Econ Theory (2009) 38:351–369
DOI 10.1007/s00199-008-0332-1

SYMPOSIUM

On non-revealing rational expectations equilibrium

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Received: 25 July 2007 / Accepted: 22 January 2008 / Published online: 26 February 2008

Abstract It is shown that a non-revealing rational expectations equilibrium may not be coalitionally Bayesian incentive compatible, may not be implementable as a perfect Bayesian equilibrium and may not belong to the weak fine core and thus may not be fully Pareto optimal. These negative results lead us to conclude the non-revealing rational expectations equilibrium is not a sensible solution concept.

Keywords Differential information economy · Rational expectations equilibrium · Coalitional Bayesian incentive compatibility · Perfect Bayesian equilibrium · Implementation · Game trees · Normal form · Weak fine core

JEL Classification C71 · C72 · D5 · D82

We wish to thank Dr A. Hadjiprocopis for his invaluable help with the implementation of Latex in a Unix environment. We also thank a referee for several, constructive suggestions.

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1 Introduction

The deterministic Arrow–Debreu–McKenzie *Walrasian Equilibrium* (WE) concept captures the idea of exchange, or contacts, or trades of goods under complete information. Since, under reasonable assumptions, such an equilibrium exists, it is Pateto optimal and implementable as Nash equilibrium of a game, one can automatically infer that the WE contacts lead to agreeable outcomes. In reality however, most contracts are made under conditions of uncertainty. As it was explained in Glycopantis and Yannelis (2005b, p. V) three main extensions of the deterministic WE notion were made to incorporate uncertainty.

The first one is due to Arrow and Debreu (see Chap. 7 of the classical treatise Theory of Value, Debreu 1959). These authors point out that once preferences and initial endowments are random, i.e. they depend on the states of nature of the world, the standard existence and optimality theorems for the deterministic WE continue to hold. This is the "state contingent model" or complete markets model (i.e. there is the same number of markets as the states of nature), and the existence, optimality and implementation results continue to hold. However, this model does not allow for asymmetric information. This was investigated by Radner and it leads to the concept of a *differential information economy.*

Indeed, in this second extension of the WE idea, Radner (1968), in addition to random preferences and initial endowments, allowed each agent to have a private information set, which is a partition of the exogenously given states of the world space. In this model agents maximize ex ante expected utility subject to an ex ante budget constraint. However, the initial endowments and all trades are required to be measurable with respect to the private information of each agents. Thus this model captures the idea of contacts made ex ante under asymmetric information. The corresponding notion is called *Walrasian expectations equilibrium* (WEE), and it exists under reasonable assumptions. One is concerned about the possible incentives that individuals might have to misreport their private information. However, it is known that without free disposal, (see for example, Hervés et al. 2005 or Podczeck and Yannelis 2007, among others), that the WEE allocation is coalitional Bayesian incentive compatible, Pareto optimal, as it belongs to the private core, and also implementable as a *perfect Bayesian equilibrium* (PBE) of an extensive form game, (see Glycopantis et al. 2001, 2003).

The third extension was also made by Radner (1979), who introduced the concept of a *rational expectations equilibrium* (REE) (see also Allen 1981). This is an interim notion and agents maximize conditional expected utility, i.e. interim expected utility, based not only on their own private information, but also on the information that the equilibrium prices have generated. Of course the resulting allocation clears the market for every state of nature.

The REE is said to be fully revealing if the equilibrium prices reveal all the private information, (in technical terms they are measurable with respect to the join of all the private information), otherwise it is called non-revealing. It follows that the fully revealing and the non-revealing REE generate different outcomes and therefore they could be considered as different equilibrium concepts.

In Glycopantis et al. (2005a) it was shown that the fully revealing REE can result in allocations (contracts) which do not satisfy *coalitional Bayesian incentive compatibility* (CBIC) and cannot be implemented as a PBE or a sequential equilibrium of an extensive form game. REE allocations might not even exist in well behaved economies, as in the Kreps (1977) example. In other words the resulting contacts may not have any of the desirable properties that we would like any reasonable contract to have.

Moreover, the fully revealing REE allocation does not necessarily belong to the *weak fine core* (WFC), and hence have the property of being fully Pareto optimal. This might not be surprising because the WFC is an ante concept and the fully revealing REE is reduced to a completely ex post concept.

According to the fully revealing REE, since agents maximize their interim expected utility based on the information that the equilibrium prices have generated, it is presumed that each one knows precisely all the primitives in the economy (i.e., random preferences, random initial endowments, private information sets and priors of all other agents). This is quite a strong assumption to the extend that it is not easy to justify.

The question now arises whether the non-revealing REE behaves any better. Prices in this case do not reveal all the information and thus the implicit assumption that agents know all the characteristics in the economy does not hold. Indeed the main purpose of this paper is to study the non-revealing REE and investigate whether it has better properties than the fully revealing one. In particular, we investigate the issues of CBIC, implementability as a PBE, and the relation to WFC. This is significant to investigate, for completeness, because a non-revealing REE is an interim concept, i.e. between an ex ante and an ex post, concept. This follows from the fact that possibly some but not all the information has been revealed to all the agents through the equilibrium prices. Furthermore, in the case in which prices reveal no information, the criticism of the fully revealing REE, namely that if prices reveal all the information then the agents must known all the characteristics of the economy, is not relevant here. This poses the question whether the non-revealing REE behaves better the fully revealing one.

We will show that the non-revealing REE, does not have any better properties than the fully revealing one. It can result to an allocation which is not consistent with Bayesian rationality. That is it is not implementable as a PBE, it is not Pareto optimal, it is not coalitionally Bayesian incentive compatible, and also, as it is known from Kreps (1977) example, it may not exist. Hence the non-revealing REE has no attractive properties which would allow us to use it as a solution concept to capture contracts (trades) under asymmetric information.

We also get as a by-product of our analysis, insights into how the extensive form, game tree formulation of a dynamic model can be replaced by a game in terms of normal form matrices. The presentation of the analysis also in terms of a more compact, normal form approach is due to the fact that the resulting game tree for our model is relatively large.

Following the ideas of Glycopantis et al. (2001), we investigate whether or not the REE can be implemented as a perfect Bayesian equilibrium (PBE) of an extensive form game. A PBE consists of optimal behavioral strategies of the players and the consistent with these decisions, beliefs attaching a probability distribution to the nodes of each information set. It is a variant of the Kreps and Wilson (1982) concept of sequential equilibrium.

The paper is organized as follows. Section 2 gives the definitions of a DIE, an REE and the WFC. Section 3 discusses briefy the idea of incentive compatibility and Sect. 4 discusses fully the non-revealing REE of a decomposable model. Section. 4.1 presents the explicit model, it explains that it can be decomposed into two sub-models, calculates its non-revealing REE and shows that it is not incentive compatible. Section. 4.2 sets up a dynamic, game tree model and shows that, under reasonable rules of behavior, the non-incentive compatible REE cannot be implemented as a PBE. Section 4.3 shows how the results of the game tree analysis can also be obtained through appropriately interpreted normal form tables. Section 4.4 discusses the relation between the non-revealing REE and the WFC and Sect. 6 concludes the discussion.

2 Differential information economy and REE

A DIE consists of set of agents each of which in characterized by a random utility function, a random consumption set, random initial endowments, a private information set and a prior probability distribution on a set of states of nature.

We consider exchange economies which are finite in all dimensions. The set of states of nature is denoted by Ω. $I = \{1, 2, \ldots, n\}$ is the set of agents and there are ℓ goods per state of nature. $\mathbb{R}^l_+$ will denote the l-fold Cartesian product of $\mathbb{R}_+$ the set of positive real numbers.

A differential information exchange economy $\mathcal{E}$ is a set

$$\{((\Omega, \mathcal{F}), X_i, \mathcal{F}_i, u_i, e_i, q_i) : i = 1, \ldots, n\}$$

where

1. $\mathcal{F}$ is a σ-algebra generated by the singletons of Ω;
2. $X_i : \Omega \to 2^{\mathbb{R}^l_+}$ is the set-valued function giving the *random consumption set* of Agent (Player) i, who is denoted by Pi;
3. $\mathcal{F}_i$ is a partition of Ω generating a sub-σ-algebra of $\mathcal{F}$, denoting the *private information*[1] of Pi. We assume that[2] $\mathcal{F} = \bigvee_{i \in I} \mathcal{F}_i$;
4. $u_i : \Omega \times \mathbb{R}^l_+ \to \mathbb{R}$ is the *random utility* function of Pi; for each $\omega \in \Omega,\ u_i(\omega, .)$ is continuous, concave and monotone;
5. $e_i : \Omega \to \mathbb{R}^l_+$ is the *random initial endowment* of Pi, assumed to be $\mathcal{F}_i$-measurable, with $e_i(\omega) \in X_i(\omega)$ for all $\omega \in \Omega$;
6. q_i is an $\mathcal{F}$-measurable probability function on Ω giving the *prior* of Pi. It is assumed that on all elements of $\mathcal{F}_i$ the aggregate q_i is strictly positive. If a common prior is assumed on $\mathcal{F}$, it will be denoted by μ.

We refer to a function with domain Ω, constant on elements of $\mathcal{F}_i$, as $\mathcal{F}_i$*-measurable*. It is assumed that the players' information partitions are common knowledge.

[1] Sometimes $\mathcal{F}_i$ will denote the σ-algebra generated by the partition, as will be clear from the context.

[2] The "join" $\bigvee_{i \in S} \mathcal{F}_i$ denotes the smallest σ-algebra containing all $\mathcal{F}_i$, for $i \in S \subseteq I$.

For any consumption $x_i : \Omega \to \mathbb{R}^l_+$ we define

$$v_i(x_i) = \sum_{\Omega} u_i(\omega, x_i(\omega))q_i(\omega). \tag{1}$$

Equation (1) gives the *ex ante expected utility* of Pi.

Let $\mathcal{G}$ be a partition of (or σ-algebra on) Ω, belonging to Pi. For $\omega \in \Omega$ denote by $E_i^{\mathcal{G}}(\omega)$ the element of $\mathcal{G}$ containing ω; in the particular case where $\mathcal{G} = \mathcal{F}_i$ denote this just by $E_i(\omega)$. Pi's conditional probability for the state of nature being ω', given that it is actually ω, is then

$$q_i\left(\omega' | E_i^{\mathcal{G}}(\omega)\right) = \begin{cases} 0 & \omega' \notin E_i^{\mathcal{G}}(\omega) \\ \frac{q_i(\omega')}{q_i\left(E_i^{\mathcal{G}}(\omega)\right)} & \omega' \in E_i^{\mathcal{G}}(\omega). \end{cases}$$

The *interim expected utility* function of Pi, $v_i(x_i|\mathcal{G})$, is given by

$$v_i(x_i|\mathcal{G})(\omega) = \sum_{\omega'} u_i(\omega', x_i(\omega'))q_i\left(\omega' | E_i^{\mathcal{G}}(\omega)\right). \tag{2}$$

Equation 2 defines a $\mathcal{G}$-measurable random variable.

We denote by $L_1(q_i, \mathbb{R}^l)$ the space of equivalence classes of $\mathcal{F}$-measurable functions $f_i : \Omega \to \mathbb{R}^l$; when a common prior μ is assumed $L_1(q_i, \mathbb{R}^l)$ will be replaced by $L_1(\mu, \mathbb{R}^l)$. L_{X_i} is the set of all $\mathcal{F}_i$-measurable selections from the random consumption set of Agent i, i.e.,

$$L_{X_i} = \{x_i \in L_1(q_i, \mathbb{R}^l) : \ x_i : \Omega \to \mathbb{R}^l \text{ is } \mathcal{F}_i\text{-measurable and } x_i(\omega) \in X_i(\omega)\ q_i\text{-}a.e.\}$$

and let $L_X = \prod_{i=1}^n L_{X_i}$.

Also let

$$\bar{L}_{X_i} = \{x_i \in L_1(q_i, \mathbb{R}^l) : \ x_i(\omega) \in X_i(\omega)\ q_i\text{-}a.e.\}$$

and let $\bar{L}_X = \prod_{i=1}^n \bar{L}_{X_i}$.

An element $x = (x_1, \ldots, x_n) \in \bar{L}_X$ will be called an *allocation*. For any subset of players S, an element $(y_i)_{i \in S} \in \prod_{i \in S} \bar{L}_{X_i}$ will also be called an allocation, although strictly speaking it is an allocation to S.

For the notion of REE we shall need the following. Let $\sigma(p)$ be the smallest sub-σ-algebra of $\mathcal{F}$ for which a price system $p : \Omega \to \mathbb{R}^l_+$ is measurable and let $\mathcal{G}_i = \sigma(p) \vee \mathcal{F}_i$ denote the smallest σ-algebra containing both $\sigma(p)$ and $\mathcal{F}_i$.

Definition 2.1 An *REE* is a pair (p, x), where $x = (x_1, \ldots, x_n) \in \bar{L}_X$ is an allocation, such that

(i) for all i the consumption function $x_i(\omega)$ is $\mathcal{G}_i$-measurable;
(ii) for all i and for all ω the consumption function maximizes $v_i(x_i|\mathcal{G}_i)(\omega)$ subject to the budget constraint at state ω,

$$p(\omega)x_i(\omega) \le p(\omega)e_i(\omega);$$

(iii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$ for all $\omega \in \Omega$.

An REE requires that we condition also on information from prices and therefore it is, in general, an interim concept. An REE is said to be *fully revealing* if $\mathcal{G}_i = \mathcal{F}$ for all $i = 1, 2, \ldots, n$. Otherwise it will be *non-revealing*.

Next we define the WFC (see Yannelis 1991; Koutsougeras and Yannelis 1993) which is a development of the fine core concept of Wilson (1978). WFC allocations always exist, provided the utility functions are concave and continuous.

Definition 2.2 An allocation $x = (x_1, \ldots, x_n) \in \bar{L}_X$ is said to be a *WFC allocation* if

(i) each $x_i(\omega)$ is $\mathcal{F}_I$-measurable;
(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$, for all $\omega \in \Omega$;
(iii) there do not exist coalition S and allocation $(y_i)_{i\in S} \in \prod_{i\in S} \bar{L}_{X_i}$ such that $y_i(\cdot) - e_i(\cdot)$ is $\mathcal{F}_S$-measurable for all $i \in S$, $\sum_{i\in S} y_i = \sum_{i\in S} e_i$ and $v_i(y_i) > v_i(x_i)$ for all $i \in S$.

It is not surprising that a fully revealing REE, which is really an ex post concept, need not be in the WFC which is ex ante "full information" Pareto optimal. We show below that an interim, non-revealing REE is also not in the WFC.

3 Incentive compatibility

Agents make contracts in the ex ante stage. In the interim stage, i.e. after they have received a signal as to what the realized state of nature could be, one considers the incentive compatibility of the contract. The basic idea is that an allocation is incentive compatible if no coalition can misreport the realized state of nature and have a distinct possibility of making its members better off.

Suppose we have a coalition S, with members denoted by i. Their pooled information $\bigvee_{i\in S} \mathcal{F}_i$ will be denoted by $\mathcal{F}_S$ and we assume that $\mathcal{F}_I = \mathcal{F}$. Let the realized state of nature be ω^*. Each member $i \in S$ sees $E_i(\omega^*)$. Obviously not all $E_i(\omega^*)$ need be the same, however all Agents i know that the actual state of nature could be ω^*.

Consider a state ω' such that for all $j \in I \setminus S$ we have $\omega' \in E_j(\omega^*)$ and for at least one $i \in S$ we have $\omega' \notin E_i(\omega^*)$. Now the coalition S decides that each member i will announce that she has seen her own set $E_i(\omega')$ which, of course, contains a lie. On the other hand we have that $\omega' \in \bigcap_{j\notin S} E_j(\omega^*)$.

The idea is that if all members of $I \setminus S$ believe the statements of the members of S then each $i \in S$ expects to gain. For CBIC of an allocation we require that this is not possible. Individually Bayesian incentive compatibility (IBIC), refers to the case when S is a singleton. In multilateral contracts the appropriate concept is that of CBIC. Contracts which are IBIC are not necessarily CBIC (see Glycopantis and Yannelis (2005b, p. VIII), Example 0.3).

The following strengthening of the concept of CBIC allows for transfers between the members of a coalition[3]

Definition 3.1 An allocation $x = (x_1, \ldots, x_n) \in \bar{L}_X$, with or without free disposal, is said to be *transfer coalitionally Bayesian incentive compatible* (TCBIC) if it is not true that there exists a coalition S, states ω^* and ω', with ω^* different from ω' and $\omega' \in \bigcap_{j \notin S} E_j(\omega^*)$ and a random, net-trade vector, among the members of S: $z = (z_i)_{i \in S}$, such that $\sum_{i \in S} z_i = 0$, such that for all $i \in S$ there exists $\bar{E}_i(\omega^*) \subseteq Z_i(\omega^*) = E_i(\omega^*) \cap (\bigcap_{j \notin S} E_j(\omega^*))$, for which

$$\sum_{\omega \in \bar{E}_i(\omega^*)} u_i(\omega, e_i(\omega) + x_i(\omega') - e_i(\omega') + z_i) q_i \left(\omega | \bar{E}_i(\omega^*)\right) >$$
$$\sum_{\omega \in \bar{E}_i(\omega^*)} u_i(\omega, x_i(\omega)) q_i \left(\omega | \bar{E}_i(\omega^*)\right). \tag{3}$$

We condition the interim expected utility on $\bar{E}_i(\omega^*)$ which implies that we require consistency between the declaration of the members of S and the observations of the agents in the complementary set. CBIC corresponds to $z = 0$ and then IBIC to the case when S is a singleton. It follows that TCBIC $\Rightarrow$ CBIC $\Rightarrow$ IBIC.

Definition 3.1 implies that no coalition of agents has an incentive to misreport the realized state of nature. That is, they do not expect that it is possible to become better off if they are believed, by adding to their initial endowment the net trade corresponding to the state they declare.

In terms of game trees, an allocation will be IBIC if there is a profile of optimal behavioral strategies and equilibrium paths along which no player misreports the state of nature he has observed. Players might lie from information sets which are not visited by an optimal play.

In general the extensive form game forces the players to consider earlier decisions by other players and what the payoffs will be if a lie is detected, through, for example, incompatibility of declarations. In this fuller description the players can make a decision to tell a lie or not.

Finally we note that, as shown in Glycopantis et al. (2005a), in a differential information economy with one good per state and monotonic utility functions any REE is TCBIC.

4 On the non-revealing REE of a decomposable model

In this section, we discuss the following. First, we present a model which can be decomposed in two sub-models, we calculate the REE and argue that it is non-revealing and

[3] For related incentive compatibility concepts see Koutsougeras and Yannelis (1993) and Hahn and Yannelis (1997).

not CBIC by explaining which agent, and under which circumstances, can misreport what he has observed. Second by considering sequential decisions, in which P1 acts first and when P2 is to act he has heard the declaration of P1, we show that, under reasonable rules, the non-revealing REE is not implementable as a PBE. The analysis is also cast in terms of normal form games which must now be interpreted with care as they correspond to a dynamic game tree. Finally we discuss the relation of the REE and the WFC.

The fact that the REE is not revealing complements the analysis of the case when the REE is fully revealing. In general an REE does not belong to the WFC. If it so happens that REE does belong to this set then a slight modification of the utility functions implies that this is no longer the case. Throughout, payoffs are given in terms of utility.

4.1 A decomposable model and its REE

The idea is to construct a model such that we have a non-revealing REE which does not belong to the WFC. At the same time we want to show how a model decomposes into two independent ones. We also use this model for comparisons between the dynamic, extensive form formulation of a game and the static, normal form approach. We explain that static tables have to be interpreted with care as they are not independent.

Example 4.1 We assume that there two agents, $I = \{1, 2\}$, two commodities, i.e. $X_i = \mathbb{R}^2_+$ for each $i \in I$, and five states of nature $\Omega = \{a, b, c, d, e\}$.

We further assume that the endowments, per state a, b, c, d and e respectively, and information partitions of the agents are given by

$$\begin{aligned} e_1 &= ((7, 1), (7, 1), (4, 1), (4, 0), (4, 0)), \quad \mathcal{F}_1 = \{\{a, b\}, \{c\}, \{d, e\}\}; \\ e_2 &= ((1, 10), (1, 7), (1, 7),\ (0, 4), (0, 4)), \quad \mathcal{F}_2 = \{\{a\}, \{b, c\}, \{d\}, \{e\}\}. \end{aligned}$$

We shall denote $A_1 = \{a, b\},\ c_1 = \{c\},\ D_1 = \{d, e\},\ a_2 = \{a\},\ A_2 = \{b, c\},\ d_2 = \{d\},\ e_2 = \{e\}$.

It is also assumed that $u_i(\omega, x_{i1}, x_{i2}) = x_{i1}^{\frac{1}{2}} x_{i2}^{\frac{1}{2}}$, where the second index refers to the good, which is a strictly quasi-concave, and monotone function in x_{ij}, and that every player expects that each state of nature occurs with the same probability, i.e. $\mu(\{\omega\}) = \dfrac{1}{5}$, for $\omega \in \Omega$. Some calculations are $u_1(7, 1) = 2.65,\ u_1(4, 1) = 2,$ $u_2(1, 10) = 3.16,\ u_2(1, 7) = 2.65\ u_1 = (4, 0) = 0,\ u_2 = (0, 4) = 0$.

The expected utilities, multiplied by 5, are given by $\mathcal{U}_1 = 7.3$ and $\mathcal{U}_2 = 8.46$.

At least from the point of view of calculating the REE allocation, the model in effect decomposes into two sub-models, one concerning states a, b and c and the other one referring to the data of d and e. The reason is that the private information of either agent does not overlap over this two sets of states. This decomposition allows us to obtain the equilibrium allocation rather easily by appealing to some previously obtained results.

The REE for states a, b and c are described in Example 3.1 of Glycopantis et al. (2005a). For states d and e the prices are equal and the allocations are (2, 2), per agent, per state. For suppose that the normalized prices were different per state. This would be contradicted by the fact that calculations per state give the same prices. On the other hand the calculations of the agents for the same prices would imply that they are equal. Since no information is provided to Player 1 concerning states d and e the REE is non-revealing.

Straightforward calculations show that there is only one, non-revealing REE.[4] The prices, the allocations and the corresponding utilities are:

$$\begin{aligned}
\text{In state a} : (p_1, p_2) &= \left(1, \frac{8}{11}\right); \quad x_{11}^* = \frac{85}{22}, \quad x_{12}^* = \frac{85}{16}, \quad x_{21}^* = \frac{91}{22}, \quad x_{22}^* \\
&= \frac{91}{16}; \quad u_1^* = 4.53, \quad u_2^* = 4.85. \\
\text{In state b} : (p_1, p_2) &= (1, 1); \quad x_{11}^* = 4, \quad x_{12}^* = 4, \quad x_{21}^* = 4, \\
& x_{22}^* = 4; \quad u_1^* = 4, \quad u_2^* = 4. \\
\text{In state c} : (p_1, p_2) &= \left(1, \frac{5}{8}\right); \quad x_{11}^* = \frac{37}{16}, \quad x_{12}^* = \frac{37}{10}, \quad x_{21}^* \\
&= \frac{43}{16}, \quad x_{22}^* = \frac{43}{10}; \quad u_1^* = 2.93, \quad u_2^* = 3.40. \\
\text{In state d} : (p_1, p_2) &= (1, 1); \quad x_{11}^* = 2, \quad x_{12}^* = 2, \quad x_{21}^* = 2, \\
& x_{22}^* = 2; \quad u_1^* = 2, \quad u_2^* = 2. \\
\text{In state e} : (p_1, p_2) &= (1, 1); \quad x_{11}^* = 2, \quad x_{12}^* = 2, \quad x_{21}^* = 2, \\
& x_{22}^* = 2; \quad u_1^* = 2, \quad u_2^* = 2.
\end{aligned}$$

The normalized expected utilities of the non-revealing REE are $\mathcal{U}_1 = 15.46$, $\mathcal{U}_2 = 16.25$.

We wish to show that the non-revealing REE allocation is not CBIC. We use and extend the static argument in Glycopantis et al. (2005a). We also give an explanation as the reason why a formulation in terms of a sequence of decisions by the agents is desirable.

Suppose that the realized state of nature is $\{a\}$. This means that P1 sees $\{a, b\}$, and P2 sees $\{a\}$ but misreports $\{b, c\}$. If P1 believes that state b has occured, he agrees to get the allocation (4, 4). P2 receives the allocation $e_2(a) + x_2(b) - e_2(b) = (1, 10) + (4, 4) - (1, 7) = (4, 7)$ with $u_2(4, 7) = 5.29 > u_2\left(\frac{91}{22}, \frac{91}{16}\right) = 4.85$.

[4] If the model contained only states a, b, c as in Example 3.1, then the REE would be fully revealing, as it can also been seen from the prices below. The addition or states d, e makes the REE of the whole model non-revealing.

Hence P2 has a possibility of gaining by misreporting and therefore the non-revealing REE is not CBIC (IBIC). He gets his initial endowments under the realized state plus his allocation under state b and he pays for this with his allocation under the assumed state b.

An alternative and probably more direct explanation of the allocation that P2 receives is as follows. P1 agrees to get (4, 4), and P2 receives the rest of the total initial endowments in state a, i.e. $e_1(a) + e_2(a) - x_1(b) = (7, 1) + (1, 10) - (4, 4) = (4, 7)$ as obtained above. The expression $e_1(a) + e_2(a) - x_1(b)$ matches up with $e_2(a) + x_2(b) - e_2(b)$. By measurability of allocations we have $e_1(a) = e_1(b)$ and by feasibility $e_1(b) + e_2(b) = x_1(b) + x_2(b)$.

On the other hand if P2 sees $\{b, c\}$ and P1 sees $\{c\}$, the latter cannot misreport $\{a, b\}$ and hope to gain. If b is believed to be the true state, the calculations now give $e_1(c) + x_1(b) - e_1(b) = (4, 1) + (4, 4) - (7, 1) = (1, 4)$ with $u_1(1, 4) = 2 < u_1\left(\frac{37}{16}, \frac{37}{10}\right) = 2.93$.

In a simple manner if P2 believes that $\{b\}$ has occured then he expects the allocation (4, 4) which subtracted from the total endowments, (5,8), under state $\{c\}$ leaves P1 with the allocation (1,4) which gives him less utility than if he had not lied. Therefore P1 will not misreport because it is not to his advantage to do so. Indeed he ends up with less utility.

Finally we mention that if P1 sees $\{d, e\}$, and P2 sees either $\{d\}$ or $\{e\}$ than the latter is indifferent in terms of utility between misreporting and not doing so.

We have considered above a case in which a player will gain if his lie is believed and one in which it is to his disadvantage if he misreports. If the conditions of incentive compatibility are satisfied then one could say that the proposed allocation is in a sense stable. However, as we have pointed out in Glycopantis et al. (2003) and above, CBIC is not concerned in detail with what happens if the lie is detected. It is in the extensive form formulation that we are forced to consider the sequence of events that will follow and therefore whether a player will risk a lie. In the fuller description of the situation in terms of a game tree describing the sequence of moves (decisions) following the players observations and the rules for calculating payoffs the analysis is more satisfactory. In this extensive form formulation to which we now turn the equilibrium notion we employ is that of a PBE. We recall that a PBE consists of optimal behavioral strategies of the players and the, consistent with these decisions, beliefs attaching a probability distribution to the nodes of each information set.

4.2 The non-implementation of the non-revealing REE allocation as a PBE

We now wish to see how the overall model decomposes into two separate ones. The decision of nature is flashed on a screen but each player sees the information sets to which this choice belongs. We assume that the players can choose to tell the truth or to lie with respect to what they have observed. We assume that P1 plays first and that when P2 is to act he knows what P1 has chosen. We specify the *rules* for calculating payoffs, i.e. the terms of the contract (payoffs are in utility terms):

(i) If the declarations of the two players are incompatible, that is one of (A_1, d_2), (A_1, e_2), (c_1, a_2), (c_1, d_2), (c_1, e_2), (D_1, a_2), (D_1, A_2), then this implies that no trade takes place.

(ii) If the state is a, b, or c and the declarations of the two players are (A_1, A_2) then this implies that state b is really declared. The player who believes it (because he has no reason to disbelieve it) gets his allocation (4, 4) and the other player gets the rest. So aA_1A_2 means P2 has lied but P1 believes it is state b and gets (4, 4). P2 gets the rest under state a that is (4, 7); bA_1A_2 means that both believe that it is the (actual) state b and each gets (4, 4); cA_1A_2 means that P2 believes it is state b and gets (4, 4) and P1 gains nothing from his lie as he gets (1, 4). If the state is d or e and the declarations of the two players are (D_1, e_2) or (D_1, d_2), respectively, then the state in the overlap is believed and the two players get their non-revealing REE allocation under this state and end up with allocation (2, 2) each, which implies $u_i = 2$.

(iii) $aA_1a_2, bA_1A_2, cc_1A_2, dD_1d_2, eD_1e_2$, imply that everybody tells the truth and the contract implements the non-revealing REE allocation under state a, b, c, d and e respectively (bA_1A_2, dD_1d_2 and eD_1e_2, in (ii) and (iii) give of course an identical result).

(iv) $ac_1A_2,\ aD_1d_2,\ aD_1e_2,\ bD_1d_2,\ bD_1e_2,\ cD_1d_2,\ cD_1e_2$, imply that both lie but their declarations are not incompatible. Each gets his non-revealing REE allocation under the overlapping state and there is free disposal of the difference between the total endowments under the true state and the allocation which the agents receive.

(v) $cA_1a_2,\ dA_1a_2,\ dA_1A_2,\ dc_1A_2,\ eA_1a_2,\ eA_1A_2,\ ec_1A_2$ means that both lie and stay with their initial endowments as they cannot get the non-revealing REE allocations under the state in the overlap of their declarations.

(vi) bA_1a_2 implies that P2 misreports and P1 believes and gets his non-revealing REE under a; P2 gets the rest under b, that is $\left(\frac{91}{22}, \frac{43}{16}\right)$. Then $u_2 = 3.33 < 4$ and $u_1 = 4.53 > 4$ and the lie of P2 really benefits P1.

(vii) bc_1A_2 means that P1 lies and P2 believes that it is state c. P2 gets his non-revealing REE allocation under c and P1 gets the rest under b, that is the allocation $\left(\frac{85}{16}, \frac{37}{10}\right)$. Then $u_1 = 4.43 > 4$ and $u_2 = 3.4 < 4$ and P1 benefits from lying.

We are looking for a PBE and we analyse Fig. 1 by considering first optimal decisions of P2. From information sets corresponding to states a, b, c, player P2 will never play d_2 or e_2, since these are dominated by strategy A_2. In states d or e, P2 never gains anything by lying and optimal decisions are to play truthfully, d_2 or e_2. Figure 2 contains only these optimal decisions of P2.

We now consider optimal decisions of P1 on the basis of Fig. 2. In states a, b, c, player P1 does not need to play D_1 since, this is dominated, for example, by c_1, and in states d or e he will always play D_1. Hence we obtain Fig. 3, in which heavy lines show plays of the game, i.e. directed paths from the initial node to a terminal node, corresponding to choices by nature and optimal behavior strategies of the players. Their beliefs are also indicated and the conditions for a PBE are satisfied.

The equilibrium paths give under state a payoffs which are different from those of the non-revealing REE allocation. Hence this is not implementable and this matches

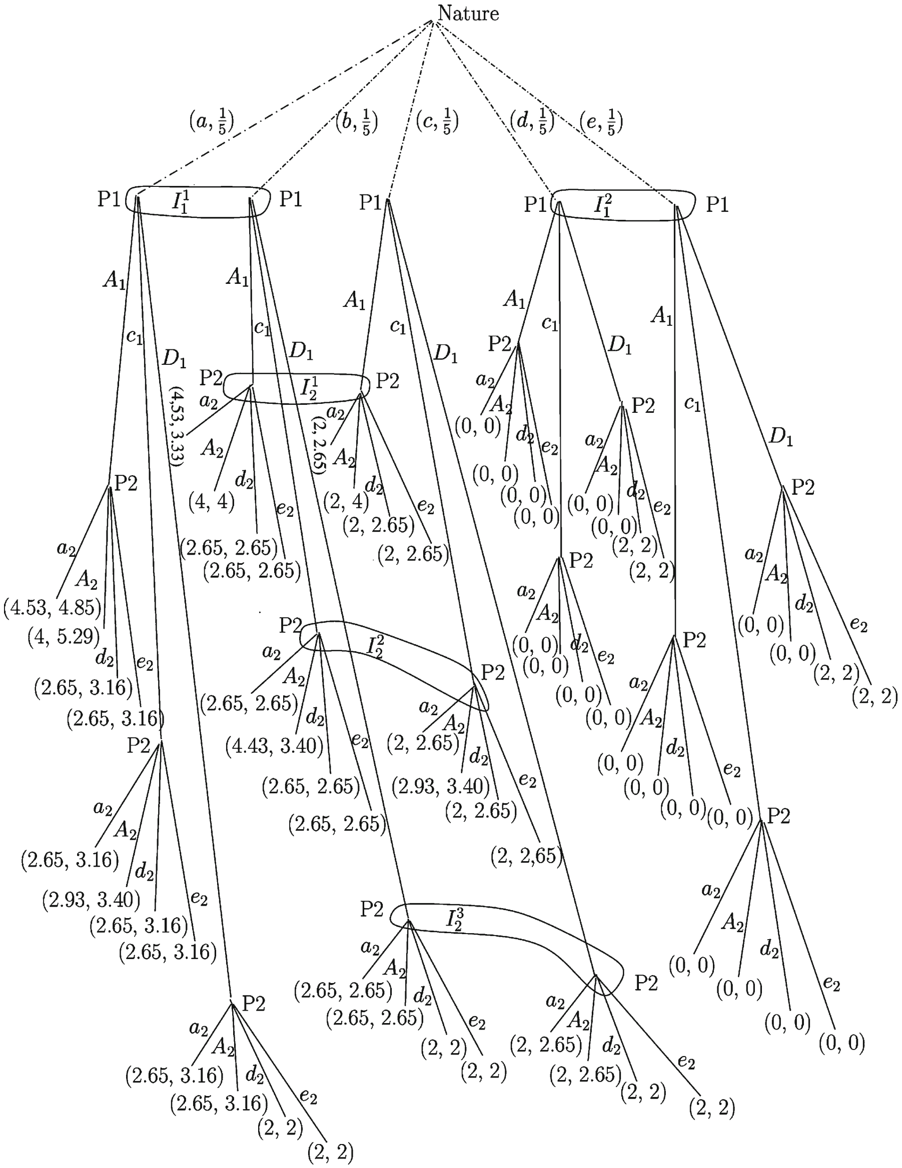

Fig. 1

up with the fact that it is not CBIC. However comparing the normalized expected utility of the PBE with those corresponding to the initial allocation we conclude that the proposed contract will be signed. This follows from the fact that both agents gain from this arrangement. On the other hand P2, because it is advantageous to him to do so, stops P1 from realizing his normalized non-revealing REE utility under state a.

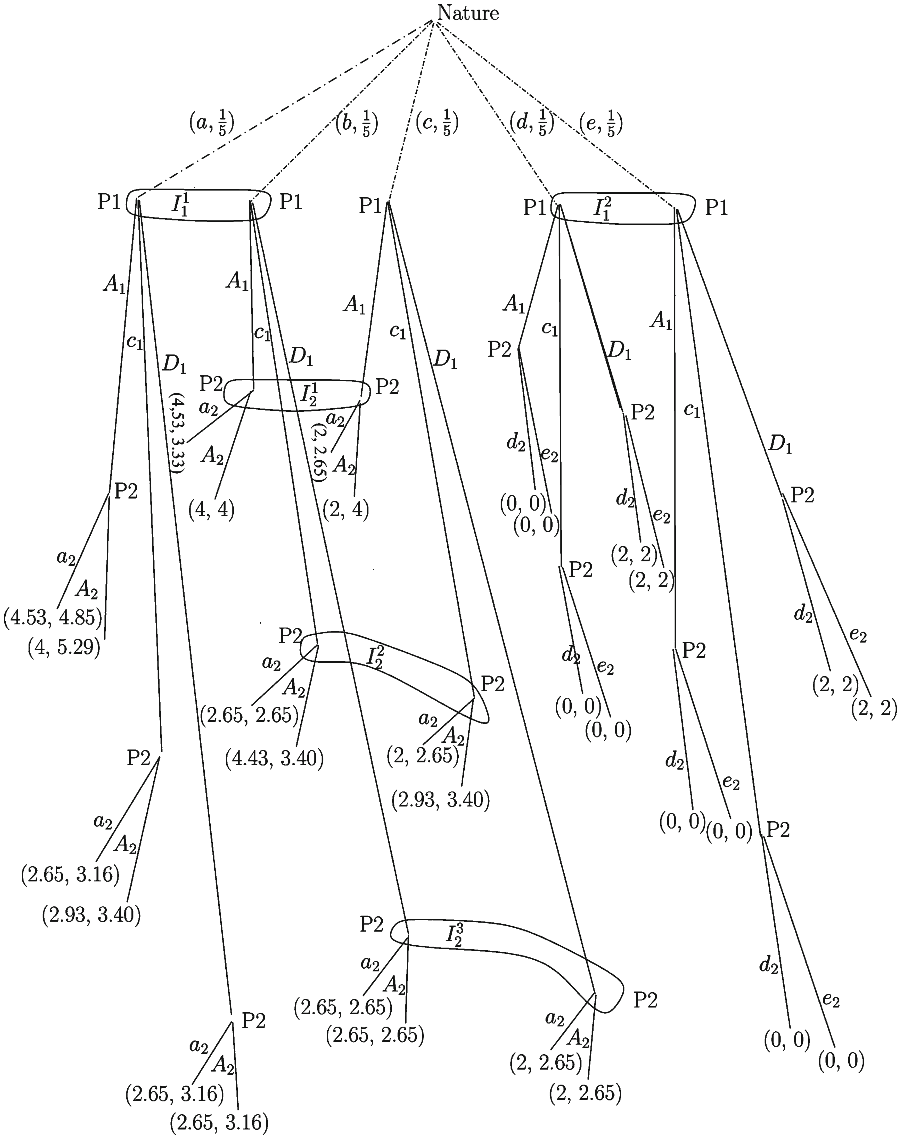

Fig. 2

4.3 The analysis in terms of normal form games

Note that instead of analyzing the example through a game tree which is larger than usual, we can tabulate the payoffs for strategy pairs in each state, as it is shown below in Tables 1 to 4. However, these tables have to be interpreted with care as they are not independent. This follows from the fact that in the game tree there exist information sets with more than one node. In general, P1 looks at his equally probable Tables 1 and 2

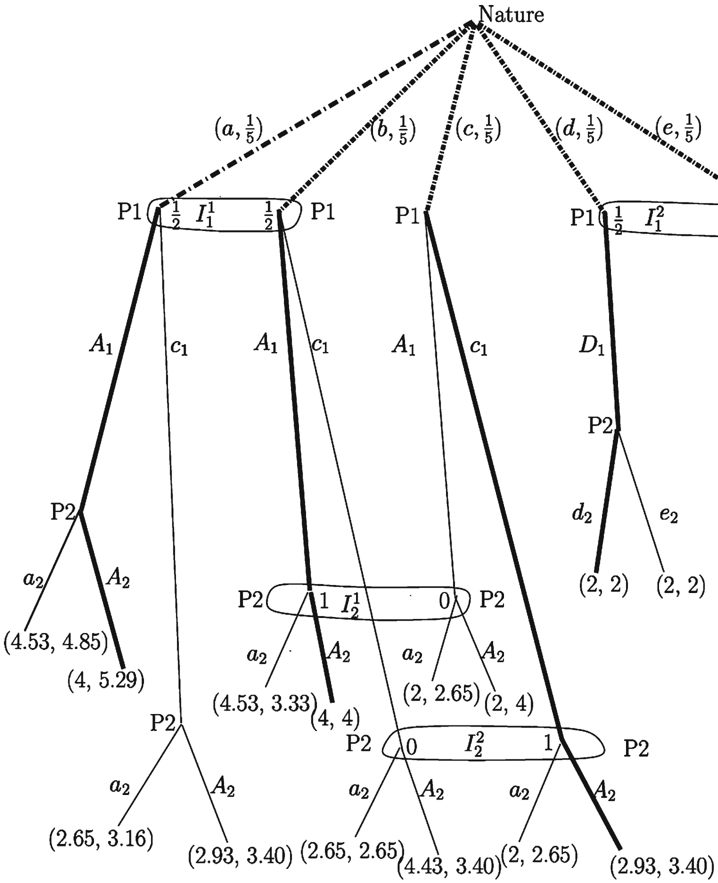

Fig. 3

Table 1 Strategies and payoffs; state of nature a

	P2			
P1	a_2	A_2	d_2	e_2
A_1	(4.53, 4.85)	(4, 5.29)	(2.65, 3.16)	(2.65, 3.16)
c_1	(2.65, 3.16)	(2.93, 3.40)	(2.65, 3.16)	(2.65, 3.16)
D_1	(2.65, 3.16)	(2.65, 3.16)	(2, 2)	(2, 2)

simultaneous and P2 considers together his own equally probable Tables 2 and 3. Table 4 puts together two equally probable states for P1 and can also be considered as referring to e and d separately for P2. The fact that the players consider tables simultaneously gives rise to behavioral strategies.

The idea is to produce optimal behavioral strategies and, consistent with these, beliefs, by analyzing the normal form matrices. We read the tables assuming P2 plays

Table 2 Strategies and payoffs; state of nature b

P1	P2 a_2	A_2	d_2	e_2
A_1	(4.53, 3.33)	(4, 4)	(2.65, 2.65)	(2.65, 2.65)
c_1	(2.65, 2.65)	(4.43, 3.40)	(2.65, 2.65)	(2.65, 2.65)
D_1	(2.65, 2.65)	(2.65, 2.65)	(2, 2)	(2, 2)

Table 3 Strategies and payoffs; state of nature c

P1	P2 a_2	A_2	d_2	e_2
A_1	(2, 2.65)	(2, 4)	(2, 2.65)	(2, 2.65)
c_1	(2, 2.65)	(2.93, 3.40)	(2, 2.65)	(2, 2.65)
D_1	(2, 2.65)	(2, 2.65)	(2, 2)	(2, 2)

Table 4 Strategies and payoffs; state of nature d or e

P1	P2 a_2	A_2	d_2	e_2
A_1	(0, 0)	(0, 0)	(0, 0)	(0, 0)
c_1	(0, 0)	(0, 0)	(0, 0)	(0, 0)
D_1	(0, 0)	(0, 0)	(2, 2)	(2, 2)

second having heard the declaration (strategy) of P1. Therefore adopting the backward induction approach we consider first the optimal decisions of P2 from the various alternative situations he might find himself in. The first observation is that in the cases of states a, b, c player P2, irrespective of what he sees on the screen and hears P1 declare, will never play d_2 or e_2, since these are dominated by behavioral strategy A_2. This does not contradict the fact that P2 cannot distinguish between states b and c. Therefore the last two columns in Tables 1, 2 and 3 are eliminated from consideration by P1 who, on the basis of these reduced tables, has to consider his decision.

It is precisely at this point that a connection exists between the two subgames, because the possibility of decision D_1 is present. If P1 plays D_1 from both $\{a, b\}$ and $\{c\}$ he will be gaining more than under decisions A_1 from c, if the game can be forced to the right hand side of I_2^1.

On the other hand consistent strategies and beliefs are for P1 to play A_1 from $\{a, b\}$ and c_1 from c and for P2 to believe that he can detect the correct states of nature, i.e. to choose each time the correct table between 2 and 3. The game never enters I_2^3 to the nodes of which P2 can attach arbitrary probabilities. Consistency between optimal decisions and beliefs is guaranteed as follows. When P2 see a he knows where he is, and he plays A_2. When he sees A_2 and he hears A_1, he believes he is definetly in state b and he plays A_2. Similarly if he sees A_2 and he hears c_1, he believes he is definitely in state c and he plays A_2.

With respect to P1, he reaches his decisions as follows. He considers Tables 1 and 2 together and chooses as optimal behavioral strategy A_1 from the (conditionally) equi-probable, to him, states a and b. This follows from the fact that his expected return will be $\frac{1}{2}(4+4)$ which is larger than $\frac{1}{2}(2.93+4.43)$, the return under c_1. From state c he plays c_1.

In states d and e, player P2 knows exactly where he is, i.e. his position is confirmed with probability 1, and, no matter what P1 does he can play either d_2 or e_2, and therefore it is also optimal to play truthfully. Hence in Table 4 there is only one column for the rational P1 to consider and he plays D_1. It does not matter that he cannot distinguish between d and e, to which he attaches equal probabilities $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Hence the game can be fully analyzed through the tables. In effect it is seen through these tables that it splits into two non-overlapping sub-games. The analysis enables us to conclude that a non-revealing REE may not be implementable as a PBE of an extensive form game. This follows from the fact that the non-revealing REE of the model considered here is not implementable as a PBE.

4.4 Non-revealing REE and the WFC

D. WFC

We now consider the relation between the non-revealing REE allocation and the WFC. Originally this allocation is in the WFC, as it is also in the sub-models which make up Example 4.1. However, as we shall see, attaching a different weight than 1 to the utility function of Player 2, for example in state d will imply that the allocation will no longer lie in the WFC.

First we show that in Example 4.1 the non-revealing REE is in the WFC. These allocations are obtained by solving the following problem, where we use superscripts to characterize the states. Superscripts 1, 2, 3, 4, 5 correspond to states a, b, c, d and e respectively. The WFC allocations are characterized as follows:

Problem

Maximize $\mathcal{U}_1 = (x_{11}^1 x_{12}^1)^{\frac{1}{2}} + (x_{11}^2 x_{12}^2)^{\frac{1}{2}} + (x_{11}^3 x_{12}^3)^{\frac{1}{2}} + (x_{11}^4 x_{12}^4)^{\frac{1}{2}} + (x_{11}^5 x_{12}^5)^{\frac{1}{2}}$ subject to

$$\begin{aligned}&((8-x_{11}^1)(11-x_{12}^1))^{\frac{1}{2}} + ((8-x_{11}^2)(8-x_{12}^2))^{\frac{1}{2}} + ((5-x_{11}^3)(8-x_{12}^3))^{\frac{1}{2}}\\ &+((4-x_{11}^4)(4-x_{12}^4))^{\frac{1}{2}} + ((4-x_{11}^5)(4-x_{12}^5))^{\frac{1}{2}} = \mathcal{U}_2(\text{fixed})\\ &\quad \mathcal{U}_1 \geq 7.3, \quad \mathcal{U}_2 \geq 8.46.\end{aligned}$$

The conditions on the utility functions imply that if there is an interior maximum per $\mathcal{U}_2$ then it is unique. Setting up the Lagrangean function we obtain the first order conditions:

1. $\dfrac{{x^1_{12}}^{\frac{1}{2}}}{{x^1_{11}}^{\frac{1}{2}}} = \ell \dfrac{(11 - x^1_{12})^{\frac{1}{2}}}{(8 - x^1_{11})^{\frac{1}{2}}}$ 6. $\left(\dfrac{{x^1_{12}}^{\frac{1}{2}}}{{x^1_{11}}^{\frac{1}{2}}}\right)^{-1} = \ell \left(\dfrac{(11 - x^1_{12})^{\frac{1}{2}}}{(8 - x^1_{11})^{\frac{1}{2}}}\right)^{-1}$

2. $\dfrac{{x^2_{12}}^{\frac{1}{2}}}{{x^2_{11}}^{\frac{1}{2}}} = \ell \dfrac{(8 - x^2_{12})^{\frac{1}{2}}}{(8 - x^2_{11})^{\frac{1}{2}}}$ 7. $\left(\dfrac{{x^2_{12}}^{\frac{1}{2}}}{{x^2_{11}}^{\frac{1}{2}}}\right)^{-1} = \ell \left(\dfrac{(8 - x^2_{12})^{\frac{1}{2}}}{(8 - x^2_{11})^{\frac{1}{2}}}\right)^{-1}$

3. $\dfrac{{x^3_{12}}^{\frac{1}{2}}}{{x^3_{11}}^{\frac{1}{2}}} = \ell \dfrac{(8 - x^3_{12})^{\frac{1}{2}}}{(5 - x^3_{11})^{\frac{1}{2}}}$ 8. $\left(\dfrac{{x^3_{12}}^{\frac{1}{2}}}{{x^3_{11}}^{\frac{1}{2}}}\right)^{-1} = \ell \left(\dfrac{(8 - x^3_{12})^{\frac{1}{2}}}{(5 - x^3_{11})^{\frac{1}{2}}}\right)^{-1}$

4. $\dfrac{{x^4_{12}}^{\frac{1}{2}}}{{x^4_{11}}^{\frac{1}{2}}} = \ell \dfrac{(4 - x^4_{12})^{\frac{1}{2}}}{(4 - x^4_{11})^{\frac{1}{2}}}$ 9. $\left(\dfrac{{x^4_{12}}^{\frac{1}{2}}}{{x^4_{11}}^{\frac{1}{2}}}\right)^{-1} = \ell \left(\dfrac{(4 - x^4_{12})^{\frac{1}{2}}}{(4 - x^4_{11})^{\frac{1}{2}}}\right)^{-1}$

5. $\dfrac{{x^5_{12}}^{\frac{1}{2}}}{{x^5_{11}}^{\frac{1}{2}}} = \ell \dfrac{(4 - x^5_{12})^{\frac{1}{2}}}{(4 - x^5_{11})^{\frac{1}{2}}}$ 10. $\left(\dfrac{{x^5_{12}}^{\frac{1}{2}}}{{x^5_{11}}^{\frac{1}{2}}}\right)^{-1} = \ell \left(\dfrac{(4 - x^5_{12})^{\frac{1}{2}}}{(4 - x^5_{11})^{\frac{1}{2}}}\right)^{-1}$

11. $((8 - x^1_{11})(11 - x^1_{12}))^{\frac{1}{2}} + ((8 - x^2_{11})(8 - x^2_{12}))^{\frac{1}{2}} + ((5 - x^3_{11})(8 - x^3_{12}))^{\frac{1}{2}}$ $+((4 - x^4_{11})(4 - x^4_{12}))^{\frac{1}{2}} + ((4 - x^5_{11})(4 - x^5_{12}))^{\frac{1}{2}} = \mathcal{U}_2$ (fixed).

These conditions are satisfied by the non-revealing REE allocations with the Lagrange multiplier $\ell = 1$. On the other hand this relation is very unstable. For example, attaching to the utility function of Player 2 in state d a weight α different than 1 leaves the REE allocation the same but this does not belong to the WFC any longer.

First it is easy to see that the non-revealing REE allocation stays the same. On the other hand with respect to the WFC we see that Eqs. 4, 5, and 11 turn into the ones below, while all other first order conditions stay the same.

4′. $\dfrac{{x^4_{12}}^{\frac{1}{2}}}{{x^4_{11}}^{\frac{1}{2}}} = \ell \times \alpha \dfrac{(4 - x^4_{12})^{\frac{1}{2}}}{(4 - x^4_{11})^{\frac{1}{2}}}$

9′. $\left(\dfrac{{x^4_{12}}^{\frac{1}{2}}}{{x^4_{11}}^{\frac{1}{2}}}\right)^{-1} = \ell \times \alpha \left(\dfrac{(4 - x^4_{12})^{\frac{1}{2}}}{(4 - x^4_{11})^{\frac{1}{2}}}\right)^{-1}$

11′. $((8 - x^1_{11})(11 - x^1_{12}))^{\frac{1}{2}} + ((8 - x^2_{11})(8 - x^2_{12}))^{\frac{1}{2}} + ((5 - x^3_{11})(8 - x^3_{12}))^{\frac{1}{2}}$
$+\alpha((4 - x^4_{11})(4 - x^4_{12}))^{\frac{1}{2}} + ((4 - x^5_{11})(4 - x^5_{12}))^{\frac{1}{2}} = \mathcal{U}_2$(fixed).

However the new set of conditions is inconsistent because 1, 2, 3, 5, 6, 7, 8, 10, require $\ell = 1$ while 4′, 9′ require $\ell \neq 1$. Therefore we obtain a corner solution. But the non-revealing REE is not in the corner of the feasible set as can be seen from inspection of the indifference curves corresponding to the u_i's. It follows that the non-revealing REE is not in the WFC.

We note that one could have obtained the result that the non-revealing REE allocation is not in general in the WFC by considering only the sub-model consisting of states a, b and c (see Glycopantis and Yannelis 2005b, pp. 44–47). On the other hand the model with five states discussed here is much richer. Also the fact that the game tree of the overall model is rather large provides an incentive to conduct the analysis in terms of a more compact, normal form formulation.

5 Concluding remarks

The fully revealing REE is completely ex post, as all states of nature are revealed to the agents through the prices, and we compared it previously with the WFC which is an ex ante concept. This follows from the fact that for the WFC all possible information is shared among the agents but the calculations take place in expectation, before any finer information becomes available.

In this paper we have enlarged our enquiry of REE by considering the non-revealing REE. This is an interim concept, because some agents perform their calculations without complete information, as not all states of nature are revealed through prices, and therefore it is of interest to investigate its properties. As we did in Glycopantis et al. (2005a); Glycopantis and Yannelis (2005b) for the fully revealing REE, we show here that the non-revealing REE is not CBIC, non-implementable, under reasonable rules, as a PBE, and in general does not belong to WFC and thus may not be fully Pareto optimal.

We have cast the analysis in terms of a decomposable model which is easy to analyze in terms of its equilibrium. As a by product of the analysis of the dynamic model we discuss a normal form representation, in which the non-independent payoff matrices have to be interpreted carefully. Strategies and payoffs of the players are presented per relevant state and in reading the tables we assume, as in the construction of the tree, that P2 plays second having his own information and knowing what P1 has declared. P1 takes into account the optimal response of P2 and the game can be fully analyzed, including the calculations of the probabilities of the alternative situations in which the players can find themselves. We can conclude that the non-revealing REE allocation is not implementable as a PBE. This approach can be useful in circumstances, as it is the case here, in which the game trees are larger than usual.

The analysis in this paper enables us to conclude that the non-revealing REE is not a sensible solution concept. This was also the case with the fully revealing REE. Both concepts lack basic properties that one wishes to have, e.g. existence, optimality, incentive compatibility and implementation. A reformulation of these concepts is needed which will enable us to advance matters more satisfactorily.

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