Optimal Fiscal Policy in the Neoclassical Growth Model Revisited[*]

Martin Gervais
University of Iowa
martin-gervais@uiowa.edu

Alessandro Mennuni
University of Southampton
alessandro.mennuni@soton.ac.uk

October 29, 2014

Abstract

This paper studies optimal taxation in a version of the neoclassical growth model in which investment becomes productive within the period, thereby making the supply of capital elastic in the short run. Because taxing capital is distortionary in the short run, the government's ability/desire to raise revenues through capital income taxation in the initial period or when the economy is hit with a bad shock is greatly curtailed. Our timing assumption also leads to a tractable Ramsey problem without state-contingent debt, which can give rise to debt-financed budget deficits during recessions.

1 Introduction

This paper studies optimal fiscal policy in a version of the neoclassical growth model in which capital is elastically supplied even in the short run. This is accomplished by letting investment in capital become productive within the period.

It is well understood that the conventional timing in the neoclassical growth model, in which the size of the capital stock today is the result of past investment decisions, implies that capital is inelastically supplied in the short run. It should be equally clear that a by-product of this conventional timing assumption—that capital is inelastically supplied in the short run—is at the heart of many well-established results within the optimal taxation literature. A prominent example is the well-known prescription to tax initial asset holdings at confiscatory rates, a result that Chamley (1986) and much of the subsequent literature tries to circumvent by imposing bounds on tax rates: without these exogenous bounds, a first-best allocation obtains, an obviously uninteresting solution. Tax rates over the business cycle are similarly dictated by the conventional timing of the neoclassical growth model. Every period, the government promises not to distort the return to investment while at the same time announcing that recessions will be financed through unusually high taxes on capital income, and *vice versa* during booms. This strategy is clearly optimal as the government can avoid distorting investment decisions *ex ante* while at the same time exploiting its ability to absorb shocks in a non-distortionary way by taxing/subsidizing the return to capital *ex post*.

This paper shows that changing the timing of events in the neoclassical growth model in such a way as to make the supply of capital elastic in the short run drastically alters the prescriptions that emanate from standard Ramsey problems. Our assumption that investment in capital becomes productive within the period, which can be interpreted as a stand-in for several factors that make capital elastic in the short run, gives individuals an alternative to supplying capital which is not present under conventional timing.[1] Knowing that this alternative exists limits the ability and desire

---

[1] Prominent factors that make capital elastic in the short run include: endogenous capital utilization; the issue of distinguishing labor and capital income; or hiding capital income altogether. While the source of capital elasticity is not explicit in our environment, our timing assumption makes the environment sufficiently tractable to study the fiscal policy implications of an elastic capital supply in deterministic/stochastic settings, with complete and incomplete markets, with and without

of the government to use capital income taxes to finance government expenditures, either in the initial period or over the business cycle.

One of our main results, already alluded to above, is that the solution to our Ramsey problem generally features a unique non-trivial level of distortions. While the level of distortions depends on individuals' initial asset holdings, it does not rely on the presence of bounds exogenously imposed on the Ramsey problem. As such, the trivial result that the solution to the Ramsey problem without imposing exogenous bounds is time-consistent does not hold in our environment, as will be clear below.[2]

Next we offer a complete characterization of the behavior of tax rates in a stochastic environment in which the government has access to state-contingent debt. Under a class of utility functions in which utility from consumption and disutility from labor are separable, we show that neither the labor nor the capital income tax varies over time, and that the tax on capital is zero in all but the initial period. Under Cobb-Douglas utility, both tax rates become pro-cyclical, that is, they are low during recessions. In either case, the government uses state-contingent debt as a shock absorber, much like the *ex post* capital income tax is used for that purpose in Chari et al. (1994).[3] As a result, debt and the primary deficit move in opposite directions, a counterfactual result which Marcet and Scott (2009) showed to be pervasive in models in which the government has access to state-contingent debt. This leads us to study a Ramsey problem under incomplete markets.

The Ramsey problem without state-contingent debt is a notoriously difficult problem to study (see Chari and Kehoe (1999)). However, this problem is quite tractable in our framework. Technically, this tractability emanates from the fact that our first order conditions can be expressed in terms of prices as functions of quantities. This allows us to write a version of the Ramsey problem, known as the primal, in which the government chooses quantities subject to a sequence of *implementability* constraints.

_____

commitment.

   [2]The conventional solution entails taxing the initial return on capital at confiscatory rates, and to finance all future government expenditures through the return on that capital. This solution turns out to be highly distortionary in our environment. The contrast in results across the two environments is reminiscent of the Lucas (1980) vs Svensson (1985) timing issue in cash-in-advance models, as shown in Nicolini (1998).

   [3]More precisely, either state-contingent debt *or* the capital income tax or combinations of these two instruments can be used to absorb shocks in Chari et al. (1994). Because of our timing assumption, there is no such indeterminacy in our environment.

Without state-contingent debt, the government resorts to taxing capital income at the outset of a recession. Indeed, even with a tax break on labor income, the government's primary deficit improves in the first period of a recession. However, the deficit increases during the latter part of a recession, and this deficit is financed by debt. Subsequently, the amount of government debt tends to revert back to its pre-recession level during good times.[4] As such, our environment can give rise to debt-financed deficits during recessions, in line with the empirical findings of Marcet and Scott (2009). In the latter paper, as well as in Scott (2007), capital income taxes are ruled out altogether in order to focus on the implications of their model with and without state-contingent debt. They argue that ruling out contingent debt is key to bring the model's prescription closer to the data. In addition, Scott (2007) shows that under incomplete markets, government debt and the labor tax rate inherit a unit root component which, as emphasized by Aiyagari et al. (2002) in a model without capital, lends some support to Barro (1979)'s conjecture. Our results confirm that these properties hold even when the government sets capital tax rates optimally. More recently, Farhi (2010) also studies optimal fiscal policy with risk-free government debt. He uses the conventional timing but imposes that the government set capital income tax rates one period ahead in order to mitigate the free lunch associated with volatile *ex post* capital income tax rates. While our Ramsey policies have qualitatively similar business cycle properties—the capital income tax rises while the labor income tax declines at the outset of a recession—the capital income tax rate is much less volatile in our environment. While Farhi (2010) emphasizes how optimal policy is affected when the government is allowed to trade capital, our focus is squarely on the fiscal policy implications of an elastic supply of capital, and the potential for these policies to lead to debt-financed deficits during recessions.

Before moving to the description of our economic environment, our central assumption that investment becomes productive within the period deserves some comments.[5] First, we show that this assumption can be viewed as the opposite from the equally extreme conventional assumption that today's investment only becomes

---

[4]Note, however, that government debt is extremely persistent in the long run. We return to this point below.

[5]Interestingly, a similar timing is commonly used in the housing literature, in which individuals move into their house in the same period in which the house is built: e.g. see Kiyotaki et al. (2011) or Fisher and Gervais (2011).

productive in the next period. Second, we view this assumption more as a way to introduce some elasticity to the supply of capital rather than a way of improving the realism of the neoclassical growth model.[6] There are countless issues for which the conventional timing assumption is either desirable or, at least, innocuous.[7] Optimal taxation is just not one of them.

The rest of the paper is organized as follows. The next section presents our general economic environment, which consists of the neoclassical growth model with an alternative timing assumption. In Sections 3 and 4 we set up and analyze a deterministic and a stochastic Ramsey problem, respectively, while Section 5 is devoted to the analysis of a Ramsey problem without state-contingent debt. A brief conclusion is offered in Section 6.

2   General Economic Environment

The economic environment we consider is similar to that of Chari et al. (1994): a stochastic version of the one-sector neoclassical growth model. As emphasized in the introduction, the main distinguishing feature of our environment is that current investment in capital becomes productive immediately. In this section, we introduce the general economic environment. We later study special cases of this environment, starting with a deterministic version, followed by stochastic versions with and without state-contingent government debt.

Time is discrete and lasts forever. Each period the economy experiences one of finitely many events $s_t \in S$. We denote histories of events by $s^t = (s_0, s_1, \ldots, s_t)$. As of date 0, the probability that a particular history s^t will be realized is denoted $\pi(s^t)$.

Households   The economy is populated by a large number of identical individuals who live for an infinite number of periods and are endowed with one unit of time every period. Individuals' preferences are ordered according to the following utility

---

[6]As already noted, an alternative would be to introduce endogenous capital utilization as in Martin (2010) or Zhu (1995). As emphasized above, the advantage of our timing assumption is that it leads to a tractable (primal) problem with non-contingent government debt, even when the government lacks commitment.

[7]In fact, the first-best allocations under both timing assumptions are essentially indistinguishable.

function

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)U\big(c(s^t),l(s^t)\big),\tag{1}$$

where $c(s^t)$ and $l(s^t)$ represent consumption and hours worked at history s^t. We assume that the felicity function is increasing in consumption and leisure $(1 - l(s^t))$, strictly concave, twice continuously differentiable, and satisfies the Inada conditions for both consumption and leisure.

Each period individuals face the budget constraint

$$c(s^t)+k(s^t)+\sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) = w(s^t)l(s^t)+r(s^t)k(s^t)+k(s^{t-1})+b(s_t|s^{t-1})\tag{2}$$

where $w(s^t) = [1 - \tau^w(s^t)]\hat{w}(s^t)$ and $r(s^t) = [1 - \tau^k(s^t)]\hat{r}(s^t)$ denote after-tax wage and interest rates, respectively. The fiscal policy instruments τ^w and τ^k, as well as government debt $b(s_{t+1}|s^t)$, will be discussed in detail below. Notice that capital and government debt are treated rather symmetrically in budget constraint (2), except of course for the fact that the size of the capital stock and its return cannot depend on tomorrow's state of the economy. In other words, today's price of one unit of capital tomorrow is $1 - r(s^t)$, much like today's price of a bond which pays one unit of consumption good tomorrow in state s_{t+1} is $q(s_{t+1}|s^t)$. As we will see later, the symmetry is even clearer without uncertainty or in the absence of state-contingent government debt. This feature is a direct consequence of our timing assumption, according to which newly installed capital is used in production within the period. We show at the end of this section that if a period is composed of many sub-periods, then this budget constraint is one way to resolve the time-aggregation problem.

Letting $p(s^t)$ denote the Lagrange multiplier on the budget constraint at history s^t, the first order necessary (and sufficient) conditions for a solution to the consumer's problem are given by (2) and

$$\beta^t\pi(s^t)U_c(s^t) = p(s^t),\tag{3}$$
$$\beta^t\pi(s^t)U_l(s^t) = -w(s^t)p(s^t),\tag{4}$$

at all dates t and histories s^t for consumption and labor,

$$-p(s^t)\big(1 - r(s^t)\big) + \sum_{s_{t+1}} p(s^{t+1}) = 0,\tag{5}$$

at all dates t and histories s^t for capital,

$$- p(s^t)q(s_{t+1}|s^t) + p(s^{t+1}) = 0, \tag{6}$$

at all dates t, histories s^t, and all states s_{t+1} tomorrow for bond holdings, as well as the transversality conditions

$$\lim_{t\to\infty} p(s^t)k(s^t) = 0, \tag{7}$$

$$\lim_{t\to\infty} \sum_{s_{t+1}} p(s^{t+1})b(s_{t+1}|s^t) = 0. \tag{8}$$

Under complete markets, it is well known that these first order conditions and the budget constraint can be conveniently combined into a single present value constraint, as stated next:

Proposition 1 *Under complete markets, an allocation solves the consumer's problem if and only if it satisfies equations* (2)–(8), *or, equivalently, if and only if it satisfies*[8]

$$\sum_{t,s^t} \beta^t \pi(s^t) \left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t) \right] = A_0, \tag{9}$$

where $A_0 = U_c(s_0)[k_{-1} + b_{-1}]$, *and* k_{-1} *and* b_{-1} *are initial amounts of capital and government debt held by individuals.*

Proof. The proof is standard. [See for example Chari et al. (1994).] ∎

Below we refer to equation (9) as an *implementability constraint*.

Production The production technology is represented by a neoclassical production function with constant returns to scale in capital (k) and labor (l):

$$y(s^t) = f\big(k(s^t), l(s^t), s_t\big) = A(s_t)k(s^t)^\alpha l(s^t)^{1-\alpha}, \tag{10}$$

where $A(s_t)$ represents the state of technology in period t, $y(s^t)$ denotes the aggregate (or per capita) level of output, and $k(s^t)$ and $l(s^t)$ denote capital and labor used in production. What distinguishes this paper from others in the literature is that capital used in production in period t is chosen in period t, which reflects the fact that the

---

[8]To obtain this equation, multiply the budget constraint (2) by $p(s^t)$, add them up, and use the first order conditions (3)–(6) to replace prices.

current accumulation of capital is used within the period. Accordingly, our law of motion for capital is defined via

$$i(s^t) = k(s^t) + \delta k(s^t) - k(s^{t-1}). \tag{11}$$

The important feature of this law of motion is that investment in capital becomes productive immediately, i.e. it is used in production and depreciates within the period. In this way, the supply of capital is elastic even in the short run.

Output can be used either for private consumption $(c(s^t))$, public consumption $(g(s^t))$, or as investment $(i(s^t))$. Using the law of motion (11), feasibility requires that

$$c(s^t) + g(s^t) + k(s^t) = f\big(k(s^t), l(s^t), s_t\big) - \delta k(s^t) + k(s^{t-1}). \tag{12}$$

The usual properties of the neoclassical growth model hold in our environment: the capital to labor ratio is independent of scale, firms make zero profits in equilibrium, and factors are paid their marginal products:

$$\hat{w}(s^t) = f_l\big(k(s^t), l(s^t), s_t\big) \qquad = f_l(s^t); \tag{13}$$

$$\hat{r}(s^t) = f_k\big(k(s^t), l(s^t), s_t\big) - \delta = f_k(s^t) - \delta, \tag{14}$$

where $\hat{w}(s^t)$ and $\hat{r}(s^t)$ denote before-tax wage and interest rates, respectively.

As with the budget constraint, we show below that if a period is composed of many sub-periods, then feasibility constraint (12) is but one of two approximations that can be used to resolve a time-aggregation problem: while one approximation implies that capital remains unused for the length of the period, the other implies that capital is used to produce itself at the beginning of the period.

The Government The government's role in this economy is to finance an exogenous stream of government expenditures, $g(s^t)$. The fiscal policy instruments available to the government consist of a proportional labor income tax $\tau^w(s^t)$; a proportional capital income tax $\tau^k(s^t)$; and issuance of new government debt $b(s_{t+1}|s^t)$.[9] At date t, the government's budget constraint is as follows:

$$g(s^t) + b(s_t|s^{t-1}) = \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) + \tau^w(s^t)\hat{w}(s^t)l(s^t) + \tau^k(s^t)\hat{r}(s^t)k(s^t). \tag{15}$$

---

[9]Evidently, we do not allow the government to tax wealth directly: doing so would render the problem trivial, as initial wealth could simply be taxed away.

The government thus has to finance government expenditures as well as debt issued yesterday that promised to pay in the event that s_t would occur today. In addition to taxing capital and labor income, the government can raise revenues by issuing new (state-contingent) debt.

Timing Assumption We mentioned above that our budget and feasibility constraints represent one of two extreme way to resolve a time-aggregation problem. To see this, imagine that any period t is divided into n sub-periods. During the first sub-period, the budget constraint is given by

$$c(s^t, 1) + k(s^t, 1) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1})$$
$$= w(s^t, 1) l(s^t, 1) + \left(1 + r(s^t, 1)\right) k(s^{t-1}) + b(s^t),$$

where $c(s^t, 1)$ denotes consumption during the first sub-period, and similarly for other variables. Note that bonds are treated in an identical fashion as in the main text, that is, they are one period instruments. For sub-periods $i = 2, \ldots, n$, the budget constraint is then given by

$$c(s^t, i) + k(s^t, i) = w(s^t, i) l(s^t, i) + \left(1 + r(s^t, i)\right) k(s^t, i - 1).$$

If we sum the sub-period budget constraints, we have

$$\sum_{i=1}^{n} c(s^t, i) + k(s^t, n) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1})$$
$$= \sum_{i=1}^{n} \left[w(s^t, i) l(s^t, i) + r(s^t, i) k(s^t, i - 1) \right] + k(s^{t-1}) + b(s^t).$$

This means that while the conventional timing assumption boils down to assuming that

$$\sum_{i=1}^{n} \left[(r(s^t, i)) k(s^t, i - 1) \right] = r(s^t) k(s^{t-1}),$$

our timing corresponds to the opposite extreme assumption that

$$\sum_{i=1}^{n} \left[(r(s^t, i)) k(s^t, i - 1) \right] = r(s^t) k(s^t).$$

Similarly, using the same logic with the feasibility constraint, one gets

$$c(s^t) + k(s^t, n) + g(s^t) = \sum_{i=1}^{n} \left[f\left(k(s^t, i-1), l(s^t, i)\right) + k(s^t, i-1)(1-\delta) \right] + k(s^{t-1}).$$

where $k(s^t, 0) = k(s^{t-1})$.

To aggregate the sub-periods, one needs to make an assumption about the sum on the right-hand-side of the previous equation. The convention is to approximate it with $f\left(k(s^{t-1}), l(s^t)\right) + k(s^{t-1})(1-\delta)$. We take the opposite extreme that

$$\sum_{i=1}^{n} \left[f\left(k(s^t, i-1), l(s^t, i)\right) + k(s^t, i-1)(1-\delta) \right] = f\left(k(s^t), l(s^t)\right) + k(s^t)(1-\delta).$$

These approximations are reminiscent of discrete time approximation of continuous time equations: while one approximation implies that capital remains unused for the length of the period, the other implies that capital is used to produce itself at the beginning of the period.

3 Deterministic Ramsey Problem

Before analyzing the general stochastic model introduced in the previous section, it will prove instructive to study a deterministic version of the model first. The intuition from this simpler model will carry over to the stochastic environment.

Accordingly, we set up a standard Ramsey problem for a deterministic version of the model. As is well known, there is an equivalence between choosing fiscal policy instruments directly and choosing allocations among an appropriately restricted set of allocations.[10] The government's problem consists of maximizing the utility of the representative individual (1) subject to the implementability constraint (9) and feasibility (12).[11] If we let λ denote the Lagrange multiplier associated with the implementability constraint, we can define a pseudo-welfare function W as

$$W\left(c_t, l_t, \lambda\right) = U\left(c_t, l_t\right) + \lambda \left(U_{c_t} c_t + U_{l_t} l_t\right).$$

---

[10]See Chari and Kehoe (1999) or Erosa and Gervais (2001).

[11]It is well known that if an allocation satisfies the implementability constraint and the feasibility constraint, it must also satisfy the government budget constraint (15)—e.g. see Chari and Kehoe (1999) or Erosa and Gervais (2001). Accordingly, we omit the proof.

The Lagrangian associated with the Ramsey problem, given k_{-1} and b_{-1}, is then given by:

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{\{c_t, l_t, k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t W\left(c_t, l_t, \lambda\right) - \lambda U_{c_0}(k_{-1} + b_{-1})$$

subject to the feasibility constraint

$$c_t + g_t + k_t = f(k_t, l_t) - \delta k_t + k_{t-1}.$$

It should be clear that one can replace the feasibility constraint into the objective function, and that the labor supply can be assumed to satisfy an optimality condition. Accordingly, slightly abusing notation, the Ramsey problem can be rewritten as

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{\{k_t\}_{t=0}^{\infty}} \left\{ W\left(k_{-1}, k_0, \lambda\right) - \lambda U_{c_0}(k_{-1}, b_{-1}) + \sum_{t=1}^{\infty} \beta^t W\left(k_{t-1}, k_t, \lambda\right) \right\}$$

Notice that the last term inside the maximand can be represented by a standard recursive problem: if we define $V(k, \lambda)$ via

$$V\left(k, \lambda\right) = \max_{k'} \left\{ W\left(k, k', \lambda\right) + \beta V(k', \lambda) \right\},$$

then the Ramsey problem becomes

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{k_0} \left\{ W\left(k_{-1}, k_0, \lambda\right) - \lambda U_{c_0} a_{-1} + \beta V\left(k_0, \lambda\right) \right\}$$

$$= \min_{\lambda} \widehat{V}(k_{-1}, b_{-1}, \lambda),$$

where $\widehat{V}$ is the value of the maximand evaluated at the optimum for any given value of λ.

Figure 1 shows, under a particular parameterization of the economy described under the figure, the shape of the value function $\widehat{V}$ as a function of λ, for a given value of initial assets. What this figure shows is that without any restrictions on the fiscal policy instruments or otherwise, the optimal level of distortions, as represented by λ, is non-zero. Indeed, labor income is taxed at a rate of 19% in the long run. Capital income is not taxed in the long run: this can be shown formally as we will see in the next section.

The fact that it is optimal to distort this economy is in sharp contrast to results obtained under the more conventional timing whereby investment made during the

11

Figure 1: Value function $\widehat{V}(\lambda)$
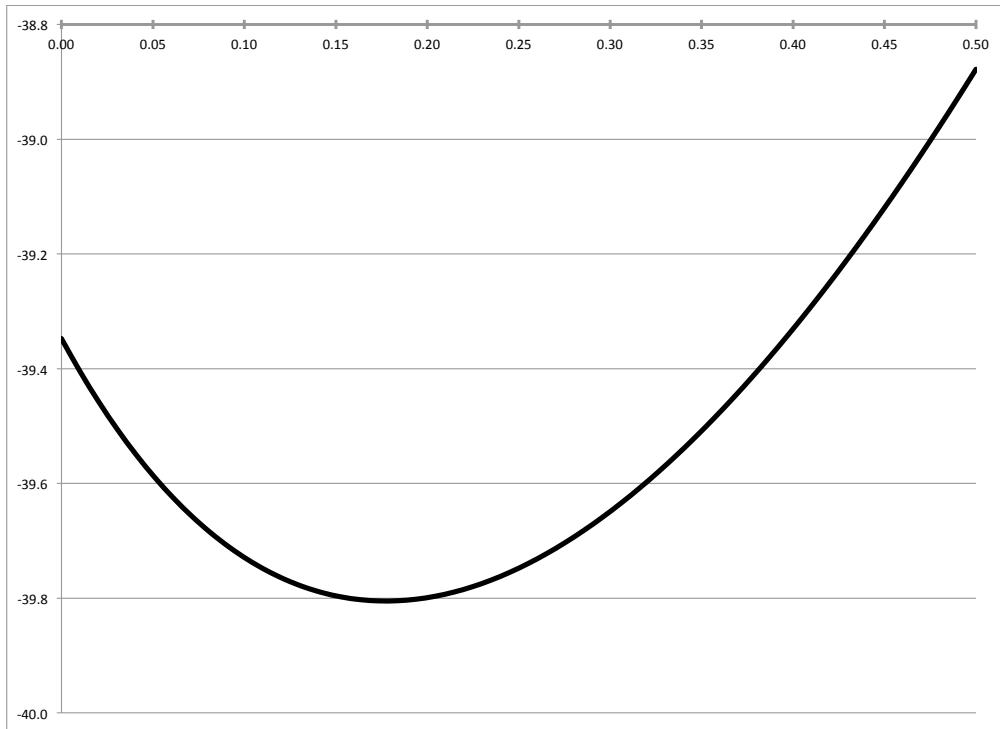
Notes: The parameterization underlying this figure is as follows: Cobb-Douglas production function with a capital share of 1/3; capital depreciates at a rate of 7% per period; utility function additively separable and logarithmic in consumption and leisure; discount factor equal to 0.958; government spending such that it represents around 17% of steady state output; initial capital is set to 1.5 (below its steady state value of 1.8) and initial debt is set to 0.

period only becomes productive the next period. The reason is well known: under conventional timing, taxing initial assets represents a lump-sum way to raise revenues for the government, as these assets were accumulated in the past. Accordingly, the optimal fiscal policy entails taxing these initial assets at 'confiscatory' rates, or just enough for the government to finance the present value of its spending. In terms of Figure 1, the value function $\widehat{V}$ would be a strictly increasing function, with its minimum at exactly zero, meaning that a first-best outcome would be attained.

The intuition for our result comes directly from our timing assumption. Since investment becomes productive immediately, and its return realized during the period, taxing capital at date zero becomes distortionary: individuals do not have to supply

capital accumulated from the past. They can, and will, consume large amounts should the government choose to tax their capital away. Realizing that fact, the government does not rely exclusively on confiscating initial assets. Nevertheless, in the numerical example underlying Figure 1, the initial tax rate on capital income is very high, close to 700%.[12] As a result, consumption at date 0 is around 50% higher than in period 1, which is itself slightly below its steady state level. The tax rate on labor at date 0, however, is negative 20%: this makes leisure relatively expensive in that period, thereby increasing the labor supply.

The general message of this analysis is that the government's ability to use capital income taxes in a lump-sum fashion disappears once the supply of capital is elastic. This simple yet powerful message will also be at the heart of our findings in a stochastic economy, to which we now turn our attention.

4   Stochastic Ramsey Problem

To study optimal policy in this environment, we proceed as in the previous section and set up a standard Ramsey problem. With λ still denoting the Lagrange multiplier on the implementability constraint, the pseudo-welfare function W now reads

$$W\big(c(s^t), l(s^t), \lambda\big) = U\big(c(s^t), l(s^t)\big) + \lambda \left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t) \right]. \qquad (16)$$

The Ramsey problem is thus as follows:

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{\{c(s^t), l(s^t), k(s^t)\}_{t,s^t}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) W\big(c(s^t), l(s^t), \lambda\big)$$
$$- \lambda U_c(s_0)[k_{-1} + b_{-1}]$$

subject to the feasibility constraint (12), keeping in mind that by Walras' Law the government budget constraint must hold and so does not constrain the solution to this problem.

The government typically has more instruments than it needs in the stochastic neoclassical growth model, in the sense that many tax codes can decentralize any given

---

[12]While the capital income tax is very high in the initial period, it is far from being sufficiently high to eliminate all future distortions, as discussed above.

allocation (e.g. see Zhu (1992) or Chari et al. (1994)). Such is not the case in our environment: our tax code is unique, in the sense that any given allocation can only be decentralized by a single tax system. Technically, this comes from the fact that the tax rate on capital income can be uniquely recovered using the marginal product of capital (14) as well as the optimality conditions (3) and (5): for any implementable allocation, there exists a single value of the capital tax which makes these equations hold. Intuitively, the indeterminacy under conventional timing comes from the fact that an allocation can, for example, be implemented with a tax rate on capital income that varies with the state tomorrow and risk-free debt, or with a flat tax on capital income tomorrow and state-contingent debt. Here, the capital income tax applies to the return to investment made during the period, so it is uniquely determined even with state-contingent debt. It follows that ruling out state-contingent debt is not innocuous in our environment, as will be clear in the next section.

The optimality conditions for this Ramsey problem are quite simple, and can be analyzed analytically. Let $\beta^t \phi(s^t)$ represent the Lagrange multiplier on the feasibility constraint (12) at history s^t. The first order conditions with respect to consumption, labor, and capital are, respectively,

$$
\begin{align}
\pi(s^t)W_c(s^t) &= \phi(s^t), \tag{17}\\
\pi(s^t)W_l(s^t) &= -f_l(s^t)\phi(s^t), \tag{18}\\
\phi(s^t)\big[1 - (f_k(s^t) - \delta)\big] &= \beta \sum_{s_{t+1}} \phi(s^{t+1}), \tag{19}
\end{align}
$$

where W_c and W_l represent the derivative of the pseudo-welfare function W (16) with respect to consumption and the labor supply, respectively.

4.1   Optimal Fiscal Policy

The rest of this section is devoted to characterizing optimal fiscal policy. Our characterization, which requires making assumptions about the form of the utility function, involves in turn the labor income tax and the capital income tax. An important note concerning state-contingent debt concludes the section.

We start by establishing that if the per-period utility function is separable between consumption and *labor*, then the labor income tax does not depend on the state of

14

the economy, nor does the capital income tax, which is zero in all but the first period in this case. We later argue that under a more general utility function in which individuals care about *leisure*, both tax rates tend to be pro-cyclical.

Proposition 2 *Assume that the felicity function is separable, $U(c,l) = u(c) + v(l)$, with $u(c)$ and $v(l)$ both exhibiting constant elasticity of substitution. Then the tax rate on labor income is invariant to the productivity shock.*

Proof. See Appendix. ■

The intuition for this result is that because the elasticity of the labor supply does not vary with the shock, there is no reason for the government to tax labor at rates that vary with the shock.[13] Note that the previous result does not apply when individuals care about leisure, as opposed to disliking labor—see Proposition 4 below.

Our next results pertain to the tax on capital income. We show that capital income should not be taxed if the utility function is separable and exhibits constant elasticity of substitution in consumption. We will argue later that under non-separable utility, the tax rate on interest income is likely to be pro-cyclical.

Proposition 3 *Assume that the felicity function is separable, $U(c,l) = u(c) + v(l)$, and that $u(c)$ exhibits constant elasticity of substitution. Then the capital income tax rate is zero at all dates and histories (other than the first period).*

Proof. See Appendix. ■

This Proposition is in sharp contrast to the results in Chari et al. (1994), where the *ex post* tax rate on capital income is extremely volatile.[14] The intuition is that in their environment, the return on investment made today is taxed tomorrow. Since the investment decision has already been made when the tax authority sets the tax rate on capital income, this instrument is extremely useful to absorb shocks to the budget of the government. For example, if the economy experiences a bad shock today, then the government will tax capital income at a high rate to absorb the loss

---

[13]Evidently, the same argument can be made using s^{t-1} and s^t as the two histories, which means that the tax rate on labor is not only state-independent, but also constant over time.

[14]As pointed out at the beginning of this section, however, one should keep in mind that this statement implicitly picks one of many potential tax codes.

in revenue. The more persistent the shock is, the higher the tax rate. In fact, under reasonable parameter values, the increase in capital income taxes is so large that the government runs a primary surplus in the period of a negative shock, thereby absorbing the future path of low government revenues with very little change to the tax rate on labor income. Of course, the tax authority always promises individuals that *on average* capital income will not be taxed. This is what Chari et al. (1994) refer to as the *ex ante* tax rate on capital income, which, under the assumptions of our proposition 3, is zero.

In our environment, the return on capital is known at the time individuals make their investment decision, thereby eliminating the distinction between *ex ante* and *ex post* taxes on capital. In particular, the tax authority no longer has the ability to absorb shocks in a non-distortionary fashion through highly volatile capital income tax rates.

Under more general preferences, the tax rate on capital income will not be equal to zero in general. We now argue that both the labor and capital income tax rates are likely to be pro-cyclical.

Proposition 4 *Assume that $\lambda > 0$ and that the felicity function is given by $U(c, l) = u(c)v(l)$, with $u(c) = (1 - \sigma)^{-1}c^{1-\sigma}$ and $v(l) = (1 - l)^{\nu(1-\sigma)} = (1 - l)^{\eta}$, with $\sigma > 1$ and $\nu > 0$, and $\ln(c) + \eta \ln(1 - l)$ for $\sigma = 1$. Assume that there exist two histories s^t and $\tilde{s}^t$ such that $l(s^t) > l(\tilde{s}^t)$. Then $\tau^w(s^t) > \tau^w(\tilde{s}^t)$ if and only if*

$$\lambda < \frac{-1}{(1 - \sigma)(1 + \nu)}. \tag{20}$$

Proof. From equations (3), (4) and (32), the tax rate on labor income can be expressed as

$$\tau^w(s^t) = \frac{\lambda\big(H_l(s^t) - H_c(s^t)\big)}{1 + \lambda + \lambda H_l(s^t)}. \tag{21}$$

Under the stated utility function, H_c and H_l are such that

$$H_l(s^t) - H_c(s^t) = \frac{-1}{1 - l(s^t)},$$

$$H_l(s^t) = -\sigma + \frac{1 - \eta l(s^t)}{1 - l(s^t)}.$$

16

Using these expression in equation (21) we have

$$\tau^w(s^t) = \frac{\lambda}{1 - \lambda(\sigma - 2) - l(s^t)\big(1 + \lambda(1 - \sigma)(1 + \nu)\big)}.$$

It follows that the tax rate is higher under state s^t than $\tilde{s}^t$ if the term multiplying labor in the denominator is positive, that is, if condition (20) holds. ∎

Note that we need to assume that the economy is distorted ($\lambda > 0$), otherwise all taxes are zero. This Proposition establishes that whenever condition (20) is satisfied, if labor is pro-cyclical, so will the tax rate on labor income. Note that under logarithmic utility, i.e. when $\sigma = 1$, the condition is always satisfied. It becomes less likely to be satisfied as individuals become more risk averse, i.e. as σ increases. As such, this Proposition is useful to interpret the finding in Chari et al. (1994) that the correlation between the shock and labor taxes changes sign as they change the risk aversion parameter. Finally, note that what is key for the cyclicality of the labor tax, or lack thereof, is whether the utility function exhibits constant elasticity of substitution (CES) in labor or in leisure. When it is CES in leisure, the labor supply elasticity varies with the *level* of the labor supply, becoming more inelastic as the labor supply increases. This is in contrast to Proposition 2, where the labor supply elasticity was invariant to the level of the labor supply.

Under the utility function stated in Proposition 4, the tax rate on capital income will also tend to be pro-cyclical: subsidized in bad times and taxed in good times. To see this, note that the function $H_c(s^t)$ under this utility function is given by

$$H_c(s^t) = -\sigma - \eta \frac{l(s^t)}{1 - l(s^t)},$$

which, since $\eta < 0$, is increasing in l. Now from equations (33) and (34), we have

$$\frac{1 - r(s^t)}{1 - \hat{r}(s^t)} = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\big(1 + \lambda + \lambda H_c(s^t)\big)U_c(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\big(1 + \lambda + \lambda H_c(s^{t+1})\big)U_c(s^{t+1})}. \qquad (22)$$

When this ratio is smaller than 1, capital income is subsidized, and capital income is taxed if the ratio is greater than 1. In particular, capital income is subsidized when $H_c(s^t)$ is relatively low, i.e. when the labor supply is relatively low. Much like the labor income tax, the capital income tax is thus likely to be pro-cyclical as long as labor is pro-cyclical.

The results of this section tell us that depending on the form of the utility function, labor and capital income taxes can either be acyclical or pro-cyclical. However, these results are silent as to the behavior of government debt over the business cycle, even if taxes are pro-cyclical. This is because with state-contingent government debt, it may be optimal for the government to commit to a policy that involves repaying a lower amount of debt during recessions—a partial default of debt in the words of Chari and Kehoe (1999). This can easily be established by deriving a present value budget constraint for the government. By substituting forward $b(s^{t+1}|s^t)$ into the government budget constraint (15), letting $ps(s^t) = \tau^w(s^t)\hat{w}(s^t)l(s^t) + \tau^k(s^t)\hat{r}(s^t)k(s^t) - g(s^t)$ denote the primary surplus, one obtains the following representation for debt:

$$b(s_t|s^{t-1}) = ps(s^t) + \sum_{\tau=t}^{\infty}\sum_{s_{\tau+1}} q(s^{\tau+1}|s^t)ps(s^{\tau+1}|s^t).$$

This equation states that a shock which reduces the present value of primary surpluses is associated with a low debt payment. In other words, the amount of debt that comes due following a shock that reduces the present value of primary surpluses must be lower than the amount of debt that comes due in the event of a shock that increases the present value of primary surpluses: state-contingent debt is used as a shock absorber. Whether this translates into an increase or a decrease in the value of debt outstanding is not clear (see equation (15)): while the government faces a primary deficit in bad times, it also wakes up with fewer bonds to repay. However, numerical results suggest that the change in the primary deficit is small relative to the relative size of debt repaid in good vs. bad times. As a result, the government issues less debt in bad times than in good times.[15]

To conclude, our model implies that while the primary deficit can be counter-cyclical (i.e. tax revenues are low in bad times and high in good times), the presence of state-contingent government debt can make government debt pro-cyclical and thus negatively correlated with the primary deficit, a phenomenon which we typically do not observe (see Marcet and Scott (2009)). Accordingly, we now turn our attention to a situation in which the government only has access to risk-free debt.

---

[15]Similar results are discussed in Chari and Kehoe (1999) and Marcet and Scott (2009).

18

5 Ramsey Problem without State-Contingent Debt

Ruling out state-contingent debt and moving to incomplete markets in the standard neoclassical growth model has proven difficult (e.g. see Chari and Kehoe (1999)). In our framework, however, this task is quite tractable. To see this, consider the consumer's budget constraint without state-contingent debt:

$$c(s^t) + k(s^t) + q(s^t)b(s^t) = w(s^t)l(s^t) + r(s^t)k(s^t) + k(s^{t-1}) + b(s^{t-1}) + T(s^t), \quad (23)$$

where T_t is a non-negative lump-sum transfer. As in Aiyagari et al. (2002) and Farhi (2010), these lump-sum transfers are required for the government to avoid rebating resources to individuals in a distortionary way. In other words, should the government find itself in a situation where it has accumulated a sufficient amount of assets (negative debt) that it can finance its spending with the return on these assets even in the worst state of the economy, then transfers will be used to rebate extra resources to individuals in better states of the world. Evidently, this situation can only occur when the government faces natural asset and debt limits: with more stringent debt limits, as will be the case in our numerical examples, these transfers will always be zero. As such, we omit these transfers in the Ramsey problem below, with the understanding that transfers would be used rather than negative distortionary taxes should that situation arise.

It should be clear that the first order conditions for consumption, labor, and capital, equations (3)–(5), remain valid under budget constraint (23). These equations imply that

$$w(s^t) = -\frac{U_l(s^t)}{U_c(s^t)};$$

$$1 - r(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{U_c(s^t)},$$

which can be replaced in the budget constraint to obtain

$$c(s^t) + (k(s^t) + b(s^t))\beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{U_c(s^t)} = -\frac{U_l(s^t)}{U_c(s^t)}l(s^t) + k(s^{t-1}) + b(s^{t-1}). \quad (24)$$

Of course, without state-contingent debt these budget constraints can no longer be expressed as a single present-value budget constraint. Ruling out state-contingent

debt amounts to imposing a sequence of budget or implementability constraints of the form above. Finally, as discussed above, we also impose debt limits: $\underline{M} \leq b(s^t) \leq \overline{M}$.

Given the form of the implementability constraint (24), we rearrange terms to obtain the following Ramsey problem in Lagrangian form:

$$
\begin{aligned}
L(k_{-1}, b_{-1}) = \min_{\{\lambda(s^t) \geq 0\}_{t, s^t}} \max_{\{c(s^t), l(s^t), k(s^t), b(s^t)\}_{t, s^t}} & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \Big\{ U\big(c(s^t), l(s^t)\big) \\
& + \lambda(s^t) \left(c(s^t) + \frac{U_l(s^t)}{U_c(s^t)} l(s^t) - k(s^{t-1}) - b(s^{t-1}) \right) U_c(s^t) \\
& + \lambda(s^{t-1})\big(k(s^{t-1}) + b(s^{t-1})\big) U_c(s^t) \Big\}
\end{aligned}
\tag{25}
$$

subject to feasibility (12) and debt limits at all dates and histories, given k_{-1} and b_{-1}, with $\lambda_{-1} = 0$.

5.1 Analysis

We first establish that the evolution of the multiplier λ, which reflects the distortionary nature of taxation over time, contains a permanent component—a result first discussed in Aiyagari et al. (2002) in a model without capital, and more recently by Scott (2007) in a model with capital in which capital income taxation is ruled out. To establish this result, notice that the first-order condition for government debt, assuming an interior solution, states that

$$
\sum_{s^{t+1} | s^t} \beta^{t+1} \pi(s^{t+1}) \big(\lambda(s^t) U_c(s^{t+1}) - \lambda(s^{t+1}) U_c(s^{t+1}) \big) = 0.
\tag{26}
$$

Since $\lambda(s^t)$ is known at history s^t, it can be taken out of the expectation, establishing that

$$
\lambda(s^t) = \frac{\sum_{s_{t+1}} \pi(s^{t+1} | s^t) U_c(s^{t+1}) \lambda(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1} | s^t) U_c(s^{t+1})},
\tag{27}
$$

so that the multiplier λ follows a risk-adjusted Martingale. An interesting special case, to which we will return below, is one where the felicity function is quasi-linear, i.e. $U(c, l) = c + v(l)$. In this case, the marginal utility of consumption is constant at unity, and so the stochastic process for the multiplier λ becomes a non-negative martingale. Indeed, Farhi (2010) shows that if the government faces natural debt

limits and the stochastic process governing the state s_t converges to a unique (non-degenerate) stationary distribution, then λ_t converges to zero, which implies that the Ramsey allocation converges to a first-best allocation (i.e. all taxes are zero in the long run). This result holds in our economy as well.

In general not much can be said analytically about the behavior of optimal taxes in this environment. In particular, nothing can be said about the labor income tax, at least as far as we can tell. For the capital income tax, we establish one special case in which it is always zero. If we let $\beta^t \pi(s^t) \phi(s^t)$ be the multiplier on the feasibility constraint at history s^t, the first order condition with respect to capital reads

$$\sum_{s^{t+1}|s^t} \beta^{t+1} \pi(s^{t+1}) \left(\lambda(s^{t+1}) - \lambda(s^t) \right) U_c(s^{t+1})$$

$$+ \beta^t \pi(s^t) \phi(s^t) \left(1 - (f_k(s^t) - \delta) \right) - \sum_{s^{t+1}|s^t} \beta^{t+1} \pi(s^{t+1}) \phi(s^{t+1}) = 0,$$

which, given (26), implies that

$$1 - (f_k(s^t) - \delta) = 1 - \hat{r}(s^t) = \frac{\sum_{s^{t+1}|s^t} \beta \pi(s^{t+1}) \phi(s^{t+1})}{\pi(s^t) \phi(s^t)}. \tag{28}$$

As usual, recalling equation (33)—which holds here as well—capital income should not be taxed ($\hat{r}(s^t) = r(s^t)$) if the shadow value of resources is proportional to the marginal utility of consumption at all dates and states, i.e. if $\phi(s^t) \propto U_c(s^t)$.[16] This will in general not be the case, even under a per-period utility function separable between consumption and leisure. In this case, the value of the multiplier ϕ, from the first order condition for consumption, is given by

$$\phi(s^t) = U_c(s^t) \left[1 + \lambda(s^t) \left(\frac{U_{cc}(s^t) c(s^t)}{U_c(s^t)} + 1 \right) \right.$$

$$\left. - \left(\lambda(s^t) - \lambda(s^{t-1}) \right) \frac{U_{cc}(s^t)}{U_c(s^t)} \left(k(s^{t-1}) + b(s^{t-1}) \right) \right]. \tag{29}$$

Clearly, the term inside the square brackets will not be constant in general. There is, however, one special case under which we can establish that capital income should not be taxed, as we state in the following proposition.

---

[16]Note that this is merely a sufficient condition, so there can be cases in which this condition does not hold yet the tax rate on capital income is nevertheless equal to zero. Indeed, this is the case in Proposition 5 below.

Proposition 5 *If the per-period utility function is quasi-linear in consumption, i.e.*
$U(c,l) = c + v(l)$, *then the tax rate on capital income is zero.*

Proof. First note that under this utility function, because the marginal utility of consumption is fixed at unity, (33) implies that $1 - r(s^t) = \beta$. From (29), the value of the multiplier on the feasibility constraint is given by $\phi(s^t) = 1 + \lambda(s^t)$. Furthermore, (27) implies that $\lambda(s^t) = \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\lambda(s^{t+1})$. Using these facts in equation (28) imply that $1 - \hat{r} = \beta$. ∎

5.2   Numerical Examples

To gain more insight into the kind of prescription that emanate from the model without state-contingent debt, we resort to numerical results. To do so, we compute solutions using a recursive formulation of the Ramsey problem (see Appendix for details). In that formulation, we use the current state of productivity (s), capital (k), debt (b), and consumption (c), to represent the state of the economy.[17] From period 1 on, the recursive Ramsey problem is as follows:

$$V(k,b,c,s) \equiv \max_{c(s'),l,k',b'} \left\{ U(c,l) + \beta E\big[V(k',b',c(s'),s')|s\big] \right\} \tag{30}$$

subject to

$$U_c c + U_l l + (k' + b')\beta \sum_{s'} \pi(s'|s)U_c(s') - U_c(k+b) = 0$$

$$f(k',l,s) - \delta k' + k - c - g - k' = 0$$

$$\underline{M} \leq b' \leq \overline{M}.$$

In turn, the problem at date zero is:

$$L(k_{-1},b_{-1},s_0) \equiv \max_{c,c(s'),l,k',b'} \left\{ U(c,l) + \beta EV(k',b',c(s'),s') \right\} \tag{31}$$

subject to

$$U_c c + U_l l + (k' + b')\beta \sum_{s'} \pi(s'|s_0)U_c(s') - U_c(k_{-1}+b_{-1}) = 0$$

$$f(k',l,s_0) - \delta k' + k_{-1} - c - g - k' = 0$$

$$\underline{M} \leq b' \leq \overline{M}.$$

---

[17]It is worth noting that consumption as a state variable is only valid if the utility function is separable. Otherwise marginal utility would have to be used instead.

We parameterize the model along the lines of Farhi (2010), who in turn follows Chari et al. (1994), with a few exceptions to be noted below. A period is taken to represent a year. The discount factor β is set to 0.958, so the pre-tax interest rate fluctuates around 4%. The utility function is given by $u(c, l) = \log(c) + \nu \log(1 - l)$. We set $\nu = 1.5$, so that individuals supply around 35% of their time endowment to the market. The production function is Cobb-Douglas with capital share α set to 0.34. Capital depreciates at a rate of 7 percent per period. Government spending g is equal to 0.1067, which implies an average spending to output ratio in the range of 17%. Our main departure from Chari et al. (1994) and Farhi (2010) concerns the process governing productivity. Like Farhi (2010), we use a two state Markov chain. However, we set the persistence such that the expected length of recessions is two years, and the expected length of booms is 5 years. We use the same standard deviation of the innovations, equal to 0.026.[18] Finally, the debt limits are set to $\pm 50\%$ of GDP in the undistorted deterministic steady state.

Perhaps the two most interesting aspects that simulations can clarify are the responses of fiscal policy instruments (tax rates and debt) to shocks and the long run properties of the economy, which we discuss in turn below.

Figures 2 displays a typical business cycle. The simulation underlying this figure consists of letting the economy repeatedly experience a cycle set to its expected length: 5 years of boom followed by a 2-year recession. The nature of this experiment is such that all variables in this figure are stationary, which need not be the case for random sequences of shocks. The first thing to note is that despite the fact that capital is elastically supplied in the short run, it is nevertheless optimal to finance part of the recession by taxing capital income at a relatively high rate (slightly less than 40%) at the outset of the recession. Indeed, even with a tax break on labor income (of about 1 percentage point), the government's primary deficit improves in the first period of the recession. However, the deficit increases substantially during the second period of the recession, and this deficit is financed by debt. Thereafter, the amount of government debt reverts back to its mean during the boom. Finally, it is worth noting that as one might expect, consumption in the first period of the recession remains fairly high: individuals choose to consume more than they otherwise would because of the

---

[18]This corresponds to the 0.04 used by Chari et al. (1994) and Farhi (2010), as they model the shock as labor augmenting.

Figure 2: Deterministic Cycles
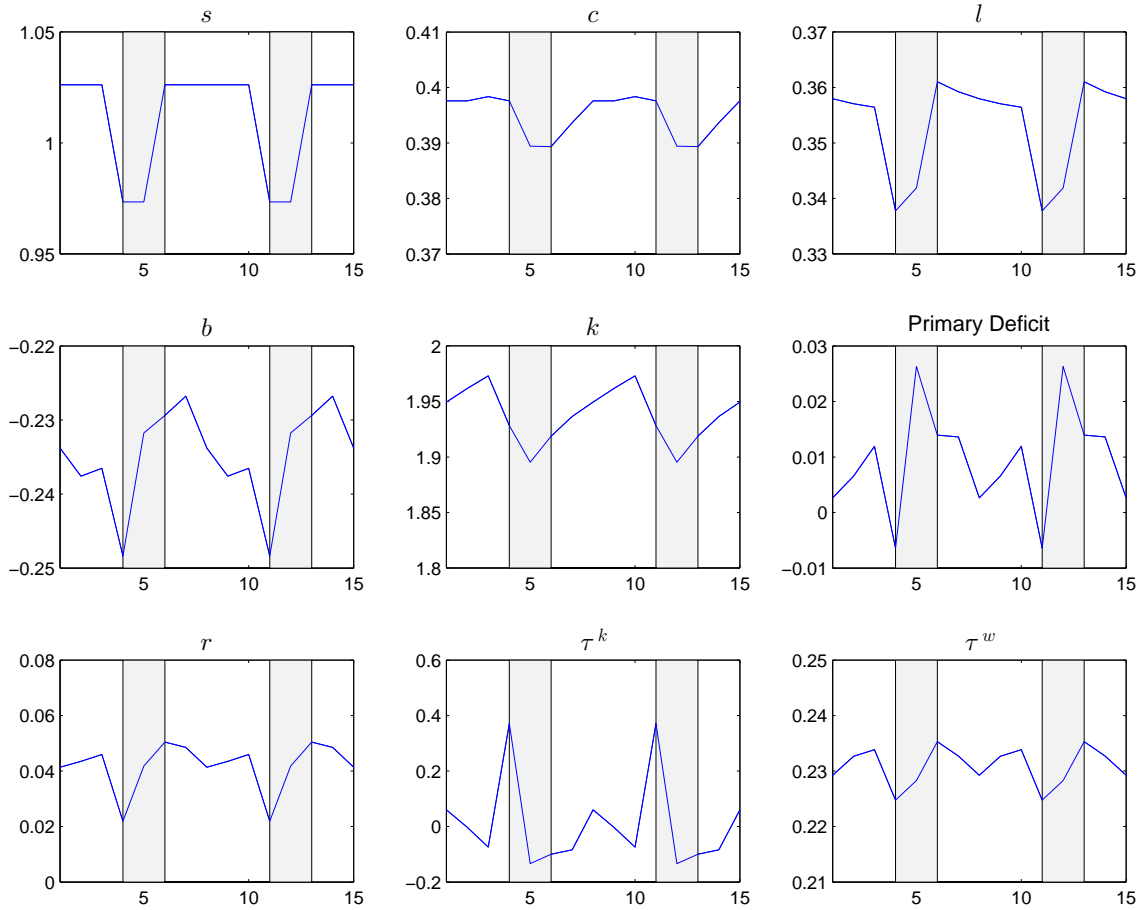
Notes: The parameterization underlying this figure is discussed in the text. Initial capital is set to 1.87 and initial debt is set to 0. *Primary deficit* refers to total tax revenues minus government spending; government debt (b) refer to the amount of debt issued; other variables are self-explanatory.

relatively high tax on capital income.

Moving to the behavior of the economy in the long run, Figure 3 displays the main variables of the economy for the last 1,000 periods of an 11,000 period simulation with random shocks. First note that while the capital income tax is highly volatile, it essentially varies between $\pm 40\%$.[19] The mean of the capital income tax is around zero, with a standard around 15%. A second interesting aspect of this long run simulation

---

[19]The capital income tax rate is outside of that range about 1% of the time: see Figure 4 below.

24

is that government debt is much more persistent that other variables. This reflects the fact that the amount of debt in the economy directly affects how distorted the economy needs to be, which is closely related to the multiplier λ discussed above (recall that λ contains a permanent component). The non-stationarity of government debt is perhaps most evident around period 650, during which an unusually large number of good shocks are realized. The trend of the labor income tax also reflects the fact that the level of distortions is highly persistent (recall that the capital income tax varies around zero). Finally, we note that the primary deficit is much less persistent than government debt, an empirical fact discussed at length in Marcet and Scott (2009).

The general pattern of tax movements over the business cycle is quite robust to the parametrization—in particular the increase (decrease) in the capital (labor) income tax at the outset of a recession. Table 1 shows some statistics for our benchmark economy in column 1 as well as an economy with tighter debt constraints ($\pm$ 20% of undistorted GDP) in column 2, an economy with longer recessions (5 years on average) in column 3, or both in column 4.[20] While these statistics are fairly robust across simulations, we note that the capital income tax tends to be more volatile either with longer recessions or under tighter debt limits. Intuitively, longer expected recessions tend to induce the government to finance more of it at the outset of a recession, especially when the government faces tight debt limits. Indeed, the histogram of the capital income tax for the same parameter configurations, displayed in Figure 4, confirms that either longer recessions or tighter debt limits tend to produce fatter tails than our benchmark economy. Finally, going back to Table 1, we also note that while the labor (capital) tax is always positively (negatively) correlated with productivity, debt is essentially acyclical, and extremely persistent. Nevertheless, the correlation between debt and the labor tax is fairly high (0.9), consistent with the results from Figure 2.

Finally, we investigate how the optimal fiscal policy changes when the economy is subject to shocks with higher volatility. The object of this exercise, which is similar in spirit to that found in Siu (2004), is to figure out which instrument, if any, becomes more relevant to absorb shocks as shocks become larger. Table 2 reports results with shocks that are 50% and 100% bigger than in the benchmark economy, which is

---

[20]The last column is meant to be comparable to results in Farhi (2010), who imposes a 20% debt limit and considers long recessions.

Figure 3: Long Run Simulation
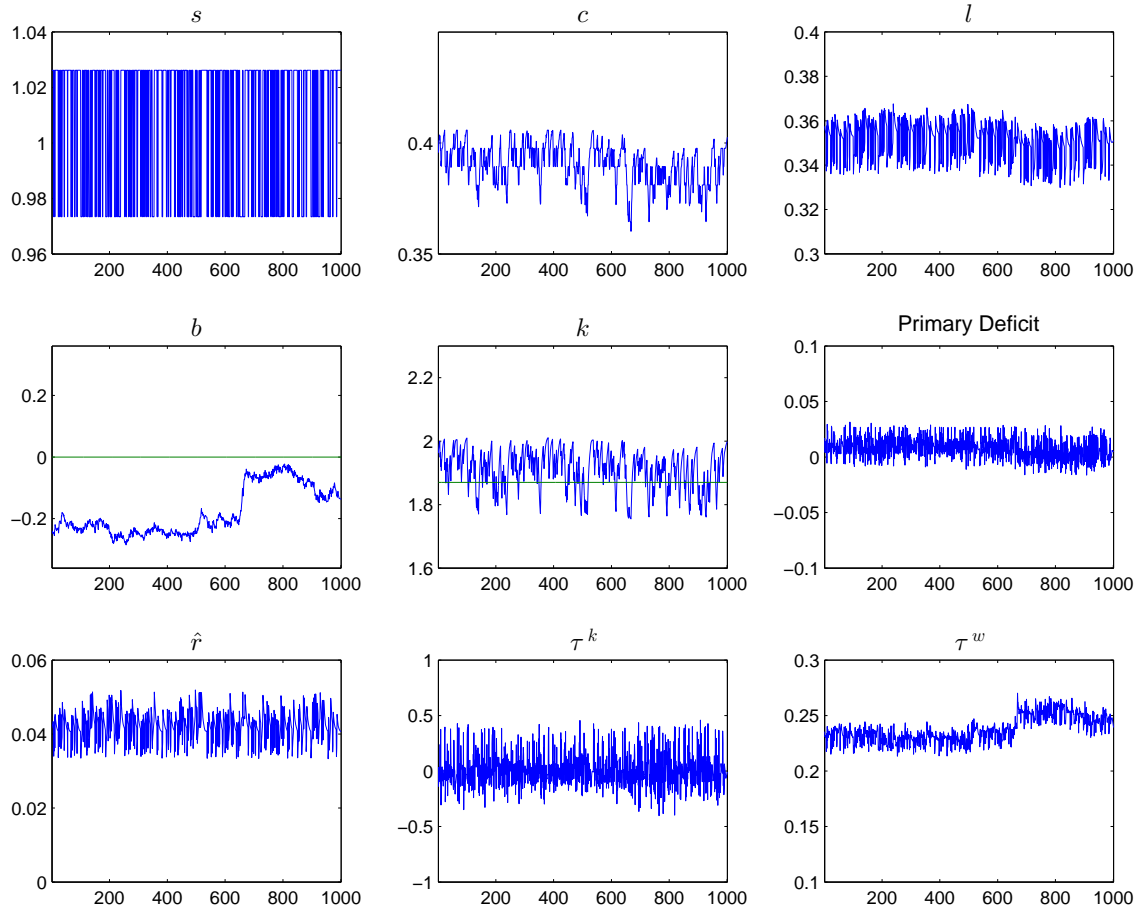
Notes: The parameterization underlying this figure is discussed in the text. Initial capital is set to 1.87 and initial debt is set to 0 (the green lines). *Primary deficit* means total tax revenues minus government spending; government debt (b) refer to the amount of debt issued; other variables are self-explanatory.

reproduced in the first column. This table shows, perhaps not surprisingly, that while mean tax rates (and debt level) do not change significantly with the size of shocks, their volatilities do. The second column of Table 2 shows that with 50% larger shocks, both the labor tax and debt become more or less 50% more volatile. By contrast, the volatility of the capital income tax only increases by about 20%. The same pattern emerges with even bigger shocks, as shown in the third column.

Table 1: Fiscal Policy Statistics

	Benchmark Economy	Tight Debt Limits	5 Year Recessions	Tight Debt + 5Y Rec.
Mean				
Labor Tax (%)	23.93	26.15	25.20	26.15
Capital Tax (%)	0.60	1.38	0.67	0.81
Debt	−0.16	0.02	−0.10	−0.02
Standard Deviation				
Labor Tax (%)	1.39	1.16	2.62	1.12
Capital Tax (%)	15.30	17.22	16.34	18.49
Debt	0.10	0.07	0.19	0.06
Autocorrelation				
Labor Tax	0.83	0.66	0.95	0.69
Capital Tax	−0.22	−0.26	−0.19	−0.22
Debt	0.99	1.00	1.00	0.98
Correlation with Productivity				
Labor Tax	0.31	0.47	0.09	0.38
Capital Tax	−0.58	−0.65	−0.53	−0.55
Debt	−0.02	0.05	−0.06	0.00

Notes: All statistics are from simulations of the model for 11,000 periods after which the first 1000 periods are dropped.

Figure 4: Histograms of Capital Income Tax Rates
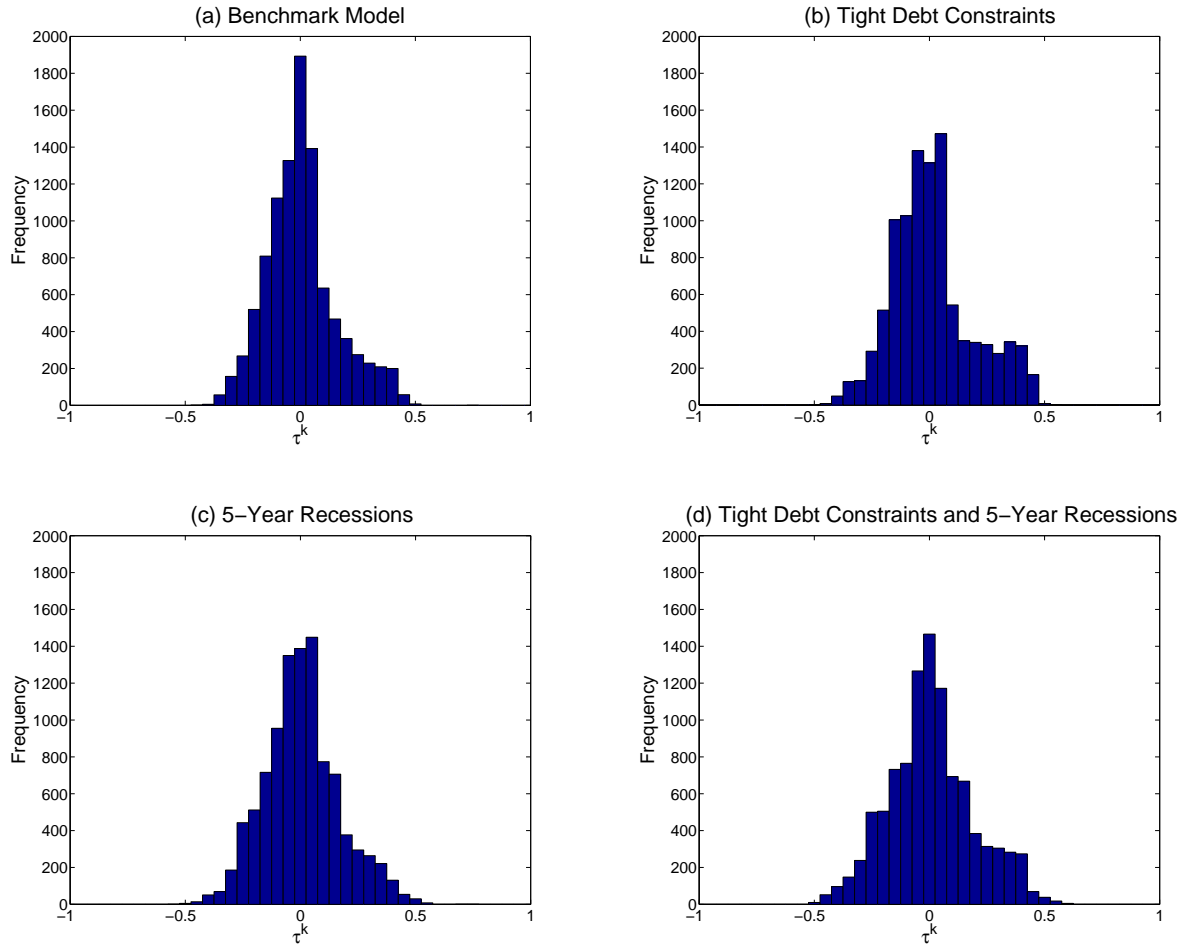
Notes: Histograms are from simulations of the model for 11,000 periods after which the first 1000 periods are dropped.

Table 2: Fiscal Policy Statistics with Bigger shocks

	Benchmark Economy	50% Bigger Shocks	100% Bigger Shocks
Mean			
Labor Tax (%)	23.93	24.19	24.31
Capital Tax (%)	0.60	1.50	2.34
Debt	−0.16	−0.13	−0.10
Standard Deviation			
Labor Tax (%)	1.39	2.00	2.60
Capital Tax (%)	15.30	18.55	25.18
Debt	0.10	0.15	0.18

Notes: All statistics are from simulations of the model for 11,000 periods after which the first 1000 periods are dropped.

6 Conclusion

This paper studies optimal fiscal policy in a neoclassical growth model in which investment becomes productive within the period. We argue that in the context of optimal taxation problems, this alternative timing is a useful assumption to avoid a perfectly inelastic supply of capital in the short run, which is at the heart of many results in the optimal taxation literature.

Our first result is that with an elastic supply of capital it is no longer optimal to confiscate initial asset holdings: the solution to the Ramsey problem features a unique non-trivial level of distortions without imposing exogenous bounds on tax instruments. A related result is that capital income taxes are no longer used as a shock absorber. However, state-contingent debt can be used for that purpose, leading to counterfactual movements between government debt and the primary deficit. This leads us to study a Ramsey problem without state-contingent debt, a typically hard problem which is considerably more tractable under our alternative timing assumption. The upshot of this problem is that the government runs debt-financed primary deficits during recessions.

A   Omitted Proofs

Proposition 2

Combining the first order conditions with respect to consumption (17) and labor (18) from the Ramsey problem and using (13), we get

$$-\frac{W_l(s^t)}{W_c(s^t)} = \hat{w}(s^t). \tag{32}$$

The derivatives W_c and W_l are given by

$$
\begin{aligned}
W_c(s^t) &= (1+\lambda)U_c(s^t) + \lambda U_c(s^t)H_c(s^t), \\
W_l(s^t) &= (1+\lambda)U_l(s^t) + \lambda U_l(s^t)H_l(s^t),
\end{aligned}
$$

where

$$
\begin{aligned}
H_c(s^t) &= \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}, \\
H_l(s^t) &= \frac{U_{l,c}(s^t)c(s^t) + U_{l,l}(s^t)l(s^t)}{U_l(s^t)}.
\end{aligned}
$$

Now pick two histories as of date t, s^t and $\tilde{s}^t$. From (32), it must be that

$$\frac{W_l(s^t)}{W_c(s^t)\hat{w}(s^t)} = \frac{W_l(\tilde{s}^t)}{W_c(\tilde{s}^t)\hat{w}(\tilde{s}^t)},$$

or, equivalently,

$$\frac{\big[1+\lambda+\lambda H_l(s^t)\big]U_l(s^t)}{\big[1+\lambda+\lambda H_c(s^t)\big]U_c(s^t)\hat{w}(s^t)} = \frac{\big[1+\lambda+\lambda H_l(\tilde{s}^t)\big]U_l(\tilde{s}^t)}{\big[1+\lambda+\lambda H_c(\tilde{s}^t)\big]U_c(\tilde{s}^t)\hat{w}(\tilde{s}^t)}.$$

Since the felicity function is separable, the functions H_c and H_l become

$$
\begin{aligned}
H_c(s^t) &= \frac{U_{c,c}(s^t)c(s^t)}{U_c(s^t)}, \\
H_l(s^t) &= \frac{U_{l,l}(s^t)l(s^t)}{U_l(s^t)}.
\end{aligned}
$$

And since the sub-utilities for consumption and labor are both from the constant elasticity of substitution class of utility functions, $H_c(s^t)$ and $H_l(s^t)$ are constants. Accordingly, the last expression reduces to

$$\frac{U_l(s^t)U_c(\tilde{s}^t)}{U_c(s^t)U_l(\tilde{s}^t)} = \frac{\hat{w}(s^t)}{\hat{w}(\tilde{s}^t)}.$$

31

But the first order conditions for consumption and labor from the household's problem (equations (3) and (4)) at histories s^t and $\tilde{s}^t$ imply

$$\frac{U_l(s^t)U_c(\tilde{s}^t)}{U_c(s^t)U_l(\tilde{s}^t)} = \frac{w(s^t)}{w(\tilde{s}^t)} = \frac{(1 - \tau^w(s^t))\hat{w}(s^t)}{(1 - \tau^w(\tilde{s}^t))\hat{w}(\tilde{s}^t)}.$$

For the last two equations to hold it must be the case that $\tau^w(s^t) = \tau^w(\tilde{s}^t)$.

Proposition 3

Recall that the first order conditions (3) and (5) from the households' problem imply that

$$(1 - r(s^t)) = \sum_{s_{t+1}} \frac{\beta \pi(s^{t+1})U_c(s^{t+1})}{\pi(s^t)U_c(s^t)}. \tag{33}$$

Similarly, combining first order conditions (17) and (19) from the Ramsey problem we have

$$\left[1 - (f_k(s^t) - \delta)\right] = (1 - \hat{r}(s^t)) = \sum_{s_{t+1}} \frac{\beta \pi(s^{t+1})W_c(s^{t+1})}{\pi(s^t)W_c(s^t)}. \tag{34}$$

But under a separable utility function and constant elasticity of substitution in consumption,

$$W_c(s^t) = (1 + \lambda + \lambda H^c(s^t))U_c(s^t) = (1 + \lambda - \lambda\sigma)U_c(s^t),$$

where σ is the inverse of the intertemporal elasticity of substitution. Hence we can replace W_c with U_c in equation (34). But then equations (33) and (34) can only hold if $\tau^k(s^t) = 0$.

B Recursive Formulation of the Ramsey Problem with Incomplete Markets

To derive a recursive formulation it is convenient to write the problem as follows

$$L(k_{-1}, b_{-1}) = \max_{\{c(s^t), l(s^t), k(s^t), b(s^t)\}_{t,s^t}} U\big(c(s_0), l(s_0)\big) + \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\big(c(s^t), l(s^t)\big) \quad (35)$$

subject to

$$U_c(s_0)c(s_0) + U_l(s_0)l(s_0) + \big(k(s_0) + b(s_0)\big) \sum_{s_1} \pi(s^1|s_0)U_c(s^1) - U_c(s_0)(k_{-1} + b_{-1}) = 0 \quad (36)$$

$$f\big(k(s_0), l(s_0), s_0\big) - \delta k(s_0) + k_{-1}) - c(s_0) - g(s_0) - k(s_0) = 0 \quad (37)$$

$$\underline{M} \le b(s_0) \le \overline{M} \quad (38)$$

$$U_c(s^t)c(s^t) + U_l(s^t)l(s^t) + \big(k(s^{t+1}) + b(s^{t+1})\big) \sum_{s_{t+1}} \pi(s^{t+1}|s^t)U_c(s^{t+1})$$
$$-U_c(s^t)(k(s^t) + b(s^t)) = 0 \quad (39)$$

$$f\big(k(s^t), l(s^t), s_t\big) - \delta k(s^t) + k(s^{t-1}) - c(s^t) - g(s^t) - k(s^t) = 0 \quad (40)$$

$$\underline{M} \le b(s^t) \le \overline{M} \quad (41)$$

where constraints (39)–(41) are imposed at $t = 1, 2, \dots$ and all s^t. This problem can be split into two parts as follows:

$$L(k_{-1}, b_{-1}) = \max_{c(s_0), l(s_0), k(s_0), b(s_0), c(s^1)_{s_1}} \left\{ \left[U\big(c(s_0), l(s_0)\big) \mid (36) - (38) \right] \right.$$

$$+ \max_{\{c(s^{t+1})_{s_{t+1}}, l(s^t), k(s^t), b(s^t)\}_{t=1,s^t}^{\infty}} \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[U\big(c(s^t), l(s^t)\big) \mid (39) - (41) \right] \right\} \quad (42)$$

Now the second part of the problem can be written recursively given state variables c, k, b and s, as in equation (30), and the problem from date 0 can then be expressed as equation (31) in the main text.

References

Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppälä (2002). Optimal taxation without state-contingent debt. *Journal of Political Economy 110*(6), 1220–1254.

Barro, R. (1979). On the determination of public debt. *Journal of Political Economy 87*(5, Part 1), 940–971.

Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica 54*(3), 607–622.

Chari, V. V., L. J. Christiano, and P. J. Kehoe (1994). Optimal fiscal policy in a business cycle model. *Journal of Political Economy 102*(4), 617–652.

Chari, V. V. and P. J. Kehoe (1999). Optimal fiscal and monetary policy. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1C of *Handbooks in Economics*, Chapter 26. North-Holland: Elsevier Science, North-Holland.

Erosa, A. and M. Gervais (2001). Optimal taxation in infinitely-lived agent and overlapping generations models: A review. *Federal Reserve Bank of Richmond Economic Quarterly 87*(2), 23–44.

Farhi, E. (2010). Capital taxation and ownership when markets are incomplete. *Journal of Political Economy 118*(5), 908–948.

Fisher, J. D. and M. Gervais (2011). Why has home ownership fallen among the young? *International Economic Review 52*(3), 883–912.

Kiyotaki, N., A. Michaelides, and K. Nikolov (2011). Winners and losers in housing markets. *Journal of Money, Credit and Banking 43*(2–3), 255–296.

Lucas, Jr., R. E. (1980). Equilibrium in a pure currecy economy. In Karaken and Wallace (Eds.), *Models of Monetary Economies*.

Marcet, A. and A. Scott (2009). Debt and deficit fluctuations and the structure of bond markets. *Journal of Economic Theory 144*, 473–501.

Martin, F. M. (2010). Markov-perfect capital and labor taxes. *Journal of Economic Dynamics and Control 34*(3), 503–521.

Nicolini, J. P. (1998). More on the time consistency of monetary policy. *Journal of Monetary Economics 41*(2), 333–350.

Scott, A. (2007). Optimal taxation and OECD labor taxes. *Journal of Monetary Economics 54*, 925–944.

Siu, H. E. (2004). Optimal fiscal and monetary policy with sticky prices. *Journal of Monetary Economics 51*, 575–607.

Svensson, L. E. O. (1985). Money and asset prices in a cash-in-advance economy. *Journal of Political Economy 93*(5), 919–944.

Zhu, X. (1992). Optimal fiscal policy in a stochastic growth model. *Journal of Economic Theory 58*(2), 250–289.

Zhu, X. (1995). Endogenous capital utilization, investor's effort and optimal fiscal policy. *Journal of Monetary Economics 36*(3), 655–677.