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Edgeworth's conjecture in economies with a continuum of agents and commodities*

Aldo Rustichini

Northwestern University, Evanston, IL 60208, USA

Nicholas C. Yannelis

University of Illinois at Urbana-Champaign, Champaign, IL 61820, USA

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Abstract: This paper contains the following results for economies with infinite dimensional commodity spaces. (i) We establish a core–Walras equivalence theorem for economies with an atomless measure space of agents and with an ordered separable Banach commodity space whose positive cone has a non-empty norm interior. This result includes as a special case the Aumann (1964) and Schmeidler–Hildenbrand [Hildenbrand (1974, p. 33)] finite dimensional theorems. (ii) We provide a counterexample which shows that the above result fails in ordered Banach spaces whose positive cone has an empty interior even if preferences are strictly convex, monotone weakly continuous and initial endowments are strictly positive. (iii) Using the assumption of an 'extremely desirable commodity' (which is automatically satisfied whenever preferences are monotone and the positive cone of the commodity space has a non-empty interior), we establish core–Walras equivalence in any arbitrary separable Banach lattice whose positive cone may have an empty (norm) interior.

1. Introduction

Two of the most widely used solution concepts in economic theory are the competitive equilibrium and the core. The first concept is usually associated with Walras, and refers to the non-cooperative allocation of resources via a price system. The essential idea behind this concept is that when agents are assumed to know only the price system (which they treat parametrically) and their own preferences and endowments, then are allowed to trade freely in a decentralized market, this process results in allocations which maximize agents' utilities (subject to their budgets) and equate supply with demand.

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The second concept is usually associated with Edgeworth, and refers to the allocation of resources via a pure quantity bargaining process. The essential idea behind this concept is that when agents are allowed to bargain freely (either multi- or bilaterally), this process leads to an allocation of resources where it is not possible for any coalition of agents to redistribute their initial endowments among themselves in any way that makes each member of the coalition better off. Thus, in contrast to the competitive equilibrium, the core allows for the possibility of cooperation among agents in the economy.

A classical conjecture about the relationship between these two concepts, attributed to Edgeworth, is that the core shrinks to the competitive equilibrium as the number of agents in the economy becomes large. This conjecture, often called the Edgeworth conjecture, and indeed the notion of the core (although not by this name) were first discussed by Edgeworth in 1881. However, the core was not the subject of modern research until it was formally introduced in the general (mathematical) theory of games by Gillies in 1953. Aumann (1964), in a pathbreaking paper, reformulated rigorously the Edgeworth conjecture by showing that in perfectly competitive economies (i.e., economies with an atomless measure space of agents) with finitely many commodities, the core coincides with the competitive (or Walrasian) equilibrium. Hence, in perfectly competitive economies, core allocations completely characterize competitive equilibrium allocations.

The formal proof of this coincidence result has come to be known as the core–Walras equivalence theorem. In the past two decades, many researchers have studied this problem extensively in economies with finitely many commodities. This research has led to very general core–Walras equivalence results and approximate core–Walras equivalence results in economies with finitely many commodities. However, since our goal in this paper is to study core–Walras equivalence results in economies with infinitely many commodities, we will not elaborate further on these finite dimensional results except where they have particular bearing on our work. However, we do refer the reader to Anderson (1986) or Emmons and Yannelis (1985) for a survey of this interesting literature.

Before proceeding to a discussion of the main results of our paper, it may be useful to discuss the general importance of infinite dimensional commodity spaces in economics. As others have observed [e.g., Court (1941), Debreu (1954), Gabszewicz (1968), Bewley (1970), Mertens (1970), Peleg and Yaari (1970)], infinite dimensional commodity spaces arise very naturally in economics. In particular, an infinite dimensional commodity space may be desirable in problems involving an infinite time horizon, uncertainty about the possibly infinite number of states of nature of the world, or infinite varieties of commodity characteristics. For instance, the Lebesgue space L_∞ of bounded measurable functions on a measure space considered by Bewley (1970), Gabszewicz (1968) and Mertens (1970) is useful in modeling uncertainty or an infinite time horizon. The space L_2 of square-integrable functions on a measure space is useful in modeling the trading of long-lived securities over time.

In this paper, we study core–Walras equivalence results for perfectly competitive economies with an infinite dimensional commodity space which is general enough to include all of the spaces that have been found most useful in equilibrium analysis.[1] The results that we obtain in this context are three-fold:

Firstly, we prove core–Walras equivalence results for perfectly competitive economies with an infinite dimensional commodity space whose positive cone has a non-empty (norm) interior. Parts of this problem have been addressed by other researchers [i.e., Gabszewicz (1968), Mertens (1970) and Bewley (1973) for the space L_∞]. However, since our assumptions are less restrictive than those adopted in these previous papers, we obtain as corollaries of our results the finite dimensional theorems of Aumann (1964) and Hildenbrand (1974, Theorem 1, p. 133). The proof of this result is similar in spirit to that of Hildenbrand (who attributes the idea of the proof to Schmeidler), except that owing to the infinite dimensional setting, we appeal to results on the integration of correspondences having values in a Banach space. The work of Khan (1985) is especially helpful in this regard.

Secondly, in infinite dimensional commodity spaces whose positive cone has an empty (norm) interior, we show that even under quite strong assumptions on preferences and endowments, core–Walras equivalence fails. In particular, we show that even when preferences are strictly convex, monotone, and weakly continuous and initial endowments are strictly positive, core–Walras equivalence fails to hold. It is interesting to note that this failure results despite the fact that these assumptions are much stronger than the standard assumptions which guarantee equivalence in either Aumann and Hildenbrand or our first theorem.

Thirdly, we obtain core–Walras equivalence for infinite dimensional commodity spaces (in particular, Banach lattices) whose positive cone may have an empty (norm) interior and are general enough to cover the spaces L_p $(1 \leqq p < \infty)$. In view of the above counterexample to core–Walras equivalence in spaces whose positive cone has an empty interior, we use the assumption of an extremely desirable commodity introduced in Yannelis and Zame

[1] Recently, substantial progress has been made in establishing existence results for the competitive equilibrium in exchange economies with finitely many agents and with a commodity space which is general enough to encompass all the spaces mentioned above [see for instance Mas-Colell (1986) or Yannelis and Zame (1986) among others]. Moreover, some progress has been made in obtaining equilibrium existence results for perfectly competitive economies, i.e., economies with an atomless measure space of agents à la Aumann (1966) and with an infinite dimensional commodity space [see for instance, Khan and Yannelis (1986), Yannelis (1987), and Zame (1987)]. However, the core has received significantly less attention in infinite dimensional settings.

(1986), which in turn is related to the condition of uniform properness in Mas-Colell (1986).[2] This assumption is essentially a bound on the marginal rates of substitution, and in practice turns out to be quite weak. For instance it is automatically satisfied whenever preferences are monotone and the positive cone of the commodity space has a non-empty (norm) interior. Hence, this assumption is implicit in any infinite dimensional commodity space whose positive cone has a non-empty interior, and is automatically satisfied in the finite dimensional work of Aumann (1964) and Hildenbrand (1974). We also wish to note that in addition to the assumption of an extremely desirable commodity, the lattice structure of the commodity space will play a crucial role in our analysis.

The remainder of the paper is organized as follows: Section 2 contains notation and definitions. The economic model is outlined in section 3. In section 4 we state and prove a core–Walras equivalence theorem for an ordered separable Banach space of commodities, whose positive cone has a non-empty (norm) interior. The failure of this result for spaces whose positive cone has an empty interior is established in section 5. In section 6, we prove a core–Walras equivalence result for a commodity space which can be any arbitrary separable Banach lattice, whose positive cone may have an empty (norm) interior. Finally, some concluding remarks are given in section 7.

2. Notation and definitions

2.1. Notation

R^l denotes the l-fold Cartesian product of the set of real numbers R.

$\text{int}\, A$ denotes the interior of the set A.

2^A denotes the set of all non-empty subsets of the set A.

$\emptyset$ denotes the empty set.

$/$ denotes the set theoretic subtraction.

dist denotes distance.

If $A \subset X$ where X is a Banach space, $\text{cl}\, A$ denotes the norm closure of A. If X is a Banach space its dual is the space X^* of all continuous linear functionals on X.

If $q \in X^*$ and $y \in X$ the value of q at y is denoted by $q \cdot y$.

2.2. Definitions

Let X, Y be sets. The *graph* of the correspondence $\phi: X \to 2^Y$ is denoted by

[2]It should be noted that a precursor of the assumption of uniform properness is in Chichilnisky and Kalman (1980). In particular, in order to apply Hahn–Banach-type separation theorems in spaces whose positive cone has an empty interior, they introduced a related assumption with that of uniform properness used by Mas-Colell.

$G_\phi=\{(x,y)\in X\times Y: y\in\phi(x)\}$. Let (T,τ,μ) be a finite measure space, and X be a Banach space [for a treatment of infinite dimensional vector spaces see Aliprantis and Burkinshaw (1978, 1985)]. The correspondence $\phi: T\to 2^Y$ is said to have a *measureable graph* if $G_\phi\in\tau\otimes\beta(X)$, where $\beta(X)$ denotes the Borel σ-algebra on X and $\otimes$ denotes product σ-algebra. A function $f: T\to X$ is called *simple* if there exist $x_1, x_2, \ldots, x_n$ in X and a_1, $a_2, \ldots, a_n$ in τ such that

$$f=\sum_{i=1}^{n} x_i\chi_{a_i} \quad \text{where} \quad \chi_{a_i}(t)=1 \quad \text{if} \quad t\in a_i \quad \text{and}$$

$$\chi_{a_i}(t)=0 \quad \text{if} \quad t\notin a_i.$$

A function $f: T\to X$ is said to be *μ-measurable* if there exists a sequence of simple functions $f_n: T\to X$ such that $\lim_{n\to\infty}\|f_n(t)-f(t)\|=0$ μ-a.e. A *μ-measurable* function $f: T\to X$ is said to be *Bochner integrable* if there exists a sequence of simple functions $\{f_n: n=1,2,\ldots\}$ such that

$$\lim_{n\to\infty}\int_T \|f_n(t)-f(t)\|\mathrm{d}\mu(t)=0.$$

In this case we define for each $E\in\tau$ the integral to be $\int_E f(t)\,\mathrm{d}\mu(t)=\lim_{n\to\infty}\int_E f_n(t)\,\mathrm{d}\mu(t)$. It can easily be shown [see Diestel and Uhl (1977, p. 45)] that if $f: T\to X$ is a μ-measurable function then f is Bochner integrable if and only if $\int_T\|f(t)\|\,\mathrm{d}\mu(t)<\infty$. We denote by $L_1(\mu,X)$ the space of equivalence classes of X-valued Bochner integrable functions $x: T\to X$ normed by $\|x\|=\int_T\|x(t)\|\,\mathrm{d}\mu(t)$. Moreover, we denote by S_ϕ the *set of all X-valued Bochner integrable selections from the correspondence* ϕ: $T\to 2^X$, i.e.

$$S_\phi=\{x\in L_1(\mu,X): x(t)\in\phi(t)\mu\text{-a.e.}\}.$$

As in Aumann (1966), the *integral of the correspondence* $\phi: T\to 2^X$ is defined as

$$\int_T \phi(t)\,\mathrm{d}\mu(t)=\left\{\int_T x(t)\,\mathrm{d}\mu(t): x\in S_\phi\right\}.$$

In the sequel we will denote the above integral by

$$\int\phi \quad \text{or} \quad \int_T\phi.$$

3. Economy, core and competitive equilibrium

Denote by E the commodity space. Throughout this section the commodity space E will be an ordered Banach space [see Aliprantis and Burkinshaw (1985)]. We will denote by E_+ and E_- the positive and negative cones of E, respectively.

An *economy* ε is a quadruple $[(T,\tau,\mu), X, >, e]$, where

(1) (T,τ,μ) is a *measure space of agents*,
(2) $X: T\to 2^E$ is a *consumption correspondence*,
(3) $>_t \subset X(t)\times X(t)$ is the *preference relation*[3] of agent t, and
(4) $e: T\to E$ is the *initial endowment*, where e is Bochner integrable and $e(t)\in X(t)$ for all $t\in T$.

An *allocation* for the economy ε is a Bochner integrable function $x: T\to E_+$. An allocation x is said to be *feasible* if $\int_T x(t)\,\mathrm{d}\mu(t)=\int_T e(t)\,\mathrm{d}\mu(t)$. A *coalition* S is an element of τ such that $\mu(S)>0$. The coalition S can *improve upon* the allocation x if there exists allocation g such that

(i) $g(t)>_t x(t)$ μ-a.e. in S, and
(ii) $\int_S g(t)\,\mathrm{d}\mu(t)=\int_S e(t)\,\mathrm{d}\mu(t)$.

The set of all feasible allocations for the economy ε that no coalition can improve upon is called the *core* of the economy ε and is denoted by $C(\varepsilon)$.

An allocation x and a price $p\in E_+^*\setminus\{0\}$ are said to be a *competitive equilibrium* (or a *Walras equilibrium*) for the economy ε, if

(i) $x(t)$ is a maximal element of $>_t$ in the budget set

$$\{y\in X(t): p\cdot y\leq p\cdot e(t)\}\,\mu-\text{a.e.,}\quad \text{and}$$

(ii) $\int_T x(t)\,\mathrm{d}\mu(t)=\int_T e(t)\,\mathrm{d}\mu(t)$.

We denote by $W(\varepsilon)$ the set of all competitive equilibria for the economy ε.

4. Core–Walras equivalence in ordered Banach spaces whose positive cone has a non-empty norm interior

We begin by stating some assumptions needed for the proof of our core–Walras equivalence result.

A.1. E is an ordered separable Banach space whose positive cone E_+ has a non-empty norm interior, i.e., $\operatorname{int} E_+\neq\varnothing$.

[3] $>$ is defined to be the asymmetric part of the weak preference relation $\gtrsim$, i.e., we say that $x>y$ if and only if $x\gtrsim y$ and not $y\gtrsim x$. This is not needed for Theorem 4.1. However, it is used in the proof of Theorem 6.1.

A.2. (*Perfect Competition*). (T, τ, μ) is a finite atomless measure space.

A.3. $X(t) = E_+$ for all $t \in T$.

A.4. (*Resource Availability*). The aggregate initial endowment $\int_T e(t)\,d\mu(t)$ is strictly positive,[4] i.e., $\int e \gg 0$.

A.5. (*Continuity*). For each $x \in E_+$ the set $\{y \in E_+ : y >_t x\}$ is norm open in E_+ for all $t \in T$.

A.6. $>_t$ is irreflexive and transitive for all $t \in T$.

A.7. (*Measurability*). The set $\{(t, y) \in T \times E_+ : y >_t x\}$ belongs to $\tau \otimes \beta(E_+)$.

A.8. (*Monotonicity*). If $x \in E_+$ and $v \in E_+ \setminus \{0\}$, then $x + v >_t x$ for all $t \in T$.

We are now ready to state our first result. We wish to note that this result for $E = C(X)$ [where $C(X)$ denotes the space of continuous functions on a compact metric space X] was first proved by Gabszewicz, and it is attributed to him.

Theorem 4.1. Under assumptions A.1–A.8, $C(\varepsilon) = W(\varepsilon)$.

Remark 4.1. Note that the assumptions of the above theorem correspond to those in Aumann (1964) in the setting of an ordered separable Banach space E of commodities. It can easily be seen that for $E = R^l$, Theorem 4.1 gives as a corollary Aumann's (1964) core equivalence result [as well as Hildenbrand's (1974, Theorem 1, p. 133) core–Walras equivalence theorem]. It may be instructive at this point to note that Bewley's (1973) infinite dimensional extension of Aumann's core equivalence theorem does not provide the above results as a corollary because it is based on stronger assumptions than those adopted by Aumann and Hildenbrand.

4.1. Proof of Theorem 4.1

The fact that $W(\varepsilon) \subset C(\varepsilon)$ is well known, and therefore its proof is not repeated here. We begin the proof by assuming that the allocation x is an element of the core of ε. We wish to show that for some price p, the pair (x, p) is a competitive equilibrium for ε.

To this end, define the correspondence $\phi: T \to 2^{E_+}$ by

[4] We will say that an element x of E is *strictly positive* (and write $x \gg 0$) if $\Pi \cdot x > 0$ whenever Π is a positive non-zero element of E_+.

$$\phi(t)=\{z\in E_+ : z >_t x(t)\} \cup \{e(t)\}. \tag{4.1}$$

We claim that:

$$\mathrm{cl}\left(\int_T \phi - \int_T e\right) \cap \operatorname{int} E_- = \varnothing, \tag{4.2}$$

or equivalently,[5]

$$\left(\int_T \phi - \int_T e\right) \cap \operatorname{int} E_- = \varnothing. \tag{4.3}$$

Suppose otherwise, i.e.,

$$\left(\int_T \phi - \int_T e\right) \cap \operatorname{int} E_- \neq \varnothing,$$

then there exists $v\in \operatorname{int} E_+$ such that

$$\int e - v \in \int \phi. \tag{4.4}$$

It follows from (4.4) that there exists a function $y: T\to E_+$ such that

$$\int_T y = \int_T e - v, \tag{4.5}$$

and $y(t)\in\phi(t)$ μ-a.e.

Let

$$S=\{t: y(t) >_t x(t)\}, \quad \text{and}$$

$$S'=\{t: y(t)=e(t)\}.$$

Since $\int y \neq \int e$ we have that $\mu(S)>0$. Define $\tilde{y}: S\to E_+$ by $\tilde{y}(t)=y(t)+v/\mu(S)$ for all $t\in S$. By monotonicity (assumption A.8) $\tilde{y}(t) >_t y(t)$. Since $y(t) >_t x(t)$ for all $t\in S$, by transitivity (assumption A.6) $\tilde{y}(t) >_t x(t)$ for all $t\in S$. Moreover, it can be easily seen that $\tilde{y}(\cdot)$ is feasible for the coalition S, i.e.,

[5]This is so since $\operatorname{int} E_-$ is an open set. In particular, if A and B are subsets of any topological space and B is open, then it can be easily seen that $A\cap B=\varnothing$ if and only if $\mathrm{cl}\, A\cap B=\varnothing$.

$$\int_S \tilde{y} = \int_S y + v = \int_T y - \int_{S'} e + v = \int_T e - \int_{S'} e = \int_S e \quad \text{[recall (4.5)]}.$$

Therefore, we have found an allocation $\tilde{y}(\cdot)$ which is feasible for the coalition S and is also preferred to the allocation x, which in turn was assumed to be in the core of ε, a contradiction which establishes the validity of (4.2).

We may now separate the set $\mathrm{cl}(\int \phi - \int e) = \mathrm{cl} \int \phi - \int e$ from $\mathrm{int}\, E_-$. Clearly the set $\mathrm{int}\, E_-$ is convex and non-empty. We wish to show that $\mathrm{cl} \int \phi - \int e$ is convex and non-empty as well. Observe first that by the definition of $\phi(\cdot)$, 0 is an element of $\int \phi - \int e$ and this shows that $\mathrm{cl} \int \phi - \int e$ is non-empty. Since (T, τ, μ) is atomless (assumption A.2) by Theorem 1 in Khan (1985) or Theorem 4.2 in Hiai and Umegaki (1977), $\mathrm{cl} \int \phi$ is convex. Thus, by Theorem 9.10 in Aliprantis and Burkinshaw (1985, p. 136) there exists a continuous linear functional $p \in E^* \setminus \{0\}$, $p \geqq 0$ such that

$$p \cdot y \geqq p \cdot \int e \quad \text{for} \quad \text{all } y \in \int \phi. \tag{4.6}$$

Since by assumption A.6, $>_t$ has a measurable graph, so does ϕ, i.e., $G_\phi \in \tau \otimes \beta(E_+)$. Therefore, it follows from Theorem 2.2 in Hiai and Umegaki (1977) that

$$\inf_{y \in \int \phi} p \cdot y = \int \inf_{z \in \phi} p \cdot z \geqq \int p \cdot e. \tag{4.7}$$

It follows from (4.7) that

$$\mu\text{-a.e. } p \cdot z \geqq p \cdot e(t) \quad \text{for} \quad \text{all } z >_t x(t). \tag{4.8}$$

To see this, suppose that for $z \in \phi(\cdot)$, $p \cdot z < p \cdot e(t)$ for all $t \in S$, $\mu(S) > 0$.

Define the function $\tilde{z}: T \to E_+$ by

$$\tilde{z}(t) = \begin{cases} z(t) & \text{if} \quad t \in S \\ e(t) & \text{if} \quad t \notin S. \end{cases}$$

Obviously, $\tilde{z} \in \phi(\cdot)$. Moreover,

$$\begin{aligned} \int_T p \cdot \tilde{z} &= \int_S p \cdot z + \int_{T \setminus S} p \cdot e \\ &< \int_S p \cdot e + \int_{T \setminus S} p \cdot e = \int p \cdot e, \end{aligned}$$

a contradiction to (4.7).

We now show that μ-a.e. $p\cdot x(t)=p\cdot e(t)$. First note that if follows directly from (4.8) that $p\cdot x(t)\geqq p\cdot e(t)$ μ-a.e. If now $p\cdot x(t)>p\cdot e(t)$ for all $t\in S$, $\mu(S)>0$, then

$$p\cdot\int_T x=p\cdot\int_{T\setminus S} x+p\cdot\int_S x$$

$$>p\cdot\int_{T\setminus S} e+p\cdot\int_S e=p\cdot\int_T e,$$

contradicting $\int_T x=\int_T e$, since $p\geqq 0$, $p\neq 0$.

To complete the proof we must show that $x(t)$ is maximal in the budget set $\{z\in E_+:p\cdot z\leqq p\cdot e(t)\}$ μ-a.e. The argument is now routine. Since $\int_T e$ is strictly positive (assumption A.4) it follows that $\mu(\{t:p\cdot e(t)\})>0$, for if $p\cdot e(t)=0$ μ-a.e., then $p\cdot\int_T e=0$ contradicting the fact that $\int_T e$ is strictly positive since $p\geqq 0$, $p\neq 0$.

Thus, we can safely pick an agent t with positive income, i.e., $p\cdot e(t)>0$. Since $p\cdot e(t)>0$ there exists an allocation x' such that $p\cdot x'<p\cdot e(t)$. Let y be such that $p\cdot y\leqq p\cdot e(t)$ and let $y(\lambda)=\lambda x'+(1-\lambda)y$ for $\lambda\in(0,1)$. Then for any $\lambda\in(0,1)$, $p\cdot y(\lambda)<p\cdot e(t)$ and by (4.8) $y(\lambda)\not>_t x(t)$. It follows from the norm continuity of $>_t$ (assumption A.5) that $y\not>_t x(t)$. This proves that $x(t)$ is maximal in the budget set of agent t, i.e., $\{w:p\cdot w\leqq p\cdot e(t)\}$. This, together with the monotonicity of preferences (assumption A.8) implies that prices are strictly positive, i.e., $p\gg 0$. Indeed, if there exists $v\in E_+\setminus\{0\}$ such that $p\cdot v=0$ then $p\cdot(x(t)+v)=p\cdot x(t)=p\cdot e(t)$ and by monotonicity $x(t)+v>_t x(t)$ contradicting the maximality of $x(t)$ in the budget set.

Thus $p\gg 0$ and $x(t)$ is maximal in the budget set whenever $p\cdot e(t)>0$. Consider now an agent t with zero income, i.e., $p\cdot e(t)=0$. Since $p\gg 0$ his/her budget set $\{z:p\cdot z=0\}$ consists of zero only, and moreover, $p\cdot x(t)=p\cdot e(t)=0$. Hence, $x(t)=0$ for almost all $t\in T$, with $p\cdot e(t)=0$; i.e., zero in this case is the maximal element in the budget set. Consequently, (p,x) is a competitive equilibrium for ε, and this completes the proof of Theorem 4.1.

5. The failure of the core–Walras equivalence in commodity spaces whose positive cone has an empty interior

In the previous section we showed that if the commodity space is an ordered separable Banach space E whose positive cone has a non-empty norm interior (i.e., $\operatorname{int} E_+\neq\varnothing$), then the standard assumptions (i.e., the assumptions of Theorem 4.1) guarantee core–Walras equivalence. We now show that if the assumption that the positive cone of the space E has a non-empty norm interior is dropped, then Theorem 4.1 fails. The following example will illustrate this.

Example 5.1. Consider the economy $\varepsilon=[(T,\tau,\mu),X,>,e]$ where,

(1) the *space of agents* is $T=[0,1]$, $\tau=$Lebesgue measurable sets, $\mu=$ Lebesgue measure,
(2) the *consumption set* of each agent is, $X(t)=l_2^+$ for all $t\in T$, where l_2 is the space of real sequences (a_n) for which the norm $\|a_n\|=(\sum|a_n|^2)^{1/2}$ is finite,
(3) the *preference relation* of each agent $>_t$, is represented by a strictly concave, monotone weakly continuous utility function, i.e., $u_t(x)=\sum_{i=1}^{\infty} i^{-2}(1-\exp(-i^2x_i))$ for all $t\in T$, and
(4) the *initial endowment* of each agent is $e(t)=e=(1/i^2)_{i=1}^{\infty}$ for all $t\in T$.

We will show that for the above economy, $C(\varepsilon)\neq\varnothing$ and $W(\varepsilon)=\varnothing$. In particular, we will show that the core of ε is unique and consists of the initial endowment e, i.e., $C(\varepsilon)=\{e\}$ and $W(\varepsilon)=\varnothing$. The latter [i.e., $W(\varepsilon)=\varnothing$] will easily follow from the fact that $C(\varepsilon)=\{e\}$. Indeed, since $W(\varepsilon)\subset C(\varepsilon)$, $W(\varepsilon)\subset\{\{e\},\varnothing\}$, but the only candidate as a supporting price p for the allocation e are multiples of $p=(1,1,\dots)$ which are not in the dual of l_2. Hence, all we need to show is that $C(\varepsilon)=\{e\}$.

To prove that $C(\varepsilon)=\{e\}$ we will first need to show that e is Pareto optimal, i.e., there does not exist a feasible allocation x such that $u(x(t))\geqq u(e)$ for all $t\in T$ and $u(x(t))>u(e)$ for all $t\in S$, $S\subset T$, $\mu(S)>0$ (note that the subscript t on u is dropped). To this end suppose by way of contradiction that there exists an allocation x such that $\int_T x=\int_T e\equiv e$, $u(x(t))\geqq u(e)$ for all $t\in T$ and $u(x(t))>u(e)$ for all $t\in S$, $\mu(S)>0$. Without loss of generality we may assume that there exist positive real numbers v, δ, with $u(x(t))\geqq u(e)+v$, $t\in S$, $\mu(S)=\delta$. Extend x to $\tilde{x}$ defined on the interval $[0,1]$ as $\tilde{x}(t)=x(-[t])$, ($[t]=$the integer part of t), and let $x^k(t)=\sum_{i=0}^{k-1}\tilde{x}(t+(i/k))/k$.
Then

$$\int_0^1 u(x^k(t))\,\mathrm{d}\mu(t)=u\left(\sum_{i=0}^{k-1}\tilde{x}\left(t+\frac{i}{k}\right)\bigg/k\right)\mathrm{d}\mu(t)$$

$$\geqq\int_0^1\sum_{i=0}^{k-1}(1/k)u\left(\tilde{x}\left(t+\frac{i}{k}\right)\right)\mathrm{d}\mu(t)$$

$$=\int_0^1 u(x(t))\,\mathrm{d}\mu(t)\geqq u(e)+v\delta. \tag{5.1}$$

Notice that each coordinate of $x^k(\cdot)$, denoted by $x_i^k(\cdot)$ (an $L_1[0,1]$ function), converges to e_i μ-a.e. (indeed in L_1), so

$$x_i^k(t)=\sum_{i=0}^{k-1}\tilde{x}\left(t+\frac{i}{k}\right)\Big/k\xrightarrow[k\to\infty]{}\int_0^1 x_i(s)\,\mathrm{d}\mu(s)=e_i$$

for almost all t in T; so $x^k(t)$ converges weakly in l_2 to e. Since u is weakly continuous it follows that $u(x^k(t))\to u(e)$ μ-a.e. Notice that by definition, u is bounded. In particular, $u(x)<\pi^2/6$ for every $x\in l_2^+$ [recall the definition of $u(\cdot)$ in (3)] and therefore by the Lebesgue dominated convergence theorem

$$\lim_{k\to\infty}\int_0^1 u(x^k(t))\,\mathrm{d}\mu(t)=\int_0^1\lim_{k\to\infty}u(x^k(t))\,\mathrm{d}\mu(t)=u(e)=u(\textstyle\int e),$$

a contradiction to (5.1). Thus, e is Pareto optimal.

We are now ready to complete the proof of the fact that $C(\varepsilon)=\{e\}$. To this end we first show that

$$C(\varepsilon)\subset\{e\}. \tag{5.2}$$

Suppose that (5.2) is false, then there exists an allocation $x\in C(\varepsilon)$ such that $x(t)\neq e$ for all $t\in S$, $\mu(S)>0$. Let $\tilde{x}=(x+e)/2$, then $\tilde{x}$ is feasible and for all $t\in T$,

$$\begin{aligned}u(\tilde{x}(t))&>\tfrac{1}{2}u(x(t))+\tfrac{1}{2}u(e)\\&\geqq u(e)\end{aligned}$$

[recall that $u(x(t))\geqq u(e)$ for all $t\in T$ since $x\in C(\varepsilon)$]. Moreover, by strict concavity of $u(\cdot)$ we have that

$$u(\tilde{x}(t))>u(e)\quad\text{for}\quad\text{all }t\in S,$$

a contradiction to the fact that e is Pareto optimal.

We now show that

$$\{e\}\subset C(\varepsilon). \tag{5.3}$$

Suppose that (5.3) is false, then there exists a coalition S and an allocation x such that $\int_S x=\int_S e$ and $u(x(t))>u(e)$ for all $t\in S$. Define the allocation $\tilde{x}(\cdot)$ as follows:

$$\tilde{x}(t)=\begin{cases}x(t) & \text{if}\quad t\in S\\ e(t) & \text{if}\quad t\notin S.\end{cases}$$

Then $u(\tilde{x}(t)) \geqq u(e)$ for all $t \in T$ and $u(\tilde{x}(t)) > u(e)$ for all $t \in S$, contradicting the fact that e is Pareto optimal.

It follows from (5.2) and (5.3) that $C(\varepsilon) = \{e\}$ and this completes the proof of the fact that $C(\varepsilon) \neq \emptyset$ and $W(\varepsilon) = \emptyset$.

Since Example 5.1 satisfies all the conditions of Theorem 4.1 except assumption A.1 (note that $\operatorname{int} l_2^+ = \emptyset$), we can conclude that if positive results are to be obtained in spaces whose positive cone has an empty norm interior some additional assumption needs to be imposed. The additional assumption we impose is that of an extremely desirable commosity introduced in Yannelis and Zame (1986) [which is related to the assumption of proper preferences introduced by Mas-Colell (1986)].

6. Core–Walras equivalence in separable Banach lattices whose positive cone has an empty interior

We begin by defining the notion of an extremely desirable commodity. Let E be a Banach lattice and denote its positive cone (which may have an empty norm interior) by E_+. Let $v \in E_+, v \neq 0$. We say that $v \in E_+$ is an *extremely desirable commodity* if there exists an open neighborhood U such that for each $x \in E_+$ and each $t \in T$, we have that $x + \alpha v - z >_t x$ whenever $\alpha > 0$, $z \leqq x + \alpha v$ and $z \in \alpha U$. In other words, v is extremely desirable if an agent would prefer to trade any commodity bundle z for an additional increment of the commodity bundle v, provided that the size of z is sufficiently small compared to the increment of v. The above notion has a natural geometric interpretation. In particular, let $v \in E_+$, $v \neq 0$, U be an open neighborhood and define the open cone C as follows:

$$C = \{\alpha v - z : \alpha > 0, z \in E, z \in \alpha U\}.$$

The bundle v is said to be an extremely desirable commodity, if for each $x \in E_+$, and each $t \in T$ we have $y >_t x$ whenever y is an element of $(C + x) \cap E_+$. This implies that v is an extremely desirable commodity if for each $x \in E_+$ we have that $((-C + x) \cap E_+) \cap \{y : y >_t x\} = \emptyset$, or equivalently $-C \cap \{y - x \in E_+ : y >_t x\} = \emptyset$.

Recall that if the preference relation $>_t$ is monotone and $\operatorname{int} E_+ \neq \emptyset$, then the assumption of an extremely desirable commodity is automatically satisfied [see for instance Yannelis and Zame (1986)].

We now state our assumptions:

A.9. E is any separable Banach lattice.

A.10. (*Extremely desirable commodity*). Let $v \in E_+/\{0\}$ and U be an open convex neighborhood. Let C be the cone spanned by $v + U$. The bundle v is said to be an extremely desirable commodity with respect to U, if for each $x \in E_+$ and each $t \in T$, we have $y >_t x$ whenever y is an element of $(C+x) \cap E_+$.

Let δ_i, $i=1,\dots,n$ be positive real numbers with $\sum_{i=1}^n \delta_i = 1$.

A.11. Let U of A.10 satisfy the following condition: if $x_i \in E_+$, $x_i \notin \delta_i U$, $i=1,2,\dots,n$, then $\sum_{i=1}^n x_i \notin \sum_{i=1}^n \delta_i U = U$.

We note that in assumption A.11 the additivity condition only concerns the neighborhood of the extremely desirable commodity, not the commodity space. To clarify this point we shall consider a specific example. Consider the space $L_p(\Omega)$, $1 \leqq p < \infty$ where Ω is a finite separable measure space. From Holder's inequality, for $f \in L_p(\Omega)$ we have that $\|f\|_1 \leqq C\|f\|_p$ for some constant C depending only on p and Ω. Suppose now that assumption A.10 is satisfied with U containing a neighborhood U' of the form:

$$U' \equiv \{f \in L_p: \|f\|_1 < \varepsilon\}.$$

Then U' is open in L_p because of the inequality mentioned above. Moreover A.11 is also satisfied. Roughly speaking we require the neighborhood U' of the extremely desirable commodity to be 'large' in the topology of E.

Finally we need:

A.12. For each $x \in E_+$, the sets $\{y \in E_+ : y >_t x\}$ and $\{y \in E_+ : x >_t y\}$ are norm open in E_+ for all $t \in T$.

We can now state the following result:

Theorem 6.1. Under assumptions A.2–A.12, $C(\varepsilon) = W(\varepsilon)$.

Proof. It can be easily shown that $W(\varepsilon) \subset C(\varepsilon)$. Hence, we will show that if $x \in C(\varepsilon)$, then for some price p, the pair (x,p) is a competitive equilibrium for ε.

Define the correspondence $\phi: T \to 2^{E_+}$ by

$$\phi(t) = \{z \in E_+ : z >_t x(t)\} \cup \{e(t)\}. \tag{6.1}$$

Let C be the open cone spanned by the set $v+U$ given by assumptions A.10–A.11, i.e., $C = \text{span}\{0, v+U\} \equiv \bigcup_{\alpha>0} \alpha(v+U)$. We claim that

$$\text{cl}\left(\textstyle\int \phi - \int e\right) \cap -C = \varnothing, \tag{6.2}$$

or equivalently,

$$(\int \phi - \int e) \cap -C = \varnothing. \tag{6.3}$$

Since $-C$ is open it suffices to show that for any $y \in S_\phi$ there exists a sequence $\{(\bar{y}^k, \bar{e}^k): k=1,2,\ldots\}$ in $L_1(\mu, E) \times L_1(\mu, E)$ such that $\bar{y}^k$ converges in the $L_1(\mu, E)$ norm to y, $\int \bar{e}^k \to \int e$, and

$$\int_T \bar{y}^k - \int_T \bar{e}^k \notin -C. \tag{6.4}$$

Let $S = \{t: y(t) >_t x(t)\}$, $S' = T/S$. Without loss of generality we may assume that $\mu(S) > 0$ [for if $\mu(S) = 0$, then $y(t) = e(t)$ μ-a.e. which implies that $\int y - \int e = 0 \notin -C$; consequently (6.3) holds]. In the argument below y and e are restricted to S. Moreover, denote by μ_S the restriction of μ to S. Since $y: S \to E_+$ is Bochner integrable and $>_t$ is norm continuous (assumption A.12) there exist $y_1^k, \ldots, y_{m_k}^k$ in E_+ and $T_1^k, T_2^k, \ldots, T_{m_k}^k$ in τ such that y^k converges in the $L_1(\mu_S, E)$ norm to y, and

$$y^k = \sum_{i=1}^{m_k} y_i^k \chi_{T_i^k} \tag{6.5}$$

$$y_i^k >_t x(t) \quad \text{for} \quad \text{all } t \in T_i^k \quad \text{and} \quad \text{all } i, i=1,\ldots,m_k, \quad \text{and} \tag{6.6}$$

$$\mu_S(T_i^k) = \xi, \quad i=1,\ldots,m_k, \quad \text{(where } \xi \text{ is a real positive number).} \tag{6.7}$$

Let

$$e^k = \sum_{i=1}^{m_k} \left(\int_{T_i^k} e(t) \, \mathrm{d}\mu(t) \right) \chi_{T_i^k}.$$

Claim 6.1. $\int_S y^k - \int_S e^k \notin -C.$

Assume that Claim 6.1 holds (a proof is given at the end of this section). We can now construct the sequence $\{(\bar{y}^k, \bar{e}^k): k=1,2,\ldots\}$. In particular, define $\bar{y}^k: T \to E_+$ by

$$\bar{y}^k(t) = \begin{cases} y^k(t) & \text{if} \quad t \in S \\ y(t) & \text{if} \quad t \notin S. \end{cases}$$

Similarly define $\bar{e}^k: T \to E_+$ by

$$\bar{e}^k(t) = \begin{cases} e^k(t) & \text{if} \quad t \in S \\ e(t) & \text{if} \quad t \notin S. \end{cases}$$

Note that $\int_T \bar{y}^k - \int_T \bar{e}^k \notin -C$ and therefore (6.4) holds.

We can now separate the convex non-empty set $\text{cl}\int\phi - \int e$ from the convex non-empty set $-C$. Proceeding as in the proof of Theorem 4.1, we can now complete the proof.

Proof of Claim 6.1. We will argue by contradiction, and for notational convenience we will drop the index k.

Suppose that Claim 6.1 is false, then

$$\sum_{i=1}^{m} y_i\xi - \sum_{i=1}^{m} e_i\xi \in -\alpha(v+U),$$

and therefore

$$\sum_{i=1}^{m} y_i + w - u = \sum_{i=1}^{m} e_i, \quad \text{where} \quad w = \frac{\alpha}{\xi} v, \quad u \in \frac{\alpha}{\xi} U. \tag{6.8}$$

Note that without loss of generality we may assume that $u \geqq 0$ [otherwise, since $u = u^+ - u^-$, we may define $\hat{y}_i = y_i + u^+/mj$ then $\hat{y}_i \geqq 0$ and $u^- \in \alpha U$ (recall that U can be assumed to be solid), $\hat{y}_i >_t x(t)$ for all $t \in T_i$ and all i and one can proceed by substituting y_i for $\hat{y}_i$].

It follows from (6.8) that for any m-tuple $(\theta_1, \dots, \theta_m)$, $\theta_i \geqq 0$ $(i = 1, \dots, m)$, $\sum_{i=1}^m \theta_i = 1$, we have that

$$\sum_{i=1}^{m} (y_i + \theta_i w) - u = \sum_{i=1}^{m} e_i \geqq 0,$$

and therefore

$$u \leqq \sum_{i=1}^{m} (y_i + \theta_i w). \tag{6.9}$$

Applying the Riesz Decomposition Property in (6.9) we obtain $u_1, \dots, u_m$ in E_+ such that

$$\sum_{i=1}^{m} u_i = u, u_i \leqq y_i - \theta_i w \quad \text{for} \quad \text{all } i. \tag{6.10}$$

(It is easy to see that the proof of the Riesz Decomposition Property provides an algorithm to choose in a unique way the u_i's above). For each i, define $\tilde{y}_i : [0,1] \to E_+$ by $\tilde{y}_i(\theta) = y_i + \theta_i w - u_i$. Moreover, for each i set $F_i(\tilde{y}_i(\theta)) = \text{dist}(\tilde{y}_i(\theta), C + y_i) = \delta_i(\theta)$. Let $\Delta = \{q \in R^m_+ : \sum_{i=1}^m q_i = 1\}$. Define the continuous mapping

$$f:\Delta\to\Delta \text{ by } f(\theta)=\left((\theta_i+\delta_i)\bigg/\left(1+\sum_{j=1}^{m}\delta_j\right)\right)_{i=1}^{m}.$$

By Brouwer's fixed point theorem, there exists $\theta^*\in\Delta$ such that $\theta^*=f(\theta^*)$, i.e.,

$$\delta_i=\theta_i^*\sum_{j=1}^{m}\delta_j,\quad i=1,\ldots,m. \tag{6.11}$$

If we show that $\sum_{j=1}^{m}\delta_j=0$ then by the definition of F_i, $\tilde{y}_i(\theta^*)\in \mathrm{cl}(C+y_i)\cap E_+$ and by continuity (assumption A.12) and the assumption of an extremely desirable commodity (assumption A.10) $\tilde{y}_i(\theta^*)\gtrsim_t y_i$ for all $t\in T_i$, and all i. Moreover, since $y_i>_t x(t)$ for all $t\in T_i$ and for all i by transitivity $\tilde{y}_i(\theta^*)_t x(t)$ for all $t\in T_i$ and for all i. Also $\xi\sum_{i=1}^{m}\tilde{y}_i(\theta^*)=\sum_{i=1}^{m}e_i\xi=\int e$. Define $\tilde{y}=\sum_{i=1}^{m}\tilde{y}_i(\theta^*)\chi_{T_i}$ and note that $\int_S\tilde{y}=\int_S e$. Therefore we have found an allocation $\tilde{y}(\cdot)$ which is feasible for the coalition S and preferred to $x(\cdot)$ which in turn was assumed to be in the core of ε, a contradiction. Consequently, we conclude that Claim 6.1 holds. Hence, all that remains to be shown is that $\sum_{j=1}^{m}\delta_j=0$.

To this end suppose that $\sum_{j=1}^{m}\delta_j>0$. Notice that by (6.11) we have that $\theta_i^*=0$ if and only if $\delta_i=0$. Define J, $K\subset I=\{1,2,\ldots,m\}$ as follows: $J=\{i\in I:\delta_i=0\}$, $K=I\backslash J$. Note that $J=\{i\in I:\theta_i^*=0\}$. Consider any $i\in J$; then by the definition of $\tilde{y}_i(\cdot)$ we have that $\tilde{y}_i(\theta^*)=y_i-u_i$. Now if $u_i\neq 0$, by monotonicity $y_i>_t\tilde{y}_i(\theta^*)$ and by virtue of continuity and extreme desirability we can conclude that $\tilde{y}_i(\theta^*)\notin \mathrm{cl}(C+y_i)$. By the definition of F_i, $\delta_i>0$, a contradiction to the fact that $i\in J$. Hence, $u_i=0$ for $i\in J$ and so

$$\sum_{i\in I}u_i=\sum_{i\in K}u_i=u. \tag{6.12}$$

Consider any $i\in K$, i.e., $\delta_i>0$, then by the definition of F_i, it follows that $\tilde{y}_i(\theta^*)=y_i+\theta_i^*w-u_i\notin C+y_i$ for every $i\in K$, and therefore, $\theta_i^*w-u_i\notin C$ for all $i\in K$ which in turn implies that $u_i\notin\theta_i^*(\alpha/\xi)U$ for all $i\in K$. It follows from (6.12), the fact that $u_i\notin\theta_i^*(\alpha/\xi)U$ for all $i\in K$, $\sum_{i\in K}\theta_i^*=1$ and assumption A.11 that $\sum_{i\in I}u_i=\sum_{i\in K}u_i=u\notin(\alpha/\xi)U$, which contradicts (6.8), (i.e., $u\in(\alpha/\xi)U$). The above contradiction establishes that $\sum_{j=1}^{m}\delta_j=0$ and this completes the proof of Claim 6.1.

7. Concluding remarks

Remark 7.1. The separability condition on our commodity space E was used in the proof of Theorems 4.1 and 6.1 at one point only. In particular, it was used to make the result of Hiai and Umegaki (1977) or Hildenbrand (1974, Proposition 6, p. 63) applicable – note that Hildenbrand's argument remains valid for correspondences taking values in a separable metric space – [recall (4.6)]. This result is proved via the measurable selection theorem, which requires separability of the range of the correspondence.

Remark 7.2. Bewley (1973) and Mertens (1970) have proved a core–Walras equivalence theorem for a commodity space, which is L_∞. Their assumptions on preferences and endowments are stronger than the ones used in the present paper. It is worth noting that Bewley and Mertens both endow L_∞ with the Mackey topology (L_∞, Mackey), and they are in a setting of a separable space whose positive cone has an empty interior. Consequently, Bewley and Mertens may be considered as predecessors of Theorem 6.1 (of course without using the extreme desirability assumption in the Mackey sense).

Remark 7.3. Subsequent to the writing of the present paper, Cheng (1987) and Zame (1987), following the coalitional approach of Vind (1964), Richter (1971) and Armstrong and Richter (1985), have obtained core–Walras equivalence theorems. Although their results are not directly comparable with ours, it appears that our assumptions on preferences are weaker than theirs.

Remark 7.4. We now indicate how our methods can cover the space $m(\Omega)$, used by Mas-Colell (1975). Specifically, Mas-Colell considers as commodity spaces the set of bounded, signed (Borel) measures on Ω, denoted by $m(\Omega)$. He endows $m(\Omega)$ witht the weak* topology. Note the weak* topology on norm bounded subsets of $m(\Omega)$ is separable and metrizable. Since preferences are also endowed with the weak* topology in order to obtain the counterpart of Theorem 6.1, one needs to work with allocations which are Gelfand integrable functions [see Khan (1985), for a definition]. The argument used to prove Theorem 6.1 remains unchanged, provided that one uses the fact the the weak* closure of the Gelfand integral of correspondence (5.1) is convex [see Khan (1985, Claim 3, p. 265)], and by noting that since we are in a setting of a locally convex, separable and metrizable linear topological space, the measurable selection theorem is applicable and therefore the counterpart of Hiai–Umegaki (1977) theorem for the Gelfand integral holds as well. Subsequent to our paper, Ostroy and Zame (1988) have provided a related argument.

Remark 7.5. It is worth pointing out that as in Aumann (1964) under the assumptions of either Theorem 4.1 or 6.1 both the Walrasian equilibrium and the core may be empty. It should also be mentioned that there is no convexification effect on aggregation, i.e., the aggregate demand set is not necessarily convex [recall that the Lyapunov theorem fails in infinite dimensional spaces, it is only approximately true and so is the Fatou Lemma, see, for instance, Yannelis (1988, 1990) and Rustichini (1989)]. However, recently Rustichini and Yannelis (1988) have shown that one can still have the convexifying effect on aggregation provided that the economy have 'many more' agents than commodities, i.e., the dimension of the measure space of agents in bigger than the dimension of the commodity space. The concept of dimension has of course to be given a rigorous formulation.

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