## Portfolio Performance and Strategic Asset Allocation Across Different Economic Conditions<sup>1</sup>

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#### **ABSTRACT**

Motivated by the theoretical results on strategic asset allocation, we examine the gains in portfolio performance when investors diversify into different asset classes, with particular focus on the *timeliness* of such gains. Although the various asset classes we analyze yield significant gains in portfolio performance, even in the presence of short sales constraints, the timeliness of the gains differs considerably across the asset classes. Our key result is that commodities and precious metals, and equity REITs are the two asset classes that deliver portfolio gains when consumption growth is low and/or volatile, i.e., when investors really care for such benefits. Consistent with these results, our examination of investor portfolio allocations using a regime switching framework reveals that during the 'bad' economic state, the mean-variance optimal risky portfolio is tilted towards equity REITs, precious metals, and Treasury bonds. Our analysis highlights an important metric by which to judge the attractiveness of an asset class in a portfolio context, namely the timeliness of the gains in portfolio performance.

Key words: Portfolio Diversification, volatility bounds, mean-variance spanning, regime switching

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# Portfolio Performance and Strategic Asset Allocation Across Different Economic Conditions

#### 1.0 Introduction

The seminal idea that investors should hold mean-variance efficient portfolios has played an important role in optimal portfolio choice decisions because it emphasized the ability of diversification to reduce risk. However, in a multi-period setting (as opposed to the static single-period setting) the portfolio choice problem can be very different. Indeed, it has long been recognized from the work of Merton (1971, 1973) that shifting investment opportunities can have important effects on the optimal portfolio choices of investors with long horizons. There is now also considerable evidence that security returns, once thought to be independent over time, are in fact predictable (see, e.g., Cochrane (1999)).

The evidence that investment opportunities are in fact time varying suggests that investors may seek to hedge their exposure to consumption shocks or hold assets that do not perform poorly during financial distress (see, e.g., Campbell and Viceira (1999) and Campbell, Chan and Viceira (2003)). Moreover, such investors may be willing to accept a portfolio with a somewhat lower average return or higher variance provided the portfolio provides a hedge against economic downturns (Cochrane (1999)).

Both the theoretical insights and the empirical facts immediately raise three questions concerning optimal portfolio allocation decisions. First, can the performance of an investor's portfolio that is invested in traditional assets such as domestic stocks and bonds be improved by the inclusion of additional assets? Second, which of the many asset classes that improve portfolio performance should be included in the optimal portfolio allocation? Furthermore, which metric or criterion should be used to judge the relative attractiveness of different asset classes for inclusion in the optimal portfolio set for investors with different expectations? This latter issue is fundamental to this analysis in light of evidence that investors care about additional sources of risks beyond co-movement with the market portfolio, and therefore will seek to hedge their exposures against financial distress.

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<sup>&</sup>lt;sup>1</sup> Building on the intuition offered by Merton, a number of recent studies solve particular long-run portfolio choice problems. See, for example, Kim and Omberg (1996), Brennan, *et al* (1997), Campbell and Viceira (1999), and Barberis (2000), among others.

The first question is related to asset spanning and has recently received considerable attention, particularly in an international context. In comparison, the second and the third questions, particularly as they relate to whether the optimal portfolio of assets will perform well when investors need it the most (i.e. in times of economic downturns), is only now attracting attention. The purpose of this paper is to help remedy the situation by empirically evaluating the extent to which different asset classes improve portfolio performance. More specifically, we are interested in knowing the timing of the improvements in performance (i.e. whether they occur during economic downturns, when they are presumably needed the most) and how the timing of the improvements influences optimal portfolio decisions.

We implement an empirical methodology that accommodates the time varying risk-reward and hedging properties of different asset classes in an environment where expected returns and volatilities vary with economic regimes. Our empirical methodology combines the asset spanning literature with the literature on optimal portfolio holdings. In this context, we evaluate improvement in portfolio performance using three separate techniques. First, we compute the shifts in the Hansen-Jagannathan (1991) volatility bounds (or equivalently changes in Sharpe ratios), and measure the reduction in the standard deviation of minimum variance portfolios, when an asset class is added to a base portfolio. Second, we regress the changes in volatility bounds or Sharpe ratios on measures of economic state or financial distress to determine which assets perform when most needed. Third, we use a regime switching framework to examine the portfolio allocation implications of the differences in the portfolio properties of several asset classes.

Our empirical analysis delivers a number of interesting results that are relevant from the standpoint of asset allocation advice to investors. Specifically, our contributions to the literature consist of the following. First, we use both unconditional and conditional spanning tests to examine whether or not investors can improve the performance of their portfolio of assets by including additional assets. We employ the Hansen-Jagannathan (1991), hereinafter H-J, volatility bound methodology to infer the greatest lower bound on the volatility of an unobserved stochastic discount factor implied by a set of benchmark asset returns. We measure the shift in the H-J bounds induced by the addition of the test assets

under consideration to a series of benchmark portfolios. The duality between the mean-variance frontier of the discount factors and asset returns, enables us to approximate the change in the portfolio Sharpe ratio with the shift in the volatility bounds for a given test asset. We find that there are real economic benefits when investors extend their portfolio of assets by including corporate bonds, small cap stocks, commodities and precious metals, international stocks, and equity REITs that are not realized by holding a sequentially augmented portfolio that initially consisted of large and medium cap stocks, and Treasury bonds. In this context, we note that the benefits from different assets are particularly pronounced when conditioning information, reflected in four business cycle related variables, is utilized. The benefits are lower when conditioning information is ignored, but they are not eliminated. Moreover, the shift in the H-J volatility bound or equivalently the improvement in the Sharpe ratio is greatest when a base portfolio is augmented by adding commodities and precious metals. Interestingly, while improvements in the Sharpe ratios occur even in the presence of short sales restrictions, there are economic costs to these restrictions. In all cases, the changes in the Sharpe ratios are unambiguously lower when short sales constraints are imposed, but they not disappear unlike the results in De Roon et al (2001).

Second, while the shift in the volatility bounds (or Sharpe ratios) provides a measure of the economic benefits of diversification afforded by a test asset (see, e.g., Bekart and Urias (1996) and De Roon et al. (2001)), from a pure diversification point of view this measure is contaminated by the impact of expected asset returns. We isolate the pure diversification effects (in terms of variance reduction) by conducting minimum variance portfolio tests. Although not identical, our minimum variance analysis is in the spirit of Green and Hollifield (1992), who study the question of when mean-variance efficiency coincides with well diversified portfolios. Our contribution here is in showing that asset classes that improve portfolio Sharpe ratios, also provide portfolio variance reduction benefits. In particular, we find that the addition of commodities and precious metals to a base portfolio consisting of government bonds, large and medium cap stocks, corporate bonds and small cap stocks yields the largest reduction in the global minimum variance portfolio's annualized standard deviation, on the order of magnitude of 3.47%.

Third, to investigate the key issue of whether the improvements in portfolio performance occur when most needed, we regress the change in the volatility bounds (Sharpe ratios) on the mean and standard deviation of the per capita consumption growth rate in the economy. We find dramatic differences in the timing of the improvements in portfolio performance yielded by different asset classes, suggesting different relative abilities of asset classes to provide a kind of insurance against adverse shocks to consumption growth opportunities.<sup>2</sup> Of the various assets tested, commodities and precious metals, and equity REITs (a proxy for real estate) are the two asset classes that possess desirable properties in terms of the timing of their respective economic benefits. For both of these asset classes the changes in the volatility bounds (Sharpe ratios) are positively correlated with the standard deviation of consumption growth. Additionally, in the case of commodities and precious metals, the improvements in the reward-risk ratios are negatively correlated with mean consumption growth. The results suggest that these two assets provide insurance against deterioration in consumption opportunities.

In contrast, similar analysis of the timing of portfolio improvements suggests that, the performance of corporate bonds and international stocks is more pronounced in favorable states of the world; that is precisely when consumption opportunities are good. In the case of small cap stocks, the relationship between portfolio performance improvements and the moments of consumption growth is insignificant. This asset class would seem a logical candidate for the portfolio of an investor who is not exposed to the consumption growth risk. Furthermore, our findings in regards to the timeliness, or rather the lack thereof, of portfolio improvements from international stocks, suggests a possible rationale for the home bias in investor portfolios that has been documented in the literature.<sup>3</sup>

By linking portfolio allocation decisions to first and second moments of consumption growth, we provide empirical support for an important implication of the theory of inter-temporal portfolio choice and contribute to the growing literature that provides advice to investors about portfolio strategies. In this

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<sup>&</sup>lt;sup>2</sup> We also experimented with the growth rate of industrial production as an alternative measure of the state of the economy. The results are qualitatively similar.

<sup>&</sup>lt;sup>3</sup> French and Poterba (1991) document that U.S. equity investors allocate nearly 94 percent of their portfolios to domestic securities even though the U.S. equity market represents less than 48 percent of the global equity market.

context, our results help explain why it may be rational for investors to have portfolios that are tilted towards certain risky assets, possibly at the expense of other assets, a point stressed by Cochrane (1999). To our knowledge this is the first empirical study of the link between the timing of diversification gains from disparate asset classes and the two moments of aggregate consumption growth rate. Our analysis highlights an important metric by which to judge the value of improvement in portfolio performance afforded by a candidate asset class, namely, the timeliness of the gains.

Finally, a natural question arises in view of our results regarding differences in the timing of the improvements in portfolio performance provided by different asset classes: "What are the portfolio allocation implications of these differences for an investor who is sensitive to potential deterioration in the economy?" To address this issue, we examine how these differences manifest themselves in the optimal portfolio choices of a mean-variance optimizing investor over the business cycle. We use the regime switching framework of Hamilton (1989) to obtain parameter estimates for two economic states labeled the "good" state and the "bad" state. We find that the implied optimal mean-variance tangency portfolio during the good state of the economy is heavily weighted in domestic stocks (56%), and also includes international stocks (21%), corporate bonds (19%), and equity REITs (4%). Conversely, the optimal tangency portfolio during the bad state is weighted heavily in government bonds (42%), equity REITs (38%), and precious metals (20%). These results further highlight the important differences in the timeliness in portfolio performance improvements offered by different asset classes.

The remainder of this paper is organized along the following lines. Section 2 reviews the H-J volatility bound framework incorporating conditioning information and short sales constraints. Section 3 describes the data used in the study. Section 4 presents the results from estimating portfolio gains from different asset classes as well as the timeliness of the gains. Section 5 discusses the implications of our findings for asset allocation strategies. Section 6 provides an interpretation of the results while Section 7 concludes.

## 2.0 The Hansen-Jagannathan Volatility Bound with Short Sales Constraints

For clarity in the derivation of the H-J volatility bound, we focus on the first order condition yielded by agents' inter-temporal optimization that asset returns have to satisfy:

$$E_{t}\left(m_{t+1}r_{t+1}|\Omega_{t}\right) = 1\tag{1}$$

The variable  $m_{t+1}$  is an unobserved stochastic discount factor,  $r_{t+1}$  represents a matrix of n (gross) asset returns, 1 is an nx1 vector of ones and  $\Omega_t$  is the time t information set of the agents. Following He and Modest (1995), we incorporate the short-sales constraint in our subsequent empirical analyses. Specifically, when short sales are ruled out, asset returns satisfy a modified version of Equation (1):

$$E_t(m_{t+1}r_{t+1}|\Omega_t) = \lambda$$
, where  $\lambda \le 1$  (2)

In order to derive a bound on the variance of m, consider its linear projection on a constant and the vector of asset returns,  $r^5$ :

$$m = c + [r - E(r)] \beta + \varepsilon$$
(3)

or,  $m = m^* + \varepsilon$  where  $\beta$  is a vector of coefficients and  $\varepsilon$  is the projection error. From the above it follows that the variance of m has to be at least as great as the variance of its linear projection onto the space of asset payoffs,  $m^*$ . In other words,

$$\sigma^2(m) \ge \sigma^2(m^*) = \beta' \Sigma \beta \tag{4}$$

where  $\Sigma$  is the unconditional covariance matrix of asset returns. Using equations (2) and (4), and the standard expression for the least squares projection coefficient,  $\beta$ , gives us the following bound:

$$\sigma(m) \ge \left\{ \left[ \lambda - E(m)E(r) \right] \mathcal{L}^{-1} \left[ \lambda - E(m)E(r) \right] \right\}^{1/2}$$
(5)

The quantity on the right hand side of (5) is the greatest lower bound on the standard deviation of the stochastic discount factor in the presence of short-sales constraints. For a given value of E(m), the lower bound is found by choosing  $\lambda$  in order to minimize the quantity on the right hand side of inequality (5).

<sup>&</sup>lt;sup>4</sup> Mean-variance portfolios are generally not utility maximizing when there are shifts in the economic state, unless the agent has log utility. Thus our use of mean variance solutions is simply a metric designed to illustrate how the diversification benefits from different asset classes vary with the state of the economy

<sup>&</sup>lt;sup>5</sup> We omit the time subscripts for the sake of notational simplicity.

The minimization is conducted subject to the restriction:  $\lambda \le 1$ . From Equation (3), also note that  $E(m) = E(m^*) = c$ .

There is also a well-known economic interpretation for the lower bound expressed in (5). To see this, note that Equation (1) and Equation (2) imply that for returns expressed in excess of the risk free return,  $E_t(m_{t+1}R_{t+1}|\Omega_t)=0$ , where R denotes the vector of excess returns. From this it follows that inequality (5) may be expressed as  $\sigma(m) \geq E(m)SR^*$ , where  $SR^*$  denotes (the absolute value of) the Sharpe ratio of the mean-variance efficient portfolio implied by the given set of asset returns (see Ferson (1995) for an extended discussion). In our empirical analysis we specify E(m) to be the mean of the inverse of the (gross) real monthly return on the one-month US T-bill. Since this value is close to one, we can interpret the lower bound in (5) as approximately equal to the Sharpe ratio of the mean-variance optimal portfolio.

#### 2.1. Conditional Volatility Bound

Let  $z_t$  be a k-dimensional vector of conditioning variables in the investor's information set at time t,  $\Omega_t$ . Consider the re-normalized vector  $r_t^z = \frac{z_t}{E[z_t]}$  that has an unconditional mean equal to 1. Then from our earlier discussion, it follows that  $E[m_{t+1}(r_{t+1} \otimes r_{t+1}^z)] = \lambda$ , where  $\lambda \leq 1$ . The conditional lower bound on the volatility of the stochastic discount factor can now be computed using the scaled asset returns and following the method described in the previous sub-section. In essence, scaling a return by the realization of a random conditioning variable amounts to changing the investment in an asset based on the value of the conditioning variable, as in a dynamic trading strategy. In this paper, we focus on the conditional volatility bounds.

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<sup>&</sup>lt;sup>6</sup> While we incorporate conditioning information using the standard 'multiplicative' approach of Hansen and Jagannathan (1991), recent studies by Ferson and Siegel (2001, 2003) and Bekaert and Liu (2004) address efficient ways of incorporating conditioning information in constructing the bounds.

## 2.2. Measuring Improvement in Portfolio Performance Induced by Different Asset Classes

As discussed earlier, the first task in our analysis is to measure the improvement in portfolio performance induced by the sequential addition of the test assets (corporate bonds, small cap stocks, commodities and precious metals, international stocks, and equity REITs) to a series of diversified benchmark portfolios. The improvement in portfolio performance from the addition of a test asset to a benchmark portfolio may be measured by the resulting increase (if any) in the reward-to-risk tradeoff of the portfolio after the addition of the test asset. There is a well-known correspondence between the bounds for the stochastic discount factor and the mean-standard deviation frontier generated by the given set of asset returns (see, e.g., Hansen and Jagannathan (1991), Ferson (1995), and De Santis (1995)). Hence, the framework discussed above allows us to make an inference about the likely improvements in portfolio performance from the various asset classes. Specifically, we measure the shift(s) in the conditional bound, discussed above, that is induced by the addition of (scaled) returns on a series of test assets, over successive 60-month sub-periods. Thus we are able to compute the time series of portfolio performance gains offered by a particular test asset. We also conduct a formal statistical test of the null hypothesis that the addition of a subset of returns (on some test assets) to the benchmark set of returns, does not lead to an increase in the bound over the entire sample period. This corresponds to a test of mean-variance spanning as suggested by De Santis (1995). We describe the test in brief below.

Let the set of asset returns r be denoted by  $r' = [r'_A \ r'_B]$  where  $r_A$  contains the returns on the  $n_A$  benchmark assets, and  $r_B$  represents the  $n_B$  test assets. Consider the linear projection of the stochastic discount factor on the  $(n_A + n_B)$  returns in  $r : m = c + [(r - E(r))'\beta]$ , where  $\beta = [\beta'_A \ \beta'_B]$ . The test involves estimating the following system of orthogonality conditions using two pre-specified values (c1 and c2) for c:

$$E\left\{ r\left[ \left( r_{A} - E(r_{A}) \right)' \beta_{A1} + c_{1} \right] - \lambda \right\} = 0$$

$$\left\{ r\left[ \left( r_{A} - E(r_{A}) \right)' \beta_{A2} + c_{2} \right] - \lambda \right\} = 0$$

$$(6)$$

Let  $b_{A1}$  and  $b_{A2}$  denote the GMM (Hansen (1982)) estimates of the coefficients associated with the benchmark assets  $r_A$  and let  $h_T(b_{A1}, b_{A2})$  represent the vector of sample moments corresponding to the system in (6). Consider the statistic  $T\{h_T(b_{A1}, b_{A2})W_T h_T(b_{A1}, b_{A2})\}$ , where T is the sample size, and  $W_T$  is an optimally chosen weighting matrix. Under the null hypothesis, in the absence of short sales restrictions (i.e., when  $\lambda = 1$ ), the above statistic has an asymptotic  $\chi^2$  distribution with  $2n_B$  degrees of freedom. In the case where scaled asset returns are utilized, the relevant degrees of freedom for the statistic is equal to  $(2n_B).k$  where k is the number of conditioning variables utilized. This allows for a formal test of the mean-variance spanning hypothesis to be carried out.

In our context, given the short sales constraint, we have the inequality restriction  $\lambda \leq 1$ . In this case the test statistic is asymptotically distributed as a mixture of  $\chi^2$  distributions. Accordingly, we assess the significance of the estimated statistic for each test asset considered by us using a simulation procedure that we briefly describe here. Consistent with the null hypothesis of mean-variance spanning, we generate simulated returns on the test assets as linear functions of the benchmark returns plus a mean-zero error term (see, e.g., Huberman and Kandel (1987), and Bekaert and Urias (1996)). The error terms are drawn with replacement from the empirical distribution of the time series of residuals based on the estimated linear model for each test asset, and are mean-centered. In order to account for potential serial correlation in the data, we adopt a block-sampling scheme based on the stationary bootstrap (Politis and Romano (1994)) with a mean block length of 60. We generate 1000 simulated samples of length 360 (our sample size) for each test asset considered. We count the number of times in 1000 replications that the computed test statistic exceeds the test statistic obtained in the original data for a particular test asset. The significance of the test of asset spanning is based on these empirical p-values.

## 3. Data

We study monthly returns of Treasury bonds, corporate bonds, domestic stocks, commodities and precious metals, international stocks, and equity REITs from January 1972 through December 2001. We use the monthly returns on the CRSP market capitalization decide portfolios of NYSE/AMEX/NASDAO

stocks to construct three size-based stock portfolios. We form a large cap stock portfolio by value-weighting the returns on the *CRSP* decile portfolios 1 through 3, and a medium cap stock portfolio by value-weighting the returns on the *CRSP* decile portfolios 4 though 7. Similarly, we form a small cap stock portfolio by value-weighting the returns on the *CRSP* decile portfolios 8 though 10.

Bond market returns for this period include the monthly return on the intermediate term Treasury bond portfolio and the intermediate term corporate bond portfolio. These data are obtained from the CRSP files and Ibbotson Associates. We use the returns on the commodity spot index provided by the Commodity Research Bureau (CRB) as the proxy for commodity returns, and the returns on the CRB precious metals sub-index as the proxy for returns on precious metals. We use the NAREIT equity REIT index total returns series as our proxy for real estate returns. The data for international stock returns are based on the Morgan Stanley Capital International (MSCI) index of stock returns for Europe, Australasia and the Far East (EAFE). These returns are measured in terms of the US dollar. We convert all return series into real terms by deflating by the CPI.

In constructing the conditional HJ bounds we use conditioning information reflected in four business cycle related variables:  $DivYld_t$  is the ratio of dividends paid during the four quarters ending at month t to the price at the end of month t on the CRSP value weighted portfolio;  $Defsprd_t$  is the yield spread at the end of month t between Moody's Baa rated corporate bonds and Aaa rated corporate bonds of similar maturity; and  $Termsprd_t$  is yield spread at the end of month t between a 10-year Treasury bond and a 3-month T-bill. Data for the two spread variables (i.e., the default spread and the term spread) are obtained from the Federal Reserve Board. The T-bill yield,  $TBYld_t$ , is the one-month T-bill yield minus its 12-month backward moving average. This stochastic de-trending method for the short rate has been used by Campbell (1991) and Hodrick (1992), among others. Monthly T-bill yields are obtained from the CRSP files. We use data on monthly, real per capita consumption (of non-durable goods and services) to measure the first and second moments of aggregate consumption growth. The consumption data are also obtained from the Federal Reserve Board.

Table 1 contains descriptive statistics for the monthly returns on the asset classes and on the CRSP value weighted index. The table also shows summary statistics for the monthly growth rate of per capita consumption of non-durable goods and services, and for the four conditioning variables utilized by us. All asset returns are expressed in real terms. As can be seen from Panel A of the table, the average return on the U.S. size-based portfolios ranged from 0.64% per month for the large cap stocks to 0.83% per month for the small cap stocks. The return on equity REITs averaged 0.66% per month during this period. For the same period, the international stock portfolio return averaged 0.60% per month. The average monthly return on the CRB commodity index was – 0.18% per month and that on the precious metals index was 0.32% per month.

Panel B of Table 1 reports return autocorrelations up to five lags. Return autocorrelations for most asset classes are low and appear to decay at longer lags suggesting the expected returns are slowly mean reverting. The first order autocorrelation for the one-month T-bill returns is large (0.91) suggesting that T-bill returns are quite persistent. Similarly, the four conditioning variables, namely, the lagged dividend yield, the detrended T-bill yield, the default premium, and the term premium, are highly persistent. In this context, Ferson, Sarkissian and Simin (2003) recommend stochastic de-trending of such variables when using them in predictive stock return regressions to guard against the possibility of uncovering spurious predictive relations.

## 4. Empirical Evidence

As discussed in Section 1, the first stage of our empirical analysis consists of a sequence of tasks. First, we use the H-J framework outlined in Section 2 to test whether a series of benchmark portfolios spans a series of test assets (corporate bonds, small cap stocks, commodities and precious metals, international stocks and equity REITs). In conducting this test we rule out short sales. We next examine the variance reduction properties of the various asset classes. We then evaluate which of the test assets deliver improvements in portfolio performance when most needed.

## 4.1. Conditional Spanning Results

In this subsection, we examine whether adding different assets to a series of benchmark portfolios leads to a significant shift in the mean-variance frontier when investors incorporate conditioning information but are not permitted to engage in short selling. Short sales constraints are particularly relevant in our case since our test assets include international stocks and real estate. Moreover, short selling can be expensive and investors may face legal and institutional constraints that prevent them from taking short positions. For each asset class, we compute the change in the volatility bound using data on asset returns for a rolling 60-month period that ends with the current month. For example, for the 60-month period that ends in January 2001, we use data from February 1996 to January 2001 to compute the volatility bounds.

Our choice of benchmark and test assets is motivated by several factors. The initial base portfolio consists of large and medium cap stocks and Treasury bonds, and is designed to mimic the portfolio of a conservatively diversified domestic investor. We then allow our hypothetical investor to augment successive portfolios by sequentially adding corporate bonds, small cap stocks, commodities and precious metals, international equities, and equity REITs, in that order. The inclusion of equity REITs helps us explore the conventional notion that equity REITs are similar to small cap stocks and also contain bond-like features. If the returns to equity REITs indeed contain only small cap and corporate bond risk factors, our tests should not find significant improvement in portfolio performance, when equity REITs are added to a portfolio that already includes corporate bonds and small cap stocks. A final consideration in the design of the benchmark portfolios is the benefits from international diversification that have been documented by previous studies. In light of the fact that international financial markets have become increasingly more integrated, it would be interesting if the findings of previous studies can be replicated in the context of our tests, which consist of many disparate domestic assets. Consequently, we include international stocks as one of the five test assets.

Table 2 reports the results for the change in the H-J bound induced by the addition of test assets to the successive benchmark portfolios. The addition of each test asset expands the benchmark portfolio

by one asset class making it more diversified than the immediate predecessor benchmark portfolio. The test assets are indicated at the top of each column. For all test assets the rows report the average shift in the H-J bound. The average figure reported for a given year represents the average shift in the H-J bound for the twelve 60-month sub-periods that end during that year. In each case, the reported shift in the H-J bound may be interpreted as the increase in the approximate conditional Sharpe ratio (i.e., the reward-to-risk tradeoff for the conditional mean-variance frontier) due to the addition of the test assets to the benchmark portfolios. Thus, the change in the volatility bounds measures the economic importance of the shift in the portfolio frontier.

The second to the last row of Table 2 reports the average shift in the H-J bound over the entire sample period, 1972-2001. To assess whether the portfolio performance improvements are statistically significant, we conduct a formal test of the null hypothesis that over the entire sample period 1972-2001, each successive benchmark spans its respective test asset. The last row of the table reports the test statistics and the associated *p*-values based on the simulation procedure described earlier in Section 2.2. On the basis of the *p*-values, we reject the hypothesis that each successive benchmark portfolio spans its respective test asset. For corporate bonds, the average change in the conditional reward-to-risk tradeoff is 0.0985 and is statistically significant at the 1 percent level or better. For small cap stocks, commodities and precious metal, and international equities, the average approximate Sharpe ratio gains are 0.1434, 0.4897, and 0.2271, respectively, and are statistically significant. It is particularly interesting to note that when equity REITs are added to the most diversified benchmark portfolio (consisting of large cap stocks, medium cap stocks, small cap stocks, corporate bonds, commodities and precious metals, and international stocks), there is a significant increase in the H-J bound. This suggests that equity REITs are not subsumed by either small cap stocks or bonds, contrary to popular view.

Based on the time series pattern of the shifts in volatility bounds the improvement in portfolio performance induced by each of the asset classes appear to be time varying. As shown in Table 2, the gains from diversifying into commodities and precious metals are particularly pronounced over the study period. For example, during the 1970s, the performance gains from commodities and precious metals

group were large and variable, followed by declining gains in the late 1980s. These gains again increase in the early 1990s before gradually declining during the latter part of the decade. There is also considerable oscillation in portfolio performance gains from exposure to equity REITs throughout the study period, with a pronounced plateau during the early part of the 1980s.<sup>7</sup>

The improvements in the conditional H-J bounds or Sharpe ratios are partly a function of the expected returns of the test assets. In order to isolate the pure diversification properties of the test assets, we further examine the impact that the inclusion of these assets has on the volatility of the respective global minimum variance portfolios. Table 3 presents the annualized standard deviation estimates for the global minimum variance portfolios constructed from various assets for different sub-periods between 1972 and 2001. We constrain portfolio weights to be non-negative when solving for the composition of the minimum variance portfolios. As can be seen from Table 3, across all sub-periods as well as over the entire sample period, the inclusion of commodities and precious metals leads by far to the largest reduction in the standard deviation of the minimum variance portfolios. For example, over the entire period 1972-2001, the addition of commodities and precious metals reduces the minimum variance portfolio's standard deviation by 3.47% (representing a proportional reduction of over 38%). In contrast, the volatility reductions achieved by the addition of other assets are much smaller. These results are thus consistent with our previous results showing that commodities and precious metals account for the largest shifts in H-J bounds or Sharpe ratio improvements.

## 4.2. Robustness Tests

In this subsection, we conduct a number of robustness tests. We begin by examining how sensitive our results are to the order in which the test assets are added to the original base portfolio. For example, it is reasonable to ask whether the large gains from the inclusion of commodities and precious metals persist when the ordering of the test assets is changed. We find that our results remain qualitatively

<sup>&</sup>lt;sup>7</sup> Although not shown, the standard deviation of the gains within each year averaged 0.0361, 0.0303, 0.0807, 0.0684, and 0.0829 in the case of corporate bonds, small cap stocks, commodities and precious metals, international stocks, and equity REITs, respectively.

unchanged when the order of addition of the test assets is altered. For example, commodities and precious metals continue to provide the highest benefit (average approximate Sharpe ratio gain of 0.68) when we start by adding small cap stocks followed by corporate bonds, international stocks, REITs, and commodities and precious metals, in that order. The gains from small cap stocks and corporate bonds are nearly identical to the original results while both international stocks and equity REITs offer higher gains under this alternative ordering scheme. The qualitative nature of our results is also unchanged when we use the S&P/BARRA value and growth stock portfolios, instead of the size-based stock portfolios, to represent the domestic equities asset class. For this case, the initial base portfolio includes government bonds, and the S&P/BARRA value portfolio. We use the S&P/BARRA growth portfolio as one of the test assets. We find that commodities and precious metals continue to provide the largest portfolio gains (average Sharpe ratio gain of 0.48) while the gains from the other asset classes are of similar magnitude as the base set of results. The average Sharpe ratio gain from the inclusion of the growth stock portfolio was 0.14 over the period 1972-2001.

As noted previously, we incorporate short sales constraints while calculating the H-J bounds. A natural question that arises is: "What is the impact of removing the short sales restrictions on the measured portfolio gains offered by the various asset classes?" To address this issue we compute the H-J bound shifts without imposing the short sales constraint. We find that while the removal of the short sales constraint does lead to improved gains in each case, the improvements in the reward-risk benefits are moderate ranging from 0.33% in the case of international stocks to 6.47% in the case of commodities and precious metals.

As mentioned earlier, the use of conditioning information when calculating the H-J bounds corresponds to the use of dynamic trading strategies by an investor. To isolate the pure asset class effects we also compute the shifts in the H-J bounds when conditioning information is not utilized. We find that in each case the shifts in the H-J bounds representing the approximate increase in the unconditional

<sup>&</sup>lt;sup>8</sup> Detailed results for the various robustness tests are omitted to conserve space but are available upon request.

Sharpe ratios are considerably smaller when conditioning information is ignored. The average approximate increases in Sharpe ratios in the case of corporate bonds, small cap stocks, commodities and precious metals, international stocks, and equity REITs are 0.0649, 0.0379, 0.0882, 0.0416, and 0.0278, respectively. Hence, the absence of conditioning information leads to a reduction in the benefits offered by the test assets ranging from 34% in the case of corporate bonds to 86% in the case of equity REITs. <sup>9</sup>

To summarize, the results from the mean-variance spanning tests suggest that there are significant gains in portfolio performance from exposure to various asset classes including corporate bonds, small cap stocks, commodities and precious metals, international stocks and real estate. The results hold despite the imposition of short sales constraints. Moreover, the shifts in the volatility bounds imply considerable variation in portfolio gains over time from each of the asset classes. The crucial question is the degree to which the variability in portfolio gains coincides with deterioration in aggregate consumption growth. We address this issue in the next section.

## 4.3 The Timeliness of Improvements in Portfolio Performance from the Asset Classes

We have shown in subsection 4.1 that portfolio diversification into disparate assets leads to significant gains, on average, for investors. Moreover, the gains in portfolio performance are time varying and persist even in the presence of market frictions such as short-sales constraints. A related and important issue for both individual and institutional investors is the ability of different asset classes to enhance portfolio performance when most needed. This issue has been of great interest in recent years in the international investments arena, due to evidence of increasing cross-country correlations conditional upon bear markets or negative shocks. In this subsection, we investigate the extent to which the various asset classes differ in their ability to smooth adverse shocks faced by investors.

We estimate an empirical model in which the changes in volatility bounds of a stochastic discount factor induced by the addition of a test asset to the benchmark portfolio (the improvements in portfolio performance) are regressed on the mean and the standard deviation of the consumption growth rate.

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<sup>&</sup>lt;sup>9</sup> Of course, our results with respect to the value of conditioning information should be interpreted with caution since we have not explicitly taken into account the cost of using the dynamic trading rules.

Specifically, we evaluate the ability of the assets to improve portfolio performance when needed the most by estimating the following regression model

$$DB_{i,t} = \alpha_0 + \alpha_1 \cdot \hat{\mu}_t + \alpha_2 \cdot \hat{\sigma}_t + \varepsilon_t \tag{7}$$

where  $DB_{t,t}$  is the measured portfolio gain (i.e., improvement in the reward-to-risk tradeoff) from asset class i measured over a 60-month interval (t-59 to t), and  $\hat{\mu}_t$  and  $\hat{\sigma}_t$  represent the estimated mean and the standard deviation of the monthly consumption growth rate over that 60-month interval. The above framework is intended to examine how the improvements in portfolio performance from different classes are related to the state of the economy. Our use of the moments of consumption growth to proxy the state of the economy follows a rich tradition in the literature. For example, Kandel and Stambuagh (1990) analyze a model in which conditional moments of asset returns are linked to the mean and standard deviation of consumption growth rate. They also show that the variation in the two moments of consumption growth is related to the business cycle. Consumption-based asset pricing models generally have the feature that changes in the moments of returns will be related to changes in the moments of consumption. Thus, our strategy of regressing the improvements in portfolio performance on the moments of consumption growth rate, in effect empirically mimics an investor who optimizes over both consumption and portfolio choice.

Based on our discussions so far, we expect the estimated gains in portfolio performance from assets that completely insure against adverse shocks to investment opportunities faced by risk-averse investors, to vary inversely with the consumption growth rate and positively with the volatility of consumption growth rate. Not all assets may, however, conform to this expectation. Some assets, for example, may provide no insurance against adverse shocks but provide risk premium, while other assets may provide a partial insurance when consumption growth is low but may not protect the portfolio against volatility shocks.

Table 4 reports, for each asset class, results from estimating Equation (7). The table reports regression estimates and the associated t-statistics. In order to account for the potential bias in the

estimated coefficients due to the persistence in the variables induced by the use of overlapping data, we implement the bias-correction procedure suggested by Stambaugh (1999). Furthermore, in view of the fact that the regressor variables are themselves estimated with sampling error, we adjust the standard errors of the estimated coefficients (based on the Newey-West covariance estimator with a lag length of 59) using the method suggested by Pagan (1984) and Murphy and Topel (1985). Panel A in Table 4 shows that in the case of corporate bonds, the coefficient on the mean consumption growth rate is significantly positive while the coefficient on the standard deviation of the consumption growth rate is significantly negative. The results are robust regardless of model specification (univariate or bivariate). We conclude that corporate bonds fail to provide gains in portfolio performance when expected consumption growth is low or when it is volatile.

Panel B in Table 4 shows that for small cap stocks the coefficient for the expected consumption growth rate is negative and insignificant in the univariate regression, but it is positive in the bivariate regression. The coefficient for the standard deviation of consumption growth rate is positive, although it is insignificant in the bivariate regression. These results suggest that small cap stocks, like corporate bonds, fail to deliver gains in portfolio performance when consumption growth opportunities are poor, i.e., when investors need such benefits the most.

Panel C presents the results for commodities and precious metals. As can be seen, the coefficient on consumption growth is significantly negative. Similarly, the coefficient on the standard deviation of consumption growth is significantly positive. These results are consistent across the univariate and the bivariate specifications. Collectively, these results suggest that commodities and precious metals provide significant gains in portfolio performance precisely during those times when investors care for such gains (i.e. in period of low consumption growth). In contrast, results for international equities, presented in Panel D of Table 4, indicate that the performance gains of this asset class are significantly more pronounced during good times i.e., in periods of high consumption growth. However, the performance

<sup>&</sup>lt;sup>10</sup> In this context, Amihud and Hurvich (2004) propose a general bias-correction method that is applicable in the case of both the single as well as the multiple regressor models.

gains from international equities do appear to increase during periods of high volatility in consumption growth which is a desirable trait. Thus, results for international equities are mixed, at best.

Panel E in Table 4 reports the results for equity REITs. The relation between the improvement in portfolio performance induced by equity REITs and consumption growth, is insignificant. On the other hand, the sign of the coefficients on the standard deviation of consumption growth is consistent with what one would expect from an asset class whose gains in portfolio performance are likely to protect the portfolio against consumption volatility. The reliably positive estimated coefficient for the standard deviation of the consumption growth suggests that the inclusion of real estate in a well-diversified portfolio helps to hedge against volatility of consumption growth. It is also worth emphasizing that the gains in portfolio performance induced by real estate are measured against the most stringent benchmark portfolio, one that includes all other assets considered by us. In this sense, the gains are quite significant from an economic standpoint.

We also examined the sensitivity of our results to the use of aggregate consumption growth data to proxy the state of the economy. Monthly consumption data are prone to measurement error and are subject to seasonal adjustments that may render the series ill-suited as a proxy for the actual consumption experience of the typical consumer. We repeated our tests by using the growth rate of the index of industrial production as an alternative proxy for the state of the economy. Although not reported, we find that our results are qualitatively unchanged. Specifically, the portfolio gains due to commodities and precious metals are significantly negatively related to the average growth rate in industrial production, and significantly positively related to its standard deviation. Similarly, equity REITs provide benefits that are significantly positively related to the volatility of industrial production. None of the other asset classes appear to be desirable in terms of the timeliness of their portfolio properties.

The results for the different asset classes in Table 4 also reveal considerable variation in the magnitude of the estimated slope coefficients as well as the regression R-square statistics. These differences appear to reflect differences in both the average volatility bound shifts or portfolio improvements across the asset classes as well differences in the timeliness of the improvements vis-à-vis

consumption growth and volatility. In summary, the results of this section suggest that the various asset classes differ dramatically in terms of their contributions to portfolio performance improvement.

## 5.0 Portfolio Allocations under Alternative Regimes

Given our findings in the previous section, a natural question that arises is: "What are the portfolio allocation implications of the differences in the timeliness of improvement in portfolio performance among the different asset classes?" In other words, how should an investor optimally choose to vary her portfolio allocation based upon the state of the economy in which she finds herself?

To provide insight into the above issue we adopt the regime switching framework of Hamilton (1989).11 Specifically, we model the asset returns and the consumption growth series as stochastic processes that are subject to changes in means and covariances due to shifts in the underlying state or regime represented by an unobserved variable  $s_t$ . We assume that  $s_t$  is described by a 2-state Markov chain. More formally, the stochastic processes governing asset returns and consumption growth can be expressed as

$$y_t | (s_t = I) \sim N(\mu_I, \Omega_I)$$
$$y_t | (s_t = 2) \sim N(\mu_2, \Omega_2)$$

where  $y_t$  is a (n+1)-dimensional vector containing the real returns on n assets as well as the real consumption growth rate. The (n+1)-dimensional vectors  $\mu_1$  and  $\mu_2$  represent the mean asset returns and the mean consumption growth rate in the two states while  $\Omega_1$  and  $\Omega_2$  represent the corresponding covariance matrices, each of dimension (n+1) x (n+1). The unobserved state variable  $s_t$  is assumed to follow a two-state Markov chain with transition probability matrix given by:

$$\begin{pmatrix} p & (1-p) \\ (1-q) & q \end{pmatrix}$$

<sup>&</sup>lt;sup>11</sup> Recent applications of the regime switching framework to the investor asset allocation decisions include Ang and Bekaert (2002, 2004), and Guidolin and Timmermann (2004), among others.

The parameters of the above processes may be represented by the vector  $\theta$  where  $\theta = (\mu_1, \mu_2, \Omega_1, \Omega_2, p, q)'$ . We estimate maximum likelihood estimates of the parameter vector  $\theta$  using the EM algorithm developed by Hamilton (1990) and illustrated in Engel and Hamilton (1990).

Note that under the above framework, the expected return and covariance of asset returns in the next period are a function of the current state. For example, if at time t the investor infers that the economy is in state 1, (i.e.,  $s_t = 1$ ) then her expected asset returns for the next period can be expressed as  $ER_{t+1}|(s_t = 1) = p\mu_1 + (1-p)\mu_2$ . On the other hand, if the investor infers the current state of the economy to be described by state 2 (i.e.,  $s_t = 2$ ), then her expected asset returns for the next period are given by  $ER_{t+1}|(s_t = 2) = (1-q)\mu_1 + q\mu_2$ . Consequently, as long as  $p \neq (1-q)$ , an investor's optimal asset allocation choices will be state-dependent.

In our context an interesting question to ask is: do the data suggest that the optimal investor asset allocations should be sensitive to the state of the economy and if so, what are the optimal allocations to the different asset classes in say, the good and the bad states of the economy? In order to answer this question we calculate the mean-variance optimal tangency portfolio weights conditional on each state, based on our estimates of the regime switching model. We employ the mean-variance optimal allocations as a metric to illustrate the improvement in portfolio performance potential of the various asset classes in the different states of the economy.

Table 5 presents the results from this exercise. Panel A of the table reports the monthly mean real returns on the nine asset classes including the one month US T-bills, in each state. For the purpose of this table, in order to reduce the dimensionality of the estimation problem, we combine the medium and large cap stocks into a single equally weighted portfolio labeled 'STKRET'. Also presented in Panel A are the estimates of the average monthly real per capita consumption growth rate in each state. The corresponding standard errors are noted in parenthesis. To avoid clutter, standard errors for elements of

the covariance matrices in panels B and C of Table 4 are reported only for the diagonal elements of the respective matrices. Based on the estimates in Panel A of Table 4, it is clear that the average consumption growth rate as well as the average asset returns are generally lower in State 2, with two notable exceptions. Both equity REITs and precious metals exhibit countercyclical behavior and have a higher average return in State 2. Furthermore, based on the estimates of the covariance matrices in the two states reported in Panels B and C, it is evident that with the exception of bonds and international stocks, asset returns are more volatile in the second state. Thus state 1, characterized by generally higher and less volatile asset returns, may be viewed as the "good" state while state 2 may be viewed as the "bad" state.

Panel D of Table 5 reports the estimated state transition probabilities. As may be seen, both states appear to be persistent as the probabilities p and q are both estimated to be above 0.50. The persistence of each state has implications for the composition of the conditionally optimal investor asset allocations in the two states. For instance, suppose that the investor knows that the economy is currently in the bad state, i.e., state 2. Since there is a 61% probability that the economy will continue to be in the bad state next period as well, the investor is likely to target assets that have higher expected payoffs in the bad state of the economy. The converse is true if the investor infers the current state to be the good state. In the latter case, there is a 79% probability of the economy continuing to be in the good state the next period as well, which would argue for the investor's allocations to be tilted towards those assets that have high payoffs in the good state of the economy.

Panel E of Table 5 reports the optimal tangency portfolio weights conditional on the current state being the good state (i.e., state 1) or the bad state (i.e., state 2). Figures 1a and 1b depict the mean-standard deviation frontiers for different portfolios of assets conditional on the good state and the bad state. The results clearly support an asset allocation policy that is state or regime-dependent. The optimal tangency portfolio in the good state includes small cap stocks (15.51%), medium and large cap stocks

<sup>&</sup>lt;sup>12</sup> Similarly, the conditional variances are given by:  $\sigma_{t+1}|(s_t = l) = p\sigma_1^2 + (l-p)\sigma_2^2 + p(l-p)(\mu_1 - \mu_2)^2$ , and  $\sigma_{t+1}|(s_t = 2) = (1-q)\sigma_1^2 + q\sigma_2^2 + q(l-q)(\mu_1 - \mu_2)^2$ .

(40.16%), corporate bonds (19.04%), international stocks (21.35%), and equity REITs (3.95%). Hence, in the good state of the economy, an investor should allocate the bulk of her portfolio to domestic stocks. Interestingly, in the bad state of the economy the optimal tangency portfolio is invested exclusively in government bonds (41.89%), precious metals (20.33%), and equity REITs (37.78%). The dominant position of precious metals and equity REITs in the tangency portfolio corresponding to the bad state, is evidence of a portfolio hedging role played by these tangible or hard assets during economic downturns. Similarly, diversification into government bonds appears to be valuable in economic downturns.

In order to further isolate the pure diversification properties of the asset classes, we examine the composition of the global minimum variance portfolio conditional on the good and the bad states. As noted previously, this calculation is not influenced by the impact of expected returns. The results appear in Panel F of Table 5. Interestingly, commodities and precious metals, equity REITs, and corporate bonds are the only asset classes that have a non-zero portfolio allocation in both states. These results serve to further underscore the portfolio diversification properties of assets such as commodities and precious metals, and equity REITs.

The optimal tangency portfolio allocations presented in Table 5 are based on the full sample of data over the period 1972-2001 and thus could not have been implemented by an investor making decisions in real time. Also, the feasibility of a real-time regime-dependent portfolio strategy will be sensitive to the transaction costs that in turn, depend on the volatility of the implied portfolio weights through time. To gauge the feasibility of such a real-time strategy, we estimated the regime switching model at the end of each year from 1988 to 2001, and calculated the corresponding optimal portfolio allocations. In each case the estimation was based on data available only up to that year. We find that over this period, the standard deviation of the asset class weights for the good economic state ranges from 6.68% in the case of small cap stocks to 18.80% in the case of medium and large cap stocks. Similarly, for the bad economic state, the standard deviation of the asset class weights ranges from 10.71% in the case of equity REITs to 17% in the case of government bonds. While these portfolio changes do not appear to be extreme, our results should not be taken as evidence in favor of the superiority of a regime

dependent allocation strategy relative to say, a passive buy-and-hold strategy. A detailed comparison involving transaction costs, position limits, and potentially parameter uncertainty, is beyond the scope of this paper.

## **6.0 Interpretation of Results**

In this section an attempt is made to interpret our results with the goal of adding to the emerging literature that seeks to provide advice to investors about their portfolio strategies. Our results suggest two broad groups of assets for the purpose of advice on portfolio strategies. Consider first a U.S. investor looking to add corporate bonds, small cap stocks, or international stocks to a portfolio that is fully invested in large cap stocks, medium cap stocks and Treasury bonds. The results of Table 4 suggest that both corporate bonds and international stocks, improve portfolio performance during good times (i.e. when consumption growth opportunities are high). Furthermore, neither corporate bonds nor small cap stocks protect the portfolio against consumption volatility shocks.

Next, consider an investor whose demand for risky assets is motivated by the risk premium, but who at the same time wishes to insure against volatility in consumption growth opportunities. Our results imply that if the investor already holds a portfolio that is diversified into stocks of various sizes, and corporate bonds, it would be appropriate for such an investor to add assets such as commodities and precious metals, as well as real estate, to her portfolio. Over the study period, 1972-2001, the evidence in Table 4 supports the claim that these asset classes are a potent hedge for diversified portfolios. First, we interpret the negative coefficient on the consumption growth rate in the case of commodities and precious metals as evidence of this asset class' ability to counterbalance deterioration in consumption opportunities. Second, these hard (tangible) assets are also powerful vehicles for hedging a diversified portfolio against the volatility of consumption growth, as evidenced by the reliably positive coefficient on the standard deviation of consumption growth rate. This conclusion is further strengthened by our results

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<sup>&</sup>lt;sup>13</sup> See for example, Campbell and Viceira (1999, 2001), and Cochrane (1999).

that suggest that the optimal mean-variance tangency portfolio is heavily weighted in equity REITs and precious metals in the bad economic state, while also including government bonds.

Our interpretation of the results leads to two main conclusions. First, highly risk averse investors who seek a stable consumption profile will want to bear a positive amount of "hard assets" risks even if the expected returns from these asset classes are lower than the returns of the other competing asset classes. Indeed, to the extent possible, equilibrium reasoning suggests that investors who dislike poor performance in their investment portfolios when consumption opportunities deteriorate, should overweight assets such as real estate, commodities and precious metals, and government bonds. This conclusion extends and complements the findings of Campbell and Viceira (1999) who recommend that a highly risk averse investor should hold government bonds, an asset class that performs well in unfavorable states of the world when consumption opportunities are poor. Here, we identify additional asset classes including real estate, and commodities and precious metals, that act as a hedge against shocks to portfolios.

The finding that real estate provides a hedge against deterioration in consumption opportunities in a portfolio context, may also illuminate the rationale (rather than the flaw) behind the popular investment advice that investors should tilt their portfolios more towards real estate, despite data that show the typical household already holds large positions in residential real estate. Our findings are, of course, subject to the usual caveat that they may be period specific and thus may not be replicated in the future.

## 7.0 Conclusion

Assessing portfolio performance as part of overall investment strategy, particularly during market downturns and increased market volatility, has been the subject of much recent debate. Most studies that examine improvements in portfolio performance from the inclusion of disparate assets in a portfolio often sidestep market frictions such as short sales constraints, and do not ask how the improvements vary with the business cycle. This paper has empirically analyzed how different asset classes contribute to portfolio performance improvements, and in particular, the timeliness of the improvements.

The results of the analysis confirm the value of portfolio diversification while providing interesting insights into the variation of gains in portfolio performance over the business cycle. A major finding of the paper is that commodities and precious metals, and real estate appear to be powerful vehicles for hedging against adverse shocks to consumption growth opportunities. Not only do these asset classes offer significant gains in portfolio performance, the gains vary directly with the standard deviation of consumption growth rate. In addition, in the case of commodities and precious metals, the performance gains vary inversely with the average consumption growth rate during a period.

We also employ Hamilton's (1989) regime switching framework to explicitly examine the asset allocation implications of the differences in the portfolio performance properties of the various asset classes. This analysis suggests that the optimal mean-variance tangency portfolio is heavily weighted in equity REITs, and precious metals in the bad state of the economy, while also including government bonds. These results underscore our earlier findings in regards to the differences in the timeliness of improvements in portfolio performance offered by different asset classes.

Our analysis also makes a methodological contribution to the literature. With few exceptions, the vast majority of asset allocation/diversification studies have not explicitly examined how the portfolio gains from diversifying into different asset classes vary over business cycle. In particular, most previous studies have not explicitly addressed the question of whether or not portfolio performance gains are most pronounced when investors really care for them. Our results suggest that this is an important metric by which to judge the usefulness of an asset class in portfolio allocation decisions. Finally, our findings support the view that the cross-sectional differences in returns across asset classes such as bonds, commodities and precious metals, international equities and real estate, may appropriately reflect the differences in their respective diversification and hedging properties.

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Table 1
Descriptive Statistics: 1972 - 2001

Panal A: Descriptive Statistics

The table presents descriptive statistics (Panel A) and autocorrelations for lags 1 through 5 (Panel B) for monthly real returns on various asset classes for the period 1972-2001. The table also shows the descriptive statistics for the four conditioning variables utilized by us. The assets include the CRSP portfolios representing small, medium, and large cap portfolios of all NYSE/AMEX/NASDAQ stocks (SMLRET, MEDRET, and LRGRET), the CRSP value-weighted index (VWRETD), Equity REITs (EREITs), the U.S. one-month T-bill return (USTRET), a portfolio of Treasury bonds (GBTRET), a portfolio of Corporate bonds (CBTRET), the CRB commodity spot index (CRB), the CRB precious-metals sub-index (CRBPM), and the MSCI index of stocks from Europe, Australasia and the Far East (EAFE). The conditioning variables include the annualized dividend yield on the value weighted index of NYSE/AMEX/NASDAQ stocks (DivYld), the detrended yield on the one month T-bill (TBYld), the default spread (Defsprd), i.e., the difference in the yield on Moody's BAA rated corporate bonds and Aaa rated corporate bonds, and the term spread (Termsprd), i.e., the difference in the yield on 10 year Treasury bonds and the yield on a 3 month T-bill. Also shown are the descriptive statistics for the monthly growth in the log real per capita consumption of non-durables goods and services (NDSGR) during the period 1972-2001.

Panel A: Descri <sub>l</sub>	Mean	Median	Std Deviation	Skewness	Minimum	Maximum
SMLRET	0.0083	0.0115	0.0659	-0.0044	-0.2919	0.3640
<b>MEDRET</b>	0.0081	0.0099	0.0568	-0.4764	-0.2720	0.2467
LRGRET	0.0064	0.0089	0.0458	-0.3869	-0.2111	0.1602
<b>VWRETD</b>	0.0065	0.0088	0.0467	-0.4904	-0.2268	0.1536
<b>EREITs</b>	0.0066	0.0074	0.0397	-0.2804	-0.1546	0.1365
USTRET	0.0054	0.0048	0.0023	1.3094	0.0015	0.0152
<b>GBTRET</b>	0.0034	0.0027	0.0306	0.4656	-0.0914	0.1395
CBTRET	0.0035	0.0028	0.0272	0.2752	-0.0962	0.1250
CRB	-0.0018	-0.0016	0.0264	0.4081	-0.0885	0.1324
CRBPM	0.0032	-0.0011	0.0596	0.5916	-0.2863	0.3453
EAFE	0.0060	0.0070	0.0499	-0.1170	-0.1494	0.1742
DivYld	0.0324	0.0337	0.0127	-0.0918	0.0097	0.0612
TBYld	0.0000	0.0000	0.0010	-0.1176	-0.0044	0.0038
Defsprd	0.0109	0.0094	0.0045	1.2162	0.0055	0.0269
Termsprd	0.0157	0.0166	0.0133	-0.5573	-0.0265	0.0442
NDSGR	0.0016	0.0018	0.0037	-0.0777	-0.0122	0.0131
Panel B: Autoco	rrelations					
		$\rho_1$	$ ho_2$	$\rho_3$	$\rho_4$	$\rho_5$
SMLRET		0.1525	-0.0683	-0.0708	-0.0680	-0.0311
MEDRET		0.1118	-0.0813	-0.0392	-0.0722	-0.0268
LRGRET		0.0078	-0.0331	0.0264	-0.0172	0.0815
VWRETD		0.0369	-0.0476	0.0136	-0.0314	0.0574
<b>EREITs</b>		0.1183	-0.0367	-0.0154	-0.0461	0.0830
USTRET		0.9098	0.8617	0.8403	0.7971	0.7730
GBTRET		0.1421	-0.0038	-0.0767	0.0321	0.0915
CBTRET		0.1932	-0.0006	-0.0303	0.0031	0.1233
CRB		0.2386	0.0448	-0.0457	-0.0325	0.1100
CRBPM		0.3210	-0.0448	-0.0331	-0.0725	-0.0825
EAFE		0.0756	-0.0360	0.0553	0.0381	0.0159
DivYld		0.9869	0.9737	0.9622	0.9498	0.9365
TBYld		0.8174	0.6365	0.4890	0.3910	0.3231
Defsprd		0.9643	0.9170	0.8821	0.8568	0.8319
Termsprd		0.9409	0.8560	0.7877	0.7232	0.6661
NDSGR		-0.2511	0.1035	0.1267	-0.0613	0.0801

Table 2
Change in conditional H-J bound: 1972-2001

This table presents the average changes in the conditional Hansen-Jagannathan bound corresponding to the approximate average change in the Sharpe ratio implied by the inclusion a set of test assets to a portfolio of base assets. The test assets are indicated at the top of each of the columns. The original base set of assets for the first column consists of: a medium capitalization and a large capitalization portfolio of NYSE/AMEX/NASDAQ stocks (obtained from CRSP files), and a government bond portfolio. For each successive column, the base set of assets is expanded by the addition of the test asset indicated in the previous column. The change in the bound is computed over (rolling) 60-month intervals using monthly data for the period 1972-2001, and when short sales are disallowed. An increase in the lower bound is a measure of the diversification gain afforded by the test asset considered. To assess whether the diversifications gains are statistically significant, we conduct a formal test of the null hypothesis that over the entire sample period 1972-2001, the base portfolio spans the test asset(s) considered. The last row of the table reports the test statistic and the associated *p-values* that are based on a simulation procedure described in Section 2.2 in the text. The conditioning variables are described in Table 1.

Year	Corp. Bonds	Small Cap	Comm & PM	Int'l Equity	Real Estate
1976	0.0912	0.1528	0.6013	0.2659	0.1529
1977	0.1193	0.2456	0.5139	0.2662	0.1024
1978	0.0875	0.1820	0.4609	0.2409	0.1223
1979	0.1612	0.2926	0.6699	0.2555	0.4134
1980	0.1633	0.2385	0.7415	0.1831	0.7323
1981	0.0811	0.0581	0.7994	0.1990	0.6875
1982	0.0485	0.1383	0.8773	0.3161	0.2910
1983	0.0837	0.0983	0.8781	0.1737	0.1768
1984	0.0748	0.0957	0.6244	0.1425	0.1284
1985	0.0291	0.0745	0.5878	0.4050	0.2651
1986	0.0418	0.0888	0.4172	0.7033	0.2199
1987	0.1053	0.0920	0.3598	0.4268	0.1470
1988	0.1205	0.0493	0.1837	0.2978	0.0896
1989	0.0592	0.1799	0.1809	0.3361	0.0736
1990	0.1128	0.2623	0.1708	0.1922	0.0641
1991	0.1043	0.2337	0.3170	0.1442	0.3834
1992	0.0473	0.2485	0.7462	0.0888	0.0663
1993	0.0410	0.2453	0.6735	0.1112	0.0718
1994	0.0375	0.2039	0.6471	0.1888	0.1161
1995	0.0777	0.1520	0.5323	0.1057	0.1104
1996	0.0872	0.0618	0.2960	0.0612	0.0840
1997	0.1085	0.0193	0.2602	0.2146	0.1359
1998	0.1803	0.0285	0.2573	0.1385	0.0959
1999	0.1601	0.0329	0.3293	0.1780	0.1607
2000	0.1165	0.0568	0.3285	0.1369	0.0671
2001	0.2210	0.1976	0.2786	0.1330	0.0409
Average	0.0985	0.1434	0.4897	0.2271	0.1923
$\chi^2$ statistic	93.04	120.09	134.81	172.36	170.05
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 3
Characteristics of Minimum Variance Portfolios

This table presents the annualized standard deviation estimates for the global minimum variance portfolios constructed from various assets for different sub-periods between 1972 and 2001. Portfolio weights are constrained to be non-negative when solving for the composition of the minimum variance portfolios. The first column contains the standard deviation estimates for the minimum variance portfolios that include a base set of assets. The base portfolio set consists of a medium capitalization and a large capitalization portfolio of NYSE/AMEX/NASDAQ stocks (obtained from CRSP files), and a government bond portfolio. Each successive column reports the standard deviation of minimum variance portfolios when the base set of assets is expanded to include the asset indicated at the top of that column. The assets are described in detail in Table 1.

Period	Annualized Standard Deviation								
renou	Base Portfolio	Corp. Bonds	Small Cap	Comm & PM	Int'l Equity	Real Estate			
1972-1976	0.0707	0.0705	0.0705	0.0532	0.0532	0.0531			
1977-1981	0.1178	0.1152	0.1152	0.0633	0.0633	0.0633			
1982-1986	0.1224	0.1050	0.1040	0.0562	0.0559	0.0538			
1987-1991	0.0955	0.0736	0.0730	0.0417	0.0414	0.0413			
1992-1996	0.0694	0.0610	0.0605	0.0348	0.0347	0.0333			
1997-2001	0.0691	0.0625	0.0625	0.0400	0.0395	0.0394			
1972-2001	0.0984	0.0909	0.0909	0.0562	0.0560	0.0556			

Table 4

Regressions of changes in conditional H-J bounds on the contemporaneous consumption growth and volatility

This table shows parameter estimates and corresponding t-statistics based on the Newey-West standard errors (in parenthesis) that are adjusted as described below, for three sets of regressions using data for the period 1972 to 2001. The coefficients are adjusted using the bias correction suggested by Stambuagh (1999) to account for the overlapinduced persistence in the variables. In calculating the t-statistics for the slope coefficients, the standard errors are corrected using the method suggested by Pagan (1984) and Murphy and Topel (1985), to account for the potential generated regressor problem. The dependent variable in each case represents the percentage increase in the conditional H-J bound upon adding to a base portfolio, a portfolio of corporate bonds (Panel A), a portfolio of small cap stocks (Panel B), a portfolio of commodities and precious metals (Panel C), a portfolio of international stocks (Panel D), or a portfolio of equity REITs (Panel E). The conditional H-J bounds are computed in the presence of short sales constraints. The base portfolio consists of large and medium capitalization stocks and government bonds in Panel A; medium and large capitalization stocks, government bonds and corporate bonds in Panel B; small, medium and large capitalization stocks, government bonds, and corporate bonds in Panel C; and small, medium and large capitalization stocks, government bonds, corporate bonds, commodities and precious metals in Panel D. In Panel E, the base portfolio consists of small, medium, and large capitalization stocks, government and corporate bonds, commodities and precious metals and international stocks. The increase in the conditional H-J bound is computed for rolling 60-month periods. Each panel contains three regression models. In Model I the independent variable is the standard deviation of the monthly real per capita consumption growth rate (continuously compounded) of non-durable goods & services (NDSDEV) during the corresponding 60-month period. In Model II the independent variable is the average growth rate of real per capita consumption (NDSGR) during the corresponding 60-month period. Model III contains both NDSDEV and NDSGR as the independent variables.

Model/Variable	Intercept	NDSGR	NDSDEV	Adj R <sup>2</sup>
I	0.529		-95.984	0.1675
	(5.52)		(-3.24)	
II	0.022	97.615		0.1162
	(0.60)	(3.05)		
III	0.363	87.225	-85.573	0.2521
	(3.53)	(3.61)	(-3.29)	
Panel B: Small cap stoci	ks added to the portfol	io in Panel A		
Model/Variable	Intercept	NDSGR	NDSDEV	Adj R <sup>2</sup>
I	0.069		39.819	0.0279
	(0.38)		(0.81)	
II	0.217	-4.997		-0.0029
	(2.72)	(-0.11)		
III	0.068	2.569	40.740	0.0247
	(0.30)	(0.05)	(0.80)	

Model/Variable	Intercept	NDSGR	NDSDEV	Adj R <sup>2</sup>
I	-0.351		259.440	0.3046
	(-1.60)		(3.00)	
II	0.974	-249.785		0.1943
	(5.23)	(-2.21)		
III	0.067	-212.956	233.512	0.4444
	(0.31)	(-2.46)	(2.81)	
Panel D: International equ	ities added to the port	folio in Panel C		
Model/Variable	Intercept	NDSGR	NDSDEV	Adj R <sup>2</sup>
I	-0.012		52.767	0. 0440
	(-0.11)		(1.34)	
П	-0.095	178.418		0.3389
	(-1.33)	(2.17)		
III	-0.389	187.964	74.871	0.4240
	(-2.12)	(2.02)	(2.00)	
Panel E: Real Estate addec	d to the Portfolio in Pa	nel D		
Model/Variable	Intercept	NDSGR	NDSDEV	Adj R <sup>2</sup>
I	-0.015		36.519	0.0377
	(-0.30)		(2.00)	
II	0.101	9.720		-0.0014
	(2.37)	(0.38)		
III	-0.045	15.717	38.613	0.0395
	(-0.54)	(0.55)	(1.94)	

Table 5
Parameter Estimates from Regime Switching Model

This table presents parameter estimates, with the corresponding standard errors in parenthesis, from a 2-state Markov Regime Switching Model. Panel A presents the mean asset returns and consumption growth rate in each state while Panel B and C present the respective covariance matrices for states 1 and 2. The estimated covariance figures are multiplied by 100. The assets include small cap stocks (SMLRET), an equally weighted portfolio of medium and large cap stocks (STKRET), Government bonds (GBTRET), corporate bonds (CBTRET), international stocks (EAFE), commodities (CRB), precious metals (CRBPM), and equity REITs (EREITs). The labels 'USTYLD' and 'CONSGR' represent the yield on a one month US Treasury bill, and the real per capita consumption growth rate. To avoid clutter, standard errors for elements of the covariance matrices in panels B and C are reported only for the diagonal elements.

Panel A: Me	ean Asset Retur	ns and Consun	nption Growth	Rate						
	SMLRET	STKRET	GBTRET	CBTRET	CRB	CRBPM	EAFE	EREITS	USTYLD	CONSGR
State 1	0.0175	0.0136	0.0038	0.0046	-0.0017	-0.0015	0.0098	0.0055	0.0014	0.0018
	(0.0044)	(0.0032)	(0.0023)	(0.0020)	(0.0015)	(0.0037)	(0.0035)	(0.0024)	(0.0002)	(0.0003)
State 2	-0.0087	-0.0045	0.0026	0.0014	-0.0020	0.0118	-0.0009	0.0086	0.0002	0.0013
	(0.0063)	(0.0052)	(0.0025)	(0.0023)	(0.0033)	(0.0067)	(0.0047)	(0.0045)	(0.0004)	(0.0003)
Panel B: Co	variance Matri	x for State 1								
	SMLRET	STKRET	GBTRET	CBTRET	CRB	CRBPM	EAFE	<b>EREITS</b>	USTYLD	CONSGR
<b>SMLRET</b>	0.4111									
	(0.0706)									
STKRET	0.2520	0.2116								
		(0.0176)								
GBTRET	0.0690	0.0760	0.1131							
			(0.0128)							
CBTRET	0.0692	0.0712	0.0919	0.0866						
				(0.0014)						
CRB	0.0024	-0.0016	-0.0123	-0.0100	0.0440					
					(0.0092)					
CRBPM	0.0116	-0.0089	-0.0350	-0.0304	0.0472	0.2761				
						(0.0013)				
EAFE	0.1043	0.0922	0.0504	0.0430	0.0010	0.0208	0.2588			
							(0.0003)			
EREITS	0.1580	0.1112	0.0504	0.0461	0.0052	0.0154	0.0590	0.1207		
								(0.0047)		
USTYLD	0.0024	0.0025	0.0018	0.0011	-0.0005	0.0014	0.0036	0.0024	0.0007	
									(0.0008)	
CONSGR	0.0056	0.0038	0.0003	0.0003	0.0001	0.0035	0.0018	0.0033	0.0002	0.0015
										(0.0001)

Panel C: Co	variance Matri	ix for State 2								
	SMLRET	STKRET	GBTRET	CBTRET	CRB	CRBPM	EAFE	EREITS	USTYLD	CONSGR
SMLRET	0.4360									
	(0.0051)									
STKRET	0.3352	0.2940								
		(0.0060)								
GBTRET	-0.0392	-0.0163	0.0583							
			(0.0185)							
CBTRET	-0.0121	0.0044	0.0495	0.0501						
~~~				(0.0011)	0.44=4					
CRB	0.0264	0.0091	-0.0251	-0.0225	0.1171					
CDDDM	0.0277	0.0207	0.0500	0.0614	(0.0143)	0.4021				
CRBPM	0.0377	0.0287	-0.0599	-0.0614	0.0339	0.4931				
EAFE	0.2025	0.1955	-0.0050	0.0100	0.0069	(0.0007) 0.0863	0.2242			
EAFE	0.2023	0.1933	-0.0030	0.0100	0.0069	0.0803	(0.0014)			
EREITS	0.1964	0.1452	-0.0183	-0.0025	0.0074	-0.0015	0.0972	0.2266		
LICLITS	0.1704	0.1432	-0.0103	-0.0023	0.0074	-0.0013	0.0712	(0.0125)		
USTYLD	0.0017	0.0028	0.0026	0.0029	0.0009	-0.0095	0.0000	0.0020	0.0013	
021122	0.0017	0.0020	0.0020	0.002	0.000	0.000	0.0000	0.0020	(0.0023)	
CONSGR	0.0015	0.0019	0.0000	0.0007	0.0018	-0.0020	0.0021	0.0020	0.0003	0.0011
										(0.0002)
Panel D: Sta	ite Transition F	Probabilities a	nd Log Likeli	hood						,
	Transition P		· ·			Log l	Likelihood Val	lue		
р		q								
0.79	916	0.612	3				-16,343.88			
(0.03	350)	(0.059	7)							
Panel E: Op	timal Tangency	y Portfolio Con	mposition (%	)						
	SMLR	ET ST	KRET	GBTRET	CBTRET	CRB	CRB	PM	EAFE	EREITS
State 1	15.51	4	0.16	0.00	19.04	0.00	0.0	00	21.35	3.95
State 2	0.00		0.00	41.89	0.00	0.00	20.3	3	0.00	37.78
Panel F: Min	nimum Varianc	ce Portfolio Co	mposition (%	(ó)						
	SMLR		KRET	GBTRET	CBTRET	CRB	CRB		EAFE	EREITS
State 1	0.00		0.00	0.00	40.00	48.04	5.2		1.74	4.97
State 2	0.00	(	0.00	6.15	44.83	33.39	8.3	0	0.00	7.34

Figure 1A: Mean-Standard Deviation Frontier Conditional on the Good State

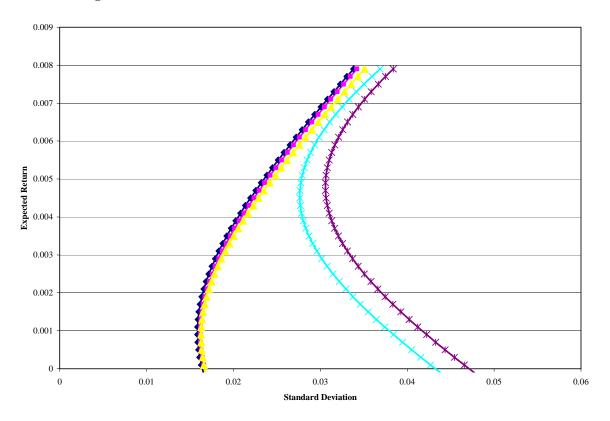


Figure 1B: Mean-Standard Deviation Frontier Conditional on the Bad State

