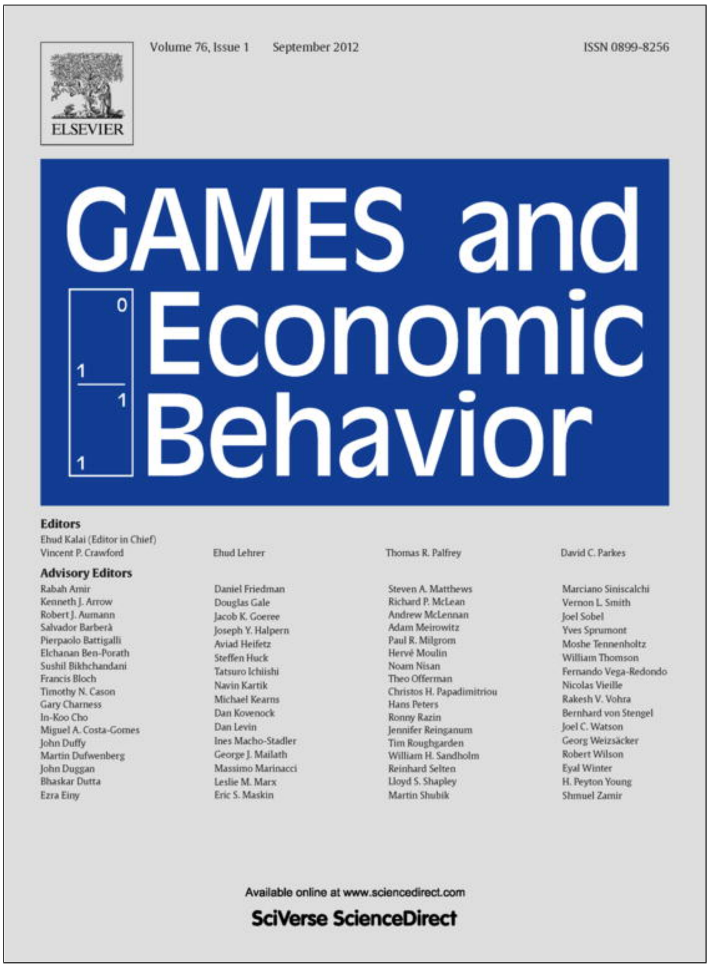

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Existence, incentive compatibility and efficiency of the rational expectations equilibrium ☆

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A B S T R A C T

The rational expectations equilibrium (REE), as introduced in Radner (1979) in a general equilibrium setting *à la* Arrow–Debreu–McKenzie, often fails to have desirable properties such as universal existence, incentive compatibility and efficiency. We resolve those problems by providing a new model which makes the REE a desirable solution concept. In particular, we consider an asymmetric information economy with a continuum of agents whose private signals are independent conditioned on the macro states of nature. For such an economy, agents are allowed to augment their private information by the available public signals. We prove the existence, incentive compatibility and efficiency for this new REE concept.

1. Introduction

In seminal papers, Radner (1979) and Allen (1981) extended the finite agent Arrow–Debreu–McKenzie economy to allow for asymmetric information, where each agent is characterized by a random utility function, random initial endowment, and private information with a prior. The equilibrium notion that Radner put forward is called rational expectations equilibrium (REE), which is an extension of the deterministic Walrasian equilibrium of the Arrow–Debreu–McKenzie model. According to the REE, each individual maximizes interim expected utility conditioned on her own private information as well as the information generated by the equilibrium price.

By now it is well known that in a finite agent economy with asymmetric information, a rational expectations equilibrium may not exist[1] (see Mas-Colell et al., 1995, p. 722, for an example due to Kreps), may not be incentive compatible, may not be Pareto optimal and may not be implementable as a perfect Bayesian equilibrium of an extensive form game (see Glycopantis et al., 2005, p. 31, and also Example 9.1.1, p. 43). Thus, if the intent of the REE notion is to capture contracts among agents under asymmetric information, then such contacts not only do they not exist universally in well behaved economies (i.e., economies with concave, continuous, monotone utility functions and strictly positive initial endowments),

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[1] It only exists in a generic sense as Radner (1979) and Allen (1981) have shown.

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but even if they exist, they fail to have desirable properties, such as incentive compatibility, Pareto optimality and Bayesian rationality.

The main difficulty that one encounters with the REE in a finite agent economy is the fact that individuals are supposed to maximize their interim expected utility conditioned not only on their own private information, but also on the information generated by the equilibrium price; at the same time, the individuals can influence the equilibrium price to their own benefit by manipulating their private information. However, this would not have been a problem if each agent's private information is negligible. This poses the following question. Is it possible to model the REE in such a way that each agent's effect on the equilibrium price is negligible and therefore the REE concept overcomes the difficulties encountered above?

We introduce a new model where the REE concept becomes free of the problems mentioned above. In particular, we consider an asymmetric information economy with a continuum of agents whose private signals are independent conditioned on the macro states of nature. For such an economy, agents are allowed to augment their private information by the available public signals. We call this new notion REE with aggregate signals. However, it is shown in Sun et al. (forthcoming) that it is not true that under a REE in a large economy, the agents will automatically report their signals truthfully. We need to consider those REE prices that capture the meaning of perfect competition, i.e., they depend only on the macro states and are not influenced by individual agents' private information. In this case, incentive compatibility is not an issue since an individual agent's private signal can influence neither the macro states nor the aggregate signals. We then prove the existence and efficiency for this new REE notion. We also show that whenever the equilibrium price fully reveals the macro states, the notion of REE with aggregate signals is equivalent to the notion of REE in the classical sense.

The paper is organized as follows. In Section 2, we present the economic model, the notions of REE, Pareto optimality and incentive compatibility, and some assumptions. The main result is stated in Section 3. In Section 4, we discuss related literature. Some concluding remarks are provided in Section 5. The proofs are given in Appendix A.

2. The economic model

In this section, we define the notion of a private information economy, followed by the definitions of rational expectations equilibrium, rational expectations equilibrium with aggregate signals, Pareto optimality and incentive compatibility.

2.1. Private information economy

We consider an atomless probability space[2] $(I, \mathcal{I}, \lambda)$ as the space of agents. Each agent receives a *private signal* of type $q \in T^0 = \{q_1, q_2, \ldots, q_L\}$.[3] Let $\mathcal{T}^0$ denote the power set of T^0, and $\Delta(T^0)$ the set of all probability distributions on T^0. A *signal profile* t is a function from I to T^0. For $i \in I$, $t(i)$ (also denoted by t_i) is the private signal of agent i while t_{-i} is the restriction of t to the set $I \setminus \{i\}$. Let $(T, \mathcal{T}, P)$ be a probability space that models the uncertainty associated with the private signal profiles for all the agents.[4] For simplicity, we shall assume that $(T, \mathcal{T})$ has a product structure so that T is the product of T_{-i} and T^0, while $\mathcal{T}$ is the product σ-algebra of $\mathcal{T}^0$ and a σ-algebra $\mathcal{T}_{-i}$ on T_{-i}. For $t \in T$ and $t'_i \in T^0$, we shall adopt the usual notation (t_{-i}, t'_i) to denote the signal profile whose value is t'_i for i and t_j for $j \neq i$.

The *private signal process* is a function from $I \times T$ to T^0 such that $f(i,t) = t_i$ for any $(i,t) \in I \times T$. For each $i \in I$, let $\tilde{t}_i$ be the projection mapping from T to T^0 with $\tilde{t}_i(t) = t_i$. The *private signal distribution* τ_i of agent i over T^0 is defined as $\tau_i(\{q\}) = P(\tilde{t}_i = q)$ for $q \in T^0$. $P^{T_{-i}}(\cdot|q)$ is the conditional probability measure on the measurable space $(T_{-i}, \mathcal{T}_{-i})$ when the private signal of agent i is $q \in T^0$. If $\tau_i(\{q\}) > 0$, then it is clear that for $D \in \mathcal{T}_{-i}$, $P^{T_{-i}}(D|q) = P(D \times \{q\})/\tau_i(\{q\})$.

We also would like to include another source of uncertainty in our model – the macro level uncertainty. Let $S = \{s_1, s_2, \ldots, s_K\}$ be the set of all possible macro states of nature, and $\mathcal{S}$ the power set of S. The S-valued random variable $\tilde{s}$ on T models the macro level uncertainty. For each macro state $s \in S$, denote the event $(\tilde{s} = s) = \{t \in T\colon\ \tilde{s}(t) = s\}$ that s occurs by C_s. The probability that s occurs is $\pi_s = P(C_s)$. Without loss of generality, assume that $\pi_s > 0$ for each $s \in S$. Let P_s be the conditional probability measure on $(T, \mathcal{T})$ when the random variable $\tilde{s}$ takes value s. Thus, for each $B \in \mathcal{T}$, $P_s(B) = P(C_s \cap B)/\pi_s$. It is obvious that $P = \sum_{s \in S} \pi_s P_s$. Note that the conditional probability measure P_s is often denoted as $P(\cdot|s)$ in the literature.

The common *consumption set* for all the agents is the positive orthant $\mathbb{R}^m_+$. Let u be a function from $I \times \mathbb{R}^m_+ \times T$ to $\mathbb{R}_+$ such that for any given $i \in I$, $u(i,z,t)$ is the *utility* of agent i at consumption bundle $z \in \mathbb{R}^m_+$ and signal profile $t \in T$. For any given $(i,t) \in I \times T$, we assume that $u(i,z,t)$ (also denoted by $u_{(i,t)}(z)$ or $u_i(z,t)$)[5] is continuous, monotonic in $z \in \mathbb{R}^m_+$.[6] For each $z \in \mathbb{R}^m_+$, $u_z(\cdot,\cdot)$ is an integrable function on $I \times T$.[7]

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[2] We use the convention that all probability spaces are countably additive.

[3] In the literature, one usually assumes that different agents have different sets of private signals and requires that agents receive each of them with positive probability. For notational simplicity, we choose to work with a common set T^0 of private signals, but allow zero probability for some of the redundant signals. There is no loss of generality in this latter approach.

[4] Thus T is a space of functions from I to T^0.

[5] In the sequel, we shall often use subscripts to denote some variable of a function that is viewed as a parameter in a particular context.

[6] The utility function $u(i,\cdot,t)$ is monotonic if for any $y, z \in \mathbb{R}^m_+$ with $y \leqslant z$ and $y \neq z$, $u(i,y,t) < u(i,z,t)$.

[7] The measure structure on the product space $I \times T$ will be specified in Section 2.3.

In our model, the *initial endowment* of an agent depends on her private signal. The *initial endowment profile* e is a function from $I \times T^0$ to $\mathbb{R}^m_+$ such that for $(i,q) \in I \times T^0$, $e(i,q)$ is the initial endowment of agent i when her private signal is q. We assume that for each $q \in T^0$, $e(\cdot,q)$ is λ-integrable over I, and $\int_I e(i,q)\,d\lambda$ is in the strictly positive cone $\mathbb{R}^m_{++}$.[8]

Formally, the private information economy is denoted by

$$\mathcal{E} = \{I \times T, u, e, f, (\tilde{t}_i,\ i \in I), \tilde{s}\}.$$

2.2. Rational expectations equilibrium via a state signal approach

As usual, a price is a normalized nonnegative vector p in Δ_m, where Δ_m is the unit simplex of $\mathbb{R}^m_+$. A *price process* $\tilde{p}$ is a measurable function from T to Δ_m. For each $t \in T$, $\tilde{p}(t)$ is the price when the signal profile is t. For notational simplicity, the letter p will be used both for a price and a price process. The terms "price" and "price process" are used synonymously in this paper.

We shall now introduce some notation for aggregate signals. For any $t \in T$, the empirical signal distribution is λf_t^{-1}, where $f_t = f(\cdot,t)$. For $q \in T^0$, $\lambda f_t^{-1}(\{q\})$ is the fraction of agents whose private signal is q when the signal profile is $t \in T$. It is clear that $\lambda f_t^{-1} \in \Delta(T^0)$. The empirical signal distribution process $\tilde{\mu}$ is a function from T to $\Delta(T^0)$ such that $\tilde{\mu}(t) = \lambda f_t^{-1}$, which is also called the *aggregate signals*.

As mentioned in the introduction, we consider the classical rational expectations equilibrium (REE) and REE with aggregate signals. We shall define a general notion of rational expectations equilibrium (REE) via a state signal formulation as in Mas-Colell et al. (1995).[9] Various related basic concepts are then defined for this general notion that covers the two special cases.

We consider a signal process F from $I \times T$ to G^0, where G^0 is either T^0 or $T^0 \times \Delta(T^0)$ in the two special cases we consider here. For agent $i \in I$, $F(i,t)$ is the derived signal of agent i from the signal profile t, which is simply t_i in the first case and $(t_i, \tilde{\mu}(t))$ in the second case. We shall allow agent i to use the information as described by F_i and the information revealed by the equilibrium price. An *allocation* x to be a measurable mapping from $I \times \Delta_m \times G^0$ to $\mathbb{R}^m_+$. For each $(i,p,g) \in I \times \Delta_m \times G^0$, $x(i,p,g)$ is the consumption bundle of agent i when the price is p and her derived signal is g.

Since an agent's initial endowment is contingent on her private signal $q \in T^0$, we denote the *budget set* for agent i by $B_i(p,q)$ when the price is p and her private signal is q. Hence, $B_i(p,q) = \{z \in \mathbb{R}^m_+ \colon\ pz \leqslant pe(i,q)\}$.

Given a consumption bundle $z \in \mathbb{R}^m_+$, a signal $g \in G^0$ and a price p, the *interim (conditional) expected utility* of agent i is defined as follows:

$$U_i(z|p,g) = E\{u(i,z,t)\,|\,\tilde{p} = p,\ F_i = g\}. \tag{1}$$

In the rational expectations equilibrium, an agent updates her belief on the distribution of signal profiles based on her signal and observation of the equilibrium price. She computes her expected utility with the updated belief and aims to maximize the interim expected utility subject to her budget constraint. The formal definition of the rational expectations equilibrium is given below.

Definition 1 *(Rational Expectations Equilibrium (REE)).* A rational expectations equilibrium for the private information economy $\mathcal{E} = \{I \times T, u, e, f, (\tilde{t}_i,\ i \in I), \tilde{s}\}$ is a pair of an allocation and a price process (x^*, p^*) such that:

1. x^* is *feasible*, i.e., $\int_I x^*(i, p^*(t), F_i(t))\,d\lambda = \int_I e(i,t_i)\,d\lambda$ for P-almost all $t \in T$;
2. for λ-almost all $i \in I$ and for P-almost all $t \in T$, $x^*(i, p^*(t), F_i(t))$ is a maximizer of the following problem:

$$\begin{array}{ll} \max & U_i(z|p^*(t), F_i(t)) \\ \text{subject to} & z \in B_i(p^*(t), t_i). \end{array}$$

The following definition of interim efficiency is self-explanatory.

Definition 2 *(Interim efficiency).* A REE (x^*, p^*) is said to be interim efficient if for P-almost all $t \in T$, there does not exist an integrable function y_t from I to $\mathbb{R}^m_+$ such that

1. $\int_I y_t(i)\,d\lambda = \int_I e(i,t_i)\,d\lambda$;
2. $U_i(y_t(i)|p^*, F_i(t)) > U_i(x^*|p^*, F_i(t))$ for λ-almost all $i \in I$.

When the signal process F is the private signal process f, Definition 1 simply defines a *REE in the classical sense*; an agent updates her belief based on her own private signal and observation of the equilibrium price.

[8] A vector z is in $\mathbb{R}^m_{++}$ if and only if all of its components are positive.

[9] The authors are indebted to an anonymous referee for suggesting this unifying approach.

On the other hand, the information on the relative frequency of private signals should be easily available. As we consider a large economy, an individual agent cannot influence such a signal distribution, which is thus a robust statistic to be used by individual agents. When the signal process F takes the form $(f, \tilde{\mu})$, the corresponding equilibrium concept as in Definition 1 is called *rational expectations equilibrium with aggregate signals*. This means that in the REE with aggregate signals, agent i updates her belief on the distribution of signal profiles upon observing the price p, her own private signal q and the empirical signal distribution μ.

2.3. Conditional independence

In order to study the relationship between the two REE notions introduced above and to prove our main result, we shall need to work with the assumption of conditional independence of private signals given the macro states.

Definition 3. The private signal process f is essentially pairwise independent conditioned on the macro state of nature $\tilde{s}$. That is, for λ-almost all $i \in I$, $\tilde{t}_i$ and $\tilde{t}_j$ are independent conditioned on $\tilde{s}$ for λ-almost $j \in I$.[10]

The above condition on f will be imposed on the rest of this paper unless otherwise noted. When such an assumption is imposed, an immediate technical difficulty arises, which is the so-called measurability problem of independent processes. In our context, a signal process that is essentially independent, conditioned on the macro states of nature is never jointly measurable in the usual sense except for trivial cases.[11] Hence, we need to work with a joint agent–probability space $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes P)$ that extends the usual measure-theoretic product $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes P)$ of the agent space $(I, \mathcal{I}, \lambda)$ and the probability space $(T, \mathcal{T}, P)$, and retains the Fubini property. Below is a formal definition of the Fubini extension in Definition 2.2 of Sun (2006).

Definition 4. A probability space $(I \times \Omega, \mathcal{W}, Q)$ extending the usual product space $(I \times \Omega, \mathcal{I} \otimes \mathcal{F}, \lambda \otimes P)$ is said to be a *Fubini extension* of $(I \times \Omega, \mathcal{I} \otimes \mathcal{F}, \lambda \otimes P)$ if for any real-valued Q-integrable function f on $(I \times \Omega, \mathcal{W})$,

(1) the two functions f_i and f_ω are integrable respectively on $(\Omega, \mathcal{F}, P)$ for λ-almost all $i \in I$, and on $(I, \mathcal{I}, \lambda)$ for P-almost all $\omega \in \Omega$;
(2) $\int_\Omega f_i \, dP$ and $\int_I f_\omega \, dP$ are integrable respectively on $(I, \mathcal{I}, \lambda)$ and $(\Omega, \mathcal{F}, P)$, with $\int_{I \times \Omega} f \, dQ = \int_I (\int_\Omega f_i \, dP) \, d\lambda = \int_\Omega (\int_I f_\omega \, d\lambda) \, dP$.[12]

To reflect the fact that the probability space $(I \times \Omega, \mathcal{W}, Q)$ has $(I, \mathcal{I}, \lambda)$ and $(\Omega, \mathcal{F}, P)$ as its marginal spaces, as required by the Fubini property, it will be denoted by $(I \times \Omega, \mathcal{I} \boxtimes \mathcal{F}, \lambda \boxtimes P)$.

Fix $s \in S$. For each $D \in \mathcal{I} \boxtimes \mathcal{T}$, let $\lambda \boxtimes P_s(D) = \lambda \boxtimes P((I \times C_s) \cap D)/\pi_s$. Then, $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes P_s)$ also extends the usual measure-theoretic product $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes P_s)$, and retains the Fubini property. Thus, one can view $\lambda \boxtimes P_s$ as the conditional probability measure on $I \times T$, given $\tilde{s} = s$.

We shall assume that f is a measurable process from $(I \times T, \mathcal{I} \boxtimes \mathcal{T})$ to T^0. For any $A \in \mathcal{T}$, the Fubini property associated with $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes P)$ implies that $P(A \cap f_i^{-1}(\{q\}))$ is measurable in $i \in I$. This means that the process f has event-wise measurable conditional probabilities, as defined in Definition 4 of Hammond and Sun (2008). By Theorem 1 of Hammond and Sun (2008), one can always find a real-valued random variable $\tilde{s}$ on $(T, \mathcal{T}, P)$ such that f is essentially pairwise independent conditioned on $\tilde{s}$. The only restriction in this paper is to assume that $\tilde{s}$ takes finitely many values. Since the σ-algebra generated by a real-valued random variable is countably generated, and a countably generated σ-algebra can be approximated by σ-algebras generated by finite partitions in terms of information as shown in Allen (1983), the assumption that the private signals are essentially pairwise independent conditioned on finitely many macro states of nature is a reasonable assumption.

When the macro state is s, the signal distribution of agent i conditioned on the macro state is $P_s f_i^{-1}$, i.e., the probability for agent i to have $q \in T^0$ as her private signal is $P_s(f_i^{-1}(\{q\}))$, where $f_i = f(i, \cdot)$. Let μ_s be the agents' *average signal distribution* conditioned on the macro state s, i.e.,

$$\mu_s(\{q\}) = \int_I P_s\left(f_i^{-1}(\{q\})\right) d\lambda = \int_I \int_T 1_{\{q\}}\left(f(i,t)\right) dP_s \, d\lambda, \tag{2}$$

where $1_{\{q\}}$ is the indicator function of the singleton set $\{q\}$. Throughout the rest of this paper, the following *non-triviality assumption* on the process f will be imposed:

[10] For a detailed discussion on pairwise (conditional) independence, see Sun (2006).

[11] See Proposition 2.1 in Sun (2006), and Proposition 4 in Hammond and Sun (2008) for detailed discussion of the measurability problem.

[12] The classical Fubini Theorem is only stated for the usual product measure spaces. It does not apply to integrable functions on $(I \times \Omega, \mathcal{W}, Q)$ since these functions may not be $\mathcal{I} \otimes \mathcal{F}$-measurable. However, the conclusions of that theorem do hold for processes on the enriched product space $(I \times \Omega, \mathcal{W}, Q)$ that extends the usual product.

$$\forall s, s' \in S, \quad s \neq s' \quad \Rightarrow \quad \mu_s \neq \mu_{s'}. \tag{3}$$

This says that different macro states of nature correspond to different average conditional distributions of agents' signals.

2.4. Incentive compatibility and revelation of macro states

The following is a notion of incentive compatibility of a REE in the classical sense. The case of a REE with aggregate signals can be similarly defined. It says that an agent cannot increase her interim expected utility by mis-reporting her private signal.

Definition 5 *(Incentive compatibility).* A REE (x^*, p^*) is said to be incentive compatible if for λ-almost all $i \in I$,

$$U_i\big(x^*\big(i, p^*(t), t_i\big)\big|p^*(t), t_i\big) \geqslant U_i\big(x^*\big(i, p^*\big(t_{-i}, t_i'\big), t_i\big)\big|p^*\big(t_{-i}, t_i'\big), t_i\big)$$

holds for P-almost all $t \in T$ and for all $t_i' \in T^0$.

As shown in the note Sun et al. (forthcoming), there exists a REE (x^*, p^*) in the classical sense in a large private information economy $\mathcal{E}^p$, where all the conditions on the information structure in Sections 2.1 and 2.3 are satisfied, such that every agent has an incentive to mis-report her signal.[13] Therefore, considering an arbitrary REE in a large economy may not lead to desirable results. As in Sun and Yannelis (2007) and Sun and Yannelis (2008), one should pay attention to those REE prices that capture the meaning of perfect competition, i.e., those prices that depend on the macro states and are not influenced by individual agents' private information.[14] This leads us to consider the following key concept in this paper.

Definition 6. Let p be a price process from T to Δ_m. (1) It is said to depend only on the macro states if the σ-algebra generated by p is essentially contained in the σ-algebra generated by $\tilde{s}$.[15] (2) It fully reveals the macro states if it depends only on the macro states, and different macro states correspond to different prices for those states up to null events. In other words, the price process p and $\tilde{s}$ generate essentially the same σ-algebras on T.[16]

Remark 1. When one focuses on those REE prices (in the classical sense or with aggregate signals) that depend only on the macro states, incentive compatibility is not an issue since an individual agent's private signal can influence neither the macro states nor the aggregate signals.

2.5. The relationship between the REE with aggregate signals and the REE

One may ask naturally what the relationship between the REE and REE with aggregate signals is. When the REE price in Definition 1 fully reveals the macro states, the following lemma, whose proof is given in Section A.2 of Appendix A shows that the two notions are equivalent in the sense that one can be converted to the other.

Lemma 1. *Let (x^*, p^*) be a REE with aggregate signals. If p^* fully reveals the macro states, then one can find an allocation $\bar{x}^*$ so that $(\bar{x}^*, p^*)$ is a REE in the classical sense. The other direction is also true.*

3. The main result

We are now ready to state the main theorem of this paper.

Theorem 1. *There exists an interim efficient REE with aggregate signals in which the equilibrium price p depends only on the macro states.*

When the economy has only idiosyncratic level information, i.e., the macro state variable $\tilde{s}$ is constant, then the conditional independence in Definition 3 becomes unconditional independence. In this case, Theorem 1 implies that the REE price with aggregate signals is constant and trivially reveals the macro states. Hence, as shown in the following corollary, Lemma 1 implies the existence of a REE in the classical sense.

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[13] As shown in Section 4 of Sun et al. (forthcoming), the private signals are independent of each other, and the macro state function $\tilde{s}$ can be regarded as constant. In this case, (x^*, p^*) is also trivially a REE with aggregate signals. Example 3 of Heifetz and Minelli (2002) considers a large finite replica economy where an agent in the first replica fails to be incentive compatible. Since we consider an atomless economy, we may ignore a negligible set of agents and thus need a stronger result on the failure of incentive compatibility.

[14] We may also point out that it may be too much to require an equilibrium price to reveal all the private information in a large economy.

[15] The strong completion of a sub-σ-algebra $\mathcal{A}$ of $\mathcal{T}$ is the σ-algebra generated by all sets in $\mathcal{A}$ and all the P-null sets in $\mathcal{T}$. A sub-σ-algebra $\mathcal{A}$ of $\mathcal{T}$ is said to be essentially contained in another sub-σ-algebra $\mathcal{D}$ of $\mathcal{T}$ if the strong completion of $\mathcal{A}$ is contained in the strong completion of $\mathcal{D}$.

[16] Two σ-algebras are said to be essentially the same if their strong completions are the same.

Corollary 1. *If the private signal process f is essentially pairwise independent in the sense that for λ-almost all $i \in I$, $\tilde{t}_i$ and $\tilde{t}_j$ are independent for λ-almost $j \in I$, then there exists an interim efficient REE in the classical sense in which the equilibrium price is constant.*

4. Discussion

In view of Kreps' example on the non-existence of REE, the earlier contributions on the REE were focused on the generic existence (Radner, 1979; Allen, 1981). By now, it is well known that with a finite number of agents, the REE not only does not exist universally but also fails to be Pareto efficient and incentive compatible; see Glycopantis et al. (2005, p. 31, and also Example 9.1.1, p. 43).

In contrast to the above, the current paper demonstrates that the REE with aggregate signals exists universally, and also is Pareto efficient and incentive compatible. Thus, the paper resolves the difficulties that the classical REE faces as a solution concept. Our results enable us to conclude that in the presence of a continuum of agents with non-trivial but negligible private information, the REE with aggregate signals becomes an appealing concept as it does have desirable properties, contrary to the finite agent case.

There are several other papers dealing with the REE with a continuum of agents that we discuss below. Let us say at the outset that our Theorem 1 proves a result about a new REE concept, which has no analog in the literature.

The early contributions by Einy et al. (2000a), Einy et al. (2000b) consider an asymmetric information economy with a continuum of agents and a finite number of states of nature. The private information of each agent is a partition of the finite state space. Since a finite state space has only finitely many different partitions, we can find a partition on the measure space of agents so that all the agents in each partitioning set of agents share the same private information. Therefore, there is little heterogeneity on the private information side of their model. In contrast, our model allows agents to have non-trivial idiosyncratic private information. Hence, their work is not directly related to ours.

Another interesting paper by Heifetz and Minelli (2002) models the idea of informational smallness without aggregate uncertainty for an asymmetric information economy with a continuum of independent replica economies. The definition of an allocation in their paper is based on the notion of Pettis integral (Definition 6, p. 213). Instead of the usual state-wise feasibility condition, the authors used the condition that the value of the aggregate consumption is equal to the value of the aggregate endowment for every price function (see pp. 212 and 213). Note that this does not imply market clearing, which requires the equality of aggregate consumption and aggregate endowment for almost all the states of nature. It seems to us that the reason they used this approach is to avoid the so-called measurability problem associated with a continuum of independent random variables.[17] Our Corollary 1 has a similar flavor of a constant REE price as in their Theorem 2 on a continuum of independent replica economies. The main differences here are (1) we work with a general continuum economy rather than a replica economy; (2) we work with a general independence condition, and a Fubini extension, where the state-wise feasibility condition is fulfilled (see part 1 of Definition 1).

McLean and Postlewaite (2002, 2003, 2005) model the idea of informational smallness (i.e., roughly speaking, approximate perfect competition) in countable replica economies. Their main objective is to show the consistency of ex post efficiency, ex ante core convergence and incentive compatibility.[18] They do not consider the REE notion in those papers.

5. Concluding remarks

As shown in Lemma 1, the REE in the classical sense and the REE with aggregate signals are equivalent when the equilibrium price fully reveals the macro states. Without such full revelation, they need not be equivalent in general. The following remark considers the case of partial revelation.

Remark 2. If we impose the additional assumptions that the utility functions $u(i, x, t)$ are strictly concave in x and depend only on the agents' private signals in the sense that $u(i, x, t) = v(i, x, t_i)$ for some function v from $I \times \mathbb{R}^m_+ \times T^0$ to $\mathbb{R}_+$, then we can claim that the REE with aggregate signals and the standard REE are again equivalent when the equilibrium price only depends on the macro states. In particular, for a REE with aggregate signals (x^*, p^*), if p^* only depends on the macro states, then one can find an allocation $\bar{x}^*$ so that $(\bar{x}^*, p^*)$ is a REE. The other direction is also true.[19] It then follows from Theorem 1 that there exists an interim efficient REE in which the equilibrium price p depends on the macro state of nature; moreover, such a REE is ex post efficient.[20]

To the best of our knowledge, this is the first paper which models the REE notion with a continuum of agents where both aggregate uncertainty and idiosyncratic private information are included. We believe that the use of aggregate signals

[17] See, for example, Sun (2006).

[18] The corresponding limiting results for a continuum of agents are considered in Sun and Yannelis (2007) and Sun and Yannelis (2008).

[19] This claim is proved in Section A.3 of Appendix A.

[20] As also pointed out by an anonymous referee, we note that with the special assumptions on the utility functions, an ex post efficient REE exists even for a finite agent economy. However, since an agent has non-negligible influence in a finite agent private information economy, the incentive compatibility remains an issue. As noted in Remark 1, our consideration of an atomless economy does amend this issue by focusing on the equilibrium prices that only depend on the macro states.

is very natural in a large economy setting. Each agent reports her private signal to a center or computer. The relative frequency of the signals is then announced. Agents can then augment their private information by this trivially available public information. The REE with aggregate signals won't make much sense in a finite agent economy since an individual agent can manipulate the aggregate signals. The well-known example of Kreps can be easily modified to show this. In the continuum setting, our new concept seems to be natural, which may also be more appropriate than the classical REE in view of the positive general results obtained in this paper.[21]

Finally, it should be noted that the exact limiting results in this paper on an atomless economy with asymmetric information have asymptotic analogs for large but finite asymmetric information economies. In principle, we can use nonstandard analysis to obtain the existence of approximate REE that is approximately incentive compatible for most agents as in Theorem 3 of Sun and Yannelis (2007).[22]

Appendix A

A.1. Proof of Theorem 1

We outline the proof first. In Step 1, we construct a large deterministic economy for each macro state of nature $s \in S$. Applying the standard Walrasian equilibrium existence results for a large deterministic economy, we can obtain a Walrasian equilibrium (y_s, p_s) for such an economy. In Step 2, we construct a price p^* and an allocation x^* from the collection of allocation–price pairs $\{(y_s, p_s)\colon s \in S\}$, and show that (x^*, p^*) is a REE with aggregate signals in which the equilibrium price depends only on the macro state of nature.

Step 1: Define a function Γ from $I \times T$ to $I \times T^0$ by letting $\Gamma(i,t) = (i, t_i)$ for $(i,t) \in I \times T$. Γ is a measurable mapping in the sense that $\Gamma^{-1}(D) \in \mathcal{I} \boxtimes \mathcal{T}$ for all $D \in \mathcal{I} \otimes \mathcal{T}^0$.

For each $s \in S$, let ν_s be the measure on $I \times T^0$ defined as

$$\nu_s(D) = (\lambda \boxtimes P_s)\big(\Gamma^{-1}(D)\big)$$

for $D \in \mathcal{I} \otimes \mathcal{T}^0$. For $(i,q) \in I \times T^0$, define a utility function $V_s(i,\cdot,q)$ on $\mathbb{R}^m_+$ by letting

$$V_s(i,z,q) = E\big\{u(i,z,t)\big|\tilde{s} = s,\ \tilde{t}_i = q\big\} \tag{4}$$

for $z \in \mathbb{R}^m_+$.

For each $s \in S$, we define a large deterministic economy $\bar{\mathcal{E}}_s = \{(I \times T^0, \mathcal{I} \otimes \mathcal{T}^0, \nu_s), V_s, e\}$, where the utility function for agent $(i,q) \in I \times T^0$ is $V_s(i,\cdot,q)$, and the initial endowment for agent (i,q) is $e(i,q)$. By the standard Walrasian equilibrium existence results (see, for example, Aumann, 1966), there is a Walrasian equilibrium allocation y_s and a strictly positive equilibrium price $p_s \in \Delta_m$ for the economy $\bar{\mathcal{E}}_s$ such that:

1. y_s is feasible, i.e., $\int_{I\times T^0} y_s(i,q)\,d\nu_s = \int_{I\times T^0} e(i,q)\,d\nu_s$, and
2. for ν_s-almost all agent $(i,q) \in I \times T^0$, $y_s(i,q)$ is a maximal element in her budget set $B_{(i,q)}(p_s) = \{y \in \mathbb{R}^m_+\colon\ p_s y \leqslant p_s e(i,q)\}$.[23]

Step 2: Define a mapping x^* from $I \times \Delta_m \times T^0 \times \Delta(T^0)$ to $\mathbb{R}^m_+$ by letting

$$x^*(i,p,q,\mu) = \begin{cases} y_s(i,q) & \text{if } p = p_s \text{ and } \mu = \mu_s \text{ for some } s \in S, \\ e(i,q) & \text{otherwise} \end{cases}$$

for each $(i,q) \in I \times T^0$.

By the non-triviality assumption in Eq. (3) in Section 2.3, there is at most one s such that $\mu = \mu_s$. Hence, x^* is well-defined.

Define the following sets

$$\forall s \in S, \quad L_s = \big\{t \in T\colon \lambda f_t^{-1} = \mu_s\big\}; \quad L_0 = T - \bigcup_{s\in S} L_s.$$

[21] In the recent papers of de Castro et al. (2011) and Condie and Ganguli (2011), the Bayesian conditional expected utility as used here is replaced by the maximin expected utility in the setting of ambiguity aversion for a finite agent economy with finitely many states. It will be interesting to know if one can consider ambiguity aversion in our setting with a continuum of agents and states to derive new insights.

[22] For discussion and references on the general methodology of obtaining asymptotic results in a similar setting, see Section 5 of Sun and Yannelis (2007). Examples 3 and 4 of Heifetz and Minelli (2002) consider a large finite replica economy where one agent in the first replica fails to be approximately incentive compatible. There is no contradiction with our results since an asymptotic version of our exact results would only imply approximate incentive compatibility for most agents, but not for a set of agents with small measure.

[23] By modifying the values of y_s on a null set (if necessary), we can assume that for every $(i,q) \in I \times T^0$, $y_s(i,q)$ is a maximal element in her budget set.

The non-triviality assumption in Eq. (3) implies that for any $s, s' \in S$ with $s \neq s'$, $L_s \cap L_{s'} = \emptyset$. The measurability of the sets L_s, $s \in S$, and L_0 follows from the measurability of f. Thus, the collection $\{L_0\} \cup \{L_s,\ s \in S\}$ forms a measurable partition of T.

By Eq. (2) and the Fubini property for $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes P_s)$, we have $\mu_s(\{q\}) = \int_{I \times T} 1_{\{q\}}(f(i,t))\, d(\lambda \boxtimes P_s)$ for any $q \in T^0$. Thus, μ_s is actually the distribution $(\lambda \boxtimes P_s) f^{-1}$ of f, viewed as a random variable on the product space $I \times T$. Since the signal process f satisfies the condition of essential pairwise conditional independence, for λ-almost all $i \in I$, the random variables f_i and f_j from $(T, \mathcal{T}, P_s)$ to X are independent for λ-almost all $j \in I$. The exact law of large numbers in Corollary 2.9 of Sun (2006) implies that $P_s(L_s) = 1$ for each $s \in S$.

Choose a strictly positive price $p_0 \in \Delta_m$ which is different from each p_s, $s \in S$. For each $t \in T$, define

$$p^*(t) = \begin{cases} p_s & \text{if } t \in L_s \text{ for some } s \in S, \\ p_0 & \text{otherwise.} \end{cases}$$

For any $s, s' \in S$, $P_s(C_{s'}) = P_s(L_{s'})$ is one when $s = s'$ and zero when $s \neq s'$. It is clear that $P(C_s \Delta L_s) = 0$. Hence $p^*(t)$ depends essentially on the macro state of nature. For this reason, we can also write the price as $p(\tilde{s})$.

We will complete the proof by showing that (x^*, p^*) is a REE with aggregate signals for the private information economy $\mathcal{E}$. By the definition of ν_s, we have

$$\int_{I \times T^0} y_s(i,q)\, d\nu_s = \int_{I \times T} y_s(i,t_i)\, d\lambda \boxtimes P_s \tag{5}$$

and

$$\int_{I \times T^0} e(i,q)\, d\nu_s = \int_{I \times T} e(i,t_i)\, d\lambda \boxtimes P_s. \tag{6}$$

Given a macro state $s \in S$, the essential pairwise independence of the private signals t_i implies the essential pairwise independence of the random variables $y_s(i, \tilde{t}_i)$, $i \in I$, and the random variables $e(i, \tilde{t}_i)$, $i \in I$, respectively. By the exact law of large numbers in Corollary 2.10 of Sun (2006), there exists $T_s \in \mathcal{T}$ with $P_s(T_s) = 1$ such that for every $t \in T_s$,

$$\int_I y_s(i,t_i)\, d\lambda = \int_{I \times T} y_s(i,t_i)\, d\lambda \boxtimes P_s \tag{7}$$

and

$$\int_I e(i,t_i)\, d\lambda = \int_{I \times T} e(i,t_i)\, d\lambda \boxtimes P_s. \tag{8}$$

By Eqs. (5)–(8), we have for any $t \in T_s$,

$$\int_I y_s(i,t_i)\, d\lambda = \int_{I \times T^0} y_s(i,q)\, d\nu_s \tag{9}$$

and

$$\int_I e(i,t_i)\, d\lambda = \int_{I \times T^0} e(i,q)\, d\nu_s. \tag{10}$$

Since y_s is a feasible allocation for the economy $\bar{\mathcal{E}}_s$, combined with Eqs. (9) and (10), we have, for any $t \in T_s$,

$$\int_I y_s(i,t_i)\, d\lambda = \int_I e(i,t_i)\, d\lambda. \tag{11}$$

Let $\Omega_0 = \bigcup_{s \in S} L_s \cap T_s$. Then $P(\Omega_0) = 1$. For any $t \in \Omega_0$, there is a unique $s \in S$ such that $t \in L_s \cap T_s$, and thus $x^*(i, p^*(t), t_i, \lambda f_t^{-1}) = y_s(i,t_i)$ and furthermore

$$\int_I x^*\big(i, p^*(t), t_i, \lambda f_t^{-1}\big)\, d\lambda = \int_I y_s(i,t_i)\, d\lambda = \int_I e(i,t_i)\, d\lambda.$$

Consequently, x^* is feasible.

Note that for each $s \in S$, $\tilde{s}(t) = s$, $\tilde{\mu}(t) = \mu_s$, and $\tilde{p}^*(t) = p_s$ hold for P-almost all $t \in L_s$. Thus, the σ-algebras generated by the random variables $\tilde{s}$ and $\tilde{\mu}$ are essentially the same while the σ-algebra generated by $\tilde{p}^*$ is a sub-σ-algebra. Hence, for any $z \in \mathbb{R}^m_+$,

$$\begin{aligned} U_i(z|p_s, q, \mu_s) &= E\{u(i, z, t)\big|\tilde{p}^* = p_s,\ \tilde{t}_i = q,\ \tilde{\mu} = \mu_s\} \\ &= E\{u(i, z, t)\big|\tilde{s} = s,\ \tilde{t}_i = q\} = V_s(i, z, q). \end{aligned}$$

It is left to show that for each agent $i \in I$, x_i^* maximizes the conditional expected utility U_i subject to her budget constraint.

As noted above, for any $t \in \Omega_0$, there is a unique $s \in S$ such that $t \in L_s \cap T_s$. The budget set for agent i is $B_i(p^*(t), t_i) = B_i(p_s, t_i) = \{z \in \mathbb{R}^m_+ \colon\ p_s z \leqslant p_s e(i, t_i)\}$. This is exactly the same as the budget set $B_{(i,t_i)}(p_s)$ of agent (i, t_i) in the deterministic economy $\bar{\mathcal{E}}_s$. Since $V_s(i, \cdot, t_i) = U_i(\cdot|p^*(t), t_i, \lambda f_t^{-1})$, the following two problems are equivalent:

$$\text{Problem (I)} \quad \begin{array}{ll} \max & V_s(i, z, t_i) \\ \text{subject to} & z \in B_{(i,t_i)}(p_s) \end{array}$$

and

$$\text{Problem (II)} \quad \begin{array}{ll} \max & U_i\big(z|p^*(t), t_i\big) \\ \text{subject to} & z \in B_i\big(p^*(t), t_i, \lambda f_t^{-1}\big). \end{array}$$

Since $y_s(i, t_i)$ is a maximizer for Problem (I), it must be a maximizer for Problem (II) as well. By definition, $x^*(i, p^*(t), t_i, \lambda f_t^{-1}) = y_s(i, t_i)$. Hence, $x^*(i, p^*(t), t_i, \lambda f_t^{-1})$ is a maximizer for Problem (II). That is to say that $x^*(i, p^*(t), t_i, \lambda f_t^{-1})$ maximizes $U_i(\cdot|p^*(t), t_i, \lambda f_t^{-1})$ subject to the budget set $B_i(p^*(t), t_i)$.

Hence, (x^*, p^*) constitutes a REE with aggregate signals for the private information economy $\mathcal{E}$.

Finally, we shall prove that (x^*, p^*) is interim efficient. Suppose the two conditions in the definition of REE with aggregate signals hold for any $t \in \bar{T}$, where $\bar{T} \in \mathcal{T}$ and $P(\bar{T}) = 1$. Fix $t \in \bar{T}$; we can construct a large deterministic economy $\mathcal{E}_t = \{(I, \mathcal{I}, \lambda), \bar{u}_t, \bar{e}_t\}$, where $\bar{u}_t(i, \cdot) = U_i(\cdot|p^*(t), t_i, \lambda f_t^{-1})$ and $\bar{e}_t(i) = e(i, t_i)$ for each agent $i \in I$. Let $\bar{p}_t = p^*(t)$ and $\bar{x}_t(i) = x^*(i, p^*(t), t_i, \lambda f_t^{-1})$ for each $i \in I$. Then, it is easy to see that $(\bar{x}_t, \bar{p}_t)$ is a Walrasian equilibrium for $\mathcal{E}_t$. It follows that $\bar{x}_t$ is efficient in the sense that there is no allocation $\bar{y}$ for the economy $\mathcal{E}_t$ such that

$$\int_I \bar{y}(i)\, d\lambda = \int_I \bar{e}_t(i)\, d\lambda, \tag{12}$$

$$\bar{u}_t\big(i, \bar{y}(i)\big) > \bar{u}_t\big(i, \bar{x}_t(i)\big) \quad \text{for } \lambda\text{-almost all } i \in I. \tag{13}$$

The above two conditions can be re-written as

$$\int_I \bar{y}(i)\, d\lambda = \int_I e(i, t_i)\, d\lambda, \tag{14}$$

$$U_i\big(\bar{y}|p^*(t), t_i, \lambda f_t^{-1}\big) > U_i\big(x^*\big(i, p^*(t), t_i, \lambda f_t^{-1}\big)\big|p^*(t), t_i, \lambda f_t^{-1}\big) \quad \text{for } \lambda\text{-almost all } i \in I. \tag{15}$$

Hence, (x^*, p^*) is interim efficient.

A.2. Proof of Lemma 1

An equilibrium price $p^*(t)$ is said to fully reveal the macro states if the price depends only on the macro states, and different macro states correspond to different prices for those states. That means $p^*(t)$ is constant p_s^* on C_s, and $p_s^* \neq p_{s'}^*$ when $s \neq s'$. In the following proof, we assume $p^*(t)$ fully reveals the macro states.

We first prove one part of the equivalence result by converting a REE with aggregate signals (x^*, p^*) to a standard REE. We can define an allocation $\bar{x}^*$ for the REE notion by letting

$$\bar{x}^*(i, p, q) = \begin{cases} x^*(i, p, q, \mu_s) & \text{if } p = p_s^* \text{ for some } s \in S, \\ e(i, q) & \text{otherwise.} \end{cases}$$

By the feasibility of x^* (see Definition 1), we have

$$\int_I x^*\big(i, p^*(t), t_i, \lambda f_t^{-1}\big)\, d\lambda = \int_I e(i, t_i)\, d\lambda$$

for P-almost all $t \in T$. As in the proof of Theorem 1, the essential pairwise independence of the private signals t_i and the exact law of large numbers in Corollary 2.9 of Sun (2006) imply that for P-almost all $t \in C_s$, $\lambda f_t^{-1} = \mu_s$. Thus, by the full revelation property of p^*, we also know that for P-almost all $t \in T$, there exists a unique $s \in S$ such that $p^*(t) = p^*_s$ and $\lambda f_t^{-1} = \mu_s$. Hence, we obtain that $\bar{x}^*(i, p^*(t), t_i) = x^*(i, p^*(t), t_i, \lambda f_t^{-1})$ for P-almost all $t \in T$. This implies the feasibility of $\bar{x}^*$ in REE (see Definition 1)

$$\int_I \bar{x}^*\big(i, p^*(t), t_i\big)\, d\lambda = \int_I e(i, t_i)\, d\lambda$$

for P-almost all $t \in T$.

Since both the REE price p^* and the aggregate signals λf_t^{-1} fully reveal the macro states, then $\tilde{s}$, p^* and λf_t^{-1} generate the same partition modulo the null events. Hence, for P-almost all $t \in T$,

$$U_i\big(\cdot|p^*(t), t_i, \lambda f_t^{-1}\big) = U_i\big(\cdot|p^*(t), t_i\big). \tag{16}$$

Therefore, it follows from condition (2) of Definition 1 that for λ-almost all $i \in I$, and for P-almost all $t \in T$, $\bar{x}^*(i, p^*(t), t_i)$ is a maximizer of the following problem:

$$\begin{array}{ll} \max & U_i\big(z|p^*(t), t_i\big) \\ \text{subject to} & z \in B_i\big(p^*(t), t_i\big). \end{array}$$

Hence, $(\bar{x}^*, p^*)$ is a REE as in Definition 1.

For the other direction of the equivalence result, fix a standard REE (x^*, p^*). We can define an allocation $\bar{x}^*$ for the REE with aggregate signals as follows:

$$\bar{x}^*(i, p, q, \mu) = \begin{cases} x^*(i, p, q) & \text{if } p = p^*_s \text{ and } \mu = \mu_s \text{ for some } s \in S, \\ e(i, q) & \text{otherwise.} \end{cases}$$

As above, we obtain that for P-almost all $t \in T$, $\bar{x}^*(i, p^*(t), t_i, \lambda f_t^{-1}) = x^*(i, p^*(t), t_i)$, and

$$\int_I \bar{x}^*\big(i, p^*(t), t_i, \lambda f_t^{-1}\big)\, d\lambda = \int_I x^*\big(i, p^*(t), t_i\big)\, d\lambda = \int_I e(i, t_i)\, d\lambda.$$

Hence, $\bar{x}^*$ is feasible.

As above, Eq. (16) still holds. It follows from condition (2) of Definition 1 that for λ-almost all $i \in I$, and for P-almost all $t \in T$, $\bar{x}^*(i, p^*(t), t_i, \lambda f_t^{-1})$ is a maximizer of the following problem:

$$\begin{array}{ll} \max & U_i\big(z|p^*, t_i, \lambda f_t^{-1}\big) \\ \text{s.t.} & z \in B_i\big(p^*(t), t_i\big). \end{array}$$

Hence, $(\bar{x}^*, p^*)$ is a REE with aggregate signals as in Definition 1.

A.3. *Proof of the claim in Remark 2*

When $p^*(t)$ depends only on the macro states, we have $p^*(t) = p^*_s$ for $t \in C_s$. In the following proof, we assume p^* possesses this property.

By the assumption in the remark, $u(i, x, t) = v(i, x, t_i)$. Thus, we can work with the utility function v. It follows from Eq. (1) that

$$U_i(\cdot|p, q, \mu) = U_i(\cdot|p, q) = v(i, \cdot, q). \tag{17}$$

As in the proof of Lemma 1, we first convert a REE with aggregate signals (x^*, p^*) to a standard REE. We define

$$\bar{x}^*(i, p, q) = \begin{cases} x^*(i, p, q, \mu_s) & \text{if } p = p^*_s \text{ for some } s \in S, \\ e(i, q) & \text{otherwise.} \end{cases}$$

As in the proof of Lemma 1, when p^* fully reveals the macro states, the choice of s is unique in the above definition. Without full revelation property of p^*, there may exist two distinct states s and s' such that $p^*_s = p^*_{s'}$. This causes ambiguity in the above definition. However, we shall justify below that such ambiguity is irrelevant.

It follows from Eq. (17) that given a private signal $q \in T^0$ and price p, agent i faces the following maximization problem in the definition of REE with aggregate signals:

$$\begin{array}{ll} \max & v_i(z, q) \\ \text{s.t.} & z \in B_i(p, q). \end{array}$$

Since u_i (and hence v_i) is strictly concave, the solution for the above problem is unique. It follows immediately that $x^*(i, p^*_s, q, \mu_s) = x^*(i, p^*_{s'}, q, \mu_{s'})$ when $p^*_s = p^*_{s'}$ in that they are the solution of the same problem. Hence, the choice of

s is irrelevant in the definition of $\bar{x}^*$. $\bar{x}^*$ is thus well-defined. This also implies that for P-almost all $t \in T$, $\bar{x}^*(i, p^*(t), t_i) = x^*(i, p^*(t), t_i, \lambda f_t^{-1})$, which in turn implies the feasibility of $\bar{x}^*$ in REE as x^* is feasible in REE with aggregate signals.

It follows from Eq. (17) and condition (2) of Definition 1 that for λ-almost all $i \in I$, and for P-almost all $t \in T$, $\bar{x}^*(i, p^*(t), t_i)$ is a maximizer of the following problem:

$$\begin{aligned} &\max && U_i\big(z|p^*(t), t_i\big) \\ &\text{subject to} && z \in B_i\big(p^*(t), t_i\big). \end{aligned}$$

Hence, $(\bar{x}^*, p^*)$ is a REE as in Definition 1.

The other direction of the equivalence result can be readily obtained by constructing a similar allocation as in the second part of the proof for Lemma 1.

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