Technological Learning and Labor Market Dynamics*

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January 25, 2014

Abstract

The search-and-matching model of the labor market fails to match two important business cycle facts: (i) a high volatility of unemployment relative to labor productivity, and (ii) a mild correlation between these two variables. We address these shortcomings by focusing on *technological learning-by-doing*: the notion that it takes workers time using a technology before reaching their full productive potential with it. We consider a novel source of business cycles, namely, fluctuations in the speed of technological learning and show that a search-and-matching model featuring such shocks can account for both facts. Moreover, our model provides a new interpretation of recently discussed "news shocks."

Journal of Economic Literature Classification Numbers: E24; E32; J63; J64
Keywords: Unemployment Volatility; Unemployment-Productivity Correlation; Technological Learning-by-Doing; News Shocks

*Manuscript received May 2013; revised December 2013.

[1] We thank Manuel Amador, David Andolfatto, Gadi Barlevy, Paul Beaudry, Aspen Gorry, Jean-Olivier Hairault, Andreas Hornstein, Patrick Kehoe, Richard Rogerson, Peter Rupert, Martin Schneider, Shouyong Shi, Robert Shimer, and workshop participants at Bocconi, CREI, LSE, Minneapolis Fed, Princeton, Richmond Fed, Tel Aviv, UC Santa Cruz, the 2011 SED Meeting, the 2011 Duke Macroeconomics Conference, the 2011 CEPREMAP Labor Market Workshop, the 2011 Vienna Macro Workshop, as well as referees and the Editor, Guido Menzio, for helpful comments. Siu and Yedid-Levi thank the Social Sciences and Humanities Research Council of Canada for support.

1 Introduction

A long-standing challenge in macroeconomics is accounting for labor market dynamics over the business cycle. This challenge is particularly acute in the seminal model of equilibrium unemployment due to Pissarides (1985) which, given related contributions by Diamond (1982) and Mortensen (1982), we hereafter refer to as the DMP framework. When applied to business cycles analysis through the introduction of stochastic shocks to technology, the model features two primary shortcomings.

First, as discussed in Andolfatto (1996), Shimer (2005a), Hall (2005), and Costain and Reiter (2008), the model fails to generate sufficient volatility of unemployment relative to that of labor productivity, in comparison to postwar U.S. data. This discrepancy indicates that the textbook DMP model embodies weak amplification of technology shocks into unemployment fluctuations, and has been referred to as the *unemployment volatility puzzle* in the literature.[2] The second shortcoming relates to the correlation between unemployment and labor productivity. In U.S. data, these two variables are only mildly negatively correlated, whereas in the model, the correlation is near minus one. We refer to this as the *unemployment-productivity correlation puzzle*. Mortensen and Nagypál (2007) argue that this discrepancy points to the omission of an important driving force in the cyclical analysis of the DMP framework.

Within this context, in Sections 2 and 3 we study a very simple search-and-matching model of the labor market that addresses both shortcomings. As in much of the literature, our model retains an important relationship between technological change and the business cycle. However, we depart by focusing on *technological learning-by-doing*: the notion that it takes workers time using a technology before reaching their full productive potential with

[2] Versions of the DMP framework where fluctuations are driven by demand shocks can naturally generate large unemployment fluctuations without any changes in productivity. See, for instance, Diamond (1982) and Kaplan and Menzio (2013).

it. Indeed, the idea that technology is subject to a learning process is well documented.[3]

The novel aspect of our framework is to model the nature of technology arrival to be stochastic in terms of its *ease* of learning-by-doing. That is, innovations are not uniform in the amount of learning time required for workers to become fully productive.[4] We embed this idea in a simple search-and-matching framework where workers have the ability to increase their proficiency through learning-by-doing. Periods in which technologies are easier to learn or more "user-friendly" imply an acceleration in the rate of technological learning, resulting in an increase in the rate of aggregate labor productivity growth. The arrival of technologies that are harder to learn generate a period of falling productivity growth.

We then study the quantitative predictions of our model in Section 4, where we show that it generates much greater volatility of unemployment relative to labor productivity, in comparison to the standard DMP framework. This is due to the fact that the volatility of the job finding rate, relative to that of labor productivity, is very close to that observed in the U.S. data. Moreover, the model delivers a correlation of unemployment and productivity that is much smaller (in absolute value) than one, and indeed, is very close to that observed in the data. To understand these results note that in the search-and-matching framework, firms create job vacancies in expectation of future profit flows when matched with a worker. In our model, potential profit gains that are due to learning-by-doing constitute an important component of this profit flow. Hence, shocks to the ease of learning generate pronounced fluctuations in expected future profit. This in turn generates fluctuations in job creation and unemployment. However, these shocks have only an indirect effect

[3] The literature documenting technological learning-by-doing has a long history in management and economics. See Wright (1936) for an early study of learning-by-doing in airplane manufacturing. For recent examples, see Argote and Epple (1990), Irwin and Klenow (1994), and the references therein.

[4] Again, the literature documenting variation in learning rates for new technologies is too vast to summarize completely; see, for instance, Argote and Epple (1990) and Balasubramanian and Lieberman (2010).

on aggregate productivity as workers gain technological proficiency. This enables the model to address the unemployment volatility puzzle. Furthermore, shocks to the learning rate of technology naturally break the tight contemporaneous correlation between unemployment and productivity, thereby addressing the correlation puzzle. To summarize, learning rate shocks generate an immediate response of job creation and unemployment; but given that the learning process takes time, the impact on labor productivity is persistent and cumulative.

In this respect, our analysis is related to the recent literature on "news shocks." Specifically, in Subsection 4.3 we relate our findings to Beaudry and Portier (2006) who find that a substantial fraction of business cycle variation in U.S. data can be attributed to shocks to long-run TFP that: (i) have immediate effects on measures such as consumption, employment, and stock market value; but (ii) have effectively no effect on TFP upon impact; instead the effect on productivity is persistent, observed over a horizon of at least 8 to 10 quarters. Beaudry and Portier refer to these as "news shocks" since they signal changes in future productivity, and find that such shocks account for at least half of the variation in hours worked at business cycle frequencies.[5]

In our model, shocks to the learning rate generate dynamic responses in unemployment, productivity and stock market value that conform with the responses to *empirically identified* news shocks. But importantly, our shock is not a news shock as *modeled* in the recent literature—that is, as signals of conventional technology shocks that are to arrive in the future.[6] Indeed, we show that conventionally modeled news shocks do not resolve either of the two shortcomings in the DMP framework. In contrast, (positive) shocks to the learning rate represent the current arrival of innovations that are easier to implement; nonetheless,

[5]See also Beaudry and Lucke (2010) and Schmitt-Grohe and Uribe (2012) who find similar results.

[6]See, for instance, Beaudry and Portier (2007), Jaimovich and Rebelo (2009), and den Haan and Kaltenbrunner (2009). See also Comin et al. (2009) who study a model in which economic activity devoted to technology adoption varies in response to the stochastic arrival rate of "frontier" technologies.

the process of learning-by-doing must still be undertaken. Naturally, and precisely because the dynamics of our model resemble those of empirically identified news shocks, unemployment in the model is bound to lead labor productivity, which suggests that other shocks must be important to explain why labor productivity leads unemployment by about two quarters in U.S. data.

At its core, our model emphasizes the idea that workers gain technological proficiency while on the job, and that the rate at which this happens varies over the business cycle. Hence, our model puts emphasis on the return to labor market experience and its cyclical properties. In Subsection 4.4, we relate our finding with those in French et al. (2006) who provide evidence for the procyclicality of the return to experience.

Our work is also related to a growing body of research studying the cyclical implications of the DMP framework. Most of this work addresses the unemployment volatility puzzle.[7] By maintaining the assumption of a technology shock-driven cycle, these papers do not make progress on the unemployment-productivity correlation puzzle.[8] By focusing on an alternative interpretation of technological change over the cycle—shocks to the ease of learning—we show that the DMP framework is able to generate substantial volatility of unemployment relative to productivity, and to deliver a muted correlation between the two

[7] The literature is too vast to provide a complete summary; see, for instance, Shimer (2004), Hall (2005), Hall and Milgrom (2008), Pries (2008), Gertler and Trigari (2009), and Menzio and Shi (2011). Hagedorn and Manovskii (2008) find that, for specific calibrations, the DMP model does not suffer from a volatility puzzle. For discussion, see Hornstein et al. (2005, 2007), Mortensen and Nagypál (2007), Reiter (2007), Costain and Reiter (2008), van Rens et al. (2008), Pissarides (2009), Eyigungor (2010), and Brügemann and Moscarini (2010).

[8] In the RBC literature, a similar puzzle exists regarding the correlation between hours worked and the real wage. Early papers addressed this by introducing shocks to labor supply (see, for example, Benhabib et al. (1991) and Christiano and Eichenbaum (1992)); unfortunately, empirical evidence for the relevance of these shocks in accounting for postwar business cycles is limited. The only paper in the search framework to address this puzzle is Hagedorn and Manovskii (2010), who follow Benhabib et al. (1991) by introducing home production/preference shocks to the DMP model.

variables.

2 Economic Environment

We study a search-and-matching model of the labor market. The matching process between unemployed workers and vacancy posting firms is subject to a search friction. The ratio of vacancies to unemployed determines the economy's match probabilities. Workers are heterogeneous in their *proficiency* with production technology. For simplicity, we assume that there are a finite number, $N \geq 2$, of proficiency levels that a worker can progress through over her lifetime. The worker's proficiency is reflected in the output in a worker-firm match. The process of improving one's proficiency with technology takes time and, crucially, occurs only while employed as emphasized in the learning-by-doing literature. Within this environment, the novel aspect of our analysis is to consider how fluctuations in the ease of learning drive the business cycle.

To isolate the role of technological learning-by-doing, we make the following assumptions on innovations to the production technology. Innovations arrive in each and every period. The cost of adopting the innovation is sufficiently small that, upon arrival, all worker-firm matches find it advantageous to adopt. Each innovation differs from the previous period's along two dimensions. First, the innovation increases the *efficiency* (and, hence, productivity when matched) of all workers by a growth factor g. This efficiency gain affects all matched workers, irrespective of their profiency level. For simplicity, we normalize $g = 0$, and focus on a model where productivity is stationary in the long-run (has no growth trend).

Secondly, innovations do not alter the proficiency level of workers *per se*; however, innovations differ in the *rate* at which workers gain technological proficiency and advance through the proficiency "ladder." This is the novel stochastic element in our analysis.

For the sake of expositional simplicity in this section, we set $N = 2$ so that a worker's proficiency is either low or high. As such, becoming fully proficient with technology is represented as a one-time level shift: a jump in the worker's proficiency from low to high.[9] As a simple example, consider the case of a worker in an office setting, where the current mode of production requires the use of personal computing technology. Full proficiency with the technology requires a complete understanding how to use the PC's operating system. For workers without full proficiency, attaining this level of understanding requires time using the PC while employed; this is represented by a hazard rate, $\lambda \in (0, 1]$, which we refer to as the *learning rate*. An employed worker currently without full proficiency in the PC technology produces output f_L. With probability λ she "figures out" the technology; in the next period, she produces $f_H > f_L$. With probability $(1 - \lambda)$ she remains at less than full proficiency, producing f_L.

Our analysis focuses on shocks to the learning rate, λ—shocks to the ease at which technology can be learned. Returning to our example, consider a PC operating system such as Microsoft Windows. Between periods $t - 1$ and t, the innovation that arrives is an update from Windows 1.1 to (say, a faster version) Windows 1.2. This innovation increases the efficiency of all workers by a factor g.[10] However, versions 1.1 and 1.2 are identical in use and the ease of learning, so that the learning rate remains at λ. In period $t + 1$, the innovation that arrives is an update from Windows 1.2 to Windows 2.0. Again, this update increases efficiency by g. However, because the new version is more "user-friendly," it increases the rate at which workers become fully proficient with the technology. In this case, innovation would represent a positive shock to the learning rate, $\lambda' > \lambda$.

[9] When we explore the quantitative predictions of the model in Section 4, we consider a more realistic environment with $N > 2$ so that becoming fully proficient is a more gradual process for the worker. See also the discussion in Section 2.6.

[10] Again, recall that we normalize $g = 0$ for simplicity.

2.1 Market Tightness

A worker's proficiency or *type* is perfectly observable. Accordingly, a firm can maintain a vacancy for workers of either type, $i \in \{L, H\}$. The cost of maintaining a vacancy for either type is κ. There is free entry into vacancy posting on the part of firms. We define market tightness in market i, θ_i, as the ratio of the number of vacancies maintained by firms to the number of workers looking for jobs of type i. While the tightness of each market is an equilibrium object, they are taken parametrically by firms and workers.

We denote the probability that a worker will meet a vacant job in market i by $p(\theta_i)$, where $p : \mathcal{R}_+ \to [0, 1]$ is a strictly increasing function with $p(0) = 0$. Similarly, we let $q(\theta_i)$ denote the probability that a firm with a vacancy meets a worker in market i, where $q : \mathcal{R}_+ \to [0, 1]$ is a strictly decreasing function with $q(\theta) \to 1$ as $\theta \to 0$, and $q(\theta) = p(\theta)/\theta$.

Allowing for market segmentation across low and high type workers is useful for a number of reasons. As will become clear, it affords analytical and computational tractability, as equilibrium is *block recursive* in the sense that agents' value functions and decision rules are independent of the distribution of workers across types and employment status (see Shi (2009) and Menzio and Shi (2010)).[11] In addition, it makes the economic mechanism transparent, highlighting the role of learning rate shocks on the incentive for job creation.

2.2 Contractual Arrangement and Timing

We specify the compensation in a match as being determined via Nash bargaining with fixed bargaining weights, as in Pissarides (1985). As such, our results do not rely on mechanisms that change the relative bargaining power of workers and firms over the cycle.[12]

[11] If we did not allow for segmented markets, the qualitative implications of technological learning would be preserved. However, the computation of equilibrium with aggregate uncertainty would be extraneously burdensome, because of the need to track the distribution of types in unemployment.

[12] See, for instance, Hall and Milgrom (2008) and Gertler and Trigari (2009).

When an unemployed worker and a firm match, they begin producing output in the following period. In all periods that a worker and firm are matched, the compensation is bargained with complete knowledge of the worker's proficiency. We let ω_i denote the compensation of a type i worker.

2.3 Technological Learning in Worker-Firm Matches

We define U_L as the value of being unemployed for a low proficiency worker:

$$U_L = z + \beta E\Big[p(\theta_L)W_L' + (1 - p(\theta_L))U_L'\Big], \tag{1}$$

where expectations are taken over tomorrow's learning rate. Here, z is the flow value of unemployment, W_L is the worker's value of being employed in a match, and primes ($'$) denote variables one period in the future. An unemployed worker transits to employment in the following period with probability $p(\theta_L)$; we refer to this as the job finding probability.

The value of being employed for a low proficiency worker is:

$$W_L = \omega_L + \beta E\Big[\lambda\left[(1-\delta)W_H' + \delta U_H'\right] + (1-\lambda)\left[(1-\delta)W_L' + \delta U_L'\right]\Big]. \tag{2}$$

During the period, the employed worker becomes proficient with probability λ, which is stochastic. At the end of the period, the match is separated with (exogenous and constant) probability $\delta \in (0, 1]$. In the case where the worker figures out the technology but is separated from her match, she enters the next period with value U_H. That is, the proficiency that the worker acquires on-the-job is retained when unemployed and can be applied to future matches. In this sense, the technology being learned is not firm- or match-specific. Note also that learning happens only when a worker is matched. That is, since technological proficiency is acquired though learning-by-doing, the worker cannot transit from type L to H while unemployed.

There is a large number of firms that can potentially maintain vacancies, as long as they pay the cost, κ. The value of maintaining a vacancy for low proficiency workers is:

$$V_L = -\kappa + \beta E\left[q(\theta_L)J'_L + (1-q(\theta_L))\max_j\left(V'_j, 0\right)\right], \tag{3}$$

where $q(\theta_L)$ denotes the firm's job filling probability. The maximization within the expectation term implies that firms who do not find a worker may choose to maintain a vacancy in either market, or be inactive in the following period. The firm's value of being matched with a type L worker is:

$$J_L = f_L - \omega_L + \beta E\Big[(1-\delta)\left[\lambda J'_H + (1-\lambda)J'_L\right] + \delta\max_j\left(V'_j, 0\right)\Big]. \tag{4}$$

This value is composed of the contemporaneous profit—output minus the worker's compensation—plus the expected discounted value from next period on. This latter part (conditional on the match surviving) consists of the value of being in a match of type H which occurs with probability λ, or being in a type L match with complementary probability.

2.4 High Proficiency Workers

To close the model description, we present the value functions associated with high proficiency workers:

$$U_H = z + \beta E\Big[p(\theta_H)W'_H + (1-p(\theta_H))U'_H\Big], \tag{5}$$

$$W_H = \omega_H + \beta E\Big[(1-\delta)W'_H + \delta U'_H\Big]. \tag{6}$$

A worker of type H transits from unemployment to employment with probability $p(\theta_H)$, and transits from employment to unemployment with probability δ.

Again, a large number of inactive firms can potentially maintain vacancies for type H workers. The value of maintaining such a vacancy is:

$$V_H = -\kappa + \beta E\left[q(\theta_H)J'_H + (1-q(\theta_H))\max_j\left(V'_j, 0\right)\right]. \tag{7}$$

Finally, J_H is simply the expected discounted value of flow profits:

$$J_H = f_H - \omega_H + \beta E\Big[(1-\delta)J'_H + \delta \max_j \big(V'_j, 0\big)\Big]. \tag{8}$$

Note that the type H market is identical to the standard DMP model.

2.5 Defining Equilibrium

An equilibrium with Nash bargaining is a collection of value functions, V_L, J_L, V_H, J_H, U_L, W_L, U_H, W_H, compensations, ω_L, ω_H, and tightness ratios, θ_L, θ_H, such that:

1. Workers are optimizing. That is, workers that are matched prefer to remain matched rather than be unemployed, $W_L > U_L, W_H > U_H$, and workers prefer to be of high proficiency as opposed to low proficiency, $W_H > W_L, U_H > U_L$.

2. Firms are optimizing. That is, the value of maintaining a vacancy is equalized across markets and is no less than the value of remaining idle, $V_L = V_H \equiv V \geq 0$, and firms that are matched must prefer to remain matched as opposed to maintaining a vacancy, $J_L, J_H > V$.

3. Compensations solve the Nash bargaining problems:

$$\omega_i = \arg\max \, (W_i - U_i)^{\tau}(J_i - V)^{1-\tau},$$

for $i \in \{L, H\}$, where τ denotes the bargaining weight of workers.

4. The free entry condition is satisfied; that is, $V = 0$.

Nash bargaining prescribes a very simple relationship between the worker's surplus, $W_i - U_i$, and firm's surplus, J_i, in a match. Let S denote the total surplus from a match:

$$S_i = W_i + J_i - U_i, \quad i \in \{L, H\}.$$

Under Nash bargaining, the worker and firm receive a constant, proportional share of the total surplus, $W_i - U_i = \tau S_i$ and $J_i = (1-\tau)S_i$.

2.6 Discussion

The model has been kept simple for exposition. In particular, we have modeled only two types of workers. Type L workers represent labor force members who have the potential to upgrade their proficiency via learning-by-doing. Type H workers are those who no longer have the ability to do so.

In our analysis, we focus attention on type L workers. This represents our presumption that, in reality, most workers have the ability to increase their proficiency while on the job. In this sense, the presence of type H workers represents an analytical device, allowing us to specify a well-defined dynamic problem for type L workers, those who represent the majority of labor force members in the economy.

However, our model naturally introduces a source of heterogeneity relative to the standard DMP model. That is, while the standard model features heterogeneity in the employment status of workers (of the same proficiency), those in our model are also distinguished by the scope of their upgrade potential. Given this, it is interesting to consider the implications of this heterogeneity in a richer framework. We do this by extending the model to include N worker types, where $N > 2$. This is done in Section 4, where we explore the quantitative properties of our model.

In our exposition, we have specified compensation as being determined by Nash bargaining. However, our results do not rely on this assumption. For example, the model's implications are identical to a version with wage posting on the part of firms, as in the competitive search framework; this is true when Hosios (1990)'s condition is met. Furthermore, when the Hosios condition is met, the model's equilibrium is efficient. We refer the reader to Appendix A.2 for details.

Note also that our model nests the standard DMP model in two cases. The first is when there is no difference in productivity across worker types, i.e., $f_L = f_H$. In this case, upgrading is meaningless, and our model collapses to the standard one. Alternatively, when $\lambda = 0$ there is no scope for proficiency upgrading for type L workers. In this case, the model features two unrelated labor markets, each of which behaves identically to the standard DMP model.

Finally, we note that while workers have the potential for proficiency upgrading, they face no chance of downgrading. Hence, our model would feature a degenerate distribution of worker types in steady state, with all workers being of type H. To address this, we introduce an exogenous probability of death: in each period, all workers (regardless of employment status or proficiency) die with probability ϕ. These workers are "re-born" to keep the measure of workers constant, in a manner to preserve a non-degenerate distribution; we elaborate on this in Subsection 4.1 where we discuss the calibration of our model with $N > 2$ worker types. As such, the discount factor, β, represents a composite of a true subjective discount factor and a survival probability, $1 - \phi$.

3 Analytical Results

In this section, we provide analytical results characterizing some key properties of our model. We begin by characterizing the model's steady state equilibrium. We then discuss the key differences between our model and the standard DMP model, and their implications for business cycle fluctuations.

3.1 Characterizing Steady State

Our analysis begins with a useful lemma.

Lemma 1 *In any steady state equilibrium, if $U_L < U_H$, then $\theta_L < \theta_H$.*

Proof. Subtracting the steady state value of being unemployed for type L, equation (1), from that of type H, equation (5), we have

$$U_H - U_L = \frac{1}{1-\beta}\Big[\beta p(\theta_H)(W_H - U_H) - \beta p(\theta_L)(W_L - U_L)\Big].$$

Using the free entry condition $V_L = V_H = 0$ together with the value of maintaining a vacancy for low, equation (3), and high proficiency workers, equation (7), it follows that

$$U_H - U_L = \frac{\tau\kappa(\theta_H - \theta_L)}{(1-\tau)(1-\beta)}.$$

■

A number of results follow immediately from this lemma, collected in the following corollary:

Corollary 2 *If $U_L < U_H$, then it follows that $p(\theta_H) > p(\theta_L)$, $q(\theta_H) < q(\theta_L)$, $J_H > J_L$, $S_H > S_L$, $W_H - U_H > W_L - U_L$, and $W_H > W_L$.*

The next proposition establishes that the steady state value of unemployment is increasing in type.

Proposition 3 *In any steady state equilibrium, $U_L < U_H$.*

Proof. The proof is by contradiction. Suppose $U_L > U_H$; from Lemma 1, this implies that $\theta_L > \theta_H$, so that $q(\theta_L) < q(\theta_H)$. From the free entry condition, this implies $J_H < J_L$. From Nash bargaining, this implies $S_H < S_L$. Total surplus is given by:

$$S_H = f_H + \beta\Big[(1-\delta)S'_H + U'_H\Big] - U_H,$$

$$S_L = f_L + \beta\Big[(1-\delta)\left[\lambda S'_H + (1-\lambda)S'_L\right] + \lambda U'_H + (1-\lambda)U'_L\Big] - U_L.$$

Using the fact that $S'_i = S_i$ and $U'_i = U_i$ in steady state, and gathering terms:

$$[1-\beta(1-\delta)(1-\lambda)](S_L - S_H) = f_L - f_H + [\beta(1-\lambda) - 1]\,(U_L - U_H). \tag{9}$$

Both terms on the LHS are positive (the first by construction, the second by assumption; $(f_L - f_H) < 0$; $[\beta(1-\lambda) - 1] < 0$ and $(U_L - U_H) > 0$ by assumption; therefore, the RHS is negative. This is a contradiction. ∎

This result is important for a number of reasons. It ensures that in steady state equilibrium, the assumptions implicit in the model exposition are verified. In particular, a high proficiency unemployed worker would prefer to maintain her type, as opposed to reverting to low proficiency (see Subsection 2.5). From Corollary 2, it also ensures that an employed worker would prefer to be of high rather than low proficiency.

More importantly, it allows us to understand the incentives for job creation in our model relative to the standard model. In steady state, the free entry condition of the standard DMP model can be expressed as:

$$\kappa = q(\theta_{DMP})\beta(1-\tau)S_{DMP},$$

where the subscript DMP refers to the standard model. Hence, the number of vacancies firms post per unemployed worker, θ, depends on the profit conditional on being matched, $\beta(1-\tau)S$, which is proportional to total surplus. Total surplus in the standard model can be expressed as:

$$S_{DMP} = f - z - \bar{\tau}\kappa\theta_{DMP} + \beta(1-\delta)S_{DMP},$$

where $\bar{\tau} \equiv \tau/(1-\tau)$. In words, the total surplus from a match consists of a contemporaneous surplus plus its continuation value; the contemporaneous surplus is the output from the match (f), net of the foregone flow value (z) and option value $(\bar{\tau}\kappa\theta_{DMP})$ of unemployment.

In our model, the analogous free entry condition must hold in each market:

$$\kappa = q(\theta_i)\beta(1-\tau)S_i, \quad i \in \{L, H\}. \tag{10}$$

Total surplus for type H matches is identical to the standard DMP model. This is not the case for type L matches. In the market for workers with the possibility of learning:

$$S_L = f_L - z - \bar{\tau}\kappa\theta_L + \beta(1-\delta)S_L + \underbrace{\lambda\beta\left[(1-\delta)(S_H - S_L) + (U_H - U_L)\right]}_{\equiv\Delta}. \tag{11}$$

Relative to the standard model, the total surplus of a type L match involves the additional term, Δ, which we refer to as the *value of learning*. This reflects a capital gain due to the fact that technological learning may occur when a worker and firm are matched.

Conditional on learning, there is an upgrade to a high productivity match in the next period. Hence, the total surplus includes the change in the worker's and firm's values, weighted by β and λ. With probability $(1-\delta)$ the match survives, so that learning reflects a change in the matched value of both the worker and the firm. With probability δ the match is separated, and the learning is reflected only in a change in the unemployed worker's value. Hence:

$$\begin{aligned}\Delta &= \lambda\beta\left[(1-\delta)(W_H - W_L + J_H - J_L) + \delta(U_H - U_L)\right] \\ &= \lambda\beta\left[(1-\delta)(S_H - S_L) + (U_H - U_L)\right].\end{aligned}$$

Moreover, given Proposition 3 and Corollary 2, $U_H - U_L > 0$ and $S_H - S_L > 0$, so that the value of learning is positive, $\Delta > 0$.

3.2 Comparative Statics

In this subsection, we provide comparative statics results for the model's steady state. These are useful in providing insight into the business cycle properties of our model.

The equations governing equilibrium job creation—more specifically, market tightness—are the free entry conditions. From (10), it is clear that the response of market tightness to a shock depends on the response of total surplus. Intuitively, a shock that causes total surplus to rise implies a rise in the flow of profits to a matched firm. Since there is free entry, firms respond by creating more vacancies per unemployed worker. Job creation occurs until the point where the rise in profit is offset by the fall in the probability that any given vacancy is filled.

The next proposition relates to the model's response to changes in the learning rate. Consider total surplus in type L matches with the possibility of learning. From equation (11), it is clear that the effect of a change in λ on total surplus operates through its influence on the value of learning, Δ. We first establish that S_L is increasing in the learning rate.

Proposition 4 *A rise (fall) in λ causes S_L to rise (fall).*

The proof is provided in the Appendix. The intuition is straightforward. An increase in the learning rate increases the value of learning, $\Delta > 0$, which is positive (see the previous subsection). As the probability that the match upgrades from low to high proficiency increases, the expected profit rises as well; that is, the "upside risk" of the match has improved. This implies an increase in total surplus. Via free entry, this causes a rise in θ_L: job creation of matches with the possibility of upgrading rises. This steady state result extends to the stochastic environment that we study in Section 4: a positive shock to λ causes θ_L and job creation to rise, and vice-versa.

It is also straightforward to see that total surplus in high productivity matches is unaffected by shocks to the learning rate. As discussed in Section 2, the H-type market is simply a standard DMP model, and independent of the type L market. Hence, job creation in this market is unresponsive to shocks to the learning rate, λ.

4 Numerical Results

In this section, we provide numerical results for our model. Subsection 4.1 discusses the calibration of the model. In subsections 4.2 and 4.3.1, we present business cycle statistics for the postwar U.S. economy and demonstrate that the standard DMP model does poorly in replicating them. Subsection 4.3.2 presents the results for our model, where we illustrate why our model improves upon the standard DMP model in terms of the volatility of unemployment relative to that of productivity, as well as the correlation between these two variables.

4.1 Calibration

As discussed in Section 2.6, in our numerical analysis we consider the implications of the model with richer heterogeneity, i.e. with $N > 2$ proficiency levels or types. In terms of notation, let f_i denote the level of output produced in a match between a firm and worker of type i, where $i = 1, \ldots, N$. For simplicity, we assume that a worker increases her proficiency by one level at a time, with λ still denoting the probability, while matched, that a worker of type $i < N$ upgrades to type $i + 1$. All other aspects of the model remain unchanged.

The characterization of the highest proficiency, type N, market is identical to that of the high proficiency market summarized by the value functions (5)–(8) (obviously, with the H-subscripts replaced by N's). The value functions for all other workers and firms with the potential for technological learning (types $i = 1, \ldots, N - 1$) are given by:

$$U_i = z + \beta E\Big[p(\theta_i)W_i' + (1 - p(\theta_i))U_i'\Big], \tag{12}$$

$$W_i = \omega_i + \beta E\Big[\lambda\left[(1-\delta)W_{i+1}' + \delta U_{i+1}'\right] + (1-\lambda)\left[(1-\delta)W_i' + \delta U_i'\right]\Big], \tag{13}$$

$$V_i = -k + \beta E\left[q(\theta_i)J_i'\right], \tag{14}$$

$$J_i = f_i - \omega_i + \beta E\Big[(1-\delta)\left[\lambda J'_{i+1} + (1-\lambda)J'_i\right]\Big]. \tag{15}$$

Many of our model features are standard to the DMP literature, so our calibration strategy is to maintain comparability wherever possible. As in Hagedorn and Manovskii (2008), the model is calibrated to a weekly frequency. As such, the discount factor is set to $\beta = 0.999$ to accord with an annual risk free rate of 5%.

We assume that the matching function in each market is Cobb-Douglas, so that:

$$\mu(\theta) = \theta q(\theta) = \theta^{\alpha}.$$

We specify $\alpha = 0.4$; this is near the mid point of the range of empirical estimates of the aggregate matching function found in the data.[13] It is important to note, however, that while these estimates are derived to match moments in the aggregate data, we impose this common value of α across each of our (type-specific) matching markets.

For comparability with previous work, we specify the parameter in the Nash bargaining problem as $\tau = 1 - \alpha$. As in the standard DMP model, this implies that the Hosios (1990) condition is met and the equilibrium is efficient (see the Appendix for details).

The vacancy cost, κ, pins down the aggregate job finding rate, $p(\theta)$. We target a weekly job finding rate of $p(\theta) = 0.139$, which corresponds with a monthly rate of 45%, as in Shimer (2005a).[14] Given this aggregate job finding rate, we set $\delta = 0.0081$ to correspond with a steady state unemployment rate of 5.5%.

Following Hall and Milgrom (2008), Mortensen and Nagypál (2007), and Pissarides (2009), we specify z, the flow value of unemployment, to equal 73% of the average return to market work. The interpretation is that z is composed of two components: a value of

[13] See, for instance, Petrongolo and Pissarides (2001), Shimer (2005a), Mortensen and Nagypál (2007), and Brügemann (2008).

[14] As discussed in Shimer (2005a), note that the exact value of κ is irrelevant. That is, by introducing a multiplicative constant, ξ, to the matching function, κ can be scaled by a factor of x and ξ by a factor of x^{α}, leaving the job finding rate unchanged.

leisure or home production, and a value associated with unemployment benefits. As in their work, the return to leisure/home production is equated to 43% of the average return to market work. Given this target, the model's Nash bargained compensation, and the steady state distribution of worker types, we set $z = 0.444$. This implies an unemployment benefit replacement rate ranging from roughly 40% for type 1 workers, and 20% for type N workers; this accords with the range of replacement rates reported by Hall and Milgrom (2008).

Relative to the standard DMP model, our model adds a number of new parameters: $\{f_i\}_{i=1}^N$, λ, and the process of 'death and rebirth'. We normalize $f_N = 1$. For the remaining parameters we follow the calibration strategy of Ljungqvist and Sargent (1998, 2004) and den Haan et al. (2005).[15] Specifically, we choose these parameters to match observations from the empirical life-cycle earnings profile estimated by Murphy and Welch (1990) and others. First, in the data, the maximal lifetime wage gain for a typical worker represents an approximate doubling of earnings. Second, in the data, this doubling occurs after the typical worker has accumulated approximately 25 years of experience. Given this, we choose $N = 25$ and set the difference between the lowest and highest productivity at $f_N/f_1 = 2$. For simplicity, we follow Ljungqvist and Sargent (1998) and specify all productivity levels to be equally spaced, so that $f_1 = 0.5$, $f_2 = 0.5208$, $\ldots$, $f_{N-1} = 0.9792$, and $f_N = 1$. We set $\lambda = 0.0185$, so that it takes the average worker 54 weeks (or approximately one year) to realize a proficiency upgrade; in this way, it takes the average worker 25 years to progress from the lowest to highest proficiency level.

Third, in the data, the average worker's earnings cease to increase from the age of

[15] Though the emphasis of their work is very different, these papers also study a DMP framework in which workers face a stochastic process of productivity upgrading (and downgrading). In particular, their papers focus on the implications of 'turbulence' in the form of a high depreciation rate on productivity or 'human capital' on steady state levels of unemployment. As such, they do not characterize the impact of learning and upgrading on incentives for job creation, nor the implications for cyclical fluctuations.

approximately 50 years old onward. In the model, the value of ϕ determines the proportion of the workforce that no longer has the potential for proficiency upgrading. As such, we calibrate ϕ so that 25% of workers are type N in steady state. This corresponds to the average fraction of the labor force over the age of 50 years in postwar U.S. data. Finally, we must specify how 'dead' workers are 'reborn' in order to maintain a constant unit mass of workers. For symmetry, we do this so that in the model's steady state, there is an equal measure of workers of types 1 through $N-1$ (specifically, $75\%/24 = 3.13\%$ of each type).[16]

To investigate the quantitative predictions of our model, we log-linearize around the steady state, simulate to obtain 250,000 observations at the weekly frequency, then time-aggregate these observations to obtain quarterly data.[17] Following Shimer (2005a), we HP filter the (logged) data with smoothing parameter 10^5 to obtain second moment statistics.

4.2 U.S. Business Cycle Facts

Column 1 of Table 1 presents selected business cycle statistics for the U.S. economy, 1953:I–2009:IV. To isolate cyclical fluctuations, we HP filter the data in the same manner as the model simulated data. See Appendix A.1 for detailed information on data sources used throughout the paper. Here, we highlight a number of well established observations and discuss their implications for the quantitative analysis of search-and-matching models.

The first is that the aggregate unemployment rate is very volatile over the business cycle relative to labor productivity. The standard deviation of unemployment relative to that of labor productivity is 9.34; unemployment is nearly an order of magnitude more volatile

[16] As a point of reference, we note that this is a close approximation to the observed age distribution of the labor force. During the postwar period, the average fraction of labor force participants in 5-year age bins between the ages of 20-24 years and 45-49 years ranges from a low of 10.3% to a high of 12.5%.

[17] We have also solved the model by obtaining (numerically) exact solutions for the case when the exogenous shock is assumed to follow a finite state Markov process. The results are essentially identical using either the approximate, log-linear solution method or the exact, non-linear approach.

than productivity. Hence, models that rely on shocks to productivity as the driving force require strong amplification.

Secondly, we report statistics relating to the cyclicality of job finding since this is the source of all unemployment volatility in the model.[18] Since the job finding rate, $p(\theta)$, is a function of the vacancy-unemployment (or tightness) ratio, θ, we present statistics for this variable as well. Column 1 of Table 1 indicates that both the job finding rate and tightness ratio are very volatile over the cycle. Relative to labor productivity, the standard deviation of these variables are 6.05 and 18.20, respectively.

Moreover, there exists a robust relationship between unemployment and vacancies over the business cycle—the "Beveridge Curve." In postwar U.S. data, this correlation is highly negative at -0.89. This summarizes the fact that recessions are periods when firms stop hiring (vacancies fall) and unemployment soars; booms are periods when hiring is brisk and unemployment is low.

Finally, we highlight the correlation between labor productivity and unemployment. Labor productivity provides a measure of the return to work effort, while unemployment measures work effort itself. Over the business cycle, these two measures are only mildly (negatively) related, with a correlation of -0.41; periods when work effort rises, and unemployment falls, are only weakly associated with higher productivity. This weak relationship is mirrored in the correlations of labor productivity with both the job finding rate (0.44) and the vacancy-unemployment ratio (0.39). These weak correlations are informative regarding the relevant business cycle impulses that should be incorporated in our models.

[18] Job separations in the model are constant at the exogenous rate δ. This simplifying assumption accords with the findings of Shimer (2005a) and Hall (2005), namely that the primary determinant of unemployment fluctuations is variation in the job finding rate. See also Fujita and Ramey (2009) and Elsby et al. (2009). They arrive at similar conclusions, though with a slightly larger contribution to job separation rates.

4.3 Cyclical Dynamics

4.3.1 Technology Shocks

We first review the properties of business cycle fluctuations in the standard DMP framework driven by technology shocks. Specifically, we consider AR(1) disturbances to the productivity of all worker-firm matches. This is done by setting $\{f_i\}_{i=1}^{N} = f$ in our model, and specifying:

$$f_t = f \exp(x_t), \quad x_t = \rho_f x_{t-1} + \varepsilon_t.$$

Calibrating the technology shock process is difficult since productivity data is not available at the weekly frequency, and extrapolating from quarterly data is problematic. As such we choose the persistence of the shock to match the correlation between vacancies and unemployment in simulated data, which we hereafter refer to as the Beveridge curve. This requires $\rho_f = 0.981$. As it is common in log-linearized models around the steady state such as ours, our model's implications for relative volatilities and correlations between different variables are invariant to the variance of the shock innovation. As such we concentrate on these statistics in what follows.

The results for the standard DMP model are presented in Column 2 of Table 1. The model delivers a standard deviation of unemployment relative to productivity of 1.21, which is approximately 8 times smaller than in the data. This large discrepancy between theory and data has been discussed extensively in the literature. Since the model's unemployment fluctuations are driven solely by the response of job creation, it is not surprising to see that the model performs poorly with respect to the job finding rate and the tightness ratio as well. On these dimensions, the model misses by a factor of approximately 5 and 6, respectively.

The standard DMP model also dramatically over-predicts the correlation of labor productivity with labor market measures. Consider first the correlation with the job finding

Table 1: Business Cycle Statistics: U.S. Data and Various Models

	U.S. data	standard DMP	tech. learning	news 1 qtr	news 1 year	news 2 years
standard deviations relative to labor productivity						
unemployment	9.34	1.21	5.54	1.32	1.33	1.33
job finding rate	6.05	1.32	6.18	1.44	1.44	1.44
tightness ratio	18.20	3.21	13.80	3.61	3.61	3.61
correlations with labor productivity						
unemployment	−0.41	−0.96	−0.31	−0.99	−0.99	−0.99
job finding rate	0.44	1.00	0.22	0.96	0.96	0.96
tightness ratio	0.39	1.00	0.22	0.96	0.96	0.96

Notes: All data are logged and HP filtered. U.S. data: 1953:I–2009:IV, various sources, see text. Models: Quarterly averages of simulated data, 250,000 observations at weekly frequency.

rate and the tightness ratio. These are 'jump' variables in the model and the correlation with productivity is perfect. In the data, these correlations are far from perfect. Unemployment is a state variable in the model. As such, its correlation with productivity is smaller than minus one, but still very close at -0.96. In the data, this correlation is only -0.41. For Mortensen and Nagypál (2007), this evidence points to the importance of other driving forces that are omitted from the standard analysis focusing solely on technology shocks. In the next subsection, we illustrate how shocks to the technological learning rate represents a potentially important omitted shock.

4.3.2 Technological Learning Rate Shocks

In this subsection, we document the cyclical properties of our benchmark model when fluctuations are driven by shocks to the technological learning rate, λ. We model the learning rate as following an AR(1) process:

$$\lambda_t = \lambda \exp(x_t), \quad x_t = \rho_\lambda x_{t-1} + \epsilon_t.$$

As with the calibration of the technology shock process in Subsection 4.3.1, the calibration of the learning rate shock process is problematic. This is because there is obviously no empirical data on the learning rate. As such, we pursue the same strategy as that of Subsection 4.3.1; namely we calibrate ρ_λ so that the model's correlation between vacancies and unemployment matches that of the data. This allows us to maintain comparability of our analysis of learning rate shocks to that of technology shocks. Matching the Beveridge Curve requires specifying $\rho_\lambda = 0.970$.[19]

The results are presented in Column 3 of Table 1. Learning rate shocks generate substantial amplification in labor market variables. The volatility of unemployment relative to productivity is more than 4 times that of the standard DMP model (displayed in Column 2); the same is true regarding the volatility of the job finding rate and the tightness ratio, relative to productivity.[20] Hence, our model makes substantial progress toward resolving the unemployment volatility puzzle, especially when viewed from the perspective of cyclicality in job creation.[21]

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[19] As discussed previously, the model's predictions for relative volatilities and correlations are invariant to the variance of the shock process. As such, we need not calibrate the standard deviation of the innovation. This leaves open the question of what fraction of the observed volatility of labor productivity (and thus of all other variables) learning rate shocks account for, relative to other sources of business cycle variation. As a reference point, in order for learning rate shocks to account for all of the variance of labor productivity, the standard deviation of the innovation would equal $\sigma_\epsilon = 0.185$. To quantify this, note that the steady state learning rate is calibrated so that the average employed worker realizes a productivity upgrade every 54 weeks. In the stochastic model, the 68% coverage region around the median learning duration would range from approximately 6 months to just over 2 years.

[20] These results are not due to the mechanisms stressed by Hagedorn and Manovskii (2008). We verify this by solving a version of our benchmark model (with $N = 25$) when driven solely by conventional technology shocks. This version of the model features the same magnification result as in the standard DMP model. For brevity we do not present the results here, but they are available from the authors upon request.

[21] In simulation experiments, we explore the robustness of our findings with respect to the value of N. We find that the model's volatility of unemployment declines monotonically as N increases from 2 to 25, but 'flattens' substantially for $N > 10$. For instance, between $N = 10$ and $N = 25$, the standard deviation of

Moreover, our model makes substantial progress toward resolving the unemployment-productivity correlation puzzle. The model generates a correlation between these two variables of -0.31. This is close to the value of -0.41 observed in the data, and far from the value near minus one generated by the standard DMP model driven by technology shocks.

This is mirrored in our model's ability to generate realistic correlations of labor productivity with the job finding rate and the tightness ratio. In the data, these correlations are 0.44 and 0.39, respectively; in the model, they are 0.22. Learning rate shocks effectively decouple the dynamics of productivity from that of the labor market. This implies business cycle behavior that is closer to that observed in the U.S. data, relative to models where fluctuations are driven by shocks to technology.

To better understand our model relative to the standard DMP model, we present impulse response functions for unemployment and labor productivity in Figure 1. The vertical scale of both panels is identical to facilitate comparison across models.

Panel A presents responses for the conventional business cycle impulse, specifically, the response to a positive one standard deviation technology shock. Technology shocks have a direct impact on matched workers' productivity. Hence, labor productivity jumps upon impact of the shock, gradually declining to steady state thereafter. This shock implies an immediate impact on firm profit. From the free entry condition, vacancies respond immediately. Because of the high empirical job finding rate that the model is calibrated to, unemployment responds quickly, peaking 15 weeks (or about 1 quarter) after the impact period of the shock.

These responses clearly illustrate the shortcomings of technology shock-driven cycles in the DMP model. Because the peak response of both unemployment and labor productivity occur in the short-run, this implies a counterfactually strong correlation of the two variables over the business cycle. Moreover, the response of unemployment is of the same order of

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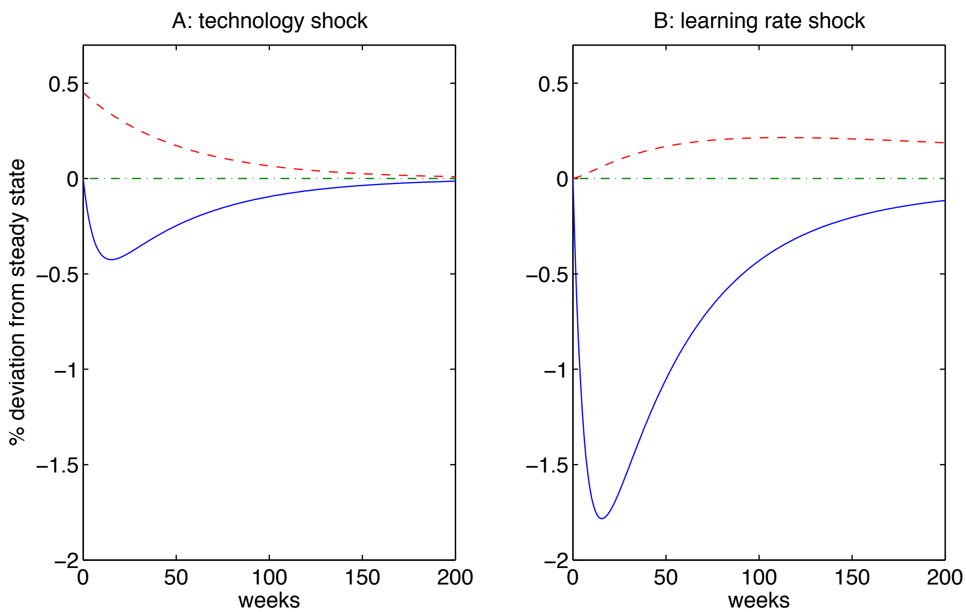
unemployment relative to productivity changes only from 5.62 to 5.54.

Figure 1: Impulse Response Functions: Standard DMP and Technological Learning Models



Notes: Response to positive, one standard deviation shock. Solid (blue) line: unemployment; dashed (red) line: labor productivity.

magnitude as that of productivity. Hence, as discussed extensively in the literature, the model displays much weaker amplification of unemployment, relative to that observed in the data.

Panel B displays the response to a positive one standard deviation learning rate shock in our model. The jump in the learning rate creates a jump in the surplus from a match. From the free entry condition, vacancies respond immediately, and unemployment soon after. The response of unemployment peaks 16 weeks (about 1 quarter) after the impact period of the shock.

In contrast to a technology shock, a learning rate shock has only an indirect effect on labor productivity via the type composition of the workforce. After a positive shock to λ, the economy-wide upgrading rate rises. This causes productivity to rise as workers shuffle from lower to higher types at a faster rate. But because the learning process must still be undertaken, the dynamic response of productivity is persistent, and only peaks about 120 periods (or about 2 years) after the shock. As a result, our model naturally decouples the dynamics between unemployment and labor productivity. While the response of unemployment peaks in the short-run, productivity peaks in the long-run. Hence, learning rate shocks generate a low correlation between these two variables, as observed in the data.

Moreover, learning rate shocks generate a substantially stronger effect on unemployment than on productivity. To understand the response of unemployment, consider the version with only two types, where the return to job creation, namely total surplus in type L matches, is displayed in equation (11). This surplus is determined largely by the value of learning, Δ, which represents the expected future profit gain from an upgrade. Since λ shocks have a direct impact on the learning value, they have strong effects on job creation and unemployment.

On the other hand, the effect of learning rate shocks on labor productivity is quantitatively weak. This can be seen analytically from the log-linearized response of productivity to a λ shock. Given the model's timing, there is no impact in the period of the shock, since upgrading is reflected in output with a one period lag. With $N = 2$, the response of labor productivity in the following period, $\widehat{LP}_{t+1}$, is given by:

$$\widehat{LP}_{t+1} = X \left\{ \left(\frac{n_L + n_H}{n_H} \right) \Big[\lambda(1-\delta) \Big] \hat{\lambda}_t - \left(\frac{\alpha \theta_L^{\alpha} u_L}{n_L} \right) \hat{\theta}_{Lt} \right\}.$$

Here, n_i (u_i) denotes the steady state measure of employed (unemployed) workers of type i, θ_L is the steady state tightness ratio in market L, X is a constant (a function of parameters and steady state values), and the circumflex represents log-linearized deviations from steady state. The first term in the curly brackets indicates the effect of the λ shock. Its strength depends on the steady state distribution of worker types, $(n_L + n_H)/n_H$ (the larger the fraction of L types with the potential to upgrade, the bigger the effect), and importantly, the level of the steady state learning rate, as captured by the term in square brackets. In order to account for life-cycle earnings dynamics, our calibration requires a small value for λ. Hence, the response of labor productivity to a learning rate shock is quantitatively small.[22]

4.3.3 Learning Rate Shocks and "News Shocks"

In this section, we provide evidence for the relevance of learning rate shocks—and, in particular, their decoupling of productivity and unemployment dynamics—for business cycle analysis by relating our model's results to the recent "news shock" literature.

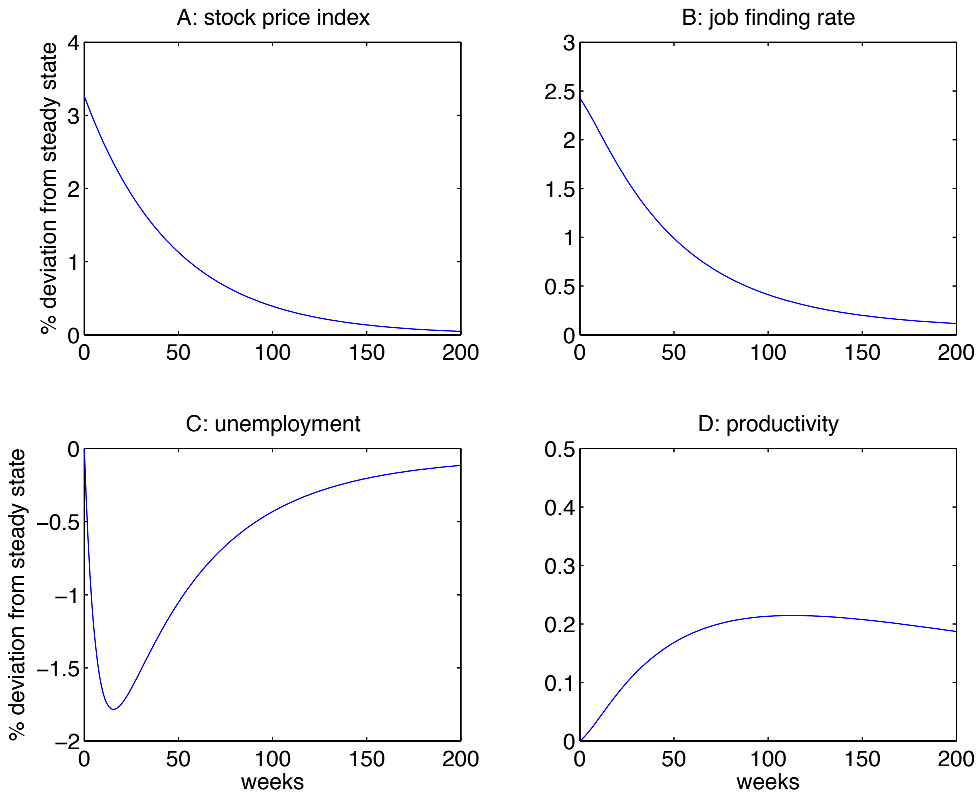
In a recent paper, Beaudry and Portier (2006) use a number of structural VAR techniques to identify shocks to productivity in U.S. data. They find that shocks to long-run productivity have essentially no effect on productivity upon impact.[23] Instead, productivity is found to respond in a gradual, persistent manner. On the other hand, measures such as the stock market index and employment are found to respond immediately (i.e., within the first quarter) to these long-run TFP shocks.

Technological learning rate shocks generate dynamic responses that share these features. Hence, shocks to the learning rate provide a theoretical interpretation of empirically identified "news shocks." This is illustrated in Figure 2, where we plot impulse response functions to a learning rate shock. In Panel A, we display the response of the model's stock price

[22] Deriving log-linearized expressions for labor productivity at longer time horizons is difficult, given the need to track the dynamic response of the distribution of workers across types and employment status. Note also that in the equation above, the second term in curly brackets is negative. This reflects the fact that a positive λ shock generates a response of job creation for L types (and no response in creation for H types). Hence, the distribution of employment shifts toward low proficiency workers. This negative composition effect offsets the positive effect of faster upgrading on the response of labor productivity. However, in our numerical experiments, this offsetting effect is small.

[23] In their benchmark bivariate system, shocks identified to have a permanent impact on productivity are found to have a small, negative effect on productivity upon impact (though the response is statistically indistinguishable from zero). And interestingly, shocks to stock market prices that are orthogonal to productivity upon impact generate a nearly identical dynamic response to TFP.

Figure 2: Impulse Response Functions to a Positive Learning Rate Shock



Notes: Response to positive, one standard deviation shock.

index. We construct this index as a weighted average of the present discounted value of firm profits in all match types, $\{J_i\}_{i=1}^N$, where the weights are the proportions of each type in the model's steady state. A positive λ shock causes the value of type $i = 1, \ldots, N-1$ matches to jump immediately as the 'upside risk' of these matches increases.[24] Hence, the stock price index jumps upon impact of the shock; the response gradually returns to zero as λ returns to its steady state value.

Panel B displays the response of the aggregate job finding rate. The learning rate shock causes firm surplus for all type $i < N$ firms to jump upon impact. From the free entry condition, vacancies and job finding rates jump. Panel C displays the response of

[24]Recall that the value of type N matches is unaffected.

the aggregate unemployment rate. Unemployment is a state variable, and therefore does not respond in the period of the shock, but responds very quickly after impact. Hence, economic activity, as measured by stock prices, job creation, and unemployment respond within the quarter of the learning rate shock.

In contrast, labor productivity responds in a persistent, protracted manner. This is evidenced in Panel D. Shocks to the learning rate induce gradual changes in the productivity composition of the workforce. As a result, the productivity response is smooth, peaking approximately 120 periods—or over 2 years—after the initial shock. Hence, the productivity response to learning rate shocks are observed in the long run. These features are consistent with the responses identified by Beaudry and Portier (2006).

Finally, while our model conforms with the *empirical evidence* for news shocks, the *theoretical mechanism* embodied by learning rate shocks are distinct from those in recent models. In those papers (e.g. Beaudry and Portier (2007), Jaimovich and Rebelo (2009), and den Haan and Kaltenbrunner (2009)), news shocks are modelled as signals of technology shocks that are to arrive a number of quarters in the future. Upon arrival, these innovations immediately affect productivity. In contrast, shocks to the learning rate represent the arrival of innovations that vary in their ease of technological learning; their effects on labor productivity are realized in a delayed manner, via the process of learning-by-doing.

This distinction is not just a matter of interpretation: while learning rate shocks make progress on rationalizing labor market dynamics in the DMP framework, conventionally modeled news shocks do not. This is illustrated in the rightmost columns of Table 1, where news shocks are introduced in the standard DMP model in the usual way—as the arrival of information at date 0 of a change in productivity in period $\hat{t}$.[25] Column 4 presents results for a one quarter ahead ($\hat{t} = 13$ weeks) news shock, while Column 5 presents the case of

[25]For brevity, we do not present details regarding this version of the model, and instead, make them available upon request.

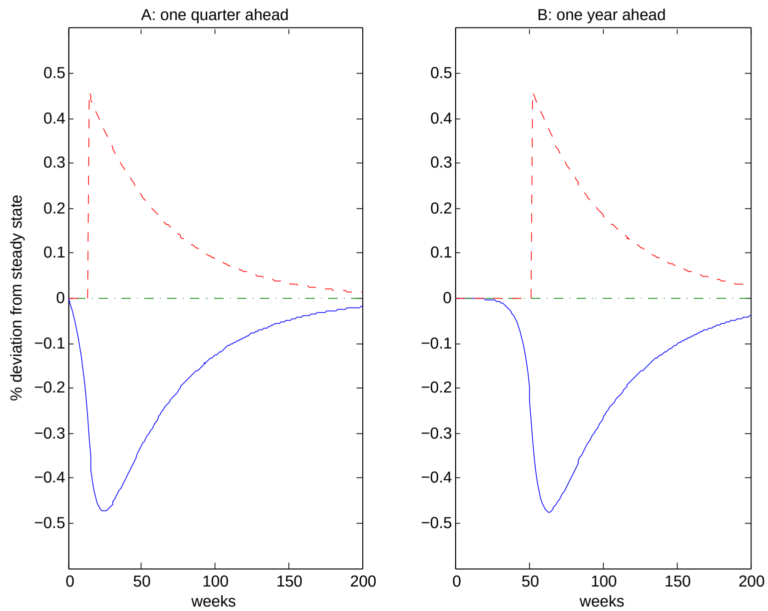
a one year ahead ($\hat{t} = 52$ weeks) news shock. The results of Beaudry and Portier (2006) indicate that the long-run effects of news shocks on productivity are evidenced at a horizon of 8 to 10 quarters (or about 2 years); this coincides with the horizon of maximal effect in our model with learning rate shocks (see, for instance, Panel D of Figure 2). Given this, Column 6 presents the results for a 2 year ($\hat{t} = 104$ weeks) ahead news shock.

As is obvious, the amplification of unemployment fluctuations relative to productivity is essentially identical to the case with standard technology shocks, discussed in Subsection 4.3.1, and presented in Column 2. To understand this, Figure 3 presents impulse response functions for unemployment and labor productivity to a positive one standard deviation news shock. Panel A presents the case for a one quarter ahead news shock, and Panel B for a one year ahead shock. A conventional news shock has a direct impact on productivity at the time the technology arrives. Vacancy creation and, therefore, unemployment respond prior to the period of arrival. Nonetheless, the maximal response of unemployment is of the same order of magnitude as the response of productivity; indeed, the maximal response of unemployment is essentially identical to the case for standard technology shocks presented in Panel A of Figure 2. Hence, conventionally modelled news shocks make little progress in solving the unemployment volatility puzzle.

Similarly, conventional news shocks make no progress on the unemployment-productivity correlation puzzle; the correlation between these variables remains near minus one in Columns 4 through 6. To understand this, we refer again to Figure 3. While vacancies and unemployment begin to respond prior to the arrival of the technological change, the peak response of both unemployment and productivity are essentially simultaneous. As a result, conventional news shocks imply a counterfactually strong correlation of the two variables.

Hence, the manner in which news shocks are modeled is important. Technological learning rate shocks generate "news shock" dynamics and help rationalize the two labor

Figure 3: Impulse Response Functions: Conventionally Modeled News Shocks



Notes: Response to positive, one standard deviation shock. Solid (blue) line: unemployment; dashed (red) line: labor productivity.

market puzzles; news shocks modelled as the arrival of information of shocks to future productivity make little to no progress on the puzzles.

Finally we note that our model, like almost all models driven solely by news shocks, cannot account for the fact that in the U.S. data labor productivity leads unemployment by two quarters. While news shocks have been found to account for a substantial fraction of business cycle variation in U.S. data, we view this lead-lag evidence as suggesting the importance of other shocks.

4.4 Learning Rate Shocks and the Return to Experience

Our analysis emphasizes the importance of learning rate shocks in accounting for the cyclical behavior of aggregate unemployment and productivity. At its core, the idea is that individual workers gain technological proficiency and productivity while on the job, and that the rate at which this happens varies over the business cycle. Shocks to the learning rate tilt the life-cycle earnings profile, making it steeper when the shock is positive and flatter when the shock is negative. Hence, our model puts emphasis on the return to labor market experience and its cyclical properties.

Relatively little empirical work has been devoted to identifying the cyclicality of the return to experience. A notable exception is that of French et al. (2006). Their work focuses on the evolution of employment and earnings for a cohort of young workers (18 to 28 year olds) in the Census Bureau's Survey of Income and Program Participation (SIPP). Using wage data for continuously employed workers who remain with the same employer, they are able to identify time variation in the return to experience. French et al. (2006) find this return to be strongly procyclical. In their baseline specification, a one percent rise in the unemployment rate generates a 1.1% fall in the return to experience, with a two standard deviation confidence band of 0.5%–1.8%.[26]

In light of this, we perform the same exercise in simulated data from our model. Specifically, we track the wages of a cohort of young workers over a 25 year period; this is done by

[26] French et al. (2006) also consider a second specification which attempts to control for a time effect that is common to all workers. This common time effect might capture, for instance, aggregate technology shocks that affect all wages, independent of a worker's experience. In this case, the cyclicality of the return to experience is significantly muted. But as French et al. (2006) point out, identifying this aggregate time effect is not at all straightforward in their empirical framework. They choose to proxy for this using the wages of new labor market entrants, with the idea that workers with no experience are not affected by changes in the return to experience. This identification assumption might hold, for instance, in a model with static wage determination, where wages are determined only by current marginal product. However, this identification assumption is clearly violated in our model. With Nash bargaining, wages are forward-looking and respond to changes in match surplus due to changes in the learning rate; and this is true for workers with no experience (type L workers in the simple $N = 2$ version, type 1 workers in the $N > 2$ version). As such, we believe future empirical work attempting to disentangle common time effects from the return to experience would be of clear value.

"interviewing" the workers at four month intervals (the same frequency of interview waves in the SIPP).[27] As in French et al. (2006), we estimate the return to experience at any point in time as the (cross-sectional) average log wage change from the previous interview, for those who were continuously employed. Repeating this period-after-period over the 25 years obtains a time series for the return to experience. We then determine its cyclicality by regressing it on a constant, time trend, and the aggregate unemployment rate, as in French et al. (2006).

We repeat this simulation exercise 100 times and report the median coefficient estimate. In our model, a one percent rise in unemployment generates a 1.6% fall in the return to experience. This is in line with the point estimate of 1.1%, and within the two standard deviation confidence band, reported in French et al. (2006). It is worth noting that our model analysis assumes that all fluctuations in unemployment are due to shocks to the learning rate. In reality, there are likely to be other shocks contributing to the cycle that do not affect the return to experience.[28] Hence, the inclusion of such shocks in a more elaborate model would also dampen the estimated covariance of unemployment with the return to experience.

[27] Our cohort of workers is initialized by presuming that all workers enter the labor force at age 18 as unemployed, type $i = 1$ workers. Applying the steady state job finding, job separation, and learning rates to these workers for a six month period allows us to generate the distribution of 18 year olds across employment statuses and types. The distribution of 19 year olds is obtained from a further 12 months of transitions for the initial cohort, and so on, until we have distributions for workers of all ages between 18 and 28 (the same age group studied in French et al. (2006)). These workers of different ages are then used to generate a representative SIPP cohort by weighting them according to the age distribution of 18 to 28 year olds in the U.S. labor force, as found in 1984 (the initial year of the SIPP) in the CPS.

[28] For instance, while our model does well in replicating the volatility of job creation, it understates that of unemployment. Recent work attributes one quarter to one third of unemployment variability to cyclical job destruction. Hence, one could imagine shocks to job separation rates as generating fluctuations in unemployment that are unrelated to the return to experience.

5 Conclusion

In this paper, we have focused on two key labor market observations in postwar U.S. data. The first is that unemployment is very volatile over the business cycle relative to labor productivity. The second is that cyclical fluctuations in unemployment and productivity are only mildly negatively correlated. The canonical model of equilibrium unemployment, when driven by technology shocks, fails to account for either of these facts.

We propose a model of technological learning that makes progress on both shortcomings. Specifically, we construct a tractable search-and-matching model in which: (a) it takes time for workers to become fully proficient with technology, and (b) shocks to the speed or ease of learning-by-doing are a source of business cycle fluctuations. Quantitative analysis indicates that our model generates substantial amplification in labor market variables and delivers a correlation between unemployment and labor productivity that is close to the data. Our model also provides a new theoretical interpretation of news shocks. Specifically, learning rate shocks generate long-run fluctuations in productivity that are associated with short-run fluctuations in stock prices and unemployment. Crucially, our model does this while simultaneously making progress on the unemployment volatility puzzle and unemployment-productivity correlation puzzle. By contrast, conventional models of news shock driven business cycles make essentially no progress on these puzzles.

A Appendix

A.1 Data Sources

Our measure of unemployment is the quarterly average of the seasonally adjusted monthly series constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). Our measure of labor productivity is (quarterly, seasonally adjusted) output divided by employment in the nonfarm business sector as constructed by the BLS Major Sector Productivity program. To construct the job finding rate, we follow the approach of Shimer (2005a,b, 2012), using monthly data on employment, unemployment, and short-term unemployment tabulated from the CPS. We use vacancy data provided by Barnichon (2010), divided by unemployment, to construct the tightness ratio. Specifically, we use his "composite Help-Wanted Index" which combines information on the number of newspaper and online job advertisements compiled by the Conference Board.

Quarterly data on aggregate output and its expenditure components are obtained from the National Income and Product Accounts. Specifically, real output, consumption, and investment refer to seasonally adjusted gross domestic product, personal consumption expenditures, and gross private domestic investment, respectively, expressed in chained 2005 dollars. Finally, employment refers to the quarterly average of the seasonally adjusted monthly series for the civilian employment-population ratio, constructed by the BLS from CPS data.

A.2 Efficiency

To show that our equilibrium is efficient, we derive equations that fully characterize the solution to a planner's problem and show that the same equations characterize equilibrium under Hosios (1990)'s condition. In a separate appendix available from the authors' webpages, we show that our equilibrium also obtains in a directed search environment in which firms post wages.

A.2.1 Compensations

To begin, we derive the compensations from generalized Nash bargaining. These are required for the characterizations below. The market for high productivity workers is identical to that of the standard DMP model. Therefore, the compensation in such matches is entirely standard:

$$\omega_H = \tau(f_H + \kappa\theta_H) + (1-\tau)z. \tag{16}$$

To obtain the compensation in low productivity matches, begin with proportionality of surplus:

$$(1-\tau)\left(W_L - U_L\right) = \tau J_L.$$

Worker surplus can be expressed as:

$$\begin{aligned}(W_L - U_L) = \omega_L - z + \beta E\Big\{&(1-\delta)\big[\lambda\left(W_H' - U_H'\right) + (1-\lambda)\left(W_L' - U_L'\right)\big] \\ &- p(\theta_L)\left(W_L' - U_L'\right) + \lambda\left(U_H' - U_L'\right)\Big\},\end{aligned}$$

or:

$$\begin{aligned}(1-\tau)\left(W_L - U_L\right) = (1-\tau)(\omega_L - z) + \beta E\Big\{&(1-\delta)\tau[\lambda J_H' + (1-\lambda)J_L'] \\ &- \tau p(\theta_L)J_L' + (1-\tau)\lambda\left(U_H' - U_L'\right)\Big\}.\end{aligned}$$

Equating this with τJ_L and using free entry, we get:

$$\omega_L = \tau(f_L + \kappa\theta_L) + (1-\tau)z - (1-\tau)\lambda\beta E\left[U_H' - U_L'\right]. \tag{17}$$

The last term is $(1-\tau)$ times the *worker's value of learning.*

The worker's value of learning can be simplified by noting that:

$$U_H - U_L = \beta E\Big[p(\theta_H)\left(W_H' - U_H'\right) - p(\theta_L)\left(W_L' - U_L'\right) + [U_H' - U_L']\Big].$$

Using the proportionality of surplus and the free entry condition again, we get:

$$
\begin{aligned}
U_H - U_L &= \bar{\tau}\kappa(\theta_H - \theta_L) + \beta E[U'_H - U'_L] \\
&= \bar{\tau}\kappa E \sum_{s=0}^{\infty} \beta^s (\theta_H^{+s} - \theta_L^{+s})
\end{aligned} \tag{18}
$$

where $\bar{\tau} = \tau/(1-\tau)$, $\theta_i^{+0} = \theta_i$, $\theta_i^{+1} = \theta'_i$, $\theta_i^{+2} = \theta''_i$, and so on.

A.2.2 A Planner's Problem

Let $V(u_L, u_H, n_L, n_H)$ denote the value function of the planner who inherits unemployment u_i for type $i \in \{L, H\}$ and employment n_i for type $i \in \{L, H\}$:

$$
\begin{aligned}
V(u_L, u_H, n_L, n_H) = \max_{\{\theta_i, u'_i, n'_i\}_{i=\{L,H\}}} \Big\{ &(u_L + u_H)z + n_L f_L + n_H f_H \\
&- \kappa(u_L\theta_L + u_H\theta_H) + \beta V(u'_L, u'_H, n'_L, n'_H) \Big\},
\end{aligned}
$$

subject to the following laws of motion:

$$u'_L \geq (1 - p(\theta_L))u_L + (1-\lambda)\delta n_L,$$

$$u'_H \geq (1 - p(\theta_H))u_H + \lambda\delta n_L + \delta n_H,$$

$$n'_L \leq p(\theta_L)u_L + (1-\lambda)(1-\delta)n_L,$$

$$n'_H \leq (1-\delta)n_H + p(\theta_H)u_H + \lambda(1-\delta)n_L.$$

Setting $u_L + u_H + n_L + n_H = 1$ and letting λ_{u_i} be the multiplier on the law of motion for u_i and λ_{n_i} be the multiplier on the law of motion for n_i, the first-order conditions with respect to θ_i, $i \in \{L, H\}$, are:

$$-\kappa + p'(\theta_L)(\lambda_{u_L} + \lambda_{n_L}) = 0,$$

$$-\kappa + p'(\theta_H)(\lambda_{u_H} + \lambda_{n_H}) = 0.$$

The first order conditions with respect to u'_i and n'_i, $i \in \{L, H\}$, are:

$$\beta V_{u_i}(u'_L, u'_H, n'_L, n'_H) + \lambda_{u_i} = 0,$$

$$\beta V_{n_i}(u'_L, u'_H, n'_L, n'_H) - \lambda_{n_i} = 0.$$

Finally, the envelope conditions are:

$$
\begin{aligned}
V_{u_L} &= z - \kappa\theta_L - \lambda_{u_L}(1 - p(\theta_L)) + \lambda_{n_L} p(\theta_L),\\
V_{u_H} &= z - \kappa\theta_H - \lambda_{u_H}(1 - p(\theta_H)) + \lambda_{n_H} p(\theta_H),\\
V_{n_L} &= f_L - \lambda_{u_L}(1-\lambda)\delta - \lambda_{u_H}\lambda\delta + \lambda_{n_L}(1-\lambda)(1-\delta) + \lambda_{n_H}\lambda(1-\delta),\\
V_{n_H} &= f_H - \lambda_{u_H}\delta + \lambda_{n_H}(1-\delta).
\end{aligned}
$$

Combining the first-order and envelope conditions, the following equations characterize a steady state:

$$\lambda_{u_L} + \lambda_{n_L} = \frac{\kappa}{p'(\theta_L)} \tag{19}$$

$$\lambda_{u_H} + \lambda_{n_H} = \frac{\kappa}{p'(\theta_H)} \tag{20}$$

$$\beta\Big(z - \kappa\theta_L + p(\theta_L)(\lambda_{u_L} + \lambda_{n_L}) - \lambda_{u_L}\Big) + \lambda_{u_L} = 0 \tag{21}$$

$$\beta\Big(z - \kappa\theta_H + p(\theta_H)(\lambda_{u_H} + \lambda_{n_H}) - \lambda_{u_H}\Big) + \lambda_{u_H} = 0 \tag{22}$$

$$\beta\Big(f_L - \delta\big[(1-\lambda)(\lambda_{u_L} + \lambda_{n_L}) + \lambda(\lambda_{u_H} + \lambda_{n_H})\big] + (1-\lambda)\lambda_{n_L} + \lambda\lambda_{n_H}\Big) - \lambda_{n_L} = 0 \tag{23}$$

$$\beta\Big(f_H - \delta(\lambda_{u_H} + \lambda_{n_H}) + \lambda_{n_H}\Big) - \lambda_{n_H} = 0 \tag{24}$$

It will prove convenient to use these equations get expressions for some of the multipliers. From equation (21), we have:

$$\lambda_{u_L} = -\frac{\beta(\alpha z + (1-\alpha)\theta_L\kappa)}{\alpha(1-\beta)}. \tag{25}$$

Similarly, from equation (22), we have:

$$\lambda_{u_H} = -\frac{\beta(\alpha z + (1-\alpha)\theta_H \kappa)}{\alpha(1-\beta)}, \tag{26}$$

and from equation (24) we have:

$$\lambda_{n_H} = \frac{\beta(\alpha f_H - \delta\theta_H \kappa / p(\theta_H))}{\alpha(1-\beta)}. \tag{27}$$

Market H Notice that equations (20), (26) and (27) fully characterize θ_H in an unrelated way to θ_L, or submarket L in general. Using (26) and (27) in (20):

$$\frac{\beta\big(\alpha f_H - \delta\theta_H\kappa/p(\theta_H) - \alpha z - (1-\alpha)\theta_H\kappa\big)}{\alpha(1-\beta)} = \frac{\kappa}{p'(\theta_H)}.$$

Using the fact that $p'(\theta) = \alpha p(\theta)/\theta$, we have:

$$\beta\left(\alpha(f_H - z) - (1-\alpha)\theta_H\kappa - \delta\frac{\theta_H\kappa}{p(\theta_H)}\right) = \frac{(1-\beta)\alpha\theta_H\kappa}{\alpha p(\theta_H)}.$$

Rearranging, θ_H is characterized by:

$$\frac{\beta\big(\alpha(f_H - z) - (1-\alpha)\theta_H\kappa\big)}{1-\beta(1-\delta)} = \frac{\theta_H\kappa}{p(\theta_H)}. \tag{28}$$

Market L Rewrite equation (23) as follows:

$$\lambda_{n_L} = \bar{\beta}(f_L - \delta(1-\lambda)(\lambda_{u_L} + \lambda_{n_L}) - \delta\lambda(\lambda_{u_H} + \lambda_{n_H}) - \lambda(\lambda_{n_L} - \lambda_{n_H})),$$

where $\bar{\beta} = \beta/(1-\beta)$. We now need an expression for $\lambda_{n_L} - \lambda_{n_H}$. From equations (19) and (20), we have

$$\lambda_{n_L} - \lambda_{n_H} = \frac{\kappa}{p'(\theta_L)} - \frac{\kappa}{p'(\theta_H)} + \lambda_{u_H} - \lambda_{u_L}.$$

Using the expressions for λ_{u_H} and λ_{u_L} from equations (25) and (26), this is

$$\lambda_{n_L} - \lambda_{n_H} = \frac{\kappa}{p'(\theta_L)} - \frac{\kappa}{p'(\theta_H)} + (\bar{\beta}/\alpha)(1-\alpha)(\theta_L - \theta_H)\kappa.$$

So λ_{n_L} becomes

$$\lambda_{n_L} = \bar{\beta}\Big[f_L - \delta(1-\lambda)(\lambda_{u_L} + \lambda_{n_L}) - \delta\lambda(\lambda_{u_H} + \lambda_{n_H}) \\ - \lambda\left(\frac{\kappa}{p'(\theta_L)} - \frac{\kappa}{p'(\theta_H)} + (\bar{\beta}/\alpha)(1-\alpha)(\theta_L - \theta_H)\kappa\right)\Big],$$

or, rearranging,

$$\lambda_{n_L} = \bar{\beta}\left[f_L - [\delta(1-\lambda)+\lambda]\frac{\kappa}{p'(\theta_L)} + (1-\delta)\lambda\frac{\kappa}{p'(\theta_H)} - \lambda(\bar{\beta}/\alpha)(1-\alpha)(\theta_L - \theta_H)\kappa\right].$$

Using this last equation together with the expression for λ_{u_L} from equation (25) in equation (19), we have

$$\bar{\beta}\Big[f_L - z - (1-\alpha)\theta_L\kappa/\alpha - [\delta(1-\lambda)+\lambda]\frac{\kappa}{p'(\theta_L)} + (1-\delta)\lambda\frac{\kappa}{p'(\theta_H)} \\ - \lambda(\bar{\beta}/\alpha)(1-\alpha)(\theta_L - \theta_H)\kappa\Big] = \frac{\kappa}{p'(\theta_L)}.$$

Grouping terms and rearranging,

$$\tilde{\beta}\left[f_L - z - (1-\alpha)\theta_L\kappa/\alpha + (1-\delta)\lambda\frac{\kappa}{p'(\theta_H)} + \lambda(\bar{\beta}/\alpha)(1-\alpha)(\theta_H - \theta_L)\kappa\right] = \frac{\kappa}{p'(\theta_L)}, \tag{29}$$

where $\tilde{\beta} = \beta/(1-\beta(1-\delta)(1-\lambda))$. Given the value of θ_H which solves equation (28), this last expression characterizes θ_L.

A.2.3 Relation to Equilibrium

Market H The steady state value of the firm in market H is

$$J_H = \frac{f_H - \omega_H}{1-\beta(1-\delta)}.$$

Using the wage equation (16) in the free-entry condition $\beta p(\theta_H)J_H = \theta_H\kappa$, we have

$$\frac{\beta\big((1-\tau)(f_H - z) - \tau\theta_H\kappa\big)}{1-\beta(1-\delta)} = \frac{\theta_H\kappa}{p(\theta_H)}.$$

Clearly, this is equivalent to its counterpart (28) from the Planner's problem if $\tau = 1-\alpha$.

Market L The free entry condition in that market reads

$$\beta p(\theta_L) J_L = \theta_L \kappa,$$

where

$$J_L = f_L - \omega_L + \beta(1-\delta)[\lambda J_H + (1-\lambda) J_L].$$

We can use the free entry condition $J_H = \theta_H \kappa / \beta p(\theta_H)$ to get

$$J_L = \frac{f_L - \omega_L + \beta(1-\delta)\lambda\theta_H\kappa/\beta p(\theta_H)}{1-\beta(1-\delta)(1-\lambda)}.$$

From equations (17) and (18), the wage in market L can be written as

$$\omega_L = \tau(f_L + \kappa\theta_L) + (1-\tau)z - \bar{\beta}\lambda\tau\kappa(\theta_H - \theta_L).$$

Using this wage in J_L, the free entry condition becomes

$$\beta p(\theta_L) \frac{f_L - \Big(\tau(f_L+\kappa\theta_L) + (1-\tau)z - \bar{\beta}\lambda\tau\kappa(\theta_H-\theta_L)\Big) + (1-\delta)\lambda\theta_H\kappa/p(\theta_H)}{1-\beta(1-\delta)(1-\lambda)} = \theta_L\kappa.$$

Rearranging, we have

$$\tilde{\beta}\left[(1-\tau)(f_L - z) - \tau\kappa\theta_L + \bar{\beta}\lambda\tau\kappa(\theta_H - \theta_L) + (1-\delta)\lambda\alpha\kappa/p'(\theta_H)\right] = \alpha\kappa/p'(\theta_L). \tag{30}$$

If $\alpha = 1-\tau$, this becomes

$$\tilde{\beta}\left[f_L - z - (1-\alpha)\theta_L\kappa/\alpha + (1-\delta)\lambda\kappa/p'(\theta_H) + \lambda(\bar{\beta}/\alpha)(1-\alpha)\kappa(\theta_H - \theta_L)\right] = \kappa/p'(\theta_L),$$

which is identical to its counterpart (29) from the Planner's problem.

A.3 Proof of Proposition 4

Proof. Start with steady state equation (11):

$$S_L = f_L - z - \bar{\tau}\kappa\theta_L + \beta(1-\delta)S_L + \lambda\beta\left[(1-\delta)(S_H - S_L) + (U_H - U_L)\right],$$

where, again, $\bar{\tau} \equiv \tau/(1-\tau)$. This can be rewritten:

$$S_L\left[1+\beta(1-\delta)\lambda-\beta(1-\delta)\right]=f_L-z-\bar{\tau}\kappa\theta_L+\lambda\beta(1-\delta)S_H+\lambda\beta\left(U_H-U_L\right).$$

Using equation (1) and Nash bargaining, we have in steady state:

$$U_L=\frac{1}{1-\beta}\left[z+\bar{\tau}\kappa\theta_L\right].$$

Substitute this into the S_L equation above:

$$S_L\left[1+\beta(1-\delta)\lambda-\beta(1-\delta)\right]=f_L-z-\bar{\tau}\kappa\theta_L+\lambda\beta\left[(1-\delta)S_H+U_H\right]-\frac{\lambda\beta}{1-\beta}\left[z+\bar{\tau}\kappa\theta_L\right].$$

Using Nash bargaining and the zero profit condition, we can express S_L as a function of θ_L:

$$S_L=\frac{\kappa}{\beta(1-\tau)q(\theta_L)}.$$

Substituting again, we have:

$$\begin{aligned}0=G\equiv-\frac{\kappa}{\beta(1-\tau)q(\theta_L)}\left[1+\beta(1-\delta)\lambda-\beta(1-\delta)\right]+f_L-z\\-\bar{\tau}\kappa\theta_L+\lambda\beta\left[(1-\delta)S_H+U_H\right]-\frac{\lambda\beta}{1-\beta}\left[z+\bar{\tau}\kappa\theta_L\right].\end{aligned}$$

Note that:

$$\frac{\partial\theta_L}{\partial\lambda}=-\frac{\frac{\partial G}{\partial\lambda}}{\frac{\partial G}{\partial\theta_L}},$$

and

$$\begin{aligned}\frac{\partial G}{\partial\lambda}&=-\frac{\kappa}{\beta(1-\tau)q(\theta_L)}\beta(1-\delta)+\beta(1-\delta)S_H+\beta U_H-\frac{\beta}{1-\beta}\left[z+\bar{\tau}\kappa\theta_L\right]\\&=\beta(1-\delta)\left[S_H-S_L\right]+\beta\left[U_H-U_L\right]>0.\end{aligned}$$

Here we use the steady state values for S_L and U_L derived above, and the results of Proposition 3 and Corollary 2 to sign both square bracketed terms. Finally:

$$\frac{\partial G}{\partial\theta_L}=-\bar{\tau}\kappa\left(1+\frac{\beta\lambda}{1-\beta}\right)-\frac{\kappa\left[1+\beta(1-\delta)\lambda-\beta(1-\delta)\right]}{\beta(1-\tau)}\left(-\frac{q'(\theta_L)}{q(\theta_L)^2}\right)<0,$$

where we use the fact that $q'(\theta)<0$. This establishes that $\partial\theta_L/\partial\lambda>0$, as desired. ■

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