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THE VALUE ALLOCATION OF AN ECONOMY WITH DIFFERENTIAL INFORMATION

BY STEFAN KRASA AND NICHOLAS C. YANNELIS[1]

We analyze the Shapley value allocation of an economy with differential information. Since the intent of the Shapley value is to measure the sum of the expected marginal contributions made by an agent to any coalition to which he/she belongs, the value allocation of an economy with differential information provides an interesting way to measure the information advantage of an agent. This feature of the Shapley value allocation is not necessarily shared by the rational expectation equilibrium. Thus, we analyze the informational structure of an economy with differential information from a different and new viewpoint.

In particular we address the following questions: How do coalitions of agents share their private information? How can one measure the information advantage or superiority of an agent? Is each agent's private information verifiable by other members of a coalition? Do coalitions of agents pool their private information? Do agents have an incentive to report their true private information? What is the correct concept of a value allocation in an economy with differential information? Do value allocations exist in an economy with differential information? We provide answers to each of these questions.

KEYWORDS: Shapley value, differential information economies, coalitional incentive compatibility.

1. INTRODUCTION

THE CONCEPT OF A (CARDINAL) VALUE allocation was first introduced by Shapley (1969). Roughly speaking, it is defined as a feasible allocation which yields to each agent in the economy a "utility level" which is equal to the sum of the agent's expected marginal contributions to all coalitions of which he/she is a member. Because the Shapley value measures the sum of an agent's expected marginal contributions to coalitions, we argue in this paper that it provides an interesting way to measure the "worth" of an agent's information advantage in an economy with differential information. This feature of the Shapley value allocation is not necessarily shared by the traditional rational expectation equilibrium. Thus, we analyze the informational structure of an economy with differential information from a different and new viewpoint. In particular, we address the following questions: How can one measure the information advantage or superiority of an agent? How do coalitions of agents share their private information? Is each agent's private information verifiable by all other members of a coalition? Do agents have an incentive to report their private information truthfully? What is the correct concept of a value allocation in the presence of differential information? Do value allocations exist in an economy with differential information? We provide answers to each of these questions.

Similar to Radner (1968), we consider a two-period economy in which each agent i is characterized by a utility function, a random second period endow-

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ment, a prior belief about the distribution of all agents' second period endowments, and private information about the actual endowment realizations after uncertainty is resolved in the second period. Although the value allocation is a cooperative solutions concept, there is a noncooperative aspect inherent in a private information economy framework. In particular, a coalition of agents might agree on how to share their own private information. Nonetheless, some agents within the coalition may have an incentive to misreport their true private information. For example, consider an economy in which agents are exposed to idiosyncratic endowment shocks (cf., Example 1). Although a coalition of agents might agree ex-ante on insuring its members if they report a low endowment realization, once the state of nature is realized, some members of this coalition may have an incentive to agree on misreporting their actual endowments to the other agents. In particular, even if they did not receive a low endowment realization, they may report differently in order to obtain the insurance payment. As long as the coalition includes all agents who would be able to verify these reports using their own private information, agents of the complementary coalition cannot detect that they are being cheated (although they might suspect it). In order to alleviate this problem, we consider two incentive compatible ways of information sharing which we refer to as weak and strong coalitional incentive compatibility, respectively. The central idea of both notions is that it should not be profitable for coalitions of agents to agree on misreporting their information in such a way that none of the remaining agents are able to detect the misreports. We then provide conditions under which strong coalitional incentive compatibility implies that no actual exchange of information is taking place. Formally, this means that each agent's net-trade must be measurable with respect to his/her own information.[2]

We define three alternative notions of a value allocation. In the weak and the strong value allocation information sharing within coalitions must fulfill weak and strong coalitional incentive compatibility, respectively. In the private value allocation each agent's net-trade must be measurable with respect to his/her information. This corresponds to the (private) core notion of Yannelis (1991). We first show that in the private value allocation information asymmetries matter. That is, an agent with superior private information can make a Pareto improvement to the economy, and he/she will be rewarded for it. This can happen, even if the agent has zero initial endowment. It also indicates that agents can benefit from the information of other agents even if they do not acquire that information for themselves. Moreover, we show that the private value allocation fulfills strong coalitional incentive compatibility. It should be noted that in the weak value allocation agents with superior information are rewarded as well. In this case agents not only benefit from the information of other agents, but they exchange information as well. It seems to us, however, that the private value is the most useful of these concepts since it is technically very tractable and also has properties which are very similar to those of the

[2] This essentially means that an agent cannot make his/her consumption contingent on two states between which he/she cannot distinguish.

weak and the strong value, i.e., agents are rewarded for superior information. In contrast, both the weak and the strong value allocation are technically not as tractable.

Before we proceed, we would like to mention two early seminal papers in cooperative game theory with incomplete information which differ from our paper. The first is by Wilson (1978) who analyzed the core of an economy with differential information. The second is by Myerson (1984) who extended the Nash bargaining solution and the NTU value to an incomplete information setting. It should be noted that Myerson analyzes a mechanism design problem whereas our approach considers an exchange economy with differential information.

The paper proceeds as follows: Section 2 contains the description of the model (i.e., the exchange economy with differential information). In Section 3 we introduce the alternative notions of value allocation with differential information. The interpretation of the private value is discussed in Section 4. Section 5 discusses other concepts of a value allocation. Finally, Section 6 contains some concluding thoughts.

2. THE ECONOMY WITH DIFFERENTIAL INFORMATION

Consider an exchange economy which extends over two time periods $t = 0, 1$ where consumption takes place at $t = 1$. Let $\mathbb{R}^l_+$ denote the commodity space.[3] At $t = 0$ there is uncertainty over the state of nature described by a probability space $(\Omega, \mathscr{F}, \mu)$. Let $I = \{1, \ldots, n\}$ denote the set of all agents. At $t = 0$ agents agree on net trades which may be contingent on the state of nature at $t = 1$. However, they have differential information with respect to the true state of nature that will be realized. This is modeled as follows: At $t = 1$ agents do not necessarily know which state $\omega \in \Omega$ has actually occurred. They know their own endowment realization, and every agent i may also have some additional information about the state. This information is described by a measurable partition $\mathscr{F}_i$ of Ω.[4] Hence if $\bar{\omega}$ is the true state of the economy at $t = 1$, agent i observes the event $E_i(\bar{\omega})$ in the partition $\mathscr{F}_i$ which contains $\bar{\omega}$. Since agents always observe their own endowment realization, we can assume without loss of generality that agent i's endowment is measurable with respect to $\mathscr{F}_i$.[5]

In summary, an *exchange economy with differential information* is given by $\mathscr{E} = \{(X_i, u_i, e_i, \mathscr{F}_i, \mu): i = 1, \ldots, n\}$ where:

(i) $X_i = \mathbb{R}^1_+$ is the *consumption* set of agent i;

(ii) $u_i: \mathbb{R}^l_+ \to \mathbb{R}$ is the *utility function* of agent i;[6]

(iii) $\mathscr{F}_i$ is a partition of Ω denoting the *private information* of agent i;

[3] $\mathbb{R}^l_+$ denotes the positive cone of $\mathbb{R}^l$.

[4] A measurable partition of Ω is a collection of sets A_j, $j \in J$, with the following properties: (a) J is finite or countable; (b) $A_j \in \mathscr{A}$ for every j, i.e., the sets are measurable; (c) $\bigcup_{j \in J} A_j = \Omega$; (d) $A_j \cap A_k = \varnothing$ for all $j \neq k$.

[5] By (slight) abuse of notation we identify the partition with the σ-algebra generated by the partition.

[6] All the results of the paper remain valid if we assume the utility function is random, i.e., u_i is a real valued function defined on $\Omega \times \mathbb{R}^l_+$.

(iv) $e_i : \Omega \to \mathbb{R}^l_+$ is the *initial endowment* of agent i, where each e_i is $\mathscr{F}_i$-measurable and $e_i(\omega) \in X_i$, μ-a.e;

(v) μ is a probability measure on Ω denoting the *common prior* of each agent.

Finally, the *expected utility* of agent i is given by $\int_\Omega u_i(x_i(\omega))d\mu(\omega)$.[7] Throughout the paper, we assume that the utility function u_i of each agent i is monotone, continuous, concave, and integrably bounded.

3. VALUE ALLOCATIONS IN ECONOMIES WITH DIFFERENTIAL INFORMATION

3.1. *Coalitional Incentive Compatibility*

When agents have differential information, arbitrary allocations are not generally viable. In particular, arbitrary allocations might not be incentive compatible in the sense that groups of agents may misreport their information without other agents noticing it, and hence achieve different payoffs ex post. To obviate this problem, we introduce in Definitions 1 and 2 below two alternative notions of incentive compatibility.

The cooperative solution concept which we apply to economies with differential information captures the idea that groups of agents come together, exchange information, or trade with each other given their informational constraints. However, this exchange of information has a noncooperative aspect. In particular, although agents might agree to exchange information, they might not have an incentive to reveal their information truthfully. We take this into account by considering two alternative definitions of coalitional incentive compatibility.

DEFINITION 1: Let $x : \Omega \to \prod_{i=1}^n X_i$ be a feasible allocation.[8] Then x fulfills *strong coalitional incentive compatibility* if and only if the following does not

[7] Bayesian updating of priors can be introduced as follows: Let q_i: $\Omega \to \mathbb{R}_{++}$ be a Radon-Nikodym derivative (density function) denoting the prior of agent i. For each $i = 1, \ldots, n$, denote by $E_i(\omega)$ the event of $\mathscr{F}_i$ containing the realized state of nature $\omega \in \Omega$ and suppose that $\int_{t \in E_i(\omega)} q_i(t)\, d\mu(t) > 0$. Given $E_i(\omega) \in \mathscr{F}_i$, define the *conditional expected utility* of agent i as follows:

$$\int_{t \in E_i(\omega)} u_i(t, x_i(t)) q_i(t | E_i(\omega))\, d\mu(t),$$

where

$$q_i(t | E_i(\omega)) = \begin{cases} 0 & \text{if } t \notin E_i(\omega), \\ \dfrac{q_i(t)}{\int_{t \in E_i(\omega)} q_i(t)\, d\mu(t)}, & \text{if } t \in E_i(\omega). \end{cases}$$

As in Yannelis (1991) all results of the paper remain valid under this conditional expected utility formulation, but we choose not to do so for the simplicity of the exposition.

[8] An allocation x: $\Omega \to \prod_{i \in I} X_i$ is said to be *feasible* if $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$ μ-a.e.

hold:

There exists a coalition $S \subset I$ and states $a, b, a \neq b$, which members of $I \setminus S$ are unable to distinguish (i.e., a and b are in the same event of the partition for every agent not in S) and such that after an appropriate endowment redistribution members of S are strictly better off by announcing b whenever a has actually occurred. Formally, strong coalitional incentive compatibility implies that there does not exist a coalition S, states a, b with $b \in E_i(a)$ for every $i \notin S$ and a net-trade vector z^i, $i \in S$ such that $\sum_{i \in S} z^i = 0$, $e^i(a) + x^i(b) - e^i(b) + z^i \in \mathbb{R}^l_+$ for every $i \in S$, and

$$\text{(C1)} \qquad u^i\big(e^i(a) + \big(x^i(b) - e^i(b)\big) + z^i\big) > u^i\big(x^i(a)\big), \quad \text{for every } i \in S.$$

Strong coalitional incentive compatibility models the idea that it is impossible for any coalition to cheat the complementary coalition by misreporting the state and making side payments to each other which cannot be observed by agents who are not members of this coalition.[9] The restriction $e^i(a) + x^i(b) - e^i(b) + z^i \in \mathbb{R}^l_+$ for every $i \in S$ is obviously required in order to ensure that the consumption of the agents is still in the commodity space.

When side payments are observable we get the following weaker notion of incentive compatibility. (Set $z_i = 0$ for every $i \in S$ in (C1) to get weak coalitional incentive compatibility.) Formally, we have the following definition.

Definition 2: Let $x: \Omega \to \prod_{i=1}^n X_i$ be a feasible allocation. Then x fulfills *weak coalitional incentive compatibility* if and only if the following does not hold:

There exists a coalition $S \subset I$ and states a, b, $a \neq b$, which members of $I \setminus S$ are unable to distinguish (i.e., a and b are in the same event of the partition for every agent not in S) and such that members of S are strictly better off by announcing b whenever a has actually occurred. Formally, $b \in E_i(a)$ for every $i \notin S$, $e^i(a) + x^i(b) - e^i(b) \in \mathbb{R}^l_+$ for every $i \in S$, and

$$\text{(C2)} \qquad u^i\big(e^i(a) + \big(x^i(b) - e^i(b)\big)\big) > u^i\big(x^i(a)\big), \quad \text{for every } i \in S.$$

Weak incentive compatibility is weaker in the sense that allocations which fulfill strong coalitional incentive compatibility must also fulfill weak coalitional incentive compatibility. It is easy to find examples where the reverse is not true. In what follows we discuss both notions. It turns out that in certain cases strong coalitional incentive compatibility is analytically more tractable since it corresponds to individual measurability of the net trade of each agent as proved in Lemma 1 below. We now introduce some notation. In particular, let $U^w(S)$ and $U^s(S)$ denote the utility allocations which coalition S can attain, and which

[9] If we want incentive compatibility to hold almost everywhere then it must be required that states in which agents misreport occur with probability zero.

fulfill weak and strong coalitional incentive compatibility, respectively. That is:

$U^w(S) = \{(w_1, \ldots, w_{|S|}) \in \mathbb{R}^{|S|}$: there exists an allocation x_i, $i \in S$, such that $\sum_{i \in S} x_i = \sum_{i \in S} e_i$, where x_i, $i \in S$ fulfills weak coalitional incentive compatibility, and where $w_i \leqslant \int u_i(x_i) d\mu$ for every $i \in S\}$.

$U^s(S) = \{(w_1, \ldots, w_{|S|}) \in \mathbb{R}^{|S|}$: there exists an allocation x_i, $i \in S$, such that $\sum_{i \in S} x_i = \sum_{i \in S} e_i$, where x_i, $i \in S$ fulfills strong coalitional incentive compatibility, and where $w_i \leqslant \int u_i(x_i) d\mu$ for every $i \in S\}$.

For both notions of incentive compatibility we will analyze the corresponding concept of a value allocation. The definitions are introduced in Section 3.3.

3.2. *Private Measurability of Allocations*

We now continue by characterizing the set $U^s(S)$. It turns out that $U^s(S)$ corresponds to the attainable utility allocations which fulfill individual measurability of the net trade of each agent for an economy with one commodity per state.

Let $U^p(S) = \{(w_1, \ldots, w_n)$: there exist net trades z_i such that $\sum_{i \in S} z_i = 0$, where z_i is $\mathscr{F}_i$-measurable, and $w_i \leqslant \int u_i(e_i + z_i) d\mu\}$.

LEMMA 1: *Assume that there is one commodity per state; then $U^p(S) = U^s(S)$.*

PROOF: We first show that $U^p(S) \subset U^s(S)$. Let $w \in U^p(S)$. Then there exists an allocation x_i, $i \in S$ such that for every agent $i \in I$ the net trades $y_i = x_i - e_i$ are $\mathscr{F}_i$-measurable, $\sum_{i \in S} y_i = 0$, and $w_i \leqslant \int u_i(x_i) d\mu$. Now assume by way of contradiction that strong coalitional incentive compatibility does not hold. Then there exist a coalition $T \subset S$, and two states a, b which members of $S \setminus T$ cannot distinguish, such that members of T are strictly better off by redistributing z_i, $i \in T$ (that is $\sum_{i \in T} z_i = 0$) and by reporting b whenever a has actually occurred. Since net trades are individually measurable it follows that $\sum_{i \in S \setminus T} y_i(a) = \sum_{i \in S \setminus T} y_i(b)$. Thus, since the feasibility constraint holds with equality we get $\sum_{i \in T} y_i(a) = \sum_{i \in T} y_i(b)$. Since there is one commodity per state and preferences are monotonic it follows that $y_i(b) + z_i > y_i(a)$ for every $i \in T$. We therefore get

$$\sum_{i \in T} y_i(b) = \sum_{i \in T} (y_i(b) + z_i) > \sum_{i \in T} y_i(a) = \sum_{i \in T} y_i(b),$$

which provides the contradiction.

It remains to prove that $U^s(S) \subset U^p(S)$. Suppose by way of contradiction that the net trade of one agent, say agent j is not $\mathscr{F}_j$-measurable. Hence, there exist two states a and b which agent j cannot distinguish and for which $z^j(a) \neq z^j(b)$. Without loss of generality assume that $z^j(a) > z^j(b)$. Then in state b the coalition $T = S \setminus \{j\}$ can announce state a and agent j is unable to verify it and they can redistribute their excess income and make all members of T better off.

This provides the contradiction to strong coalitional incentive compatibility. Hence $U^s(S) \subset U^p(S)$. This concludes the proof.

Lemma 1 implies that when there is one commodity per state of nature the sets $U^s(S)$ and $U^p(S)$ coincide for every S. The fact that strong coalitional incentive compatibility—i.e., the assumption that coalitions of agents can trade with each other in such a way that agents of the complementary coalition cannot observe that such trades are taking place—implies individual measurability of net trades, means that no exchange of information takes place. That is, after the state of nature is realized agents still only know the event of their own information partition which contains the true state. Nevertheless, agents can benefit from each other's information as we show in Examples 1 and 2 below. In contrast, we show in Example 4 that agents may exchange information *and* benefit from each others' information if we require only weak coalitional incentive compatibility. Which of the two incentive compatibility assumptions is more appropriate will clearly depend on the specific economic question which we want to analyze.

In the case of more than one commodity, Lemma 1 does not apply, and we can therefore define a third concept of information sharing. In particular, we assume that all net trades are individually measurable, i.e., no actual exchange of information takes place. As mentioned earlier agents can nevertheless benefit from each others' information. More importantly, however, private information sharing is analytically more tractable than weak or strong coalitional incentive compatibility. At the same time the results are qualitatively not very different. This seems to indicate that private information sharing is the most useful concept to analyze economies with private information.

3.3. *Value Allocations*

The strategy in this section is to derive a game with transferable utility from the economy with differential information, $\mathscr{E}$, in which each agent's utility is weighted by a factor λ_i which allows interpersonal utility comparisons. In the value allocation itself no side payments are necessary. This claim is justified by appealing to the *principle of irrelevant alternatives:* "If restriction of the feasible set, by eliminating side payments, does not eliminate some solution point, then that point remains a solution" (Shapley (1969)).[10] We thus get a game with side payments as follows:

DEFINITION 3: A *game with side payments* $\Gamma = (I, V)$ consists of a finite set of agents $I = \{1, \ldots, n\}$ and a superadditive, real valued function V defined on 2^I such that $V(\varnothing) = 0$. Each $S \subset I$ is called a coalition and $V(S)$ is the "worth" of the coalition S.

[10]See Emmons and Scafuri (1985, p. 60) or Shafer (1980, p. 468) for further discussion.

The Shapley value of the game Γ (Shapley (1953)) is a rule which assigns to each agent i a "payoff" Sh_i given by the formula[11]

$$Sh_i(V) = \sum_{\substack{S \subset I \\ S \supset \{i\}}} \frac{(|S|-1)!(|I|-|S|)!}{|I|!}\left[V(S) - V(S \setminus \{i\})\right].$$

The Shapley value has the property that $\sum_{i \in I} Sh_i(V) = V(I)$, i.e., the Shapley value is Pareto optimal. Moreover, it is individually rational, i.e., $Sh_i(V) \geqslant V(\{i\})$ for all $i \in I$.

We now start by defining the private value allocation. For each economy with differential information $\mathscr{E}$ and each set of weights $\{\lambda_i : i = 1, \ldots, n\}$, we can now associate a game with side-payments (I, V_λ^p) (we also refer to this as a "transferable utility" (TU) game) according to the rule:

For $S \subset I$ let

$$(3.1) \qquad V_\lambda^p(S) = \max_{x_i} \sum_{i \in S} \lambda_i \int u_i(x_i(\omega))\, d\mu(\omega)$$

subject to: (i) $\sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega)$, μ-a.e.; (ii) the net trades $x_i - e_i$ are $\mathscr{F}_i$-measurable for every $i \in S$.

We now define the private value allocation for economies with differential information.

Definition 4: An allocation $x\colon \Omega \to \prod_{i=1}^n X_i$ is said to be a *private value allocation* of the economy with differential information $\mathscr{E}$ if the following holds:

(i) The net trade $e_i - x_i$, is $\mathscr{F}_i$ measurable for all $i \in I$.

(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$, μ-a.e.

(iii) There exist $\lambda_i \geqslant 0$, for every $i = 1, \ldots, n$ which are not all equal to zero, with $\lambda_i \int u_i(x_i(\omega))\, d\mu(\omega) = Sh_i(V_\lambda^p)$ for every i, where $Sh_i(V_\lambda^p)$ is the Shapley value of agent i derived from the game (I, V_λ^p), defined in (3.1).

Condition (i) says that net trades must be measurable with respect to private information; (ii) is a market clearing condition; and (iii) says that the expected utility of each agent multiplied by his/her weight λ_i must be equal to his/her Shapley value derived from the TU game (I, V_λ^p).

Alternatively, we can also define the weak and the strong value allocation. In order to define the weak value allocation, first replace (ii) in (3.1) by:

(ii′) x_i, $i \in S$ fulfills weak coalitional incentive compatibility.

We thus derive a TU-game V_λ^w in Definition 4 and replace (i) in Definition 4 by:

(i′) x_i, $i \in I$, fulfills weak coalitional incentive compatibility.

Similarly, we can define a strong value allocation by replacing (ii) in (3.1) by:

(ii″) x_i, $i \in S$ fulfills strong coalitional incentive compatibility.

[11] The Shapley value is the sum of the expected marginal contributions an agent can make to each coalition of which he/she is a member (see Shapley (1953)).

This defines the TU-game V_λ^s. We now define the strong value allocation in the obvious way. However, by Lemma 1 the strong and the private value allocation coincide if there is one commodity per state. We therefore only discuss the properties of the private and the weak value allocations in the examples in the subsequent sections.

3.4. *Existence and Incentive Compatibility of the Private Value Allocation*

A private value allocation always exists for a differential information economy. This follows from the existence results in Emmons and Scafuri (1985) or Shapley (1969). Specifically, these papers show that if agents' utility functions are concave, and continuous; and if the consumption set of each agent is bounded from below, closed and convex; then a value allocation exists.

In order to apply their existence results, we first define the consumption space to be the set of all functions $x_i\colon \Omega \to \mathbb{R}_+^l$ which are $\mathscr{F}_i$-measurable. One can then show that the expected utility is (weakly) continuous on this consumption set. Thus, the above results can be applied (for details see Krasa and Yannelis (1994)).

We next show that the private value fulfills strong coalitional incentive compatibility. In the case of one commodity per state this follows from Lemma 1. In the case of more than one commodity, we add to Definition 1 the requirements that agents in coalition S must agree on the state a that they misreport, i.e., equation (C1) must hold for every $s \in \bigwedge_{i \in S} E_i(a)$.[12] In other words the event in which the agents in coalition S cheat must be common knowledge to its members.[13]

Lemma 2: *The private value allocation fulfills strong coalitional incentive compatibility.*

Proof: Let $x_i, i \in I$ be a private value allocation. Assume by way of contradiction that the allocation does not fulfill strong coalitional incentive compatibility. Let $T \subset I$ be a coalition for which it is optimal to report state b whenever state a has actually occurred. Members in $I \setminus T$ cannot distinguish state a from state b. Since net trades are individually measurable it follows that $x_i(a) = e_i(a) + (x_i(b) - e_i(b))$ for every $i \in I \setminus T$. Let z_i, $i \in T$, denote the redistribution within coalition T. Then we define a new allocation x_i^*, $i \in I$ as follows: For every $i \in T$ let

$$x_i^*(s) = \begin{cases} e_i(s) + (x_i(b) - e_i(b)) + z_i & \text{for every } s \in \bigwedge_{i \in T} E_i(a); \\ x_i(s) & \text{otherwise.} \end{cases}$$

[12] $\bigwedge_{i \in S} E_i(a)$ is the event in $\bigwedge_{i \in S} \mathscr{F}_i$ which contains a, where the symbol $\bigwedge_{i \in S} \mathscr{F}_i$ denotes the "meet," i.e., the intersection of all $\mathscr{F}_i$, which is interpreted as the "common knowledge" information of the coalition S.

[13] For related incentive compatibility results for the core of an economy with differential information, see Koutsougeras and Yannelis (1993).

Let $x_i^* = x_i$ for every $i \in I \setminus T$. Then x_i^* is a feasible allocation and all net trades $x_i - e_i$ are $\mathscr{F}_i$-measurable. Furthermore, members of T are strictly better off with the new allocation than with the original allocation. Members of $I \setminus T$ are indifferent. This, however, is a contradiction to the Pareto efficiency of the value allocation. Thus, the value allocation must fulfill strong coalitional incentive compatibility.

4. INTERPRETATION OF THE VALUE ALLOCATION

4.1. *The Private Value Allocation*

In this section we discuss the properties of the private value allocation. We illustrate these properties by way of simple examples. In the first example, we consider an economy in which two agents, denoted by I and J are subject to independent endowment shocks in the second period. Specifically, each agent's endowment can be either high or low. Neither of the agents, however, is able to verify whether or not the other agent's endowment is high or low. We assume that there is a third agent, denote by K, in the economy whose superior information potentially allows verification of the endowment shocks. Example 1 addresses the question to what extent the superior information of agent K makes an insurance arrangement between agents I and J possible. Moreover, it is also shown that agent K is rewarded for his/her superior information.

EXAMPLE 1: Consider an economy with three agents denoted by I, J, and K, and four states of nature a, b, c, and d. Assume there is only one consumption good in each state. The random endowment of the agents are given by $(4,4,0,0)$ for I; $(4,0,4,0)$ for J; and $(1,1,1,1)$ for K. Agent K has an information set $\mathscr{F}_K$. We consider the case where $\mathscr{F}_K$ is trivial—i.e., $\mathscr{F}_K = \{\{a,b,c,d\}\}$—and the case where $\mathscr{F}_K$ corresponds to full information—i.e., $\mathscr{F}_K = \{\{a\}, \{b\}, \{c\}, \{d\}\}$. Further, assume that agent I cannot distinguish state a from b, and state c from d; and finally, agent J cannot distinguish a from c, and b from d, i.e., $\mathscr{F}_I = \{\{a,b\},\{c,d\}\}$ and $\mathscr{F}_J = \{\{a,c\},\{b,d\}\}$. Let all agents have the same von Neumann-Morgenstern utility function $\sqrt{x}$, and assume that each state occurs with the same probability.

Consider first the case where agent K has full information. We now derive the set of all attainable utility allocations for the private value allocation. For the one-agent coalitions we get: $U^p(\{i\}) = \{w_i\colon w_i \leqslant 1\}$, for $i = I, J, K$. Further, all trades between agents I and J must be state independent.[14] However, since each agent's consumption must be nonnegative, it follows that no trade between agents I and J is possible. Thus, $U^p(\{I,J\}) = \{(w_I, w_J)\colon w_I \leqslant 1, \text{ and } w_J \leqslant 1\}$. This, does not apply to the other two agent coalitions: $U^p(\{I,K\}) = U^p(\{J,K\}) = \{(w_1, w_2)\colon w_1 \leqslant (1/2)\sqrt{4+t_1} + (1/2)\sqrt{t_2},\ w_2 \leqslant (1/2)\sqrt{1-t_1} + (1/2)\sqrt{1-t_2},$

[14] This follows immediately since the net trade z between both agents must be measurable with respect to the information of each agent. However, only state independent net trades are measurable with respect to both agents' information.

such that $-1 \leqslant t_1 \leqslant 1$ and $0 \leqslant t_2 \leqslant 1$, for $i = 1,2\}$. Similarly, $U^p(\{I, J, K\}) = \{(w_1, w_2, w_3)$: there exist state independent net trades z_i, $i = I, J, K$ where z_i is $\mathscr{F}_i$-measurable for $i = J, K$, where $\sum_{i=I,J,K} z_i = 0$, and $w_i \leqslant \sum_{s=a,b,c,d} (1/4)\sqrt{e_i + z_i}$, for $i = I, J, K\}$. In view of Section 3.4, a private value exists. Further, it is obvious that $V_\lambda^p(\{I, J, K\}) \geqslant V_\lambda^p(\{I, J\}) + V_\lambda^p(\{K\})$ for all $\lambda > 0$.[15] Moreover, $V_\lambda^p(\{i, K\}) > V_\lambda^p(\{i\}) + V_\lambda^p(\{K\}), i = I, J$ for all $\lambda > 0$.[16] Hence, $Sh_K(V_\lambda^p) > \int u_K(e_K)\, d\mu$, i.e., agent K must get a higher utility than he/she derives from the initial endowment.[17]

Next consider the case where $\mathscr{F}_K$ is trivial. Now only constant net trades are possible within all coalitions. Since the private value allocation is individually rational, all agents must consume their initial endowment.

Example 1 shows that agent K's information clearly matters. It shows that when agent K has full information and I, J do not, he/she can use this information advantage to act as an intermediary to allow trade between agents I and J that would not otherwise be possible. This becomes clear when looking at $U^p(\{I, J, K,\})$ and $U^p(\{I, J\})$. Without agent K, agents I and J cannot make any beneficial trades. This changes when agent K enters the coalition. Now all trades basically go through agent K since $U^p(\{I, J, K,\})$ is essentially the union of $U^p(\{I, K\})$ and $U^p(\{J, K\})$. It is important to note that agents do not exchange their private information. Nonetheless, using the superior information of agent K, trade makes everybody in the economy better off. Moreover, agent K is compensated for his/her intermediation service by getting a strictly higher utility than in the case where he/she is less well informed and is unable to facilitate trade. However, it is essential for agent K to have a strictly positive endowment in every state. If agent K's endowment is for example 0, then he/she is not able to trade with agents I and J, i.e., to increase their consumption in the low-income state and decrease it in the high-income state since this would require agent K to hold a positive initial endowment in state d. The private value allocation in such a case assigns to every agent the initial

[15] This inequality is true in general since the game derived from the differential information economy is superadditive. In particular, the sum of the utilities of the agents must be at least as great when utility is jointly maximized than when utility is only maximized for agents I and J, and agent K consumes his/her initial endowment.

[16] This follows immediately from the first order condition. Note that $V_\lambda^p(\{i, K\})$ can be found by solving

$$\max_{t_i, t_i'} \frac{\lambda_i}{2}\sqrt{4 + t_i'} + \frac{\lambda_i}{2}\sqrt{t_i} + \frac{\lambda_K}{2}\sqrt{1 - t_i'} + \frac{\lambda_K}{2}\sqrt{1 - t_i},$$

where $i = I, J$. The first order conditions imply that it is optimal to choose the transfer t_i to be strictly positive. Thus the agents are strictly better off by trading than by consuming their initial endowment which implies that the strict inequality holds.

[17] The Shapley value agent K is a weighted sum of K's marginal contribution $V_\lambda^p(S) - V_\lambda^p(S \setminus \{K\})$ to coalitions S of which agent K is a member. By the above arguments we have $V_\lambda^p(S) - V_\lambda^p(S \setminus \{K\}) \geqslant V_\lambda^p(\{K\})$ for all coalitions S which contain K. Moreover, the strict inequality holds if $S = \{I, K\}$ and if $S = \{I, J\}$. Thus, agent K must receive a higher utility in the value allocation than $V_\lambda^p(\{K\})$.

endowment. This changes immediately if we consider endowments which are not independent.[18]

We now consider the case where endowments are not independent in the context of another simple example. This modification of the previous example demonstrates that the information superiority of an agent can matter, even though this agent may have no initial endowment at all. As long as this agent's private information is useful to the rest of the economy, he/she can always trade his/her superior information for actual goods.

EXAMPLE 2: Consider an economy as in Example 1 but assume that the initial endowments of agents I and J are given by $(4,4,0,4)$ and $(4,0,4,4)$, respectively. Assume that $\mathscr{F}_I=\{\{a,b,d\},\{c\}\}$, $\mathscr{F}_J=\{\{a,c,d\},\{b\}\}$, and that agent K has full information. We also assume that agent K has a zero endowment in all states. The derivation of $U^p(S)$ is similar to the derivation in Example 1. We therefore omit any specification of the set of attainable utility allocations and show directly that $Sh_K(V_\lambda^p)>0$ for every $\lambda \geqslant 0$.[19]

We now show that $V_\lambda^p(\{I,J,K\})>V_\lambda^p(\{I,J\})$. Let t_i be the net trade of agent $i=I,J$ in the low income state, and let t_i' denote the net trade of agent $i=I,J$ in the high-income state.

We first consider the case where $\lambda_K>0$. The first order conditions imediately imply that it is never optimal to choose $t_i=t_i'$ for $i=I,J$.[20] This, however, means that the weighted sum of the expected utilities will always be lower if we restrict agents I, J, and K to state-independent net trades. However, state-independent trades are the only ones which are individually measurable in the two agent coalition $\{I,J\}$. Thus, $V_\lambda^p(\{I,J,K\})>V_\lambda^p(\{I,J\})$.

[18] Let L and H denote the endowment realizations in the low and in the high states respectively. Then independence of endowments means that $P(\{e_I=L\})=P(\{e_I=L|e_J=L\})=P(\{e_I=L|e_J=H\})$, and similarly for the high state and for the other agents (where P denotes the probability).

[19] By $\lambda \geqslant 0$, we mean that some, but not all, of the weights can be zero. In particular, we must consider the case where $\lambda_K=0$.

[20] The agents solve

$$\max_{t_i,t_i'}\sum_{i=I,J}\left[\frac{3\lambda_i}{4}\sqrt{4+t_i'}+\frac{\lambda_i}{4}\sqrt{t_i}\right]+\frac{\lambda_K}{2}\sqrt{-t_I'-t_J'}+\frac{\lambda_K}{4}\sqrt{-t_I'-t_J}+\frac{\lambda_K}{4}\sqrt{-t_I-t_J'}.$$

Without loss of generality assume that $\lambda_I>0$. Then the first order conditions with respect to t_I and t_I' are

$$\frac{3\lambda_I}{\sqrt{4+t_I'}}=\frac{2\lambda_K}{\sqrt{-t_I'-t_J'}}+\frac{\lambda_K}{\sqrt{-t_I'-t_J}},$$

$$\frac{\lambda_I}{\sqrt{t_I}}=\frac{\lambda_K}{\sqrt{-t_I-t_J'}}.$$

If $t_I'=t_I$ and $t_J'=t_J$, then the first order conditions yield $\sqrt{4+t_I}=\sqrt{t_I}$, which is clearly impossible. Thus, there does not exist a solution if $t_I'=t_I$ and $t_J'=t_J$.

We now consider the case where $\lambda_K = 0$. The first order conditions imply that it is never optimal to choose $t_i = t_i' = 0$.[21] In addition, agents I and J cannot receive a negative net transfer in all states since this would imply negative consumption. Thus, it can never be the case that $t_i = t_i' \neq 0$.

Since there cannot be state-independent net transfers in the value allocation, it follows that $V_\lambda^p(\{I, J, K\}) - V_\lambda^p(\{I, J\}) > 0$ for all weights λ which are candidates for utility comparison weights. Consequently agent K must have a positive Shapley value,[22] and he/she must get positive consumption in the value allocation. Moreover, it is also the case that the value allocation assigns agent K a strictly positive consumption in all states. This follows since the value allocation must be a solution to the Pareto problem in which each agent i's utility is multiplied with weight λ_i. Since $\lambda_K > 0$, the first order conditions stated in footnote 20 immediately implies the result, i.e., that agent K's consumption is strictly positive in all states.

Now compare this result with the case where agent K has no information (i.e., $\mathscr{F}_k = \{\{a, b, c\}\}$). Then his/her Shapley value is 0, and the initial allocation is the only equilibrium. This demonstrates that information superiority matters. Finally, note that any notion of a rational expectation equilibrium in this economy gives zero consumption to agent K since his/her budget set is zero.

When agent K has useful private information the value allocation assigns positive consumption to agent K if the endowments of agents I and J are not independent, but assigns zero consumption if the endowments are independent. This occurs for the following reason: In both cases agents I and J attempt to insure against low-income realizations. Because of differential information, however, they need agent K as an intermediary to execute the correct trades. This arrangement works even if agent K has a zero endowment as long as only one of the agents has a low endowment realization, because the claim of this particular agent can then be covered by the agent who has the high endowment realization. This is the essence of Example 2. If both agents have low endowment realizations at the same time (which can occur if endowments are independent as in Example 1), then they both want a positive net transfer. Agent K cannot fulfill his/her payment obligations because his/her endowment is zero, and K claims insolvency. However, this claim is problematic because agents I and J cannot verify whether agent K is in fact insolvent. Thus,

[21] The argument is similar to the one above. Just consider the first order conditions of

$$\max_{t_I, t_I'} \frac{3\lambda_I}{4}\sqrt{4 + t_I'} + \frac{\lambda_I}{4}\sqrt{t_I} + \frac{3\lambda_J}{4}\sqrt{4 - t_I} + \frac{\lambda_J}{4}\sqrt{-t_I'},$$

and choose $t_I = t_I' = 0$. We can substitute the constraints $t_I + t_J' \leqslant 0$ and $t_I' + t_J \leqslant 0$ in the maximization problem, since they must obviously hold with equality.

[22] This follows immediately, since $(1/3)(V_\lambda^p(\{I, J, K\}) - V_\lambda^p(\{I, J\}))$ is one of the summands in the formula for the Shapley value. None of the summands is negative since the game is superadditive (see footnote 17).

the problem is to find an incentive compatible way to let agent K announce bonafide insolvencies. Clearly this is possible if agents I and J are able to observe state d.[23]

An alternative way to permit bonafide insolvencies by agent K is to weaken the incentive compatibility requirements. We do this in Example 4.

4.2. *Risk Sharing Versus Informational Effects: The Shafer Example*

Readers familiar with the Roth (1980) and Shafer (1980) examples (as well as the debate on the value allocation—Aumann (1985, 1987), Roth (1983), Scafuri and Yannelis (1984), Yannelis (1983)) will notice that our Example 2 has a similar flavor in the sense that an agent with a zero endowment ends up with positive consumption. In the Roth and Shafer examples this effect may be attributed to risk aversion: the agent with a zero initial endowment is less risk averse. However, this is clearly *not* the case in our setting since all agents have *identical* utility functions. Also notice that in our Example 2 agent K gets zero consumption in the value allocation if he/she has no information and this is also the *only* Shapley value allocation. However, the agent gets positive consumption if he/she has full information. In fact, when agents I and J implicitly "use" agent K's information, this leads to a Pareto improvement for the whole economy. Thus, it is solely informational effects that drive our examples rather than risk sharing. To illustrate this point we introduce differential information in the Shafer example (1980, Example 2, pp. 471–472).

EXAMPLE 3: Assume there are three agents denoted by I, J, K, two possible states of nature a, b, and one commodity per state. The endowments of agents I and J are given by $(4,0)$ and $(0,4)$, respectively. Of course, I and J have full information. That is $\mathscr{F}_I = \mathscr{F}_J = \{\{a\},\{b\}\}$. They have the same utility function given by $W^i(x_a, x_b) = ((1/2)\sqrt{x_a} + (1/2)\sqrt{x_b})^2$ for $i = I, J$. Agent K's endowment is $(0,0)$ and he/she is risk neutral, i.e., the utility function is given by $W^K(x_a, x_b) = (1/2)x_a + (1/2)x_b$. Shafer's example corresponds in our differential information framework to the complete information case, i.e., $\mathscr{F}_I = \mathscr{F}_J = \mathscr{F}_K = \{\{a\},\{b\}\}$. It can be shown[24] that for $\lambda_I = \lambda_J = \lambda_K = 1$ there exists a value allocation which gives positive consumption to the agent with zero initial endowment. In particular, in the value allocation, agents I and J receive $(11/6, 11/6)$ and agent K receives $(2/6, 2/6)$.

Now consider the case where agent K has trivial information, i.e., $\mathscr{F}_K = \{\{a, b\}\}$ and agents I and J have full information, i.e., $\mathscr{F}_I = \mathscr{F}_J = \{\{a\},\{b\}\}$. We will show that the private value allocation assigns zero consumption to agent K (despite the fact that agent K is less risk averse). Assume by way of contradic-

[23] This can be done simply by assuming in Example 1 that the information partitions of agents I and J are given by $\{\{a,b\},\{c\},\{d\}\}$ and $\{\{a,c\},\{b\},\{d\}\}$, respectively. The proof that agent K's Shapley value (and hence consumption) is strictly positive is along the same lines as Example 2.

[24] For explicit computations, see Yannelis (1983, pp. 291–292).

tion that there exists a private value allocation which assigns positive consumption to agent K. In such a value allocation K must also have a positive weight λ_K. However, agent K cannot enter into any trades with agent I or with agent J separately, since those trades would have to be state-independent and would therefore assign negative consumption to one of the agents in each state (so agents I and J both prefer their initial endowment in a two coalition with K). Thus, $V_\lambda(\{I,K\}) = V_\lambda(\{J,K\}) = V_\lambda(\{I\}) = V_\lambda(\{J\})$, where the first and the third equality follow from symmetry. Furthermore, $V_\lambda(\{K\}) = 0$, since K has a zero initial endowment. Thus, K's Shapley value is given by

$$(4.1) \qquad Sh_K(V_\lambda) = \tfrac{1}{3}(V_\lambda(\{I,J,K\}) - V_\lambda(\{I,J\})).$$

Let (x^I, x^J, x^K) be the private value allocation. Then since no side payments are necessary in the value allocation we must have

$$(4.2) \qquad V_\lambda(\{I,J,K\}) = \sum_{i=I,J,K} \lambda_i W^i(x^i).$$

Furthermore, since $x^I + x^J \leqslant e^I + e^J = (4,4)$, it follows that

$$(4.3) \qquad V_\lambda(\{I,J\}) \geqslant \lambda_I W^I(x^I) + \lambda_J W^J(x^J).$$

(4.1), (4.2), and (4.3) imply

$$Sh_K(V_\lambda) \leqslant \tfrac{1}{3}\lambda_K W^K(x^K) < \lambda_K W^K(x^K).$$

However, this means that agent K gets strictly more than his/her Shapley value, a contradiction to the fact that (x^I, x^J, x^K) is assumed to be a value allocation. Thus, the *only* value allocation which exists in this example assigns zero consumption and a weight of zero to agent K.

The introduction of differential information in the Shafer example enables us to draw the following two conclusions.

(a) It resolves the problem noted by Roth and Shafer that a "dummy player" ends up with positive consumption.

(b) More importantly, the example indicates that there is an essential difference between the risk aversion effect which drives the Roth-Shafer examples and the informational asymmetries which drive our results.

It is important to note that in *all* our examples all agents have the *same* utility function and therefore the same risk attitude. Nonetheless, in situations where an agent with zero consumption ended up with positive consumption this was due purely to the information superiority of the agent. Furthermore, if we consider the economy as a transferable utility game (i.e., we fix the weights λ and allow side payments in equilibrium), our Examples 1 and 2 show that the agent with superior information gets strictly positive consumption. This is independent of the agent's risk aversion and holds for *any choice* of λ, i.e., even if the agent with a zero endowment has zero weight. The agent with a zero endowment still receives a strictly positive Shapley value due to his/her superior information.

5. OTHER VALUE CONCEPTS

5.1. *The Weak Value Allocation*

In all of our examples, the private and the strong value coincide. In contrast, weak incentive compatibility increases in general the set of attainable utility allocations, thus resulting in a different value allocation. Similar to the private value allocation, the weak value allocation rewards agents with superior information. On the other hand, however, in the weak value allocation agents do not only benefit from each others' information but they can also exchange information. Consider the following example.

EXAMPLE 4: Consider the economy of Example 1, except assume that agent K's endowment is given by $(0,0,0,0)$ and that the agent has full information. We now analyze the weak coalitional incentive compatible value.

It is clear that the $U^p(S) = U^w(S)$ for the coalitions $S = \{I\}, S = \{J\}$, and $S = \{I, J\}$. Further, for the coalitions $S = \{I, K\}$ and $S = \{J, K\}$ we can derive $U^w(S)$ by a similar procedure as $U^p(S)$, taking into account that agent K has zero endowment. The attainable utility allocations differ in an interesting way when we consider the grand coalition. We show that $U^w(\{I, J, K\})$ corresponds to the attainable utility allocations under full information:

Consider an allocation (x_I, x_J, x_K) which is Pareto optimal under full information. Let $x_i(s)$ denote the consumption of agent i in state s. We now show that this allocation fulfills weak coalitional incentive compatibility. Clearly, $x_i(b) = x_i(c)$ for $i = I, J, K$, since the aggregate endowment in states b and c coincides. Further, $x_K(a) \geq x_K(b) \geq x_K(d)$ since the aggregate endowment in state a is higher than the aggregate endowment in state b, and since the aggregate endowment in state b is higher than the aggregate endowment in state d. Note that agent K cannot misreport if state d occurs because one of the other agents will disagree.[25] The same is true if state b or state c occurs. Finally, agent K has no incentive to misreport in state a since this is the state where he/she gets the highest net transfer.

The remainder of the argument is similar to that in Example 2. We must show that agent K must get a strictly positive consumption of the good in each state of nature in the value allocation.[26] This, however, follows immediately since the above computations imply that $V_\lambda^w(\{I, J, K\}) - V_\lambda^w(\{I, J\}) > 0$ for every λ and hence $Sh_K(V_\lambda^w) > 0$.[27] Since $Sh_K(V_\lambda^w) = \lambda_k \int u_K(x_K)\, d\mu$, where x_K is the consumption assigned to agent K in the value allocation, x_K must be strictly positive.

[25] Agent K can only announce either states b or c when attempting to misreport the occurrence of d. Hence either agent I or J must agree. Without loss of generality assume it is agent I. This, however, is impossible since I would have to pay a net transfer corresponding to a high income state. Such a transfer is always strictly higher than the net transfer in a low income state.

[26] Note that the sets $U^w(S)$ are compact and convex, and thus superadditivity is fulfilled. Hence the conditions for the existence of a value allocation hold (cf., Shapley (1969) or Emmons and Scafuri (1985)). Hence a value allocation exists.

[27] The inequality follows since the grand coalition can attain all unconstrained Pareto efficient allocations. None of them can be attained via state-independent net transfers.

The weak, and the private value allocations explicitly take into account an agent's information superiority in an economy with differential information. Example 1 shows that the information of agent K matters (as does the relative lack of information of all other agents) and Examples 2 and 4 show that agent K can be an "intermediary" between agents I and J even without having a positive endowment (agent K simply announces the true state of nature and as compensation gets a positive net transfer for this service). However, in the weak value allocation exchange of information takes place. Consider the above example. Agent I's and agent J's net-trades assigned in the weak value allocation are not individually measurable. Specifically, the agents' net trades are different in the four states. Thus, once the state of nature is realized, there is no differential information ex-post. In this sense, agents I and J have "received information" from agent K.

The role of agent K in Examples 2 and 4 is also interesting in connection with the literature on financial intermediation. For example, Boyd and Prescott (1986) argue that coalitional structures, i.e., cooperative games with differential information, are important for understanding financial intermediation. As our examples indicate, the Shapley value can be an interesting tool for such an analysis and might provide an alternative to the core which Boyd and Prescott use. It is important to point out, that we do not need to assume the existence of a "state verification technology" at the outset as it is standard in this literature. Rather, verification evolves endogenously in our model. Whenever there is "doubt" about the state of the economy, agent I and agent J can turn to agent K who then announces the true state. Further, agent K is compensated for this service by a positive net transfer. This net transfer to agent K can be interpreted as a "cost of state verification" paid by agent I and agent J, and the magnitude of this cost is determined endogenously. For further work on intermediation and the relationship between intermediation and media of exchange in a differential information economy with pairwise trade, see Huggett and Krasa (1993).

5.2. *The Coarse and the Fine Value*

We now define the coarse and the fine value allocation.

(a) A coalition of agents S always pools all the information of its members. Formally, let $\mathscr{F}_i$ be the information partition of agent i; then the *pooled* information $\bigvee_{i \in S} \mathscr{F}_i$ is given by the union of the information partitions of the individual agents.[28]

(b) A coalition of agents makes their trades contingent solely on common knowledge information. This can be interpreted as assuming that trades within a coalition must be verifiable by all members of the coalition. Formally, *common*

[28] Strictly speaking, we must take the coarsest partition which contains the information partition of every agent. Thus, two states a and b are indistinguishable for the coalition S if and only if for all $A_i \in \mathscr{F}_i$ it is the case that $\bigcap_{i \in S} A_i \neq \varnothing$ implies $\{a, b\} \subseteq \bigcap_{i \in S} A_i$.

knowledge information is given by $\bigwedge_{i \in S} \mathscr{F}_i$ which is the intersection of the information partitions $\mathscr{F}_i$ of all agents.[29]

Wilson (1978) refers to the core with information sharing as in (a) as the *fine core* and to the core with information sharing as in (b) as the *coarse core*. In this section we want to discuss briefly the analogous definitions for the Shapley value, and show that they are problematic.[30] By exchanging constraint (ii) in (3.1) for "$x_i - e_i$ is $\bigvee_{i \in S} \mathscr{F}_i$-measurable for every $i \in S$" and (i) in Definition 6 for "$x_i - e_i$ is $\bigvee_{i \in I} \mathscr{F}_i$-measurable for every $i \in I$" we get the definition of the "fine value." Similarly, we can define a "coarse value."

The first obvious problem of the fine value is that allocations will in general not be incentive compatible. Consider, for example two agents I, J with the same endowments and the same utility functions as I and J in Example 1. Since agents pool their information, there is full information in coalition $S = \{I, J\}$. Consequently arbitrary trades can be achieved. For example, agent I can promise agent J the net-trade $(-1, -1, 1, 1)$. However, since J's information partition is $\mathscr{F}_j = \{\{a, c\}, \{b, d\}\}$, agent I always has the incentive to announce state a. Such trades are ruled out by weak as well as strong coalitional incentive compatibility as well as by the private value allocation. However, they are admissible in the fine value.

The second problem of the fine value is that information asymmetries are not taken into account. In particular, consider again the economy of Example 1. Since the coalition $S = \{I, J\}$ always has full information, the information of K is irrelevent. In contrast to the value concepts discussed above, agent K is not needed since there are no informational problems between I and J any more. Similarly, the value allocation will not change if we change the information of I and J.[31] It is relatively easy to see that this observation is true in general. Thus, the fine value does not seem to be a useful concept.

On the other hand, although the coarse value takes information asymmetries into account, it has some rather strange features. For example, add to an arbitrary economy an agent who has zero initial endowment and no information. Then the common knowledge information of the grand coalition immediately becomes trivial, and hence only trivial net trades are possible. Thus, an agent who should be irrelevant in the economic allocation process, influences the outcome in the coarse value in a major way. The reason is that "bad" information of one agent poses a negative externality on all other agents in the economy. Thus, rather than measuring the marginal contributions of an agent, the coarse value measures this negative externality of an agent on all other coalitions. This seems to us to be very much against the general idea of what the Shapley value is supposed to describe.

[29] Thus, two states a and b are indistinguishable with respect to the common knowledge information of coalition S, if and only if there exists an agent $i \in S$ and a set $A_i \in \mathscr{F}_i$ such that $\{a, b\} \subset A_i$.

[30] For related work on the coarse and the fine core see Koutsougeras and Yannelis (1993).

[31] In this particular example this means that the agent has more information, since the initial endowment of an agent must always be measurable with respect to his/her information.

For a more thorough discussion of these two concepts, see Krasa and Yannelis (1994). There the existence of the fine value as well as examples of nonexistence of the coarse value are presented. Independently of our work, Allen (1991) has also examined the existence and the nonexistence of the coarse, the private, and the fine value.

6. CONCLUDING REMARKS

In this paper we study several value allocation concepts in an economy with differential information. We show that the private value allocation provides an interesting way to measure the "worth" of an agent's information advantage. In particular, the Shapley value provides an explicit way not only to measure the information superiority of an agent, but also to reward the agent for making a Pareto improvement for the economy as a whole by using his/her informational advantage. Moreover, this concept ensures truthful revelation of information within a coalition because incentive compatibility is inherent in it. Furthermore, our examples suggest that the value allocation may be suitable for analyzing problems of financial intermediation.

It should be noted that our different value allocation concepts do not provide a dynamic procedure which explains how the final equilibrium outcome is reached. However, we know from the work of Winter (1994) that the Shapley value of a TU game can be rationalized as a solution to a noncooperative game in extensive form.[32] His results require convex games and risk neutrality which are stronger conditions than those adapted in our modeling. It would be of interest to see if one can provide noncooperative foundations of our results, and explain the dynamics of reaching equilibrium outcomes. This seems to be an important open question.

Finally, it is well known that cardinal value allocations characterize Walrasian equilibrium allocations in large economies. However, in a differential information economy framework it is not only unknown to us whether such a result can be obtained, but it is not even clear what should be the correct definition of a Walrasian equilibrium in a differential information economy.

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[32] Gul (1989) has also provided a noncooperative (bargaining) foundation for the Shapley value of a TU-game. However, his model is based on bilateral bargaining rather than multilateral, contrary to Winter's (1992) model. The restriction to bilateral bargaining appears to make his results inapplicable to our framework.

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