The Market for Crash Risk

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Abstract

This paper examines the equilibrium when negative stock market jumps (crashes) can occur, and investors have heterogeneous attitudes towards crash risk. The less crash-averse insure the more crash-averse through the options markets that dynamically complete the economy. The resulting equilibrium is compared with various option pricing anomalies reported in the literature: the tendency of stock index options to overpredict volatility and jump risk, the Jackwerth (2000) implicit pricing kernel puzzle, and the stochastic evolution of option prices. The specification of crash aversion is compatible with the static option pricing puzzles, while heterogeneity partially explains the dynamic puzzles. Heterogeneity also magnifies substantially the stock market impact of adverse news about fundamentals.

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The markets for stock index options play a vital role in providing a venue for redistributing and pricing various types of equity risk of concern to investors. Investors who like equity but are concerned with crash risk can purchase portfolio insurance, in the form of out-of-the-money put options. Direct bets on (or hedges against) future stock market volatility are feasible; most simply by buying or selling straddles, more exactly by the option-based bet on future realized variance proposed by Britten-Jones and Neuberger (2000) and analyzed further by Jiang and Tian (2005). By creating a market for these risks, the options markets should in principle permit the dispersion of these risks across all investors, until all investors are indifferent at the margin to taking on more or less of these risks given the equilibrium pricing of these risks. This idealized risk-pooling underlies our theoretical construction of representative-agent models, and our pricing of risks from aggregate data sources; for instance, estimating the consumption CAPM based on aggregate consumption data.

How well do the stock index option markets operate? Evidence from the observed order flow through the options markets and from option returns suggests that our idealized models of the trading of crash and volatility risks may be far from realistic. First, most investors do not routinely use options to manage risks associated with equity investments. Although stock index options are among the most actively traded options, the stock positions hedged by exchange-traded options on the S&P index or futures represented at most 2.6% of the S&P 500 market capitalization in 1998.[1] Furthermore, there appears to be a fundamental dichotomy between buyers and sellers. A broad array of individual and institutional investors buy index options as part of their overall risk management strategies, while a relatively concentrated group of option market makers and proprietary traders predominantly write them and delta-hedge their positions.[2] This may reflect

[1]This is computed based upon the open interest in 1998 for CBOE options on the S&P 100 and S&P 500 indexes, and for CME options on S&P 500 futures. It represents an upper limit in assuming every option corresponds one-for-one to an underlying stock position. Strategies involving multiple options (vertical spreads, collars, straddles, etc.) would substantially reduce the estimate of the stock positions being protected.

[2]See Pan and Poteshman (2005, Table 1) for a breakdown of option order flow at the CBOE.

market frictions; individual investors can easily buy stock index options, but face hurdles at the broker level to writing naked calls or puts.

Second, empirical evidence on option returns suggests that stock index options markets are operating inefficiently. Such evidence is in essence based on substantial divergences between the "risk-neutral" distributions compatible with observed post-'87 option prices, and the conditional distributions estimated from time series analyses of the underlying stock index. Perhaps most important has been the substantial disparity between implicit standard deviations (ISD's) inferred from at-the-money options, and the subsequent realized volatility over the lifetime of the option. As illustrated below in Figure 1, ISD's have generally been higher than realized volatility. Furthermore, regressing realized volatility upon ISD's almost invariably indicates that ISD's are informative but biased predictors of future volatility, with bias increasing in the ISD level.

While the level of at-the-money ISD's is puzzling, the *shape* of the volatility surface across strike prices and maturities also appears at odds with estimates of conditional distributions. It is now widely recognized that the "volatility smirk" implies substantial negative skewness in risk-neutral distributions, and various correspondingly skewed models have been proposed: implied binomial trees, stochastic volatility models with "leverage" effects, and jump-diffusions. And although these models can roughly match observed option prices, the associated implicit parameters do not appear especially consistent with the absence of substantial negative skewness in post-'87 stock index returns. To paraphrase Samuelson, the option markets have predicted nine out of the past five market corrections. A further puzzle is that implicit jump risk assessments are strongly counter-cyclical. As shown below in Figure 2, implicit jump risk over 1988-98 was highest immediately after substantial market drops, and was low during the bull market of 1992-96.

It is of course possible that the pronounced divergence between objective and risk-neutral measures represents risk premia on the underlying risks. The fundamental theorem of asset pricing states that provided there exist no outright arbitrage opportunities, it is possible to construct a "representative agent" whose preferences are compatible with any observed divergences between the two distributions. However, Jackwerth (2000) and Rosenberg and Engle (2002) have pointed out that the preferences necessary to reconcile the two distributions appear rather oddly shaped, with sections that are locally risk-loving rather than risk-averse. Furthermore, the post-'87 Sharpe ratios from writing put options or straddles seem extraordinarily high – two to six times that of investing directly in the stock market. These speculative opportunities appear to have been present in the stock index options markets for almost 20 years.

I believe the stock index options markets are functioning more as insurance markets, rather than as genuine two-sided markets for trading financial risks. The view of options markets as an insurance market for crash risk may be able to explain some of the option pricing anomalies – especially if there exist barriers to entry. If crash risk is concentrated among option market makers, calibrations based upon the risk-taking capacity of all investors can be misleading.[3] Speculative opportunities such as writing more straddles become unappealing when the market makers are already overly involved in the business. Furthermore, the dynamic response of option prices to market drops resembles the price cycles observed in insurance markets: an increase in the price of crash insurance caused by the contraction in market makers' capital following losses.[4]

This paper represents an initial attempt to model the dynamic interaction between option buyers and sellers. A two-agent dynamic general equilibrium model is constructed in which relatively crash-tolerant option market makers insure crash-averse investors. Heterogeneity in attitudes towards crash risk is modeled via heterogeneous state-dependent utility functions – an approach roughly equivalent to heterogeneous beliefs about the frequency of crashes. Crashes can occur in the model, given occasional adverse jumps in news about fundamentals. Derivatives are consequently not redundant in the model and serve the important function of dynamically completing the market. Given complete markets, equilibrium can be derived using an equivalent central planner's problem, and the corresponding dynamic trading strategies and market equilibria are identified. Those equilibria are compared to styled facts from options markets.

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[3]Basak and Cuoco (1998) make a similar point regarding calibrations of the consumption CAPM when most investors don't hold stock.

[4]Froot (2001, Figure 3) illustrates the strong, temporary impacts of Hurricane Andrew in 1992 and the Northbridge earthquake in 1994 upon the price of catastrophe insurance.

There have been previous papers exploring heterogeneous-agent dynamic equilibria, some of which have explored implications for option pricing. These papers diverge on the types of investor heterogeneity, the sources of risk, and the choice between production and exchange economies. Back (1993) and Basak (2000) focus on heterogeneous beliefs. Grossman and Zhou (1996) explore the general-equilibrium implications of heterogeneous preferences (in particular, the existence of portfolio insurers) in a terminal exchange economy, given only one source of risk (diffusive equity risk). Options are redundant in this framework, but the paper does look at the implications for option pricing. Weinbaum (2001) has a somewhat similar model, in which power utility investors differ in risk aversion. Bardhan and Chao (1996) examine the general issue of market equilibrium in exchange economies with intermediate consumption, with heterogeneous agents under jump-diffusions with discrete jump outcomes. Dieckmann and Gallmeyer (2005) use a special case of the Bardhan and Chao structure to explore the general-equilibrium implications of heterogeneous risk aversion.

This paper assumes a terminal exchange economy, and sufficient sources of risk that options are not redundant. Perhaps the major divergence from the above papers is this paper's focus on options markets. Whereas Bardhan and Chao (1996) and Dieckmann and Gallmeyer (2005) assume there are sufficient financial assets to dynamically complete the market, this paper focuses on the plausible hypothesis that options are the relevant market-completing financial assets. The paper develops some tricks for computing competitive equilibria using the short-dated options with overlapping maturities that we actually observe. Finally, the hypothesized source of heterogeneity – divergent attitudes towards crash risk – is plausible for motivating trading in options markets.

The objective of the paper is not to develop a better option pricing model. That can be done better with "reduced-form" option pricing models tailored to that objective; e.g., multi-factor option pricing models such as the Bates (2000) affine model or Santa-Clara and Yan (2005) quadratic model. Furthermore, this paper ignores stochastic volatility, which is assuredly relevant when building option pricing models. Rather, the objective of this paper is to build a relatively simple model of the role of options markets in financial intermediation of crash risk, in order to examine the theoretical implications for prices and dynamic equilibria. Key issues include: what fundamentally determines the price of crash risk? Can we explain the sharp shifts we observe in the price of crash risk? The ultimate objective is to explore the impact of plausible market frictions, such as assuming that only option market makers can write options, but that issue is not explored in this paper.

Section 1 of the paper recapitulates specific various stylized facts from empirical options research that influence the model construction. Section 2 introduces the basic framework, and identifies a benchmark homogeneous-agent equilibrium. Section 3 explores the implications of heterogeneity in agents. Section 4 concludes.

1. Empirical option pricing anomalies and stylized facts

Three categories of discrepancies between objective and risk-neutral measures will be kept in mind in the theoretical section of the paper: volatility, higher moments, and the implicit pricing kernel that in principle reconciles the objective and risk-neutral probability measures. Furthermore, each category can be decomposed further into average discrepancies, and conditional discrepancies.

The unconditional volatility puzzle is that implicit standard deviations (ISD's) from stock index options have been higher on average over 1988-98 than realized volatility over the options' lifetimes. For instance, ISD's from 30-day at-the-money put and call options on S&P 500 futures have been 2% higher on average than the subsequent annualized daily volatility over the lifetime of the options.[5] This discrepancy has generated substantial post-'87 profits on average from writing at-the-money puts or straddles, with Sharpe ratios roughly double that of investing in the stock market. See, e.g., Fleming (1998) or Jackwerth (2000).

The conditional volatility puzzle is that regressing realized volatility upon ISD's generally yields slopes that are significantly positive, but significantly less than one. For instance, the regressions using the 30-day ISD's and realized volatilities mentioned above yield volatility and variance results

[5]The puzzle is slightly exacerbated by the fact that at-the-money ISD's are in principle *downwardly* biased predictors of the (risk-neutral) volatility over the lifetime of the options.

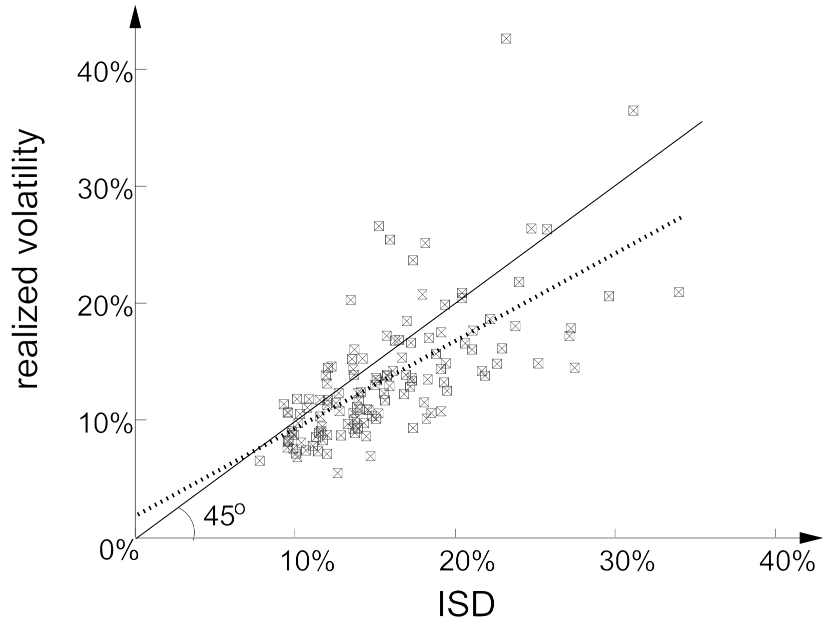

Figure 1. ISD's and realized volatility, 1988-98. ISD's are from 30-day S&P 500 futures options. Realized volatility is annualized, from daily log-differenced futures prices over the lifetime of the options.

$$\sqrt{\frac{365}{T}\sum_{\tau=t+1}^{T}(\Delta \ln F_\tau)^2} = \underset{(.0142)}{.0160} + \underset{(.102)}{.756}\, ISD_t + \varepsilon_{t+T}, \quad R^2 = .45 \tag{1}$$

$$\frac{365}{T}\sum_{\tau=t+1}^{T}(\Delta \ln F_\tau)^2 = \underset{(.0033)}{.0027} + \underset{(.161)}{.681}\, ISD_t^2 + \varepsilon_{t+T}, \quad R^2 = .33 \tag{2}$$

with heteroskedasticity-consistent standard errors in parentheses.[6] Since intercepts are small, the regressions imply that ISD's are especially poor forecasts of realized volatility when high. Straddle-trading strategies conditioned on the ISD level achieved Sharpe ratios almost triple that of investing directly in the stock market over 1988-98.

[6]Jiang and Tian (2005) find similar results from regressions using the "model-free" implicit variance measure of Britten-Jones and Neuberger (2000).

The skewness puzzle is that the levels of skewness implicit in stock index options are generally much larger in magnitude than those estimated from stock index returns – whether from unconditional returns (Jackwerth, 2000) or conditional upon a time series model that captures salient features of time-varying distributions (Rosenberg and Engle, 2002). Furthermore, implicit skewness falls off only slightly for longer maturities of stock index options of, e.g., 3-6 months.[7] By contrast, the distribution of log-differenced stock indexes or stock index futures converges rapidly towards near-normality as one progresses from daily to weekly to monthly holding periods.

Table 1
Implicit jump parameters, and (risk-neutral) cumulants at 1- and 6-month horizons, 1988-98 estimates.

Average jump size: -6.6%
Jump standard deviation: 11.0%
Jump intensity: $\lambda_t = 81.41\, V_{1t} + .01\, V_{2t}$

1-month cumulants

$$K_2 = 1.76e{-4} + .2053\, V_{1t} + .0795\, V_{2t}$$
$$K_3 = -1.01e{-5} - .0371\, V_{1t} - .0012\, V_{2t}$$
$$K_4 = 2.47e{-6} + 10.54e{-3}\, V_{1t} + .06e{-3}\, V_{2t}$$

6-month cumulants

$$K_2 = .0058 + 1.3080\, V_{1t} + .3802\, V_{2t}$$
$$K_3 = -.0012 - .8112\, V_{1t} - .0336\, V_{2t}$$
$$K_4 = .0007 + .7556\, V_{1t} + .0083\, V_{2t}$$

Average factor realizations: $Avg(V_1) = .0092$; $Avg(V_2) = .0143$.
Conditional variance = K_2; skewness = $K_3/K_2^{3/2}$; excess kurtosis = K_4 / K_2^2.

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[7]In options research, implicit skewness is roughly measured by the shape of the volatility "smirk," or pattern of ISD's across different strike prices ("moneyness"). The skewness/maturity interaction can be seen by examined by the volatility smirk at different horizons conditional upon rescaling moneyness proportionately to the standard deviation appropriate at different horizons. See, e.g., Bates (2000, Figure 4). Tompkins (2001) provides a comprehensive survey of volatility surface patterns, including the maturity effects.

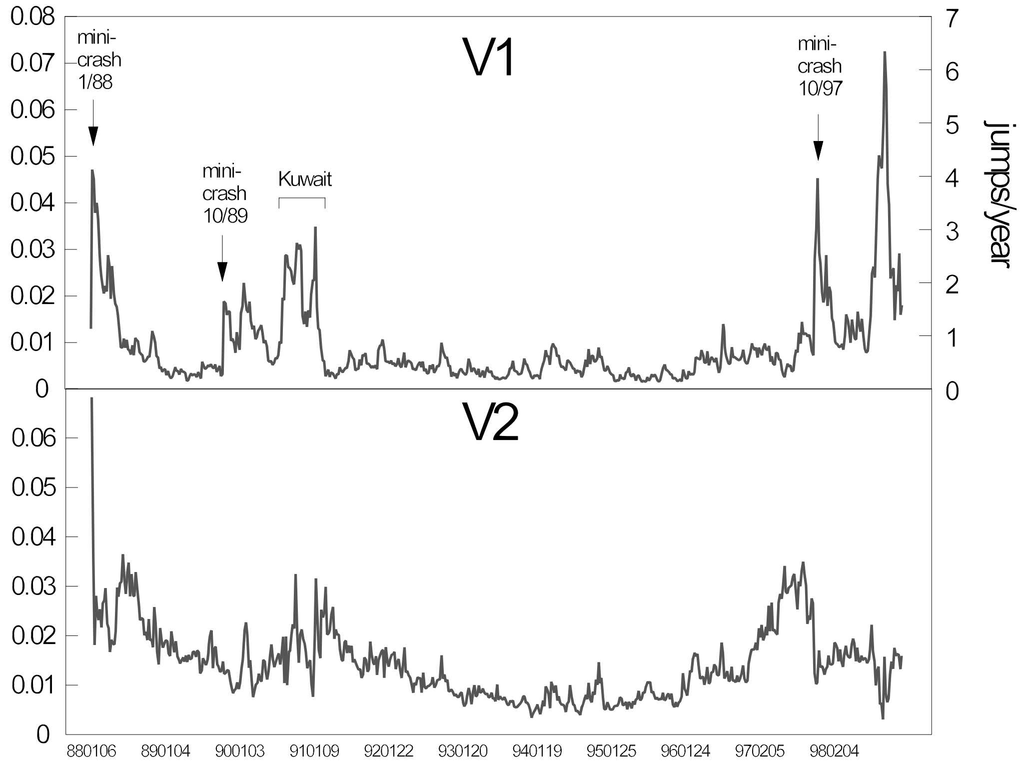

Figure 2. **Implicit factor estimates from S&P 500 futures options, 1988-98.** V1 affects all cumulants, at all maturities. V2 affects conditional variance but has little impact on higher cumulants. Units are in instantaneous variance per year conditional on no jumps (left scale), or in implicit jump frequency (right scale). See Bates (2000) for estimation details.

A further puzzle is the *evolution* of distributions implicit in option prices. Figure 2 summarizes that evolution using updated estimates of the Bates (2000) 2-factor stochastic volatility/jump-diffusion model with time-varying jump risk. The affine structure of that model permits a factor representation of implicit cumulants in terms of two underlying state variables. The first factor (V1) affects variance directly and also determines the jump intensity, thereby affecting cumulants at all maturities. The second factor (V2) influences instantaneous variance (with roughly half the variance loading of V1; see Table 1 above), but has relatively little impact on higher cumulants.

The graph indicates that the sharp market declines over 1988-98 (in January 1988, October 1989, August 1990, November 1997, and August 1998) were accompanied by sharp increases in implicit jump risk. The puzzles here are the abruptness of the shifts (Bates (2000) rejects the hypothesis that implicit jump risk follows an affine diffusion), and the magnitudes of implicit jump risk achieved following the market declines. Since affine models assume the risk-neutral and objective jump intensities are proportional, these models imply objective crash risk is highest immediately following crashes. And while assessing the frequency of rare events is perforce difficult, Bates (2000) finds no evidence that the occasionally high implicit jump intensities over 1988-93 could in fact predict subsequent stock return jumps.

Finally, there is the implicit pricing kernel puzzle discussed in Jackwerth (2000) and Rosenberg and Engle (2002). The sharp discrepancy between the negatively skewed risk-neutral distribution and roughly lognormal objective distribution at monthly horizons causes the risk-neutral mode to be to the right of the estimated objective mode, even though the risk-neutral mean is perforce to the left of the objective mean. If the level of the stock index is viewed as a reasonably good proxy for overall wealth of the representative agent, this discrepancy in distributions implies marginal utility of wealth is locally increasing in areas – implying utility functions that are locally convex in areas, rather than globally concave.[8]

It is possible that a standard representative agent/pricing kernel model can explain the above puzzles. Coval and Shumway (2001) and Bakshi and Kapadia (2003) attribute the substantial speculative opportunities from writing stock index options to a volatility risk premium. Pan (2002), by contrast, finds a substantial risk premium on time-varying jump risk is a promising candidate. The risk premium raises implicit jump risk, volatility, and skewness relative to the values from the objective distribution, while the time variation in jump risk can explain the conditional volatility

[8]Jackwerth's results are disputed by Aït-Sahalia and Lo (2000), who find no anomalies when comparing average option prices from 1993 with the unconditional return distribution estimated from overlapping data from 1989-93. The difference in results perhaps highlights the importance of using conditional rather than unconditional distributions, as in Rosenberg and Engle (2002). For instance, both conditional variance and implicit standard deviations are time-varying; and a substantial divergence between the two can produce anomalous implicit utility functions even in a lognormal environment.

bias. Bates (2000) finds that this model can also match the maturity profile of implicit skewness better than models with constant implicit jump risk.

The challenges for these explanations are devising theoretical models of compensation for risk consistent with the *magnitude* of the speculative opportunities. The stochastic evolution of implicit jump risks from option prices also appears difficult to explain. The apparent magnitude and evolution of the crash risk premium are the two central styled facts that I will attempt to match, in the models below.

2. A jump-diffusion economy

I consider a simple continuous-time endowment economy over $[0, T]$, with a single terminal dividend payment D_T at time *T*. News about this dividend (or, equivalently, about the terminal value of the investment) arrives as a univariate Markov jump-diffusion of the form

$$d\ln D = \mu_d dt + \sigma_d dZ + \gamma_d dN \qquad \textbf{(3)}$$

where Z is a standard Wiener process,

N is a Poisson counter with constant intensity λ, and

$\gamma_d < 0$ is a deterministic jump size or announcement effect, assumed negative.

$D_t = E_t D_T$ is the current signal about the terminal payoff and follows a martingale, implying $\mu_d = -\frac{1}{2}\sigma_d^2 - \lambda(e^{\gamma_d} - 1)$.

Financial assets are claims on terminal outcomes. Given the simple specification of news arrival, any three non-redundant assets suffice to dynamically span this economy; e.g., bonds, stocks, and a single long-maturity stock index option. However, it is analytically more convenient to work with the following three fundamental assets:

1) a riskless numeraire bond in zero net supply that delivers one unit of terminal consumption in all terminal states of nature;
2) an equity claim in unitary supply that pays a terminal dividend D_T at time *T*, and is priced at S_t at time *t* relative to the riskless asset; and

3) a jump insurance contract in zero net supply that costs an instantaneous and endogenously determined insurance rate λ_t^* per period, and pays off 1 additional unit of the numeraire asset conditional on each jump. The terminal payoff of one insurance contract held to maturity is $N_T - \int_0^T \lambda_t^* dt$.

Other assets such as options are redundant given these fundamental assets, and are priced by no arbitrage given equilibrium prices for the latter two assets. Equivalently, the jump insurance contract can be synthesized from the short-maturity options markets with overlapping maturities that we actually observe. The equivalence between optionsand jump insurance contracts is discussed below in section 3.4.2.

Agents are assumed to have crash-averse utility functions over terminal outcomes of the form

$$U(W_t, N_t, t) = E_t\left[e^{YN_T} \frac{W_T^{1-R} - 1}{1-R}\right] \text{ for } R > 0 \tag{4}$$

where W_T is terminal wealth, N_T is the number of jumps over $[0, T]$, and $Y > 0$ is a parameter of crash aversion. As this state-dependent generalization of power utility has not previously appeared explicitly in the finance literature, some motivation is necessary.

First, this specification makes explicit in utility terms what is implicit in the affine pricing kernels routinely used in the affine asset pricing literature. A typical affine approach for the pricing kernel η_t specifies a linear structure in the underlying sources of risk:

$$d\ln\eta = \mu_m dt + \sigma_m dZ + \gamma_m dN \tag{5}$$

see, e.g., Ho, Perraudin, and Sørensen (1996). Affine models place state-dependent restrictions on the functional forms of the coefficients σ_m and γ_m; they must generate covariances between $d\ln\eta$ and any state variable innovations that are linear in those state variables. However, the magnitudes of the coefficients are unrestricted; and those magnitudes determine the risk premia on the underlying shocks. The absence of such restrictions is equivalent to introducing state dependency into the utility function of the representative agent, in a exponentially affine form similar to (4).

A related justification is revealed preference – the derivation of utility functions consistent with observed risk premia. The prices of all risks in a traditional representative-agent power utility specification depend upon the risk aversion parameter R, constraining the ability of such models to simultaneously match the equity premium and the crash risk premium. The above utility function can be derived as the entropy-minimizing pricing kernel that generates specific instantaneous equity and jump risk premia, when returns are generated by an i.i.d jump-diffusion process. Conversely, the ability of the above utility specification to generate equity and crash risk premia will be apparent below.

Perhaps the most intuitive justification is that the crash aversion parameter Y can be viewed a utility-based proxy for subjective beliefs about crash risk. Investors with crash-averse preferences ($Y > 0$) are equivalent to investors with state-independent preferences and a subjective belief that the jump intensity is λe^{Y}:

$$\begin{aligned} E_0\left[e^{YN_T} u(W_T)\right] &= \sum_{N=0}^{\infty} \frac{e^{-\lambda T}\left(\lambda T e^{Y}\right)^N}{N!} E_0[u(W_T) | N \text{ } jumps] \\ &= e^{\lambda T(e^{Y}-1)} E_0^*\left[u(W_T) \mid \lambda^* = \lambda e^{Y}\right]. \end{aligned} \qquad \textbf{(6)}$$

This reflects the general proposition that preferences and beliefs are indistinguishable in a terminal exchange economy. It should be recognized, however, that this interpretation involves very strong subjective beliefs, in that investors do not update their subjective jump intensities λe^{Y} based on learning over time, or based on trading with other investors in the heterogeneous-agent equilibrium derived below.[9]

A final justification is provided by Liu, Pan, and Wang (2004), who derive the utility specification (4) from robust control methods given uncertainty aversion to the estimate of the jump intensity. It is also worth noting that (4) is a utility specification with convenient properties. It retains the homogeneity of standard power utility, and the myopic investment strategy property of the log utility subcase $(R = 1)$.

[9]As illustrated in Basak (2000), models with heterogeneous subjective beliefs can be substantially harder to solve than the utility-based approach used here.

The above is a model of "external" crash aversion: investors are averse to bad news shocks. An alternate "internal" crash aversion model could be constructed assuming investors' aversion to crashes depends on the degree to which their own investments are directly affected:

$$U(W_T) \;=\; u(W_T)\exp\left[-y\sum^{\tilde{N}_T}\gamma_w\right] \tag{7}$$

where γ_w is the jump in log wealth conditional upon a jump occurring, and conditional upon the investor's portfolio allocation. The major advantage to the external crash aversion in (4) is its analytic tractability. While it is possible to work out homogeneous-agent equilibria using internal crash aversion, deriving heterogeneous-agent equilibria is trickier. The difference in specifications echoes the analytic advantages of external over internal habit formation models discussed in Campbell, Lo and MacKinlay (1997, p. 327-8).

2.1 Equilibrium in a homogeneous-agent economy

The fundamental equations for pricing equity and crash insurance are

$$\begin{aligned} \eta_t &= E_t\,\eta_T \\ S_t &= \frac{E_t\,\eta_T D_T}{\eta_t} \\ \lambda_t^* &= \lambda\left[\frac{\eta_{t+dt}|_{dN=1}}{\eta_t}\right] \end{aligned} \tag{8}$$

where η_T/η_t is a nonnegative pricing kernel. The first two equations are standard; see, e.g., Grossman and Zhou (1996). The last is derived in Bates (1988, 1991).[10] If all agents have identical crash-averse preferences of the form given in (4) above, the pricing kernel can be derived from the terminal marginal utility:

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[10]A crash insurance contract with instantaneous cost $\lambda_t^*\,dt$ that pays off 1 unit of the numeraire conditional upon a jump occurring in $(t, t+dt]$ is priced at

$$\eta_t\lambda_t^*\,dt \;=\; E_t\left[\eta_{t+dt}1|_{dN=1}\right] \;=\; \lambda\,dt\,\eta_{t+dt}|_{dN=1}$$

yielding the above expression.

$$\eta_T = U_W(W_T, N_T)\,|_{W_T = D_T} \\ = D_T^{-R}\, e^{YN_T}. \tag{9}$$

The following lemma is useful for computing relevant conditional expectations.

Lemma: *If* $d_t \equiv \ln D_t$ *follows the jump-diffusion in* (3) *above and* N_t *is the underlying jump counter with intensity* λ*, then*

$$E_t e^{\Phi d_T + \psi N_T} = \exp\left\{\Phi d_t + \psi N_t + (T-t)\left[\Phi\mu_d + \tfrac{1}{2}\Phi^2\sigma_d^2 + \lambda\left(e^{\Phi\gamma_d + \psi} - 1\right)\right]\right\}. \tag{10}$$

Proof: For $\tau \equiv T - t$, there is a probability $w_n = e^{-\lambda\tau}(\lambda\tau)^n / n!$ of observing $n \equiv N_T - N_t$ jumps over $(t,\ T]$. Conditional upon n jumps, $\Delta d \equiv \ln D_T / D_t \sim N[\mu_d\tau + \gamma_d n,\ \sigma_d^2\tau]$, and

$$\begin{aligned} E_t e^{\Phi d_T + \psi N_T} &= e^{\Phi d_t + \psi N_t} E_t \exp[\Phi\Delta d + \psi n] \\ &= e^{\Phi d_t + \psi N_t} E_t \exp[\Phi\Delta d|_{n=0} + n(\Phi\gamma_d + \psi)] \\ &= e^{\Phi d_t + \psi N_t}\exp\left[(\Phi\mu_d + \tfrac{1}{2}\Phi^2\sigma_d^2)\tau + \lambda\tau\left(e^{\Phi\gamma_d + \psi} - 1\right)\right]. \end{aligned} \tag{11}$$

The last line follows from the independence of the Wiener and jump components, and from the moment generating functions for Wiener and jump processes. ■

Using the lemma, equations (8), and $\mu_d = -\tfrac{1}{2}\sigma_d^2 - \lambda(e^{\gamma_d} - 1)$ yields the following asset pricing equations:

$$\lambda^* = \lambda e^{Y - R\gamma_d} \tag{12}$$

$$\eta_t = D_t^{-R} e^{YN_t}\ e^{(T-t)[-R\mu_d + \frac{1}{2}R^2\sigma_d^2 + (\lambda^* - \lambda)]} \tag{13}$$

$$S_t = D_t \exp\left\{(T-t)\left[-R\sigma_d^2 + (\lambda^* - \lambda)(e^{\gamma_d} - 1)\right]\right\} \tag{14}$$

The last equation implies that the price of equity relative to the riskless numeraire follows roughly the same i.i.d. jump-diffusion process as the underlying news about terminal value, with identical instantaneous volatility and jump magnitudes:

$$dS/S = \mu\,dt + \sigma_d dZ + k(dN - \lambda\,dt) \tag{15}$$

for $k \equiv e^{\gamma_d} - 1 < 0$. The instantaneous equity premium

$$\begin{aligned} \mu &= R\sigma_d^2 + (\lambda - \lambda^*)k \\ &\approx R(\sigma_d^2 + \lambda\gamma_d^2) + (-\lambda\gamma_d)Y \end{aligned} \tag{16}$$

reflects required compensation for two types of risk. First is the required compensation for stock market variance from diffusion and jump components, roughly scaled by the coefficient of relative risk aversion. Second, the crash aversion parameter $Y \geq 0$ increases the required excess return when stock market jumps are negative.

Crash aversion also directly affects the price of crash insurance relative to the actual arrival rate of crashes:

$$\log(\lambda^*/\lambda) = -R\gamma_d + Y. \tag{17}$$

Finally, derivatives are priced *as if* equity followed the risk-neutral martingale

$$dS/S = \sigma_d dZ^* + k(dN^* - \lambda^* dt) \tag{18}$$

where N^* is a jump counter with constant intensity λ^*. The resulting (forward) option prices are identical to the deterministic-jump special case of Bates (1991), given the geometric jump-diffusion.

2.2 Consistency with empirical anomalies

The homogeneous crash aversion model can explain some of the stylized facts from section 1. First, unconditional bias in implied volatilities is explained by the potentially substantial divergence between the risk-neutral instantaneous variance $\sigma_d^2 + \lambda^*\gamma_d^2$ implicit in option prices, and the actual instantaneous variance $\sigma_d^2 + \lambda\gamma_d^2$ of log-differenced asset prices. Second, the difference between λ^* and λ is consistent with the observation in Bates (2000, pp. 220-1) and Jackwerth (2000, pp.

446-7) of too few observed jumps over 1988-98 relative to the number predicted by stock index options. The extra parameter Y permits greater divergence in λ^* from λ than is feasible under standard parameterizations of power utility.

To illustrate this, consider the following calibration: a stock market volatility $\sigma_d = 15\%$ annually conditional upon no jumps, and adverse news of $\gamma_d = -10\%$ that arrives on average once every four years ($\lambda = .25$). From equations (16) and (17), the equity premium and crash insurance premium are

$$\begin{aligned} \mu &\approx .025R + .025Y \\ \ln(\lambda^*/\lambda) &= .10R + Y \end{aligned} \qquad \textbf{(19)}$$

For $R = 1$ and $Y = 1$, the equity premium is 5%/year, while the jump risk λ^* implicit in option prices is three times that of the true jump risk. Thus, the crash aversion parameter Y is roughly as important as relative risk aversion for the equity premium, but substantially more important for the crash premium. Achieving the observed substantial disparity between λ^* and λ using risk aversion alone ($Y = 0$) would require levels of R that most would find unpalatable, and which would imply an implausibly high equity premium.

Since returns are i.i.d. under both the actual and risk-neutral distribution, the homogeneous-agent model is not capable of capturing the dynamic anomalies discussed in section 1. The standard results from regressing realized on implicit variance cannot be replicated here, because neither is time-varying in this model. Were there a time-varying volatility component in the news process, however, the difference between λ^* and λ would affect the intercept from such regressions but could not explain why the slope estimate is less than 1. Second, the model cannot match the observed tendency of λ_t^* to jump contemporaneously with substantial market drops. Finally, the i.i.d. return structure implies that implicit distributions should rapidly converge towards lognormality at longer maturities, which does not accord with the maturity profile of the volatility smirk.

Furthermore, Jackwerth's (2000) anomaly cannot be replicated under homogeneous crash aversion. As discussed in Rosenberg and Engle (2002), Jackwerth's implicit pricing kernel involves the projection of the actual pricing kernel upon asset payoffs. E.g., stock index options with terminal payoff $V(S_t)$ have an initial price

$$\begin{aligned} v_0 &= \frac{E_0[\eta_t V(S_t)]}{\eta_0} \\ &= E_0\left[V(S_t)\,\frac{E_0[\eta_t \mid S_t]}{E_0[\eta_t]}\right] \\ &\equiv E_0[V(S_t)\,M(S_t)]\,, \end{aligned} \tag{20}$$

where $M(S_t)$ has the usual properties of pricing kernels: it is nonnegative, and $E_0[M(S_t)] = 1$.

It is shown in the appendix that for crash-averse preferences, this projection takes the form

$$M(S_t) = \kappa(t)\; S_t^{-R}\;\frac{p\left(S_t \mid \lambda e^{Y}\right)}{p(S_t \mid \lambda)} \tag{21}$$

where $\kappa(t)$ is a function of time and $p(S_t \mid \lambda)$ is the probability density function of S_t conditional upon a jump intensity of λ over (0, t). Implicit relative risk aversion is given by $-\partial \ln M(S) / \partial \ln S$. For $Y = 0$, one observes the strictly decreasing pricing kernel and constant relative risk aversion associated with power utility. For $Y > 0$, it is proven in the appendix that $\ln M(S_t)$ is a strictly decreasing function of $\ln S_t$ that is illustrated below in Figure 3. Thus, this pricing kernel cannot replicate the negative implicit risk aversion (positive slope) estimated by Jackwerth (2000) and Rosenberg and Engle (2002) for some values of S_t. However, crash-averse preferences can replicate the higher implicit risk aversion (steeper negative slope) for low $\ln S_t$ values that was estimated by those authors and by Aït-Sahalia and Lo (2000).

Jackwerth (2000, p.446) conjectures that the negative risk aversion estimate may be attributable to investors overestimating the crash risk relative to the observed *ex post* crash frequency. Within this model, such overestimation is equivalent to a positive value of Y, and cannot generate the required divergences between objective and risk-neutral distributions. In equilibrium the equity premium (16) is also positively affected by Y, shifting the mode of the objective distribution sufficiently to the right to preclude observing Jackwerth's anomaly. Of course, there could still be an anomalous disparity between the risk-neutral distribution and the *estimate* of the objective distribution.

Jackwerth's exploration of whether the divergence between the risk-neutral and estimated objective distributions is implausibly profitable is a separate issue. Within this framework, crash aversion can generate investment opportunities with high Sharpe ratios. For instance, the instantaneous Sharpe ratio on writing crash insurance is

$$\frac{\lambda^* dt - E_t[1|_{dN=1}]}{\sqrt{Var_t[1|_{dN=1}]}} = \frac{(\lambda^* - \lambda)dt}{\lambda dt(1 - \lambda dt)} = \frac{\lambda^*}{\lambda} - 1 \tag{22}$$

which can be substantially larger than the instantaneous Sharpe ratio $\mu / \sqrt{\sigma^2 + \lambda k^2}$ on equity given investors' aversion to this type of risk. The put selling strategies examined in Jackwerth implicitly involve a portfolio that is instantaneously long equity and short crash insurance. Since adding a high Sharpe ratio investment to a market investment must raise instantaneous Sharpe ratios, this model is consistent with the substantial profitability of option-writing strategies reported in Jackwerth (2000) and elsewhere.

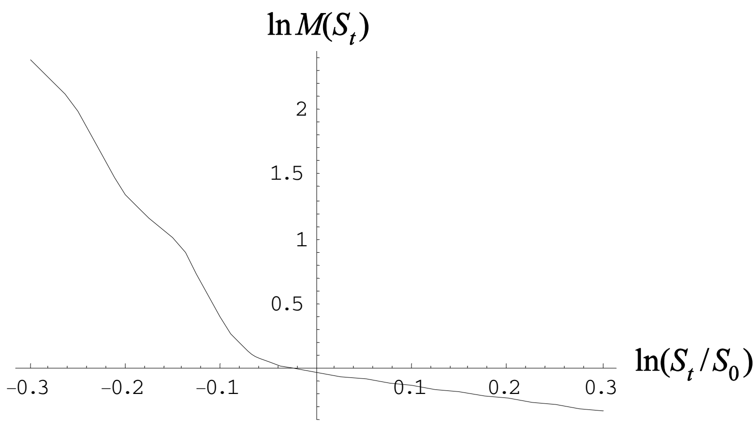


Figure 3. Log of the implicit pricing kernel conditional upon realized returns. Calibration: $t = 1/12$, $\sigma_d = .15$, $R = Y = 1$, $\gamma_d = -.10$, $\lambda = .25$.

3. Equilibrium in a heterogeneous-agent economy

As this model is dynamically complete, equilibrium in the heterogeneous-agent case can be identified by examining an equivalent central planner's problem in weighted utility functions. The solution to that problem is Pareto-optimal, and can be attained by a competitive equilibrium for traded assets in which all investors willingly hold market-clearing optimal portfolios given equilibrium asset price evolution. Section 3.1 below outlines the central planner's problem, while Section 3.2 discusses the resulting asset market equilibrium. Section 3.3 identifies the supporting individual wealth evolutions and associated portfolio allocations, and confirms the optimality of the equilibrium. Section 3.4 discusses the implications for option prices, while Section 3.5 compares the equilibrium with the stylized facts discussed above in Section 1.

3.1 The central planner's problem

For analytic tractability, I will assume all investors have common risk aversion R, but differ in crash aversion Y. Under common beliefs about state probabilities, the central planner's problem of maximizing a weighted average of expected state-dependent utilities is equivalent to constructing a representative state-dependent utility function in terminal wealth (Constantinides 1982, Lemma 2):

$$
\begin{aligned}
&\boldsymbol{U}(W_T, N_T; \boldsymbol{\omega}) \equiv \max_{\{W_{YT}\}} \sum_Y \omega_Y f^Y(N_T) \frac{W_{YT}^{1-R} - 1}{1-R}, \quad R > 0 \\
&\textit{subject to } W_T = \sum_Y W_{YT}, \; W_{YT} \geq 0 \; \forall Y
\end{aligned}
\tag{23}
$$

for fixed weights $\boldsymbol{\omega} \equiv \{\omega_Y\}$ that depend upon the initial wealth allocation in a fashion determined below in Section 3.3. Since the individual marginal utility functions $U_W(W_{YT}, N_T; Y) = +\infty$ at $W_{YT} = 0$ and the horizon is finite, the individual no-bankruptcy constraints $W_{YT} \geq 0$ are non-binding and can be ignored. Optimizing the Lagrangian

$$
\max_{\{W_{YT}\}, \eta_T} \sum_Y \omega_Y f^Y(N_T) \frac{W_{YT}^{1-R} - 1}{1-R} + \eta_T \left[W_T - \sum_Y W_{YT} \right] \tag{24}
$$

yields a terminal state-dependent wealth allocation

$$w_Y(N_T, T; \boldsymbol{\omega}) \equiv \frac{W_{YT}}{W_T} = \frac{[\omega_Y f^Y(N_T)]^{1/R}}{\sum_Y [\omega_Y f^Y(N_T)]^{1/R}} \tag{25}$$

and a Lagrangian multiplier

$$\begin{aligned} \eta_T &= W_T^{-R}\left\{\sum_Y \left[\omega_Y f^Y(N_T)\right]^{\frac{1}{R}}\right\}^R \\ &\equiv W_T^{-R}\bar{f}(N_T; \boldsymbol{\omega}) \end{aligned} \tag{26}$$

where $\bar{f}$ is a CES-weighted average of individual crash aversion functions f^Y's. The Lagrangian multiplier $\eta_T = U_W(W_T, N_T; \boldsymbol{\omega})$ is the shadow value of terminal wealth, and therefore determines the pricing kernel when evaluated at $W_T = D_T$. From the first-order conditions to (24), all individual terminal marginal utilities of wealth are directly proportional to the multiplier:

$$U_W(W_{YT}, N_T; Y) = \frac{\eta_T}{\omega_Y}. \tag{27}$$

3.2 Asset market equilibrium

As in equations (8) above, the pricing kernel η_T/η_t can be used to price all assets. That asset market equilibrium depends critically upon expectations of average crash aversion. Define

$$\begin{aligned} g(N_t, t; \lambda') &\equiv E_t\left[\bar{f}(N_t + n) \mid \lambda'\right] \\ &= \sum_n \frac{e^{-\lambda'(T-t)}[\lambda'(T-t)]^n}{n!}\bar{f}(N_t + n) \end{aligned} \tag{28}$$

as the conditional expectation of $\bar{f}(N_T)$ given jump intensity λ' over $(t, T]$ for future jumps $n \equiv N_T - N_t$. It is shown in the appendix that the resulting asset pricing equations are

$$\eta_t = e^{\kappa_\eta(T-t)} D_t^{-R} g\left(N_t, t, \lambda e^{-R\gamma_d}\right) \tag{29}$$

$$\frac{S_t}{D_t} = e^{\kappa_S(T-t)} \frac{g\left(N_t, t, \lambda e^{(1-R)\gamma_d}\right)}{g\left(N_t, t, \lambda e^{-R\gamma_d}\right)} \equiv e^{\kappa_S(T-t)} m(N_t, t) \tag{30}$$

$$\lambda^*(N_t, t) = \lambda e^{-R\gamma_d} \frac{g\left(N_t + 1, t, \lambda e^{-R\gamma_d}\right)}{g\left(N_t, t, \lambda e^{-R\gamma_d}\right)} \tag{31}$$

where $\kappa_\eta = -R\mu_d + \frac{1}{2}R^2\sigma_d^2 + \lambda(e^{-R\gamma_d} - 1)$ and $\kappa_S = -R\sigma_d^2 + \lambda\left(e^{-R\gamma_d} - 1\right)(e^{\gamma_d} - 1)$.

The equilibrium equity price follows a jump-diffusion of the form

$$\frac{dS}{S} = \mu(N_t, t)\,dt + \sigma_d dZ + k(N_t, t)\,(dN - \lambda\,dt) \tag{32}$$

where

$$\mu(N_t, t) = -E_t\left[\frac{dS}{S}\frac{d\eta}{\eta}\right] = R\sigma_d^2 + [\lambda - \lambda^*(N_t, t)]\,k(N_t, t) \tag{33}$$

and

$$1 + k(N_t, t) = e^{\gamma_d} \frac{m(N_t + 1, t)}{m(N_t, t)} \tag{34}$$

for $m(N, t)$ defined above in equation (30). The risk-neutral price process follows a martingale of the form

$$\frac{dS}{S} = \sigma_d dZ + k(N_t^*, t)\,[dN^* - \lambda_t^* dt] \tag{35}$$

for N_t^* a risk-neutral jump counter with instantaneous jump intensity $\lambda^*(N_t^*, t)$, the functional form of which is given above in equation (31).

Several features of the equilibrium are worth emphasizing. First, conditional upon no jumps the asset price follows a diffusion similar to the news arrival process D_t – i.e., with identical and constant instantaneous volatility σ_d. This property reflects the assumption of common relative risk aversion R, and would not hold in general under alternate utility specifications or heterogeneous risk aversion.[11] A further implication discussed below is that all investors hold identical equity positions.

Second, the equilibrium price process and crash insurance premium depend critically upon the heterogeneity of agents. This is simplest to illustrate in the $R = 1$ case, for which equilibrium values can be expressed directly in terms of the weighted distribution of individual crash aversions. Define pseudo-probabilities

$$\pi_{Yt} \equiv \frac{\omega_Y \exp[YN_t + \lambda e^{-\gamma_d}(T-t)(e^{Y}-1)]}{\sum_Y \omega_Y \exp[YN_t + \lambda e^{-\gamma_d}(T-t)(e^{Y}-1)]} \tag{36}$$

as the weight assigned to investors of type Y at time t, and define cross-sectional average $E_{CS}(\bullet)$, variance $Var_{CS}(\bullet)$, and covariance with respect to those weights. It is shown in the appendix that the asset market equilibrium takes the form

$$\begin{aligned} \ln(\lambda_t^*/\lambda) &= -\gamma_d + \ln E_{CS}\left[e^{Y}\right] \\ &\approx -\gamma_d + E_{CS}[Y] + \tfrac{1}{2} Var_{CS}[Y] \end{aligned} \tag{37}$$

$$\begin{aligned} \frac{\ln(S_t/D_t)}{T-t} &= -\kappa_s + \left\{\ln E_{CS}\left[e^{\Phi(e^{Y}-1)} \mid \Phi = \lambda(T-t)e^{-\gamma_d}(e^{\gamma_d}-1)\right]\right\} / (T-t) \\ &\approx -\tfrac{1}{2}\sigma_d^2 + \left[\lambda e^{-\gamma_d} E_{CS}\left(e^{Y}\right) - \lambda\right](e^{\gamma_d}-1) \end{aligned} \tag{38}$$

$$\ln(1+k_t) \approx \gamma_d\left[1 + \lambda e^{-\gamma_d}(T-t)\, Cov_{CS}\left(Y, e^{Y}\right)\right]. \tag{39}$$

To a first-order approximation, jump insurance premia in (37) and equity prices in (38) replicate the homogeneous-agent equilibria of (12) and (14) at $R = 1$, using average values for Y and e^{Y}, respectively. However, heterogeneity introduces second- and higher-order effects, as well, depending upon the dispersion of agents. In particular, the size of log equity jumps $\ln(1+k_t)$ in (34) and (39) can be substantially magnified relative to the adverse news shock γ_d about terminal value

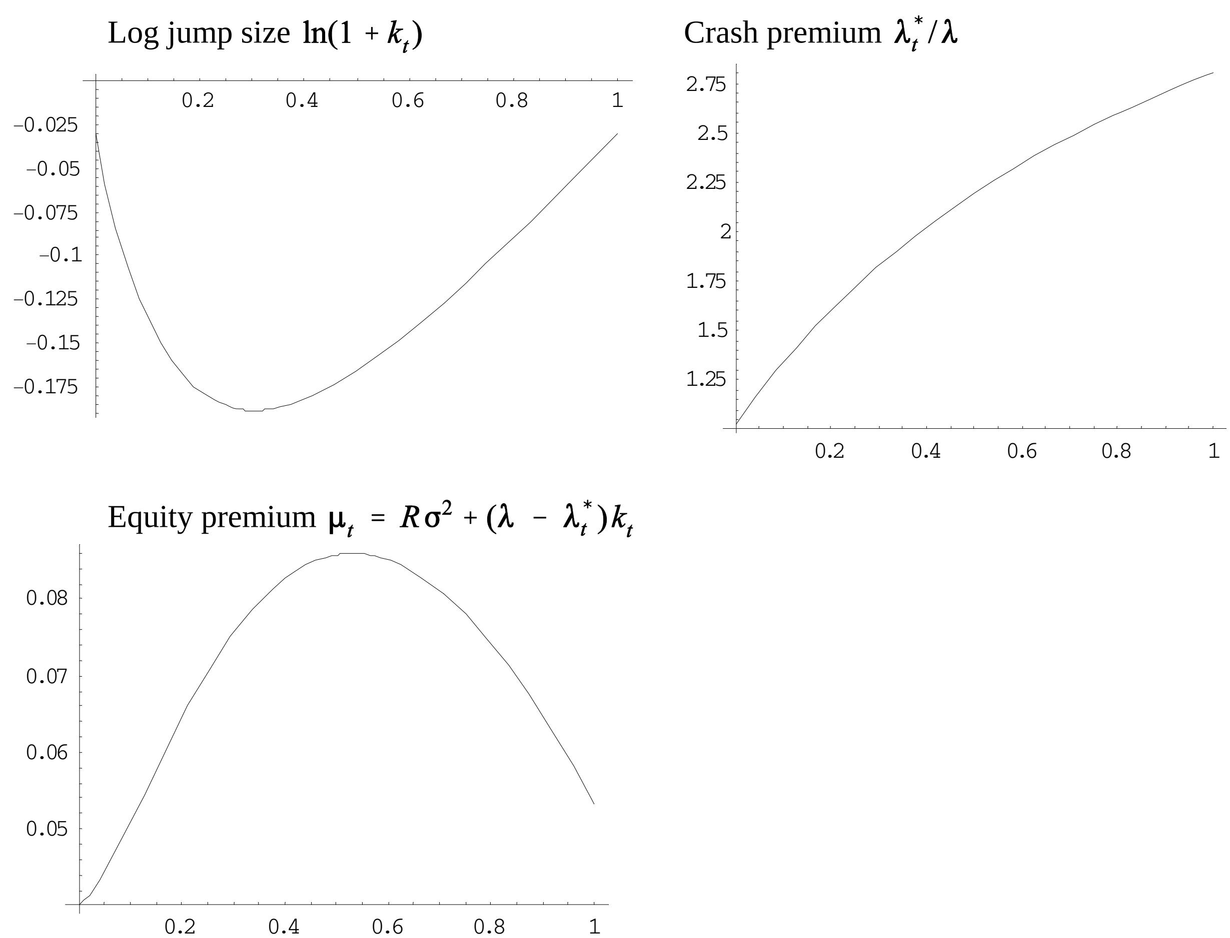
[11] Dieckmann and Gallmeyer (2005) find that heterogeneous risk aversion increases stock market volatility relative to the underlying sources of risk.

when there is substantial heterogeneity in agents.

Figure 4 below illustrates these impacts in the case of only two types of agents, conditional upon the initial wealth distribution and its impact on social weights $\boldsymbol{\omega}$ (given below in equation (42)) and conditional upon a small adverse news shock $\gamma_d = -.03$. The calibration does involve substantial heterogeneity; crash-averse preferences with $Y = 1$ are equivalent to subjective beliefs regarding jump frequencies that are $e^Y \approx 2.7$ times higher than those of the crash-tolerant agents with $Y = 0$.

When both types of agents are well represented, in the central areas of wealth distribution,

Figure 4: Impact of initial relative wealth share $w_1 = W_1(0)/W(0)$ upon initial equilibrium quantities. Two agents, with crash aversion $Y = 0, 1$, respectively. Calibration: $\sigma = .20$, $\lambda = .25$, $\gamma_d = -.03$; $R = 1$, $T = 50$, $t = 0$.



there is a substantial impact of small announcements upon jumps in log equity prices. The divergence of preferences implies substantial trading of crash insurance, and substantial wealth redistribution and shifts in the investment opportunity set conditional upon a jump. The result is that a modest 3% drop in the terminal value signal can induce a 3% to 18% drop in the log price of equity. Crashes redistribute wealth, making the "average" investor more crask-averse and exacerbating the impact of adverse news shocks. As indicated in Table 2 below, this magnification is also present for alternate values of the risk aversion parameter *R*.

The crash insurance rate λ_t^* is always between the $\lambda e^{-R\gamma_d}$ value of the crash-tolerant investors ($Y = 0$), and the $\lambda e^{Y - R\gamma_d}$ value of the crash-averse investors. Its value depends monotonically upon the relative weights of the two types of investors, and is biased upward relative to the wealth-weighted average by the variance term in equation (37). The equity premium μ varies somewhat with the magnitude of crash risk, in a non-monotonic fashion.

Table 2. Average log jump size $\ln(1 + k_t)$ *conditional* upon initial wealth allocation $w_1 = W_1(0)/W(0)$ and risk aversion *R*. Calibration: $\sigma = .20$, $\lambda = .25$, $\gamma_d = -.03$, $T = 50$.

$\ln(1 + k_t)$ *given:*	*R*				
w_1	.5	1	2	4	8
0	-.030	-.030	-.030	-.030	-.030
.0001	-.030	-.030	-.030	-.030	-.030
.001	-.032	-.032	-.030	-.030	-.030
.01	-.052	-.045	-.036	-.031	-.030
.1	-.157	-.136	-.090	-.044	-.034
.2	-.189	-.178	-.135	-.061	-.038
.3	-.187	-.189	-.163	-.079	-.043
.4	-.171	-.183	-.177	-.097	-.048
.5	-.149	-.166	-.178	-.114	-.053
.6	-.125	-.144	-.169	-.128	-.059
.7	-.101	-.118	-.149	-.135	-.066
.8	-.076	-.090	-.119	-.129	-.072
.9	-.053	-.060	-.079	-.096	-.065
.99	-.032	-.033	-.035	-.035	-.033
.999	-.030	-.030	-.030	-.030	-.030
1	-.030	-.030	-.030	-.030	-.030

A final observation is that the asset market equilibrium depends upon the number of jumps N_t, and is consequently nonstationary. This is an almost unavoidable feature of equilibrium models with a fixed number of heterogeneous agents. Heterogeneity implies agents have different portfolio allocations, implying their relative wealth weights and the resulting asset market equilibrium depend upon the nonstationary outcome of asset price evolution.[12] In this model, the number of jumps N_t and time t are proxies for wealth distribution. Crashes redistribute wealth towards the more crash-averse, making the representative agent more crash-averse. An absence of crashes has the opposite effect through the payment of crash insurance premia.

3.3 Supporting wealth evolution and portfolio choice

An investor's wealth at any time t can be viewed as the value (or cost) of a contingent claim that pays off the investor's share of terminal wealth $W_T = D_T$ conditional upon the number of jumps:

$$\begin{aligned} W_Y(t) &= E_t\left[\frac{\tilde{\eta}_T}{\eta_t}\tilde{D}_T\, w_Y(N_T, T; \boldsymbol{\omega})\right] \\ &= S_t\,\frac{E_t\left[\bar{f}(N_T;\boldsymbol{\omega})\ \dfrac{\omega_Y^{1/R}e^{YN_T/R}}{\sum_Y \omega_Y^{1/R}e^{YN_T/R}}\ \Big|\ \lambda e^{(1-R)\gamma_d}\right]}{E_t\left[\bar{f}(N_T;\boldsymbol{\omega})\ \big|\ \lambda e^{(1-R)\gamma_d}\right]} \\ &\equiv S_t\, w_Y(N_t, t; \boldsymbol{\omega}), \end{aligned} \tag{40}$$

see equation (A.16) in the appendix for details. The quantity $w_Y(N_t, t; \boldsymbol{\omega})$ is the current share of current total wealth $W(t) = S_t$, and appropriately sums to 1 across all investors. The weights $\boldsymbol{\omega}$ of the social utility function are implicitly identified up to an arbitrary factor of proportionality by the initial wealth distribution:

$$\begin{aligned} w_Y\big|_{t=0} &= w_Y(0,0;\boldsymbol{\omega}) \\ &= \kappa\, E_0\left[\omega_Y^{1/R}e^{YN_T/R}\,\bar{f}(N_T;\boldsymbol{\omega})^{1-\frac{1}{R}}\ \big|\ \lambda e^{(1-R)\gamma_d}\right] \end{aligned} \tag{41}$$

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[12] See Dumas (1989) and Wang (1996) for examples of the predominantly nonstationary impact of heterogeneity in a diffusion context. An interesting exception is Chan and Kogan (2002), who show that external habit formation preferences can induce stationarity in an exchange economy with heterogeneous agents.

for $\kappa \equiv E_0\left[\bar{f}(N_T;\boldsymbol{\omega}) \mid \lambda e^{(1-R)\gamma_d}\right]$. In the $R = 1$ case the mapping between $\boldsymbol{\omega}$ and the initial wealth distribution is explicit, and takes the form

$$w_Y\big|_{t=0} = \kappa\,\omega_Y\, e^{\lambda T(e^{Y}-1)}. \tag{42}$$

The investment strategy that dynamically replicates the evolution of $W_Y(t)$ can be identified using positions in equity and crash insurance that mimic the diffusion- and jump-contingent evolution:

$$\begin{aligned} X_Y &= \frac{\partial W_Y(S_t, N_t, t)}{\partial S} = w_Y(N_t, t;\boldsymbol{\omega}) \\ Q_Y &= [\Delta W_Y - N_S \Delta S]_{dN=1} \\ &= S(1+k_t)[w_Y(N_t+1, t, \boldsymbol{\omega}) - w_Y(N_t, t, \boldsymbol{\omega})]. \end{aligned} \tag{43}$$

where $k_t = k(N_t, t)$ is the percentage jump size in the equity price given above in equations (34) and (39). Thus, each investor holds $X_Y = W_Y(t)\,/\,S_t$ shares of equity (i.e., is 100% invested in equity), and holds a relative crash insurance position of

$$q_Y(t) \equiv \frac{Q_Y(t)}{W_Y(t)} = (1+k_t)\left[\frac{w_Y(N_t+1, t;\boldsymbol{\omega})}{w_Y(N_t, t;\boldsymbol{\omega})} - 1\right]. \tag{44}$$

The wealth-weighted aggregate crash insurance positions $\sum_Y w_Y(N_t, t;\boldsymbol{\omega})\, q_Y(t)$ appropriately sum to 0.

Figure 5 below graphs the individual crash insurance demands (q_0, q_1) given crash aversions $Y = 0$ and 1, respectively, conditional upon the initial wealth allocation $w_1 = W_1(0)\,/\,W(0)$ and its impact upon equilibrium (λ_t^*, k_t). The aggregate demand for crash insurance $w_1 q_1$ is also graphed, using the same calibration as in Figure 4 above. At $w_1 = 0$, crash-tolerant investors $(Y = 0)$ set a relatively low market-clearing price $\lambda_t^* = \lambda e^{-\gamma_d}$ and sell little insurance. Crash-averse investors $(Y = 1)$ insure heavily individually, but are a negligible fraction of the market. As w_1 increases, λ_t^* does as well (see Figure 4 above) and the crash insurance positions of both investors decline. Aggregate crash insurance volumes are heaviest in the central regions where both types of investors are well represented. As w_1 approaches 1, the high price of crash insurance induces crash-tolerant investors to sell insurance that will cost them 60% of their wealth conditional upon a crash.

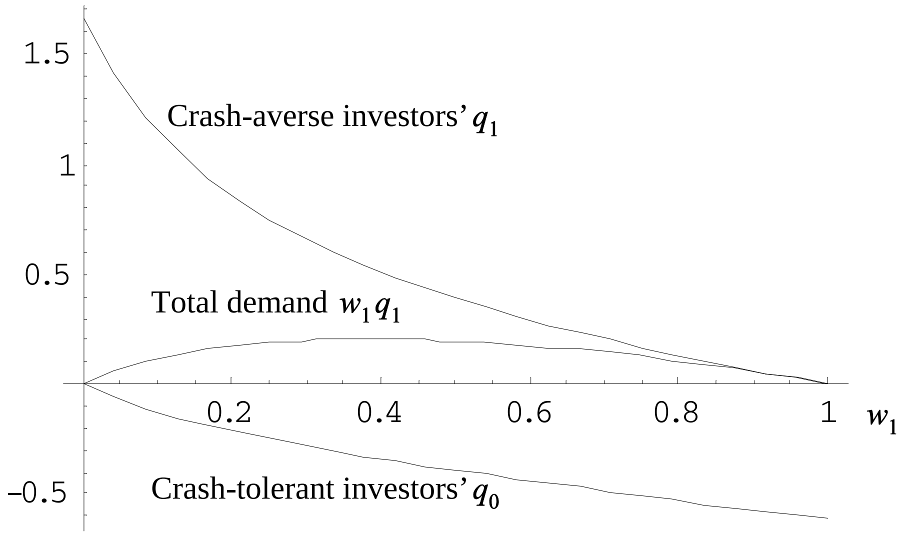

Figure 5. Equilibrium crash insurance positions and aggregate demand for crash insurance, as a function of $w_1 = W_1(0)/W(0)$. Calibration is the same as in Figure 4.

3.3.1 Optimality

The individual's investment strategy yields a terminal wealth W_{YT}, and an associated terminal marginal utility of wealth $U_W(W_{YT}, N_T; Y)$ that (from equation (27)) is proportional to the Lagrangian multiplier η_T that prices all assets. Therefore, no investor has an incentive to perturb his investment strategy given equilibrium asset prices and price processes. Furthermore, as noted above, the markets for equity and crash insurance clear, so the markets are in equilibrium. Since all individual state-dependent marginal utilities are proportional at expiration, the market is effectively complete. All investors agree on the price of all Arrow-Debreu securities, so their introduction would not affect the equilibrium.

3.3.2 Comparison with myopic investment strategies

The equilibrium asset price evolution in Section 3.2 involves considerable and stochastic evolution over time of the instantaneous investment opportunity set. Since Merton (1973), hedging against such shifts has been identified as the key distinction between static and dynamic asset market equilibria. As there are conflicting results even in a diffusion setting as to the quantitative importance of such hedging,[13] and as there has been little exploration of the issue in a jump-diffusion context, a comparison with the myopic investment strategies characteristic of static equilibria may be useful. Furthermore, myopic strategies are optimal when investors have unitary risk aversion ($R = 1$), or when returns are i.i.d. – e.g., in the case of investor homogeneity.

The myopic portfolio allocation is defined as the position that maximizes terminal expected utility

$$J(W_t,\ N_t,\ t)\ \equiv\ \max\ E_t\ e^{YN_T}\frac{W_T^{1-R}-1}{1-R} \tag{45}$$

conditional upon assuming instantaneous investment opportunities will remain unchanged at the current level over the investor's lifetime. Those opportunities are summarized by the instantaneous cost of crash insurance λ^*, and the price process

$$dS/S\ =\ \mu\,dt\ +\ \sigma\,dZ\ +\ k(dN\ -\ \lambda\,dt).$$

No assumptions are made at this stage regarding the values of $(\mu,\ k,\ \lambda^*)$.

It is shown in the appendix that the myopic investor will choose constant portfolio proportions

$$\begin{aligned} x^{myopic} &= \frac{1}{R\sigma^2}\left[\mu\ +\ (\lambda^*\ -\ \lambda)k\right] \\ q^{myopic} &= \left(\frac{\lambda e^{Y}}{\lambda^*}\right)^{\frac{1}{R}}\ -\ (1+w^*k) \end{aligned} \tag{47}$$

where $x\ \equiv\ S_t X/W$ is the portfolio share in equity, and

$q\ \equiv\ Q/W$ is the number of insurance contracts as a fraction of overall wealth.

If investors are homogeneous, the market-clearing conditions $(x^{myopic}, q^{myopic})\ =\ (1,\ 0)$ yield the equilibrium and time-invariant $(\mu,\ \lambda^*)$ given above in equations (12) and (16). The above myopic portfolio weights are also optimal under time-varying $(\mu_t,\ k_t,\ \lambda_t^*)$ when $R = 1$, but not for general

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[13]Campbell and Viceira (1999) and Campbell, Chacko, Rodriguez and Viceira (2004) find substantial hedging against stochastic shifts in expected returns, while Chacko and Viceira (2005) find little hedging against stochastic volatility. The two approaches diverge in the specification and calibration of shifts in the investment opportunity set.

R.

The myopic portfolio allocation equations (47) indicate that equity and crash insurance are complements when jumps are negative ($k < 0$). An increase in the price of crash insurance λ^* lowers the demand for both equity and crash insurance, while an increase in the expected excess return μ on equity raises both. The equations also indicate that myopic crash insurance positions but *not* equity positions are directly affected by the investor's idiosyncratic crash aversion parameter Y. Furthermore, at the *equilibrium* equity premium (33), myopic investors duplicate the optimal investment strategy of holding 100% in equity, and diverge from that optimum only in their holdings of crash insurance.

Table 3 compares the optimal and myopic crash insurance strategies at the equilibrium values for (k_t, λ_t^*) resulting from various initial wealth allocations and risk aversion. The two strategies are broadly similar across different asset market equilibria, and are identical either when risk aversion $R = 1$, or when a preponderance of one type of individual ($W_1(t) / W(t) \approx 0$ or 1) yields a homogeneous-agent equilibrium with a time-invariant investment opportunity set.

The table indicates that a myopic strategy can be a poor approximation to the optimal strategy in other cases. The divergence is most pronounced for the large positions achieved under low levels of risk aversion (R = ½), but is also present for larger R values. For instance, when crash-tolerant and crash-averse investors are equally represented (W_1 / W = ½) and R = 2, a 3% adverse news shock will induce a 17.8% stock market crash (from Table 2). The crash-averse buy crash insurance contracts from the crash-tolerant that pay off 36.5% of current wealth conditional on a crash. The myopic positions $(q_0^{myopic}, q_1^{myopic})$ = (-16.9%, 26.4%) in Table 3 substantially understate the magnitude of those optimal insurance positions.

Table 3. Optimal and myopic crash insurance positions, at equilibrium asset prices determined by idiosyncratic crash aversions $Y = 0, 1$, initial wealth allocation $w_1 = W_1(0) / W(0)$, and common risk aversion R. Equilibrium values for $\ln(1 + k_t)$ and parameter values are in Table 2 above. Entries indicate the payoff of insurance positions conditional on a crash, as a fraction of investor's wealth.

w_1	Crash aversion $Y = 0$; R = 0.5	1	2	4	8	Crash aversion $Y = 1$; R = 0.5	1	2	4	8
Optimal positions q_Y^*										
0.0	.000	.000	.000	.000	.000	6.200	1.667	.630	.276	.129
.0001	.000	.000	.000	.000	.000	1.852	1.667	.652	.276	.129
.001	-.002	-.002	-.001	.000	.000	1.773	1.660	.793	.277	.129
.01	-.017	-.016	-.011	-.003	-.001	1.648	1.598	1.134	.299	.131
.1	-.128	-.128	-.119	-.062	-.017	1.148	1.152	1.069	.557	.152
.2	-.215	-.214	-.205	-.140	-.045	.858	.856	.822	.559	.179
.3	-.285	-.282	-.270	-.204	-.080	.666	.657	.629	.477	.187
.4	-.347	-.339	-.321	-.255	-.115	.520	.509	.482	.382	.173
.5	**-.402**	**-.391**	**-.365**	**-.293**	**-.145**	**.402**	**.391**	**.365**	**.293**	**.145**
.6	-.452	-.440	-.405	-.322	-.166	.301	.293	.270	.214	.111
.7	-.499	-.485	-.444	-.341	-.177	.214	.208	.190	.146	.076
.8	-.542	-.529	-.482	-.352	-.175	.136	.132	.121	.088	.044
.9	-.584	-.572	-.518	-.345	-.153	.065	.064	.058	.038	.017
.99	-.629	-.609	-.497	-.241	-.118	.006	.006	.005	.002	.001
.999	-.651	-.613	-.418	-.217	-.114	.001	.001	.000	.000	.000
1.0	-.839	-.613	-.382	-.215	-.114	.000	.000	.000	.000	.000
Myopic positions q_Y^{myopic}										
0.0	.000	.000	.000	.000	.000	6.200	1.667	.630	.276	.129
.0001	.000	.000	.000	.000	.000	6.195	1.667	.629	.276	.129
.001	-.004	-.002	-.001	.000	.000	6.153	1.660	.628	.275	.129
.01	-.041	-.016	-.007	-.002	-.001	5.764	1.598	.615	.272	.128
.1	-.284	-.128	-.056	-.028	-.010	3.362	1.152	.500	.236	.117
.2	-.429	-.214	-.091	-.051	-.021	2.122	.856	.418	.201	.104
.3	-.522	-.282	-.117	-.066	-.032	1.440	.657	.358	.178	.091
.4	-.592	-.339	-.142	-.073	-.041	1.012	.509	.309	.164	.080
.5	**-.648**	**-.391**	**-.169**	**-.077**	**-.048**	**.717**	**.391**	**.264**	**.154**	**.071**
.6	-.695	-.440	-.199	-.080	-.053	.501	.293	.220	.147	.066
.7	-.737	-.485	-.234	-.087	-.055	.334	.208	.173	.137	.062
.8	-.774	-.529	-.275	-.103	-.058	.201	.132	.122	.117	.058
.9	-.808	-.572	-.324	-.142	-.072	.092	.064	.065	.075	.043
.99	-.836	-.609	-.376	-.209	-.110	.008	.006	.007	.006	.004
.999	-.839	-.613	-.381	-.214	-.114	.001	.001	.001	.001	.000
1.0	-.839	-.613	-.382	-.215	-.114	.000	.000	.000	.000	.000

3.4 Option markets

3.4.1 Option prices

At time 0, European call options of maturity t are priced at expected terminal value weighted by the pricing kernel:

$$\begin{aligned} c(S_0, t; X) &= E_0\left[\frac{\eta_t}{\eta_0}\max(S_t - X,\ 0)\right] \\ &\equiv E_0^*[\max(S_t - X,\ 0)]\,. \end{aligned} \tag{48}$$

Conditional upon N_t jumps over $(0, t]$, η_t and S_t have a joint lognormal distribution that reflects their common dependency on D_t given above in equations (29) and (30). Consequently, it is shown in the appendix that the risk-neutral distribution for S_t is a weighted mixture of lognormals, implying European call option prices are a weighted average of Black-Scholes-Merton prices:

$$\begin{aligned} c(S_0, t; X) &= \sum_N w_N^*\, c^{BS}(S_0, t; X,\ b_N, r=0) \\ &= \sum_N w_N^*\left[S_0 e^{b_N t} N(d_{1N}) - XN(d_{2N})\right] \end{aligned} \tag{49}$$

where $\lambda' \equiv \lambda e^{-R\gamma_d}$,

$$w_N^* \equiv \frac{e^{-\lambda' t}(\lambda' t)^N}{N!}\,\frac{g(N, t; \lambda')}{g(0, 0; \lambda')},$$

$$b_N = -\lambda'(e^{\gamma_d} - 1) + \left\{n\gamma_d + \ln[m(N,t)/m(0,0)]\right\}/t,$$

$$d_{1N} = [\ln(S_0/X) + b_N t + ½\sigma_d^2 t]\,/\,\sigma_d\sqrt{t} \text{ and}$$

$$d_{2N} = d_{1N} - \sigma_d\sqrt{t}.$$

Put prices can be computed from call prices using put-call parity:

$$p(S_0,\ t,\ X) = c(S_0,\ t,\ X) + X - S_0. \tag{50}$$

Since jumps are always negative, the distribution of log-differenced equity prices implicit in option prices is always negatively skewed. The maturity profile of implicit skewness is quite sensitive to the initial distribution of wealth, given the nonmonotonic dependency of $\ln(1 + k_t)$ on wealth distribution shown above in Table 2 and Figure 4. For small values of w_1, a second jump will be larger than the first. The increasing probability of multiple jumps at longer maturities causes implicit skewness to fall slower than the $1/\sqrt{t}$ rate of i.i.d. returns, implying slower flattening out of the implicit volatility smirk. For larger values of w_1 the size of sequential jump sizes is reversed,

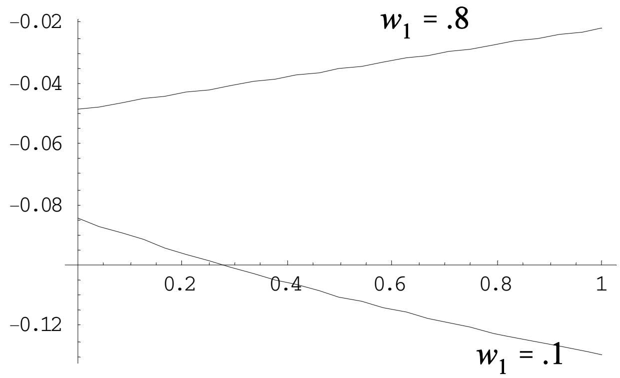

Figure 6. Annualized risk-neutral skewness $Skew[t] \times \sqrt{t}$, as a function of *t*.

and implicit skewness can fall *faster* than the $1/\sqrt{t}$ rate of i.i.d. returns; see Figure 6.

However, model-specific estimates from option prices such as in Table 1 above indicate that implicit $Skew[t]$ (rather than $Skew[t] \times \sqrt{t}$) is roughly flat across option maturities t. This stylized fact appears common to a broad array of futures options, as indicated in the Tompkins (2000) survey of volatility smiles and smirks.[14] Thus, although Bates (2000) argues that stochastic implicit jump intensities λ_t^* are needed to match the volatility smirk at longer maturities, it does not appear that the stochastic variation of (λ_t^*, k_t) in this model generates the correct maturity profile of implicit skewness.

3.4.2 Option replication and dynamic completion of the markets

Options can be dynamically replicated using positions in equity and crash insurance. Instantaneously, each call option has a price $c(S_t, N_t, t)$, and can be viewed as an instantaneous bundle of c_S units of equity risk, and $[\Delta c - c_S \Delta S]_{dN=1} > 0$ units of crash insurance.

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[14]Tompkins examines implicit volatility patterns from various countries' futures options on currency, stock index, bonds and interest rates, with the moneyness dimension appropriately scaled by maturity-specific volatility estimates from at-the-money options. He finds some maturity variation in implicit volatility patterns, but not much by comparison with the strong inverse pattern predicted by i.i.d. returns.

This equivalence between options and crash insurance indicates how investors replicate the optimal positions of section 3.3 dynamically using the call and/or put options actually available. Crash-averse investors choose an equity/options bundle with unitary delta overall and positive gamma (e.g., hold 1½ stocks and buy one at-the-money put option with a delta of -½), while crash-tolerant investors take offsetting positions that also possess unitary delta (e.g., hold ½ stock, and write 1 put option). Equity and option positions are adjusted in a mutually acceptable and offsetting fashion over time, conditional upon the arrival of news. As options expire, new options become available and investors are always able to maintain their desired levels of crash insurance. All investors recognize that the price of crash insurance implicit in option prices will evolve over time, conditional on whether crashes do or do not occur, and take that into account when establishing their positions.

A further implication is that the crash-tolerant investors who write options actively delta-hedge their exposure, which is consistent with the observed practice of option market makers. As λ_t^* / λ increases (e.g., because of wealth transfers to the crash-averse from crashes) , the market makers respond to the more favorable prices by writing more options as a proportion of their wealth.[15] They simultaneously adjust their equity positions to maintain their overall target delta of 1. This strategy is equivalent to market makers putting their personal wealth in an index fund, and fully delta-hedging every index option they write.

3.5 Consistency with empirical option pricing anomalies

The heterogeneous-agent model explains unconditional deviations between risk-neutral and objective distributions analogously to the homogeneous-agent model. The divergence in the jump intensity λ_t^* implicit in options and the true jump frequency λ can reconcile the average divergence between risk-neutral and objective variance, and between the predicted and observed frequency of jumps over 1988-98. The heterogeneous-agent model can also be somewhat more consistent with the maturity profile of implicit skewness than the homogeneous-agent model, although still appears inadequate relative to observed patterns.

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[15]As indicated above in Figure 4, the total volume (open interest) in crash insurance and therefore in options can either rise or fall as the wealth distribution varies.

The advantage of the heterogeneous-agent model is that it can explain some of the conditional divergences as well. First, the stochastic evolution of λ_t^* is qualitatively consistent with the evolution of jump intensity proxy V1 shown above in Figure 2. λ_t^* depends directly upon the relative wealth distribution, which in turn follows a pure jump process given above in (41) for the $R = 1$ case. Consequently, market jumps cause sharp increases in λ_t^*, while an absence of jumps generates geometric decay in λ_t^* towards the lower level of crash-tolerant investors.

Figure 7 below illustrates the resulting evolution of instantaneous risk-neutral variance ($R\sigma^2 + \lambda_t^* \gamma_t^2$) *conditional* on the five major shocks over 1988-98, and conditional on starting with $w_1 = .1$ at end-1987. This behavior is qualitatively similar to the actual impact of jumps on overall variance and on jump risk shown above in Figure 2. However, the absence of major shocks over 1992-96 and the resulting wealth accumulation by crash-tolerant investors/option market makers implies that the shocks of 1997 and 1998 should not have had the major impact that was in fact observed.

It is possible the heterogeneous model can explain the results from ISD regressions as well. The analysis is complicated by the fact that instantaneous objective and risk-neutral variance are

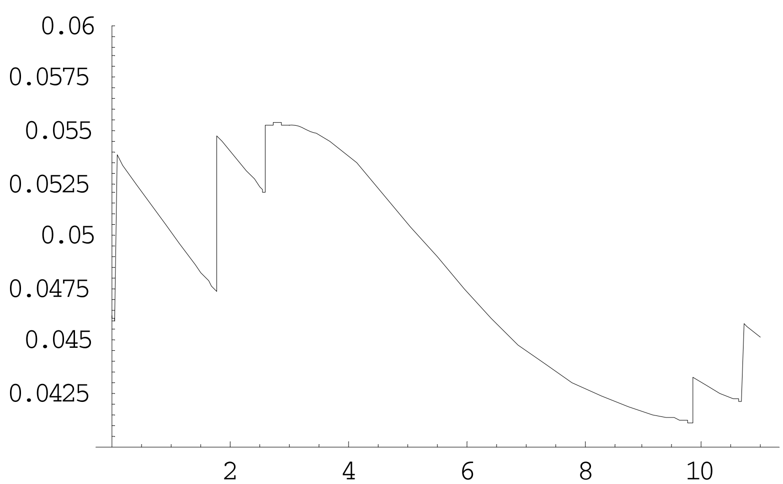

Figure 7. Simulated instantaneous risk-neutral variance $R\sigma^2 + \lambda_t^* \gamma_t^2$ *conditional* upon jump timing matching that observed over 1988-98. Calibration: $w_1(0) = 10\%$; i.e., crash-averse investors own 10% of total wealth at end-1987.

nonstationary, with a nonlinear cointegrating relationship from their common dependency on the nonstationary variable N_t:

$$\begin{aligned} Var_t[d\ln S] &= [\sigma^2 + \lambda\gamma_t^2]\,dt \\ Var_t^*[d\ln S] &= [\sigma^2 + \lambda^*(N_t,t)\,\gamma_t^2]\,dt \end{aligned} \qquad \textbf{(51)}$$

for $\gamma_t \equiv \ln[1 + k(N_t, t)]$ and $\lambda_t^* > \lambda$. It is not immediately clear whether regressing realized on implied volatility is meaningful under nonlinear cointegration. However, the fact that implicit variance does contain information for objective variance but is biased upwards suggests that running this sort of regression on post-'87 data would yield the usual informative-but-biased results reported above in equation (2), with estimated slope coefficients less than 1 in sample.

It does not appear that the heterogeneous-agent model can explain the implicit pricing kernel puzzle. Using the same projection as in (20) above , the projected pricing kernel is

$$\begin{aligned} M(S_t) &\equiv \frac{E_0[\eta_t \mid S_t]}{\eta_0} = \kappa S_t^{-R}\frac{\sum_{N=0}^{\infty} w_N^{**}\,p(S_t \mid N)}{p(S_t)} \quad \textit{where} \\ w_N &= \frac{e^{-\lambda t}(\lambda t)^N}{N!},\; w_N^{**} = \frac{w_N\; m(N,t)^R\, g\!\left(N, t, \lambda e^{(1-R)\gamma_d}\right)}{\sum_{N=0}^{\infty} w_N\; m(N,t)^R\, g\!\left(N, t, \lambda e^{(1-R)\gamma_d}\right)}. \end{aligned} \qquad \textbf{(52)}$$

As illustrated in Figure 8, this implicit pricing kernel appears to be a strictly decreasing

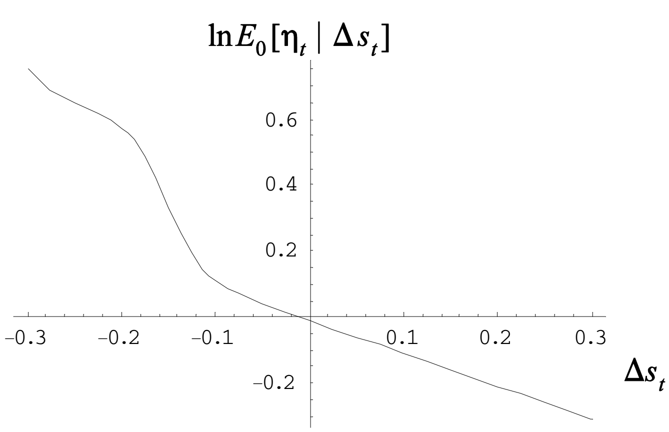

Figure 8. Log of the implicit pricing kernel conditional upon realized asset returns.
Calibration: $w_1 = .3$, $t = 1/12$.

function of S_t – in contrast to the locally positive sections estimated in Jackwerth (2000) and Rosenberg and Engle (2002). However, the above implicit kernel can replicate those studies' high implicit risk aversion for large negative returns, as indicated by the slope of the line in Figure 8 for Δs in the -10% to -20% ranges.

4. Summary and conclusions

This paper has proposed a modified utility specification, labeled "crash aversion," to explain the observed tendency of post-'87 stock index options to overpredict realized volatility and jump risk. Furthermore, the paper has developed a complete-markets methodology that permits identification of asset market equilibria and associated investment strategies in the presence of jumps and investor heterogeneity. The assumption of heterogeneity appears to have stronger consequences than observed with diffusion models. Jumps can cause substantial reallocation of wealth, and the resulting shifts in the investment opportunity set can be substantial. Small announcement effects regarding the terminal value of the market can have substantially magnified instantaneous price impacts when investors are heterogeneous.

The model has been successful in explaining some of the stylized facts from stock index options markets. The specification of crash aversion is compatible with the tendency of option prices to overpredict volatility and jump risk, while heterogeneity of agents offers an explanation of the stochastic evolution of implicit jump risk and implicit volatilities. In this model, the two are higher immediately after market drops not because of higher objective risk of *future* jumps (as predicted by affine models), but because crash-related wealth redistribution has increased average crash aversion. Crash aversion is also consistent with the implicit pricing kernel approach's assessment of high implicit risk aversion at low wealth levels, although the approach cannot replicate the locally risk-loving behavior reported in Jackwerth (2000) and Rosenberg and Engle (2002).

While motivated by empirical option price regularities, the model in the paper is not suitable for direct estimation. First, jump risk is not the only risk spanned in the options markets. Stochastic variations in conditional volatility occur more frequently, and are also important to option market makers. Second, the nonstationary equilibrium derived here and characteristic of most heterogeneous-agent models hinders estimation. The purpose of the paper is to provide a theoretical framework for exploring the trading of jump risk through the options markets, as an initial model of the option market making process. A more plausible model that might also resolve the nonstationarity issue would be to include profit-taking by market makers, to limit their size in the market.

The framework in this paper can be expanded in various ways. For simplicity, this paper has focused on deterministic jumps and an "external" crash aversion specification insensitive to the impact of crashes upon individual wealth. Extending the model to random jumps and/or "internal" crash aversion should be relatively straightforward, although feedback effects in the latter case could require additional restrictions to achieve an equilibrium. A particularly interesting extension could be to explore the implications of portfolio constraints on positions in options and/or jump insurance. Selling crash insurance requires writing calls or puts – a strategy that individual investors cannot easily pursue. Further research will examine the impact of such constraints upon equilibria in equity and options markets.

Appendix

Section A.1 of the appendix prices assets when agents are heterogeneous. Section A.2 derives the myopic investment strategies. Section A.3 derives the objective and risk-neutral probability density functions under heterogeneity. Section A.4 derives properties of the implicit pricing kernel under homogeneous and heterogeneous agents.

A.1 Asset market equilibrium in a heterogeneous-agent economy (Section 3.2)

Lemma: If the log-dividend $d_t \equiv \ln D_t$ follows the jump-diffusion given above in equation (3) and $h(N_T)$ is an arbitrary function, then

$$E_t\left[D_T^m\, h(N_T) \mid \lambda\right] = D_t^m e^{(T-t)[m\mu_d + \frac{1}{2}m^2\sigma_d^2 + \lambda(e^{m\gamma_d}-1)]}\, E_t\left[h(N_t+n) \mid \lambda e^{m\gamma_d}\right] \qquad \textbf{(A.1)}$$

where $n \equiv N_T - N_t$ and $E_t[\bullet \mid \lambda]$ denotes expectations conditional upon a jump intensity λ over $(t, T]$.

Proof: Define $\Delta d \equiv \ln(D_T/D_t)$ and $\tau \equiv T - t$. Then

$$\begin{aligned}
E_t\left[D_T^m\, h(N_T)\right] &= D_t^m\, E_t\left[e^{m\Delta d}\, h(N_T)\right] \\
&= D_t^m\, E_t\left[e^{m(\Delta d|_{n=0} + n\gamma_d)}\, h(N_t+n)\right] \\
&= D_t^m\, e^{\tau[m\mu_d + \frac{1}{2}m^2\sigma_d^2]} E_t\left[e^{mn\gamma_d}\, h(N_t+n)\right] \\
&= D_t^m\, e^{\tau[m\mu_d + \frac{1}{2}m^2\sigma_d^2]} \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}\left(\lambda e^{m\gamma_d}\right)^n}{n!}\, h(N_t+n) \\
&= D_t^m e^{(T-t)[m\mu_d + \frac{1}{2}m^2\sigma_d^2 + \lambda(e^{m\gamma_d}-1)]}\, E_t\left[h(N_t+n) \mid \lambda e^{m\gamma_d}\right].
\end{aligned} \qquad \textbf{(A.2)}$$

The asset pricing equations (29)-(31) follow directly from the lemma:

$$\begin{aligned}
\eta_t &= E_t \eta_T \\
&= E_t\left[D_T^{-R}\, \bar{f}(N_T)\right] \\
&= D_t^{-R}\, e^{\kappa_\eta (T-t)}\, E_t\left[\bar{f}(N_T) \mid \lambda e^{-R\gamma_d}\right] \\
&\equiv D_t^{-R}\, e^{\kappa_\eta (T-t)}\, g\left(N_t, t; \lambda e^{-R\gamma_d}\right)
\end{aligned} \qquad \textbf{(A.3)}$$

$$\begin{aligned} S_t &= \frac{E_t\left[D_T\,\eta_T\right]}{\eta_t} \\ &= \frac{E_t\left[D_T^{1-R}\,\bar{f}(N_T)\right]}{E_t\left[D_T^{-R}\,\bar{f}(N_T)\right]} \\ &= D_t\,e^{\kappa_S(T-t)}\,\frac{E_t\left[\bar{f}(N_T)\mid \lambda e^{(1-R)\gamma_d}\right]}{E_t\left[\bar{f}(N_T)\mid \lambda e^{-R\gamma_d}\right]} \\ &\equiv D_t\,e^{\kappa_S(T-t)}\,\frac{g\left(N_t, t; \lambda e^{(1-R)\gamma_d}\right)}{g\left(N_t, t; \lambda e^{-R\gamma_d}\right)} \end{aligned} \qquad \textbf{(A.4)}$$

$$\begin{aligned} \lambda_t^* &= \lambda\,\frac{\eta_t\,|_{jump}}{\eta_t} \\ &= \frac{(D_t\,e^{\gamma_d})^{-R}\,E_t\left[\bar{f}(N_T)\mid \lambda e^{-R\gamma_d},\ N_t+1\right]}{D_t^{-R}\,E_t\left[\bar{f}(N_T)\mid \lambda e^{-R\gamma_d},\ N_t\right]} \\ &= e^{-R\gamma_d}\,\frac{g\left(N_t+1, t; \lambda e^{-R\gamma_d}\right)}{g\left(N_t, t; \lambda e^{-R\gamma_d}\right)} \end{aligned} \qquad \textbf{(A.5)}$$

for $\kappa_\eta = -R\mu_d + \tfrac{1}{2}R^2\sigma^2 + \lambda(e^{-R\gamma} - 1)$ and $\kappa_S = (\mu_d + \tfrac{1}{2}\sigma_d^2) - R\sigma^2 + \lambda e^{-R\gamma_d}(e^{\gamma_d} - 1)$. Given $\mu_d = -\tfrac{1}{2}\sigma_d^2 - \lambda(e^{\gamma_d} - 1)$, κ_S can be written as $\kappa_S = -R\sigma^2 + \lambda(e^{-R\gamma_d} - 1)(e^{\gamma_d} - 1)$.

In the special case $R = 1$ and for arbitrary λ',

$$\begin{aligned} g(N_t, t, \lambda') &\equiv E_t\left[\bar{f}(N_T)\mid \lambda'\right] \\ &= E_t\left[\sum_Y \omega_Y\,e^{YN_T}\mid \lambda'\right] \\ &= \sum_Y \omega_Y \exp\left[YN_t + \lambda'(T-t)\left(e^Y - 1\right)\right]. \end{aligned} \qquad \textbf{(A.6)}$$

Define $\lambda' \equiv \lambda e^{-R\gamma_d}$ and $\lambda'' \equiv \lambda e^{(1-R)\gamma_d}$, and define pseudo-probabilities

$$\pi_{Yt} \equiv \frac{\omega_Y \exp[YN_t + \lambda'(T-t)(e^Y - 1)]}{\sum_Y \omega_Y \exp[YN_t + \lambda'(T-t)(e^Y - 1)]} \qquad \textbf{(A.7)}$$

Using (A.6) for g, the equity pricing equation (A.4) becomes

$$\frac{S_t}{D_t} = e^{\kappa_S(T-t)} \frac{\sum_Y \omega_Y \exp\left[YN_t + \lambda''(T-t)\left(e^{Y}-1\right)\right]}{\sum_Y \omega_Y \exp\left[YN_t + \lambda'(T-t)\left(e^{Y}-1\right)\right]}$$

$$= e^{\kappa_S(T-t)} \sum_Y \pi_{Yt} \exp\left[(\lambda'' - \lambda')(T-t)\left(e^{Y}-1\right)\right] \qquad \textbf{(A.8)}$$

$$= e^{\kappa_S(T-t)} E_{CS}\left[e^{\Phi(e^{Y}-1)}\right]$$

for the cross-sectional expectation $E_{CS}(\bullet)$ defined with regard to probabilities (A.7), and for $\Phi \equiv (\lambda'' - \lambda')(T-t) = \lambda e^{\gamma_d}(e^{-R\gamma_d} - 1)(T-t)$. From (A.5), the jump risk premium has a similar representation:

$$\frac{\lambda_t^*}{\lambda} = e^{-R\gamma_d} \frac{\sum_Y \omega_Y \exp\left[Y(N_t+1) + \lambda'(T-t)\left(e^{Y}-1\right)\right]}{\sum_Y \omega_Y \exp\left[YN_t + \lambda'(T-t)\left(e^{Y}-1\right)\right]}$$

$$= e^{-R\gamma_d} \sum_Y \pi_{Yt} e^{Y} \qquad \textbf{(A.9)}$$

$$= e^{-R\gamma_d} E_{CS}\left(e^{Y}\right).$$

The approximation for the log jump size follows from the following approximations:

$$\ln m(N_t, t) \equiv \ln\left[\frac{g(N_t, t; \lambda'')}{g(N_t, t; \lambda')}\right]$$

$$\approx \frac{\partial \ln g(N_t, t; \lambda')}{\partial \lambda'} (\lambda'' - \lambda') \qquad \textbf{(A.10)}$$

$$\ln(1 + k_t) = R\gamma_d + \ln\left[\frac{m(N_t+1, t)}{m(N_t, t)}\right]$$

$$\approx R\gamma_d + \frac{\partial \ln m(N_t; t)}{\partial N_t} \qquad \textbf{(A.11)}$$

$$\approx R\gamma_d + \frac{\partial^2 \ln g(N_t; t; \lambda')}{\partial N_t \partial \lambda'} (\lambda'' - \lambda').$$

For $R = 1$, the partial derivatives of $\ln g$ are

$$\frac{\partial \ln g(N_t, t; \lambda')}{\partial N_t} = \frac{\sum_Y \omega_Y Y \exp\left[YN_t + \lambda'(T-t)\left(e^Y - 1\right)\right]}{\sum_Y \omega_Y \exp\left[YN_t + \lambda'(T-t)\left(e^Y - 1\right)\right]} \qquad \textbf{(A.12)}$$

$$= E_{CS}(Y)$$

$$\frac{\partial \ln g(N_t, t; \lambda')}{\partial \lambda'} = \frac{\sum_Y \omega_Y (T-t)\left(e^Y - 1\right) \exp\left[YN_t + \lambda'(T-t)\left(e^Y - 1\right)\right]}{\sum_Y \omega_Y \exp\left[YN_t + \lambda'(T-t)\left(e^Y - 1\right)\right]} \qquad \textbf{(A.13)}$$

$$= (T-t)\, E_{CS}\left(e^Y - 1\right)$$

while the cross-derivative is

$$\frac{\partial^2 \ln g(N_t, t; \lambda')}{\partial N_t \partial \lambda'} = (T-t)\left\{\sum_Y \pi_{Yt} Y\left(e^Y - 1\right) - \sum_Y \pi_{Yt} Y \sum_Y \pi_{Yt}\left(e^Y - 1\right)\right\} \qquad \textbf{(A.14)}$$

$$= (T-t)\, Cov_{CS}\left(Y, e^Y\right).$$

Consequently (from (A.11)),

$$\ln(1 + k_t) \approx R\gamma_d + (\lambda'' - \lambda')(T-t)\, Cov_{CS}\left(Y, e^Y\right) \qquad \textbf{(A.15)}$$

$$\approx R\gamma_d + \lambda e^{\gamma_d}(e^{-R\gamma_d} - 1)(T-t)\, Cov_{CS}\left(Y, e^Y\right).$$

Section 3.3, equation (40)

$$V_t = E_t\left[\frac{\tilde{\eta}_T}{\eta_t} \tilde{D}_T \left(\frac{e^{YN_T}}{\bar{f}(N_T)}\right)^{\frac{1}{R}}\right]$$

$$= \frac{E_t\left[D_T^{1-R} e^{YN_T/R} \bar{f}(N_T)^{1-\frac{1}{R}}\right]}{E_t\left[D_T^{-R} \bar{f}(N_T)\right]} \qquad \textbf{(A.16)}$$

$$= \frac{D_t^{1-R}}{D_t^{-R}} e^{\kappa_S(T-t)} \frac{E_t\left[e^{YN_T/R} \bar{f}(N_T)^{1-\frac{1}{R}} \mid \lambda e^{(1-R)\gamma_d}\right]}{E_t\left[\bar{f}(N_T) \mid \lambda e^{-R\gamma_d}\right]}.$$

Substituting in $S_t = D_t e^{\kappa_S(T-t)} \dfrac{E_t\left[\bar{f}(N_T) \mid \lambda e^{(1-R)\gamma_d}\right]}{E_t\left[\bar{f}(N_T) \mid \lambda e^{R\gamma_d}\right]}$ from (A.4) yields (40).

A.2 Myopic portfolio choice (Section 3.3.2)

The myopic portfolio allocation strategy (x, q) in equity and crash insurance maximizes the Hamilton-Jacobi-Bellman equation

$$\begin{aligned} 0 &= \max_{\{x,q\}} E_t\, dJ(W_t, N_t, t) \\ &= \max_{\{x,q\}} E_t\, J_t + W J_W[x(\mu - \lambda k) - \lambda^* q] + W^2 J_{WW}\, x^2 \sigma^2 \\ &\qquad + \lambda\left[J\big(W(1 + xk + q), N_t + 1, t\big) - J\right] \end{aligned} \tag{A.17}$$

under the assumption of constant $(\mu, \sigma, \lambda, \lambda^*, k)$, and subject to the terminal boundary condition

$$J(W_T,\ N_T,\ T) = e^{YN_T}\, \frac{W_T^{1-R} - 1}{1 - R}\,. \tag{A.18}$$

The first-order conditions to (A.17) with respect to q and x are

$$\begin{aligned} \lambda^* &= \lambda\, \frac{J_W[W(1 + xk + q), N + 1, t]}{J_W(W, N, t)} \equiv \lambda\, \frac{J_W^*}{J_W} \\ x &= \left(\frac{-W J_{WW}}{J_W}\right)^{-1} \frac{\mu + (\lambda^* - \lambda)k}{\sigma^2} \end{aligned} \tag{A.19}$$

Given the terminal utility specification, it is straightforward to show that the value function J is of the form

$$J(W_t, N_t, t) = g_1(T - t)\, \frac{W_t^{1-R}}{1 - R}\, e^{YN_t} - \frac{e^{YN_t} g_2(T - t)}{1 - R} \tag{A.20}$$

with an associated marginal utility function

$$J_W(W_t, N_t, t) = g_1(T - t)\, W_t^{-R}\, e^{YN_t}. \tag{A.21}$$

Since $(-W J_{WW}/J_W) = R$ and $J_W^*/J_W = e^{Y}(1 + xk + q)^{-R}$, this marginal utility function yields constant portfolio proportions that satisfy

$$\begin{aligned} x^{myopic}k + q^{myopic} &= \left(\frac{\lambda e^{Y}}{\lambda^*}\right)^{\frac{1}{R}} - 1 \\ x^{myopic} &= \frac{1}{R\sigma^2}\left[\mu + (\lambda^* - \lambda)k\right] \end{aligned} \tag{A.22}$$

under a constant investment opportunity set. Furthermore, the value function and these portfolio proportions satisfies the Hamilton-Jacobi-Bellman equation for some functions g_1 and g_2 that appropriately converge to 1 as $t \to T$.

If $R = 1$, myopic investment strategies are optimal even if investment opportunities $(\mu_t, \sigma_t, \lambda_t^*, k_t)$ are stochastic. Defining $\tau \equiv T - t$, the objective function becomes

$$\begin{aligned} J(W_t,\ N_t,\ t) &\equiv \max\ E_t\ e^{YN_T} \ln W_T \\ &= \max \sum \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} e^{Y(N_t + n)} E_t[\ln W_T \mid N_t + n\ jumps] \\ &= e^{YN_t} e^{\lambda\tau(e^{Y}-1)} \max\ E_t^*\left[\ln W_T \mid \lambda e^{Y}\right] \\ &= e^{YN_t} e^{\lambda\tau(e^{Y}-1)} \left\{\ln W_t + \int_{s=t}^{T} \max\ E_t^*\left[(d\ln W_s) \mid \lambda e^{Y}\right]\right\} \end{aligned} \tag{A.23}$$

where E_t^* is a modified expectation conditional upon a jump intensity λe^{Y} over $(t, T]$. Consequently, the marginal utility

$$J_W(W_t, N_t, t) = \frac{e^{YN_t} e^{\lambda(T-t)(e^{Y}-1)}}{W_t} \tag{A.24}$$

is again of the form (A.21) above, and optimal portfolio proportions are given by (A.22) with $R = 1$.

A.3 Objective and risk-neutral distributions

Stock prices and pricing kernels are jump-dependent multiples of the dividend signal, which is in turn a draw from jump-dependent mixture of lognormals. From (30), gross stock returns are

$$\frac{S_t}{S_0} = e^{-\kappa_S t} \frac{D_t}{D_0} \frac{m[N_t, t]}{m[0, 0]} \tag{A.25}$$

for $\kappa_S = (\mu_d + ½\sigma_d^2) - R\sigma^2 + \lambda e^{-R\gamma}(e^{\gamma} - 1)$. The density function for $\Delta d \equiv \ln[D_t/D_0]$ is

$$p_d(\Delta d) = \sum_{N=0}^{\infty} w_N\, n(\Delta d \mid \mu_d t + N\gamma_d,\ \sigma_d^2 t) \ \textit{for}\ w_N \equiv \frac{e^{-\lambda t}(\lambda t)^N}{N!} \tag{A.26}$$

for $n(z \mid m, \sigma^2)$ equal to the normal density function with mean m and variance σ^2. Consequently, log-differenced stock prices $\Delta s \equiv \ln[S_t/S_0]$ are also drawn from a mixture of normals:

$$\begin{aligned} p(\Delta s) &= \sum_{N=0}^{\infty} w_N\ n\Big(\Delta s \mid (R - \tfrac{1}{2}\sigma_d^2)t + \lambda e^{-R\gamma_d}(e^{\gamma_d} - 1)t + N\gamma_d + \ln[m(N,t)/m(0,0)],\ \sigma_d^2 t\Big) \\ &\equiv \sum_{N=0}^{\infty} w_N\ n\Big(\Delta s \mid \mu_N, \sigma_d^2 t\Big). \end{aligned} \tag{A.27}$$

Define $1(\Delta s = z)$ as the delta function that takes on infinite value when $\Delta s = z$, zero value elsewhere, and integrates to 1. The objective density function $p(z) = E_0[1(\Delta\tilde{s} = z)]$, while the risk-neutral density function is

$$\begin{aligned} p^*(z) &= E_0^*[1(\Delta\tilde{s} = z)] \\ &= E_0\left[\frac{\eta_t}{\eta_0} 1(\Delta\tilde{s} = z)\right] \\ &= \sum_{n=0}^{\infty} \frac{w_N\, E_0\big[\eta_t 1(\Delta\tilde{s} = z) \mid N\, jumps\big]}{\eta_0}. \end{aligned} \tag{A.28}$$

For any two normally distributed variables $\tilde{x}$ and $\tilde{y}$ and any arbitrary function $h(y)$,

$$E\big[e^x h(y)\big] = E\big[e^x\big]E\big[h(y^*)\big] \tag{A.29}$$

where y^* is also normally distributed with mean $E(y) + Cov(x, y)$ and variance $Var(y)$. Conditional upon n jumps, $\ln \eta_t$ and Δs are both normally distributed with covariance $-R\sigma_d^2$. Consequently, (A.28) can be re-written as

$$\begin{aligned} p^*(z) &= \sum_{N=0}^{\infty} \frac{w_N\, E_0(\eta_t \mid N\ jumps)\ E_0\big[1(\Delta\tilde{s}^* = z) \mid N\, jumps\big]}{\eta_0} \\ &= \sum_{N=0}^{\infty} \frac{w_N\, E_0(\eta_t \mid N\ jumps)\ n(\Delta s \mid \mu_N - R\sigma_d^2, \sigma_d^2)}{\eta_0} \\ &\equiv \sum_{N=0}^{\infty} w_N^*\ n(\Delta s \mid \mu_N - R\sigma_d^2, \sigma_d^2) \end{aligned} \tag{A.30}$$

Since $\eta_0 = E_0\,\eta_t = \sum_N w_N E_0[\eta_t \mid N\ jumps]$, the weights w_n^* sum to 1. Furthermore, since

$$\eta_t = e^{-\kappa_\eta (T-t)} D_0^{-R} \exp[-R(\Delta \tilde{d}|_{N_t=0} + N_t\,\gamma_d)]\, g\left(N_t, t, \lambda e^{-R\gamma_d}\right), \tag{A.31}$$

it is straightforward to show that

$$\begin{aligned} w_N^* &= \frac{w_N e^{-R\gamma_d N} g\left(N, t; \lambda e^{-R\gamma_d}\right)}{\sum_{N=0}^{\infty} w_N e^{-R\gamma_d N} g\left(N, t; \lambda e^{-R\gamma_d}\right)} \\ &= \frac{e^{-\lambda' t}(\lambda' t)^N}{N!}\, \frac{g(N, t; \lambda')}{g(0, 0; \lambda')} \end{aligned} \tag{A.32}$$

for $\lambda' \equiv \lambda e^{-R\gamma_d}$.

A.4 Implicit pricing kernels (equations (21) and (52))

Using equations (13) and (14), the projection of the pricing kernel upon the asset price in the homogeneous-agent case is

$$\begin{aligned} M(S_t) &\equiv \frac{E_0[\eta_t \mid S_t]}{\eta_0} \\ &= E_0\left[D_t^{-R} e^{YN_t} \kappa_0(t) \mid S_t\right] \\ &= S_t^{-R} E_0\left[\left(\frac{S_t}{D_t}\right)^R e^{YN_t} \kappa_0(t) \mid S_t\right] \\ &= \kappa_1(t)\, S_t^{-R} E_0\left[e^{YN_t} \mid S_t\right] \end{aligned} \tag{A.33}$$

where $\kappa_0(t)$ and $\kappa_1(t)$ capture time-dependent terms irrelevant to implicit risk aversion. The distribution of $s_t \equiv \ln S_t$ is an N_t-dependent mixture of normals:

$$p(s_t \mid N_t) \equiv p_{N_t}(s_t) \sim N(\mu_0 + N_t \gamma_d^2,\ \sigma^2 t)\ \textit{with probability}\ w_{N_t} = \frac{e^{-\lambda t}(\lambda t)^{N_t}}{N_t!}. \tag{A.34}$$

Consequently, the conditional expectation in (A.33) can be evaluated using Bayes' rule to evaluate the conditional probabilities

$$Prob[N_t = n \mid S_t] = \frac{w_n p(s_t \mid n)}{\sum_{n=0}^{\infty} w_n p(s_t \mid n)}, \tag{A.35}$$

yielding an implicit pricing kernel

$$M(S_t) = \kappa_1(t)\, S_t^{-R} \frac{\sum_{n=0}^{\infty} w_n p(s_t \mid n)\, e^{Yn}}{\sum_{n=0}^{\infty} w_n p(s_t \mid n)} \tag{A.36}$$

$$= \kappa(t)\, S_t^{-R} \frac{p(s_t \mid \lambda e^{Y})}{p(s_t \mid \lambda)}$$

where $p(s_t \mid \lambda)$ denotes the unconditional density of s_t given a jump intensity of λ over $(0, t]$. Taking partials with respect to s_t and using the fact that $p_s(s_t \mid n) = -p(s_t \mid n) \frac{s_t - (\mu_0 + n\gamma_d)}{\sigma_d^2}$ yields (after some tedious calculations) an implicit risk aversion value

$$-\frac{\partial \ln M(S_t)}{\partial s_t} = R + \frac{-\gamma_d}{\sigma_d^2 t} \frac{Cov_0^{**}(e^{Y\tilde{n}}, \tilde{n})}{E_0^{**}[Y\tilde{n}]} \tag{A.37}$$

where E_0^{**} and Cov_0^{**} are defined with regard to the probabilities in (A.35). Since e^{Yn} and n are both increasing functions of n, the covariance term is positive. Consequently, the implicit risk aversion is everywhere positive given $\gamma_d < 0$.

The heterogeneous-agent case is similar. From (29) and (30), the Lagrange multiplier is

$$\eta_t = e^{\kappa_\eta (T-t)} S_t^{-R} \left(\frac{S_t}{D_t} \right)^R g\left(N_t, t; \lambda e^{-R\gamma_d}\right)$$

$$= e^{(\kappa_\eta + R\kappa_s)(T-t)} S_t^{-R} m(N_t, t)^R g\left(N_t, t; \lambda e^{-R\gamma_d}\right). \tag{A.38}$$

This is of the same form as (A.33), with $m(N_t, t)^R g(N_t, \bullet)$ replacing e^{YN_t}. Consequently, the implicit pricing kernel becomes

$$M(S_t) \;=\; \kappa_1(t)\, S_t^{-R}\; \frac{\sum_{n=0}^{\infty} w_n\, p(s_t \mid n)\, m(n,t)^R\, g\big(n, t, \lambda\, e^{-R\gamma_d}\big)}{\sum_{n=0}^{\infty} w_n\, p(s_t \mid n)}$$

$$= \kappa(t)\, S_t^{-R}\; \frac{\sum_{n=0}^{\infty} w_n^{**}\, p(s_t \mid n)}{\sum_{n=0}^{\infty} w_n\, p(s_t \mid n)} \tag{A.39}$$

for $w_n^{**} \equiv \dfrac{w_n\, m(n,t)^R\, g(n, t; \lambda\, e^{-R\gamma_d})}{\sum_{n=0}^{\infty} w_n\, m(n,t)^R\, g(n, t; \lambda\, e^{-R\gamma_d})}$.

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