Optimal Mortgage Contract Choice Decision in the Presence of Pay Option Adjustable Rate Mortgage and the Balloon Mortgage

Comments Welcome

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Abstract

The unprecedented run-up in global house prices of the 2000s was preceded by a revolution in U.S. mortgage markets in which borrowers faced a plethora of mortgages to choose from collectively known as nontraditional mortgages (NTMs), whose poor performance helped ignite the global financial crisis in 2007. This paper studies the choice of mortgage contracts in an expanded framework where the menu of contracts includes the pay option adjustable rate mortgage (PO-ARM), and the balloon mortgage (BM), alongside the traditional long horizon fixed rate mortgage (FRM) and the short horizon regular ARM. The inclusion of the PO-ARM is based on the fact it is the most controversial and perhaps the riskiest of the NTMs, whereas the BM has not been analyzed in the literature despite its different risk-sharing arrangement and long vintage. Our inclusive model relates the structural differences of these contracts to the horizon risk management problems and affordability constraints faced by the households that differ in terms of expected mobility. The numerical solutions of the model generates a number of interesting results suggesting that households select mortgage contracts to match their horizon, manage horizon risk and mitigate liquidity or affordability constraints they face. From a risk management and welfare perspectives, we find that the optimal contract for households with shorter horizons, specifically households who expect to move house once every one to two years, is the PO-ARM. The welfare advantage of the PO-ARM diminishes when the household's horizon extends beyond 2 years at which point the BM becomes the more optimal contract up to 5-year horizon. The FRM is found to be the most suitable contract for relatively sedentary households who expect to move house once every six years and beyond. While the PO-ARM is found to dominate the FRM and BM, the dominance is not absolute. Overall, the results suggest that households are neither as risk averse as the selection of the FRM would suggest, nor are they as risk-seeking as the selection of PO- ARM or regular ARM would suggest. The results also suggest that the exuberance demonstrated for NTMs, especially PO-ARM, during the 2000s mortgage revolution may be both *rational* and *irrational*.

JEL: G12, G18, G21, G32
KEYWORDS: Mortgage choice, balloon mortgages, risk management, household mobility

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1. Introduction

For decades the U.S. mortgage market was dominated by the standard long horizon fixed rate mortgage (FRM), short horizon regular adjustable rate mortgage (ARM), and the intermediate horizon balloon mortgage (BM)' The latter contract has been largely ignored in the extant mortgage choice literature despite its obviously different lender-borrower risk-sharing arrangement. Starting in the early 2000s the U.S. mortgage market experienced extensive and eclectic innovations that collectively became known as *nontraditional mortgages* (NTMs). The NTM innovations greatly expanded the menu of contracts that households use to create leverage position in the dominant asset in their portfolio – the house.

In this context, the most intriguing and controversial NTM was the *pay option adjustable rate mortgage* henceforth PO-ARM, whose complexity and inherent risk layering belie its presumed flexibility and affordability. [1] Indeed, the PO-ARM introduced other risk variables that hitherto have not been analyzed in a broader choice framework of competing mortgage contracts. Thus, from a household horizon risk management and financial advice points of view, the mortgage choice paradigm of the 1980s and 1990s, which was typically couched as a horserace between the long horizon FRM and short horizon ARM, has little to say about the *suitability* of other mortgage contracts that can be used to leverage the housing asset, such as the BM, let a alone the PO-ARM, which is of a relatively recent vintage. [2]

This paper studies the choice of mortgage contracts in an expanded framework where the menu of contracts includes the BM and the PO-ARM alongside the standard FRM and regular ARM contracts. The focus on the BM and PO-ARM is of course both deliberate and timely. The

[1] The transformation in U.S. mortgage market in the 2000s reflects a confluence of factors including innovation in the structure, underwriting and marketing of mortgages, rising house prices, declining affordability, rising mortgage demand to support of homeownership, historically low interest rate, intense lender competition and abundance of capital from mortgage backed securities investors.

[2] The introduction of PO-ARM is U.S. mortgage market dates back to 1981 when thrifts were allowed to originate this mortgage type to help them manage interest rate risk that had contributed to S&L crisis in which taxpayers lost about $140 billion. Back then the PO-ARM was marketed largely to wealthy and sophisticated households as financial management tool to permit such household to manage their monthly cash flows. However, during the 2000s mortgage revolution a new group of households, many of questionable credit risk, entered the home ownership market largely because products such as PO-ARM significantly enhanced their ability (affordability) to buy high-priced homes they could not have qualified for using more traditional mortgages such as FRM and regular ARM. Some observers have likened PO-ARM to "neutron bomb" that will financially kill people but leave their houses standing.

BM with its intermediate maturity possess some of the essential benefits of the FRM and ARM without some of their disadvantages, which we presume gives it a different risk-sharing profile, but has not really been analyzed in the mortgage choice literature despite its long history of existence. [3] From a household risk management perspective the greatest challenge posed by PO-ARM lies in its risk layering arising out of its key features including the borrower option to decide how much monthly payment to make, substantial deep teaser rates, potential for negative amortization and significant payment shock. The so-called flexibility and affordability features of PO-ARM, purportedly to make for better management of monthly cash flow, have introduced other risk factors such as expectations of house price appreciation, volatility of borrower cash flows and varying time preferences that calls for fresh analysis in a richer choice framework than the 1980s and 1990s paradigm.[4] To the best of our knowledge the effects of these risk variables on mortgage choice have not been analyzed in an expanded choice framework of competing contracts that includes the BM and PO-ARM alongside the standard mortgage products of 1980s an 1990s as we do in this paper.

A major concern is that depending on the type of mortgage chosen, households are exposed to various risk combinations. These risks include house price volatility risk as in PO-ARM, wealth risk and overpayment for prepayment option as in FRMs, and refunding risk as in BM. Also in varying degrees both the standard ARM and PO-ARM carry the risk of payment shock and cash flow volatility risk, which may be substantial in the case of PO-ARM. Indeed cash flow risk is an inherent attribute of PO-ARM because the periodic adjustment of interest rates are largely uncapped which means that when the mortgage recast to fully amortize the loan for the remaining term the payment shock can be significant. Under these circumstances higher-risk and/or financially savvy borrowers may be more likely to default.

Intuitively, since households' risk aversion should influence mortgage choice decision there is a clear need to take a fresh look at mortgage choice decision with the view to understanding how the new paradigm of NTMs may have reshuffled the deck of mortgage choice. [5] Moreover,

[3] While the demand for balloon mortgages in the US have waned and waxed overtime it is nonetheless an important contract, especially when one takes into account the fact it has been the typical mortgage contract used by our neighbors to the north, Canadians, to lever their investment in housing asset. Typically the balloon mortgage is amortized over a period of say, 20, 25, 30, etc, a period longer than the term of the mortgage, resulting in balloon payment at the end of the contract, which highlights it cash flow and refunding risks.

[4] The flexibility and affordability features of PO-ARM made it the dominant contract of the 2000s. These features essentially camouflaged the complexity and riskiness of the contract which may have led to uneducated choices on the part of mortgage borrowers, especially financially unsophisticated households.

[5] Campbell and Coco (2003) who show that households with lower risk aversion are more likely to choose regular ARM over standard FRM, but their analysis did not include the BM and PO-ARM. They also consider inflation-indexed FRM as one solution to household risk management problem. Although the

the subprime financial crisis that erupted in 2007 has led to growing calls for *suitability standard* for mortgage borrowers, presumably to help borrowers better manage the financial risks associated with different mortgages. If a *suitability standard* is to be instituted for determining the types of mortgage contracts that are appropriate or suitable for various borrowers, what should be the basis of this standard? Our framework allows us to contribute to this debate by analyzing the *partition* or *indifference* points between and among four alternative contracts, the standard FRM, regular ARM, PO-ARM and BM that compete for market shares.

The goal is to analyze how borrowers self-select among the competing mortgage contracts that differ in lender-borrower risk sharing arrangements on the basis of characteristics such as mobility, attitude towards risk (risk aversion), liquidity or affordability constraints, and other market factors such as slope of the yield curve, level of interest rates and expectation about future house prices. To what extent does the introduction of BM and PO-ARM rearranges the choices made by borrowers in a market previously dominated by FRMs and ARMs? How does mobility determine the mortgage choices made by borrowers in this expanded menu context? To what extent do changes in factors such as borrower preferences, expectations of future house prices, lender preferences, mortgage features such as affordability and market conditions affect the optimal mortgage contract choice? Under what circumstances would NTMs exemplified by the PO-ARM dominate the standard FRM, regular ARM and BM as the optimal contract for horizon risk management? Our aim is to provide answers to these questions using numerical analysis that is sufficiently general to accommodate risk factors mentioned above.

The main contributions of our framework are: (1) demonstrate the importance of how expanding the mortgage menu to include the BM and PO-ARM, which expands the lender-borrower risk-sharing space, influence the type of mortgage contract chosen by borrowers, (2) show how the specific circumstances of the borrower (e.g. mobility, attitude toward risk, income volatility, wealth risk. etc) affects the optimal mortgage contract choice decision, (3) how the risk variables introduced by the mortgage innovation of the 2000s, such as expectations about future house prices, borrower cash flow volatility and different notions of time preferences influence mortgage choice, and (4) whether or not the PO-ARM dominates the traditional mortgage products of the 1980s and 1990s and if so to what extent. Since the model is complicated to permit tractable solution, we use plausible values to calibrate and numerically solve the model to address the key questions raised above.

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merits of the inflation-indexed FRM have been noted in the academic literature, it has not really been offered as competing alternative contract in U.S. mortgage market. Dunn and Spatt [1988] and Sa-Aadu and Sirmans [1995] suggest that lumping mortgage contracts may limit our understanding of how private information affects optimal mortgage contact choice

We summarize the main implications of the results of our model's numerical analysis as follows;

Based on the features of the contracts the partition points or market shares we simulated suggest that both the regular ARM and PO-ARM dominate the BM and FRM mortgages in a three way horserace where the menu consists of either PO-ARM, BM and FRM, or regular ARM, BM and FRM. The degree of dominance of PO-ARM and regular ARM depends on the size of the teaser rate. With substantial teaser rate of 1% the regular ARM appears to dominate the BM and FRM more than the PO-ARM does. However, when the teaser rate narrows to say 3% (shallow teaser), the PO-ARM becomes more dominant than the regular ARM in a three-way horserace. Indeed, when the teaser rate is shallow (at 3%) the market share of the regular ARM declines by about -18% relative to its dominance at the deep teaser rate of 1%. In contrast the market share of the PO-ARM contract goes down by only -5% for the same benchmark comparison. We attribute this finding to the negative amortization and recast effects that become less aggravated when the teaser rate is shallow.

The partition points or market shares of the contracts are not static; they are dynamic in that they are affected by changes in borrower characteristics and market conditions. We isolate the characteristics of the household and market conditions that tilt preference for one form of contract over others. For example, a decrease in borrower discount factor, or rising income, tend to push borrowers more towards the PO-ARM and BM. However, rising risk aversion and increases in interest rate volatility together tend to push households more towards the FRM and away from the PO-ARM as well as the BM, although not by much. The main reason for the tilt towards FRM is that it provides protection against the risk of rising interest rates, which the regular ARM and PO-ARM do not. However, the BM does provide protection against rising interest albeit a partial one relative to PO-ARM and regular ARM. Hence, one would expect that the tilt in mortgage preference when interest rates are expected to rise should also favor the BM, given that it is a "cheap FRM".

The magnitudes of utility or welfare delivered by the three mortgage contracts (FRM, PO-ARM, and BM) differ substantially in terms of borrower horizon or tenure choice. Households who expect to move house once every 1 to 2 years (highly mobile households) are clearly better off with PO-ARM. Such households would be able to manage their horizon risk much more effectively if they use the PO-ARM. This result confirms a ubiquitous finding in the literature. On the other hand, we find that the optimal contract for households with intermediate mobility, those who expect to move house say once every 2 to 4 or 5 years is unquestionably the BM; this finding is new. From a utility or welfare gain perspective, the FRM is the most advantageous mortgage contract for households who expect to move house less frequently, e.g. anywhere from once every five or six years and beyond.

The duration over which a contract maintains its optimality with regard to household mobility or tenure, especially in the case of PO-ARM, is largely insensitive to changes in conditioning variables. This implies that mobility could be the main driver of mortgage contract choice. Further the results underscore the need for borrower financial education and counseling as mechanisms to help mitigate investment and financing mistakes of the sort uncovered in Stanton and Wallace (1998). They find that shorter horizon households who select FRM are apt to prepay more often than they would in the presence of symmetric information, and thus incur deadweight cost associated with such prepayment.[6]

There is a basic tradeoff between liquidity or affordability needs and risk aversion of borrowers We find that higher levels of income growth and housing wealth tend to raise the share of both PO-ARM and regular ARMs largely at the expense of the standard FRM and to some extent the BM. However, rising risk aversion as proxied by wealth uncertainty effect tilts borrowers towards the FRM. This result highlights the importance of liquidity or affordability constraints in mortgage choice. From an affordability perspective, the PO-ARM greatly mitigates the liquidity problem encountered by borrowers in inflationary and high interest rate environments given its payment flexibility and initial deep teaser rate. Moreover, rising income and increasing housing wealth coupled with payment flexibility afforded by PO-ARM may provide financially sophisticated households with greater ability to manage monthly cash flow volatility risk and payment shock when the loan recast. Consequently, the augmentation of mortgage menu to include the PO-ARM acts as additional separating device that induces borrowers to further reveal their types as they self-select mortgage contracts that more closely match their liquidity needs, degree of risk aversion and financial sophistication.

Consistent with Brueckner [1992] the choices made by borrowers do influence the relative price of the contracts chosen as borrower preferences and market factors change, in a manner that may seem counterintuitive at first. We analyze the equilibrium conditions for the interest rate on alternative mortgage contracts consistent with the lender earning zero expected profit. Then we test Brueckner's proposition that an increase in the mobility of the FRM pool reduces the price risk of the contract, and thus lowers its interest rate, by varying the average mobility of the FRM and BM contracts. The interest rate on the both the FRM and BM contracts decline consistently as the average mobility of the two pools increases. We also find that an

[6] See Campbell [2006] for an in-depth discussion of the household finance problem in general and in particular the notion that resolving the so-called investment mistakes is central to advancing household finance.

increase in the lenders' discount factor, but a *decrease* in borrowers' discount factor increases the interest rate of both the FRM and the BM. To make sense of this result note that as the borrower's discount factor falls borrower preferences for future consumption declines; this reduces the attractiveness of both the FRM and BM. Intuitively, this outcome in turn reduces their average mobility which increases their relative price.

A decrease in borrower's discount factor tilts the preference of borrowers towards the short horizon PO-ARM and away from standard FRM and BM. One implication of this finding is that as housing markets become more populated by impatient households, households with higher time preference who care more about current rather future consumption, we should expect more households to enter the housing market sooner rather than later. Such household are likely to use risky but initially affordable mortgages such as PO-ARM to accelerate their entry into housing markets.

Households are perhaps neither as risk averse as the selection of FRM suggests, nor are they as risk-loving as the selection of PO-ARM would suggest. The solution of our model using baseline parameterization yields" partition points" or "separation points" at which borrowers are indifferent between and among the four alternative contracts. From the partition points or market shares we infer households disproportionately self-select the short horizon PO-ARM or regular ARM, if the mortgage menu consists of the PO-ARM, BM and FRM or the regular ARM, BM and FRM. For example with teaser rate at the substantial rate of 1% the simulation results suggest the following preference ordering or proportion of borrowers self-selecting alternative contracts in three-contract horserace: PO-ARM (47.14%), BM (33.34%) and FRM (19.51%). However, in a two-contract horserace with either PO-ARM and BM or regular ARM and BM borrowers, and teaser rate at 3%, borrowers tend to slightly prefer the BM over PO-ARM (54.57%) and over regular ARM (55.22%). Hence, if households are perceived as selecting mortgage contracts to manage their horizon risks, they are clearly better off when the menu of mortgage contracts include enough variety to facilitate effective hedging and speculation. In this regard we suspect mobility and affordability constraints to be the main drivers of these outcomes.

The remainder of the paper is organized as follows. The next section provides a brief conceptual overview of the nature of balloon mortgages and pay option ARMs and discusses the risk associated with their key features to further motivate our work Section 3 briefly discusses the related literature. Section 4 develops a model of mortgage choice that details how the contract rate and borrower utility under each mortgage contract is determined. Section 5 presents and discusses the result of our numerical analysis. Section 6 concludes the paper.

2.0 The Nature and Risk of Balloon Mortgage (BM) and Pay Option Adjustable Rate Mortgage (PO-ARM)

2.1 Balloon Mortgage

A balloon mortgage is a relatively short horizon loan compared to a traditional FRM that is amortized over a longer period of say 25-30 years. Because it has shorter term-to-maturity and not fully amortizing there is always a positive outstanding balance when the loan matures in say, 5, 10 or 15 years. Since a BM typically has a shorter term-to-maturity than the FRM, its contract rate should be lower because the lender is exposed to less interest rate risk than on an otherwise FRM that amortizes over an equivalent period. Hence, all else equal the BM is more affordable than standard FRM. Relative to regular ARM, the BM carries less risk of rising interest because its term-to-maturity, equivalent to ARM's interest rate adjustment period, is much longer

In addition to its relative affordability feature the BM is in effect an intermediate horizon contract situated between the long-horizon FRM and the short-horizon ARM, which implies a different lender-borrower risk sharing arrangement between the borrower and lender. For example, the BM provides the borrower insurance against the risk of rising interest rate of the ARM, while mitigating the extreme wealth risk of the FRM when inflation is high.[7] Moreover, because it has shorter term-to-maturity (shorter than the FRM) the cost of the prepayment option is reduced leading to lower contract rate. This effectively makes the BM contract a "cheap" FRM in states of the world where the yield curve is upward sloping. Moreover, a household with a more certain expectation of when to move house or who expects a large infusion of capital before the maturity of the loan should be able to use the BM to undertake a more effective duration or asset-liability matching to manage its horizon risk.

In spite of the foregoing virtues balloon mortgages carry with them some extreme risks. As state above balloon mortgages are by definition partially amortizing; i.e. a balloon payment is due to the lender when the mortgage matures. Borrowers usually fund the balloon payment through refinancing, use of proceeds from sale of the housing asset or through some infusion of large capital (e.g. large settlement, bequest or inheritance) on or before maturity. Of these refunding options the most likely is refinancing. However, even a borrower in good credit standing during the life of the mortgage might be unable to refinance at maturity due higher interest rates, tighter underwriting standards, or deteriorating collateral value among other factors. So BM like PO-ARM carries with it risk associated with future expectations about house prices,

[7] Rising unexpected inflation results in wealth transfer from lenders to borrowers because the inflation premium included in FRM contract rate only reflects expected inflation.

term structure and changing risk aversion. These situations translate into real refunding risk that cannot be discounted.

Historical volume of mortgage purchases by Freddie Mac provides indirect evidence of the importance of balloon mortgages (BMs) in the U.S. mortgage markets. For example, the number of BMs purchased by Freddie Mac grew from 15,000 in 1990 to over 225,000 units by 1992.[8] Moreover, in 1992 BMs constituted about 11.00% of Freddie Mac's total mortgage purchases. However, in recent years the volume of BM originations has shrunk significantly to just under 1% of all mortgage originations in the U.S.[9] The precipitous decline in the volume of BM may be related to the recent flattening of the yield curve, making such mortgages less competitive relative to FRMs. In fact the FRM-BM effective interest rate spread was only 17 basis points as of October 26, 2007. More intriguing is the fact that for the same period, PO-ARMs comprised 15% of all mortgages originated compared to less than 1% for BM, even though the BM-ARM spread was only 3 basis points.

However, since the BM exposes borrowers to less interest-rate risk compared to the ARM one would expect that more borrowers will gravitate towards BMs given the narrow spread of only 3 basis points. This observation raises the issue of whether or not the average prospective borrower understands the relative risk exposures and the comparative advantages (and disadvantages) of different mortgage contracts for horizon risk management. These stylized facts may hint at the general lack of financial education by many borrowers, which has given rise to the call for *suitability standards* for mortgage borrowers to minimize or avoid mistakes in mortgage choice decision.

As noted earlier, the literature on mortgage choice tends to cast the choice decision in a FRM-ARM context, without considering other contracts. Hence, at this point, we do not fully understand how market factors and borrower characteristics (including mobility) interact to determine households' contract choice when the mortgage menu is expanded to include the PO-ARM and BM. For example, does the relatively higher origination of PO-ARM during the mortgage revolution of the 2000s imply that the average household is risk-loving? This study seeks to shed light on this issue in the context of household risk management as reflected by the type of mortgage contract chosen by borrowers. In passing, we note that unlike the U.S. the Canadian mortgage market is dominated by balloon mortgages. Data from Canada Mortgage and

[8] See MacDonald and Holloway [1996] for additional discussion on the volume of BMs origination overtime.
[9] See Mortgage Bankers Association, Weekly Mortgage Application Survey week ending 10/26/2007. Our guess is that the precipitous decline in the origination of BM may be related to the flat term structure.

Housing Corporation show that as September 2007, 58% of all mortgages outstanding were BMs of various maturities of up to 5 years, 29% were ARMs, and the rest were FRMs.

2.2 Pay Option Adjustable Rate Mortgage (PO-ARM)

A PO-ARM belongs to a class of mortgages collectively known as nontraditional mortgages (NTMs) that came into vogue in the 2000s U.S. mortgage revolution.[10] Since 2003 there have a growing use of MTNs including PO-ARM. Appendix table 1 presents data on the volume of NTMs issued, from the June 15, 2007 issue of *Inside Mortgage Finance*, the most frequently cited private available data on NTMs.[11] In 2006, at the peak of U.S. house price boom PO-ARM constituted about 27% of all NTMs originated and NTMs constituted about 32% of roughly $3.0 billion mortgages originated that year. A distinguishing feature of NTM products is that a borrower faces two payment regimes: an initial low payment regime followed by a second regime where payments increase to fully amortize the loan by a certain period. It should be noted the increase in payment is to compensate the lender for pure cost of capital and risk premium.

For various reasons including affordability, flexibility, complexity and riskiness of the mortgages our analysis focuses on PO-ARM as a representative instrument of the NTMs. A typical PO-ARM has several distinguishing features: (1) *teaser rate* which is below market interest rate usually 1% to 2%, *contract interest rate* that changes monthly based on an index plus margin, (2) *payment option* that allows the borrower to decide how much payment to make each month, and (3) *negative amortization*, which results when the borrower makes minimum monthly payment less than the amount of accrued interest.[12] The payment option in essence gives the borrower several payment methods during the option period: (1) a minimum payment based on a substantial low teaser rate, (2) an interest only payment based on the fully indexed contract rate, and (3) conventional fully amortized payment based on 15 year or 30-year amortization period.

In the 2000s PO-ARMs were marketed largely on the basis of their “flexibility and affordability” which conceal their complexity and riskiness. In general, the low introductory teaser rate and multiple monthly payment options permit borrowers, especially first time home

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[10] On the basis of features of the mortgage two other classes of NTMs that became popular in the 2000s are interest only mortgage (fixed and variable) and hybrid ARMs.

[11] In this context it is worth noting that PO-ARM and other NTMs are really not new innovations as the popular press seems to imply. They have been in existence as far back as the early1980s when regulators in response to the S&L crisis that cost taxpayers $140 billion encouraged S&Ls to shift to originating various forms of ARMs to mitigate their interest rate exposure. However, back then PO-ARM were issued primarily by financially sophisticated borrowers as a financial management tool.

[12] Effectively, the negative amortization trigger acts as pseudo line of credit which permits the household to automatically borrow additional amount any time the monthly payment made by the borrower is below the accrued interest.

buyers, to buy more expensive homes than they could have qualified for using more traditional mortgages. In reality, the instrument is fraught with many risks, the primary risk being payment shock, which occurs when the loan recasts and monthly payments increase dramatically, due to several factors acting alone or in concert including cessation of teaser rate, rising interest rate at end of teaser period and negative amortization.

The major concern is that the PO-ARM may be ill suited to some borrowers due its inevitable risk layering and that higher-risk borrowers are more likely to be affected by a major payment shock leading to delinquency and ultimately default. Indeed, persistent negative amortization could cause the value of the mortgage to exceed the value of the house especially as home price appreciation slows and dramatically falls as happened after home prices peaked in 3rdQ of 2006. The net result is the "put" is pushed in-the-money which may cause borrowers to default, especially financially savvy households. Appendix table 2 shows delinquency rates on NTMs, and Fannie and Freddie prime loans. The table shows that the PO-ARM is one of the lowest credit quality mortgage types among the NTMs. Of the 1.1 million PO-ARM outstanding as of June 30, 2008, 31% were 30+ days past due and in foreclosure. The contrast in quality based on delinquency rates between the Fannie and Freddie prime loans and NTMs is striking.

It is clear that PO-ARM carry significant risk (risk layering) and have introduced risk variables including expectations of house price appreciation, borrower cash flow volatility, wealth uncertainty effect and varying time preference that have not been analyzed fully in an expanded mortgage choice framework.[13] We intend to shed light on the effects of these risk factors on the type of mortgage contract chosen by borrowers in an expanded framework that include the PO-ARM and BM mortgages alongside the workhorse FRM and regular ARM.

3.0 Related Literature

Our paper is related to a growing body of literature on mortgage choice decision and mortgage pricing. However, as previously stated the focus of the extant literature has typically been restricted to a choice menu consisting of only the long horizon FRM and short horizon ARM.[14] See for example the empirical work of Dhillon et.al. [1987], Brueckner and Follain [1988], Phillips and VanderHoff [1991], Sa-Aadu and Sirmans [1995], Stanton and Wallace [1998], etc.

[13] Given the complexity and the often confusing features of the PO-ARM a legitimate question is whether borrowers understand the risk associated with this type of mortgage. In a study entitled "Do Homeowners Know Their House Values and Mortgage Terms, Brian Bucks and Karen Pence , Federal Reserve Board, (2006), show that a significant number of borrowers do not understand the terms of their ARMs, particularly the percent by which the interest rate can change, whether there is a cap on increases and the index to which the rate is tied.

[14] For a review of the literature see for example Brueckner [1993], Follain [1990], Stanton and Wallace [1998].

These papers have examined the factors that influence mortgage choice, but in a limited framework consisting of only the FRM and ARM contracts. The general advice emanating from this literature is that more mobile borrowers should select adjustable rate mortgage (ARMs) or mortgage contracts with combinations of high coupons and low points, and less mobile borrowers should select Fixed Rate Mortgages (FRMs) or contracts with higher points and low coupon rates.

Although these important works have enhanced our understanding of the determinants of mortgage choice, the analytical framework is limiting in the sense that it is generally restricted to a choice menu consisting of the two stylized contracts, FRM and ARM. Dunn and Spatt [1988] discuss a number of factors potentially influencing mortgage contract choice and pricing. They argue rather persuasively that in a framework where contract design, prepayment and mortgage pricing are determined simultaneously, the choice made by a borrower facing a variety of mortgage contracts is a maximizing one, consistent with his/her circumstance, although they did not offer a specific model.

Recently, two influential works have sought to broaden the focus of the literature. In a framework that incorporates refinancing decision, Brueckner [1992] studies the effects of self-selection on the pricing of FRM. An insightful result in Brueckner[1992], which at first glance may appear counter intuitive, is that increase in demand for FRM actually reduces its cost. This is because an increase in demand also increases the mobility of the FRM pool which shortens its duration which in turn reduces its price risk. Despite this major contribution the framework was as usual confined to FRM and ARM contracts. Campbell and Coco [2003] study a model of life-cycle with consumption and the decision of how to finance the purchase of house, but focus their analysis on the long horizon FRM and short horizon ARM. Interestingly, when their framework is broadened to include inflation-indexed FRM it is shown to be a superior vehicle for horizon risk management. MacDonald and Holloway [1996] study the default performance of BMs relative to FRMs and ARMs, but without investigating how the presence of the BM in a mortgage menu affects the optimal choices made by households.

Thus the extant literature is largely silent on the key issue of whether and how the household choice behavior would be affected when the choice menu is expanded to include mortgage contracts with different risk-sharing arrangements. While the choice of optimal mortgage contract has many special features, two key features are that the household must plan over a specific horizon and manage its monthly cash flows to assure monthly mortgage payments. Thus the proliferation and diversity of mortgage contracts that differ in maturity, affordability and flexibility in monthly payments spawned by the 2000s innovations in the U.S. mortgage markets may be construed as a strategic attempt by lenders to induce households to reveal their type. That is borrowers self-select into contracts that more closely match their horizons, financial management skills, and their affordability or liquidity constraints.

The challenge then is how best to match households with different horizon, financial management skills and affordability problem with specific mortgage instruments that would enhance their ability to hedge their risks as well speculate. Proper resolution of this challenge should help minimize investment mistakes in mortgage choice decisions and ultimately lead to more efficient household risk management. Consistent with this notion, this study supplements and extends the literature on mortgage choice by analyzing the suitability of various mortgage contracts for horizon and cash flow volatility risk management. We are particularly interested in isolating which mortgage contract(s) is best suited for a household on basis of several key risk variables and borrower characteristics in an expanded choice framework that includes the BM and PO-ARM alongside the stylized FRM and FRM. The risk variables include factors such as borrower mobility, time preferences, future expectation of house prices, cash flow volatility, wealth uncertainty effects and housing wealth effect.

4. Model Specification

4.1 Basic assumption

Our model consists of five periods and six dates, *t*=0, 1, 2,3,4 and 5. Each period in the model is in reality a compression of 72 months, which translates into a 30-year or 360-month mortgage. The 5-period compressed model was employed to simplify the extensive computation involved in the subsequent numerical analysis and very importantly to incorporate key properties of both the PO-ARM and BM contracts.

For example, assume a BM with 30-year amortization period and monthly payments that are fixed for the first twelve years (i.e. 12/18 BM). This structure translates into a 2/3 BM in our compressed model where the periodic payments are fixed for the first two periods and the mortgage payments are amortized over 5 periods. At the end of period two the borrower can either prepay the outstanding balance or re-contract into another fixed rate fully amortized loan for the remaining 3 periods.

In the case of PO-ARM the model allows monthly interest rate to adjust every period with no limit except life of loan interest rate cap, typical of PO-ARMs that were prevalent in the 2000s. However, period to period payments are allowed to increase by no more than 7.5% with two exceptions. The first is that every 3 periods the loan will “recast” to replicate a fully amortizing loan at the full indexed rate (index value + margin). The second is that when the contemporaneous outstanding loan amount reaches a negative amortization maximum of 115% of the original loan amount (a key feature of PO-ARM), the periodic payment will immediately increase (or recast) to the fully amortizing level, regardless of the size of the increase.[15]

In our model borrowers face a menu consisting of four alternative contracts, FRM, ARM, BM and PO-ARM. The FRM and the ARMs have extreme rate-risk combinations while the BM has an intermediate rate-risk combination. At $t = 0$, the borrower selects one of the four contracts and at time $t = 1, 2, 3$, or 4, the borrower either moves house and prepays the loan as required by due-on-sale clause, or continues with the loan contract for another period. Except for differences in the probability of moving, borrowers are assumed to be identical. The borrower has an exogenous probability of moving, λ, at the end of each period which lies between 0 and 1 and is assumed to have a uniform distribution, $f(\lambda)$. Mortgage prepayment occurs exogenously due to a move by the borrower[16].

4.2 Interest Rates on Alternate Mortgage Contracts

To establish the interest rates on the alternative mortgage contracts, we assume lenders are risk-neutral and face a competitive market with zero expected profit whenever they invest in each of the four alternative contracts. Short-term interest rates for each period: r_0, r_1, r_2, r_3, and r_4 are assumed to have an upward drift with the same variance and have cumulative distribution functions of $F(r_0)$, $F(r_1)$, $F(r_2)$, $F(r_3)$, and $F(r_4)$ respectively. This means that $r_0<E(r_1)<E(r_2)<E(r_3)<E(r_4)$, *and* $Var(r_1)=Var(r_2)=Var(r_3)=Var(r_4)=\sigma^2$.[17]

Let $\theta < 1$ denote lender's discount factor, then the expected discounted FRM profit at time zero, per dollar of loan, can be written as

$$\begin{aligned}&(i_f - r_0) + \theta(1-E(\lambda))\int(i_f - r_1) f(r_1)dr_1 \\ &\quad + \theta^2(1-E(\lambda))^2\int(i_f - r_2) f(r_2)dr_2 + \theta^3(1-E(\lambda))^3\int(i_f - r_3) f(r_3)dr_3 \\ &\quad + \theta^4(1-E(\lambda))^4\int(i_f - r_4) f(r_4)dr_4 \qquad (1),\end{aligned}$$

[15] All PO-ARMs have negative amortization trigger that ranges from 110% to 125% of the original loan balance and a loan recast rule. For a borrower who has chosen the minimum payment option the combination of these two features means that the probability of payment shock is greater the larger is the increase in the interest rate index; the larger is the margin and the lower (or deeper) the initial teaser rate that determined the minimum payment.

[16] We exclude interest-rate motivated prepayment from the discussion to focus on the effect of mobility on mortgage choice.

[17] This assumption implies that the interest-rate yield curve is upward sloping. Under this assumption, a borrower with high probability of moving will prefer an ARM and a borrower with low probability of moving will choose an FRM. If the yield curve is downward-sloping, the choice preference will be reversed.

where $E(\lambda)$ is the average probability of moving by borrowers who choose FRMs. The profit from borrowers who do not prepay has a random component realized in periods 1, 2, 3, and 4 that depends on the difference between the contract rate on the mortgage and the prevailing market rate in the respective periods.

Setting the profit on FRM equal to zero, the solution for the FRM interest rate, i_f, consistent with zero expected profit is,

$$i_f = \frac{r_0 + \theta(1-E(\lambda))E(r_1) + \theta^2(1-E(\lambda))^2 E(r_2) + \theta^3(1-E(\lambda))^3 E(r_3) + \theta^4(1-E(\lambda))^4 E(r_4)}{1+\theta(1-E(\lambda)) + \theta^2(1-E(\lambda))^2 + \theta^3(1-E(\lambda))^3 + \theta^4(1-E(\lambda))^4} \quad (2)$$

As in Brueckner [1992] our regular ARM and PO-ARM have no interest-rate caps and thus the borrower absorbs all interest rate risk. Then consistent with the lender earning zero expected profit, the interest rate on the regular ARM and PO-ARM is simply equal to the prevailing short-term interest rate. Alternatively, the markup on the ARM contract rate, α_1, is shown to be zero[18]:

$$\alpha_1 + \theta(1-E(\lambda))\alpha_1 + \theta^2(1-E(\lambda))^2\alpha_1 + \theta^3(1-E(\lambda))^3\alpha_1 = 0 \implies \alpha_1 = 0 \quad (3)$$

Our stylized BM has two interest rates. The initial interest rate, i_b, which prevails over the initial term of the BM is established at time zero, and the second interest rate, i_d, which is random, will prevail during the second term of the loan if the borrower re-contracts.[19] The expected discounted value of the balloon mortgage contract in period zero, per dollar of loan for the first two periods can be written as

$$(i_b - r_0) + \theta(1-E(\lambda))\int (i_b - r_1) f(r_1) dr_1 = 0 \quad (4)$$

Setting (4) equation equal to zero, the interest rate for the initial term of the BM contract, i_b,, is

$$\Rightarrow i_b = \frac{r_0 + \theta(1-E(\lambda))E(r_1)}{1+\theta(1-E(\lambda))} \quad (5)$$

---

[18] Brueckner [1993] shows that uncapped ARMs can exist in the market even if lenders are risk-neutral, and the interest rate will be $\alpha_1 + r_i$

[19] Balloon mortgages are structured such that the borrower can either payoff the remaining balance or re-contract when the loan matures. The re-contracting option allows the borrower to reset the interest rate to the current market rate for the remainder of the amortization period. Two typical balloon mortgages are 5/25, and 7/23. The first number (5 or 7) is balloon maturity date and the second (25 or 23) is the remaining amortization period.

Similarly, the expected discounted profit and interest rate for the second term of the BM loan if the borrower re-contracts (refinances) are respectively

$$\int (i_d - r_2) f(r_2) dr_2 + \theta(1 - E(\lambda)) \int (i_d - r_3) f(r_3) dr_3 + \theta^2 (1 - E(\lambda))^2 \int (i_d - r_4) f(r_4) dr_4 = 0 \quad (6)$$

$$\Rightarrow E(i_d) = \frac{E(r_2) + \theta(1 - E(\lambda))E(r_3) + \theta^2 (1 - E(\lambda))^2 E(r_4)}{1 + \theta(1 - E(\lambda)) + \theta^2 (1 - E(\lambda))^2} \quad (7)$$

Equation (5) shows that interest rate for the initial term of the BM i_b is a function of the market interest rate, r_0, and the expected market interest rate at $t = 1$, $E(r_1)$, while equation (7) shows that BM interest rate for the second term if the borrower re-contracts, i_d, is a function of the expected market interest rates in periods 2, 3, and 4, $E(r_2)$, $E(r_3)$, and $E(r_4)$.

Equations (2), (4), and (7) also show that the interest rate on both FRMs and BMs are determined by the lender's discount factor, θ, and the average mobility of borrowers who choose these contracts, $E(\lambda)$. Indeed, it can be shown that $\frac{\partial i}{\partial \theta} > 0$ and $\frac{\partial i}{\partial E(\lambda)} < 0$. The first inequality suggests that an increase in the lenders discount factor increases the interest rate on these contracts, while the second inequality implies that the price of these contracts decrease as the average mobility of the their respective pools increases. The reason is that as average mobility increases the duration of the pools shortens which lowers the lender's price risk and therefore the price of these contracts must decrease. This is the key insight in Brueckner [1992] concerning the unusual price behavior of the FRM. We verify the validity of this insight for both the FRM and the BM using numerical analysis.

Now consider a mortgage market in which all four mortgage contracts, FRM, BM, regular ARM, and PO-ARM are arranged in order of term-to-maturity long, medium, and short, etc. In such a world it is easy to see that the expected yield on the ARMs should be the lowest at the origination date assuming an upward sloping yield curve. Further, the total risk premium, and hence the expected yield to the lender, increases as we move from the pure ARMs to the partially-amortizing BM and to the fully-amortizing FRM (see Figure 1). If households can borrow and lend on the same terms as lenders, they can essentially duplicate the effects of a FRM on their own. Consequently, the household with average mobility should be indifferent between any loan priced on the line segment AB, as shown in Figure 1.

Next, assume that borrowers have better information than lenders with respect to their actual mobility. Further assume that there are three types of borrowers: high, medium and low mobility borrowers. Now we can make some general observations about this situation. First, realizing that they are unlikely to pay off their mortgages any time soon, low mobility borrowers will expect to incur a high payment shock due to variability in interest rates and the effect of negative amortization when a PO-ARM and to some extent standard ARM is selected. However, low mobility borrowers who choose a FRM will find the built-in prepayment option underpriced. The second observation is that the converse should hold for high mobility borrowers. Their high mobility pattern should offset the default risk premium associated with an ARM to some extent, thus reducing the effective cost of borrowing. In contrast, if such households select FRM, their mobility pattern will increase the effective cost of borrowing, because the cost of the prepayment option will be excessive.

The third observation is that the partially-amortizing BM may or may not be an optimal loan contract for high mobility borrowers depending on the length of the partial amortization period. If the partial amortization period i.e. the term-to-maturity is short enough, high mobility borrowers should find BMs a better choice. Likewise if the term-to-maturity of the BM is long enough, low mobility borrowers should find BMs to be optimal. For such households BMs become cheap FRMs. The fourth observation is that it is an empirical question which type of low or high mobility borrowers should prefer to issue the partially amortizing BM. That is what should be the level of a household mobility in order for the BM contract to be optimal for that household. This is one of the key issues addressed in this paper. Finally, there should, in theory, be partition or separation points that make the average borrower indifferent to the selection of any of the four mortgage contracts. We estimate these partition points and interpret them as relative market shares.

4.3 Utility function

The household is assumed to have Von-Neumann-Morgenstern utility function, *V(.)*, and discount factor δ. The wealth endowment of the household for each of the five periods is denoted z_0, z_1, z_2, z_3, and z_4 which for simplicity are assumed to be known with certainty. Thus the only source of uncertainty is the randomness of short-term interest rates. Here, we have standardized the mortgage principal to equal 1. In each period, borrowers make the mortgage payment based on contract rate and the outstanding mortgage balance. If the borrower moves house, he/she must prepay the mortgage outstanding.

Although there is a large academic literature on mortgage choice, the typical assumption is that the borrower uses an interest only mortgage. Unlike the extant literature, we allow for the amortization of principal in order to capture a critical and distinguishing property of the balloon mortgage contract i.e. the balloon amount at the end of first term which must be repaid or refinanced by the borrower. Like Brueckner [1992] we assume borrowers will refinance into ARM contracts for the rest of their homeownership where ever they prepay a mortgage. [20]

Then the expected utility of FRM borrowers can be specified as:

$$
\begin{aligned}
V_{FRM}(\lambda) = \lambda[&\mathrm{V}(\mathrm{Z}_0 - X_1^1(1+i_f)) + \delta\int V(Z_1 - X_2^1(r_1)) f(r_1)dr_1 + \delta^2\int V(Z_2 - X_3^1(r_2)) f(r_2))dr_2 \\
&+ \delta^3\int V(Z_3 - X_4^1(r_3)) f(r_3)dr_3 + \delta^4\int V(Z_4 - X_5^1(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)[&V(Z_0 - X_1^2(i_f)) + \delta V(Z_1 - X_2^2(1+i_f)) + \delta^2\int V(Z_2 - X_3^2(r_2)) f(r_2)dr_2 \\
&+ \delta^3\int V(Z_3 - X_4^2(r_3)) f(r_3)dr_3 + \delta^4\int V(Z_4 - X_5^2(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)^2[&V(Z_0 - X_1^3(i_f)) + \delta V(Z_1 - X_2^3(i_f)) + \delta^2 V(Z_2 - X_3^3(1+i_f)) \\
&+ \delta^3\int V(Z_3 - X_4^3(r_3)) f(r_3)dr_3 + \delta^4\int V(Z_4 - X_5^3(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)^3[&V(Z_0 - X_1^4(i_f)) + \delta V(Z_1 - X_2^4(i_f)) + \delta^2 V(Z_2 - X_3^4(i_f)) \\
&+ \delta^3 V(Z_3 - X_4^4(1+i_f)) + \delta^4\int V(Z_4 - X_5^4(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)^4[&V(Z_0 - X_1^5(i_f)) + \delta V(Z_1 - X_2^5(i_f)) + \delta^2 V(Z_2 - X_3^5(i_f)) \\
&+ \delta^3 V(Z_3 - X_4^5(i_f)) + \delta^4 V(Z_4 - X_5^5(i_f))]
\end{aligned}
$$

Similarly, the expected utility from choosing the BM can be written as:

$$
\begin{aligned}
V_{BM}(\lambda) = \lambda[&\mathrm{V}(\mathrm{Z}_0 - X_1^1(1+i_b)) + \delta\int V(Z_1 - X_2^1(r_1)) f(r_1)dr_1 + \delta^2\int V(Z_2 - X_3^1(r_2)) f(r_2))dr_2 \\
&+ \delta^3\int V(Z_3 - X_4^1(r_3)) f(r_3)dr_3 + \delta^4\int V(Z_4 - X_5^1(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)[&V(Z_0 - X_1^2(i_b)) + \delta V(Z_1 - X_2^2(1+i_b)) + \delta^2\int V(Z_2 - X_3^2(r_2)) f(r_2)dr_2 \\
&+ \delta^3\int V(Z_3 - X_4^2(r_3)) f(r_3)dr_3 + \delta^4\int V(Z_4 - X_5^2(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)^2[&V(Z_0 - X_1^3(i_b)) + \delta V(Z_1 - X_2^3(i_b)) + \delta^2\int V(Z_2 - X_3^3(1+i_d)) f(i_d)di_d \\
&+ \delta^3\int V(Z_3 - X_4^3(r_3)) f(r_3)dr_3 + \delta^4\int V(Z_4 - X_5^3(r_4)) f(r_4)dr_4] \\
+ \lambda(1-\lambda)^3[&V(Z_0 - X_1^4(i_f)) + \delta V(Z_1 - X_2^4(i_f)) + \delta^2\int V(Z_2 - X_3^4(i_d)) f(i_d)di_d \\
&+ \delta^3\int V(Z_3 - X_4^4(1+i_d)) f(i_d)di_d + \delta^4\int V(Z_4 - X_5^4(r_4)) f(r_4)dr_4]
\end{aligned}
$$

[20] We thank Brueckner for suggesting this idea which simplified and made the model more tractable..

$$+\lambda(1-\lambda)^4[V(Z_0-X_1^5(i_b))+\delta V(Z_1-X_2^5(i_b))+\delta^2\int V(Z_2-X_3^5(i_d))f(i_d)di_d$$
$$+\delta^3\int V(Z_3-X_4^5(i_d))f(i_d)di_d+\delta^4\int V(Z_4-X_5^5(i_d))f(i_d)di_d]$$

Finally, the expected utility of borrowers who choose either regular ARMs or PO-ARMs is:

$$V_{PO\_ARM}\ or_{ARM}(\lambda)=\lambda[\mathrm{V}(\mathrm{Z}_0-X_1^1(1+r_0^I))+\delta\int V(Z_1-X_2^1(r_1))f(r_1)dr_1+\delta^2\int V(Z_2-X_3^1(r_2))f(r_2))dr_2$$
$$+\delta^3\int V(Z_3-X_4^1(r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^1(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)[V(Z_0-X_1^2(r_0^I))+\delta\int V(Z_1-X_2^2(1+r_1))f(r_1)dr_1+\delta^2\int V(Z_2-X_3^2(r_2))f(r_2)dr_2$$
$$+\delta^3\int V(Z_3-X_4^2(r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^2(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)^2[V(Z_0-X_1^3(r_0^I))+\delta\int V(Z_1-X_2^3(r_1))f(r_1)dr_1+\delta^2\int V(Z_2-X_3^3(1+r_2))f(r_2)dr_2$$
$$+\delta^3\int V(Z_3-X_4^3(r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^3(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)^3[V(Z_0-X_1^4(r_0^I))+\delta\int V(Z_1-X_2^2(r_1))f(r_1)dr_1+\delta^2\int V(Z_2-X_3^4(r_2))f(r_2)dr_2$$
$$+\delta^3\int V(Z_3-X_4^4(1+r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^4(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)^4[V(Z_0-X_1^5(r_0^I))+\delta\int V(Z_1-X_2^2(r_1))f(r_1)dr_1+\delta^2\int V(Z_2-X_3^5(r_2))f(r_2)dr_2$$
$$+\delta^3\int V(Z_3-X_4^5(r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^5(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)[V(Z_0-X_1^2(i_f))+\delta V(Z_1-X_2^2(1+i_f))+\delta^2\int V(Z_2-X_3^2(r_2))f(r_2)dr_2$$
$$+\delta^3\int V(Z_3-X_4^2(r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^2(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)^2[V(Z_0-X_1^3(i_f))+\delta V(Z_1-X_2^3(i_f))+\delta^2 V(Z_2-X_3^3(1+i_f))$$
$$+\delta^3\int V(Z_3-X_4^3(r_3))f(r_3)dr_3+\delta^4\int V(Z_4-X_5^3(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)^3[V(Z_0-X_1^4(i_f))+\delta V(Z_1-X_2^4(i_f))+\delta^2 V(Z_2-X_3^4(i_f))$$
$$+\delta^3 V(Z_3-X_4^4(1+i_f))+\delta^4\int V(Z_4-X_5^4(r_4))f(r_4)dr_4]$$
$$+\lambda(1-\lambda)^4[V(Z_0-X_1^4(i_f))+\delta V(Z_1-X_2^4(i_f))+\delta^2 V(Z_2-X_3^4(i_f))$$
$$+\delta^3 V(Z_3-X_4^4(i_f))+\delta^4 V(Z_4-X_5^4(i_f))]$$

4.4 Optimization

We study the optimal mortgage choice of households as defined by their probability of moving conditional on their horizon risk management problems. The borrower's optimization problem is solved assuming market equilibrium, each time with a mortgage menu consisting three contracts: FRM, PO-ARM and BM, or FRM, regular ARM and BM. The borrower solves a dynamic problem by maximizing expected utility subject to the incentive compatibility and the lender's zero expected profit constraints.

Now suppose the market consists of a continuum of borrowers with different probabilities of moving λ where all λ in the interval [0, 1] are represented in the pool of borrowers. Let λ* and λ** denote critical partition points such that the marginal borrower is indifferent among three contracts or between a pair of contracts each. The borrower's optimization problem when the menu includes three contracts, is derived by choosing interest rates, i_f, i_b, and i_d, and two partition points, λ* and λ**such that the borrower's expected utility is maximized subject to incentive compatibility constraints and lender's zero expected profit constraint. The preceding utility equations show that in all cases the household discount factor, wealth endowment, and the relevant contract rate affect expected utility of the household for a given mortgage contract.

The optimization problem is reduced to the following simultaneous equations

$V_{FRM}(\lambda^*) = V_{BM}(\lambda^*)$

$V_{BM}(\lambda^{**}) = V_{ARM}(\lambda^{**})$

The *partition points* (or *indifference points*) λ* and λ** determine the equilibrium interest rates for three mortgage contracts, with either regular ARM or PO-ARM among three. These partition or indifference points can also be interpreted as market shares of the mortgages included in the menu.

The borrower's optimization problem is complicated to permit tractable closed-form solutions for equilibrium analysis. Hence we use standard numerical technique to obtain a solution for the problem. To implement the numerical analysis, we assume borrower preferences are captured by exponential utility function: $V(Z - PMT) = 1 - e^{-R(Z-PMT)}$ *, V′> 0, V′′< .0,* where *Z* is the wealth endowment of the household, *PMT* is mortgage payment, and R $= (1-\lambda)\beta\left[Z_0 - \frac{r_0}{1+(1+r_0)^{-5}} - H\right]$ is the Arrow-Pratt measure of risk-aversion. The Pratt measure of risk-aversion is a function of mobility (λ), wealth uncertainty effect (β), borrowers' initial income (Z_0), initial mortgage payment ($\frac{r_0}{1+(1+r_0)^{-5}}$) and housing wealth effect (H).

In this context an exponential utility function with (1-λ) implies that more mobile borrowers have a small CARA. The parameter, β, measures the sensitivity of borrowers' risk aversion to wealth uncertainty effect, particularly due to income volatility; Z_0, borrowers are likely to be less risk averse when initial income is high. Likewise when initial interest rates, r_0, are high, borrower's affordability is low and borrowers become more risk averse. The parameter H in the risk aversion coefficient gauges housing wealth effect (HWE), by means of consumption change induced by house price appreciation.

Recall that in this paper, we allow the mortgage loans to amortize instead of assuming interest only mortgages. The cost of allowing the mortgages to amortize is that it increases the difficulty of solving an expected utility function such as the following: $\int V(z_1 - \frac{r_1}{1-(1+r_1)^{-3}} f(r_1) dr_1$. We employ Taylor's expansion to approximately estimate $\frac{r_1}{1-(1+r_1)^{-3}}$, and then use the moment generation function of normal distribution to derive the answer. We use numerical methods to calculate expected utility from each contract. Appendix 4 shows the derivation.

Also recall our 5-period model is compressed to represent a 360-month mortgage. Table 1 shows the parameter assumptions in our baseline simulation. The annual contract rate in this setting is denoted, $R_{A,}$ and the corresponding monthly contract rate is then R_A/12. Then the equivalent periodic rate in the 5-period compressed model, R_P = ((1+R_A/12)^360)^(1/5)-1.

We experiment with teaser rate of 1% (deep or substantial teaser) and 3% (shallow teaser) for both the regular ARM and PO-ARM. The teaser rate for typical PO-ARM is 1% to 2%, which implies a much deeper subsidy than that for the standard ARM, which is consistent with industry practice. We proxy interest rate volatility with the variance of interest rate which is time varying. Also we assume borrowers are more risk sensitive than lenders; hence the borrower discount rate is set to be larger than the discount rate of lenders.

5.0 Results

This section presents the results of our analysis and their implications. Our analysis consists of first estimating the *partition or indifference points* between and among the three contracts. From these results we infer borrowers' preference or market share for each of the four mortgage types. Next, we proceed to analyze the welfare implications of optimal mortgage choice by calculating the expected utility associated with each mortgage contract under different probabilities of moving. Based on the magnitude of expected utility delivered by each contract, we infer the optimal mortgage contract for managing horizon risk consistent with household expected mobility. Finally, we consider the effects of mortgage choice, borrower characteristics and market factors on mortgage contract rates.

5.1. Solution of the Partition Points (Market Shares)

The literature on housing wealth effect (HWE) suggests that households may be more willing to use affordable but risky mortgage contracts such as PO-ARM to create leverage position in the housing asset.[21] But since house prices are volatile and the house is a risky asset households with different degrees of risk aversion may react differently in their mortgage selection. Because of this our model as stated above includes the Pratt-Arrow risk aversion measure to gauge the effect of households' attitude towards on mortgage choice

Table 2 shows the self-selection behavior of households under different composition of the mortgage menu, based on the solution of the model using baseline parameters shown in table 1. We are particularly interested in the effects of the introduction of the BM and PO-ARM on the partition points or the choices households should make in mortgage markets previously dominated by standard FRM and regular ARM. In particular, PO-ARMs became prevalent contracts during the mortgage revolution of the 2000s. Some experts argue that this innovation replaced the dominant paradigm of the 1980s and 1990s, when the standard FRM and regular ARM dominated the U.S. mortgage market. We shed light on the extent to which the flexibility and affordability features of PO-ARMs may have tilted self-selection more towards this controversial contract in an expanded framework consisting of four mortgage contracts.

Accordingly, we first consider how households self-select in a three-way horserace with the menu consisting of three contracts: FRM, regular ARM and BM; FRM, PO-ARM and BM. As shown in Panel A of the table 2 when the menu consists of three contracts made up of FRM, regular ARM and BM and teaser rate set at 1%, a majority of the households overwhelmingly prefer the regular ARM over the FRM and BM. With similar composition of three contracts but with PO-ARM in place of the regular ARM and a deep teaser rate of 1%, our simulation results suggest PO-ARM dominates both the FRM and BM, but the extent of its dominance over these contracts is slightly below that of the regular ARM (53.9% versus 47.1%). (See panels A and C of table 2).

Next, in a world with the same composition of contracts (i.e. FRM, PO-ARM and BM or FRM, regular ARM and BM) but reduced teaser rate, for example teaser rate set at 3%, the results

[21] Studies of HWE show that rise in house prices increases the level of wealth which causes household to consume more. In a study that covers 14 western countries Case, Quigley and Shiller (2005) find that aggregate housing wealth has a significant effect on aggregate consumption and that the effect dominates that of financial wealth. Thus it stands to reason that as house price rise, which ceteris paribus reduces affordability, households may gravitate towards relatively more affordable mortgages to enable them to consume more housing. Invariably mortgages that are structured to increase affordability by means of lower initial contract rats such as standard ARMs, PO-ARMs and BM tend to be who more risky for the borrower in that the burden of risk-sharing tilts more towards the borrower than the lender to make the reduced interest rate rational.

of the numerical solution of our model suggests that 45.2% of borrowers will self-select into PO-ARM, compared 44.5% for regular ARM (See panels B and D of table 2). Note that the relative dominance of the regular ARM is reduced by -18% (.4448-.5393)/.5393 compared to a reduction of only -5.0% for the PO-ARM, as we go from a deep teaser rate of 1% to a shallow teaser rate of 3%. Interestingly, when the choice is strictly between PO-ARM and BM or PO-ARM and FRM our simulation results suggest both FRM and BM dominate PO-ARM, regardless of the size teaser rate, although the dominance is not overwhelming (See panels C and D columns 2, and 4).

The preceding results leave the answer to the question of whether or not NTMs, as represented by PO-ARMs in this analysis, dominate the standard mortgage contracts of the 1980s and 1990s somewhat murky. In summary, however, our simulation results lead to the conclusion that PO-ARMs in fact dominate the standard FRM and BM, especially in a three way contest, but with some qualifications. The qualifications hinges on both the composition of the mortgage menu at any point in time and the complex features of the PO-ARM that tend to elevate its risk. First, in regard to menu composition we see that when the menu consists of all three contracts concurrently, the PO-ARM is undeniably the dominant contract. Second, the substantial teaser rate of 1%, typical during the mortgage revolution of the 2000s, clearly makes PO-ARMs more affordable especially during periods of rising house prices, but also reduces the dominance of the contract due to increase chance of negative amortization.

Further, the deeper is the teaser rate the more substantial is the mortgage payment when the loan "recast" even if the index value does not go up. Also, for most PO-ARMs the initial teaser rate lasts for only one to three months and then adjusts freely with no limit on size of interest rate increase except a maximum over the life of the loan. These outcomes either severally or jointly may lead to significant payment shock which should dampen the appetite or enthusiasm for PO-ARMs contracts causing both the standard FRM and BM to dominate the PO-ARMs in a two-way horserace as we have demonstrated above.

So why might the BM dominate the FRM? To understand borrower preference for the BM at the expense of the FRM as shown in Table 2, recognize that while the insurance protection against the risk of rising interest rate provided by the BM contract is not as extensive as that of the FRM, both the term premium and the cost of prepayment option embedded in the BM are also lower. The combination of these factors makes the BM cheaper and more competitive than the FRM in certain states of the world. Thus households, especially those with medium term horizon, should find the BM more efficient than the FRM for managing horizon risk. Such households seek to only partially self-insure against rising interest rates and do not need a longer window of protection to exercise the refinancing option.

The preceding narrative of course begs the question as to why borrowers enthusiastically embraced PO-ARM contracts during the 2000s run-up in house prices. First, the affordability and flexibility features of the PO-ARM may have masked or caused borrowers to develop myopia about the complexity and risk of the contracts, the consequences of which borrowers may not have understood.[22] Indeed, a 2006 report to Congress by the FDIC argues that while financial institutions generally appeared to be making disclosures required by regulation, the disclosures were not designed to address the features of NTMs such as PO-ARMs.[23] Second, retrospectively, it is reasonable to surmise that some financially sophisticated households may have acted strategically in their choice of PO-ARMs contracts based on its affordability and payment flexibility. Such financially savvy household can then take advantage of the payment flexibility and initial affordability and use the PO-ARM as a financial management tool, knowing full well that when the option to default is "in-the-money" they can walk away from the house. Additionally, it is entirely possible that borrowers may have assumed based on "story telling" that house price appreciation will enable them to escape payment shock and avoid foreclosure by refinancing. In retrospect this falsehood appears wildly apparent in 2007.

While the numerical analyses are sensitive to parameter values, at this point there are two striking observations from the baseline analysis. The first observation is that PO-ARMs dominate both the standard FRM and BM if the menu consists of all three contracts, regardless of whether the teaser rate is deep at 1% or shallow at 3%. The second observation is that in cases where the choice is strictly between say the PO-ARM and FRM, or PO-ARM and BM some borrowers may switch to either the FRM or BM. We interpret this result to mean that the average household is not as highly mobile or highly immobile as the respective structures of the PO-ARM and FRM would suggest. Rather, the average mobility of the marginal borrower is somewhere in between the two extreme contract maturities and is shaped by other factors.

Alternatively, the implication of the second observation from the baseline results is that the marginal borrower is neither extremely risk averse nor extremely risk-loving as the selection of the FRM and the PO-ARM (or regular ARM), respectively, would suggest. Hence, if households are perceived to be attempting to manage their horizon risks, i.e. select mortgage contracts that match their holding period, and/or manage their monthly cash flows, households would be clearly better off when the menu of contracts has enough variety to allow for meaningful separation, positioning and interest rate risk hedging by borrowers.

[22] Additionally Angell and Williams (2005) raised the possibility post-2003 rise in house prices might be related to the rising share of PO-ARMs and other NTMs.

[23] See Sandra L. Thompson (2006), Statement on Nontraditional Mortgage Products, Subcommittee on Housing and Transportation of the Committee on Banking, Housing and Urban Affairs U.S. Senate

5.2 Sensitivity Analysis

Obviously, the preceding results would be sensitive to changes in our baseline assumptions. To gain further insight into borrower self-selection of mortgage type we vary the baseline parameters. We are particularly interested on how changes in the lender discount factor, borrower discount factor, risk aversion, slope of the yield curve, initial interest rate level, initial borrower income level, and borrower income tilt, affect the partition points or implied market shares as households self-select into specific contracts.

Panel A of table 3 suggests that a change in the lender discount rate has clear effect on the proportion of borrowers that self-select into various contracts. With the lender's discount rate at 4%, alternatively a lender discount factor of 0.96, the proportion of borrowers choosing PO-ARM, BM, and FRM are 47.14%%,33.34% and 19.51%, respectively. As the lender's discount rate rises, correspondingly as the lender's discount factor declines, the proportion of borrowers choosing both the PO-ARM and BM declines with the FRM being the beneficiary contract. Indeed as the lender's discount rate doubles from 4% to 8% preference for the BM declines precipitously by -69%, where as the preference for the FRM increase by more than 100% with the same doubling of lender discount rate. To interpret this self-selection behavior, recall that as the lender's discount factor declines, the lender cares less about future consumption and the average premium on FRM contract declines, which increases the attractiveness of this contract over both the PO-ARM and BM.

In contrast, increasing borrower discount rate (decreasing borrower discount factor) decreases borrower preferences for the FRM, the contract that offers complete protection against the risk of rising interest rate, while preference for both BM an PO-ARM rise (see Panel B of table 3. As the borrower discount rate increases from 4% to 6% the preference for the BM and PO-ARM rise by 30.6% and 7.75%, respectively. Unconditionally, one would expect that the mortgage contract likely to increase its market share in response to rising borrower discount rate should be the PO-ARM with substantial teaser rate; rather the BM appears to be the beneficiary. Overall, the results suggest that under the right circumstances, e.g. higher interest rate and higher inflation environments, or lower affordability, relatively long horizon households may be willing to take on more interest rate risk to enhance affordability so as to increase the odds of purchasing a house. Parenthetically, we also note that borrowers may not be willing to completely give up the protection against rising interest rate offered by the FRM as evidenced by the lower increase in the market share of the PO-ARM in comparison with that of the BM.

Panel C in table 3 shows that rising borrower risk aversion, as measured by wealth uncertainty effect, unambiguously reduces borrower preference for the PO-ARM, the contract with the most uncertain future payment, but surprisingly not by much. With risk aversion factor at 1.18 the initial proportion of borrowers selecting, PO-ARM, BM, and FRM are 47.56%, 33.76% and 18.67%, respectively. Indeed, a rise in the coefficient of risk aversion from 1.18 to 1.22 reduces the choice of PO-ARM and BM by only -1.74% and -2.43%, respectively. In contrast the preference for FRM rises from 18.76% to 20.33% as the risk rises from 1.18 to 1.22. We suspect that the relatively limited but clear increase in preference for the FRM as risk aversion rises, +8.89%, [(0.2033-0.1867)/0.1867] may be due to the attenuating effects of HWE, household mobility, and possibly lower payment on PO-ARM and BM, relative to FRM. One interpretation of this result is that expectations of house price appreciation may cause households to gravitate towards risky mortgages such as PO-ARM, if it increases likelihood of longing the housing asset despite its volatility. Additionally, since our risk aversion parameter is also a function of household income this result captures the cash flow volatility of the borrower which is a relevant risk variable that may influence the choice of PO-ARM. Indeed panels D and F suggest that HWE and increasing household income may in fact moderate the impact of rising risk aversion.

Panel E of table 3 shows that a steepening of the yield curve accompanied by rising interest rate volatility increases the proportion of households who self-select FRM relative to both the BM and the PO-ARM. However, the reduction in preference for both the PO-ARM and BM is relatively minor. Hence the PO-ARM is still competitive relative to FRM even though it does not offer protection against rising interest rate as the former contract does. Going back Panel D the results suggest that borrower's with rising income are less likely to use the FRM and more likely to prefer PO-ARMs and BM. In this case one would like to tell an ability-to-bear interest rate risk story. It is clear that rising income allows households to better handle the payment shock that may accompany increases in PO-ARM payments.

Table 4 shows the effects of changing market interest rate level and the size of the teaser on the choice of mortgage contract type. Panel C shows that keeping the teaser rate at the substantial level of 1% in the presence of rising market rate slightly discourage borrowers from selecting PO-ARM, although the proportion of borrowers selecting this contract dominates those of the FRM and BM contract at every level of market interest rate. In fact the proportion of borrowers selecting the BM first drops significantly and then flattens out as market interest rate rise. We interpret the gradual reduction in the demand for the PO-ARM as reflecting the dampening effect of negative amortization which becomes exacerbated as market interest rate rise. Incidentally, when the PO-ARM is replaced with the regular ARM that does not allow for negative amortization the impact of rising interest rate is positive.

Together, the above results suggest that households attempt to select mortgage contracts to mitigate liquidity needs and affordability problems consistent to with the need to manage the monthly cash flow volatility and horizon risk problems they face. Hence, when faced with changing circumstances such as volatile interest rates, steepening of yield curve, rising risk aversion and rising income, the contract choice made by households are likely to change as well. Thus, the answer to the question of which mortgage contract is "suitable" for a particular household is a moving target that may change with time and circumstances. Households are likely to consider a host of factors including their contemporaneous situation such as mobility, preferences, and changing market factors, in deciding which mortgage to use to leverage the housing asset.

5.3 Welfare Analyses of Mortgage Contracts

We now use our model to analyze the welfare implications of optimal mortgage contract choice. We do so by calculating the expected utility associated with each mortgage contract under different probabilities of moving. Based on the magnitude of the expected utilities delivered by the alternative contracts, we infer the optimal mortgage contract for managing horizon risk and cash flow volatility consistent with the household's expected mobility or holding period. As a consequence, we consider a range of expected probabilities of moving, from 0.99 to 0.1, i.e. from extremely mobile to less mobile households. This allows us to study the impact of probability of moving or household mobility on optimal mortgage choice. In calculating the expected utilities, we take into account the effect of specific level of mobility conditional on changes in factors such as lender discount factor, household discount factor, household risk aversion, slope of yield curve and other household characteristics.

Table 5 shows the magnitudes of the realized horizon utilities conditional on certain household characteristics and market factors. By comparing expected utility across different probabilities of moving, we gain further insight into how alternative mortgage contracts can be used by households to manage their specific horizon risks. As a first step in our welfare analysis, consider a household with probability of moving between 0.99 and 0.66, meaning the household is likely to move house once every 1 to 1.67 years. Conditional on the changes in lender's discount factor, our simulated results suggest that the optimal contract for this highly mobile household is the PO-ARM, which is not surprising. This is the contract that delivers the highest utility everywhere within the expected mobility range, 1 to 1.67 years, for different levels of lender discount factor ranging from 0.96=1/(1+0.4%) down to 0.93=1/(1+10%). Incidentally, when the PO-ARM is replaced with regular ARM in the choice menu the result is dramatically different. Only households who plan to move house in one year or less prefer the regular ARM to complete their capital structure.

The welfare advantage of PO-ARM diminishes as the probability of moving declines. For example, if the household expects to move house once every 2.0 years up to about 3.0 years, the BM is clearly the best contract for managing horizon risk for such households conditional on lender discount rate of to up 7%. For households planning to stay in their homes for periods beyond 4 years, the FRM becomes the optimal contract for managing horizon risk and cash flow volatility risk. For sedentary households with longer horizon or housing tenure the FRM provides full protection against the risk that real interest rate will increase and its prepayment option may be underpriced, which is more likely to permit future capital structure adjustment in place.

The results in Panel B are interesting in the following sense. When mortgage choice is conditioned on changes in borrower discount rate, mobile borrowers seem to prefer the same contract as when the choice is conditioned on changes in lender discount rate, i.e. PO-ARM, over the mobility range of 1 to 1.67 years. However, unlike the previous case when the choice is conditional on lender discount rate, the BM contract now becomes the optimal contract chosen by borrowers over a relatively longer horizon than before, 2 to 5 years, after which the optimal contract switches to the FRM, irrespective of changes in borrower discount rate. Also it is important to stress that the results in Panel B are largely replicated regardless of the conditioning variable (see Panels B to F of Table 5). That is the mobility range over which a particular mortgage contract dominates as the optimal choice, from a welfare or utility perspective, is the same regardless of the market factor and/or borrower characteristic used to condition the choice.

Overall the fundamental insight from the results of this section is that the degree of mobility is a key driver of the type of mortgage chosen by borrowers to leverage the housing asset despite its volatility, although other factors may also influence the maximizing choices made by borrowers. This conclusion supports the contention that if borrowers are well informed about their expected probability of moving, or holding period, they would self-select mortgage contracts that more closely match their horizon to allow them manage affordability constraints, monthly cash flow volatility, wealth uncertainty effect, housing wealth risk, and deadweight cost of refinancing. In this regard, the proliferation of non-traditional mortgages during the 2000s mortgage revolution, while potentially confusing to borrowers, do contribute to market completion that facilitate hedging, speculation and positioning by borrowers. Hence, financial education and financial counseling on the essentials of mortgage finance and the associated consequences are warranted to minimize potential investment mistakes by borrowers.

5.4 Effects of Mortgage Choice, Borrower Characteristic and Market Factors on Mortgage Contract Rates

We now consider the effects of changes in key parameters on the relative price of the alternative mortgage contracts. Specifically, we analyze the equilibrium condition for interest rate on the alternative mortgage contracts consistent with the lender earning zero expected profit. The equilibrium condition is determined primarily by the lender's discount factor,θ, and the average mobility of the contract, $E(\lambda)$. Brueckner [1992] finds that an increase in the mobility of pool of FRM contracts shortens their duration which reduces the price risk of the contract, and thus lowers their interest rates.

Consistent with above intuition our comparative static analysis shows that $\partial i \ / \partial E(\lambda) < 0$. To test this proposition, we vary the average mobility of the pool of the FRM and the BM borrowers from 0.99, meaning the average household moves house once every year, to an average mobility of 0.1, meaning the average household moves house once every 10 years. As shown in panel A of Table 6, changes in the average mobility of the pool have clear and unambiguous impact on the expected interest rates of both the FRM and the BM contracts. Specifically, the interest rates on both the FRM and the BM contracts decline consistently as the average mobility of the two pools increase. This result confirms and extends the key finding in Brueckner [1992].

Since there are other factors besides mobility that are likely to influence the choice and hence relative price of mortgage contracts as demonstrated earlier, we also consider how changes in lender preference, borrower preference, risk aversion, and slope of the yield curve affects the interest rate on the FRM and BM contracts. Our comparative static analysis shows that, $\partial i \ / \partial \theta > 0$, i.e. an increase in the lender's discount factor, θ, *ceteris paribus*, increases the interest rate on mortgage contracts. Our numerical results are shown in panel C of the table, which suggest that as the lender's discount factor, θ, rises the interest rates on both the FRM and BM contracts increase. Although, a rise in the discount factor of patient lenders (correspondingly lenders with lower time preference rate) appear to have a more pronounced effect on the interest rate of the FRM than that of the BM.

For example, an increase in the lender's discount factor from 0.9259 to 0.9615 increases the interest rate on the FRM by 74 basis points compared to only 13 basis points for the BM during its first maturity leg. This result makes intuitive sense because under normal conditions, such as our assumed upward sloping yield curve, the term and prepayment premia on the FRM are likely to be higher than those of the BM. More over an increase in lender's discount rate causes the lender to care more about future consumption than current consumption. This outcome heightens the lender's sensitivity to interest rate risk causing the lender to demand additional premium for investing in both FRMs and BMs, but the premium increase should be relatively smaller for BM given its shorter term-to-maturity.

In contrast, a decrease in borrower's discount factor, δ , increases the interest rate of both the FRM and the BM. To make sense of this result note that as the discount factor falls, the borrower preference for future consumption declines, which reduces the attractiveness of both the FRM and BM (see also panel B of Table 3). Intuitively, the altering of partition points or realignment in market share lowers the average mobility of both the FRM and BM pools, as borrowers shift to shorter term ARM contracts. Correspondingly, this outcome lengthens the durations of both mortgage contracts. In turn this raises their interest rates because they have become more risky contracts.

This result has potential implications for mortgage and housing markets. One implication is that as markets become more populated by impatient households, households with higher time preference who care more about current than future consumption, we should expect more households to enter the housing market sooner rather than later. To a larger extent this was what happened in the 2000s which increased the U.S.homeownership rate to almost 70%. However, since affordability is a potential constraint, such impatient households are more likely to gravitate more towards regular ARMs and the more risky PO-ARM as observed during the run-up in housing prices of the 2000s. Indeed this view is consistent with the excessive use of other exotic contracts such as the NTMs, other than standard FRMs or BM, to make housing purchase more likely, as happened between 2002 and 2006, the period preceding the subprime mortgage financial crisis. Hence, under this circumstance, we should expect households to be affected more by the income risk of PO-ARMs. Indeed we do find that growth in borrower income tends to tilt the choice of mortgage contract more towards the PO-ARM contracts, again highlighting the importance of the effects of borrowing constraints on mortgage choice.

In panels D and E of the table, we consider the effects of increasing risk aversion and slope of the yield curve on mortgage contract rate. We vary the risk aversion coefficient from 1.18 to 1.22. With regard to slope of the yield curve we take into account both the growth rate and volatility of interest rate. Our numerical analysis shows that as the risk aversion coefficient increases the interest rate on both the FRM and BM decrease. The intuition for this result can be stated as follows: As the risk aversion coefficient increases the income or interest rate risk of the PO-ARM becomes exacerbated. This causes households to shift their preferences away from

PO-ARM and more towards mortgage contracts that provides protection against increases in future interest rates. Unconditionally, the beneficiaries of this reordering of preferences are the FRM and BM contracts. This result complements our earlier results showing that an increase in risk aversion increases the market share of the FRM and BM at the expense of the PO-ARM. The effect of this reshuffling is to increase the average mobility of both the FRM and ARM, which reduces their duration and hence their contract rates.

6. Summary and Conclusions

The U.S. mortgage revolution that preceded the unprecedented run-up in house prices in 2000s introduced a plethora of mortgage contracts that differ in several dimensions or features. The diversity of the mortgage contracts suggests that in some states of the world other contracts may dominate both the standard FRM and the regular ARM, as household self-select mortgage contracts that best match their affordability needs and at the same time allow them to better manage their risk exposures over the holding periods. Despite the diversity of mortgage contracts, previous studies typically model the mortgage choice problem as a choice between the long horizon FRM and the short horizon regular ARM, two contracts at the opposite ends of the risk-sharing spectrum. As a result the mortgage choice literature has little to say about the role of contracts such as non-traditional mortgages (NTMs) that were prevalent during the 2000s mortgage revolution or for that matter balloon mortgage (BM) that predated the NTMs.

Indeed, the most controversial NTM is the so-called pay-option adjustable rate mortgage (PO-ARM) which on the surface looks very attractive due to its flexibility and initial affordability. But the same contract is also complex and risky due to its negative amortization and significant payment shock that confronts the borrower when the monthly mortgage payments recast to fully amortize the mortgage. Further, NTMs have introduced other variables whose effects on mortgage choice have at best been partially analyzed in previous literature. Thus it is not clear which of the many contracts that prevail in the market will dominate and the circumstances under which each will dominate when the mortgage menu is expanded to include the BM and PO-ARM alongside the standard FRM and regular ARM.

In this paper, we have studied the effects of the introduction of PO-ARM and BM on optimal mortgage choice in a framework where the menu includes four contracts, the FRM, the regular ARM, BM and the PO-ARM. We have related the structural differences in these mortgage contracts to differences in borrower mobility and horizon risk management problems to determine the optimal mortgage contract households should use in leveraging the housing asset.

We find important differences in the suitability of these alternative contracts for managing the horizon risk that households face. Our key conclusions distilled from the numerical solutions of our model can be summarized as follows: First, in a three item menu consisting of either the regular ARM, BM and the FRM or the PO-ARM, BM and FRM the inference from the *partition point (or indifference points*) is that households would overwhelmingly prefer the regular ARM and PO-ARM over the intermediate horizon BM contract and long horizon FRM. Second, when the menu consisted of only two contracts (regular ARM, BM, and PO-ARM and BM) the *partition points* among the contracts imply preference orderings that tilts choice towards the BM contract; in fact the slightly BM dominates the PO-ARM. From this result, we conclude that households are neither as risk averse as the selection of FRMs would suggest, nor are they as risk-loving as the selection of regular ARM and PO-ARM would suggest, in their attempt to solve the horizon risks or and affordability problems they face. The risk aversion of the marginal households falls somewhere in between.

Third, in terms of mortgage dominance, we find interestingly that with deep or substantial tear rate of 1% the regular ARM dominates both the FRM and BM more than the PO-ARM dominates these contracts. We attribute this instructive outcome to both the negative amortization and payment recast effects both of which are likely to become excercebated when the teaser rate is substantial or deep. In fact with shallow teaser rate of 3% the dominance switches in favor of the PO-ARM; this contract now dominates both the FRM and BM more than regular ARM does in a three way horserace. From these results we conclude that part of euphoria or affinity for PO-ARM exhibited by households during the 2000s run-up in house prices are either due to affordability problems, degree of mobility of borrowers and financial sophistication . In this context we find that mortgage contract dominance is in fact influenced by several factors including borrower mobility, discount rate, lender discount rate, general wealth effect, housing wealth, slope of yield curve, and income volatility.

Forth, and very significantly, from a welfare or utility perspective the PO-ARM is the contract of choice for households with probability of moving from 0.99 to 0.60, or alternatively households who expect to move house once every 1.0 to 2.0 years. Such households would be able to manage their horizon risks much more effectively if they use the PO-ARM. Fifth, in terms of suitability or appropriateness of a contract for horizon risk management, the BM picks up where the PO-ARM drops off and the FRM picks up where the BM taper off. In general, households who expect to move house once every 2 to 5 years should select the BM, and those who expect to move house once every 6 years and beyond are better off with the standard long horizon FRM. Therefore, the most suitable contract for managing horizon risks and addressing affordability problems for a highly mobile household is the PO-ARM, and the most suitable contract for a somewhat sedentary household is the FRM. Households with intermediate horizons should take out BMs.

We also investigated how changes in the *partition points* (market share) and household mobility affect the prices of the alternative contracts. Here our results both confirm and extend the work of Brueckner [1992]. In particular, we do find the unusual FRM price behavior established in Brueckner [1992], namely that an increase in demand of the FRM or BM in our case reduce relative prices. The explanation for this unusual price behavior hinges on how increase in mobility of the pool of the FRM and BM contracts shortens their duration which reduces their price risk which in turn reduces their prices.

Some of our findings and conclusions with regard to regular ARM are similar to those of Campbell and Cocco [2003], Sa-Aadu and Sirmans [1995] and Dhillon, Sirmans and Shilling [1987], among others. However, our findings in connection with the PO-ARM and BM, contracts that have been largely ignored in the extant literature on mortgage choice, are novel and constitute the primary contribution of this paper. Here, we have identified the PO-ARM and the BM as superior instruments for households with short and intermediate horizons, respectively. This finding supports the hypothesis that the diversity of mortgage contracts, especially during the mortgage revolution of the 2000s, suggests that other contracts may dominate both the FRM and regular ARM in some states of the world.

Interestingly, changes in household characteristics and market factors considered in our numerical analyses do not qualitatively change our conclusions about the superiority of PO-ARM and BM for short horizon and intermediate horizon risk management, respectively. In fact, except in few instances, this finding is largely unaffected by conditioning variables used such borrower and lender discount factors, risk aversion, housing wealth effect, slope of the yield curve, income volatility, teaser rate and the level of market interest rate.

Our finding in regard to BM raises the question of why the BM is the superior contract for managing intermediate horizon risk. Here, it bears repeating some of what we stated earlier concerning the positive attributes of the BM. The structural characteristics of BM make it a straddled contract between the long horizon FRM and the short horizon ARM in the risk-sharing spectrum. By virtue of this position the BM has inherited some of the advantages of the FRM, regular ARM and PO-ARM while at the same time mitigating some of their disadvantages. This contract protects the household against the risk of rising interest rates as does the FRM, but it does so at a lower cost, i.e. BM is effectively a "cheap" FRM. The PO-ARM has the advantage of substantial teaser interest rate which the BM lacks, but it provides no protection against the risk of rising real interest rate, where as the BM does, while still limiting wealth risk of FRM, albeit at a slightly higher cost.

Finally, we should stress that the dominance of PO-ARM over the traditional mortgage, especially for households who expects to move house once every one to two years is quite pervasive. Nevertheless, when the choice is strictly between the PO-ARM and FRM or between the PO-ARM, the long horizon FRM and the intermediate horizon BM tend to have slightly higher market shares. Thus "investment mistakes" in mortgage choice of the type discussed in Stanton and Wallace [1998] are likely. Therefore, we suggest financial education and financial advice to mitigate investment mistakes, especially by first time home buyers and/or during periods of extreme diversity of mortgage contracts.

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Table 1: Parameter Values for Baseline Solution of Model Using Numerical Analysis.

Parameters	Description	Assumptions Annual rate (A)	 Periodical rate (B)	Notes
r_L	lenders' discount rate	4%	27.07%	B=((1+A/12)^360)^(1/5)-1
r_B	borrowers' discount rate	5%	34.90%	B=((1+A/12)^360)^(1/5)-1
β	Wealth uncertainty effect	1.2	1.2	
H	House wealth effect	0.5	0.5	
z_0	borrower's initial wealth	2	2	
g_z	income growth rate	0.1%	0.60%	B=((1+A/12)^360)^(1/5)-1
r_0^I	Teaser rate at period 0	1%	6.18%	B=((1+A/12)^360)^(1/5)-1
r_0	marker rate at period 0	6%	43.20%	B=((1+A/12)^360)^(1/5)-1
g_r	interest rate growth rate	2%	12.74%	B=((1+A/12)^360)^(1/5)-1
σ^2	interest rate volatility	2.00%	175.47%	B=(((1+A^0.5/12)^360)^(1/5)-1)^2

Lender's discount factor is $\theta = 1/(1+r_L)$ and. borrower's discount factor is $\delta=1/(1+r_B)$.

Table 2: Proportion of Households Self-Selecting Alternative Mortgage Contracts Implied by the *Partitions Points* (indifference points) among the Contracts

This table shows the self-selection behavior of households under different composition of the mortgage menu from the solution of the model using numerical analysis using baseline parameters in Table 1
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Panel A: Regular ARMs with teaser rate=1% (Deep teaser rate).

Mortgage menus	(FRM, BM, regular ARM)	(FRM, regular ARM)	(FRM, BM)	(BM, regular ARM)
Regular ARM borrowers	0.5393	0.5643	n.a.	0.5411
BM borrowers	0.2702	n.a.	0.3984	0.4589
FRM borrowers	0.1905	0.4357	0.6016	n.a.

Panel B: Regular ARMs with teaser rate=3% (Shallow Teaser Rate).

Mortgage menus	(FRM, BM, regular ARM)	(FRM, regular ARM)	(FRM, BM)	(BM, regular ARM)
Regular ARM borrowers	0.4448	0.4834	n.a.	0.4478
BM borrowers	0.3579	n.a.	0.3984	0.5522
FRM borrowers	0.1973	0.5166	0.6016	n.a.

Panel C: Pay option ARMs with teaser rate=1%.(Deep Teaser Rate)

Mortgage menus	(FRM, BM, PO_ARM)	(FRM, PO_ARM)	(FRM, BM)	(BM, PO_ARM)
PO_ARM borrowers	0.4714	0.4918	n.a.	0.4729
BM borrowers	0.3334	n.a.	0.3984	0.5271
FRM borrowers	0.1951	0.5082	0.6016	n.a.

Panel D: Pay option ARMs with teaser rate=3% (Shallow Teaser Rate).

Mortgage menus	(FRM, BM, PO_ARM)	(FRM, PO_ARM)	(FRM, BM)	(BM, PO_ARM)
PO_ARM borrowers	0.4517	0.4863	n.a.	0.4543
BM borrowers	0.3516	n.a.	0.3984	0.5457
FRM borrowers	0.1967	0.5137	0.6016	n.a.

Table 3: The Impact of Changes in Key Parameter on the Proportion of Households Self-Selecting Alternative Mortgage Contracts

Panel A: The effect of lenders' discount rate on the choice of (PO_ARM, BM, FRM)

Lender's discount rate	4.00%	5.00%	6.00%	7.00%	8.00%
PO_ARM borrowers	0.4714	0.4704	0.4695	0.4687	0.4681
BM borrowers	0.3334	0.2638	0.2023	0.1487	0.1021
FRM borrowers	0.1951	0.2659	0.3283	0.3826	0.4298

Panel B: The effect of borrowers' discount rate on the choice of (PO_ARM, BM, FRM)

Borrowers' discount rate	4.00%	4.50%	5.00%	5.50%	6.00%
PO_ARM borrowers	0.4539	0.4626	0.4714	0.4803	0.4891
BM borrowers	0.2914	0.3118	0.3334	0.3564	0.3807
FRM borrowers	0.2547	0.2256	0.1951	0.1634	0.1302

Panel C: Wealth uncertainty effect on the choice of (PO_ARM, BM, FRM)

By changing the parameter, β, in $R = (1-\lambda)\beta[(Z_0 - \frac{r_0}{1+(1+r_0)^{-5}} - H]$, we can see how the change in wealth uncertainty can affect mortgage choice

Wealth uncertainty effect	1.18	1.19	1.2	1.21	1.22
PO_ARM borrowers	0.4756	0.4735	0.4714	0.4694	0.4673
BM borrowers	0.3376	0.3355	0.3334	0.3314	0.3294
FRM borrowers	0.1867	0.1910	0.1952	0.1993	0.2033

Panel D, The effect of housing wealth effect on the choice of (PO_ARM, BM, FRM)

Housing wealth effect	0.4	0.45	0.5	0.55	0.6
PO_ARM borrowers	0.4513	0.4614	0.4714	0.4828	0.4949
BM borrowers	0.4019	0.4157	0.3334	0.3450	0.3581
FRM borrowers	0.1468	0.1229	0.1952	0.1722	0.1470

Panel E: Slope of the Yield Curve on the choice of (PO_ARM, BM, FRM)

(Interest rate growth, Interest rate volatility)	(1.98%,1.98%)	(1.99%,1.99%)	(2.0%,2.0%)	(2.01%,2.01%)	(2.02%,2.02%)
PO_ARM borrowers	0.4749	0.4732	0.4714	0.4697	0.4680
BM borrowers	0.3358	0.3346	0.3334	0.3323	0.3311
FRM borrowers	0.1893	0.1922	0.1952	0.1980	0.2009

Panel F: Borrower’s Income Growth Rate on the choice of (PO_ARM, BM, FRM)

Income growth rate	0.06%	0.08%	0.10%	0.12%	0.14%
PO_ARM borrowers	0.4705	0.4710	0.4714	0.4719	0.4724
BM borrowers	0.3289	0.3312	0.3334	0.3357	0.3381
FRM borrowers	0.2006	0.1979	0.1951	0.1924	0.1896

Panel G: Initial Interest Rate on the choice of (PO_ARM, BM, FRM)

Initial interest rate	5.00%	5.50%	6.00%	6.50%	7.00%
PO_ARM borrowers	0.4774	0.4755	0.4714	0.4659	0.4596
BM borrowers	0.3780	0.3401	0.3334	0.3304	0.3304
FRM borrowers	0.1446	0.1845	0.1951	0.2036	0.2100

Panel H: Teaser Rate on the choice of (PO_ARM, BM, FRM)

Teaser rate	1.00%	1.50%	2.00%	2.50%	3.00%
PO_ARM borrowers	0.4714	0.4681	0.4641	0.4589	0.4517
BM borrowers	0.3334	0.3365	0.3402	0.3449	0.3516
FRM borrowers	0.1951	0.1954	0.1957	0.1961	0.1967

Table 4: Lower Teaser Rate Effect and Negative Amortization Effect

Panel A: Lower teaser rate effect on the choice of (PO_ARM, BM, FRM)

Initial interest rate	6.00%	6.00%	6.00%	6.00%	6.00%
Teaser rate level	1.00%	1.50%	2.00%	2.50%	3.00%
PO_ARM borrowers	0.4714	0.4681	0.4641	0.4589	0.4517
BM borrowers	0.3334	0.3365	0.3402	0.3449	0.3516
FRM borrowers	0.1951	0.1954	0.1957	0.1961	0.1967

Panel B: Negative amortization effect on the choice of (regular ARM, BM, FRM)

Initial interest rate	5.00%	5.50%	6.00%	6.50%	7.00%
Teaser rate level	1.00%	1.00%	1.00%	1.00%	1.00%
Regular ARM borrowers	0.4886	0.5138	0.5393	0.5650	0.5914
BM borrowers	0.3711	0.3046	0.2702	0.2375	0.2058
FRM borrowers	0.1404	0.1816	0.1905	0.1975	0.2028

Panel C: Negative amortization effect on the choice of (PO_ARM, BM, FRM)

Initial interest rate	5.00%	5.50%	6.00%	6.50%	7.00%
Teaser rate level	1.00%	1.00%	1.00%	1.00%	1.00%
PO_ARM borrowers	0.4774	0.4755	0.4714	0.4659	0.4596
BM borrowers	0.3780	0.3401	0.3334	0.3304	0.3304
FRM borrowers	0.1446	0.1845	0.1951	0.2036	0.2100

Table 5: Welfare Analysis of Alternative Mortgage Contracts and their Suitability for Household Horizon Risk Management

Panel A: Optimal Mortgage for Horizon Risk Management Period Conditional on Lender's Discount Rate

Mobility	4.00%		5.00%		6.00%		7.00%		8.00%	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.03964	ARM	0.03964	ARM	0.03964	ARM	0.03964	ARM	0.03964	ARM
0.9	0.35975	ARM	0.35975	ARM	0.35975	ARM	0.35975	ARM	0.35975	ARM
0.8	0.64041	ARM	0.64041	ARM	0.64041	ARM	0.64041	ARM	0.64041	ARM
0.75	0.75042	ARM	0.75042	ARM	0.75042	ARM	0.75042	ARM	0.75042	ARM
0.7	0.83801	ARM	0.83801	ARM	0.83801	ARM	0.83801	ARM	0.83801	ARM
0.66	0.88961	ARM	0.88961	ARM	0.88961	ARM	0.88961	ARM	0.88961	ARM
0.6	0.92940	ARM	0.92940	ARM	0.92940	ARM	0.92940	ARM	0.92940	ARM
0.5	0.88753	BM	0.88790	BM	0.88823	BM	0.88852	BM	0.88878	BM
0.4	0.66724	BM	0.66734	BM	0.66737	BM	0.66735	BM	0.67068	FRM
0.33	0.09304	BM	0.09301	BM	0.09309	BM	0.09838	FRM	0.10383	FRM
0.25	-3.54566	BM	-3.54498	FRM	-3.53880	FRM	-3.53308	FRM	-3.52783	FRM
0.2	-31.21949	BM	-31.21478	FRM	-31.20910	FRM	-31.20385	FRM	-31.19900	FRM
0.1	-1.75E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM

Panel B: Optimal Mortgage Contract for Horizon Risk Management Conditional on Borrower's Discount Rate

Mobility	4.00%		4.50%		5.00%		5.50%		6.00%	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.04351	ARM	0.04150	ARM	0.03964	ARM	0.03793	ARM	0.03636	ARM
0.9	0.39362	ARM	0.37597	ARM	0.35975	ARM	0.34484	ARM	0.33111	ARM
0.8	0.69791	ARM	0.66793	ARM	0.64041	ARM	0.61514	ARM	0.59190	ARM
0.75	0.81590	ARM	0.78175	ARM	0.75042	ARM	0.72167	ARM	0.69525	ARM
0.7	0.90864	ARM	0.87178	ARM	0.83801	ARM	0.80703	ARM	0.77858	ARM
0.66	0.96204	ARM	0.92424	ARM	0.88961	ARM	0.85788	ARM	0.82877	ARM
0.6	0.99980	ARM	0.96303	ARM	0.92940	ARM	0.89863	ARM	0.87043	ARM
0.5	0.96311	BM	0.92358	BM	0.88753	BM	0.85465	BM	0.82461	BM
0.4	0.71853	BM	0.69147	BM	0.66724	BM	0.64556	BM	0.62615	BM
0.33	0.09798	BM	0.09459	BM	0.09304	BM	0.09298	BM	0.09421	BM
0.25	-3.78220	FRM	-3.66231	BM	-3.54566	BM	-3.43174	BM	-3.32043	BM
0.2	-33.16992	FRM	-32.18085	FRM	-31.21949	BM	-30.28538	BM	-29.37899	BM
0.1	-1.86E+12	FRM	-1.81E+12	FRM	-1.75E+12	FRM	-1.70E+12	FRM	-1.65E+12	FRM

Panel C: Optimal Mortgage Contract for Horizon Risk Management Conditional on Wealth Uncertainty Effect

Mobility	1.18		1.19		1.2		1.21		1.22	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.03899	ARM	0.03931	ARM	0.03964	ARM	0.03997	ARM	0.04029	ARM
0.9	0.35451	ARM	0.35713	ARM	0.35975	ARM	0.36236	ARM	0.36496	ARM
0.8	0.63269	ARM	0.63656	ARM	0.64041	ARM	0.64424	ARM	0.64804	ARM
0.75	0.74255	ARM	0.74651	ARM	0.75042	ARM	0.75430	ARM	0.75813	ARM
0.7	0.83088	ARM	0.83447	ARM	0.83801	ARM	0.84148	ARM	0.84488	ARM
0.66	0.88388	ARM	0.88679	ARM	0.88961	ARM	0.89234	ARM	0.89498	ARM
0.6	0.92767	ARM	0.92862	ARM	0.92940	ARM	0.93002	ARM	0.93048	ARM
0.5	0.89584	BM	0.89190	BM	0.88753	BM	0.88272	BM	0.87744	BM
0.4	0.71482	BM	0.69204	BM	0.66724	BM	0.64022	BM	0.61076	BM
0.33	0.25507	BM	0.17897	BM	0.09304	BM	-0.00436	BM	-0.11523	BM
0.25	-2.14910	BM	-2.76083	BM	-3.54566	BM	-4.56874	BM	-5.92600	BM
0.2	-14.55444	BM	-20.97810	BM	-31.21949	BM	-48.28890	BM	-78.23888	FRM
0.1	-3.81E+07	FRM	-2.94E+09	FRM	-1.75E+12	FRM	-5.41E+16	FRM	-1.76E+25	FRM

Panel D: Optimal Mortgage Contract for Horizon Risk Management Conditional of Housing Wealth Effect

Mobility	0.4		0.45		0.5		0.55		0.6	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.04310	ARM	0.04137	ARM	0.03964	ARM	0.03791	ARM	0.03617	ARM
0.9	0.38717	ARM	0.37355	ARM	0.35975	ARM	0.34578	ARM	0.33163	ARM
0.8	0.67980	ARM	0.66045	ARM	0.64041	ARM	0.61969	ARM	0.59827	ARM
0.75	0.78954	ARM	0.77055	ARM	0.75042	ARM	0.72918	ARM	0.70682	ARM
0.7	0.87159	ARM	0.85571	ARM	0.83801	ARM	0.81853	ARM	0.79733	ARM
0.66	0.91378	ARM	0.90305	ARM	0.88961	ARM	0.87362	ARM	0.85519	ARM
0.6	0.92698	ARM	0.93076	ARM	0.92940	ARM	0.92343	ARM	0.91326	ARM
0.5	0.80872	BM	0.85632	BM	0.88753	BM	0.90607	BM	0.91448	BM
0.4	0.20728	BM	0.49240	BM	0.66724	BM	0.77786	BM	0.84806	BM
0.33	-2.23473	BM	-0.61423	BM	0.09304	BM	0.45264	BM	0.65368	BM
0.25	-1.24E+02	BM	-15.31430	BM	-3.54566	BM	-0.87757	BM	0.03382	BM
0.2	-3.82E+05	FRM	-6.17E+02	FRM	-31.21949	BM	-5.09155	BM	-1.12827	BM
0.1	n.a.	n.a.	n.a.	n.a.	-1.75E+12	FRM	-1.41E+04	FRM	-68.76371	FRM

Panel E: Optimal Mortgage Contract for Horizon Risk Management Conditional on Yield Curve Slope

Mobility	(1.98%,1.98%)		(1.99%,1.99%)		(2.0%,2.0%)		(2.01%,2.01%)		(2.02%,2.02%)	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.03980	ARM	0.03972	ARM	0.03964	ARM	0.03956	ARM	0.03948	ARM
0.9	0.36153	ARM	0.36064	ARM	0.35975	ARM	0.35886	ARM	0.35796	ARM
0.8	0.64456	ARM	0.64249	ARM	0.64041	ARM	0.63832	ARM	0.63622	ARM
0.75	0.75615	ARM	0.75330	ARM	0.75042	ARM	0.74753	ARM	0.74461	ARM
0.7	0.84576	ARM	0.84190	ARM	0.83801	ARM	0.83407	ARM	0.83010	ARM
0.66	0.89947	ARM	0.89457	ARM	0.88961	ARM	0.88460	ARM	0.87953	ARM
0.6	0.94368	ARM	0.93660	ARM	0.92940	ARM	0.92210	ARM	0.91468	ARM
0.5	0.91129	BM	0.89957	BM	0.88753	BM	0.87518	BM	0.86249	BM
0.4	0.72413	BM	0.69639	BM	0.66724	BM	0.63654	BM	0.60418	BM
0.33	0.24447	BM	0.17214	BM	0.09304	BM	0.00623	BM	-0.08939	BM
0.25	-2.35277	BM	-2.88878	BM	-3.54566	BM	-4.36022	BM	-5.38330	BM
0.2	-16.64767	BM	-22.53509	BM	-31.21949	BM	-44.45167	BM	-65.37263	FRM
0.1	-1.23E+08	FRM	-6.42E+09	FRM	-1.75E+12	FRM	-9.33E+15	FRM	-2.46E+22	FRM

Panel F: Optimal Mortgage Contract for Horizon Risk Management Conditional on Borrower's Income Growth Rate

Mobility	0.06%		0.08%		0.10%		0.12%		0.14%	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.03936	ARM	0.03950	ARM	0.03964	ARM	0.03978	ARM	0.03992	ARM
0.9	0.35728	ARM	0.35851	ARM	0.35975	ARM	0.36099	ARM	0.36223	ARM
0.8	0.63594	ARM	0.63818	ARM	0.64041	ARM	0.64265	ARM	0.64490	ARM
0.75	0.74504	ARM	0.74773	ARM	0.75042	ARM	0.75312	ARM	0.75582	ARM
0.7	0.83171	ARM	0.83486	ARM	0.83801	ARM	0.84116	ARM	0.84431	ARM
0.66	0.88256	ARM	0.88609	ARM	0.88961	ARM	0.89314	ARM	0.89667	ARM
0.6	0.92107	ARM	0.92524	ARM	0.92940	ARM	0.93356	ARM	0.93772	ARM
0.5	0.87766	BM	0.88260	BM	0.88753	BM	0.89246	BM	0.89738	BM
0.4	0.65465	BM	0.66095	BM	0.66724	BM	0.67351	BM	0.67977	BM
0.33	0.07680	BM	0.08493	BM	0.09304	BM	0.10113	BM	0.10920	BM
0.25	-3.58080	BM	-3.56321	BM	-3.54566	BM	-3.52815	BM	-3.51068	BM
0.2	-31.40207	FRM	-31.31024	BM	-31.21949	BM	-31.12889	BM	-31.03844	BM
0.1	-1.76E+12	FRM	-1.76E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM	-1.74E+12	FRM

Panel G: Optimal Contract for Horizon Risk Management Conditional on Initial Interest Rate Level

Mobility	5.00%		5.50%		6.00%		6.50%		7.00%	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.04484	ARM	0.04226	ARM	0.03964	ARM	0.03700	ARM	0.03434	ARM
0.9	0.40454	ARM	0.38236	ARM	0.35975	ARM	0.33679	ARM	0.31359	ARM
0.8	0.71675	ARM	0.67905	ARM	0.64041	ARM	0.60101	ARM	0.56099	ARM
0.75	0.83884	ARM	0.79522	ARM	0.75042	ARM	0.70463	ARM	0.65800	ARM
0.7	0.93669	ARM	0.88807	ARM	0.83801	ARM	0.78666	ARM	0.73420	ARM
0.66	0.99551	ARM	0.94344	ARM	0.88961	ARM	0.83417	ARM	0.77729	ARM
0.6	1.04494	ARM	0.98846	ARM	0.92940	ARM	0.86778	ARM	0.80370	ARM
0.5	0.99739	BM	0.94385	BM	0.88753	BM	0.82856	BM	0.76719	BM
0.4	0.62534	BM	0.65651	BM	0.66724	BM	0.66158	BM	0.64275	BM
0.33	-0.70216	BM	-0.19963	BM	0.09304	BM	0.26507	BM	0.36334	BM
0.25	-3.29E+01	BM	-9.49599	BM	-3.54566	BM	-1.46487	BM	-0.57513	BM
0.2	-9.88E+03	BM	-2.55E+02	BM	-3.12E+01	BM	-7.70769	FRM	-2.64400	FRM
0.1	n.a.	n.a.	n.a.	n.a.	-1.75E+12	FRM	-5.90E+04	FRM	-2.63E+02	FRM

Panel H: Optimal Contract for Horizon Risk Management Conditional on Teaser Rate

Mobility	1.00%		1.50%		2.00%		2.50%		3.00%	
	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice	E(U)	Choice
0.99	0.03964	ARM	0.03921	ARM	0.03876	ARM	0.03830	ARM	0.03783	ARM
0.9	0.35975	ARM	0.35588	ARM	0.35188	ARM	0.34774	ARM	0.34347	ARM
0.8	0.64041	ARM	0.63367	ARM	0.62670	ARM	0.61950	ARM	0.61204	ARM
0.75	0.75042	ARM	0.74269	ARM	0.73471	ARM	0.72647	ARM	0.71795	ARM
0.7	0.83801	ARM	0.82969	ARM	0.82114	ARM	0.81233	ARM	0.80323	ARM
0.66	0.88961	ARM	0.88125	ARM	0.87269	ARM	0.86390	ARM	0.85481	ARM
0.6	0.92940	ARM	0.92203	ARM	0.91458	ARM	0.90697	ARM	0.89908	ARM
0.5	0.88753	BM	0.88755	BM	0.88758	BM	0.88761	BM	0.88766	BM
0.4	0.66724	BM	0.66725	BM	0.66725	BM	0.66726	BM	0.66728	BM
0.33	0.09304	BM	0.09304	BM	0.09303	BM	0.09302	BM	0.09301	BM
0.25	-3.54566	BM	-3.54566	BM	-3.54566	BM	-3.54566	BM	-3.54566	BM
0.2	-3.12E+01	BM	-3.12E+01	BM	-3.12E+01	BM	-3.12E+01	BM	-3.12E+01	BM
0.1	-1.75E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM	-1.75E+12	FRM

Table 7: Effects of Changes in Key Variables on Contract Rates on Alternative Mortgage Contract Rates

Panel A: The Effect of Changes in Expected Mobility of Households on Mortgage Interest Rates

Mobility	0.1	0.2	0.25	0.33	0.4	0.5	0.6	0.66	0.7	0.75	0.8	0.9	0.99
FRM *if*	7.89%	7.63%	7.50%	7.29%	7.12%	6.87%	6.65%	6.53%	6.45%	6.36%	6.28%	6.13%	6.01%
BM *ib*	6.61%	6.57%	6.54%	6.51%	6.47%	6.42%	6.35%	6.31%	6.28%	6.24%	6.20%	6.11%	6.01%
BM *id*	9.70%	9.61%	9.57%	9.49%	9.42%	9.31%	9.20%	9.13%	9.08%	9.02%	8.97%	8.85%	8.76%

Panel B: Effects of Changes in Lender's Discount Rates on Mortgage Interest Rates

Lender's discount rate	4.00%	5.00%	6.00%	7.00%	8.00%
FRM *if*	7.90%	7.68%	7.48%	7.31%	7.16%
BM *ib*	6.49%	6.45%	6.42%	6.39%	6.36%
BM *id*	9.46%	9.38%	9.32%	9.26%	9.21%

Panel C: Effects of Changes in Borrower's Discount Rates on Mortgage Interest Rates

Borrowers' discount rate	4.00%	4.50%	5.00%	5.50%	6.00%
FRM *if*	7.82%	7.86%	7.90%	7.94%	7.98%
BM *ib*	6.47%	6.48%	6.49%	6.50%	6.51%
BM *id*	9.42%	9.44%	9.46%	9.48%	9.50%

Table 7 continue

Panel D: Effects of Changes in Wealth Uncertainty Effect on Mortgage Interest Rate

Wealth uncertainty effect	1.18	1.19	1.2	1.21	1.22
FRM *if*	7.91%	7.90%	7.90%	7.89%	7.89%
BM *ib*	6.49%	6.49%	6.49%	6.49%	6.49%
BM *id*	9.46%	9.46%	9.46%	9.45%	9.45%

Panel E: Effects of Changes in Housing Wealth Effect on Mortgage Interest Rate

Housing wealth effect	0.4	0.45	0.5	0.55	0.6
FRM *if*	7.85%	7.87%	7.90%	7.93%	7.96%
BM *ib*	6.48%	6.48%	6.49%	6.50%	6.51%
BM *id*	9.43%	9.44%	9.46%	9.48%	9.49%

Panel F: Effects of Changes in Yield Curve Slope on Mortgage Interest Rate

(Interest rate growth, Interest rate volatility)	(1.98%,1.98%)	(1.99%,1.99%)	(2.0%,2.0%)	(2.01%,2.01%)	(2.02%,2.02%)
FRM *if*	7.89%	7.89%	7.90%	7.90%	7.91%
BM *ib*	6.49%	6.49%	6.49%	6.49%	6.49%
BM *id*	9.43%	9.44%	9.46%	9.47%	9.49%

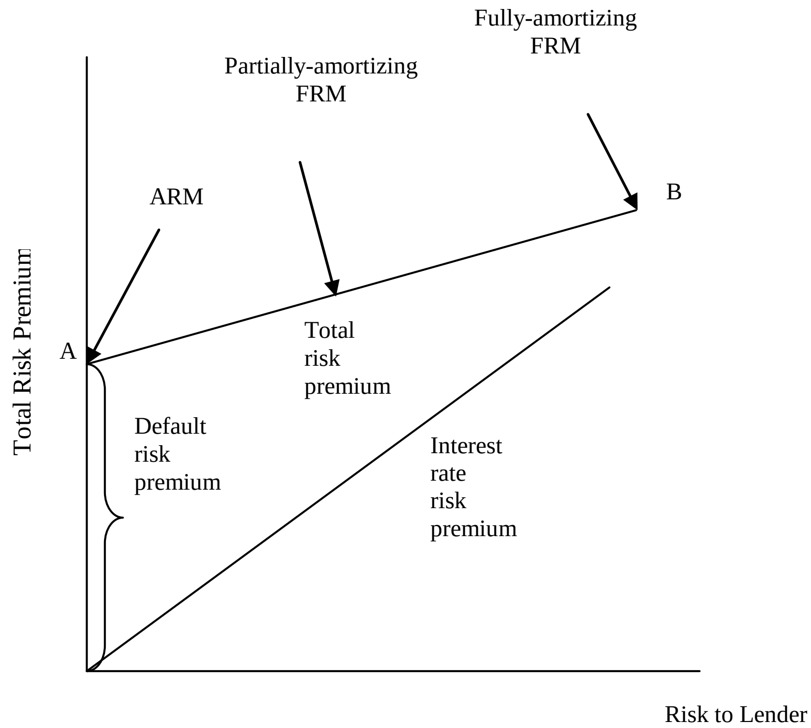
Panel G: Effects of Changes in Borrower's Income Growth Rate on Mortgage Interest Rate

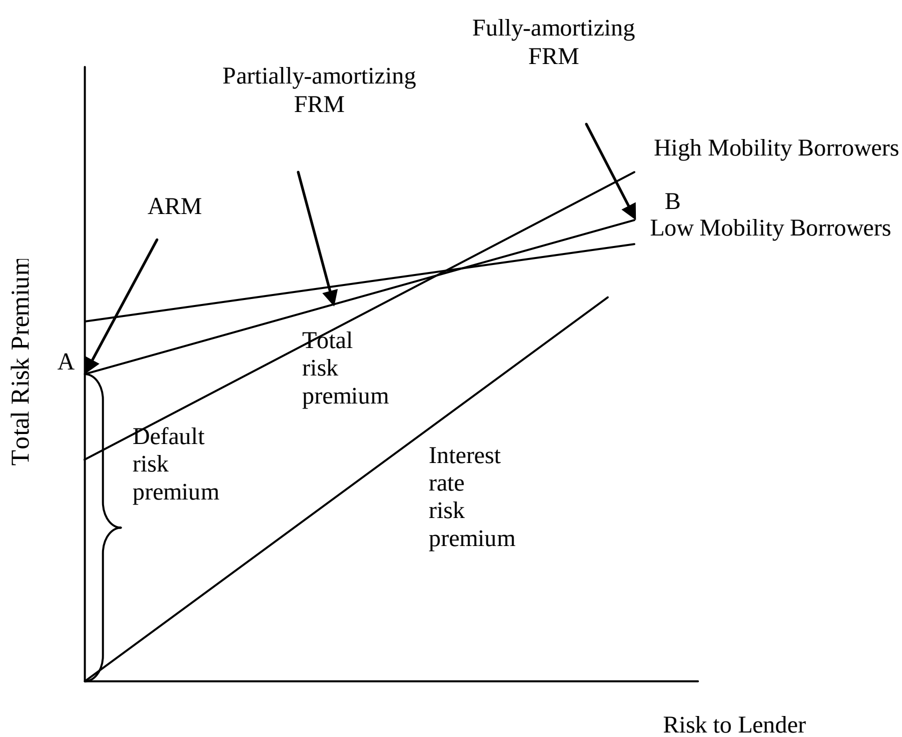
Income growth rate	0.06%	0.08%	0.10%	0.12%	0.14%
FRM *if*	7.89%	7.89%	7.90%	7.90%	7.91%
BM *ib*	6.49%	6.49%	6.49%	6.49%	6.49%
BM *id*	9.45%	9.46%	9.46%	9.46%	9.46%

Panel H: Effects of Changes in Initial Interest Rate Level on Mortgage Interest Rate

Initial interest rate	5%	5.5%	6%	6.5%	7%
FRM *if*	7.07%	7.47%	7.90%	8.34%	8.78%
BM *ib*	5.54%	6.01%	6.49%	6.97%	7.46%
BM *id*	8.68%	9.06%	9.46%	9.86%	10.27%

Figure 1: Optimal Mortgage Contract Selection for Borrowers with Different Mobility





Appendix Table 1: Volume of Non-Traditional Mortgages (NTMs)

	2004	2005	2006
Total Originations	$2,920	$3,120	$2,980
Non-traditional products	$365	$866	$958
All Interest-only	$60	$481	$526
Interest-only fixed rate	$55	$418	$387
Interest-only ARMs	$5	$63	$139
Option ARMs	$145	$280	$255

Source: *Inside Mortgage Finance*

Appendix Table 2: Delinquency Rates on Nontraditional Mortgages

Loan Type	Estimated # of Loans	Total Delinquency Rate (30+ Days and in Foreclosure)
1. High Rate Subprime (including Fannie/Freddie private MBS holdings)	6.7 million	45.0%
2. Option Arm	1.1 million	30.5%
3. Alt-A (inc. Fannie/Freddie/FHLBs private MBS holdings)	2.4 million	23.0%
4. Fannie Subprime/Alt-A/Nonprime	6.6 million	17.3%
5. Freddie Subprime/Alt-A/Nonprime	4.1 million	13.8%
6. Government	4.8 million	13.5%
Subtotal # of Loans	**25.7 million**	
7. Non-Agency Jumbo Prime	9.4 million	6.8%
8. Non-Agency Conforming Prime		5.6%
9. Fannie Prime	11.2 million	2.6%
10. Freddie Prime	8.7 million	2.0%
Total # of Loans	**55 million**	

Sources:
Lender Processing Services, LPS Mortgage Monitor, June 2009: 1, 2, 3, 6,7 &8.
Fannie Mae 2009 2Q Credit Supplement: 4 &9.
Based on Freddie Mac 2009 2Q Financial Results Supplement: 5 & 10

Appendix Table 3
Mortgage payment schedule

Regular ARM and PO-ARM payment schedule

	t=1	t=2	t=3	t=4
Prepay at t=1	$1+r_0$	$\frac{r_1}{1-(1+r_1)^{-3}}$	$\frac{r_2}{1-(1+r_2)^{-2}}*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})$	$(1+r_3)((\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})$
Prepay at t=2	$\frac{r_0}{1-(1+r_0)^{-4}}$	$(1+r_1)*(\frac{(1+r_0)^4-(1+r_0)}{(1+r_0)^4-1})$	$\frac{r_2}{1-(1+r_2)^{-2}}$	$(1+r_3)*(\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})$
Prepay at t=3	$\frac{r_0}{1-(1+r_0)^{-4}}$	$(\frac{r_1}{1-(1+r_1)^{-3}})(\frac{(1+r_0)^4-(1+r_0)}{(1+r_0)^4-1})$	$(1+r_2)*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})(\frac{(1+r_0)^4-(1+r_0)}{(1+r_0)^4-1})$	$(1+r_3)$
No prepayment	$\frac{r_0}{1-(1+r_0)^{-4}}$	$(\frac{r_1}{1-(1+r_1)^{-3}})(\frac{(1+r_0)^4-(1+r_0)}{(1+r_0)^4-1})$	$(\frac{r_2}{1-(1+r_2)^{-2}})(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})*(\frac{(1+r_0)^4-(1+r_0)}{(1+r_0)^4-1})$	$(1+r_3)((\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})(\frac{(1+r_0)^4-(1+r_0)}{(1+r_0)^4-1})$

Balloon payment schedule

	t=1	t=2	t=3	t=4
Prepay at t=1	$1+i_b$	$\frac{r_1}{1-(1+r_1)^{-3}}$	$\frac{r_2}{1-(1+r_2)^{-2}}*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})$	$(1+r_3)((\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})$
Prepay at t=2	$\frac{i_b}{1-(1+i_b)^{-4}}$	$(1+i_b)*\frac{(1+i_b)^4-(1+i_b)}{(1+i_b)^4-1}$	$\frac{r_2}{1-(1+r_2)^{-2}}$	$(1+r_3)*(\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})$
Prepay at t=3	$\frac{i_b}{1-(1+i_b)^{-4}}$	$\frac{i_b}{1-(1+i_b)^{-4}}$	$(1+i_d)*(\frac{(1+i_b)^4-(1+i_b)^2}{(1+i_b)^4-1})$	$(1+r_3)$
No prepayment	$\frac{i_b}{1-(1+i_b)^{-4}}$	$\frac{i_b}{1-(1+i_b)^{-4}}$	$(\frac{i_d}{1-(1+i_d)^{-2}})(\frac{(1+i_b)^4-(1+i_b)^2}{(1+i_b)^4-1})$	$(\frac{i_d}{1-(1+i_d)^{-2}})(\frac{(1+i_b)^4-(1+i_b)^2}{(1+i_b)^4-1})$

FRM payment schedule

	t=1	t=2	t=3	t=4
Prepay at t=1	$1+i_f$	$\frac{r_1}{1-(1+r_1)^{-3}}$	$\frac{r_2}{1-(1+r_2)^{-2}}*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})$	$(1+r_3)((\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})*(\frac{(1+r_1)^3-(1+r_1)}{(1+r_1)^3-1})$
Prepay at t=2	$\frac{i_f}{1-(1+i_f)^{-4}}$	$(1+i_f)*\frac{(1+i_f)^4-(1+i_f)}{(1+i_f)^4-1}$	$\frac{r_2}{1-(1+r_2)^{-2}}$	$(1+r_3)*(\frac{(1+r_2)^2-(1+r_2)}{(1+r_2)^2-1})$
Prepay at t=3	$\frac{i_f}{1-(1+i_f)^{-4}}$	$\frac{i_f}{1-(1+i_f)^{-4}}$	$(1+i_f)*\frac{(1+i_f)^4-(1+i_f)^2}{(1+i_f)^4-1}$	$(1+r_3)$
No prepayment	$\frac{i_f}{1-(1+i_f)^{-4}}$	$\frac{i_f}{1-(1+i_f)^{-4}}$	$\frac{i_f}{1-(1+i_f)^{-4}}$	$\frac{i_f}{1-(1+i_f)^{-4}}$

Appendix 4,

We assume the utility function as: V(Z-x)=1- $e^{-(1-\lambda)R(Z-x)}$

Z is income, x is payment, λ is probability of moving and R is a constant

We now solve

$\int_{-\infty}^{\infty}\left\{1-\exp(-(1-\lambda)R[Z-B\frac{r}{1-(1+r)^{-T}}]\right\}f(r)dr$ Where $r \sim N(\mu,\sigma^2)$, B is the mortgage outstanding.

$$= \int_{-\infty}^{\infty} f(r)dr - \exp(-(1-\lambda)RZ)\int_{-\infty}^{\infty}\exp[(1-\lambda)RB\frac{r}{1-(1+r)^{-T}}]f(r)dr \quad \ldots\ldots\ldots\ldots \quad (1)$$ and

$\int_{-\infty}^{\infty}\exp[(1-\lambda)RB\frac{r}{1-(1+r)^{-T}}]f(r)dr$ in equation (1).

We define $g(r)=\frac{r}{1-(1+r)^{-T}}$, and use Taylor Expansion around r=k to the second differentiation to approximately estimate the function.

$g(r)=g(k)+g'(k)(r-k)+\frac{g''(k)}{2}(r-k)^2$. This equation is then substituted into $\int_{-\infty}^{\infty}\exp[(1-\lambda)RB\frac{r}{1-(1+r)^{-T}}]f(r)dr$.

$$\approx\int_{-\infty}^{\infty}\exp\{(1-\lambda)RB[g(k)+g'(k)(r-k)+\frac{g''(k)}{2}(r-k)^2]\}f(r)dr$$

$$=\exp((1-\lambda)RB[g(k)-\frac{g'(k)^2}{2g''(k)}]\int_{-\infty}^{\infty}\exp\{(1-\lambda)RB\frac{g''(k)}{2}[r-(k-\frac{g'(k)}{g''(k)})]^2\}f(r)dr \text{ ……………..…. (2)}$$

To solve for $\int_{-\infty}^{\infty}\exp\{(1-\lambda)RB\frac{g''(k)}{2}(r-(k-\frac{g'(k)}{g''(k)}))^2\}f(r)dr$ in equation (2),

we define $v=k-\frac{g'(k)}{g''(k)}$ and $t=(1-\lambda)RB\frac{g''(k)}{2}$. We also have $f(r)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-\mu)^2}{2\sigma^2}}$

$$\int_{-\infty}^{\infty}e^{t(r-v)^2}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(r-\mu)^2}{2\sigma^2}}dr$$

$$=\int_{-\infty}^{\infty}e^{tr^2-2trv+tv^2}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{r^2-2r\mu+\mu^2}{2\sigma^2}}dr$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(1-2t\sigma^2)r^2-2(\mu-2tv\sigma^2)r+(\mu^2-2tv^2\sigma^2)}{2\sigma^2}}dr$$

$$=e^{-\frac{(\mu^2-2tv^2\sigma^2)}{2\sigma^2}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(1-2t\sigma^2)r^2-2(\mu-2tv\sigma^2)r+\frac{(\mu-2tv\sigma^2)^2}{(1-2t\sigma^2)}}{2\sigma^2}}e^{\frac{\frac{(\mu-2tv\sigma^2)^2}{(1-2t\sigma^2)}}{2\sigma^2}}dr$$

$$= e^{-\frac{(\mu^2 - 2tv^2\sigma^2)}{2\sigma^2}} e^{\frac{\frac{(\mu - 2tv\sigma^2)^2}{(1-2t\sigma^2)}}{2\sigma^2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[\sqrt{1-2t\sigma^2}\, r - \frac{(\mu - 2tv\sigma^2)}{\sqrt{1-2t\sigma^2}}]^2}{2\sigma^3}} dr$$

Let = $y = \sqrt{1-2t\sigma^2}\, r$, then $dy = \sqrt{1-2t\sigma^2}\, dr$

$$= e^{-\frac{(\mu^2 - 2tv^2\sigma^2)}{2\sigma^2}} e^{\frac{\frac{(\mu - 2tv\sigma^2)^2}{(1-2t\sigma^2)}}{2\sigma^2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[y - \frac{(\mu - 2tv\sigma^2)}{\sqrt{1-2t\sigma^2}}]^2}{2\sigma^3}} \frac{1}{\sqrt{1-2t\sigma^2}} dy$$

$$= \frac{1}{\sqrt{1-2t\sigma^2}} e^{-\frac{(\mu^2 - 2tv^2\sigma^2)}{2\sigma^2}} e^{\frac{\frac{(\mu - 2tv\sigma^2)^2}{(1-2t\sigma^2)}}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} \left\{ 1 - \exp(-(1-\lambda)R[Z - B\frac{r}{1-(1+r)^{-T}}] \right\} f(r) dr$$

$$= 1 - \exp(-(1-\lambda)RZ) \int_{-\infty}^{\infty} \exp[(1-\lambda)RB\frac{r}{1-(1+r)^{-T}}] f(r) dr$$

$$= 1 - \exp(-(1-\lambda)RZ)\ \exp((1-\lambda)RB[g(k) - \frac{g'(k)^2}{2g''(k)}] \frac{1}{\sqrt{1-2t\sigma^2}} \exp(-\frac{(\mu^2 - 2tv^2\sigma^2)}{2\sigma^2}) \exp(\frac{(\mu - 2tv\sigma^2)^2}{2\sigma^2(1-2t\sigma^2)})$$

Where, $v = k - \frac{g'(k)}{g''(k)}$, and $t = (1-\lambda)RB\frac{g''(k)}{2}$, $|t| < 1$.

Note,

Where $g(r) = \frac{r}{1-(1+r)^{-T}} = \frac{r(1+r)^T}{(1+r)^T - 1}$,

$$g'(r) = (\frac{H_1(r)}{H_2(r)})' = \frac{(H_1(r))' H_2(r) - H_1(r)(H_2(r))'}{(H_2(r))^2}$$

Where

$$H_1(r) = r(1+r)^T$$

$$H_2(r) = (1+r)^T - 1$$

$$H_1'(r) = (1+r)^T + Tr(1+r)^{T-1}$$

$$H_2'(r) = T(1+r)^{T-1}$$

$$g'(r) = \frac{[(1+r)^T + Tr(1+r)^{T-1}][(1+r)^T - 1] - [r(1+r)^T][T(1+r)^{T-1}]}{[(1+r)^T - 1]^2}$$

$$= \frac{[(1+r)^{2T} - (1+r)^T - rT(1+r)^{T-1}]}{[(1+r)^T - 1]^2}$$

$$g''(r) = (\frac{H_3(r)}{H_4(r)})' = \frac{(H_3(r))' H_4(r) - H_3(r)(H_4(r))'}{(H_4(r))^2}$$

Where

$$H_3(r) = [(1+r)^{2T} - (1+r)^T - rT(1+r)^{T-1}]$$

$$H_4(r) = [(1+r)^T - 1]^2$$

$$H_3'(r) = 2T(1+r)^{2T-1} - T(1+r)^{T-1} - T(1+r)^{T-1} - rT(T-1)(1+r)^{T-2}$$

$$H_4'(r) = 2T[(1+r)^T - 1](1+r)^{T-1}$$

$$g''(r) = \frac{[2T(1+r)^{2T-1} - T(1+r)^{T-1} - T(1+r)^{T-1} - rT(T-1)(1+r)^{T-2}][(1+r)^T - 1]^2}{[(1+r)^T - 1]^2}$$
$$- \frac{[(1+r)^{2T} - (1+r)^T - rT(1+r)^{T-1}]2T[(1+r)^T - 1](1+r)^{T-1}]}{[(1+r)^T - 1]^2}$$

(II)
We also need to solve for the following expected utility when borrowers prepay.

$$\int_{-\infty}^{\infty} \{1 - \exp(-(1-\lambda)R[Z - B(1+r)]\} f(r) dr$$

$$=1\text{-} \exp[-(1-\lambda)R(Z-B)] \exp(\mu t + \frac{t^2\sigma^2}{2})$$

Where $t = (1-\lambda)RB$, and $|t| < 1$.