THE STATE BUDGET PROCESS: AN ANALYSIS OF FORECASTING ERROR AND OPTIMAL RESERVES

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Project Commissioned by

THE IOWA LEGISLATIVE EXTENDED ASSISTANCE GROUP

Funded by

THE NORTHWEST AREA FOUNDATION

Report Published by

The Legislative Extended Assistance Group (LEAG)

The University of Iowa

Oakdale, Iowa 52319

April, 1989

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PREFACE

Iowa's Legislative Extended Assistance Group (LEAG) was formed in 1978 to encourage interaction between state government and university researchers. Each year LEAG has identified issues of public policy where research is needed to aid Iowa's legislature. Specific projects for research have been solicited and LEAG has funded practical projects of policy research which have been undertaken by college and university faculty throughout Iowa. The results of the research work are given peer review prior to being published and made available to all members of the Iowa General Assembly. In addition, from time to time LEAG publishes research in conjunction with other legislative activities.

This report, by Professors Barnard and Horowitz of The University of Iowa's Economics Department, and economics graduate students Rietz and Vigdor, deals with producing economic forecasts for state expenditure purposes and with establishing a state reserve fund to cover possible budget shortfalls. In part I of the report, the researchers provide background information on the development of tax forecast models and on the extent of forecast error in national models and in Iowa's state models. In Part II they analyze types of reserve funds that could be established on the basis of several criteria; the criteria include fund stability, the net cost of providing a fund, and the expected chance of exhausting a fund. The information given in this report should be of interest to those who wish to investigate the performance of forecasting models, and to those who wish to consider alternative ways of establishing state reserve funds.

The research performed for this report was competitively funded with a special award from the Northwest Area Foundation. That Foundation also supported publication of the findings and distribution of the final report to members of the Iowa General Assembly. Copies of this LEAG report, and a price list of other reports available from LEAG, can be obtained by telephone or by mail from:

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TABLE OF CONTENTS

Page

PREFACE.. i

INTRODUCTION.. 1

PART I. THE TAX FORECASTING PROJECT.................................. 4

1. Methods of Developing Forecasts.............................. 6
   Hybrid Model.. 6
   Structural Model.. 8

2. Tax Receipts Forecasting Models.............................. 10
   Sales Tax... 11
   Use Tax... 12
   Income Tax.. 14
   Forecast Error Simulation..................................... 16

3. Forecasting Error.. 18
   Tax Model Forecast Error...................................... 20
   GNP Forecast Error.. 20

4. Forecasting Error and Budget Making.......................... 23

REFERENCES.. 26

APPENDIX I.. 27

PART II. MODELING OPTIMAL RESERVES.................................. 34

1. The Methods.. 34

2. Basis of Comparison.. 37
   Fund Stability.. 37
   Allocation Stability.. 38
   Initial Fund and Annual Allocation Levels..................... 38
   Expected Net Cost... 40
   Chance of Exhausting Fund..................................... 40

3. Conclusions.. 45

APPENDIX II.. 47

LIST OF TABLES

Page

Table I.1 Estimated Mean Error for Sales Tax Forecasts 19

Table I.2 Estimated Mean Absolute Percent Error for Sales Tax Forecasts 19

Table II.1 Initial Fund Levels, Allocations and Expected Net Costs for Achieving a Five-Percent Chance of Bankruptcy in Ten Years 39

LIST OF FIGURES

Figure I.1 Sales Tax Net Deposits: Actual and Forecast, Iowa, 1962-1976 13

Figure I.2 Iowa Tax Receipts Forecast Error, Fiscal Years 1975-1986 ... 21

Figure I.3 DRI Forecast Error on GNP, 1977-1985 22

Figure II.1 Method 1 Without Interest Accumulation 41

Figure II.2 Method 1 With Interest Accumulation 41

Figure II.3 Method 2 Without Interest Accumulation 42

Figure II.4 Method 2 With Interest Accumulation 42

Figure II.5 Method 3 Without Interest Accumulation 43

Figure II.6 Method 3 With Interest Accumulation 44

Figure II.7 Methods with 2.5% to 4% Chance of Exhaustion in 10 Years 44

INTRODUCTION

The magnitude of economic fluctuations at both national and regional levels has increased over the past two decades. The two most recent U.S. recessions in 1974-75 and 1980-82 are recorded as the most severe since the great depression of the 1930s. The Iowa economy, historically identified with agriculture and agriculture-related industries, was relatively insensitive to national business cycles until the most recent recession.[1] Iowa ranked 42nd among the states in sensitivity to the national business cycle covering the years 1948-72 and 46th in the 1973-75 recession. In the 1980-82 recession, Iowa moved up in sensitivity ranking to 19th [Bretzffelder (1973), Bretzfelder and Brown (1982), and Bretzfelder and Friedenberg (1980)]. Research by Barnard and Kennedy (1986) indicates the Iowa economy has become more sensitive to fluctuations in the U.S. economy, especially from changes in interest rates and industrial production.

Increasing fluctuation in the national economy and changing economic structure have also changed the sensitivity ranking of other states and regions. These fluctuations and structural changes have had a major impact on the tax revenues of state governments. Some states have accumulated large surpluses in recent years while other states have encountered shortfalls

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[1]The sensitivity of regions and states to the national business cycle measures their relative change during expansions and recessions of the national business cycle. The larger the change in regional or state income relative to the change at the level of national economy, the more sensitive (and higher ranked) is the region or state to the national business cycle. States in the Great Lakes region (Michigan, Indiana, Ohio), which are heavily engaged in the manufacture of durable goods, show the greatest sensitivity to the business cycle. Bretzfelder's, et. al., sensitivity analysis uses changes in nonfarm personal income to relate change at the regional and state level to national business cycles. Farm income is excluded from the analysis because changes in farm income are more closely tied to commodity cycles, weather, and changes in international trade than to the national business cycle.

necessitating mid-year budget cuts and/or tax increases.[2] The state of Iowa's budget history over the past fifteen yéars has involved both unexpectedly large surpluses, and shortfalls necessitating mid-year budget cuts.[3] This by necessity puts increased emphasis on forecasting economic activity and tax receipts. Where forecasting of current trends would once suffice, more complex models to forecast levels and changes and gather economic information are now needed.

There are three major issues connected with the requirement for a balanced budget at the state level. The first deals with generating accurate forecasts of expected revenues upon which the expenditure budget is prepared. The second deals with explicit considerations of expected forecasting error in the preparation of the budget. The third issue deals with the determination of the appropriate level of reserve that a state government should carry, given the statistical fact that revenue forecasts will undoubtedly be in error.

The Institute for Economic Research has been involved in forecasting state economic activity and tax revenues since 1973. These forecasts have been incorporated into the budget-making process. Part I of this report discusses the Iowa tax revenue forecasting project, the process of developing

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[2] A recent article by Bernard L. Weinstein (1987) characterizes current state government finances as being in turmoil as a result of federal tax changes, and regional prosperity and decline that find some states dealing with major tax windfalls while other states are cutting budgets and proposing to raise taxes.

[3] Iowa ran budget surpluses exceeding $200 million in three consecutive fiscal years, 1974 -- $207 million, 1975 -- $261 million, and 1976 -- $207 million. In fiscal years 1981 and 1984, budget reversions were required to meet revenue shortfalls.

forecasts, various forecasting models employed over this period, and the forecasting error of the tax revenue forecasts.

Part II of this report addresses the issue of establishing a fund that would cover budget shortfalls. Three methods of establishing and maintaining a reserve fund to cover possible revenue shortfalls are developed and examined by simulating government revenue forecasting errors.

I

THE TAX FORECASTING PROJECT

The forecasting of tax revenues is a fundamental part of the budgeting process. The operations of state government are carried out on a fiscal year basis (July 1 to June 30) with expenditures budgeted according to expected or forecast revenues. Potential deficits at the end of the year, or unplanned surpluses, are typically viewed as problems for elected officials. A surplus indicates too much was taken from the private sector, while a deficit indicates that government did not have sufficient funds to produce the desired level of public services. The State of Iowa Code essentially requires that state government must not incur a current-account deficit. Accordingly, it is important that forecasts of tax revenues for budget making be accurate, otherwise once a budget is approved by the legislative and executive branches, if it appears there will be a shortfall of revenues relative to the budget the Governor must cut the budget to bring expenditures in line with receipts.

From the standpoint of economic efficiency there is more to balancing the budget than making mid-year budget adjustments. Once government operational budgets have been approved and spending has commenced, mid-year budget adjustments can be viewed as leading to distortions and inefficiency. Mid-year budget adjustments can interfere with the government manager's planned least-cost-input combinations for producing public goods and services. They can also interfere with the planned, optimal combination of public goods and services to be produced. The point is that cutting budgets is not an inconsequential matter once budgets have been approved, if we view the original planned budget allocation as optimal for producing the desired level of public goods and services at least-cost, and as producing the optimal combination of public goods and services.

Another aspect to the issue of state government tax revenue shortfalls ties in with state government efforts to promote economic development. Simply stated, business investment and economic development are promoted by public services that represent high value relative to their cost in taxes, and by tax rates that are stable and remove uncertainty from the business planning and investment process [(Barnard, Forkenbrock and Pogue (1987)]. Shortfalls in tax revenues which are covered by increasing state taxes run the risk of undermining the economic development effort. S.P.A. Brown (1988) cautions that tax increases to cover revenue shortfalls run the risk of not being rescinded once adopted, and may eventually induce excessive growth of state government and retard economic development. Brown also argues for an economic stabilization or reserve fund to avoid the cycle of increasing taxes to cover revenue shortfalls and unwarranted growth of state government.

The Iowa Tax Forecasting Project was started in 1974 and involved a University of Iowa research team and the Comptroller's Office (now the Department of Management). Also, at this same time, the Iowa Economic Advisory Council was appointed by Governor Robert Ray to provide economic information and advise state government on economic affairs. In 1982, the Iowa Economic Forecasting Council was appointed by Governor Branstad and given the charge of focusing on economic forecasting for the state.

This section of the report discusses the process of developing the forecasts, the forecasting error on tax revenues, and budget making.

1. Methods of Developing Forecasts

The method of forecasting Iowa tax revenues has continued to evolve since its beginning. The project began using a structurally simple model with modest data requirements. In 1982 the project shifted to a more structurally complete model which is driven by the Data Resources, Inc. (DRI) forecasting model of the U.S. economy. The model generates forecasts of sectoral employment, income, tax revenues and other variables. This section reviews the forecasting models used and presents standard measures of their performance.

Hybrid Model

A complete account of the original model (designated as a hybrid model) was published earlier [(see Barnard and Dent (1979)]. Accordingly, this model will be discussed only briefly in this report.

The term "hybrid model" is used to describe a model that uses an ARIMA, (Autoregressive-integrated moving average) process (which is an intrinsic, or extrapolative approach) to forecast nonfarm personal income components. Then forecast components are then linked with an extrinsic (or associative) model to forecast individual income, sales, and use taxes. The logic of such an approach is that of using the intrinsic model to forecast the income variables, because earned income and consumer spending lead the various tax receipt variables.

In Iowa, ARIMA processes were estimated for the various components of personal income: nonfarm personal income (N_t), nonfarm wages and salary income (WS_t), and nonfarm proprietors and property income (NPP_t). Forecasting farm income was a major challenge given the pattern of fluctuations because of crop and livestock production cycles, marketing patterns, weather, and changing government programs. A mixture of field and theoretical experience was employed in developing and using a simulation model of farm production and income accounts to make farm income forecasts. The computer simulation model allowed alternative scenarios to be examined as a means of homing in on forecasts of farm income.

Tax revenue forecasting equations to estimate income, use, sales and corporate income taxes were developed using the various components of farm and nonfarm income. The approach will be demonstrated by reporting only the sales tax equation and a graph of actual and fitted values.

A quarterly sales tax receipts equation was estimated over the period 1962-1976 in the following form:

$$\text{(I.1)}\quad S_t = \underset{(1.79)}{3.72C_1} + \underset{(1.95)}{0.27Q_{12}} + \underset{(2.62)}{1.66Q_{13}} - \underset{(0.98)}{1.65Q_{14}} + \underset{(5.61)}{.01037N_1(-1)}$$

$$+ \underset{(0.52)}{.00326F_1(-1)} + \underset{(2.36)}{4.77C_2} + \underset{(3.07)}{1.35Q_{22}} + \underset{(2.81)}{0.80Q_{23}} - \underset{(1.61)}{1.47Q_{24}}$$

$$+ \underset{(18.38)}{.0176513N_2(-1)} + \underset{(2.39)}{.0041716F_2(-1)} - \underset{(-2.23)}{4.45G_1} - \underset{(-4.19)}{6.57G_2}$$

$$- \underset{(-4.87)}{8.94G_3} - \underset{(-4.53)}{8.57G_4} + \underset{(5.09)}{7.67E_1} - \underset{(-7.22)}{10.65E_2} + \underset{(3.25)}{4.71E_3} + \underset{(3.76)}{5.30E_4}$$

$R^2 = 0.9962$, D/W = 1.18, $\hat{\sigma} = 1.301$, period 1961.3 - 1975.3.

Student t values in parentheses,

where,

S_t = quarterly sales tax receipts,

C_1 = intercept for period 1 (1961.3-1967.3),

C_2 = intercept for period 2 (1967.4-1975.1),

Q_{1t} = seasonal dummy variates for period 1961.3-1967.3,

Q_{2t} = seasonal dummy variates for period 1967.4-1975.1,

E_j = dummy terms for changes in accounting procedures, reporting and deposit requirements (1966.1, 1967.4, 1969.3, 1971.2),

G_k = dummy terms for food and drug exclusions in period 1974.3-1975.3,

N_{t-1} = non-farm income,

F_{t-1} = farm income.

The estimated equation takes into account quarterly seasonal variation in consumption. Income slope coefficients are permitted to vary due to the increase in the sales tax rate from 2 to 3 percent in 1967. Student-t values on non-seasonal terms are 18.4 for nonfarm income (N) and 2.38 for farm income (F) in the current tax period. The seasonal terms jointly have a significant contribution. The coefficients on nonfarm and farm income indicate the marginal propensity to collect sales tax out of nonfarm income is approximately four times as large as corresponding propensity out of farm income, reflecting different purchase patterns between the farm and nonfarm groups.

Structural Model

The forecasting project moved to a structural model in 1982 with its first forecasts of tax receipts made for fiscal year 1983. In part, the shift to a structural model linked to the U.S. economy was motivated by the impact of the 1980-82 recession on the Iowa economy and the desire for greater information on how the Iowa economy and its various sectors relate to the U.S. economy. In 1982 the Iowa econometric model was developed with linkage to DRI's U.S. macro model and forecast system. The Iowa model brings together both national and state data series that are maintained and updated by DRI as new data become available from various U.S. government and state government agencies. Initially the Iowa model was maintained on DRI's interactive computer system at Lexington, Massachusetts. In late 1986 it was transferred to the Institute's PC computer.

The basic theoretical construct underlying the Iowa forecasting model is the export-base model. In an export-base model, the economy is divided into two major sectors, those sectors producing goods and services for export outside the region, and those sectors which produce for the local economy. For example, Iowa exports large amounts of agricultural commodities and processed food products, while the retail trade sector is part of the local sector providing the retailing services for local consumption. In the export-base model, a change in export activity increases (or decreases) income and employment in the export sector. Workers in the exporting sectors spend the additional income for purchase of goods and services from the local sectors, thereby stimulating an increase in output and employment in the local sectors. The economic interdependence between the export and local sector which results in a multiplier effect is expressed in the following equation:

$$\Delta Y = \frac{1}{1 - \frac{L}{T}} \times \Delta E \qquad \text{(I.2)}$$

where,

ΔY = change in regional employment,
L = employment the in local sector,
T = total employment,
ΔE = change in export employment.

In the Iowa model, the agricultural sector and the manufacturing sectors are treated as export sectors. Changes in national agricultural production and commodity markets will determine the levels of production, prices, and cash marketings in Iowa. Levels of employment in the Iowa manufacturing sectors are determined by levels of national sectorial demand as expressed in changes in the index of industrial production of the counterpart national sector. Additionally, national/regional comparative cost factors and regional inter-industry linkages are modeled as determinants of regional sectoral demand and employment levels.

Employment in the local producing sectors classification--transportation, communication and utilities, trade, services, and government--is basically determined by levels of employment and income in the export sectors.

Wages and salaries in the export sectors are viewed as being determined in a national labor market. Wage and salary levels in the local sectors are dependent upon the levels of wages and salaries in the export sectors. Population is also part of the Iowa model. Population levels are determined in part by population growth trends and employment levels, which act as an inducement to migration.

2. Tax Receipts Forecasting Models

Tax receipts forecasting models have evolved over the course of the project but still have basically the same structure as when they were first developed. They are re-estimated each quarter once new quarterly data become available, and the functional form of an equation may be altered to deal with specific forecasting or data problems.

The basic structure of the tax models captures the fact that income from production and consumer spending lead tax receipts. A forecast from the economic model generates the independent variables that are used to forecast tax receipts. To the extent possible, each tax is linked to the income stream to which the tax is applied. This point will become clearer as each tax model is examined in more detail.

Sales Tax

The Iowa sales tax is levied on sales of selected goods and services, currently at a rate of 4 percent. From time to time legislated changes in the rate and base have occurred, such as a deletion of the sales tax on food and prescription drugs effective July 1, 1974, and an increase in the rate from 3 to 4 percent effective March 1, 1983.

Sales tax receipts are modeled as dependent on personal income; however, we have found that certain components of personal income are weak in explaining sales tax receipts. In the sales tax model presented in the earlier section, a satisfactory result was obtained by using the two major components of personal income, nonfarm and farm income, along with dummy variables for seasonal variation, and rate and base changes. With the onset of the 1980 recession, we found the best forecasting model was based on only wage and salary income because other income components, such as farm proprietors income, nonfarm proprietors income, and property income, became weekly associated with sales tax receipts (current spending).[4] See Figure I.1 for actual versus forecast sales tax deposits in Iowa from 1962-76. A representation of a recent sales tax model is reported in Appendix I. In this model, quarterly sales tax receipts are modeled as a function of: wage and

[4]This is consistent with household consumption theory that the marginal and average propensities to consume of both farm and nonfarm proprietors are less than those of wage and salary workers. Households whose major sources of income come from variable sources of income, such as proprietors income and dividends, interest and rents, cut back strongly on spending during periods of economic uncertainty and recession.

salary income for the current period and lagged one quarter; seasonal dummy variables, and; dummy variables for the 1983.1 rate change and 1985.3 change in the tax base. Included in Appendix I are the estimated parameters of the model, and plots of the actual and fitted values, and residuals. The value of $R^2 = 0.9853$, the Durbin-Watson statistic is 1.9903, and the standard error is 4.2755.

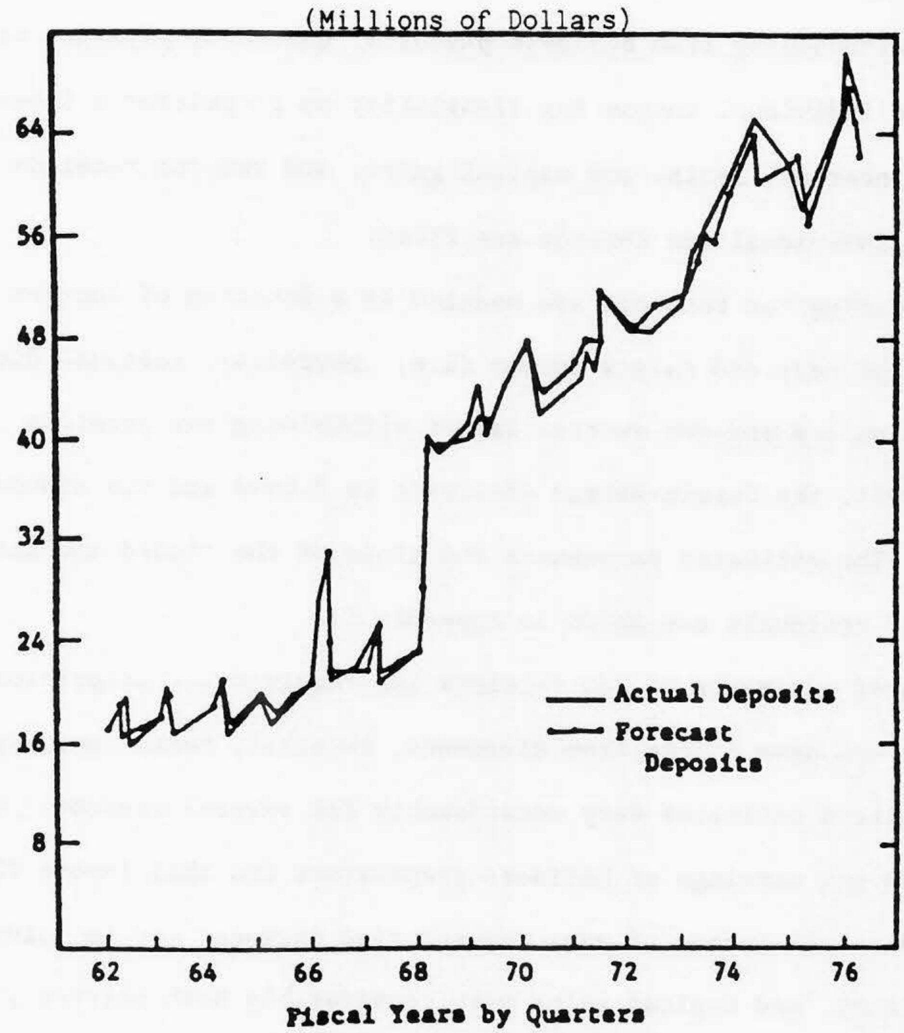
Use Tax

The Iowa use tax is currently imposed at the rate of 4 percent on the purchase of tangible personal property, or services, for use in Iowa. The tax is collected under three categories: retailer use, consumer use, and motor vehicle use. Only retailer's and consumer's use taxes are forecast because they go into the general fund; the motor vehicle use tax goes to the highway fund.

Use taxes would include such items as office paper supplies purchased by a manufacturer, and sales in Iowa by out-of-state firms. Use tax receipts are related to a one-quarter lag of the use tax, a time variable to capture growth, and a four-quarter moving average of wage and salary income to represent current economic activity.

The reported statistics for the model and plots of fitted and actual values, and a plot of the residuals are shown in Appendix I. The value of $R^2 = 0.9349$, Durbin-Watson = 1.8808, and the standard error = 1.4554.

Figure I.1. Sales Tax Net Deposits: Actual and Forecast, Iowa, 1962-1976



Income Tax

Individual income tax receipts are collected from three sources: individual withholding from business payrolls; quarterly payments of declared estimates of individual income tax liabilities on proprietor's income, dividends, interest, rents, and capital gains; and returns receipts collected at the time individual tax returns are filed.

Withholding tax receipts are modeled as a function of current and a one-quarter lag of wage and salary income (i.e., payrolls), seasonal dummy variates, plus one-and-two-quarter lagged withholding-tax receipts. The value of $R^2 = 0.9961$, the Durbin-Watson statistic is 2.0998 and the standard error is 2.9860. The estimated parameters and plots of the fitted and actual value and plots of residuals are shown in Appendix I.

Declared estimates of tax receipts are required for proprietors and individuals who have income from dividends, interest, rents, and capital gains. Declared estimates vary considerably for several reasons: first, they are based on net earnings of business proprietors and this income fluctuates more than does the income of wage and salaried persons; second, dividends, interest, rents, and capital gains vary considerably both quarterly and annually; and, third, they involve an estimate made by an individual, and/or the person's accountant at the time of annual tax liability. There is a strong seasonal element in declared estimates receipts, with quarterly collections increasing in each successive quarter as the information on an individual's tax liability becomes clearer as the person approaches the tax filing deadline.

Declared estimates receipts are modeled as a function of a five-quarter moving average of property income (dividends, interest, and rents) lagged one quarter, nonfarm proprietor's income, and seasonal dummy variates. The value of $R^2 = 0.9043$, the Durbin-Watson statistic is 1.9666, and the standard error is 3.4998. The estimated parameters, graphs of fitted and actual values, and the plots of residuals are given in Appendix I.

The return tax receipts are the final payments that are received when individual annual tax returns are filed. Most individual tax returns are filed on a calendar-year basis. Accordingly, payments for the third and fourth quarters of the calendar year are very small. Farmers do not have to file declared estimates, provided they file their tax returns by March 1. Accordingly, the returns receipts for the first calendar quarter are mostly farm returns. The returns received during the second quarter reflect final tax liabilities of nonfarm tax payers, principally on proprietor's incomes, property income, and capital gains.

Returns tax receipts are modeled as a function of a four-quarter moving average of farm proprietor's income lagged one quarter with a dummy for payment during the first calendar quarter, a five-quarter moving average of nonfarm proprietors income and property income lagged one quarter, seasonal dummy variates, and a time-trend variable. The value of $R^2 = 0.9030$, the Durbin-Watson statistic is 2.0364, and the standard error is 10.2120. The estimated parameters of the model along with plots of actual and fitted values and residuals are reported in Appendix I.

Forecast Error Simulation

Forecast error simulations are carried out using the sales tax model to demonstrate the forecasting error that can be expected when forecasting future values of tax receipts, and to show the need for formulating expectations of errors. The simulation involves generating forecasts of sales tax receipts for eight quarters by using actual values of the independent variables in the sales tax equation SSTR. The procedure is known as ex post forecasting, or measuring the out-of-sample forecast error.

Two measures of the forecast are estimated which provide insight into the nature of the error pattern for sales tax forecasts. The first measure is the mean error (ME) which is simply the average error measured in millions of dollars as computed in the following equation:

$$ME = \frac{\Sigma(A-F)}{n} \qquad \text{(I.3)}$$

where a is the actual value, F is the forecast value and n is the number of observations. This measure provides an indication of whether the forecast over- or under-predicts the actual value and the sign on ME indicates the direction of bias. Bias in a forecasting model is generally undesirable, but in certain cases where under- or over-prediction has high costs, a model with a preferred bias may be chosen. For example, in tax forecasting the preferred bias is to under-predict.

The second measure of forecast error computed is the mean absolute percent error. The important characteristic of this measure is that it focuses on point accuracy where over- or under-prediction are equally undesirable. The mean absolute percent error (MAPE) measures the mean of the absolute error as a percentage of the actual value:

(I.4) $$MAPE = \frac{\Sigma|\%E|}{n}$$

where $|\%E|$ is the absolute-value percent error, and n is the number of observations.

The simulation procedure estimates the SSTR equation over a sample period, then uses actual values of the independent variables, to forecast SSTR for eight quarters (two years). Subsequent sample forecasts are generated by extending the sample period by one quarter and repeating the process. For example, the first equation is fit over the period 1974.1 to 1980.1; then SSTR is forecast for eight quarters, using actual values of the independent variables. The second equation is fit over the period 1974.1 to 1980.2 and forecast for eight quarters, then the equation is estimated over the period 1974.1 to 1980.3, and so on for the eight equations.

Table I.1 reports the estimated mean error for sales tax forecasts. The average error for a one-quarter-ahead forecast is $0.243 million. The minus signs on the estimated mean errors for quarters two to eight indicate the model over-predicts. The standard deviation measures the variability, or disbursement, of forecast error among the various samples for each quarter. For one quarter ahead the standard deviation is $3.806 million. The mean and standard deviations are sufficient statistics to enable estimates to be made of the population of forecast errors and probability statements for the one-to-eight-quarter forecast error. For example, the probability of the forecast error for eight quarters ahead being within one standard deviation above the mean and one standard deviation below the mean is 68.3 percent. The probabilities the forecast error will be within either two or three standard deviations above or below the mean are 95.4 percent and 99.7 percent.

Table I.2 reports the mean absolute percent error for the set of simulations over the same period as the preceding analysis. The mean absolute percent error ranges from 2.763 percent for one quarter ahead to 3.219 percent for four quarters ahead. Probability statements similar to those given above can be made, i.e., the probability of the mean absolute forecast error being within one standard deviation above or below the mean is 68.3 percent.

3. Forecasting Error

Ex ante forecast error analysis involves measuring the error when using forecast values of the independent variables to produce a forecast of the dependent variables. The source of error is greater than **ex post** error and originates from errors in the estimated relationship and the independent variable estimation. There are three levels or sources of forecast error in the model currently used: the error that comes from the Data Resources, Inc. forecast of the national economic variables that drive the Iowa model; the forecast error that comes from the forecasts of the Iowa econometric model that drives the tax models; and the forecast error that comes from the tax receipts forecasting models. We will not attempt to explicitly partition the error associated with the tax forecasts that originate from the Iowa and national models, for it is of no great importance for our purposes here; however, it is of course of great importance to the forecast analyst. In this section we report the tax receipts forecast error since the beginning of the

Table I.1 Estimated Mean Error
For Sales Tax Forecasts

Variable	OBS	Mean	Std. Error of Mean	T	Std. Dev.	Min. Value	Max. Value
SSTR1Q	24	0.243	0.776	0.31	3.806	-11.192	8.044
SSTR2Q	23	-0.158	0.922	-0.17	4.425	-16.008	6.868
SSTR3Q	22	-0.102	1.001	-0.10	4.699	-15.803	9.132
SSTR4Q	21	-0.232	1.048	-0.22	4.807	-15.769	8.976
SSTR5Q	20	-0.128	1.094	-0.12	4.895	-16.059	9.035
SSTR6Q	19	-0.295	1.122	-0.26	4.893	-15.547	9.132
SSTR7Q	18	-0.434	1.148	-0.38	4.875	-15.233	8.895
SSTR8Q	17	-0.307	1.189	-0.26	4.875	-15.105	8.735

Table I.2 Estimated Mean Absolute Percent Error
For Sales Tax Forecasts

Variable	OBS	Mean	Std. Error of Mean	T	Std. Dev.	Min. Value	Max. Value
ASSTR1Q	24	2.763	0.523	5.28	2.56	0.085	11.192
ASSTR2Q	23	2.801	0.704	3.98	3.37	0.018	16.008
ASSTR3Q	22	3.093	0.740	4.18	3.47	0.090	15.803
ASSTR4Q	21	3.219	0.764	4.21	3.50	0.151	15.769
ASSTR5Q	20	3.071	0.838	3.67	3.74	0.046	16.059
ASSTR6Q	19	3.177	0.839	3.79	3.65	0.097	15.547
ASSTR7Q	18	3.073	0.880	3.49	3.73	0.104	15.233
ASSTR8Q	17	3.081	0.909	3.39	3.75	0.135	15.105

project. We also report the error on Data Resources, Inc. forecasts of GNP (Gross National Product) primarily to put the level of forecast error of the tax receipts forecast in perspective.

Tax Model Forecast Error

So far in this discussion no mention has been made of the common practice in forecasting of making judgmental adjustments to the actual forecast produced by the forecast models. This may take place at all stages of the modeling process. The adjustments typically involve smoothing of the data, constraining the forecast, or explicitly adding additional information to the model which is not in the historical data (such as a strike or the settlement of a strike). The Iowa economic forecast is adjusted by the Institute prior to meetings with the Forecasting Council and may be further adjusted based on consensus judgments by the Council. Similarly, the tax forecast is an adjusted forecast.

Plots of the forecast error for one and two years ahead are shown in Figure I.2. The largest error for the one-year-ahead forecast is 6.4 percent. The mean-absolute-forecast error for the twelve-year period is 2.1 percent. The largest error for the two-year-ahead forecast is 11.6 percent and occurred in 1981. The mean-absolute-forecast error is 3.4 percent and is larger, as expected, than the one-year-ahead forecast error. The largest errors in both the one- and two-year-ahead forecast occurred during the 1980-82 recession and involved over-forecasts of tax revenues. This clearly points out the difficulty and hazards of forecasting during turning points.

Figure I.2

Iowa Tax Receipts Forecast Error, Fiscal Years 1975-1986

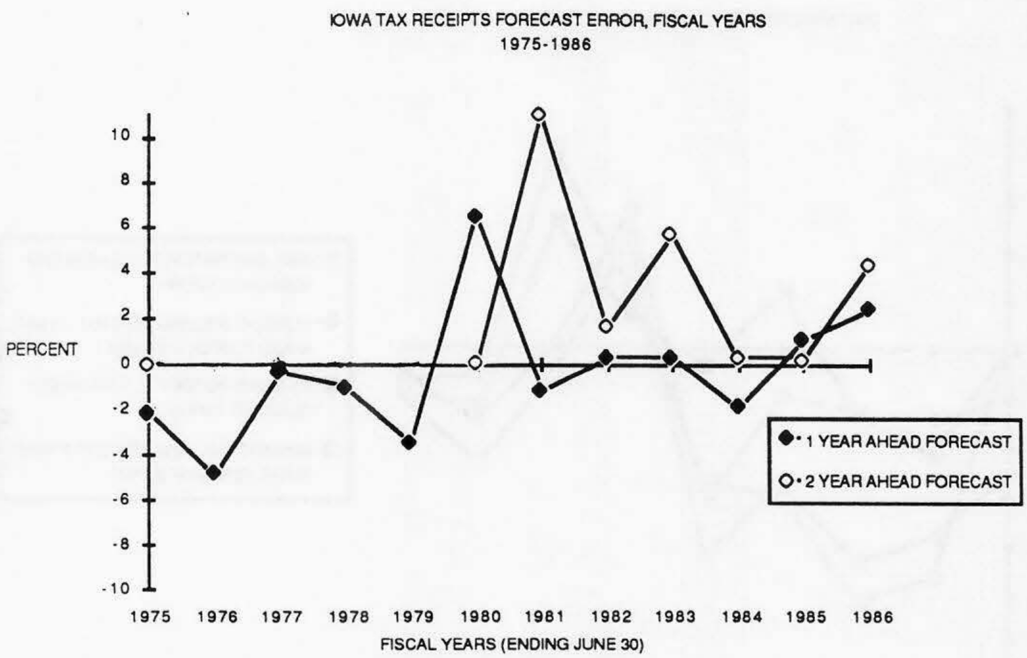
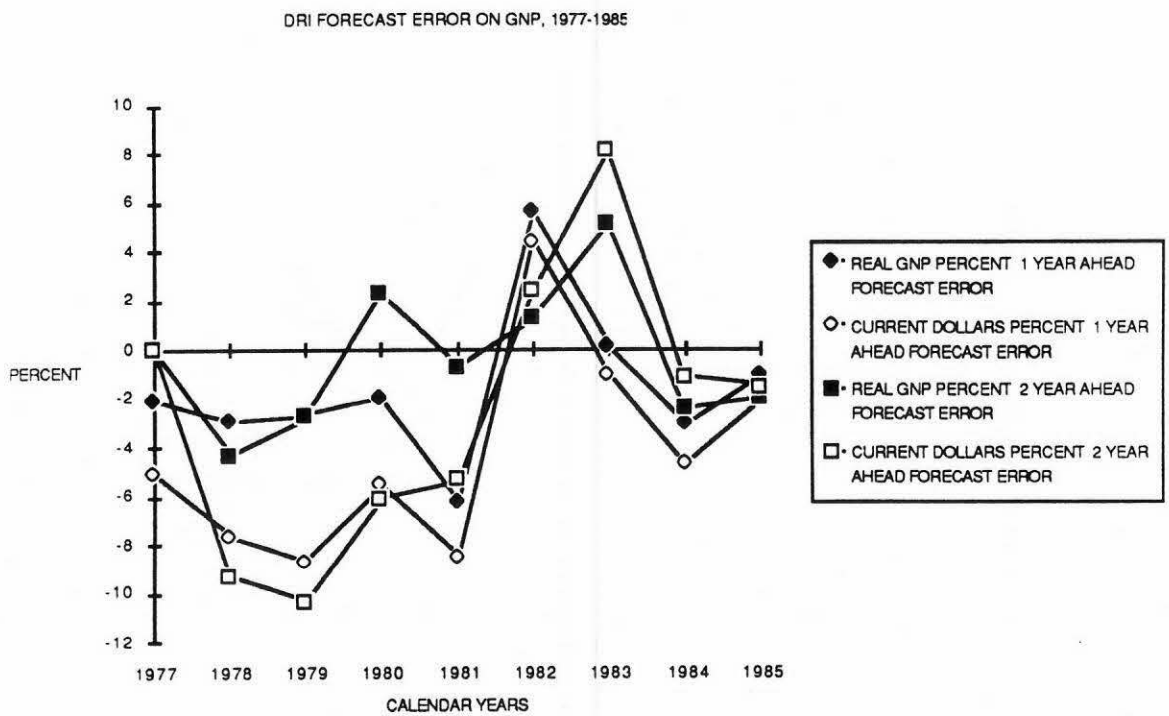


Figure I.3

DRI Forecast Error on GNP, 1977-1985



GNP Forecast Error

The forecast error from Data Resources, Inc.[5] forecasts of real and nominal GNP for one-and two-year-ahead forecasts are presented in Figure I.3. In both the one- and two-year-ahead forecasts, the error on real GNP is about one-half the error of the nominal GNP series. This arises because in forecasting nominal GNP the forecaster is also forecasting changes in prices. The range of error in forecasting real GNP one year ahead is from -6.1 percent in 1981 to 0.1 percent in 1983 with a mean absolute value of 2.8 percent. The range of error for one-year-ahead forecasts of nominal GNP was from -8.6 percent in 1979 to -0.9 in 1983 with a mean absolute value of 5.2 percent. The mean absolute value forecast error for real GNP two-years ahead is 2.6 percent. The two-year-ahead forecast of nominal GNP has a -10.2 percent error for 1979, indicating forecasters considerably under-forecast the rate of increase in prices in 1979. The 8.1 percent error over-forecast in 1983 indicates forecasters likewise over-forecast the level of price increases for 1983.

4. Forecasting Error and Budget Making

The observed forecasting error and the statistical parameters of the models provide important information that can be incorporated into the final forecast of tax revenue used for budget making.[6] To explicitly take into account the statistical parameters of the model, and especially the standard

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[5]In a recent article in the Wall Street Journal, Charles Wolf, Jr., Dean of the RAND Graduate School, ranked major forecasting firms. Over the four-year period 1983 through 1986, Data Resources, Inc., ranked number one.

[6]Litterman and Supel (1983) also stress this point in an article dealing with Minnesota's revenue forecasts.

errors of the tax models, is to recognize that it is quite possible that there could be a revenue shortfall or surplus. Given the recent period of recession and shift of the state economy to a lower growth path, it seems particularly important to recognize that there is error in a forecast because of the tremendous pressure for available funds. It is when economic times are the most unfavorable and uncertain that there is pressure on state government to push the budget higher, while at the same time the probability of the forecast of tax revenue coming in on target is pushed to a lower level.

The estimated standard error of the tax forecast models provides important information that should be evaluated in settling on a final forecast for budget making. The one-standard-error bands of the residual error for the tax equations in Appendix I can be used to give an interval estimate with a probability assuming no error in the independent variables. For example, the summation of the standard errors of the tax equations totals $22.430 million for a given quarter. Accordingly, it can be inferred that roughly two-thirds of the time tax receipts will be within ±$22.430 million of the forecast value. To apply a probability of 0.95 percent implies widening the interval to ±2 standard errors. The tax-revenue-forecast error history provides relevant information and a reason for caution when there is concern that the economy is entering a major turning point.

The above discussion, along with the fact that budget shortfalls tend to cause more problems than surpluses, leads public decision makers to choose what is deemed to be a conservative forecast of tax revenue for budget planning. Nevertheless, as mentioned above there is a strong tendency to push the budget up in periods of economic adversity and at the same time to want the forecast upon which the budget is based to have a high probability of being accurate. This is, of course, inconsistent with sound budget planning and needs to be recognized as being so.

The fact that revenue shortfalls from time to time are a reality, in spite of diligent efforts directed at accurate forecasting, calls for a reserve fund to be used for such shortfalls. The size of the fund is significant and involves balancing the costs of the fund--i.e., the taxes to create it--against the benefits to be derived from not having to cut the budget and government service levels. This problem is addressed in Section II.

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APPENDIX I

The tax receipt equations, statistics, plots of actual and predicted values, and definitions of variables are reported below.

Definition of Variables

Sales Tax

SSTR	=	quarterly sales tax receipts, time t;
WSD@IA	=	quarterly wage and salary income, time t;
WSD@IA(-1)	=	quarterly wage and salary income, time t-1;
D1, D2, D3	=	seasonal dummy variates for the first, second, and third quarters;
DS	=	dummy variable representing the change in tax rate from 3 to 4 percent, 1983.1;
DST	=	dummy variable representing the change in the tax base during 1985.3.

Use Tax

USETX	=	quarterly use tax receipts, time t;
USETX(-1)	=	quarterly use tax receipts, time t-1;
MAVG(4,WSD@IA(-1))	=	moving average over four quarters of wage and salary income, time t-1.

Income Tax

PYWTHTX	=	quarterly withholding tax receipts, time t;
PYWTHTX(-1)	=	quarterly withholding tax receipts, time t-1;
PYWTHTX(-2)	=	quarterly withholding tax receipts, time t-2;
D1, D2, D3	=	seasonal dummy variates for first, second, and third quarters;
WSD@IA	=	quarterly wage and salary income; time t;
WSD@IA(-1)	=	quarterly wage and salary income; time t-1.

Declared Estimates

PYDECESTTX	=	declared estimates tax receipts, time t;
MAVG(5,YPPROP@IA(-1))	=	moving average over five quarters of property income (dividends, interest, and rent), time t-1;
MAVG(5,YENTAFR@IA(-1))	=	moving average over five quarters of farm proprietors income, time t-1.
D1, D2, D3	=	seasonal dummy variates for first, second, and third quarters.

Returns

PYRETTX	=	quarterly tax return receipts, time t;
MAVG(5,YENTEAF@IA(-1) + YPPROP@IA(-1)	=	moving average over five quarters of nonfarm proprietors income plus property income (dividends, interest, and rent), time t-1;
MAUG(4,YENTAFR@IA(-1)	=	moving average over four quarters of farm proprietors income, time t-1;
D1, D2, D3	=	Dummy variates for first, second, and third quarters;
TIME	=	time variable.

Sales Tax Equation

```
SSTR=C1*WSD@IA+C2*(WSD@IA(-1))+C3*D1+C4*D2+
C5*D3+C6*DS+C7*DST+C8
```

	PARAMETER	T-STATISTIC	STD ERROR	MEAN	ELASTICITY	PARTIAL
C1)	7.25431	1.98038	3.66309	14.120	.9671	.29224
C2)	-1.17872	-.32979	3.57411	13.935	-.1551	-.05082
C3)	-4.15395	-2.41834	1.71769	.240	-.0094	-.34961
C4)	-11.34698	-6.58339	1.72358	.240	-.0257	-.71264
C5)	-5.35480	-3.13087	1.67838	.260	-.0129	-.43500
C6)	33.93983	17.09737	1.98509	.300	.0961	.93508
C7)	8.20740	3.49126	2.35084	.120	.0093	.47427
C8)	13.82927	3.33132	4.15128	1.000	.1306	.45717

```
VARIANCE=  .18279710E+02          DEPENDENT MEAN= 105.92
STANDARD ERROR=    4.27547800      PERCENT ERROR=  4.0
R-SQUARE=  .9853             R-BAR-SQUARE=  .9829
LLF=  -139.23288
F TEST(  7, 42)=      402.6156
DW STATISTIC=  1.99034  WALLIS-DW( 4) STATISTIC=  1.56473
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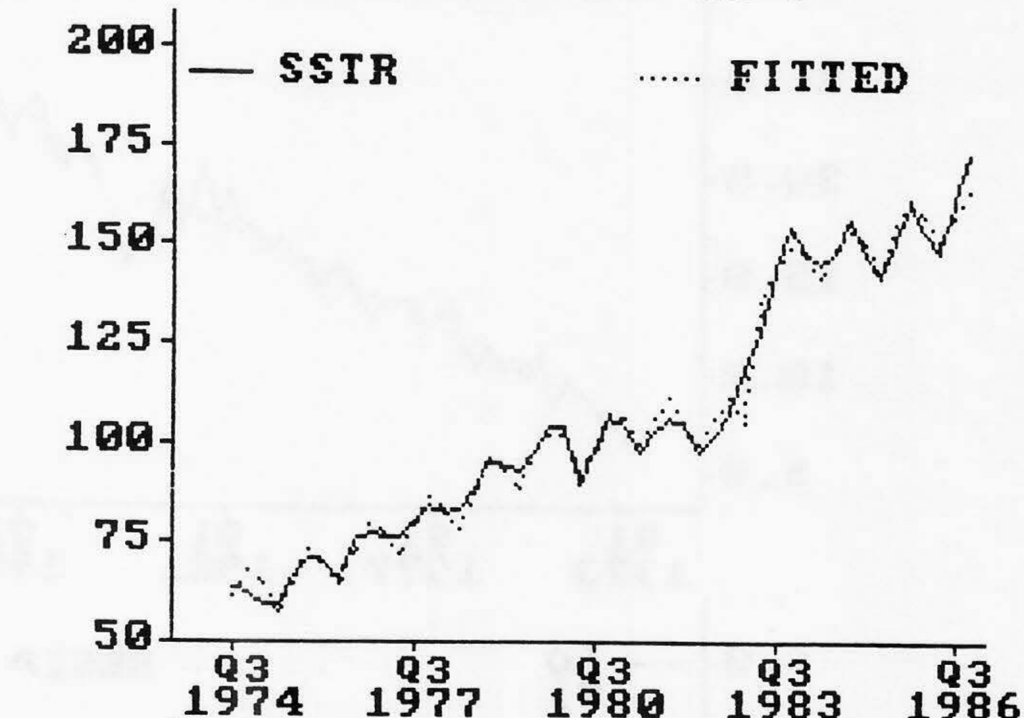
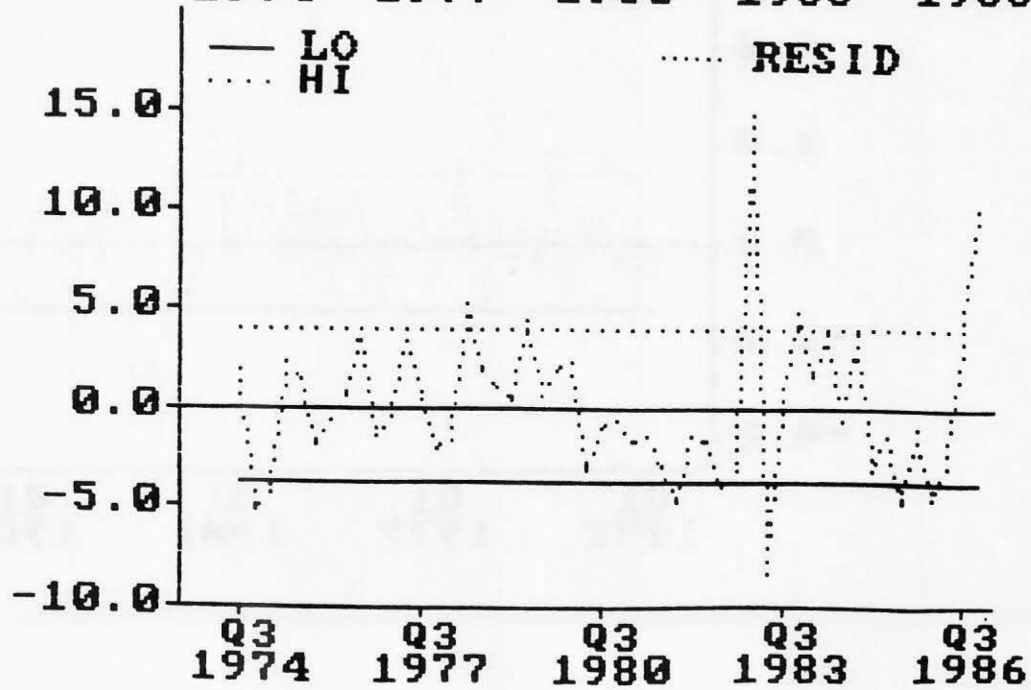


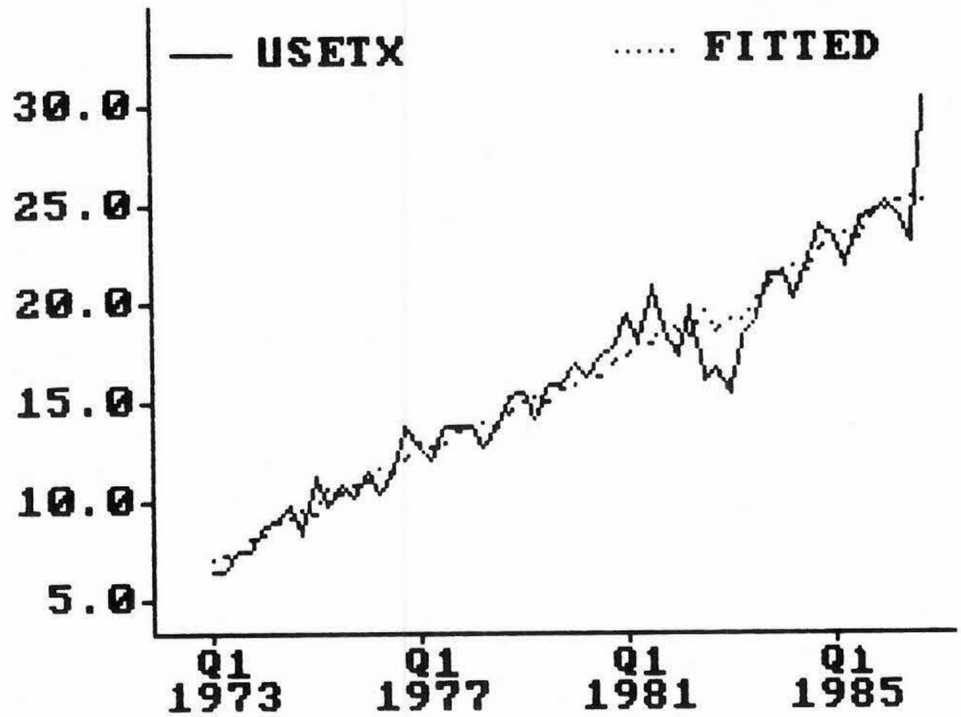


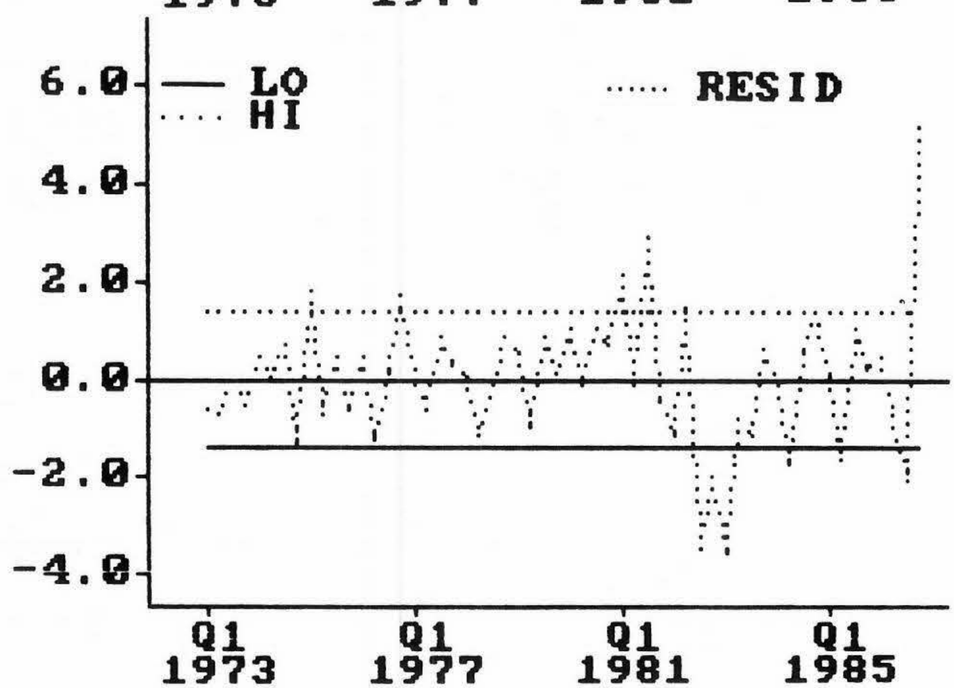
Use Tax Equation

```
USETX=C1*(USETX(-1))+C2*(MAVG(4,WSD@IA(-1)))+C3*TIME+
C4
```

	PARAMETER	T-STATISTIC	STD ERROR	MEAN	ELASTICITY	PARTIAL
C(1)	.31226	2.03513	.15343	15.484	.3040	.27161
C(2)	-.36310	-1.23331	.29441	12.950	-.2957	-.16858
C(3)	.30718	3.92359	.07829	32.500	.6277	.47794
C(4)	5.[illegible]8811	3.02236	1.91510	1.000	.3639	.38655

```
VARIANCE=  .21181440E+01          DEPENDENT MEAN= 15.904
STANDARD ERROR=     1.45538400      PERCENT ERROR=  9.2
R-SQUARE=  .9349             R-BAR-SQUARE=  .9311
LLF=    -98.40066
F TEST(  3, 52)=     248.7661
DW STATISTIC=  1.88082  WALLIS-DW( 4) STATISTIC=  1.47127
```





Withholding Tax Equation

```
PYWTHTX=C1*(PYWTHTX(-1))+C2*(PYWTHTX(-2))+C3*D1+C4*D2+
C5*D3+C6*WSD@IA+C7*(WSD@IA(-1))+C8
```

	PARAMETER	T-STATISTIC	STD ERROR	MEAN	ELASTICITY	PARTIAL
C1)	.60510	4.70858	.12851	115.140	.5925	.55425
C2)	.31452	2.59924	.12100	112.775	.3016	.34502
C3)	3.10357	2.78096	1.11601	.241	.0064	.36600
C4)	-3.29786	-2.83437	1.16353	.241	-.0068	-.37206
C5)	-.86491	-.68528	1.26213	.259	-.0019	-.09646
C6)	6.73182	2.75458	2.44387	13.201	.7557	.36299
C7)	-5.65331	-2.15157	2.62753	13.012	-.6256	-.29110
C8)	-2.58606	-.91000	2.84182	1.000	-.0220	-.12764

```
VARIANCE=  .89159723E+01          DEPENDENT MEAN= 117.59
STANDARD ERROR=     2.98596300      PERCENT ERROR=  2.5
R-SQUARE=  .9961            R-BAR-SQUARE=  .9956
LLF=  -141.44174
F TEST(  7, 50)=    1832.7690
DW STATISTIC=  2.09981  WALLIS-DW( 4) STATISTIC=  2.18604
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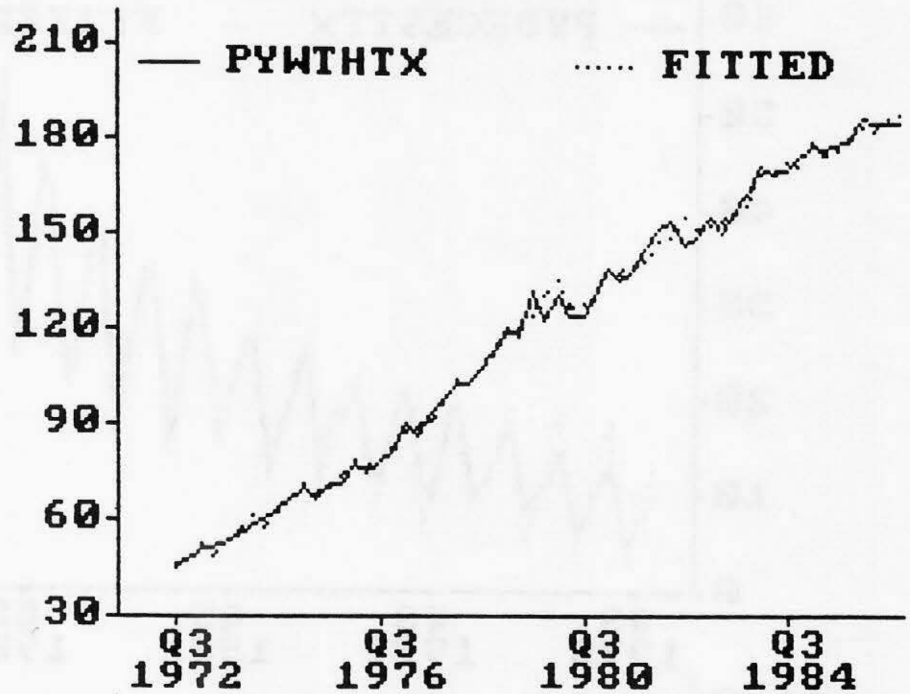

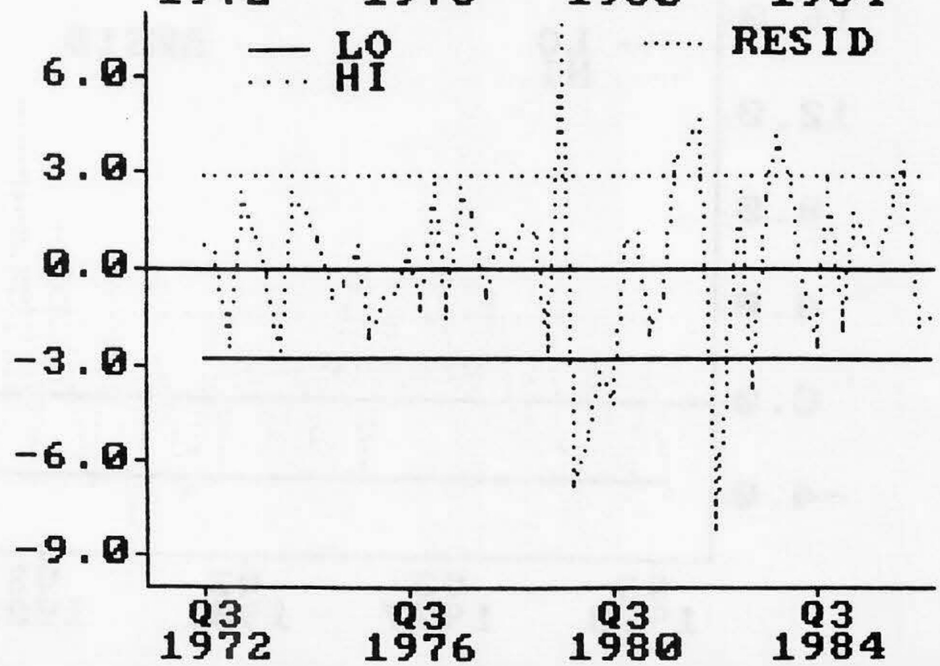

Declared Estimates Equation

```
PYDECESTTX=C1*(MAVG(5,YPPROP@IA(-1))*5)+C2*(MAVG(5,YENTAFR@IA(-1))*
5)+C3*D1+C4*D2+C5*D3+C6
```

	PARAMETER	T-STATISTIC	STD ERROR	MEAN	ELASTICITY	PARTIAL
C1)	.68292	12.80137	.05335	27.589	.9306	.87739
C2)	.00213	.01421	.14956	3.103	.0003	.00203
C3)	14.45460	10.29901	1.40349	.236	.1687	.82705
C4)	11.82974	8.71509	1.35739	.255	.1487	.77965
C5)	4.12236	3.11568	1.32310	.255	.0518	.40664
C6)	-6.07785	-3.11784	1.94938	1.000	-.3002	-.40687

```
VARIANCE=  .12248794E+02          DEPENDENT MEAN= 20.247
STANDARD ERROR=     3.49982800     PERCENT ERROR=  .17E+02
R-SQUARE=  .9043            R-BAR-SQUARE=  .8946
LLF=  -143.76427
F TEST(  5,  49)=      92.6406
DW STATISTIC=  1.96665  WALLIS-DW( 4) STATISTIC=    .66852
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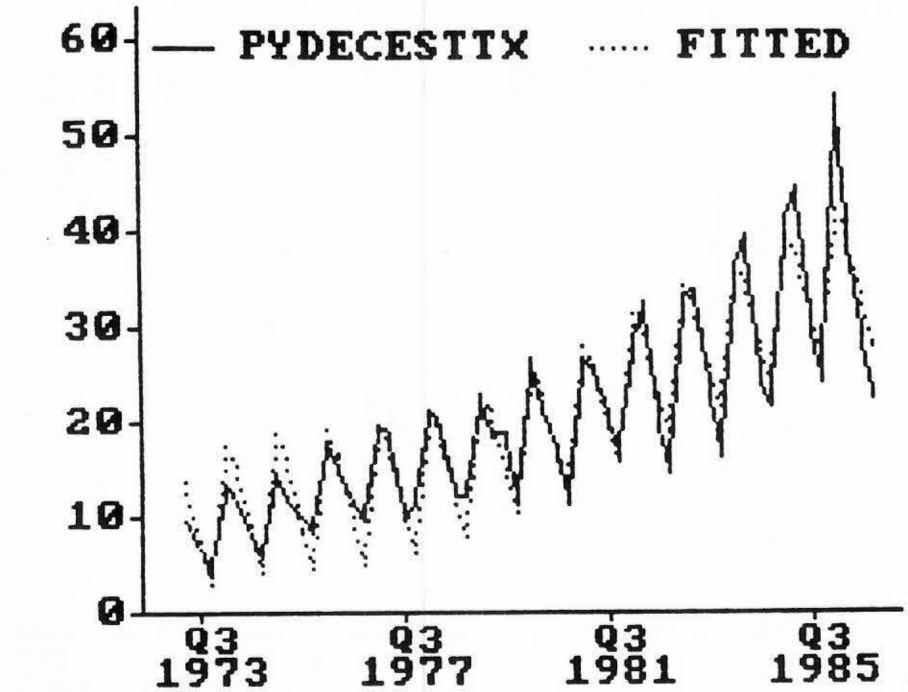

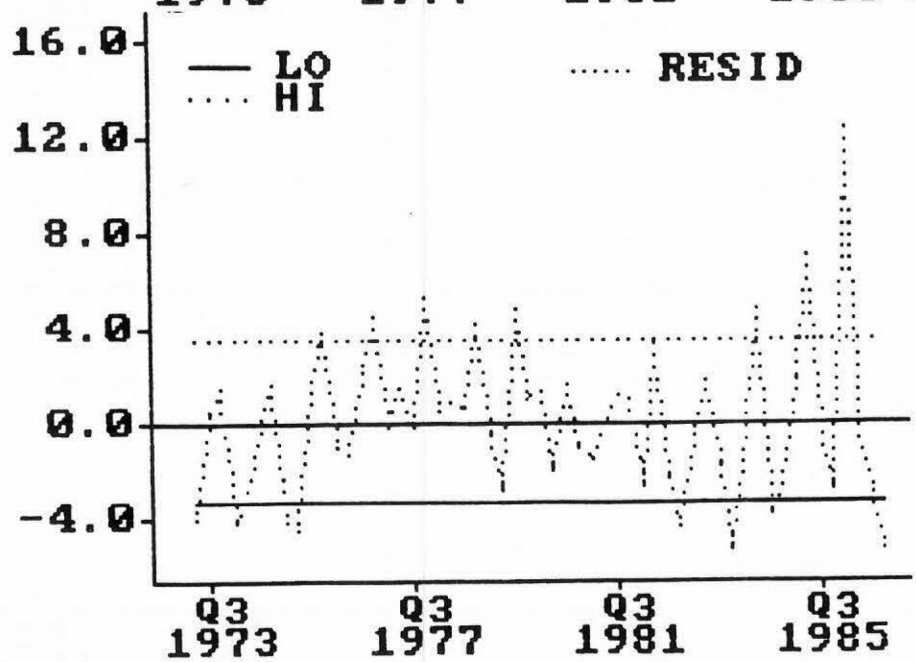

```
PYRETTX=C1*(D3*MAVG(5,YENTEAF@IA(-1)+YPFROF@IA(-1))*5)+C2*(D3*MAVG(4,
YENTAFR@IA(-1))*4)+C3*D1+C4*D2+C5*D3+C6*TIME+
C7

        PARAMETER      T-STATISTIC     STD ERROR       MEAN    ELASTICITY    PARTIAL
C1)      -.25290        -.90924          .27815        9.134     -.0715      -.13012
C2)       .05222         .05503          .94897         .595      .0010       .00794
C3)     59.30732       15.07329         3.93460         .236      .4339       .90862
C4)     57.27731       14.81940         3.86502         .255      .4513       .90589
C5)     10.08251         .82706        12.19074         .255      .0794       .11853
C6)       .36480        3.62709          .10058       33.000      .3726       .46381
C7)     -8.61533       -1.96912         4.37521        1.000     -.2667      -.27339

     VARIANCE=  .10428554E+03              DEPENDENT MEAN= 32.308
     STANDARD ERROR=  10.21203000           PERCENT ERROR=  .32E+02
     R-SQUARE=  .9030                R-BAR-SQUARE=  .8909
     LLF=  -202.09413
     F TEST(  6, 48)=       74.5127
     DW STATISTIC=  2.03637  WALLIS-DW( 4) STATISTIC=  .53434
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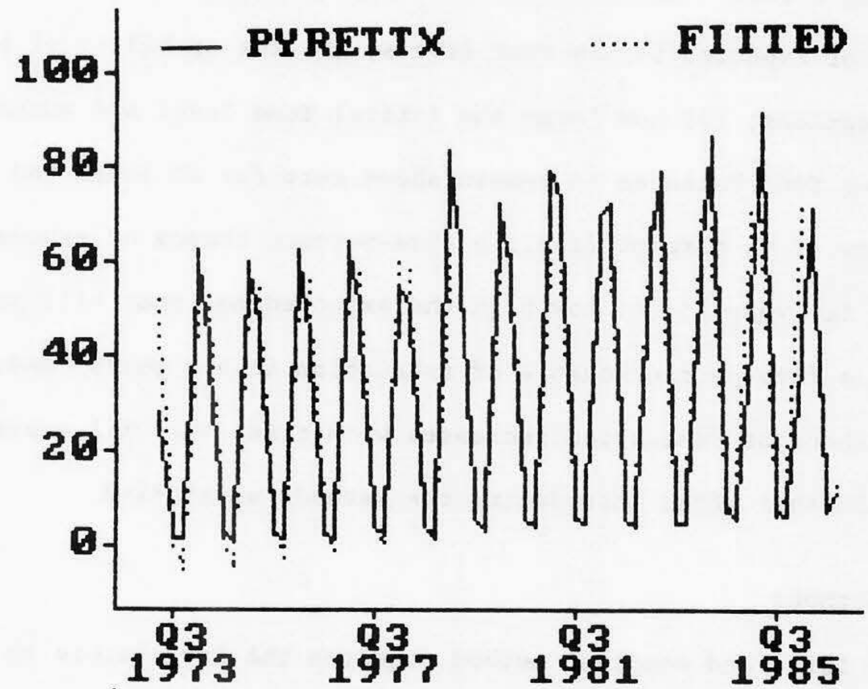

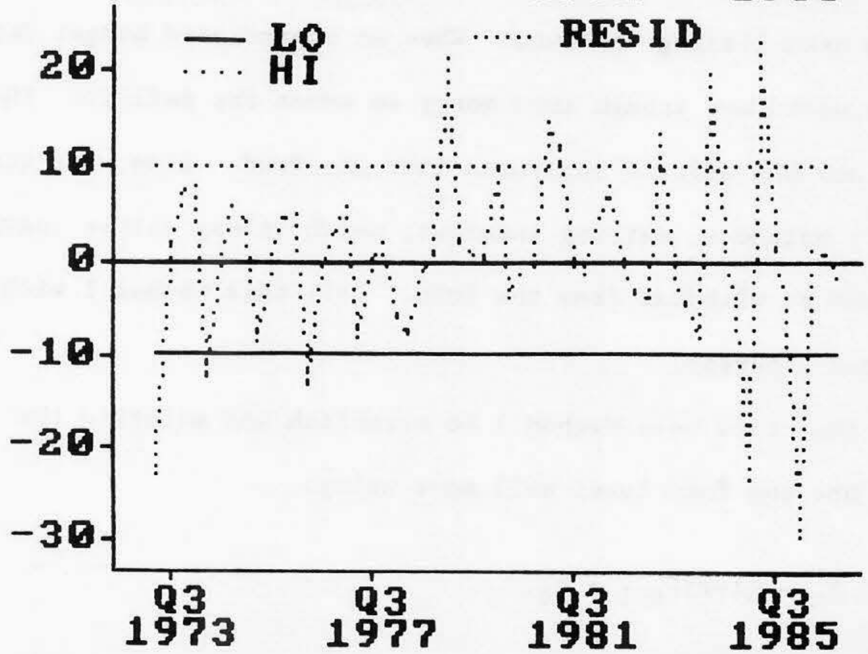

II.

MODELING OPTIMAL RESERVES

This section discusses three methods of establishing and maintaining a reserve fund to be used in the event of revenue shortfalls. These methods are compared using a computer program that simulates government revenue forecasting errors. The methods are compared using five criteria: (1) the stability of expected future fund levels; (2) the stability of expected future fund allocations; (3) how large the initial fund level and annual allocations must be for fund balances to remain above zero for at least ten years with a probability of 95 percent (i.e., a five-percent chance of exhaustion over a period of ten years); (4) how high the expected net cost will be to maintain a fund with a five-percent chance of exhaustion in ten years, and; (5) how fast the likelihood of exhaustion increases with time. We will explain these criteria further after introducing the methods we studied.

1. The Methods

The first and simplest method requires the legislature to allocate money only when establishing the fund. When an unpredicted budget deficit develops, the state withdraws enough fund money to cover the deficit. The state deposits any unpredicted surpluses into the fund. Also, the state will invest the fund's balances, earning interest, which it may either leave to accumulate in the fund or withdraw from the fund. Call this Method 1 with, or without, accumulated interest.

If the state uses Method 1 to establish and maintain the fund, we can describe how the fund level will move using:

(II.1) $F_t = (1+r)F_{t-1} + e_t$.

Here, F_t is the fund's balance at the end of period t, r is the real interest rate earned by and left in the fund and e_t is period t's budget forecast error. Deficit has developed when $e_t < 0$, and when $e_t > 0$ surplus has developed.[1] If the state allows the earned interest to accumulate, $r > 0$; if not, $r = 0$.

The second method requires lower initial fund levels, but annual legislative commitments. The state allocates money to establish the fund and withdraws money when paying for budget deficits. However, the state does not deposit surpluses into the fund. Instead, it maintains the fund by allocating a predetermined amount annually. Again, interest earnings may or may not be withdrawn from the fund. Call this Method 2 with, or without, accumulated interest.

If the state uses Method 2 to establish and maintain the fund, we can describe its movement using:

$$\text{(II.2)} \qquad F_t = (1+r)F_{t-1} + \text{MIN}(e_t, 0) + \text{ALOC}.$$

Here, F_t, e_t and r represent the same items as above. The annual fund allocation is ALOC and the function MIN {.,.} takes the smaller of the two arguments' values. Thus, when $e_t < 0$, the state withdraws money, covering the deficit, but the state does not deposit the surplus when $e_t > 0$. Again, if the state allows the interest to accumulate, $r > 0$; if not, $r = 0$.

The third method is more complicated than the first two, requiring even greater legislative commitment. The initial fund level is even lower, but the

[1]For this paper, we assumed that the budget error, e_t, is distributed normally with a mean of zero and a standard deviation of 3 percent of the annual state budget.

annual allocation varies. The state sets a target fund level and initially allocates enough money to achieve the target level. Then it withdraws money when covering deficits and deposits any surpluses.

The state maintains the fund by comparing the projected fund level with its target level and making an annual allocation or withdrawal, partly offsetting the fund's deviation from its target. For example, the government may choose to allocate or withdraw half of the fund's deviation from target. Then if the fund is \$5 million above the target, the state will withdraw \$2.5 million. If the fund is \$5 million below its target, the state will allocate \$2.5 million.

When using this method, the state may both allocate and withdraw fund money in a single year. The state may first withdraw money to partly offset a deviation and later deposit money from a budget surplus. Likewise, it may deposit money and later make a withdrawal.

If the state uses this method to establish and maintain the fund, we can describe its movement using:

$$(II.3) \qquad F_t = (1+r)F_{t-1} + p(F_0 - (1+r)F_{t-1}) + e_t.$$

Here, we define F_t, e_t and r the same as above. The initial fund level, and also its target level, is F_0. The projected fund level in period t is $(1+r)F_{t-1}$, so $(F_0 - (1+r)F_{t-1}$ is the fund's projected deviation from its target level and p is the proportion of this deviation that the state offsets annually. If the state chooses to offset none of the fund's deviation from target, or $p = 0$, this method becomes Method 1. In this paper, we assumed that the state allocates or withdraws half of the deviation annually, or $p = 0.5$.

Again, if the state allows accumulated interest, $r > 0$; and if not, $r = 0$. Call this Method 3 with, or without, accumulated interest.

2. **Basis of Comparison**

Fund Stability

We evaluate the fund's future stability using the fund's expected value and variance. While we cannot predict any future fund's level with certainty, we can make a best guess at its value. This guess is the fund's expected value. The fund is just as likely to fall above as below its expected value, but on the average, it will equal its expected value. For stability, we would like the fund's expected value to remain constant over time. In finding the expected fund values, we assumed the government forecasts revenues and expenses correctly on the average and tries to balance the budget based on these forecasts.

Method 1's expected value is stable if the government withdraws the fund's interest earnings. If not, the fund will tend to grow over time as these earnings accumulate.

Using past budget forecast errors, we can predict how large Method 2's annual allocations must be to expect a stable fund level. If the government withdraws the fund's interest earnings, the annual allocation will necessarily be larger than if the interest earnings are not withdrawn.

Using Method 3, the fund level will be stable at the target level if the government withdraws the interest earnings. If not, the fund will stabilize at a slightly higher level.

The fund's variance is a measure of how closely the fund reaches its expected value on the average. A low variance means that we are reasonably sure the fund will fall close to its expected value and so is better than a high variance. Because we cannot predict the distant future as accurately as the near, all the funds show increased variance with time. However, Method 3 will have a smaller variance than Method 1 and will likely have a smaller variance than Method 2.[2]

Allocation Stability

Because the government must include future fund allocations in predicted expenses, known annual allocations may be preferred to unknown ones. The allocations of Methods 1 and 2 are fixed and known in advance. Method 3's allocations vary with the fund level and cannot be known in advance. This is Method 3's major disadvantage.

Initial Fund and Annual Allocation Levels

Using the computer simulation, we found initial fund levels and annual allocation levels that were large enough to give each method a five-percent chance of exhaustion in ten years. Table II.1 shows these values. Because Method 3's annual allocations vary, Table II.1 shows the value to which the expected annual allocation converges. We quote each allocation as a percent of the current state budget, not an actual dollar figure. These initial fund levels are large fractions of the state budget, from 7.52 to 16.82 percent.

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[2]In all methods, the fund levels follow a random walk, their variances increasing with time. In Method 1, the fund level deviates from its stable value by a sum of the budget errors. In Method 3, it deviates from its stable value by a weighted sum of these errors. Method 3 has a lower variance than Method 1 if $0 < p < 1$, where p is the proportion of the projected fund error offset annually. If p is large enough, Method 3 will also have a lower variance than Method 2.

To achieve the same chance of exhaustion, Method 3 requires the lowest initial fund level, while Method 1 requires the highest. Method 2 requires the highest expected annual allocation, but caution should be exercised when judging this figure. It is higher because the government does not deposit budget surpluses into the fund. On the average, the budget surpluses would just offset the required annual allocation. Method 3 with interest accumulation has an expected annual withdrawal equalling p times the expected interest accumulation.

One must also exercise caution when comparing initial fund levels between methods that accumulate interest and those that do not. Recall that the state will withdraw and use the earned interest if interest is not allowed to accumulate. Thus, we cannot clearly choose between methods with interest accumulation and methods without interest accumulation based solely on the initial allocation levels.

Table II.1 Initial Fund Levels, Allocations and Expected Net Costs for Achieving a Five-Percent Chance of Bankruptcy in Ten Years.

Method	Init. Alloc.	Annual Alloc.	Exp. 10 Year Fund Level **	Exp. 10 Year Net Cost ***
1 w/o Acc. Interest	16.82%	0.00%	16.82%	0.00%
1 w/ Acc. Interest	14.14%	0.00%	19.01%	0.00%
2 w/o Acc. Interest	11.00%	1.20%	11.00%	10.21%
2 w/ Acc. Interest	12.25%	0.87%	12.25%	10.21%
3 w/o Acc. Interest	7.90%	0.00%*	7.90%	0.00%
3 w/ Acc. Interest	7.52%	-0.12%*	7.75%	0.00%

* Expected Annual Allocation
** The funds expected value in the tenth year
*** The total expected net cost for creating the fund and maintaining the fund for ten years.

Expected Net Cost

Creating and maintaining a fund for a period will cost the state the initial allocation plus each annual allocation's value. The state will receive the value of any interest withdrawals and the fund's value at the period's end. We discount each annual allocation, interest withdrawals and the final fund level for time, recognizing that future income (or expenses) are not as valuable (or costly) as current income (or expenses). As we cannot know any final fund level, interest payments, not Method 3's future allocations, we use their expected values to calculate these costs.

To compare methods, we find the initial fund levels and allocations that allow each method to achieve a five-percent chance of exhaustion in ten years according to the computer simulation. We then find each method's expected net costs by totaling the method's expected cost (discounting expected future allocations) and subtracting its discounted expected interest earnings and final fund level.

Table II.1 gives each method's expected net costs. Notice that the expected net costs of Methods 1 and 3 are zero, showing that the expected income and final values of these methods exactly offset their costs. Method 2's expected net cost is above zero. This difference results from the state not depositing surpluses. If we considered the benefit of using these surpluses, they would just offset Method 2's extra cost on the average.

Chance of Exhausting Fund

As time passes, the chances that the state will exhaust the fund increase. We would like to minimize this rate of increase. Figures II.1 through II.6 show how the chances of exhaustion increase with time for various

FIGURE II.1: METHOD 1 WITHOUT INTEREST ACCUMULATION

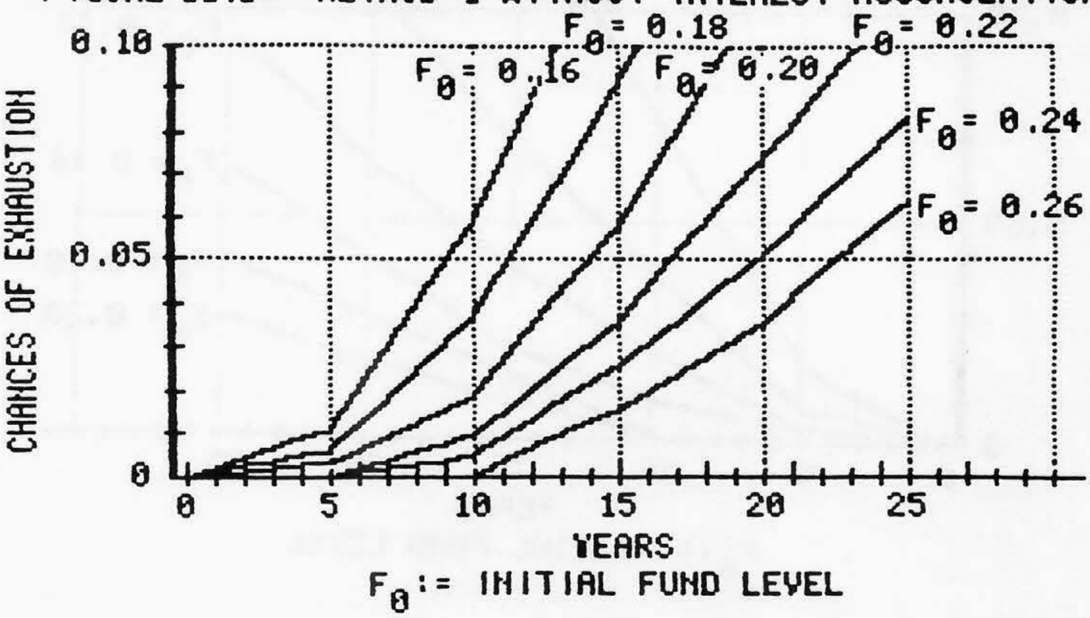


FIGURE II.2: METHOD 1 WITH INTEREST ACCUMULATION

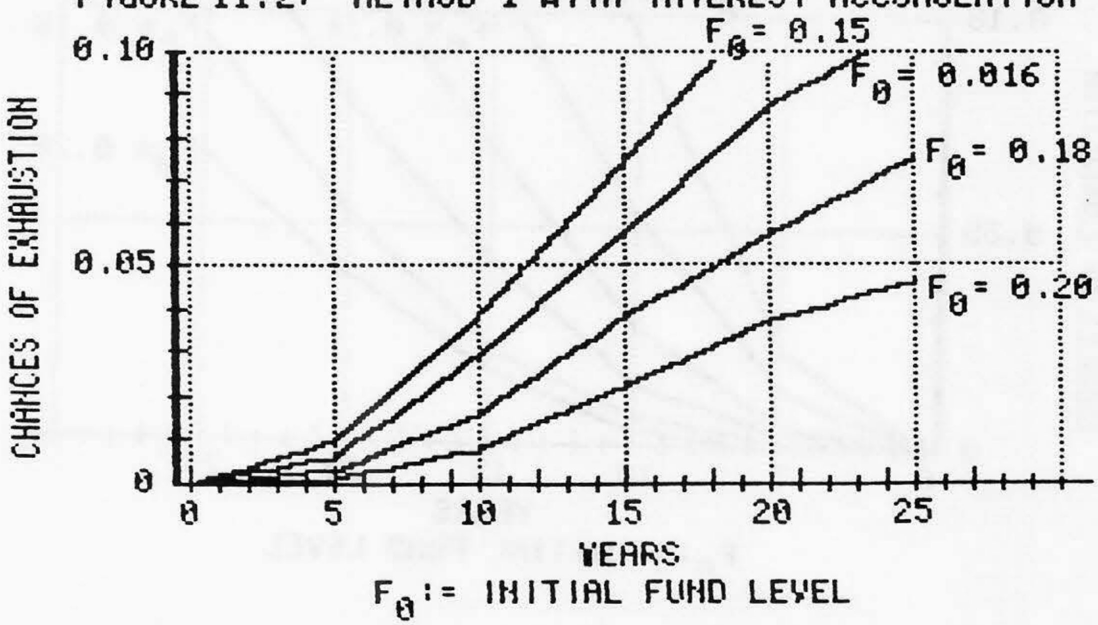


FIGURE II.3: METHOD 2 WITHOUT INTEREST ACCUMULATION

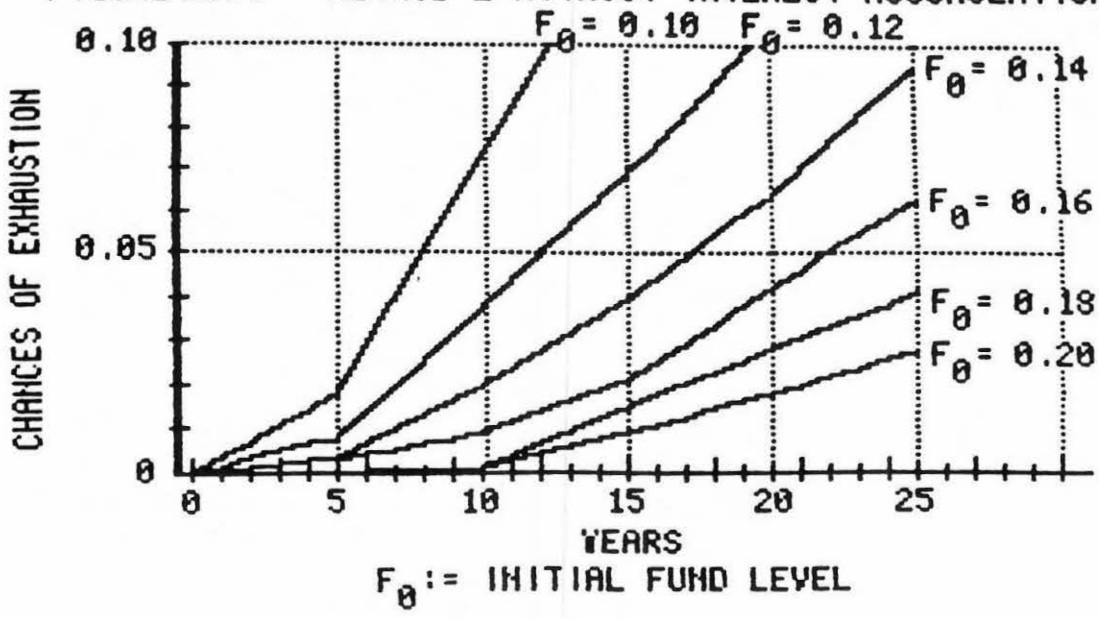
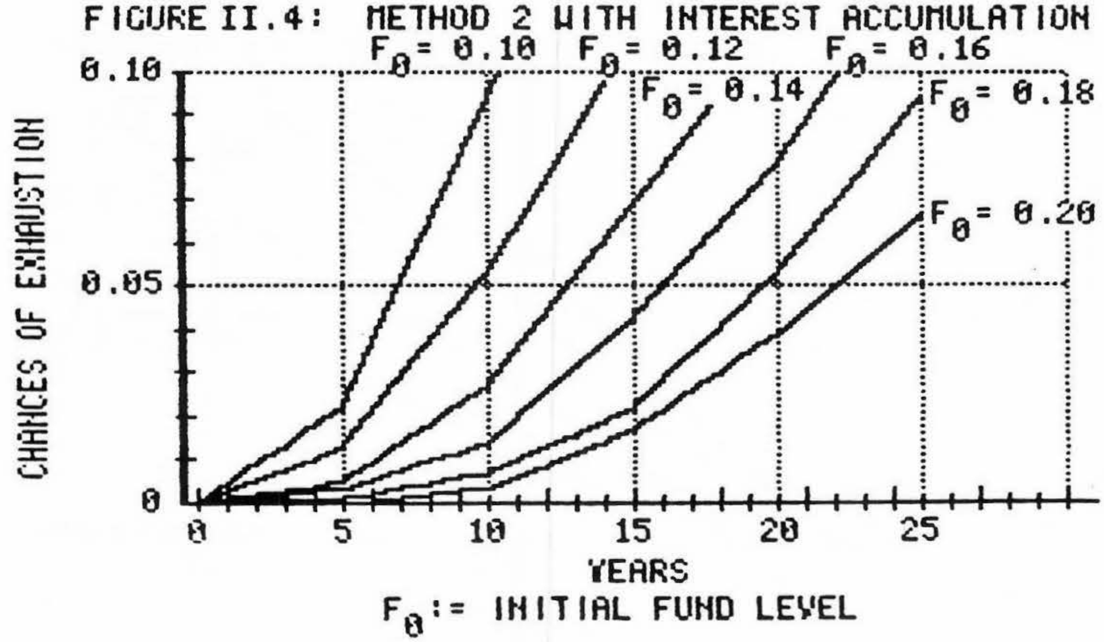


FIGURE II.4: METHOD 2 WITH INTEREST ACCUMULATION

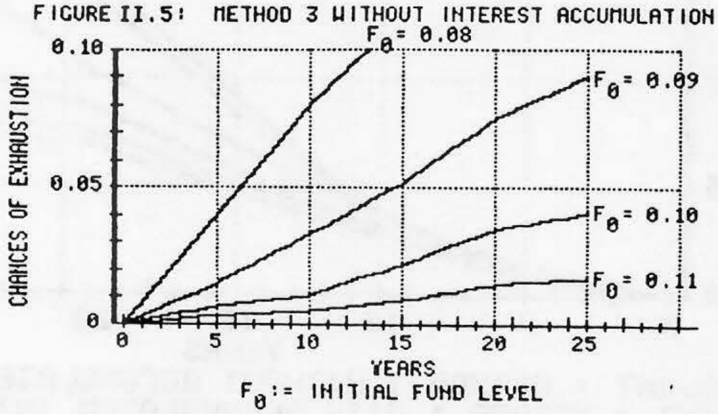


configurations of all methods. Figure II.1 shows the chances of exhaustion for Method 1 without interest accumulation. Figure II.2 shows the chances for Method 1 with interest accumulation. Figures II.3 and II.4 show Method 2 without and with interest accumulation, and Figures II.5 and II.6 do the same for Method 3.

Figures II.1 and II.2 show that Method 1's chances of exhaustion increase more rapidly when the state does not accumulate interest. Figures II.3 and II.4 show that Method 2's chances of exhaustion increase more rapidly when the state does accumulate interest.[3] Figures II.5 and II.6 show that Method 3 with accumulated interest differs little from Method 3 without accumulated interest.

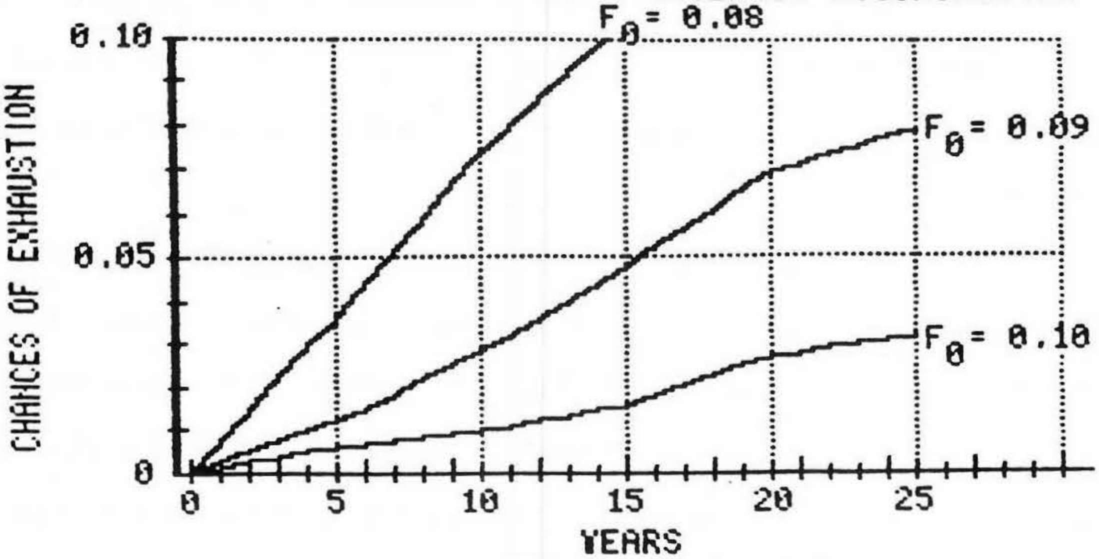
In Figure II.7, we have plotted sample configurations for each method. Each plotted configuration has a ten-year chance of exhaustion of between 2.5 and 4.0 percent. The plots how that for similar ten-year chances of exhaustion, Method 1's chances of exhaustion when interest is not accumulated grow the fastest, followed by Method 1's with accumulated interest. The next fastest are method 2's with, and without, accumulated interest. Method 3's chances of exhaustion when interest is accumulated grows the slowest and, when interest is not accumulated, the next slowest.

FIGURE II.5: METHOD 3 WITHOUT INTEREST ACCUMULATION



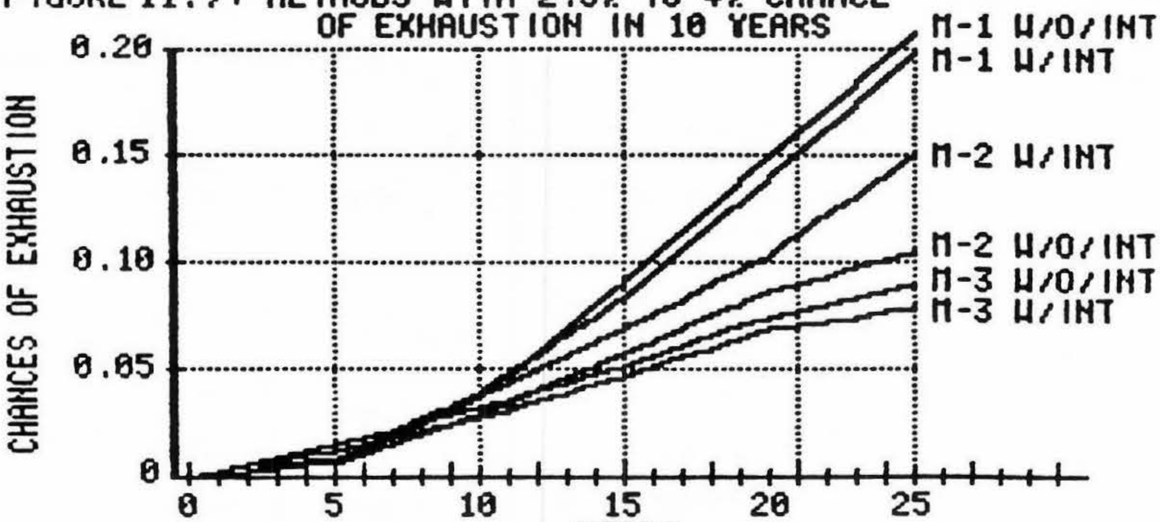
3

FIGURE II.6: METHOD 3 WITH INTEREST ACCUMULATION



F_0 := INITIAL FUND LEVEL

FIGURE II.7: METHODS WITH 2.5% TO 4% CHANCE OF EXHAUSTION IN 10 YEARS



M-1 W/O/INT = METHOD 1 WITHOUT ACCUMULATED INTEREST
M-1 W/INT = METHOD 1 WITH ACCUMULATED INTEREST, ETC.

3. Conclusions

While there is no clearly best way to establish and maintain a fund to pay for unpredicted budget deficits, some methods we studied are better than others. Recall that, using method 1, the state allocates money only when establishing the fund and then withdraws money for deficits and deposits money from surpluses. Using Method 2, the state allocates money once to establish the fund and annually thereafter to maintain the fund. It withdraws money for deficits, but does not deposit money from surpluses. Using Method 3, the state allocates money to establish the fund and deposits or withdraws money annually thereafter to partly offset the fund's deviations from target. It withdraws money for deficits and deposits money from surpluses.

In order to maximize stability, the state should choose Method 1 without interest accumulation over Method 1 with interest accumulation. The state should also choose Method 2 without interest accumulation over Method 2 with accumulation, because the former requires lower initial fund levels and its chances of exhaustion increase at slower rates. On the average, the interest withdrawn offsets the higher annual allocation. The state should probably choose Method 3 with interest accumulation over Method 3 without accumulation because the former requires lower initial allocations and its chances of exhaustion increase at slightly slower rates. However, the expected annual withdrawal is only half of the expected interest withdrawal when using method 3 without interest accumulation.

The state should choose between the remaining methods (or some variation thereof) based on its own funding preferences and commitment.[4] After

[4]An example of a variation would be if the state decides to spread a method's initial allocation over several years.

considering that the state does not deposit surpluses when using Method 2, the expected net cost of all methods is zero. Also, considering this and interest withdrawals, the expected net annual allocations are very similar. If the state prefers to make only one allocation, it should choose Method 1. If it prefers lower initial allocations and stable annual allocations with the chances of exhaustion increasing more slowly, it should choose Method 2. Finally, if the state does not mind variable annual allocations, it may choose Method 3 because it requires even lower initial allocations, and its chances of exhaustion increase even more slowly.

APPENDIX II

1. The Variables

We define the variables used in this paper as:

F_t: = the fund level at the end of period t.

F_0: = the initial or target fund level.

r: = the real interest rate allowed to accumulate in the fund.

e_t: = the unpredicted state budget error.

$e_t > 0$ denotes an unpredicted surplus.

$e_t < 0$ denotes an unpredicted deficit.

p: = the proportion of the fund's deviation from target that the state offsets annually.

ALOC: = the annual fund allocation.

E(.): = the argument's unconditional expectation.

VAR(.): = the argument's unconditional variance.

v: = the variance of the budget error, e_t.

s: = the standard deviation of the budget error, e_t.

MIN(.,.1): = the smaller of the arguments' values.

2. Method 1

If the state uses Method 1 to establish and maintain the fund, we can describe the fund's movement by:

$$F_t = (1+r)F_{t-1} + e_t.$$

Future expected fund levels are given by:

$$E(F_t) = (1+r)E(F_{t-1}) = (1+r)^t F_o,$$

Future Fund variances are given by:

$$VAR(F_t) = v \left| \sum_{n=1}^{t} (1+r)^{t-n} \right|$$

3. **Method 2**

If the state uses Method 2 to establish and maintain the fund, we can describe its movement using:

$$F_t = (1+r)F_{t-1} + MIN\ (e_t, 0) + ALOC.$$

Future expected fund levels are given by:

$$E(F_t) = (1+r)E(F_{t-1}) + ALOC - \frac{s}{(2\pi)^{0.5}}.$$

For stability, ALOC is chosen to be:

$$ALOC = \frac{s}{(2\pi)^{0.5}} - rF_0.$$

Then $E(F_t) = E(F_{t-1}) = F_0$.

4. **Method 3**

If the state uses Method 3 to establish and maintain the fund, we can describe the fund's movement using:

$$F_t = (1+r)F_{t-1} + p(F_0 - (1+r)F_{t-1}) + e_t$$

$$= (1+r)^t(1-p)^t F_0 + \{ \sum_{n=1}^{t} [(1+r)(1-p)]^{(t-n)} pF_0 \}$$

Future expected fund levels are given by:

$$E(F_t) = (1+r)(1-p)E(F_{t-1}) + pF_0$$

$$= (1+r)^t(1-p)^t F_0 + \{ \sum_{n=1}^{t} [(1+r)(1-p)]^{(t-n)} (pF_0 + e_n) \}$$

Allowing t to increase, the limit of the expected fund level is:

$$\frac{pF_0}{1 - (1+r)(1-p)}.$$

Future fund level variances are given by:

$$VAR\ (F_t = v\{ \sum_{n=1}^{t} [(1+r)(1-p)]^{(t-n)} (pF_0 + e_n) \}$$

5. Computer Simulation Results

Method 1: $F_t = (1+r)F_{t-1} + e_t$

		r=0			r=0.03		
F0	Years	Mean Fund Level	Exhaustion Frequency	Exhaustion Variance	Mean Fund Level	Exhaustion Frequency	Exhaustion Variance
.05	5	.05	.343	.00023	.0580	.311	.00021
.05	10	.05	.504	.00025	.0672	.452	.00025
.05	15	.05	.580	.00024	.0779	.52	.00025
.05	20	.05	.644	.00023	.0903	.546	.00025
.05	25	.05	.685	.00022	.1047	.572	.00024
.1	5	.1	.083	.00008	.1159	.068	.00006
.1	10	.1	.226	.00017	.1344	.162	.00014
.1	15	.1	.333	.00022	.1558	.24	.00018
.1	20	.1	.400	.00024	.1806	.283	.0002
.1	25	.1	.461	.00025	.2094	.315	.00022
.12	5	.12	.046	.00004			
.12	10	.12	.159	.00013			
.12	15	.12	.252	.00019			
.12	20	.12	.325	.00022			
.12	25	.12	.388	.00024			
.13	5				.1507	.02	.00002
.13	10				.1747	.062	.00006
.13	15				.2025	.107	.0001
.13	20				.2348	.155	.00013
.13	25				.2722	.18	.00015
.14	5	.14	.025	.00002	.1623	.013	.00001
.14	10	.14	.102	.00009	.1881	.052	.00005
.14	15	.14	.193	.00016	.2181	.099	.00009
.14	20	.14	.252	.00019	.2529	.136	.00012
.14	25	.14	.313	.00022	.2931	.164	.00014
.15	5	.15	.014	.00001	.1739	.01	.00001
.15	10	.15	.080	.00007	.2016	.038	.00004
.15	15	.15	.155	.00013	.2337	.074	.00007
.15	20	.15	.222	.00017	.2709	.111	.0001
.15	25	.15	.277	.00020	.3141	.13	.00011
.16	5	.16	.011	.00001	.1855	.006	.00001
.16	10	.16	.059	.00006	.2150	.029	.00003
.16	15	.16	.129	.00011	.2493	.058	.00005
.16	20	.16	.201	.00016	.2890	.087	.00008
.16	25	.16	.249	.00019	.3350	.105	.00009

Method 1: $F_t = (1+r)F_{t-1} + e_t$

		r=0			r=0.03		
F0	Years	Mean Fund Level	Exhaustion Frequency	Exhaustion Variance	Mean Fund Level	Exhaustion Frequency	Exhaustion Variance
.18	5	.18	.006	.00001	.2087	.003	0
.18	10	.18	.037	.00004	.2419	.016	.00002
.18	15	.18	.091	.00008	.2804	.038	.00004
.18	20	.18	.149	.00013	.3251	.057	.00005
.18	25	.18	.207	.00016	.3769	.075	.00007
.2	5	.2	.003	0	.2319	.001	0
.2	10	.2	.019	.00002	.2688	.008	.00001
.2	15	.2	.058	.00005	.3116	.022	.00002
.2	20	.2	.112	.0001	.3612	.037	.00004
.2	25	.2	.1525	.00013	.4188	.046	.00004
.22	5	.22	0	0			
.22	10	.22	.01	.00001			
.22	15	.22	.036	.00003			
.22	20	.22	.074	.00007			
.22	25	.22	.112	.001			
.24	5	.24	0	0			
.24	10	.24	.005	0			
.24	15	.24	.026	.00003			
.24	20	.24	.051	.00005			
.24	25	.24	.084	.00008			
.26	5	.26	0	0			
.26	10	.26	0	0			
.26	15	.26	.016	.00002			
.26	20	.26	.036	.00003			
.26	25	.26	.064	.00006			
.28	5	.28	0	0			
.28	10	.28	0	0			
.28	15	.28	.009	.00001			
.28	20	.28	.028	.00003			
.28	25	.28	.045	.00004			

Method 2: $F_t = (1+r)F_{t-1} + MIN\{e_t,0\} + ALOC$

Initial and Mean Fund Level	Years	r=0 ALOC	r=0 Exhaustion Frequency	r=0 Exhaustion Variance	r=0.03 ALOC	r=0.03 Exhaustion Frequency	r=0.03 Exhaustion Variance
.06	5	.0120	.106	.00009			
.06	10	.0120	.224	.00017			
.06	15	.0120	.31	.00021			
.06	20	.0120	.385	.00024			
.06	25	.0120	.435	.00025			
.1	5	.0120	.018	.00002	.0090	.022	.00002
.1	10	.0120	.073	.00007	.0090	.093	.00008
.1	15	.0120	.129	.00011	.0090	.167	.00014
.1	20	.0120	.187	.00015	.0090	.239	.00018
.1	25	.0120	.23	.00018	.0090	.287	.0002
.11	5	.0120	.015	.0001			
.11	10	.0120	.05	.00005			
.11	15	.0120	.096	.00009			
.11	20	.0120	.144	.00012			
.11	25	.0120	.188	.00015			
.12	5	.0120	.008	.00001	.0084	.013	.00001
.12	10	.0120	.037	.00004	.0084	.053	.00005
.12	15	.0120	.069	.00006	.0084	.108	.0001
.12	20	.0120	.104	.00009	.0084	.171	.00014
.12	25	.0120	.149	.00013	.0084	.227	.00018
.135	5				.0079	.01	.00001
.135	10				.0079	.047	.00004
.135	15				.0079	.095	.00009
.135	20				.0079	.157	.00013
.135	25				.0079	.211	.00017
.13	5				.0081	.008	.00001
.13	10				.0081	.04	.00004
.13	15				.0081	.084	.00008
.13	20				.0081	.139	.00012
.13	25				.0081	.198	.00016
.14	5	.0120	.003	0	.0078	.005	0
.14	10	.0120	.02	.00002	.0078	.027	.00003
.14	15	.0120	.04	.00004	.0078	.069	.00006
.14	20	.0120	.064	.00006	.0078	.111	.0001
.14	25	.0120	.095	.00009	.0078	.169	.00014

Method 2: $F_t = (1+r)F_{t-1} + MIN\{e_t,0\} + ALOC$							
Initlial and Mean Fund Level	Years	r=0			r=0.03		
		ALOC	Exhaustion Frequency	Exhaustion Variance	ALOC	Exhaustion Frequency	Exhaustion Variance
.15	5	.0120	.003	0	.0075	.003	0
.15	10	.0120	.009	.00001	.0075	.022	.00002
.15	15	.0120	.023	.00002	.0075	.052	.00005
.15	20	.0120	.05	.00005	.0075	.096	.00009
.15	25	.0120	.077	.00007	.0075	.151	.00013
.16	5	.0120	.003	0	.0060	.003	0
.16	10	.0120	.009	.00001	.0060	.014	.00001
.16	15	.0120	.021	.00002	.0060	.043	.00004
.16	20	.0120	.042	.00004	.0060	.079	.00007
.16	25	.0120	.063	.00006	.0060	.127	.00011
.17	5	.0120	.001	0			
.17	10	.0120	.003	0			
.17	15	.0120	.018	.00002			
.17	20	.0120	.032	.00003			
.17	25	.0120	.05	.00005			
.18	5	.0120	0	0	.0060	.001	0
.18	10	.0120	.001	0	.0060	.007	.00001
.18	15	.0120	.015	.00001	.0060	.022	.00002
.18	20	.0120	.028	.00003	.0060	.053	.00005
.18	25	.0120	.041	.00004	.0060	.094	.00009
.2	5	.0120	0	0	.0060	0	0
.2	10	.0120	.001	0	.0060	.003	0
.2	15	.0120	.009	.00001	.0060	.017	.00002
.2	20	.0120	.018	.00002	.0060	.039	.00004
.2	25	.0120	.028	.00003	.0060	.067	.00006

Method 3: $F_t = (1+r)F_{t-1} + 0.5[F_t - (1+r)F_{t-1}] + e_t$

		r=0			r=0.03		
F0	Years	Mean Fund Level	Exhaustion Frequency	Exhaustion Variance	Mean Fund Level	Exhaustion Frequency	Exhaustion Variance
.05	5	.05	.249	.00019	.0514	232	.00018
.05	10	.05	.43	.00025	.0515	.413	.00024
.05	15	.05	.574	.00024	.0515	.548	.00025
.05	20	.05	.684	.00022	.0515	.66	.00022
.05	25	.05	.771	.00018	.0515	.747	.00019
Avg ALOC:			.00731			-.02919	
.06	5	.06	.139	.00012	.0617	.125	.00011
.06	10	.06	.275	.0002	.0618	.248	.00019
.06	15	.06	.381	.00024	.0618	.346	.00023
.06	20	.06	.493	.00025	.0618	.456	.00025
.06	25	.06	.588	.00024	.0618	.551	.00025
Avg ALOC:			.00731			-.03656	
.07	5	.07	.073	.00007	.0720	.065	.00006
.07	10	.07	.173	.00012	.0721	.132	.00011
.07	15	.07	.209	.00017	.0721	.194	.00016
.07	20	.07	.297	.00021	.0721	.275	.0002
.07	25	.07	.362	.00023	.0721	.337	.00022
Avg ALOC:			.00731			-.04395	

Method 3: $F_t = (1+r)F_{t-1} + 0.5[F_t - (1+r)F_{t-1}] + e_t$

FO	Years	r=0 Mean Fund Level	r=0 Exhaustion Frequency	r=0 Exhaustion Variance	r=0.03 Mean Fund Level	r=0.03 Exhaustion Frequency	r=0.03 Exhaustion Variance
.08	5	.08	.04	.00004	.0823	.036	.00003
.08	10	.08	.08	.00007	.0824	.073	.00007
.08	15	.08	.111	.0001	.0824	.104	.00009
.08	20	.08	.159	.00013	.0824	.148	.00013
.08	25	.08	.198	.00016	.0824	.184	.00015
Avg ALOC:			.00731			-.051535	
.09	5	.09	.015	.00001	.0926	.012	.00001
.09	10	.09	.033	.00003	.0927	.028	.00003
.09	15	.09	.052	.00005	.0927	.048	.00005
.09	20	.09	.075	.00007	.0927	.069	.00006
.09	25	.09	.09	.00008	.0927	.079	.00007
Avg ALOC:			.00731			-.05874	
.1	5	.1	.007	.00001	.1029	.006	.00001
.1	10	.1	.011	.00001	.1030	.01	.00001
.1	15	.1	.022	.00002	.1030	.016	.00002
.1	20	.1	.035	.00003	.1030	.027	.00003
.1	25	.1	.041	.00004	.1030	.032	.00003
Avg ALOC:			.00731			-.06615	
.11	5	.11	.003	0			
.11	10	.11	.005	0			
.11	15	.11	.009	.00001			
.11	20	.11	.015	.00001			
.11	25	.11	.017	.00002			
Avg ALOC:			.00731				