + \tilde{\epsilon}_{i,t}$$  \hspace{1cm} (2)

involving a single risky asset and neither study examines the performance of the strategies in the context of repeated samples.

\(^9\) Samuelson (1970) provides a theoretical justification for the mean-variance criterion. For recent applications of this utility specification see Solnik (1993) and Pástor and Stambaugh (2000), among others.
where \( \mathbf{X}_{t-1} = (1, x_{1,t-1}, x_{2,t-1}, \ldots, x_{k,t-1})' \) is a \( k \)-dimensional vector of explanatory (predictor) variables observed at the end of month \( t-1 \), \( \mathbf{B} \) is a \( k \times 1 \) vector of regression coefficients, and \( \tilde{\varepsilon}_{i,t} \sim i.i.d. N(0, \sigma^2_t) \). The excess return on a stock portfolio, \( r_{i,t} \), is defined as the portfolio’s monthly return in excess of the return on the one-month treasury bill. The vector \( \mathbf{X}_{t-1} \) consists of variables such as the dividend yield, one month T-bill yield, default spread and the term spread in addition to a constant term.\(^{10}\) At the start of each month the investor chooses the optimal combination of predictor variables from the full set. We describe the predictor variables and the process by which the investor chooses the appropriate variables to use in the forecasting model, in more detail in Section III.

The i.i.d. investor, believes that stock returns are independently and identically distributed. In particular, the investor believes that the excess return on a portfolio \( i \) has the following distribution:

\[
\tilde{r}_{i,t} = \mu_i + \tilde{\varepsilon}_{i,t}
\]

where \( \tilde{\varepsilon}_{i,t} \sim i.i.d. N(0, \sigma^2_t) \). Hence, the expected next period excess return for the i.i.d. investor is given by equation (2) where the set of predictor variables \( \mathbf{X}_{t-1} \) includes just the constant 1, and the estimated regression coefficient represents the sample mean excess return. This yields a set of optimal portfolio allocations for the i.i.d. investor for that month. The portfolio allocation is updated every month.

**B. Portfolio Optimization Problem**

Consider an investor who forms a portfolio \( P \) at the start of month \( t \) by choosing the optimal proportions to invest in multiple risky assets and a risk-free asset. With multiple risky assets, an investor chooses the optimal allocation to the risky assets by solving the following problem:

\(^{10}\) Our framework is similar to Campbell (1991), Hodrick (1992), Kandel and Stambaugh (1996) and Barberis (2000), among others.
\[
\max w' Er - \left( \frac{1}{2} \lambda \right) w' S w - \sum_{i=1}^{n} \kappa \left( w_{i,t} - w_{i,t-1} \right)
\]

\[\text{s.t. } \sum_{i=1}^{n} |w_i| \leq 2\]

where \( Er \) denotes a \( nx1 \) vector of expected excess returns on the \( n \) risky assets, \( S \) denotes the \( nxn \) covariance matrix of returns and \( w \) denotes the \( nx1 \) vector of weights on the risky assets chosen by the investor at the start of month \( t \). Note that the proportion of the portfolio invested in the risk-free asset is given by \( 1 - \sum_{i=1}^{n} w_{i,t} \). In the above equation \( \kappa \) is the proportional transaction cost incurred by the investor. We assume that the value of \( \kappa \) is 0.0025; this is equivalent to a proportional transaction cost of 50 basis points for a round trip trade in stocks.\(^{11}\) We further assume that investors can costlessly alter their positions in the risk-free asset. In addition, we allow investors to take short or long positions consistent with a 50% margin requirement.\(^{12}\) In equation (4) an investor’s expected portfolio return, \( E(r_{p,t}) \) and variance of return, \( \sigma_{p,t}^2 \), are given by:

\[
E(r_{p,t}) = r_{f,t} + w' Er ; \quad \sigma_{p,t}^2 = w' \Sigma w
\]

where \( r_{f,t} \) is the risk-free rate of return for month \( t \) observed at the start of the month.

Both the \textit{i.i.d.} investor and the \textit{mutual fund} investor form an estimate of the expected excess return matrix, \( Er \) and the covariance matrix, \( S \). In our analysis we consider Bayesian investors who account for parameter uncertainty. Hence, in order to solve for the optimal portfolio allocation they use the moments of the \textit{predictive} distribution of returns based on their respective beliefs.\(^{13}\) This process is outlined in more detail below.

\(^{11}\) This is consistent with estimates used in the literature (for example, Balduzzi and Lynch, 1999).

\(^{12}\) As an example, under a 50% margin requirement, a $100 stock investment may be financed with $50 in capital, which corresponds to a portfolio weighting of 200% in the stock. A similar calculation applies to short-sales subject to a 50% margin requirement. We also allow for margin purchases by borrowing at the risk-free rate.

\(^{13}\) The impact of parameter uncertainty on portfolio choice has been studied by Klein and Bawa (1976), Bawa, Brown and Klein (1979), and more recently, in our context by Kandel and Stambaugh (1996) and Barberis (2000). In related work Lewellen and Shanken (2002) show that Bayesian learning in the presence of parameter uncertainty can lead to stock return predictability, although investors can neither perceive it nor exploit it.
B.1. Predictive Distribution for the Mutual Fund Investor

As previously discussed, the mutual fund investor relies on a set of predictive variables in forming expectations of future stock returns at the start of each month \( t \). Consider the case where the investor forms a portfolio by investing in a single risky asset, in addition to the risk free asset.\(^{14}\) We can express the predictive model described in equation (2) as:

\[
R = X \beta + \epsilon ; \quad \epsilon \mid X \sim N(0, h^{-1}I_T)
\]

(6)

where in this context \( R \) is a \( T \times 1 \) matrix of excess returns, \( X \) is a \( T \times k \) matrix of \( k \) predictor variables, and \( B \) is a \( k \times 1 \) matrix of regression coefficients. The parameter \( h \) is the precision of the i.i.d. disturbances and equals the inverse of \( \text{Var}(\epsilon) = \sigma^2 \). The mutual fund investor faces uncertainty about his estimate of the return variance and also his estimate of the regression coefficient vector, \( B \). We assume that the investor has the standard normal-gamma conjugate prior:

\[
\begin{align*}
\beta \mid h & \sim N(\beta, h^{-1}H^{-1}) \\
\epsilon \mid X & \sim N(0, h^{-1}I_T)
\end{align*}
\]

(7)

Similar to Pástor and Stambaugh (2000), we specify the degrees of freedom \( \nu \) to be 15 so that the prior contains only as much information as 15 observations. The value of \( \beta \) is set equal to the OLS estimate of \( \beta \) using data for the period 1946 – 1953. Similarly, we set \( s^2 \) equal to the average of the sample estimate of the variance over the period 1946 – 1953. We further specify \( H = s^2(\nu - 2) \). The posterior distribution for the parameters of interest is then given by:

\[
\begin{align*}
(\epsilon^2 + Q)h \mid R & \sim \chi^2(T + \nu) \\
\beta \mid R, h^{-1} & \sim N(\bar{\beta}, h^{-1}H^{-1})
\end{align*}
\]

(8)

where

\(^{14}\) The multiple risky assets case is similar in spirit and we omit the details for the sake of brevity.
\[
\bar{H} = H + X'X; \quad \bar{\beta} = \bar{H}^{-1}(H\beta + X\beta'), \quad \text{and} \\
Q = (\bar{\beta} - \bar{\beta})'X'X(\bar{\beta} - \bar{\beta}) + (\bar{\beta} - \bar{\beta})'H(\bar{\beta} - \bar{\beta})
\] (9)

The term \(\bar{\beta}\) is the \(k \times 1\) vector of estimated OLS coefficients using the T-period sample of data available to the investor. The parameters of the posterior distribution given in equation (9) have an intuitive form. For example, note that the precision of the posterior distribution, \(\bar{H}\), is the sum of the prior precision and the term \(XX'\) which would be the precision if the prior precision were 0. In similar fashion the mean of the posterior distribution is a weighted average of the prior \(\bar{\beta}\) and the sample estimate, \(\hat{\beta}\). In other words, the sample estimate is “shrunk” towards the prior.

The predictive return distribution for \(\tilde{r}' = (r_{t+1}, \ldots, r_{T})\) at time \(T\) is in the form of a multivariate t-distribution i.e.,

\[
\tilde{r}'|R, X \sim t(\bar{\beta}X\beta', \sigma^2 + Q)(\bar{X}H^{-1}\bar{X}' + I_f); v + T
\] (10)

At the start of each month, the mutual fund investor uses the mean and the variance of the one-step ahead predictive distribution as his expectation for the expected excess returns, and the expected variance, of the risky assets. Using these inputs, the investor solves the problem in equation (4) and chooses the investment proportions in the risky assets and the risk-free asset, respectively.

**B.2. Predictive Distribution for the i.i.d. Investor**

The i.i.d. investor is also assumed to have a conjugate prior similar to the mutual fund investor. However, the matrix of explanatory variables \(X\) used by this investor consists solely of a vector of ones. The predictive return distribution is again of the Student \(t\)-form. The i.i.d. investor uses the moments of the predictive return distribution as inputs in solving for the optimal allocation to the equity portfolio, \(w\), in equation (4).

**C. Alternative Predictive Models**

As a robustness check we allow the mutual fund investor to employ two alternative forecasting models in forming his expectations. We discuss each of these cases below.
C.1. A VAR specification

Following Kandel and Stambaugh (1996), we allow the mutual fund investor to model the stock returns and the predictor variables as a first-order vector autoregression. That is, the mutual fund investor believes that

\[ Y = XB + U \]

where \( r_t \) is the first element of \( Y_t \) and \( X_t \) is a \( 1 \times k \) vector of predictor variables, and \( U \) is a vector of independent mean-zero error terms.

C.2. Expectations based on an asset pricing model

We extend our analysis to allow our investors to invest in ten risky assets along with the risk-free assets. Additionally, we allow the mutual fund investor to use the Capital Asset Pricing Model (CAPM) to form estimates of expected returns rather than relying on the predictor variables considered earlier. We adopt the framework suggested by Pástor and Stambaugh (2000) who study the optimal portfolio allocations resulting from prior beliefs in different asset pricing models as a way of comparing the models. The framework is described in more detail in Section VI. As a further extension of the standard CAPM based framework, we consider a setting where the alphas, betas and the market risk premium are allowed to potentially vary with the conditioning variables considered by us.

III. Research Design

A. Data

Data for the study from the period January 1954 to December 2002, are obtained from the CRSP database. As a proxy for the risk-free asset, we use the 1-month US T-bill yield from the CRSP files. For the case involving investment in a risk-free asset and one risky asset, we use the CRSP value-weighted index of Nyse/Amex/Nasdaq stocks as the risky asset. For the case involving a risk-free asset and three risky assets, we use three CRSP cap-based portfolios. We form a large cap portfolio by value-weighting the returns on the CRSP decile portfolios 1 through 3. Similarly, we form a medium cap portfolio by value-weighting the returns on the CRSP decile portfolios 4 through 7, and we form a small cap portfolio by value-weighting the returns on the CRSP decile portfolios 8 through 10. We construct monthly excess returns by
subtracting the one-month T-bill return from the monthly returns on the respective stock portfolios. In subsequent analysis (reported in Section VI) we utilize returns on ten industry-based portfolios provided by Ken French.

B. Predictor Variables

At the start of each month we allow the mutual fund investor to choose from a base set of four predictor variables. These include the dividend yield, the one month T-bill yield, a measure of the default spread and a measure of the term spread. Our primary motivation for focusing on these variables is that they are business-cycle related variables (see, for example, the discussion in Fama and French (1989)) and have therefore attracted a lot of attention in the literature. The dividend yield series is constructed as the natural log of the cumulated dividends on the CRSP value-weighted market index (covering NYSE, AMEX and NASDAQ stocks) over the previous 12 months divided by the current level of the index. The T-bill yield is the one-month T-bill yield minus its 12-month backward moving average. This stochastic detrending method for the short rate has been used by Campbell (1991) and Hodrick (1992), among others. The default spread is the yield difference between Moody’s BAA rated corporate bonds and AAA rated corporate bonds. The term spread is the difference between the 10-year and the 3-month treasury yields. Data for the default spread and the term spread are obtained from the Federal Reserve Bulletin while the relative T-bill yield is constructed from CRSP data. All conditioning variables are lagged by one month.

C. Methodology

The study is conducted on a monthly basis and spans the period January 1954 to December 2002. In our tests we contrast the performance of the mutual fund investor with that of the i.i.d. investor in two cases: (i) the single risky asset case when investors have the choice of investing in one risky asset and a risk-free asset, and (ii) the multiple risky asset case where investors have the choice of investing in three risky assets and a risk-free asset. Specifically, we

track the certainty equivalent rates of return (CER) and the adjusted Sharpe ratios for the two investors. Both investors make their portfolio choices at the start of a month and are assumed to incur a proportional transaction cost on the change in equity holdings equivalent to 50 basis points for a round trip trade. We conduct tests using three levels of investor risk-aversion, i.e., low risk-aversion (when the coefficient of relative risk aversion is set at 2), moderate risk-aversion (when the coefficient of relative risk aversion is set at 5), and high risk-aversion (when the coefficient of relative risk aversion is set at 10). Henceforth, we use the terms low, moderate and high to refer to risk aversion values of 2, 5 and 10, respectively.

As the mutual fund strategy and the i.i.d. strategy both require an initial “learning” phase, we reserve the period January 1954 to December 1958 (60 months) for this purpose. Hence, the actual estimation starts from January 1959 to December 2002 (528 months). As we move forward from January 1959 to December 2002, the investors “learn” from historical market performance. Hence, an important feature of the empirical methodology is that we allow the investors to use all historical data from an expanding window from January 1954 to the month preceding the month in which estimation is carried out. As a robustness check we also allow investors to consider non-expanding, moving estimation windows. We divide the performance evaluation period into three sub-periods, where the first sub-period extends from January 1959 to December 1973 (180 months), the second from January 1974 to December 1988 (180 months) and the last from January 1989 to December 2002 (168 months).

An important research design issue is the choice of the forecasting model employed by the mutual fund investor. Clearly, the investor is faced with the problem of model uncertainty in real time. While a full Bayesian treatment of model uncertainty is beyond the scope of this paper, we address this issue using the recursive modeling framework advocated by Pesaran and Timmermann (1995). Specifically, at the start of each month the mutual fund investor is allowed to search over the base set of the four predictor variables and use only historical data to choose an optimal forecasting model based on a predefined model selection criterion. The investor chooses one model out of the set of 16 (\(2^4\)) possible models, that includes the i.i.d. model, and uses it to predict the excess stock returns. While a number of statistical model selection criteria may be used to select the appropriate forecasting model, we present results
based on a recursive utility maximization criterion that is consistent with the investor’s objective (expressed in equation (4)) and that takes into account constraints such as transaction costs and short sales limits.\textsuperscript{17} To implement this model selection rule, the mutual fund investor tracks the realized performance to date of the portfolio allocations resulting from the use of each of the 16 candidate models, and chooses the model that provides the maximum realized utility.

\textit{D. Performance Measures}

The primary measure of performance that we use in our analysis is the certainty equivalent rate (CER) of return for a portfolio defined as:

\[ CER = \bar{r}_p - \frac{1}{2} \lambda \sigma^2_p \]  

(17)

where $\bar{r}_p$ denotes the mean realized return on a portfolio net of transaction costs. The CER of a risky portfolio represents the rate of return earned with certainty that would provide the investor with the same utility as the expected utility derived from the risky portfolio. The CER is a widely used measure of performance in the literature [see, for example, Kandel and Stambaugh (1996)]. We also report the adjusted Sharpe ratio measure proposed by Graham and Harvey (1997). The adjusted Sharpe ratio is computed as the difference between the mutual fund portfolio’s return and the return on a volatility-matched portfolio obtained by combining the \textit{i.i.d.} portfolio with T-bills, over the same evaluation period. Specifically, we first compute the standard deviation of the mutual fund portfolio over a particular evaluation period. We next lever or unlever the \textit{i.i.d.} portfolio by combining it with T-bills in order to match the standard deviation of the mutual fund portfolio. We report the difference in the two portfolio’s returns as the adjusted Sharpe ratio. The measure reported here is analogous to the GH1 measure of Graham and Harvey (1997). Intuitively, the difference represents the gain (or loss) in return from investing in the mutual fund portfolio relative to a passive strategy, for an investor with a target level of volatility equal to the mutual fund portfolio’s volatility. We note that in our context the CER is the most relevant performance measure as it is consistent with the investor’s utility maximization objective presented in Equation (4).

\textsuperscript{16} We thank the referee for his or her suggestions in this regard.

\textsuperscript{17} We confirm that our results are robust to alternative model selection criteria such as the adjusted $R^2$, among others.
We report the difference in the CER of the mutual fund and the i.i.d. portfolio that we refer to as the CER differential, along with the adjusted Sharpe ratio. We assess the significance of the CER differential and the adjusted Sharpe ratio using the stationary bootstrap technique proposed by Politis and Romano (1994). The technique is particularly suited to weakly dependent stationary time series. The stationary bootstrap involves re-sampling from the original time series of returns using a block-sampling scheme. Briefly, the steps involved in the bootstrap procedure are: (1) randomly select an observation, say, $R_t$, from the original time series, (2) with a fixed probability $q$, select the next observation randomly from the original time series, and with probability $(1-q)$ select it as the next observation to $R_t$ (i.e., select $R_{t+1}$) from the original time series, (3) repeat this process to generate a pseudo time series of desired length. This procedure ensures that the data are re-sampled in blocks where the block length has a geometric distribution with a mean of $1/q$. In our procedure we use a value of $q$ that corresponds to a mean block length of 60 months (5 years). We replicate the above process 1000 times. For each such replication, we compute the optimal allocations for each investor through 528 months. At every month, the investors are allowed to utilize just the information available up to that point in time. We calculate the difference in certainty equivalent between the two strategies and the adjusted Sharpe ratio for each replication.

We count the proportion of times in 1000 replications that these differences exceed the certainty equivalent and adjusted Sharpe ratio based on the original data for a given set of results. The significance of the original differences in these statistics is inferred based upon these empirical $p$-values. The bootstrap $p$-values provide us with a measure of the statistical significance of the relative performance of the two strategies in the context of repeated samples. We believe this is important to consider, both for a potential investor as well as for the academic community, since we have access to only a single sample path of the U.S. economy. As emphasized by Carlson, et al. (2001), it is hazardous to base inference on estimates from a single sample path. Of course, it is also important to pay attention to the economic magnitude of the performance differences when making

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18 Sullivan, Timmerman and White (1999) employ a similar bootstrap technique to assess the performance of technical trading strategies.

19 Our choice of $q$ is consistent with slow mean-reversion in returns over long horizons documented in Fama and French (1988a), for example. We tested for robustness by varying the value of $q$ and found that our results are not sensitive to our choice of $q$. 
an inference about the value of the predictor variables being examined. In our discussion of the results we pay attention to both of these aspects of performance evaluation.\(^{20}\)

We examine the relative performance of the *mutual fund* strategy for three sub-periods and in each sub-period we consider three levels of risk aversion. This gives us a total of nine sub-period/risk aversion permutations and correspondingly nine observations on the CER differentials. A positive CER differential indicates that the *mutual fund* strategy outperforms the *i.i.d.* strategy and vice versa. Instead of basing inference on bootstrap *p-values*, an alternate way to conduct inference about the performance of the *mutual fund* strategy is to ask the question whether positive CER differentials are equally likely as negative differentials. In order to assess this issue, we perform a non-parametric sign test with the null hypothesis that positive performance is equally likely as negative performance. The sign test is a robust, non-parametric test, well suited for this task. Rejection of the null hypothesis would indicate that the relative performance of the *mutual fund* strategy is superior.

Yet another way to judge the performance of the *mutual fund* strategy is to examine the quality of market timing it achieves. Superior market timing ability is a natural outcome of the *mutual fund* strategy if the forecasting model and the predictors it relies on have value, and is a necessary condition that it should satisfy. To assess market timing ability (implied by the predictor variables) we allow the *mutual fund* investor to follow a pure switching strategy.\(^{21}\) At the start of each month, he shifts his portfolio entirely into stocks or into the risk-free asset, based on his expected return on these assets for the next month. This is equivalent to assuming risk neutrality on the part of the investor. We restrict this analysis to the case where the *mutual fund* investor is allowed to invest in a single risky asset and a risk-free asset, since this setting allows for a more natural market timing interpretation. Perfect market timing would mean that the investor is fully invested in stocks in up-markets (when the value-weighted stock index outperforms the T-bills) and fully invested in T-bills in down-markets. Hence, perfect market timing would require an investor to correctly predict market direction (up versus down market) and then fully shift his portfolio from stocks to bonds or vice versa. We measure the percent of

\(^{20}\) We are grateful to the referee for pointing out the need for interpreting the *p*-values in the context of the economic significance of the observed performance differences.

\(^{21}\) Typically, the literature has tested market timing ability using such a switching strategy. See, for example, Pesaran and Timmerman (1995).
months the *mutual fund* investor correctly predicts an up-market and similarly, the percent of times he correctly predicts a down-market. The weighted average gives his overall market timing ability.

**IV. Preliminary Analysis**

**A. Descriptive Statistics**

Table 1 presents descriptive statistics for monthly stock portfolio excess returns, 1-month T-bill yield, dividend yield on a value-weighted market portfolio, 1-month detrended T-bill yield, default spread and the term spread for the period 1954-2002. The mean value-weighted portfolio excess return was 0.55% per month with a standard deviation of 4.38%. The small cap, medium cap and the large cap portfolio excess returns averaged 0.77%, 0.70% and 0.53%, respectively. The annualized log dividend yield on the value-weighted market portfolio averaged -3.4913 with a standard deviation of 0.4019. The summary statistics reported in the table also include the first order autocorrelations for each series. As expected, the dividend yield series is quite persistent with an autocorrelation coefficient of 0.9893.

Let us consider the overall sample period 1959-2002 used in the study as well as the three sub-periods we examined i.e., 1959-73, 1974-88 and 1988-2002. The first sub-period from 1959 to 1973 was characterized by three economic expansions and two economic contractions, according to the *National Bureau of Economic Research (NBER)*. The average monthly equity risk premium over this sub-period was 0.30%. The second sub-period from 1974 to 1988 was more volatile, consisting of three expansions and three contractions. It included two severe downturns from December 1973/January 1974 to March 1975 (16 months), and August 1981 to November 1982 (16 months). The average monthly equity risk premium over the sub-period was 0.45%. The final sub-period from 1989 to 2002 consisted of a long expansion broken only by two eight-month downturns from August 1990 to March 1991 and from April 2001 to November 2001. It included the longest post-war expansion in our sample, spanning April 1991 to March 2001 (120 months). The average monthly equity risk premium over the period was 0.56%. In summary, the first sub-period (1959-73) was a period of low to moderate expansion coupled with low volatility, the second period (1974-88) was a period of high volatility and moderate
economic activity, and the final period (1989-2002) was characterized by strong expansion and high equity premium.

**B. In-sample Regression Evidence**

Our framework allows for four predictor variables: dividend yield, T-Bill yield, default spread and term spread. Our mutual fund investor chooses the model for predicting the next period returns from the 16 \(2^4\) candidate models given by these four predictor variables. These four business cycle related predictor variables have been proposed and tested in the literature by various researchers who have found them to be of high predictive value for stock returns. In order to allow for comparisons to previous results in the literature and for the sake of completeness, we present evidence on predictive regressions using these variables. We estimate each of the 16 regression models, labeled A through P, over the sample period 1959-2002 for the value-weighted market portfolio of all NYSE, AMEX and NASDAQ stocks. These results appear in Table 2. Note that model P is simply the \textit{i.i.d.} model. The table presents the estimated coefficients, the corresponding t-statistics based on the Newey-West covariance matrix, and the adjusted \(R^2\) values for each regression.\(^{22}\)

Dividend yield appears in 8 of the 16 regressions individually and in combination. Its coefficient is significant at the 5% level in 6 of the 8 regressions and at the 10% level for the univariate regression with an \(R^2\) of 0.58%.

The coefficient for T-bill yield is negative and significant at the 1% level for all 8 regressions in which it appears. The coefficient is -0.689 with a t-statistic of -4.30 in the univariate regression with an \(R^2\) of 2.99%. In the multivariate regression with all variables (Model O), the coefficient is -0.706 with a t-statistic of -3.11.

Interestingly, default spread is insignificant over the full sample period in all the 8 regressions it appears in, including the univariate regression. Of course, there is every reason to believe default spread is significant in sub-samples across time and across specific portfolios.

The term spread is significant in 4 of the 8 regressions. It is significant in the univariate regression with a coefficient of 0.351 and a t-statistic of 2.48. This is not true in the multivariate

\(^{22}\) Biases in predictive regressions involving stock returns have been analyzed by Stambaugh (1999), and Ferson, Sarkission, and Simin (2002), among others.
regression with all variables (Model O) where it is insignificant with a coefficient of 0.096 and a t-statistic of 0.52.

In the univariate regressions, three variables out of four are significant at conventional levels, the only exception being default spread. In the multivariate regression with all variables, i.e., Model O, only dividend yield and T-Bill yield are significant.

V. Out-of-sample results

In this section we present our main results relating to the out-of-sample performance of the mutual fund and the i.i.d. strategies.

A. Portfolio Performance

Panel A of Table 3 presents the performance of the i.i.d. and mutual fund strategies when both strategies adjust allocations every month between the riskfree asset and an equity portfolio. Performance results are presented for the three sub-periods January 1959 – December 73, January 1974 – December 88, January 1989 – December 2002, and for the overall sample period January 1959 – December 2002, for three coefficients of relative risk aversion (2, 5 and 10). We report the differences in the average return and certainty equivalent rate of return between the mutual fund strategy and the i.i.d. strategy. We also report the adjusted Sharpe ratio for the mutual fund strategy. For both the certainty equivalent rate of return and the adjusted Sharpe ratio, we report the bootstrapped p-values.

It is clear from the results in Panel A that the relative performance of the mutual fund strategy is unstable across sub-periods. The results are generally positive for the 1959-73 and 1974-88 sub-periods, and generally negative for the last sub-period, 1989-2002. Also, the results in the third sub-period suggest that an aggressive mutual fund strategy is more likely to fail, compared to a conservative strategy.

For the three sub-periods and for the three risk-aversion cases (i.e., for the nine possible sub-period/risk aversion permutations), judging by the bootstrap p-values, the differences in performance of the two strategies are not significant at the conventional levels in each case. Notwithstanding the p-values, the mutual fund strategy tends to outperform the i.i.d. strategy in the first two sub-periods and conversely, the i.i.d. strategy generally outperforms the mutual fund
strategy in the last sub-period. For the full sample period 1959-2002, the mutual fund strategy does better than the i.i.d. strategy for the low and high risk-aversion cases.

The CER differences and the adjusted Sharpe ratios are positive in six of the nine cases. We subject this hypothesis to a non-parametric sign test. The test is that the finding of six out of nine positive performances is consistent with the null hypothesis that positive performance is equally likely as negative performance. The sign test yields a p-value of 0.2539, failing to reject the null hypothesis at conventional levels of significance.

The bootstrap p-values provide us with a measure of the statistical significance of the difference in performance between the mutual fund and the i.i.d. strategies in the context of repeated samples. However, as we stated earlier, we are also interested in the economic significance of the performance difference. In the low risk aversion case, for the full sample period of 44 years, the monthly difference in CER between the mutual fund and the i.i.d. strategy is 37 bps per month that is clearly significant in economic terms. However, consider the high risk aversion case where the monthly CER differential of 4 basis points has to be viewed in context of the corresponding bootstrap p-value suggesting that a similar improvement in performance could have been obtained with a 74.4% probability even if the data had no predictability but had a similar correlation structure as the observed returns in the economy. It would be safe to say that the mutual fund investor would balk at giving up much utility to get access to the predictor variables if there was only a 25.6% level of confidence in outperforming the i.i.d. investor.

Panel B reports results for the i.i.d. and mutual fund strategies when the strategies adjust allocations every month between the riskfree asset and three equity portfolios. The results are qualitatively similar to the single risky asset case. There continues to be instability across the three sub-periods. In the first sub-period the mutual fund strategy tends to outperform the i.i.d. strategy while in the third sub-period it tends to underperform. For the full sample period, as before, in the low risk aversion case the mutual fund strategy enjoys an monthly CER performance advantage of 38 basis points while in the other two risk aversion cases the difference in performance is negligible.

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23 The implicit assumption that we make is that there are nine independent draws of which six are positive.
Overall, the results suggest that the relative performance of the *mutual fund* strategy is unstable across sub-periods. In the first sub-period it clearly outperforms in economic terms while in the second sub-period it outperforms only in the low risk-aversion case only. In the third sub-period the *mutual fund* strategy is generally outperformed by the *i.i.d.* strategy. Clearly, there is little evidence of steady improvement in performance over time which is indicative of the unstable nature of stock return predictability. We explore this issue further in sub-section E below, by examining the choices made by the *mutual fund* investor and how they evolve over time in this environment.

**B. Portfolio Allocation**

Figure 1 shows the equity allocations for the two strategies in the single risky asset case when investors have moderate risk aversion. The solid vertical lines on the graphs depict the three sub-periods analyzed by us. It is evident from the graph that the *mutual fund* strategy is characterized by extremely volatile portfolio allocations. Recall that we have allowed for a margin requirement of 50% that translates to an equity weight between 200% and – 200%. The *mutual fund* strategy shows particularly wide swings in the period 1974-1988, moving between extremes of shorting equity between –160% to –200% and taking long positions in equity ranging from 160% to 200%. In contrast, the *i.i.d.* strategy is much more consistent. It adheres to an equity exposure ranging between 120% and 200% in the first sub-period, and thereafter stabilizes to a more moderate equity exposure of about 80% for most of the second and third sub-periods.

**C. Market Timing Performance (Single Risky Asset)**

Table 4 presents results on the market timing ability of the *mutual fund* strategy in the case with a single risky asset that lends itself to a more natural market timing interpretation. As we previously noted, the first sub-period (1959-73) is characterized by low to moderate economic growth and market performance. In total, the market risk premium was positive in 58.89% of the months representing up-markets. The *mutual-fund* investor is able to correctly predict up-markets 47.17% of the times and is able to predict down-markets 63.51% of the times,

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24 By definition, the investor following the *i.i.d.* strategy does not time the market and hence, we do not measure his market timing ability.
yielding a weighted average prediction accuracy rate of 53.89% over the sub-period. It is interesting to compare this performance to that of a naïve investor who predicts an up-market or a down-market based on a random coin toss. Such a naïve investor would have an overall predictive accuracy of 50% suggesting that our mutual fund investor does only somewhat better than a random coin toss.

The second sub-period (1974-88) coincides with low economic growth and poor stock market performance and in total only 52.22% of the months correspond to up-markets. The mutual-fund investor is able to correctly predict up-markets 74.47% of the times but is much less capable of predicting down-markets (correctly predicting it only 40.70% of the times), averaging 58.33%.

Finally, the last sub-period represents a period of high growth for the stock market and the economy. In total, 61.31% of the months represent up-markets. The mutual-fund investor is able to correctly predict up-markets 36.89% of the times but is able to predict down-markets 46.15% of the times, averaging a meager 40.48%, i.e., worse than a random coin toss. Overall, for the entire period 1959-2002, the mutual fund investor predicts up-markets 52.15% of the times and down markets 49.78% of the times, for an average prediction rate of 51.14%.

More in-depth analysis, not reported in Table 4, indicates that the realized market risk-premium is negative in 225 months out of the 528 months examined. Examination of the mutual fund strategy reveals that it is fully invested in T-bills in 57 of these 225 months (being incorrectly fully invested in T-bills in 63 of the remaining 303 months).

D. Sensitivity Analysis

We performed a set of tests to confirm the robustness of our basic findings. One such test involved the use of an alternative VAR framework, described in Section II.C.1, by the mutual fund investor. Our results were qualitatively similar to the base case discussed above. For example, in the single risky asset case with moderate risk aversion, the CER differentials are 34 bps, 27 bps and -26 bps in the three sub-periods. For the entire sample period the CER differentials are 11 bps, 12 bps, and 4 bps for the low, moderate and high risk aversion cases.

25 Detailed tables on the robustness tests are not included in the paper for the sake of brevity, but are available upon request.
Recall that our base set of results allow investors to engage in limited short selling consistent with a 50% margin requirement. It is interesting to examine whether our results are significantly altered if short sales are disallowed. Short sales restrictions are realistic as in practice many investors face institutional or regulatory barriers to taking short positions. While restricting short sales mitigates some of the extreme performance differences observed earlier, we find that our results are qualitatively unchanged when short sales are ruled out. The mutual fund investor continues to perform better than the i.i.d. investor in the first two sub-periods and underperforms in the third sub-period. For example, in the single risky asset case with moderate risk aversion, the CER differentials are 30 bps, 43 bps and -25 bps in the three sub-periods. Overall, for the entire sample period the CER differentials are 4 bps, 17 bps, and 9 bps for the low, moderate and high risk aversion cases.

We also confirm that our results are qualitatively unchanged when transaction costs are eliminated. For example, in the single risky asset case with zero transaction costs, the CER differentials are 45 bps, -6 bps, and 7 bps for the low, moderate and high risk aversion cases. These figures are qualitatively similar to those reported in our base case (Table 3). A final sensitivity check conducted by us concerns the choice of the sample period used by the investors when forming prior beliefs and the prior degrees of freedom, $n$. Recall that in our base set of results the investors base their priors on data from the period 1946-1953 and that the value of $n$ is specified to be 15. We ran our tests under the alternative assumption that the priors are based on data from the period 1926-1953 and that the degrees of freedom are based on the conventional length of the sample period. While as expected, the model choices of the mutual fund investor and the resulting asset allocations are altered under this scenario, we find that the qualitative nature of our results continues to hold. The performance of the mutual fund strategy is unstable across time and it continues to be poor during the most recent sub-period, 1989-2002.

E. Mutual Fund Investor’s Model Choice

The results presented here raise some natural questions. First, recall that the mutual fund investor has the choice of using any one of the sixteen predictive models that includes the i.i.d. model. Given that the i.i.d. model performs better in the third sub-period, one might ask why the mutual fund investor fails to use it. In order to explore this issue, we examine the model choices made by the mutual fund investor across time. Recall, that each month the mutual fund investor
chooses an optimal forecasting model from the full set of 16 potential models. We refer to the 16 models by their labels A through P. The models are fully described in Table 2.

Table 5 presents the results for the single risky asset case with moderate risk aversion. The table analyzes the model usage pattern of the mutual fund investor for the three sub-periods as well as the entire sample period, 1959-2002. Also shown is the best performing model in each period, i.e., the model which when used consistently (i.e., continuously) over the period leads to the highest realized CER in that period. Panel A of the table presents results for the base case where the mutual fund investor selects a forecasting model using an expanding window. As is evident from the identity of the models that performed best ex-post, there is a clear lack of consistency in the predictive ability of the models over time. For example, while Model B (that includes the T-bill yield as the sole predictor) was the best performer over the full sample period, it was outperformed by models K, L, and D during the first three sub-periods. To see why the mutual fund investor may underperform the i.i.d. investor in a given sub-period, we can focus on the reasons for his choices during the third sub-period (1989-2002).

When models are evaluated based upon the realized utility to-date, Model B starts to dominate partway through the third sub-period and then retains its ranking to the end. This results in the mutual fund investor relying on model B most frequently during the third sub-period. In turn, this causes him to miss out on the fact that Model D is the best performing model in the third sub-period environment. As it turns out, the i.i.d. model (Model P) performs second-best while Model B, the model most frequently chosen by the mutual fund investor in this sub-period, is the third-best performer during the third sub-period. It is clear that if the underlying economic environment and hence, the rankings of the models are stable across time, then the mutual fund investor’s performance may be positive. It is reasonable to expect that the underlying changes in the economic environment and hence, the instability in the rankings of the models, are likely to persist in the future. The model choices of the mutual fund investor in turn will reflect these changes making the task of uncovering the “true” model for a particular sub-period, a challenging one. Therefore, it is not surprising that in this study when the mutual fund

\[26\] Although not shown, the monthly CER figures corresponding to the consistent (i.e., continuous) use of Model D, Model P (the i.i.d. model), and Model B during the third sub-period are 62.45 bps, 52.21 bps, and 3.75 bps, respectively.
investor selects a model using an expanding window, he has difficulty in uncovering the appropriate model in the changed environment of the third sub-period.

One question that arises is that if the mutual fund investor had a shorter horizon for model selection, would his performance improve? We present evidence on this issue in Panel B of Table 5. We consider three model selection windows of length 60 months, 36 months, and 12 months. The performance of the mutual fund investor during the third sub-period indeed improves as the model selection window is shortened. The monthly CER differential improves from –32 basis points for the 60 month window to 5 basis points for the 12 month window. In contrast, the performance of the mutual fund investor during the second sub-period declines monotonically as the model selection window is shortened. The monthly CER differential goes from zero for the 60 month window to –57 basis points for the 12 month window. Evidently, the optimal length of the model selection window is not easy to identify, ex-ante.

To summarize, the following conclusions emerge:

- The performance of the mutual fund strategy is unstable across time for both the single risky asset case and the multiple risky assets case.

- The performance of the mutual fund strategy is strong in the first sub-period (1959–1973) for both the single and multiple asset cases. The economy in this sub-period experienced low to moderate expansion and low volatility,

- In the second sub-period (1974-1988), which is characterized by volatile economic performance, the mutual fund strategy shows strong performance for the low risk aversion case only. For the moderate and high risk-aversion cases, the two strategies’ performance is not appreciably different.

- The performance of the mutual fund strategy is noticeably poor during the third sub-period (1989-2002).

- Judging by the mutual fund investor’s infrequent use of the i.i.d. model, there is support for predictability in the data. However, the shifts in the economic environment make it difficult to uncover the appropriate model for a particular time period.

- The mutual fund strategy demonstrates more extreme portfolio swings as compared to the i.i.d. strategy that has more stable portfolio allocations.
• Overall, the timing ability of the mutual fund strategy is only marginally better than a simple coin toss.

VI. A Model-Based Approach to Asset Allocation

A. CAPM Framework with Varying Confidence in the Model

Pástor and Stambaugh (2000) and Pástor (2000) advocate a model-based approach to study the optimal asset allocations of an investor whose beliefs about expected asset returns are centered on an asset pricing model and who has varying degrees of prior confidence in the model’s pricing ability. Stated differently, under this framework the investor is assumed to have an informative prior on the degree of assets’ mispricing (\( \alpha \)) under the model. Prior confidence in the model i.e., confidence that \( \alpha \) is indeed zero, as implied by the model, is represented by \( \sigma_\alpha \), the prior (annualized) standard deviation of \( \alpha \). The observed sample mispricing \( \hat{\alpha} \) is shrunk towards the prior mean of \( \alpha \) to yield the posterior mean of \( \alpha \) which is utilized in making the asset allocation decision.

In this section we adopt the above framework and allow the mutual fund investor to rely on the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) to determine expected returns. Furthermore, we allow both investors access to an expanded set of risky assets that consists of ten industry-based portfolios. Appendix A provides a brief description of the Pastor and Stambaugh methodology utilized by us.

Table 6 presents results for three cases where the mutual fund investor’s prior beliefs about the standard deviation of the CAPM pricing error, \( \sigma_\alpha \), are represented by: \( \sigma_\alpha = 0\% \) (Panel A), \( \sigma_\alpha = 2\% \) (Panel B), and \( \sigma_\alpha = \infty \) (Panel C). In practice the results for the case \( \sigma_\alpha = 0\% \) are obtained by specifying a very low value for \( \sigma_\alpha \) and similarly, results for \( \sigma_\alpha = \infty \) are obtained

\[ \text{Equation} \]

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\[ \text{Reference} \]
by specifying a large value for $\sigma_a$. Results in Panel A correspond to the case where the investor’s prior beliefs reflect full confidence in the CAPM. Under this scenario, the mutual fund strategy’s performance during the first sub-period 1959-73 as measured by CER differentials, is quite impressive. It outperforms the i.i.d. strategy across the board and the difference is statistically and economically significant for the moderate and high risk aversion cases where it outperforms by 89 basis points per month and 125 basis points per month, respectively. In contrast, the mutual fund strategy’s performance is decidedly poor during the second sub-period, 1974-88, particularly for the low risk aversion case where it underperforms the i.i.d. strategy by 56 basis points per month. Finally, during the last sub-period, 1989-2002, the mutual fund strategy outperforms the i.i.d. strategy with particularly strong performance for the moderate and high risk aversion cases, even though the differences are not statistically significant. Overall, for the full sample period, the mutual fund strategy underperforms the i.i.d. strategy by 10 basis points (p-value 0.777) in the low risk aversion case, but significantly outperforms by 45 basis points (p-value 0.016) in the moderate risk aversion case, and by 51 basis points (p-value 0.034) in the high risk aversion case. In contrast to the CER differentials, none of the adjusted Sharpe ratios are significant whether one looks at the individual sub-periods or the overall sample period.

The results in Panel B correspond to the case where the mutual fund investor’s prior beliefs realistically reflect less than perfect confidence in the CAPM, and admit a moderate degree of pricing error. Once again, the mutual fund strategy significantly outperforms the i.i.d. strategy in terms of the CER differentials during the first sub-period, for the moderate and high risk aversion cases. However, the strategy’s performance is mixed in the second sub-period while it weakly dominates the i.i.d. strategy during the third sub-period. Overall, for the full sample period, the mutual fund strategy outperforms the i.i.d. strategy by 4 basis points (p-value 0.645) in the low risk aversion case, 23 basis points (p-value 0.136) in the moderate risk aversion case, and 41 basis points (p-value 0.130) in the high risk aversion case. Based on the magnitude of the CER differentials the improvement in performance in the moderate and high risk aversion cases is economically significant. Once again the adjusted Sharpe ratios are insignificant across the board.

Panel C presents results for the case when the mutual fund investor’s prior beliefs reflect extreme skepticism about the CAPM’s ability to explain asset returns. Despite the initial
skepticism, reliance on the CAPM leads the *mutual fund* strategy to outperform the *i.i.d. strategy* during the first sub-period for the moderate and high risk aversion cases. For instance, in the case with high risk aversion, the CER differential equals 89 basis points per month which is economically meaningful and also statistically significant at the 10% level. The performance of the *mutual fund* strategy is mixed in the second and third sub-periods during which it underperforms in the cases with low risk aversion and outperforms in the moderate and high risk aversion cases. Overall, for the full sample period, the *mutual fund* strategy underperforms the *i.i.d.* strategy by 11 basis points (p-value 0.269) in the low risk aversion case, and outperforms by 18 basis points (p-value 0.202) in the moderate risk aversion case, and by 36 basis points (p-value 0.152) in the high risk aversion case. The adjusted Sharpe ratios are either zero or negative in six of the nine cases during the three sub-periods as well as in each case for the overall sample period.

Further insight into the relative performance of the *mutual fund* strategy can be gained by examining its portfolio allocations. Figure 2 presents the equity allocation weights for the moderately risk averse *i.i.d.* and *mutual fund* strategies in the case where the *mutual fund* strategy’s beliefs admit a moderate degree of pricing error ($\sigma_a = 2\%$). While the *mutual fund* strategy generally has a more conservative allocation to equities, it is interesting to note that the differences in allocations of the two strategies appear to be considerably smaller than in the base case involving a single risky asset depicted in Figure 1. Furthermore, the volatility in the equity allocation weights of the *mutual fund* strategy is roughly comparable to that of its *i.i.d.* counterpart.

Two facts emerge from the analysis in this section. First, the use of a model-based approach translates into superior performance for the *mutual fund* investor in cases with moderate and high risk aversion. In contrast to the earlier results involving the use of business cycle related predictor variables, the performance advantage is significant in both economic and statistical terms. Second, the *mutual fund* investor’s performance advantage is the strongest when the investor has the highest confidence in the model and it gets moderated with an increase in model mispricing uncertainty.
B. CAPM Framework with Time-Varying Alphas and Betas

We next allow the mutual fund investor to adopt a CAPM-based framework within which the alphas and betas are allowed to vary over time with the four predictor variables considered by us. Specifically, following Shanken (1990) and Ferson and Harvey (1999) we model the alphas and betas for the ten industry-based portfolios as linear functions of the four predictor variables. In a similar vein, we also model the market risk premium as a linear function of the four predictor variables. Such a framework is designed to capture the potential time series variation, related to macroeconomic variables, in model mispricing (i.e., alphas), and the risk premia implied by the model. The variation in risk premia is captured by the time series variation in betas and the market risk premium.\(^3\)

Under this setting, the beliefs of the mutual fund investor regarding the process governing asset returns may be represented as

\[ r_{t+1} = \alpha_t + \beta_t r_{m,t+1} + \nu_{t+1} \] (18)

\[ \alpha_t = \alpha_0 + \alpha_1 z_t \] (19)

\[ \beta_t = \beta_0 + \beta_1 z_t \] (20)

\[ r_{m,t+1} = a_m + A_m z_t + \nu_{m,t+1} \] (21)

\[ z_{t+1} = a_z + A_z z_t + \nu_{z,t+1} \] (22)

where, in this context, \( r_{t+1} \) is a \( N \)-vector of excess returns on the available risky assets, \( r_{m,t+1} \) represents the excess returns on the market portfolio, and \( z_t \) is a \( L \)-dimensional vector of information variables known at time \( t \). The alpha parameters, \( \alpha_0 \) and \( \alpha_1 \), representing fixed and potentially time-varying model mispricing, are of dimensions \( N \times 1 \) and \( N \times L \), respectively. Similarly, \( \beta_0 \) and \( \beta_1 \) represent fixed and potentially time-varying market betas, and have dimensions \( N \times 1 \) and \( N \times L \) respectively. In the above framework, the predictive variables are themselves modeled as a vector AR(1). Such a specification has been adopted by Kandel and

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\(^3\) Recent studies that allow for time variation in model mispricing and/or risk parameters in a Bayesian context include Shanken and Tamayo (2001), Avramov (2004), and Chordia and Avramov (2004).
Stambaugh (1996), Barberis (2000), Avramov (2004), and Avramov and Chordia (2004), among others. The residual terms $v_{t+1}$, $v_{m,t+1}$, and $v_{z,t+1}$ are assumed to have i.i.d. normal distributions. Note that the above framework yields the following expression for the asset excess returns:

$$r_{t+1} = \alpha_0 + \alpha_1 z_t + \beta_0 r_m, t+1 + \beta_1 (r_{m,t+1} \otimes z_t) + v_{t+1}$$  \hspace{1cm} (23)$$

The **mutual fund** investor is assumed to have non-informative prior beliefs about the parameters characterizing the asset returns. Appendix B outlines the moments of the resulting predictive return distribution used by the **mutual fund** investor to make his asset allocation decision.

Table 7 presents results for two polar cases under the above framework. Panel A of the table presents results for the case when the alphas are allowed to be unrestricted signifying a lack of confidence in the model, while Panel B presents results for the case when the alphas are restricted to be zero as implied by a conditional version of the CAPM. Consistent with the results under the standard CAPM framework presented in Section VI.A., the performance of the **mutual fund** strategy is poor in the second sub-period while it is mixed in the first and the third sub-periods. When alphas are unrestricted (Panel A), the performance of the **mutual fund** strategy is most favorable in the low risk aversion case. Interestingly, imposing the restriction that the alphas equal zero, appears to worsen results for the low risk aversion case. Also, under this restriction, the adjusted Sharpe ratio differentials are negative in eight of the nine sub-period/risk aversion cases and zero in the remaining case. Similarly, the CER differentials are positive and economically significant during the first sub-period but consistently negative during the second and the third sub-periods.

Figure 3 presents the equity allocations for the **mutual fund** and the i.i.d. strategies for the moderate risk aversion case when the alphas are unrestricted. Interestingly, the **mutual fund** strategy maintains a much more aggressive equity allocation during the second sub-period compared to the i.i.d. strategy. It maintains an equity exposure ranging between 120% and 200% during this sub-period. The large equity exposure of the **mutual fund** strategy during the second sub-period which was marked by relatively low and volatile stock returns helps explain the strategy’s poor performance during this sub-period.

As a robustness check we also examine the case where, at each month, the **mutual fund** investor optimally chooses which of the four predictor variables to include in the vector $z_t$ in order to estimate the alphas, betas, and the market risk premium. The optimal choice of predictor
variables at each month is based on the recursive utility maximization criterion described earlier in Section III.C. We further examine the robustness of the results to different estimation windows, i.e., expanding versus non-expanding windows of fixed length. The results of these robustness tests are qualitatively similar to those reported in Table 7. For example, when the choice of predictor variables is based on an expanding window, the relative performance of the mutual fund strategy continues to be inconsistent across the sub-periods. It is generally favorable in the first sub-period, mixed in the second sub-period, and poor in the third sub-period. The results for non-expanding estimation windows are similar in spirit.

In summary, it is evident from the above results that allowing for time variation in the CAPM alphas and betas does not lead to consistently superior performance for the mutual fund strategy.

VII. Conclusion

It is worth asking whether belief in predictability of returns translates into economically significant gains that are sustainable. This paper provides evidence on this issue in the context of an asset allocation framework with Bayesian investors. We examine the performance of two strategies, in real time, as a way to answer this question. The first strategy (the mutual fund strategy) makes portfolio allocations based on the belief that stock returns are predictable while the second strategy (the i.i.d. strategy) bases allocations on the belief that stock returns are i.i.d. Our framework allows for both a data-based approach to portfolio allocation and an alternative asset pricing model-based approach.

The data-based approach relies on the use of sample data on stock returns and predictor variables to form expectations of next month’s stock returns. The mutual fund strategy uses predictor variables such as lagged values of the dividend yield, T-bill yield, default spread and term spread to predict returns one month ahead. An important feature of our research design is that at the start of each month the mutual fund strategy is allowed to select an optimal forecasting model based on a recursive utility maximization criterion. The mutual fund and the i.i.d. strategies are allowed to form portfolios by investing in a risk-free asset and (a) a single stock portfolio, or (b) three stock portfolios.

We also employ a model-based approach where the mutual fund strategy is allowed to predict next month stock returns based on varying degrees of prior confidence in the Capital
Asset Pricing Model. Under this framework, we allow both strategies to allocate across ten industry-based portfolios in addition to the risk-free asset. As an extension of this framework, we allow the *mutual fund* strategy to incorporate time-varying alphas, betas, and the market risk premium.

We compare the portfolio performance of the two strategies using measures that include the certainty equivalent rate of return, the adjusted Sharpe ratio of Graham and Harvey (1997), a non-parametric sign test and market timing performance. Our test design allows for out-of-sample performance evaluation, transaction costs, short sales and margin purchases, parameter uncertainty, and partly addresses model uncertainty. Our major findings are summarized below.

When relying on the data-based approach to asset allocation, the gains to the *mutual fund* strategy are impressive in economic terms, during some sub-periods. However, its performance is unstable across time. The *mutual fund* strategy’s performance is generally good during the sub-periods 1959-1973 and 1974-1988 but is noticeably poor during the most recent sub-period, 1989-2002. Furthermore, the portfolio allocations of the *mutual fund* strategy tend to be quite volatile. We also find that the strategy is able to do only marginally better than a random coin toss in terms of its market timing ability. When judged in terms of statistical significance, for the full sample period, the performance of the *mutual fund* strategy is not significantly different than that of the *i.i.d.* strategy.

Interestingly, the results from the model-based approach utilizing the standard CAPM framework are more favorable for the *mutual fund* strategy. We find that under this approach, the *mutual fund* strategy significantly outperforms the *i.i.d.* strategy for moderate and high risk aversion cases. The performance advantage is significant in both statistical and economic terms, especially when the investor has a high degree of confidence in the model. Allowing for time varying parameters does not lead to significant improvement relative to the standard CAPM framework.

Our results suggest that variables such as the T-bill yield, dividend yield, default spread, and term spread are valuable for asset allocation decisions. This is supported by the fact that the *mutual fund* investor continues to rely on these variables instead of opting for the *i.i.d.* model in most cases. Furthermore, as noted above, there are significant economic gains to using these models in some sub-periods. However, we also find that these gains are not sustainable over time. To gain further insight into the performance of the *mutual fund* strategy, we analyze the
model choices made by the strategy over time. We find that shifts in the economic environment often make it difficult to identify the model that works best during a particular sub-period. In this respect our results serve to underscore the fact that translating the predictability that exists in the data into consistently superior performance is a challenging task and therefore investment strategies that rely on data-based predictor variables should be tempered with caution. Our analysis also helps explain the gap between the findings of stock return predictability and the lack of superior performance of the average professional money manager, noted in the literature. Interestingly, our results also serve to underscore the value of a model-based approach motivated by theoretical considerations, for asset allocation choices.
REFERENCES


Appendix A

For the sake of completeness we present here, in brief, Pástor and Stambaugh’s (2000) derivation of the first two moments of the predictive distribution for asset returns based on the posterior moments of an asset pricing model’s parameters. We retain their notation throughout to the extent possible.

Let \( r_t = (r_{1,t}^r, r_{2,t}^r)' \) represent a matrix of \( n \) excess asset returns where \( r_{2,t}^r \) contains the returns on \( k \) benchmarks or factors while \( r_{1,t}^r \) contains the returns on the \( m \) (\( = n - k \)) non-benchmark assets. Let the mean vector the variance-covariance matrix of \( r_t \) be partitioned as:

\[
E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \quad V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}
\]  \hspace{1cm} (A 1)

Note that in the context of the CAPM there is a single market factor and accordingly \( r_{2,t}^r \) contains the excess return on the market portfolio. Consider the multivariate regression:

\[
r_{1,t} = \alpha + B r_{2,t}^r + u_t
\]  \hspace{1cm} (A 2)

where \( u_t \) obeys a multivariate normal distribution with zero mean and variance-covariance matrix equal to \( \Sigma \). Note that the vector of betas for the non-benchmark assets, \( B \), is defined by

\[
B = V_{12} V_{22}^{-1}
\]  \hspace{1cm} (A 3)

Similarly, the other parameters of interest for the regression in equation (A 1), can be expressed as: \( \alpha = E_1 - B E_2 \), and \( \Sigma = V_{11} - B V_{22} B' \).

The CAPM imposes the restriction, \( \alpha = 0 \). On the other hand the model does not impose any restrictions on \( B, \Sigma, E_2, \) and \( V_2 \), so the prior distributions for these parameters are noninformative. The prior distribution for \( \Sigma \) is specified as inverse Wishart,

\[
\Sigma^{-1} \sim W(H^{-1}, v)
\]  \hspace{1cm} (A 4)

Following Pástor and Stambaugh (2000) we specify the degrees of freedom \( v = 15 \), and set the value of \( s^2 \) to be equal to the average of the diagonal elements of the sample estimate of \( \Sigma \).

The mutual fund investor’s prior confidence in the CAPM’s pricing ability can be represented by the prior distribution of \( \alpha \) which is expressed as a normal distribution,
\[
\alpha | \Sigma \sim N \left( 0, \sigma^2_a \left( \frac{1}{s^2} \Sigma \right) \right) \tag{A 5}
\]

Uncertainty about the model’s pricing ability is captured by the unconditional prior variance of \( \alpha , \sigma^2_a \). Complete confidence in the model on the part of the investor is captured by specifying \( \sigma^2_a \) to be equal to zero. Similarly, \( \sigma^2_a = \infty \) corresponds to the situation where the mutual fund investor is skeptical about the CAPM’s validity, and essentially regards it as useless.

Let \( Y = (r_{t,1}, \ldots, r_{t,T})' \), \( X = (r_{m,1}, \ldots, r_{m,T})' \), and \( Z = (t, X) \), where \( t \) denotes a \( T \)-dimensional vector of ones. Also let the \((2 \times m)\) matrix \( A = (\alpha \quad B)' \) and let \( a = \text{vec}(A) \). Then the regression model may be written as

\[
Y = ZA + U, \text{ vec}(U) \sim N(0, \Sigma \otimes I_T)
\]

where \( U = (u_1, \ldots, u_T)' \). Consider the statistics

\[
\hat{A} = (Z'Z)^{-1}ZY, \quad \hat{a} = \text{vec}(\hat{A}), \quad \hat{\Sigma} = (Y - Z\hat{A})'(Y - Z\hat{A})/T, \quad \hat{E}_2 = X't/T, \quad \text{and}
\]

\[
\hat{V}_{22} = (X - t\hat{E}_2)'(X - t\hat{E}_2)/T
\]

Pástor and Stambaugh (2000) show that the posterior moments of the parameters of interest are given by:

\[
\tilde{a} = E(a|R) = \left( I_m \otimes F^{-1}Z'Z \right) \hat{a}
\]

\[
\tilde{\Sigma} = E(\Sigma|R) = \frac{1}{T + \nu - m - k - 1} \left( H + T\hat{\Sigma} + \hat{A}'Q\hat{A} \right) \tag{A 6}
\]

where \( Q = Z'(I_T - ZF^{-1}Z')Z \), \( F = (D + Z'Z) \) and in our context \( D \) is \( 2 \times 2 \) matrix whose \((1,1)\) element is \( \frac{s^2}{\sigma^2_a} \), and all other elements are zero.

The moments of the benchmark returns can be expressed as:

\[
\tilde{E}_2 = (E_2|R) = \hat{E}_2, \quad \tilde{V}_{22} = E(V_{22}|R) = \frac{T}{T - k - 2}\hat{V}_{22}
\]

The mean of the predictive return distribution, \( E^* \) is given by:
\[ E^* = E \begin{pmatrix} \alpha + BE_2 \\ E_2 \end{pmatrix}^{' \times} R = \begin{pmatrix} \tilde{\alpha} + \tilde{B}E_2 \\ \tilde{E}_2 \end{pmatrix} \quad \text{(A 7)} \]

where \( \tilde{\alpha} \) and \( \tilde{B} \) are obtained from equation (A 6). The predictive covariance matrix \( V^* \) may be partitioned as \( V^* = \begin{bmatrix} V_{11}^* & V_{12}^* \\ V_{21}^* & V_{22}^* \end{bmatrix} \). The matrix \( V_{11}^* \) is defined by its \((i,j)\) element:

\[
Cov(r_{i,T+1}, r_{j,T+1})^{' \times} R = \tilde{b}_i^* V_{22}^* \tilde{b}_j^* + tr[V_{22}^* Cov(b_i, b_j)^{' \times} R] + \tilde{\Sigma}_{i,j} \\
+ \lfloor \begin{array}{c} 1 \\ \tilde{E}_2^* \end{array} \rfloor Cov(a_i, a_j)^{' \times} R \lfloor \begin{array}{c} 1 \\ \tilde{E}_2^* \end{array} \rfloor \quad \text{(A 8)}
\]

where \( b_i^* \) is the \( i^{th} \) row of the beta matrix, \( B \), \( a_i \) is the \( i^{th} \) column of \( A \), and \( \tilde{\Sigma}_{i,j} \) denotes the \((i,j)\) element of \( \tilde{\Sigma} \). Finally, the sub-matrix \( V_{22}^* = \tilde{V}_{22} + Var(E_2^{' \times} R) \).

Equations (A 7) and (A 8) allow us to obtain the first two moments of the predictive return distribution for the non-benchmark portfolios. The mutual fund investor relies on these moments to solve the optimal asset allocation problem.
Appendix B

Avramov and Chordia (2004) derive the moments of the predictive distribution for asset returns corresponding to the case where the mutual fund investor’s beliefs reflect time variation in the CAPM alphas and betas, and the market risk premium. We present these results here in brief retaining their notation to the extent possible.

Let \( r_i \) represent the T-vector of excess returns on asset \( i \). Also, let \( G_i = [G'_i, ..., G''_i] \), where \( G_i = [I, z'_{r-1}, r_{m,t}, r_{m,t} \otimes z'_{r-1}] \), let \( B_i = [\alpha_{i0}, \alpha'_i, \beta_{i0}, \beta'_i] \), let the matrix of predictor variables \( Z = [z'_1, ..., z'_T] \), and let \( F = [r_{m,1}, ..., r_{m,T}] \) represent the vector of excess returns on the market portfolio. Also, let \( X = [x_0, x'_1, ..., x'_{T-1}] \) where \( x_j = [I, z'_j] \), let \( V_f = [v_{m,1}, ..., v_{m,T}] \), let \( V_z = [v'_1, ..., v'_{T}] \), and let \( V_{rz} \) be a \( T \times N \) matrix whose \( i \)-th column contains \( T \) values of \( v_{z_i} \), the residual for asset \( i \) in regression (18). Further let, \( A_z = [\alpha_z, A'_z] \), \( A_f = [\alpha_m, A'_m] \), \( Q_{G_i} = I_T - G_i (G'_i G_i)^{-1} G'_i \), \( Q_{r} = I_T - X (XX)^{-1} X \), \( W_z = [X, V_f, V_{rz}] \), and \( Q_{Z} = I_T - W_z (W'_z W_z)^{-1} W'_z \). Also, let the variance-covariance matrix of asset returns, market portfolio returns, and the predictor variables be denoted by \( \Sigma_{\Psi}, \Sigma_{gf}, \) and \( \Sigma_{zz} \), respectively. The stochastic processes for excess returns for asset \( i \), the market factor, and predictor variables may then be expressed as \( r_i = G_i B_i + v_i \), \( F = X A_f + V_f \), and \( Z = X A_z + V_z \).

Under the assumption that the prior beliefs of the investor about the parameters governing the above stochastic processes are uninformative and the residuals \( v_{i,t+1}, v_{m,t+1}, \) and \( v_{z,t+1} \) are distributed as normal i.i.d. variates, the predictive moments for asset returns are given by

\[
\mu_{r,T+1} = \hat{B} [I_{T+1}, \hat{A}_f, \hat{A}_f (z'_T)]' x_T \tag{B 1}
\]

\[
\Sigma_{r,T+1} = A_f + (1 + \delta_r) \hat{\beta} (z'_T) \tilde{\Sigma}_{gf} \hat{\beta} (z'_T)' \tag{B 2}
\]

where in this context the \( N \times N \) matrix \( \hat{B} = (G'G)^{-1} G' R \) represents the maximum likelihood estimates of the alphas and betas, \( R \) is the \( T \times N \) matrix of excess asset returns, \( \hat{A}_f = (XX)^{-1} X F \), and...
\[
\hat{\Sigma}_{ff} = \frac{T}{(T-L-2)-3} \hat{\Sigma}_{ff}, \quad \hat{\Sigma}_{ff} = \frac{F'Q_xF}{T}, \ A_r \text{ is an } N \times N \text{ diagonal matrix whose } i\text{-th entry is } \\
\psi_i(1+\delta_{it}), \text{ and} \\
\delta_r = \frac{1}{T} \left[ 1 + (\bar{z} - z_T)'V_z^{-1}(\bar{z} - z_T) \right], \quad (B\ 3)
\]

\[
\delta_{it} = x_i'\Omega_i^{11}x_T + 2x_i'[\Omega_i^{12} + \Omega_i^{13}(z_T)]A_r'x_T + tr\left[A_r'x_Tx_T'A_r\tilde{\Omega}_i\right] \\
+ (1+\delta_r)tr\left[\hat{\Sigma}_{ff}\tilde{\Omega}_i\right], \quad (B\ 4)
\]

\[
\tilde{\Omega}_i = \Omega_i^{22} + \Omega_i^{23}(z_T) + (z_T')\Omega_i^{32} + (z_T')\Omega_i^{33}(z_T), \quad (B\ 5)
\]

\[
\hat{\psi}_i = \frac{T}{(T - L - 1 - KL) - 3} \hat{\psi}_i, \quad (B\ 6)
\]

where \( \hat{\psi} \) is a diagonal matrix whose \((i,i)\) element is \( \psi_i = \frac{r_i'Q_{G_i}r_i}{T} \) and \( \Omega_i^{jk} \) represents the following partitions based on \( G_i = [I, z'_{i-1}, r_{m,i}, r_{m,i} \otimes z'_{i-1}] \)

\[
(G'G_i)^{-1} = \begin{pmatrix}
\Omega_i^{11} & \Omega_i^{12} & \Omega_i^{13} \\
\Omega_i^{21} & \Omega_i^{22} & \Omega_i^{23} \\
\Omega_i^{31} & \Omega_i^{32} & \Omega_i^{33}
\end{pmatrix}
\quad (B\ 7)
\]

The predictive moments given in equations (B 1) and (B 2) are used by the mutual fund investor when determining the optimal asset allocations for month \( T+1 \).
Summary statistics for monthly equity portfolio excess returns, 1-month T-bill yield, dividend yield on a value-weighted market portfolio, 1-month detrended T-bill yield, default spread and term spread for the period 1954-2002 are reported. The equity portfolios are based on CRSP data covering all stocks on the NYSE, AMEX and NASDAQ. These include the value-weighted market index and three market capitalization based portfolios. The small cap portfolio is a value-weighted portfolio of all stocks in the bottom three market cap deciles (deciles 8 through 10), the medium cap portfolio is a value-weighted portfolio of all stocks in deciles 4 through 7 and the large cap portfolio is a value-weighted portfolio of all stocks in the top three market cap deciles (deciles 1 through 3). The dividend yield series is constructed as the natural log of the cumulated dividends on the CRSP value-weighted market index (covering NYSE, AMEX and NASDAQ stocks) over the previous 12 months divided by the current level of the index. The 1-month detrended T-bill yield is reported as the yield on a one-month T-bill minus its 12-month backward moving average. The default spread is the yield difference between Moody’s BAA rated corporate bonds and AAA rated corporate bonds. The term spread is the difference between the 10-year T-bond yield and the 3-month T-bill yield. We report the mean, median, standard deviation as well as the minimum and maximum values using monthly data over the period 1954-2002. The last column reports the first order autocorrelation for each series.

<table>
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<th>Summary Statistics</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
<th>$\rho_1$</th>
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<td>Value-weighted market portfolio return</td>
<td>0.0055</td>
<td>0.0090</td>
<td>0.0438</td>
<td>-0.2308</td>
<td>0.1598</td>
<td>0.0534</td>
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<td>0.0101</td>
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<td>-0.2961</td>
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<td>0.0070</td>
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<td>0.0528</td>
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<td>0.2461</td>
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<td>0.0081</td>
<td>0.0428</td>
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<td>-3.4913</td>
<td>-3.4366</td>
<td>0.4019</td>
<td>-4.6390</td>
<td>-2.7931</td>
<td>0.9893</td>
</tr>
<tr>
<td>1-month detrended T-Bill yield</td>
<td>0.0000</td>
<td>0.0006</td>
<td>0.0113</td>
<td>-0.0571</td>
<td>0.0496</td>
<td>0.8274</td>
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<tr>
<td>Default Spread</td>
<td>0.0095</td>
<td>0.0081</td>
<td>0.0043</td>
<td>0.0032</td>
<td>0.0269</td>
<td>0.9711</td>
</tr>
<tr>
<td>Term Spread</td>
<td>0.0140</td>
<td>0.0137</td>
<td>0.0137</td>
<td>-0.0301</td>
<td>0.0441</td>
<td>0.9444</td>
</tr>
</tbody>
</table>
Table 2
Regression Estimates

This table presents the coefficients (with the corresponding t-statistics based on the Newey-West covariance matrix in italics) and the adjusted $R^2$ values from all combinations of univariate and multivariate regressions on the monthly excess returns on the value-weighted market portfolio of all NYSE, AMEX and NASDAQ stocks using the lagged values of following explanatory variables: (a) the annualized log dividend yield on the value-weighted CRSP market index (Div. Yield), (b) the one month T-bill yield net of its 12-month backward moving average (T-bill Yield), (c) a measure of the default spread, i.e., the difference between the yield on a 10-year T-bond and a 3-month T-bill (Def. Spread), and (d) a measure of the term spread, i.e., the difference between the 10-year T-bond yield and the 3-month T-bill yield (Term Spread). The sample period is January 1954 to December 2002.

<table>
<thead>
<tr>
<th>Variable</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
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<tr>
<td>Intercept</td>
<td>0.038</td>
<td>0.005</td>
<td>-0.001</td>
<td>0.001</td>
<td>0.041</td>
<td>0.031</td>
<td>0.009</td>
<td>0.039</td>
<td>0.006</td>
<td>0.005</td>
<td>-0.003</td>
<td>0.053</td>
<td>0.041</td>
<td>0.040</td>
<td>0.005</td>
<td>0.053</td>
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<tr>
<td></td>
<td>2.11</td>
<td>3.03</td>
<td>-0.09</td>
<td>0.21</td>
<td>2.29</td>
<td>1.43</td>
<td>1.26</td>
<td>1.55</td>
<td>-0.62</td>
<td>1.24</td>
<td>1.87</td>
<td>2.46</td>
<td>2.29</td>
<td>0.96</td>
<td>2.48</td>
<td>2.95</td>
</tr>
<tr>
<td>Div. Yield</td>
<td>0.009</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<td>0.010</td>
<td>0.008</td>
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<tr>
<td></td>
<td></td>
<td>1.81</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>T- Bill Yield</td>
<td>-0.689</td>
<td>-0.703</td>
<td>-0.690</td>
<td>-0.667</td>
<td>-0.771</td>
<td>-0.645</td>
<td></td>
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<tr>
<td></td>
<td>-4.30</td>
<td>-4.44</td>
<td>-4.10</td>
<td>-3.02</td>
<td>-4.61</td>
<td>-2.91</td>
<td></td>
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<tr>
<td>Def. Spread</td>
<td>0.617</td>
<td></td>
<td></td>
<td>-0.006</td>
<td>0.398</td>
<td>-0.500</td>
<td>-0.046</td>
<td>-0.08</td>
<td>-0.02</td>
<td>-0.08</td>
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<td></td>
<td>1.34</td>
<td>0.338</td>
<td>-0.01</td>
<td>0.89</td>
<td>-1.03</td>
<td>-0.09</td>
<td>0.01</td>
<td>0.07</td>
<td>0.09</td>
<td>0.09</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Term Spread</td>
<td>0.351</td>
<td></td>
<td></td>
<td>-0.325</td>
<td>0.083</td>
<td>0.399</td>
<td>0.031</td>
<td>0.096</td>
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<tr>
<td></td>
<td>2.48</td>
<td>2.83</td>
<td>0.17</td>
<td>2.28</td>
<td>0.45</td>
<td>2.81</td>
<td>0.17</td>
<td>0.52</td>
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</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.58%</td>
<td>2.99%</td>
<td>0.19%</td>
<td>1.05%</td>
<td>3.71%</td>
<td>0.51%</td>
<td>1.93%</td>
<td>2.83%</td>
<td>1.02%</td>
<td>3.72%</td>
<td>3.59%</td>
<td>1.76%</td>
<td>2.67%</td>
<td>3.62%</td>
<td>0.00%</td>
<td></td>
</tr>
</tbody>
</table>
Table 3
Portfolio Performance of the Bayesian i.i.d. and Mutual fund Strategies

Monthly out-of-sample return performance and certainty equivalents are tracked for the i.i.d. strategy and the mutual fund strategy when (a) both strategies adjust allocations every month between the riskfree asset and an equity portfolio (Panel A), and (b) both strategies adjust allocations every month between the riskfree asset and three equity portfolios (Panel B). The period of evaluation is January, 1959 to December, 2002. Both strategies account for parameter uncertainty and incur a transaction cost equivalent to 0.5% for a round trip trade. A holdout period of 60 months (January 1954 to December 1958) is used for initial allocations. The mutual fund strategy predicts next-month returns by using one of the sixteen models (including the i.i.d. model) that incorporate one or more of the following predictive variables: lagged values of dividend yield on the value-weighted market index, the detrended yield on a 30 day T-bill, the default spread and the term spread. At the start of each month in the sample period the mutual fund strategy selects the optimal forecasting model based on a recursive utility maximization criterion. In contrast, the i.i.d. strategy, although using a similar Bayesian framework, ignores the information content of the predictor variables. The table reports the differences in the average return, adjusted Sharpe ratio and certainty equivalent rate of return between the mutual fund strategy and the i.i.d. strategy. Also reported, in parenthesis, are the bootstrap p-values related to the adjusted Sharpe ratio and CER differentials, based on a bootstrap procedure described in the text. The adjusted Sharpe ratio represents the difference in the average return earned by the mutual fund portfolio and the average return earned by a volatility-matched portfolio formed by combining the i.i.d. portfolio with T-bills and is identical to the GH1 measure of Graham and Harvey (1997).

<table>
<thead>
<tr>
<th>Sub-period</th>
<th>$\lambda^a$</th>
<th>Average Return Differential</th>
<th>Adjusted Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
<th>Adjusted Return Differential</th>
<th>Adjusted Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-59</td>
<td>2</td>
<td>0.001</td>
<td>0.002</td>
<td>(0.818)</td>
<td>28</td>
<td>(0.720)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.003</td>
<td>-0.001</td>
<td>(0.898)</td>
<td>38</td>
<td>(0.414)</td>
<td></td>
</tr>
<tr>
<td>Dec-73</td>
<td>10</td>
<td>-0.001</td>
<td>0.000</td>
<td>(1.000)</td>
<td>17</td>
<td>(0.518)</td>
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</tr>
<tr>
<td>Jan-74</td>
<td>2</td>
<td>0.012</td>
<td>0.013</td>
<td>(0.235)</td>
<td>136</td>
<td>(0.168)</td>
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<tr>
<td></td>
<td>5</td>
<td>0.007</td>
<td>0.005</td>
<td>(0.384)</td>
<td>12</td>
<td>(0.776)</td>
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</tr>
<tr>
<td>Dec-88</td>
<td>10</td>
<td>0.003</td>
<td>0.002</td>
<td>(0.396)</td>
<td>-7</td>
<td>(1.000)</td>
<td></td>
</tr>
<tr>
<td>Jan-89</td>
<td>2</td>
<td>-0.007</td>
<td>-0.006</td>
<td>(0.554)</td>
<td>56</td>
<td>(0.599)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.004</td>
<td>-0.005</td>
<td>(0.417)</td>
<td>-59</td>
<td>(0.185)</td>
<td></td>
</tr>
<tr>
<td>Dec-02</td>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>(1.000)</td>
<td>1</td>
<td>(0.898)</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>5</td>
<td>0.002</td>
<td>0.003</td>
<td>(0.670)</td>
<td>37</td>
<td>(0.574)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
<td>(1.000)</td>
<td>-3</td>
<td>(0.922)</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

$^a$Co-efficient of relative risk aversion is assumed to be 2, 5 or 10.
Table 4  
Market Timing Ability of the Bayesian Mutual fund Strategy

Monthly out-of-sample market timing ability is tracked for the mutual fund strategy when the strategy switches between the risk-free asset and an all-equity portfolio depending on whether the expected risk-premium at the start of a month is positive or negative. The performance of the strategy is assessed for the period January 1959 to December 2002. The strategy incurs a transaction cost equivalent to 0.5% for a round trip trade and accounts for parameter uncertainty. A holdout period of 60 months (January 1954 to December 1958) is used for initial allocations. The mutual fund strategy predicts next-month returns by using one of the sixteen models (including the i.i.d. model) that incorporate one or more of the following predictive variables: lagged values of dividend yield on the value-weighted market index, the detrended yield on a 30 day T-bill, the default spread and the term spread.

<table>
<thead>
<tr>
<th>Year</th>
<th>% Up-markets(^a)</th>
<th>% correct predictions in(^b)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Up-markets</td>
<td>Down markets</td>
</tr>
<tr>
<td>Jan-59</td>
<td>58.89%</td>
<td>47.17%</td>
<td>63.51%</td>
</tr>
<tr>
<td></td>
<td>Dec-73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan-74</td>
<td>52.22%</td>
<td>74.47%</td>
<td>40.70%</td>
</tr>
<tr>
<td></td>
<td>Dec-88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan-89</td>
<td>61.31%</td>
<td>36.89%</td>
<td>46.15%</td>
</tr>
<tr>
<td></td>
<td>Dec-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>57.39%</td>
<td>52.15%</td>
<td>49.78%</td>
</tr>
</tbody>
</table>

\(^a\) The percentage number of months when the market risk premium is positive.

\(^b\) The percent of times the mutual fund strategy predicts the sign of the risk premium correctly, i.e., positive in up-markets and negative in down markets.
Table 5
Forecasting Model Choice of the Mutual Fund Strategy: Single Risky Asset, Moderate Risk Aversion Case

The table shows the model choices and certainty equivalent return (CER) differential between the mutual fund strategy and the i.i.d. strategy in basis points per month based on their out-of-sample performance for the period January, 1959 to December, 2002, for the moderate risk aversion case. Both strategies adjust allocations every month between the riskfree asset and an equity portfolio. Both strategies account for parameter uncertainty and incur a transaction cost equivalent to 0.5% for a round trip trade. A holdout period of 60 months (January 1954 to December 1958) is used for initial allocations. The mutual fund strategy predicts next-month returns by using one of the sixteen models (including the i.i.d. model) that incorporate one or more of the following predictive variables: lagged values of dividend yield on the value-weighted market index, the detrended yield on a 30 day T-bill, the default spread and the term spread. At the start of each month the mutual fund strategy chooses an optimal forecasting model using a recursive utility maximization criterion applied to each model's performance over a historical window whose length is indicated in the first column. Also shown is the best performing model in each period, i.e., the model which when used consistently (i.e., continuously) over the period leads to the highest realized CER in that period.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expanding window</td>
<td>CER Differential</td>
<td>38</td>
<td>12</td>
<td>-59</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>Frequency of i.i.d. Model Use (%)</td>
<td>10.56</td>
<td>0.00</td>
<td>0.00</td>
<td>3.60</td>
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<tr>
<td></td>
<td>Most Frequently Used Model</td>
<td>G</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>Best Performing Model</td>
<td>K</td>
<td>L</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>Panel A: Model Usage Analysis, Base Case (Expanding Model Selection Window)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>60 months</td>
<td>CER Differential</td>
<td>36</td>
<td>0</td>
<td>-32</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Frequency of i.i.d. Model Use (%)</td>
<td>7.22</td>
<td>5.00</td>
<td>4.17</td>
<td>5.49</td>
</tr>
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<td>Most Frequently Used Model</td>
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<td>E</td>
<td>D</td>
<td>B</td>
</tr>
<tr>
<td>Panel B: Model Usage Analysis, Alternative Model Selection Windows</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36 months</td>
<td>CER Differential</td>
<td>63</td>
<td>-21</td>
<td>-22</td>
<td>7</td>
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<tr>
<td></td>
<td>Frequency of i.i.d. Model Use (%)</td>
<td>8.33</td>
<td>10.00</td>
<td>0.00</td>
<td>6.25</td>
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<td>Most Frequently Used Model</td>
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<td>E</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>12 months</td>
<td>CER Differential</td>
<td>44</td>
<td>-57</td>
<td>5</td>
<td>-3</td>
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<tr>
<td></td>
<td>Frequency of i.i.d. Model Use (%)</td>
<td>19.44</td>
<td>7.22</td>
<td>5.36</td>
<td>10.79</td>
</tr>
<tr>
<td></td>
<td>Most Frequently Used Model</td>
<td>P</td>
<td>N</td>
<td>B</td>
<td>B</td>
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</table>
Table 6
Portfolio Performance of the Bayesian i.i.d. and Mutual fund Strategies: CAPM Results

Monthly out-of-sample return performance and certainty equivalents are tracked for the i.i.d. strategy and the mutual fund strategy when both strategies adjust allocations every month between the risk-free asset and ten industry-based equity portfolios. The period of evaluation is January, 1959 to December, 2002. Both strategies account for parameter uncertainty and incur a transaction cost equivalent to 0.5% for a round trip trade. A holdout period of 60 months (January 1954 to December 1958) is used for initial allocations. The mutual fund strategy’s expectations of next-month returns are based on the CAPM. The mutual fund investor’s prior beliefs reflect varying degrees of uncertainty in the cross-sectional pricing ability of the CAPM. The pricing uncertainty is captured by the prior specification of the annualized standard deviation of the pricing errors (alphas), $\sigma_\alpha$, represented by: $\sigma_\alpha = 0\%$ (Panel A), $\sigma_\alpha = 2\%$ (Panel B), and $\sigma_\alpha = \infty$ (Panel C). The i.i.d. strategy, uses a Bayesian framework but does not use the model-based approach of the mutual fund investor. The table reports the differences in the certainty equivalent rate of return between the mutual fund strategy and the i.i.d. strategy. Also reported is the adjusted Sharpe ratio for the mutual fund strategy. The adjusted Sharpe ratio represents the difference in the average return earned by the mutual fund portfolio and the average return earned by a volatility-matched portfolio formed by combining the i.i.d. portfolio with T-bills and is identical to the GH1 measure of Graham and Harvey (1997). The table also reports, in parenthesis, the bootstrap p-values related to the adjusted Sharpe ratio and CER differentials, based on a bootstrap procedure described in the text.

<table>
<thead>
<tr>
<th>Sub-period</th>
<th>$\lambda$</th>
<th>Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
<th>Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
<th>Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A: Std Dev of Pricing Error = 0%</td>
<td>B: Std Dev of Pricing Error = 2%</td>
<td>C: Std Dev of Pricing Error = infinite</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan-59</td>
<td>2</td>
<td>0.001 (0.754)</td>
<td>25 (0.606)</td>
<td>0.001 (0.513)</td>
<td>11 (0.506)</td>
<td>0.000 (1.000)</td>
<td>0 (1.000)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.001 (0.665)</td>
<td>89 (0.016)</td>
<td>0.001 (0.407)</td>
<td>41 (0.099)</td>
<td>0.000 (1.000)</td>
<td>26 (0.285)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.000 (1.000)</td>
<td>125 (0.004)</td>
<td>0.000 (1.000)</td>
<td>102 (0.051)</td>
<td>0.000 (1.000)</td>
<td>89 (0.068)</td>
</tr>
<tr>
<td>Jan-74</td>
<td>2</td>
<td>-0.004 (0.250)</td>
<td>-56 (0.248)</td>
<td>-0.002 (0.131)</td>
<td>-15 (0.267)</td>
<td>-0.003 (0.039)</td>
<td>-29 (0.066)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.001 (0.656)</td>
<td>2 (0.951)</td>
<td>-0.001 (0.288)</td>
<td>4 (0.798)</td>
<td>0.000 (1.000)</td>
<td>12 (0.428)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.001 (0.554)</td>
<td>-1 (0.956)</td>
<td>0.000 (1.000)</td>
<td>5 (0.727)</td>
<td>-0.001 (0.091)</td>
<td>2 (0.887)</td>
</tr>
<tr>
<td>Jan-89</td>
<td>2</td>
<td>0.001 (0.798)</td>
<td>1 (0.987)</td>
<td>0.001 (0.415)</td>
<td>17 (0.213)</td>
<td>-0.001 (0.405)</td>
<td>-4 (0.738)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.003 (0.309)</td>
<td>44 (0.209)</td>
<td>0.001 (0.217)</td>
<td>24 (0.154)</td>
<td>0.000 (1.000)</td>
<td>17 (0.266)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.002 (0.321)</td>
<td>29 (0.225)</td>
<td>0.000 (1.000)</td>
<td>16 (0.238)</td>
<td>0.000 (1.000)</td>
<td>15 (0.261)</td>
</tr>
<tr>
<td>Dec-02</td>
<td>2</td>
<td>-0.001 (0.743)</td>
<td>-10 (0.777)</td>
<td>0.000 (1.000)</td>
<td>4 (0.645)</td>
<td>-0.001 (0.257)</td>
<td>-11 (0.269)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.001 (0.587)</td>
<td>45 (0.016)</td>
<td>0.000 (1.000)</td>
<td>23 (0.136)</td>
<td>0.000 (1.000)</td>
<td>18 (0.202)</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.001 (0.363)</td>
<td>51 (0.034)</td>
<td>0.000 (1.000)</td>
<td>41 (0.130)</td>
<td>0.000 (1.000)</td>
<td>36 (0.152)</td>
</tr>
</tbody>
</table>

$^a$Coefficient of relative risk aversion.
Table 7
Portfolio Performance of the Bayesian *i.i.d.* and Mutual fund Strategies: CAPM with Time Varying Parameters

Monthly out-of-sample return performance and certainty equivalents are tracked for the *i.i.d.* strategy and the mutual fund strategy when both strategies adjust allocations every month between the riskfree asset and ten industry-based equity portfolios. The period of evaluation is January, 1959 to December, 2002. Both strategies account for parameter uncertainty and incur a transaction cost equivalent to 0.5% for a round trip trade. A holdout period of 60 months (January 1954 to December 1958) is used for initial allocations. The mutual fund investor predicts next month returns using a CAPM based framework in which alphas, betas, and the market risk premium are allowed to vary with the following variables: lagged values of dividend yield on the value-weighted market index, the detrended yield on a 30 day T-bill, the default spread and the term spread. Results are presented for the case when the alphas are unrestricted (Panel A), and when alphas are restricted to be zero in accordance with the model’s restrictions (Panel B). The *i.i.d.* strategy, uses a Bayesian framework but does not use the model-based approach of the mutual fund investor. The table reports the differences in the certainty equivalent rate of return between the mutual fund strategy and the *i.i.d.* strategy. Also reported is the adjusted Sharpe ratio for the mutual fund strategy. The adjusted Sharpe ratio represents the difference in the average return earned by the mutual fund portfolio and the average return earned by a volatility-matched portfolio formed by combining the *i.i.d.* portfolio with T-bills and is identical to the GH1 measure of Graham and Harvey (1997). The table also reports, in parenthesis, the bootstrap p-values related to the adjusted Sharpe ratio and CER differentials, based on a bootstrap procedure described in the text.

<table>
<thead>
<tr>
<th>Sub-period</th>
<th>( \lambda^a )</th>
<th>Average Return Differential</th>
<th>Adjusted Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
<th>Average Return Differential</th>
<th>Adjusted Sharpe Ratio Differential</th>
<th>Certainty Equivalent Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-59</td>
<td>2</td>
<td>-0.001</td>
<td>0.000 (1.000)</td>
<td>15 (0.710)</td>
<td>-0.003</td>
<td>-0.001 (0.782)</td>
<td>11 (0.848)</td>
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<tr>
<td>-</td>
<td>5</td>
<td>0.000</td>
<td>0.001 (0.820)</td>
<td>32 (0.463)</td>
<td>-0.002</td>
<td>0.000 (1.000)</td>
<td>61 (0.102)</td>
</tr>
<tr>
<td>Dec-73</td>
<td>10</td>
<td>0.000</td>
<td>0.000 (1.000)</td>
<td>-14 (0.858)</td>
<td>-0.002</td>
<td>-0.001 (0.739)</td>
<td>71 (0.127)</td>
</tr>
<tr>
<td>Jan-74</td>
<td>2</td>
<td>-0.001</td>
<td>0.000 (1.000)</td>
<td>-1 (0.972)</td>
<td>-0.013</td>
<td>-0.005 (0.199)</td>
<td>-64 (0.329)</td>
</tr>
<tr>
<td>-</td>
<td>5</td>
<td>0.006</td>
<td>0.002 (0.530)</td>
<td>-54 (0.197)</td>
<td>-0.006</td>
<td>-0.004 (0.183)</td>
<td>-30 (0.374)</td>
</tr>
<tr>
<td>Dec-88</td>
<td>10</td>
<td>0.008</td>
<td>0.007 (0.110)</td>
<td>-234 (0.043)</td>
<td>-0.004</td>
<td>-0.005 (0.056)</td>
<td>-60 (0.089)</td>
</tr>
<tr>
<td>Jan-89</td>
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<td>0.004</td>
<td>0.005 (0.176)</td>
<td>56 (0.239)</td>
<td>-0.009</td>
<td>-0.003 (0.439)</td>
<td>-42 (0.583)</td>
</tr>
<tr>
<td>-</td>
<td>5</td>
<td>0.004</td>
<td>0.003 (0.374)</td>
<td>8 (0.840)</td>
<td>-0.006</td>
<td>-0.003 (0.284)</td>
<td>-9 (0.805)</td>
</tr>
<tr>
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<td>0.008</td>
<td>0.004 (0.289)</td>
<td>-90 (0.432)</td>
<td>-0.004</td>
<td>-0.003 (0.215)</td>
<td>-17 (0.571)</td>
</tr>
<tr>
<td>All</td>
<td>2</td>
<td>0.001</td>
<td>0.002 (0.326)</td>
<td>23 (0.361)</td>
<td>-0.008</td>
<td>-0.003 (0.180)</td>
<td>-31 (0.545)</td>
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<tr>
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<td>5</td>
<td>0.003</td>
<td>0.002 (0.367)</td>
<td>-5 (0.829)</td>
<td>-0.005</td>
<td>-0.002 (0.269)</td>
<td>7 (0.686)</td>
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<tr>
<td></td>
<td>10</td>
<td>0.005</td>
<td>0.002 (0.484)</td>
<td>-113 (0.266)</td>
<td>-0.003</td>
<td>-0.002 (0.248)</td>
<td>-2 (0.934)</td>
</tr>
</tbody>
</table>
Figure 1
Figure 2
Figure 3