Contracting in Delegated Portfolio Management: The Case of Alternative Assets

WEI LI[a] and ASHISH TIWARI[b]

Abstract

This study explores the implications of portfolio opacity for the design of asset management contracts. This is a case of particular interest given the growing importance of ‘alternative’ assets like hedge funds and private equity funds in institutional portfolios. The alternative assets are often characterized by relatively illiquid or non-publicly traded holdings, proprietary strategies, and exemption from mandatory reporting requirements. These features contribute to the relative opacity or lack of transparency of the underlying portfolios. We analyze the link between portfolio opacity and the optimal portfolio management contract in this context and demonstrate that the second-best optimal contract features a convex component. The importance of the convex component is an increasing function of the portfolio’s opacity. Furthermore, the principal’s utility loss from restricting the weight of the convex component to zero is increasing in the portfolio’s opacity.

Keywords: *Alternative assets, Delegated portfolio management, Optimal contract, Portfolio opacity, Hedge funds, Private equity funds*

---

[a] Department of Finance, Henry B. Tippie College of Business, The University of Iowa, Iowa City, IA 52242-1000. Phone: (319) 335-0911; Fax: (319) 335-3690; Email: wei-li@uiowa.edu.

[b] Department of Finance, Henry B. Tippie College of Business, The University of Iowa, Iowa City, IA 52242-1000. Phone: (319) 353-2185; Fax: (319) 335-3690; Email: ashish-tiwari@uiowa.edu.

We thank Juan Pedro Gomez, Konrad Raff, and the seminar participants at the 2012 European Finance Association Meetings (Copenhagen), and the University of Iowa for their helpful comments.

Contracting in Delegated Portfolio Management: The Case of Alternative Assets

Alternative assets including hedge funds and private equity funds have attracted increasing interest from institutional investors in recent years. In contrast to traditional investments such as mutual funds, the 'alternative' investments are often characterized by relatively illiquid or non-publicly traded portfolio holdings.[1] Furthermore, they engage in proprietary trading and investment strategies and are generally exempt from the type of mandatory reporting requirements that traditional investment funds are subject to. These characteristic features in turn, contribute to the relative opacity or lack of transparency of the alternative portfolios. This makes the task of performance evaluation/benchmarking considerably harder for alternative asset investors. A question of interest in this context is: "How does the opaque nature of the underlying portfolio impact the portfolio management contract in the case of alternative assets." The goal of this paper is to provide some answers to the question by analyzing the link between portfolio opacity and the optimal portfolio management contract.

Specifically, the paper seeks to examine how the form of the optimal portfolio management contract is influenced by the opacity of the underlying portfolio, in a sense to be made precise below. The opacity of the alternative asset portfolios makes it considerably harder for an investor to benchmark their performance, making the principal-agent problem harder to resolve. With an increase in the difficulty of benchmarking performance, a more flexible form of contract becomes desirable.

We consider a prototypical delegated portfolio management problem in which the agent (the portfolio manager) manages a portfolio that is invested in a single risky asset and a risk free asset. As discussed above, a particular setting of interest is the case when the portfolio manager invests in illiquid or non-public assets or is engaged in proprietary strategies that are not subject to disclosure, making the portfolio relatively 'opaque'. To examine how the asset opacity impacts the form of the contract, we model the degree of transparency or opacity of a portfolio of assets via the correlation of the portfolio's returns with the appropriate publicly observed benchmark.[2]

[1] Current SEC guidelines allow a mutual fund to invest up to a maximum of 15% of its assets in illiquid securities. Many funds voluntary adopt even lower limits on such investments, and most mutual funds avoid them entirely.
[2] For example, in the extreme case, an equity index fund is likely to have a correlation close to one with respect to say, the S&P 500 index.

The case when the investor's information about the returns of the underlying assets in the portfolio is less than perfect is modeled by treating the correlation between the risky asset and the benchmark as being strictly less than one. We show that in this case the agent's allocation to the actively managed risky asset is less than in the first-best case. In other words, the presence of a noisy benchmark, which is itself a byproduct of the characteristics of the underlying assets, leads to effort shirking by the portfolio manager relative to the first-best outcome.

Our analysis suggests two major results. One, the second-best optimal contract in this setting features a convex component in addition to a component that is linear in performance. Two, the relative importance of the convex component is an increasing function of the asset's opacity. In particular, the weight of the convex component in the optimal second-best contract increases with asset opacity. Furthermore, the principal's utility loss from restricting the weight of the convex component to zero is increasing in the asset's opacity.

To understand the intuition behind the above result, note that the key effect of the convex component of the contract is to dampen the performance sensitivity of the manager's compensation when the portfolio underperforms the benchmark. Consequently, in the case of opaque assets, the underperformance penalty on the manager in the form of lower compensation is less severe. Intuitively, as information about the manager's action becomes less reliable (due to portfolio opacity), it becomes more desirable to moderate the underperformance penalty imposed on the manager while still providing the appropriate outperformance incentive.

To highlight the important effect of portfolio opacity, we contrast our main model against the case without opacity, i.e., the case in which the underlying portfolio is transparent. We show that in the latter case, the moral hazard problem is resolved trivially and the first-best solution is achieved. This comparison helps illustrate how portfolio opacity drives a wedge between the first-best solution and the second-best solution, and is therefore the source of the welfare loss due to the moral hazard problem.

Holmström and Tirole (1993) emphasize the importance of the liquidity of a firm's stock for structuring managerial incentives within the firm. However, to our knowledge, our study is the first to explicitly examine the impact of portfolio opacity on the contracting solution in the delegated portfolio management context.[3] More broadly, our study is related to an extensive

[3]Examples of other studies that focus on fund transparency, or lack thereof, include Gervais and Strobl (2012) who analyze a model in which fund managers choose the transparency of their fund at inception to signal their

literature that explores the principal-agent problem in this particular setting. Following the pioneering work of Holmström and Milgrom (1987), a number of studies have demonstrated the optimality of linear contracts in the generic principal-agent setting. By contrast, Stoughton (1993) highlights an important shortcoming of linear contracts in the delegated portfolio management setting. Based on a similar intuition as Stoughton (1993), Admati and Pfleiderer (1997) highlight the limitations of benchmark-linked linear contracts.[4] Similarly, Starks (1987) finds that a symmetric contract does not eliminate the effort underinvestment problem in a setting where the appropriate risk sharing and effort incentives are both of concern. Stoughton (1993) and Bhattacharya and Pfleiderer (1985) explore quadratic contracts within a security-analyst context in which the portfolio manager simply reveals his information to the investor. However, as noted by Stoughton (1993), such contracts are not feasible in the more realistic delegated portfolio management setting.

In a recent paper Ou-Yang (2003) demonstrates the optimality of linear contracts in a particular form of the delegated management problem. On the other hand, Li and Tiwari (2009) adopt Stoughton's (1993) framework to show that an appropriately designed option-type bonus fee contract can in fact be used to improve efficiency and such a contract dominates all symmetric contracts. The present paper extends the literature along two dimensions. One, our study highlights the link between portfolio opacity and the contract form. This analysis helps explain the observed differences in the qualitative nature of the portfolio management contracts across different kinds of investment vehicles. Notably, it helps explain the existence of asymmetric option-type contracts in the alternative asset universe including hedge funds and private equity funds, without appealing to market frictions such as high entry costs and imperfect competition in the industry.[5, 6] Two, it helps clarify the conditions under which the first-best

---

investment skill. Easley, O'Hara, and Yang (2012) examine the impact of ambiguity regarding hedge fund trading strategies on market efficiency and aggregate welfare.

[4]Interestingly, Gómez and Sharma (2006) show that in the presence of short-selling constraints, linear performance-adjusted contracts do provide portfolio managers with the appropriate incentives.

[5] In related work, Das and Sundaram (2002) show that in a framework with differential managerial ability and imperfect competition in the market for managers/advisors, investor welfare is generally higher under a regime where only the option type "bonus" performance incentive fee is allowed in the contract relative to a regime where only the "fulcrum" fee is allowed. Studies that focus exclusively on the risk taking incentives include Carpenter (2000) and Grinblatt and Titman (1989) while Palomino and Prat (2003) explore a setting which abstracts from the risk sharing concern.

[6] While it is not the focus of the present paper, a number of studies have explored the economics of the commonly observed high-water mark contracts in hedge funds. These include Goetzmann, Ingersoll, and Ross (2003), Panageas and Westerfield (2009), and Lan, Wang, and Yang (2012).

outcome may be achieved in the delegated portfolio management setting. As an application, since our framework accommodates Ou-Yang's (2003) model as a special case, we are able to demonstrate that his result arises naturally due to the observability of asset returns in the model. Importantly, his result is *not* due to the dynamic nature of the model, as has often been interpreted in the literature. The rest of the paper is organized as follows. Section I contains our main theoretical results. We examine the contracting problem in the case where the manager's portfolio is invested in relatively opaque assets. The closed form solution for the second best contract is derived. The contract solution is compared with the case without opacity. Section II presents results from a numerical analysis that examines how the form of the optimal contract changes and how the principal's utility varies with a change in the underlying portfolio's opacity. Concluding remarks are presented in Section III.

I. The Case of Opaque Assets

I.A. Model Setup and the Second-best Solution

For simplicity, we use a single risky asset and a risk free asset to represent the manager's investment opportunity set. Further, we assume that the risky asset is 'opaque.' Specifically, we assume that instead of observing the return on the risky asset, the investor can only observe the return on a benchmark reference asset that has the same marginal return distribution, but is only imperfectly correlated with the risky asset. As described below, the degree of opacity of the manager's portfolio is captured by the correlation, ρ, between the risky asset and a publicly observed benchmark. Assets that are illiquid or privately held are likely to have a low correlation with external benchmarks. Conversely, assets that are publicly traded in liquid markets are likely to display a high correlation with the relevant benchmarks. By examining a range of values for the correlation we are able to address the wide range of investment options available to investors that are characterized by varying degrees of opacity.

The model analyzed here is a stationary one. A principal contracts with an agent to manage her wealth. We normalize the initial investment to $1. With this assumption, the return and the terminal value of the investment are identical. We assume that the rate of return on the risky asset follows the geometric Brownian motion, so that the asset payoff is never negative. At the beginning of the period, the manager decides the portfolio weights, A, which are held constant throughout the period. With the above assumption, the risky asset return has a log-normal distribution. The log rate of return for the portfolio, w, is:

$$w = A(r - r_f) + r_f \ , \quad (1)$$

where r is the log return of the risky asset and r_f is the log risk free rate of return. We denote the terminal value of the portfolio by W, and we have $W = e^w$. We assume that there is a benchmark asset, whose return is denoted by r_b, and this benchmark asset helps the investor track the performance of the risky asset. Specifically, we assume that r and r_b, are jointly normally distributed with the identical marginal distribution, $N(\mu, \sigma^2)$. We denote the correlation between r and r_b by ρ, where $0 < \rho \le 1$. Given this distributional assumption, the rate of return on the risky asset may be expressed as: $r = (1-\rho)\mu + \rho r_b + \sqrt{1-\rho^2}\sigma\varepsilon$, where the noise term, ε, is independent of r_b and has a standard Normal distribution.

The principal is assumed to be risk neutral while the agent is risk averse with the log utility function: $U_a(C) = \log(C)$, where C denotes the agent's compensation. The agent suffers a disutility, $V(A)$, of managing the portfolio. Overall, the agent's utility may be expressed as $U_a(C) - V(A)$, where the agent's compensation, C, is subject to the restriction $C \ge c$ to account for the limited liability feature. The constant c is set to be a small positive number.[7] The agent's reservation utility is assumed to be, $U_a(p)$, where the constant p is to be interpreted as the agent's opportunity cost of entering the contract with the principal.

The first-best portfolio allocation is determined by the following maximization problem:

$$A_{FB} = \arg\max_A R_f \exp\left(A(\mu - r_f) + \frac{1}{2}A^2\sigma^2 \right) - pe^{V(A)}, \quad (2)$$

We assume that the agent's cost function is of the form, $V(A) = \frac{1}{2}kA^2$. We note that such a function has the following properties: $V(0) = V'(0) = 0$, and with a sufficiently large choice of k, the objective function in (2) is strictly concave.

The question we focus on in this section is: what happens when the principal's information is less than perfect, i.e., when $\rho < 1$. We begin by noting that in this case the benchmark will not be perfect and the first-best solution will not be achieved. Below we examine the form of the second-best optimal contract. The contract is dependent on the observables. In particular, we

[7] To avoid the singularity of the log function at zero, we bound the compensation away from zero.

write the contract as $C(r_b, w)$, a function of the log return on the benchmark asset, r_b, and the log portfolio value, w. A key to the analysis is the question of how the probability distribution of the log portfolio return, w, depends on the portfolio allocation, A, and the observed benchmark log return, r_b. Note that, conditional on observing r_b, the distribution of the risky asset's return, r, is $N((1-\rho)\mu+\rho r_b, 1-\rho^2)$. We denote $\mu_b=(1-\rho)\mu+\rho r_b$. The conditional distribution of the log return of the portfolio, w, is given by $N(r_f + A(\mu_b - r_f), A^2(1-\rho^2)\sigma^2)$. The log p.d.f. of the terminal portfolio value, W, is given by:

$$\log f(W, r_b; A) = -\frac{1}{2}\log\left(2\pi(1-\rho^2)\sigma^2\right) - w - \log(A) - \frac{\left(w - r_f - A(\mu_b - r_f)\right)^2}{2A^2(1-\rho^2)\sigma^2}. \tag{3}$$

By the likelihood ratio principle (see Holmström (1979)), the solution of the optimal contract problem is characterized by the following set of equations that jointly solve for the compensation, C, portfolio allocation, A, and parameters, λ, and ϕ. First, the optimal contract should set the compensation C to satisfy the following equation:

$$\frac{U_p^{'}\left(W-C\right)}{U_a^{'}\left(C,A\right)} = \lambda + \phi \cdot \frac{\partial}{\partial A}\left(\log f(W, r_b; A)\right). \tag{4}$$

Then, the shadow price of the incentive compatibility constraint, ϕ is the solution to the adjoint equation,

$$\begin{aligned} 0 = {} & E\left[\left(e^w - C(r_b, w; A)\right)\frac{\partial\left(\log f(W, r_b; A)\right)}{\partial A}\right] \\ & + \phi \cdot \left\{ E\left[\left(\left(\frac{\partial\left(\log f(W, r_b; A)\right)}{\partial A}\right)^2 + \frac{\partial^2\left(\log f(W, r_b; A)\right)}{\partial^2 A}\right)\log C(r_b, w; A)\right] - V''(A)\right\}, \end{aligned} \tag{5}$$

and the value of the parameter λ is the solution to the participation constraint:

$$E\left[\log\left(C(r_b, w; A)\right)\right] - V(A) = \log(p). \tag{6}$$

Finally, the portfolio allocation in the second-best solution is given by the first order condition of the agent's problem:

$$E\left[h_1 \log\left(C(r_b, w; A)\right)\right] - V'(A) = 0. \tag{7}$$

Note that the contract compensation, C, depends only on r_b, and w, and not directly on the manager's portfolio allocation, which is of course unobservable to the investor and therefore cannot be directly contracted on. However, through Equation (4), the portfolio allocation, A, in the second-best solution, does influence C as a parameter. Therefore, we use the notation $C(w, r_b; A)$ to track this dependence in the above equation system. After the portfolio allocation A is determined, its value is plugged in to solve for C. From Equation (4), we have that the optimal contract takes the form:

$$\begin{aligned} C(w, r_b) &= c + \left[\lambda + \phi \cdot \frac{\partial}{\partial A}\left(\log f(W, r_b; A^*)\right) - c \right]^+ \\ &= c + \left\{ \lambda + \phi \cdot \left[-\frac{1}{A^*} + \frac{\left(w - r_f - A^*(\mu_b - r_f)\right)(\mu_b - r_f)}{A^{*2}(1-\rho^2)\sigma^2} + \frac{\left(w - r_f - A^*(\mu_b - r_f)\right)^2}{A^{*3}(1-\rho^2)\sigma^2} \right] - c \right\}^+ . \end{aligned} \tag{8}$$

A notable feature of the above contract is that the manager's compensation is based on a comparison of the portfolio's performance, w, against the benchmark: $r_f + A^*(\mu_b - r_f)$. Here, A^* is the portfolio allocation in the second-best solution, and it does not vary with the manager's off-equilibrium portfolio allocation.

For ease of interpretation, it is helpful to rewrite the contract in (8) in the following form:

$$C(w, r_b) = F + \beta\left(w - r_f - A^*(\mu_b - r_f)\right)(\mu_b - r_f) + \gamma\left(w - r_f - A^*(\mu_b - r_f)\right)^2 \tag{9}$$

with the constraint that the manager's compensation is no less that a predetermined small number, c. Furthermore, $\beta / \gamma = A^*$. Of course, by definition, the second best contract in (8) is also the best contract among all contracts that take the form in Equation (9). Therefore, we can identify the second best contract by choosing the parameters F, β and γ in Equation (9) to maximize the principal's objective function while being subject to the manager's participation and incentive compatibility constraints. It is clear from the form of the contract in Equation (9) that the second-best contract is convex in the portfolio performance, w. We can interpret the first term, F, in the above contract as the fixed salary component. The second component can be interpreted as a linear component, and the third component as a convex component. We note that the linear component is linear in the portfolio's performance, w. However, the component is in the form of the interaction between the portfolio performance and the performance of the benchmark. Therefore, the pay-performance sensitivity in this case varies with the benchmark performance.

Proposition 1. *When the principal's information is not perfect (i.e.,* $\rho < 1$*), the shadow price of the incentive compatibility constraint is positive. That is,* $\phi > 0$ *. The portfolio allocation is lower than the first-best allocation, i.e.,* $A^* < A_{FB}$ *.*

Proof. See Appendix II.

The above proposition shows that due to imperfect information (i.e., when ρ is strictly less than 1), the manager's allocation to the actively managed portfolio will be less than that in the first-best solution, which is a form of effort shirking in our setting. Hence, the first-best outcome is achieved only in the special case with perfect observability of the risky asset's return, i.e., when $\rho = 1$. This case is discussed in more detail in Section I.B.

It is worth noting the role of the limited liability assumption in the agent's compensation. Given that the agent has log utility, the limited liability constraint on the agent's part will always be satisfied endogenously in equilibrium in any contractual agreement, because the agent's utility would otherwise be negative infinity.

In the case of opaque assets, the simple limited liability assumption will in general not be sufficient. To see this, consider for example a benchmarked contract under which due to the limited liability assumption, the manager will receive zero payment if his performance, measured by the log return of the portfolio, is below the benchmark by an amount, X. Given that there is the noise term, ε, in the manager's active portfolio that the investor cannot benchmark on, for any positive investment in the active portfolio, compared to any feasible benchmark, there is a positive probability that the manager's performance will be lower than the benchmark by X. With log utility, such an outcome implies that the manager's realized utility will be negative infinity. This, in turn, means that the contractual relation cannot be established due to the manager's participation constraint. The only exception is the case where the manager invests only in the risk-free asset and the benchmark is also based only on the risk-free rate. However, in this case, there is no benefit for the investor to delegate the investment decision to the manager. We face such a technical complication if we set c equal to zero in the contract specified in Equation (8). In order to bypass such a technical complication, we need something stronger than the bare minimum limited liability assumption. Accordingly, we choose to bound the agent's compensation, C, away from zero by imposing the condition, $C > c$.

I.B. The Case without Opacity and the First-best Solution

It is worth pointing out that if the effect of opacity is removed, the moral hazard problem will be completely resolved. That is, when $\rho = 1$, there is a contract that will solicit the first-best behavior from the manager and achieve the first-best outcome for the principal and the agent. Such a result provides a natural benchmark to better understand our main result in Section I.A. Specifically, comparing the case with $\rho < 1$, where there is a real cost due to the moral hazard problem, as shown in Proposition 1, with the case where $\rho = 1$, allows us to highlight the key driver of the moral hazard problem in our setting, namely the opacity of the managed portfolio.

Intuitively, when $\rho = 1$, we have, $r_b = r$, and thus the investor can in fact observe the risky asset return, r. Based on Equation (1), the investor can then infer the manager's portfolio allocation weight, A. Therefore, the manager's "hidden action" is no longer hidden, and thus the first-best solution is achieved. We note that the ability for the investor to infer the manager's allocation, A, from the log of the portfolio wealth (w) and the asset returns (r and r_f) relies on the assumption that there is only one risky asset. Interestingly, however, the conclusion that the first-best solution is achieved when the asset returns are observed does not depend on such an assumption.

It is worthwhile to demonstrate such a result in a more general case when there are multiple risky assets and thus both the asset return, r, and the manager's allocation choice, A are vectors, and the cost function is $V(A) = \frac{1}{2} kA^T A$. With the same condition as noted in the single risky asset case above, the objective function for the first-best solution is concave. There is a unique first-best solution. We denote the agent's action in the first-best solution by A^*, the portfolio's total value by W^*, and the principal's payoff by W_p^*. Therefore, the agent's payoff in the first-best solution is $W_A^* = W^* - W_p^*$. We denote the agent's compensation contract by S. We have the following proposition.

Proposition 2. *The first-best solution is achieved by the following compensation contract:*

$$S^* = W - W_P^*, \tag{10}$$

Proof. The proof is trivial. However, we include it in the detailed derivation in Appendix II.

Since the first-best outcome is achieved, we have, $S^* = W_A^*$, but only in equilibrium. However, note the crucial difference between S^* and W_A^*, where S^* contains the realized portfolio value W which directly depends on the agent's action, A, while W_A^* contains the term W^*. As a result, if the manager considers an off equilibrium allocation, A (i.e., A is different from A^*), the agent's payoff under the first-best outcome, W_A^*, is by definition, unaffected by the manager's allocation choice. However, the term S^* does vary with the allocation, A, through its dependence on W.

Implicit in the above result are two key assumptions: (a) the manager's portfolio allocation is not based on private information, and (b) the principal is able to observe the asset returns. These assumptions ensure that the principal can determine the appropriate first-best payoff, W_P^* , to be demanded from the agent as part of the portfolio management contract, which in turn ensures that the first-best portfolio allocation is realized. It is important to note that the proposition's assumptions still allow for the possibility that the manager's actions are unobserved by the investor. In general, the investor cannot infer the manager's portfolio allocation ex post. Further, the manager alone bears the private cost of his actions. Nevertheless, the moral hazard problem is completely resolved under the above assumptions. Intuitively, with perfect observability of asset returns, the investor can infer the terminal value of the (first-best) benchmark portfolio and therefore can make the agent to bear the full consequence of any deviation from this benchmark value. Also, note that the contract specified in (10) is linear in the final outcome, W. In particular, under the contract, the manager's compensation is determined by comparing the portfolio's performance, W, to a benchmark, W_P^*.

We next examine how the second-best outcomes converge to the first-best as the correlation between the risky asset and the reference benchmark asset approaches 1.To be comparable with the second-best contracts in Section II.A, we constrain the contract $C_{FB}(w, r_b)$ away from zero. That is, we consider the contract $C'_{FB}(w, r_b) = c + \left[W - R_f \exp\left(A_{FB}(\mu_b - r_f)\right) + \lambda - c\right]^+$. Note that, given this contract, the benefit to the manager from deviating from the first-best solution, is bounded by a function of the deviation of his portfolio allocation from the first-best allocation, $A - A_{FB}$. The cost to the manager of deviating from the first-best is the possibility of getting the minimum payment, c. For any $\varepsilon > 0$ and $|A - A_{FB}| > \varepsilon$, we have the cost increasing to infinity as c approaches zero uniformly. Therefore, as $c \to 0$, the agent's portfolio allocation choice when facing the above contract also approaches the first-best. In other words, when we choose c to be sufficiently small, the agent's portfolio choice is practically the same as the first-best solution. Given that the second-best solutions for the cases with $\rho < 1$ should weakly dominate the outcome from the contract, C'_{FB}, the second-best solutions will converge to the first-best solution as ρ approaches 1. We summarize the conclusion in the following corollary.

Corollary 1. *As $\rho \to 1$, and $c \to 0$, the second-best solution approaches the first-best.*

Remarks on the literature: While the intuition behind Proposition 2 is straightforward, it helps shed light on some of the recent results in the literature on delegated portfolio management. Ou-Yang (2003) derives a closed form solution of the optimal contracting problem in a continuous-time delegated portfolio management setting. His result is often interpreted in the literature as justification of the optimality of linear contract in the delegated portfolio management, and the tractability that leads to the closed form solution is often attributed to the continuous-time setup, an insight originally highlighted by the well-known Holmstrom and Milgrom (1987) study. However, we show below that Ou-Yang's (2003) main results can be quickly derived as corollaries to a fuller version of Proposition 2, where the intuition is conveyed in a more general setup so that Ou-Yang's model is strictly a special case. The full derivation is included in the Technical Appendix (Appendix I). As is clear from our derivation, the continuous-time setting in Ou-Yang (2003) plays no essential role in the key intuition. Consequently, the optimality of linear contract in this setting is not due to the intuition illuminated by Holmstrom and Milgrom (1987).

Note that in the setting examined by Ou-Yang (2003), all essential assumptions underlying Proposition 2 are satisfied. In particular, the first-best portfolio allocation is based on public information and the principal can observe the asset returns. While Ou-Yang does not present the first-best solution, we provide a complete solution in the Technical Appendix. In particular, the first-best solution of the optimal portfolio policy at time *t* in his setting, denoted as A_t, is given by

$$A_t^* = f(t)\left[k_t + \frac{R_a R_p}{R_a + R_p} f^2(t)\sigma\sigma^T\right]^{-1} h\,, \tag{11}$$

where $f(t)=(1-\frac{\gamma}{r})e^{r(T-t)}+\frac{\gamma}{r}$.[8] The investor's payoff is

$$W_P^* = -F + \frac{R_a}{R_a + R_p}\int_0^T f(t) A_t^{*T} diag(P_t)^{-1} dP_t, \tag{12}$$

where F is a constant. The optimal contract follows as a corollary to Proposition 2.[9] Combining Equation (10) and Equation (12), an optimal contract in this setting can be expressed as

$$S_T = W_T - W_P^* = F + \frac{R_p}{R_a + R_p} W_T + \frac{R_a}{R_a + R_p}\left[W_T - \int_0^T f(t) A_t^{*T}\mathrm{diag}(P_t)^{-1} dP_t\right], \tag{13}$$

which is exactly the same as Equation (10) in Theorem 1 in Ou-Yang (2003, page 185).

Ou-Yang further develops some other forms of the optimal contract in the case where the cost function c is constant. These results are contained in his Theorem 2 (p. 188). For these cases, consider the following compensation schedule for the agent:

$$S = S^* + \lambda\left[(A - A^*)'(R - R_f)\right], \tag{14}$$

where λ is a non-zero constant. It can be readily shown that all contracts in the form of (14) are optimal. The derivation of the above result is provided in Appendix I (See Proposition A.2 in Appendix I). By restricting the value of the parameter, λ in Equation (14) to the interval $(0, 1)$, we arrive at Theorem 2 in Ou-Yang. It is interesting to note, however, that there is no need to restrict the parameter λ in an interval as Ou-Yang did, as long as λ is not zero.

From the above discussion, it is evident that the optimal contract derived by Ou-Yang (2003) is in principle identical to the contract we derive in Proposition 2. Importantly, the moral hazard problem is completely resolved under these assumptions. As a key point of departure, in the framework adopted by Stoughton (1993) and Li and Tiwari (2009), the manager expends effort to collect private information that is correlated with the asset returns. In the Technical Appendix, we use Stoughton's framework to illustrate the point. Also, it is important to note that the insight of Holmström and Milgrom (1987) is based on a framework characterized by asymmetric

[8] Following Ou-Yang's notation, the terms k_t and γ are the coefficients in the cost function of managing the portfolio, R_a and R_p are the risk aversion coefficients for the agent and the principal, respectively, and h is the vector of the expected excess returns for the risky assets.

[9] While Proposition 2 is derived under some specific assumptions that are not directly applicable to the current situation, the same result can also be derived in a completely general setting (Proposition A1 in Appendix I) that indeed embeds Ou-Yang's setup as a specific case. The detailed derivation is provided in Appendix I.

information. In the continuous-time game analyzed by them, the agent incurs a cost and controls the mean (μ) of the output diffusion process (Z_t):

$$dZ_t = \mu_t dt + dB_t, \tag{15}$$

where B_t is a driftless N-dimensional vector standard Brownian motion. An important condition that leads to the optimality of a linear contract is the assumption that the principal has coarser information about the multi-dimensional Brownian process compared to the agent. This is in contrast to the requirement of Proposition 2, where a term in the contract specified by Equation (10) is a contingent payoff, W_P^*, which serves as the benchmark. It is important to note that W_P^* does not depend on the agent's *actual* actions while it can be written in the form of the agent's first-best actions. The only contactable benchmark payoff in the Holmström and Milgrom (1987) framework that does not depend on the agent's actions is a constant payoff. This is due to the fact that the principal in this framework does not observe anything beyond the output process, and this output is the result of the agent's *actual* actions. Clearly, such a constant payoff cannot be equal to W_P^* in the case in which the agent is risk averse. This sufficiently demonstrates that the contract in (10), while applying trivially to Ou-Yang (2013), does not apply to the Holmström and Milgrom setting.

In a recent study Edmans and Gabaix (2009) show that the Holmström and Milgrom result on the optimality of linear contracts can be achieved in settings that do not rely on assumptions such as exponential utility, continuous time framework, and Gaussian noise. It is natural to ask whether this generalization extends to the delegated portfolio management setting. A key aspect of the Edmans and Gabaix generalization is that it relies on a framework in which information is revealed to the agent *before* the agent's action is chosen. Furthermore, while the model has a multiple period structure, the potential complexity from such a structure is minimized. This is achieved through two key assumptions. First, the cost function of the agent's action is assumed to take a pecuniary form. Therefore, the aggregation of the overall cost to the agent of his actions in multiple periods can be achieved by adding up the pecuniary cost incurred in each individual period. The second assumption is that at the end of each period, the realization of the outcome, jointly determined by nature and the agent's action, is publicly observed. By these assumptions, each period's game is sufficiently independent, and the aggregation through multiple periods can be achieved rather mechanically. In stark contrast, as emphasized by Stoughton (1993), the interesting feedback effect of actions in different stage of the game in delegated portfolio management is what makes the contracting problem particularly challenging. More specifically, the agent in such an environment undertakes costly effort *prior* to the realization of a noisy signal related to future asset payoffs, and he is then required to decide on the asset allocation under imperfect information. Hence, the limitations of the Homstrom-Milgrom framework in the context of delegated portfolio management, first highlighted by Stoughton (1993), are still valid.

II. Numerical Analysis

How do the parameters in the optimal contract, the manager's portfolio allocation, and the principal's utility change with a change in the correlation between the risky asset and the benchmark asset, ρ? In particular, how do the contract parameters, the portfolio allocation and the principal's utility change as ρ declines substantially below 1, i.e., as the manager's portfolio progressively becomes more opaque? We rely on numerical analyses to address these questions. To calibrate the model, we assume that the asset return R in the model has the same statistical characteristics as the broad market index. Using the monthly U.S. T-bill and value-weighted market index returns for the period 1963:01-2011:09, we get the following annualized statistics: average risk free rate of 5.19%, average market excess return equal to 5.186%, and market volatility of 15.6%.[10] We further assume that the agent's reservation utility is equal to 2% of the initial assets under management. The cost function parameter, k, is set equal to 1.

For the purpose of comparison, we study the outcomes under the following three contracts as the correlation coefficient, ρ, varies between 0 and 1: (a) the second-best optimal contract, (b) the optimal linear contract, and (c) the practical incentive contract with an option-like bonus fee. The last contract is a contract that is similar to the kind of incentive contract observed in practice. The second-best optimal contract is as described in (9).

We impose the constraint, as in our analytical study, that the manager's compensation is no less that a predetermined small number, c, which is set equal to 0.0001. Such a constraint is imposed in all three contracts. To solve for the second best contract, we let γ in (9) be the free parameter to be determined through optimization. For any given γ, the parameter F is determined by the manager's participation constraint, and the parameter β is determined by

[10] As a robustness check we also calibrated the model using the market statistics for the following sub-periods: (a) 1926:07 – 2012:03; (b) 1995:01 – 2012:03; (c) 2000:01 – 2012:03; (d) 1963:01 – 1987:12; and (e) 1988:01 – 2012:03. In each case the results are qualitatively similar to the results for our base case presented here.

$\beta = \gamma \cdot A^*$, where A^* is solution of the manager's incentive compatibility constraint for a given contract. We iterate on the solutions for A^* and β till they converge, by initially setting A^* to be at the first-best solution. Finally, we optimize the principal's objective function by choosing the parameter, γ. This leads to the final identification of the second best contract.

The linear contract we consider is a contract that relates the manager's compensation to the portfolio performance in a linear fashion. That is,

$$C(w, r_b) = F + \beta(w - w_b)\,, \tag{16}$$

where F is the fixed salary, β is the pay-performance sensitivity, and $w_b = A_b(r_b - r_f) + r_f$. We set A_b at the equilibrium allocation. As before, for any given β, the parameter F is determined by the manager's participation constraint. We then iterate on the solution for A from the manager's incentive compatibility constraint, and set A_b equal to the equilibrium allocation in the benchmark, till they converge. We then optimize the principal's objective function by choosing β. This leads to the identification of the optimal linear contract.

Finally, for the practical incentive contract, we take the above linear contract as the starting point and add a component that resembles the usual option-type bonus fee:

$$C(w, r_b) = F + \beta(w - w_b) + \gamma(w - w_b)^+. \tag{17}$$

For any given pair of (β, γ), the parameter F is determined by the manager's participation constraint. We iterate, as in the case of linear contract, the solution A from the manager's incentive compatibility constraint and set it to be A_b in the benchmark, till they converge. Finally, we optimize the principal's objective function by choosing the pair (β, γ). This leads to the identification of the optimal practical incentive contract.

The results of the numerical analysis are reported in Table 1. In each panel of the table, the first column lists the value of the correlation between the benchmark and the risky asset return, while the second column reports the principal's utility in excess of the initial investment and as a percentage of the initial investment. The third column shows the corresponding asset allocation to the actively managed asset induced by the contract. In the following two (Panel B) or three (Panels A and C) columns, the contract parameters (F, β, γ) are listed under the title "salary," "linear", and "quadratic" or "option". The last two columns of Panel A report the relative weights of the two contract components: linear versus quadratic. The relative weights are based on the relative variation in the linear and quadratic component of the manager's compensation as the performance of the actively managed asset and the benchmark asset varies across the joint distribution of the risky asset return and the benchmark return. For easy comparison, the last row of each panel lists the first-best outcomes in terms of the portfolio allocation and the principal's utility.

Before reporting the results of the numerical analysis we note that, because we have the explicit probability density function by assumption, we can base all the numerical calculations on numerical integration rather than relying on simulations based on random draws. This is important because of the potential concern with respect to the left tail of the agent's payoff. Given that the manager has logarithmic utility and the lower bound on his compensation, c, is close to zero, the manager has to suffer a very large negative utility in the left tail of the payoff distribution. This raises the potential concern that with a random sampling approach, insufficient draws will result in substantial sampling error, rendering the outcome unreliable. However, this is not a problem with the numerical integration approach because the left tail, as the lower point in the integration interval, is always appropriately accounted for in the integration.

From Panel A, it is clearly seen that under the second-best contract, when the correlation coefficient, ρ , is less than one, there is an underinvestment in the actively managed risky asset. In the first-best outcome, the allocation to the risky asset is 146.77% with the resulting principal's utility at 8.75%. The induced allocation in the second-best case when $\rho = 0$ is 89.08%. The principal's utility in this case drops to 6.51%, i.e., the principal suffers a utility loss of more than twenty five percent as the benchmark asset's correlation drops from a perfect 1 to zero. The underinvestment problem is less severe as the correlation, ρ increases. Indeed, as ρ increases, both the allocation to the risky asset and the resulting principal's utility increase gradually. Furthermore, as ρ approaches 1, both the risky asset allocation and the principal's utility converge to their corresponding values in the first-best outcome.

The second-best contract needs to achieve the twin objectives of providing the appropriate effort incentive to the manager and achieving the appropriate risk sharing between the two parties. As ρ decreases, the performance of the benchmark reference asset is less informative about the manager's actions, i.e., his risky asset allocation. As a result, as ρ decreases, i.e., as the portfolio becomes increasingly opaque, the linear component of the contract becomes less effective, and the contract relies more on the convex component to motivate the manager to invest in the risky asset. Indeed, as seen from Panel A of the table, the relative weight of the convex component increases substantially as ρ decreases.

When comparing the linear contract outcomes (Panel B) with the second-best outcomes (Panel A), we can see that there is substantial utility loss for the principal when ρ is low. For instance, when $\rho = 0$, the utility decreases from 6.51% in the second-best case to 3.86% in the linear contract case – a utility loss of over 40%. The loss of utility in the case of linear contracts is largely due to the underinvestment in the risky asset. For instance, when $\rho = 0$, the allocation to the risky asset is 89.08% in the case of the second-best solution, while it is only 12.65% in the case of the linear contract. As ρ increases, the allocation to the risky asset in the linear contract case increases substantially with the resulting decline in the principal's utility loss. For example, when $\rho = 0.98$, the utility loss under the linear contract is less than 1% relative to the second-best case.[11] This result highlights the fact that as the underlying portfolio becomes less opaque the linear contract becomes more attractive. By Corollary 1, the principal's utility converges to first-best in both cases when c approaches zero.

We next consider the outcomes under the practical incentive contract (Panel C) which features the option-like component in addition to the linear contract component. We note that the underinvestment problem in the linear contract is alleviated to a certain degree by the inclusion of the option-type component in the practical incentive contract. In the case when $\rho = 0$, the portfolio allocation to the risky asset increases from 12.65% in the linear contract case to 27.04% in the case of the contract with the option-like component. The corresponding investor's utility increases from 3.86% in the linear contract case to 4.33% in the case of the practical incentive contract. The gain in the investor's utility is much less for higher values of ρ. Indeed, when ρ gets close to 1, the option-like component is no longer useful. In fact, as seen from Panel C of Table I, when $\rho = 0.90$, the option-like component coefficient is close to zero, and the coefficient becomes zero when $\rho = 0.95$ or when $\rho = 0.98$. This is an interesting result as it confirms that as the 'opacity' of the manager's portfolio declines, the option-like component is no longer needed to motivate the manager, and the linear contract component suffices.

[11] Some residual difference remains due to the constraint that all contract payments have to be larger than a positive constant, c.

Conversely, it is precisely in the case of non-traditional, relatively ‘opaque’ assets that the option-type component is a desirable contract feature.

III. Concluding Remarks

This paper analyzes the implications of portfolio opacity for the design of asset management contracts. This is an issue that is increasingly relevant given the growing interest in non-traditional investments such as hedge funds and private equity funds on the part of institutional investors. The ‘alternative’ investment portfolios are often invested in relatively illiquid or non-publicly traded assets while implementing proprietary strategies, and are generally exempt from public disclosure requirements. The relative opacity of the portfolios in these cases contributes to the benchmarks for such assets being relatively noisy. We examine how the optimal contract changes as the correlation between the managed portfolio and the reference benchmark portfolio declines. The analysis suggests that in the absence of perfect observability, i.e., when the correlation between the managed portfolio and the benchmark portfolio is less than 1, the first-best outcome is no longer feasible. The second-best optimal contract in this setting features a convex component in addition to a component that is linear in performance. Moreover, the relative importance of the convex component is an increasing function of the portfolio’s opacity. Further, the principal’s utility loss from restricting the weight of the convex component to zero is increasing in the asset’s opacity.

References

Admati, A. R., and P. Pfleiderer. 1997. Does It All Add Up? Benchmarks and the Compensation of Active Portfolio Managers. *Journal of Business* 70, 323-350.

Bhattacharya, S., and P. Pfleiderer. 1985. Delegated Portfolio Management. *Journal of Economic Theory* 36, 1-25.

Carpenter, J. N. 2000. Does Option Compensation Increase Managerial Risk Appetite? *Journal of Finance* 50, 2311-2331.

Easley, D., M. O'Hara, and L. Yang. 2012. Opaque Trading, Disclosure and Asset Prices: Implications for Hedge Fund Regulation. *Review of Financial Studies,* forthcoming.

Gervais, S., and G. Strobl. 2012. The Industrial Organization of Money Management. Working paper, Duke University.

Goetzmann, W.N., J. Ingersoll, and S.A. Ross. 2003. High-water marks and hedge fund management contracts. *Journal of Finance* 58, 1685-1717.

Das, S. R., and R. K. Sundaram. 2002. Fee Speech: Signaling, Risk-Sharing, and the Impact of Fee Structures on Investor Welfare. *Review of Financial Studies* 15, 1465-1497.

Edmans, A., and X. Gabaix. 2011. Tractability in Incentive Contracting. *Review of Financial Studies* 24, 2865-2894.

Gómez, J.P., and T. Sharma. 2006. Portfolio delegation under short-selling constraints. *Economic Theory* 28, 173-196.

Grinblatt, M., and S. Titman. 1989. Adverse Risk Incentives and the Design of Performance-Based Contracts. *Management Science* 35, 807-822.

Holmström, B., 1979, Moral hazard and Observability. *The Bell Journal of Economics,* 10, 74-91.

Holmström, B., and P. Milgrom. 1987. Aggregation and Linearity in the Provision of Intertemporal Incentives. *Econometrica* 55, 303-328.

Holmstrom, B., and J. Tirole. 1993. Market Liquidity and Performance Monitoring. *Journal of Political Economy*, 101, 678-709.

Lan, Yingcong, N. Wang, and J. Yang. 2013. The Economics of Hedge Funds. *Journal of Financial Economics* 110, 300-323.

Li, C. W., and A. Tiwari. 2009. Incentive Contracts in Delegated Portfolio Management. *Review of Financial Studies* 22, 4681-2714.

Massa, M., and R. Patgiri. 2009. Incentives and Mutual Fund Performance: Higher Performance or Just Higher Risk Taking? *Review of Financial Studies* 22, 1777-1815.

Ou-Yang, H. 2003. Optimal Contracts in a Continuous-Time Delegated Portfolio Management Problem. *Review of Financial Studies* 16, 173-208.

Palomino, F., and A. Prat. 2003. Risk Taking and Optimal Contracts for Money Managers. *RAND Journal of Economics* 34, 113-137.

Panageas, S., and M.M. Westerfield. 2009. High-water marks: High risk appetites? Concave compensation, long horizons, and portfolio choice. *Journal of Finance* 64, 1-36.

Shavell, S. 1979. Risk Sharing and Incentives in the Principal and Agent Relationship. *Bell Journal of Economics* 10, 55-73.

Starks, L. T. 1987. Performance Incentive Fees: An Agency Theoretic Approach. *Journal of Financial and Quantitative Analysis* 22, 17-32.

Stracca, L. 2006. Delegated Portfolio Management: A Survey of the Theoretical Literature. *Journal of Economic Surveys* 20(5), 823-848.

Stoughton, N. 1993. Moral Hazard and the Portfolio Management Problem. *Journal of Finance* 48, 2009-2028.

Table 1

The table documents the numerical results on the outcomes of using three types of contracts under different scenarios where the correlation, ρ , between the return on benchmark observed by the principal, and the return on the portfolio managed by the agent, varies from 0 to 0.98. The table reports, for each contract, the principal's expected utility net of the initial investment (second column) expressed as a percentage of the initial investment, the agent's allocation weight on the actively managed risky portfolio (third column), and the contract components. Panels A, B and C correspond to the second-best contract, the linear contract, and the practical incentive contract, respectively. The contracts are described in more detail in Section II. in the text. Columns 4, 5 and/or 6, under the subtitle "Contract", report the coefficients in the contract: coefficient F under "Salary", coefficient β under "Linear", and/or coefficient γ under "Quadratic" in Panel A and under "Option" in Panel C. To facilitate interpretation, in Panel A, we also report the relative weights of the two variable contract components. The relative weights are based on the relative (absolute) variation in the linear and quadratic component of the manager's compensation as the performance of the actively managed risky asset and the benchmark asset varies across the joint distribution of the risky asset return and the benchmark return. The relative weights are reported in Columns 7 and 8 under the subtitle "Relative Weights". The key inputs for the calibration are based on the following annualized U.S. market statistics for the period 1963:01-2011:09: average risk free rate of 5.19%, average market excess return equal to 5.186%, and market volatility equal to 15.6%.

Panel A. Second-best Contract

			Contract			Relative Weights	
ρ	Utility	Allocation	Salary	Linear	Quadratic	Linear	Quadratic
0	6.51%	89.08%	0.016	0.261	0.293	17%	83%
0.25	6.54%	90.42%	0.016	0.278	0.307	20%	80%
0.5	6.63%	92.23%	0.019	0.310	0.336	31%	69%
0.75	7.00%	104.02%	0.026	0.398	0.382	44%	56%
0.9	7.61%	119.50%	0.038	0.491	0.411	59%	41%
0.95	8.07%	127.26%	0.043	0.541	0.425	67%	33%
0.98	8.41%	127.63%	0.044	0.573	0.449	77%	23%
First-best	8.75%	146.77%					

Panel B. Linear Contract

ρ	Utility	Allocation	Contract: Salary	Contract: Linear
0	3.86%	12.65%	0.020	0.120
0.25	3.88%	13.09%	0.020	0.123
0.5	3.95%	15.07%	0.021	0.138
0.75	4.23%	23.21%	0.022	0.185
0.9	5.06%	44.94%	0.025	0.253
0.95	6.20%	83.99%	0.039	0.424
0.98	7.45%	128.36%	0.060	0.579
First-best	8.75%	146.77%		

Panel C. Practical Incentive Contract

ρ	Utility	Allocation	Contract: Salary	Contract: Linear	Contract: Option
0	4.33%	27.04%	0.019	0.120	0.165
0.25	4.33%	27.26%	0.019	0.123	0.173
0.5	4.33%	27.22%	0.019	0.138	0.175
0.75	4.42%	29.34%	0.020	0.186	0.201
0.9	5.06%	43.08%	0.024	0.253	0.097
0.95	6.20%	83.99%	0.039	0.424	0.000
0.98	7.45%	128.36%	0.060	0.579	0.000
First-best	8.75%	146.77%			

Appendix I （The technical appendix)

Derivation of the results of Section I under a general framework.

Below, we derive the results corresponding to those of Section I under a general framework. As will be demonstrated, our results do not rely on (a) the specific form of a utility function for the principal and the agent, (b) assumptions regarding how the agent's action affects the distribution of a project's payoff, and therefore the distributional assumptions regarding the state variables, and (c) the choice of a dynamic framework – namely, whether it is a continuous-time or a discrete-time model.

Consider an economy in which a principal hires an agent to carry out a business endeavor. The outcome of the endeavor is a wealth distribution along state space and time. We assume that the pair of a measure space and a sigma algebra, denoted by (Ω, F), captures all the dimensions and all the relevant distinctions in the state space and time within the model. To capture the idea that the agent's action contributes to the outcome, we denote it in functional form as $W(\cdot): \mathbf{A} \to \mathbf{D}$, where $\mathbf{A}$ is the space of all feasible actions by the agent, and $\mathbf{D}$ is the space of all possible wealth distributions along state space and time. That is, $\mathbf{D}$ consists of real valued F- measurable functions with domain on Ω, and a feasible action $A \in \mathbf{A}$ by the agent leads to a wealth distribution $W(A) \in \mathbf{D}$. We assume that the principal has a utility function, $U_P : \mathbf{D} \to R$. Such a general utility function ranks all feasible wealth distributions the principal potentially gets out of the joint venture.

The agent has a utility function over the compensation he receives and the action he chooses. Formally, $U_A : \mathbf{D} \times \mathbf{A} \to R$. In the principal-agent setting, the agent's action has to be voluntarily chosen. Therefore, an appropriately designed contract is needed to elicit the proper action from the agent. We view the contract from the agent's perspective. As the outcome of any action $A \in \mathbf{A}$ the agent chooses, there are the resulting distributions (on the measure space (Ω, F)) of the compensation, which we denote by $S(A) \in \mathbf{D}$. That is, similar to $W(\cdot)$, $S(\cdot)$ is a function from $\mathbf{A}$ to $\mathbf{D}$. We denote the space of all feasible compensation-based contracts by $\mathbf{S}$.[12] The generic contracting problem can be stated as the following optimization problem:

$$\sup_{S \in \mathbf{S}, A^e \in \mathbf{A}^e(S)} U_p\left(W(A^e) - S(A^e)\right), \tag{A.1}$$

[12] For a contract to provide motivation, it has to link the compensation with the agent's action. Given that the agent's action is not observable, this link is in general, not direct. For instance, S may be determined by A indirectly through $W(A)$. Therefore, depending on the ex post information the principal has, the feasible contract space $\mathbf{S}$ is in general a proper subset in the space of all functionals from $\mathbf{A}$ to $\mathbf{D}$.

$$s.t.\ \ \mathbf{A}^e(S) = \arg\max_{A\in\mathbf{A}} U_a(S(A), A)\,, \tag{A.2}$$

$$U_a(S(A^e), A^e) \geq \bar{U}_a\,. \tag{A.3}$$

In the above, $\bar{U}_a$ is the agent's reservation utility, and Inequality (A.3) is the agent's participation constraint. Here, we assume that the principal takes the entire surplus and that the agent's utility is driven to the reservation level. The additional constraint (A.2) is the agent's incentive compatibility constraint. The notion of $\arg\max$ in (A.2) is to be understood as the set of all solutions to the maximization problem. Therefore, $\mathbf{A}^e$ is a set-value function such that for any $S \in \mathbf{S}$, $\mathbf{A}^e(S) \subset \mathbf{A}$. As a special case, if $\mathbf{A}^e(S)$ is always reduced to a single point for all candidate contracts, S, then A^e is uniquely determined by S through constraint (A.2). In this case, the only control variable for the optimization problem will be S, the choice of the contract. In the way we state the problem in (A.1), we adopt the convention that among all actions for which the agent is indifferent, he will choose the one that is most beneficial to the principal. Such a convention becomes unnecessary if we adopt the assumption that the agent keeps the entire surplus and the principal's utility is driven to reservation level.

We assume that a first-best solution exists. That is, the following optimization problem has at least one solution.

$$\max_{A\in\mathbf{A}, S\in\mathbf{D}} U_p\left(W(A) - S\right), \tag{A.4}$$

$$s.t.\ \ U_a(S, A) \geq \bar{U}_a\,. \tag{A.5}$$

For easy reference, we denote the set of first-best solutions by $\mathbf{Q}_{FB}$:

$$\mathbf{Q}_{FB} \equiv \left\{ (A^*, S^*) \mid A^* \in \mathbf{A}, S^* \in \mathbf{D}, (A^*, S^*) \text{ solves the optimization problem in (A.1) and (A.2)} \right\} \tag{A.6}$$

For the following discussion, we will in general use the notations A^* and S^* for the resulting quantities in the first-best solution. The first-best solution is typically understood as the optimal solution that can be achieved when the agent's action can be ex post verified and therefore contracted on. Although this case is well understood in the literature, we make some remarks in order to facilitate our later comparison. Note that with a slight abuse of notation, S in (A.4) and (A.5) is a point in the space in $\mathbf{D}$, and it determines only one compensation distribution for the agent. In contrast, a contract $S(A)$ in (A.1)-(A.3) is a function from $\mathbf{A}$ to $\mathbf{D}$, and different actions by the agent can potentially lead to different compensation distributions for him. However, this inconsistency can be easily resolved following the general protocol of viewing a constant number as a constant function, by viewing the compensation distribution S (therefore, a point in $\mathbf{D}$) as equivalent to the constant function that maps any action

$A \in \mathbf{A}$ to the same compensation distribution S, with the understanding that there is an implicit dimension of the contract that threatens to punish the manager when his action deviates from the optimal action, A^*. This dimension of the contract is not explicitly stated in the problem, but it is viewed as trivial in intuition.

Comparing the optimization problems in (A.4)-(A.5) and (A.1)-(A.3), we note that the difference stems from the additional constraint, namely, the incentive compatibility constraint (A.2), for the contracting problem in (A.1)-(A.3). This constraint is the key factor that can lead to a potential moral hazard problem. The potential moral hazard leads to a loss in efficiency if and only if constraint (A.2) is binding. For our purpose, a particular set of contracts are of interest. They take the form:

$$S^{**}(A) = W(A) - (W(A^*) - S^*) \tag{A.7}$$

for some pair $(A^*, S^*) \in \mathbf{Q}_{FB}$. We denote the set of contracts taking the form in (A.7) by $\mathbf{S}^{**}$. That is,

$$\mathbf{S}^{**} = \left\{ S(\cdot) \mid S(\cdot) \in \mathbf{S}, \text{ with } S(A) = W(A) - (W(A^*) - S^*) \text{ for some pair } (A^*, S^*) \in \mathbf{Q}_{FB} \right\}. \tag{A.8}$$

It will become clear later that this set of contracts can be intuitively understood as being equivalent to the principal "selling" the project to the agent. The key question about set $\mathbf{S}^{**}$ is whether it is empty. In case it is not empty, we have the following result:

Proposition A.1. *If there exists a contract,* $S^{**} \in \mathbf{S}^{**}$*, then the incentive compatibility constraint in (A.2) is not binding and the first-best solution is achieved by this contract. In fact, for any* A^e *in the set of* $\mathbf{A}^e(S^{**})$, *the pair* $(A^e, S^{**}(A^e))$ *is a first-best solution. If we assume, in addition, that both* $U_p(\cdot)$ *and* $U_a(\cdot,\cdot)$ *are continuous and strictly increasing in wealth, we have that* $A^* \in \mathbf{A}^e(S^{**})$. *That is,* A^* *is an optimal response to contract* S^{**} *for the agent.*

Proof. Given the definition of $\mathbf{S}^{**}$, there exists $(A^*, S^*) \in \mathbf{Q}_{FB}$ such that

$$W(A) - S^{**}(A) = W(A^*) - S^*, \text{ for any } A. \tag{A.9}$$

That is, the distribution of the principal's payoff, $W(A) - S^{**}(A)$, does not depend on the agent's action A, and we have $U_p\left(W(A^e) - S(A^e)\right) = U_p(W^* - S^*)$, where the right-hand side is by definition the solution of the optimization in (A.4)-(A.5). The principal achieves the first-best outcome regardless of the agent's action. For the agent, because A^e is determined by (A.2), we have $U_a(S^{**}(A^e), A^e) \geq U_a(S^{**}(A^*), A^*) \geq \overline{U}_a$. In summary, the pair $(A^e, S(A^e))$ satisfies the constraints in (A.5) and achieves the maximum value of the objective function in (A.4). It thus qualifies as a solution for the optimization problem in (A.4)-(A.5). The proof of the last claim is also straightforward, but is omitted for the sake of briefness. Q.E.D.[13]

If the principal can ex post demand the payoff that always matches the first-best outcome, she can effectively "sell" the project to the agent in return for a "price" equal to $W^* - A^*$. In general, however, $W^* - A^*$ is a random variable. Indeed, when the agent is risk averse, the quantity $W^* - A^*$ should vary with the underlying states in such a way that it achieves the optimal risk sharing between the principal and the agent at the equilibrium. Nevertheless, this contingent payoff is in no way dependent on the agent's action, A, and is not directly related to the project's outcome, $W(A)$. Therefore, in this sense, the principal-agent relation is severed at the point when the contract is signed by both parties, and it is also in this sense that we may interpret it as the principal "selling" the project to the manager in return for a contingent payment of $W^* - A^*$ at the terminal date. By doing so, the agent bears all the consequences of his action, and therefore the incentive problem is completely addressed.

As is typical in the moral hazard literature, the manager's utility can be specified as a function of two variables, the wage the manager receives and a private cost due to his action. That is, the utility takes the form, $U_P(S(A), c(A))$, where $c(A)$ is the private cost incurred by action, A. We next analyze the case in which the cost of action to the manager is a constant – that is, $c(A) \equiv c_0$ for some constant, c_0. Under this assumption, the manager's utility is reduced to $U_P(S(A))$, which is dependent on his action A only through the payoff he receives. We denote the set of all feasible wealth distribution of the project by $W(\mathbf{A})$ – that is, $W(\mathbf{A}) \equiv \{W(A) \mid A \in \mathbf{A}\}$. We call the set $W(\mathbf{A})$ a manifold if for any $\lambda \in R$ and any two attainable outcomes, $W_1, W_2 \in W(\mathbf{A})$, the linear combination $W_3 = \lambda W_1 + (1-\lambda) W_2$ is an element in $W(\mathbf{A})$. We have the following proposition:

Proposition A.2. *If, in addition to the assumptions in Proposition A.1, we assume that $c(A) \equiv c_0$ for some constant, c_0, and the set $W(\mathbf{A})$ is a manifold, then all of the following contracts will achieve the first-best outcome and therefore are optimal contracts:*

$$S(A) = S^* + \lambda(W(A) - W(A^*)), \textit{ for any fixed } \lambda \in R, \textit{ and any } (A^*, S^*) \in \mathbf{Q}_{FB}. \qquad \text{(A.10)}$$

Proof. Fix the pair $(A^*, S^*) \in \mathbf{Q}_{FB}$. Note that when the manager chooses action, A^*, the payoffs for the manager and the investor exactly match the respective payoffs under the first-best solution. Therefore, we need only to show that, under the contract in (A.10), there is no action $A \in \mathbf{A}$ that the manager would

[13] As this proposition does not assume the uniqueness of the first-best solution, it makes no claim about whether $(A^e, S(A^e)) = (A^*, S^*)$.

prefer over A^*. Assume otherwise, that there exists $A^e \in \mathbf{A}$ such that $U_a(S(A^e)) > U_a(S(A^*))$. Because $W(\mathbf{A})$ is a manifold, $\lambda W(A^e) + (1-\lambda)W(A^*)$ can be implemented by a certain action of the manager, which we denote as A^λ. Notice that

$$S(A^e) = S^* + [\lambda W(A^e) + (1-\lambda)W(A^*)] - W(A^*) = S^* + W(A^\lambda) - W(A^*) = S^{**}(A^\lambda), \qquad \text{(A.11)}$$

and $S(A^*) = S^* = S^{**}(A^*)$. Therefore, $U_a(S^{**}(A^\lambda)) = U_a(S(A^e)) > U_a(S(A^*)) = U_a(S^{**}(A^*))$. This contradicts the conclusion in Proposition A.1 that A^* is the optimal strategy for the manager when facing the contract, $S^{**}(A)$. Q.E.D.

The results in Propositions A.1 and A.2 generalize the results in Section I of the text.

Derivation of the first-best solution in Ou-Yang's (2003) setting.

The fully specified model in Ou-Yang's paper includes the following assumptions for the price processes, the wealth process, the cost function, and the utility functions. The risk-free rate is constant and denoted by *r*. The price-process of the risky assets is described by the following geometric Brownian motion, $dP_t = \text{diag}[P_t](\mu dt + \sigma dB(t))$, where μ is a constant vector in R^N, σ is a constant matrix in $R^{N\times d}$ with linearly independent rows, and B is a $d \geq N$ dimensional standard Brownian motion. The wealth process $\{W_t\}$ for the portfolio strategy $\{A_t\}$ is then given by $dW_t = [rW_t + A_t^T h]dt + A_t^T \sigma\, dB_t$, where $h \equiv \mu - r \cdot \mathbf{1}$, and $\mathbf{1}$ denotes the unit vector, and A_t is the dollar amount invested in the risky asset at time *t*. The instantaneous cost function is specified as $c(t, A_t, W_t) = \frac{1}{2} A_t^T k(t) A_t + \gamma W_t$, where $k(t)$ is an $N \times N$ matrix, and γ is an constant. The agent's preference over wealth is described by $U_a(W) = -\frac{1}{R_a} e^{-R_a W}$, and the principal's utility is $U_p(W) = -\frac{1}{R_p} e^{-R_p W}$.

We state a more detailed version of the result regarding the first-best solution below:

Proposition A3. *The first-best solution of the optimal portfolio policy and the optimal payment to the agent are given by*

$$A_t^* = f(t)\left[k_t + \frac{R_a R_p}{R_a + R_p} f^2(t)\sigma\sigma^T\right]^{-1} h\,, \qquad \text{(A.12)}$$

and

$$S_T^* = \int_0^T \left(c(A_t^*, W_t^*, t) + \frac{1}{2} R_a g_t^{*T} g_t^* \right) dt + \int_0^T g_t^{*T} dB_t\,, \qquad \text{(A.13)}$$

where $f(t) = (1-\frac{\gamma}{r})e^{r(T-t)} + \frac{\gamma}{r}$, $g_t^* = \frac{R_p}{R_a + R_p} f(t)\sigma^T A_t^*$ *, and the process* $\{W_t^*\}$ *,* for $t \in [0,T]$ *is given by*

$$W_t^* = e^{rT}W_0 + \int_0^t \left(e^{r(T-s)} \cdot A_s^{*T} h\right) ds + \int_0^t \left(e^{r(T-s)} \cdot A_s^{*T} \sigma\right) dB_s . \tag{A.14}$$

We further have the investor's payoff in the first-best solution as

$$W_P^* = W_T^* - S_T^* = -F + \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \mathrm{diag}(P_t)^{-1} dP_t \ , \tag{A.15}$$

where F *is a constant.*

Proof: Consider the portfolio manager's expected utility,

$$x_t \equiv E_t\left[U_a\left(S_T - \int_0^T c(A_t, W_t, t)dt\right)\right] < 0 \ , \tag{A.16}$$

which is a martingale over the Brownian fields generated by B_t. A theorem of Meyer (see Jacod (1977)) holds that every martingale over the Brownian fields can be represented as an Ito stochastic integral with respect to the driftless Brownian motion: $dx_t = \theta_t^T dB_t$, where $\{\theta_t; t \geq 0\}$ is a d-dimensional adopted stochastic process and $\int_0^T \theta_t^T \theta_t dt$ is almost surely finite. Define the process $\{Z(t)\}$ as

$$Z(t) = U_a^{-1}(x_t) = -\frac{1}{R_a} \ln\left(-R_a x_t\right). \tag{A.17}$$

By Ito's lemma, we have,

$$Z(T) = Z(0) + \frac{1}{2} R_a \int_0^T g_t^T g_t dt + \int_0^T g_t^T dB_t \ , \tag{A.18}$$

where $g_t \equiv -\frac{\theta_t}{R_a x_t}$. We further have that $Z(T) = S_T - \int_0^T c(A_t, W_t, t)dt$, and from the participation constraint, $Z(0) = Z_0$. Hence, we get the following equivalent representation of the participation constraint:

$$\begin{aligned} S_T &= Z(T) + \int_0^T c(A_t, W_t, t)dt \\ &= Z_0 + \int_0^T c(A_t, W_t, t)dt + \frac{1}{2} R_a \int \ g_t^T g_t dt + \int_0^T g_t^T dB_t \ . \end{aligned} \tag{A.19}$$

With the participation constraint noted in Equation (A.19), we can state the first-best problem as follows:

$$\begin{aligned}
&\sup_{\{A_t\},\{g_t\}} E[U_p(N_T)] \\
&s.t. \\
&dP_t = P_t^d(\mu dt + \sigma dB_t); \\
&dW_t = (rW_t + A_t^T h)dt + A_t^T \sigma dB_t; \\
&dN_t = dW_t - c(A_t, W_t, t)dt - \frac{1}{2} R_a g_t^T g_t dt - g_t^T dB_t,
\end{aligned} \tag{A.20}$$

where $N_t \equiv W_t - S_t$. Given that A_t and g_t are adapted stochastic processes, $\{P_t; W_t; N_t\}$ are controlled Markov processes, with A_t and g_t being the controls. We define a value function process $V(t, W_t, N_t, P_t)$ for the above optimal control problem as

$$V(t, W_t, N_t, P_t) = \sup_{\{A_u\},\{g_u\}} E_t\left[U_p(N_T)\right]. \tag{A.21}$$

The Bellman equation for this dynamic programming problem is as follows:

$$0 = \sup_{A,g} A^{A,g} V, \tag{A.22}$$

where $A^{A,g}$ stands for the backward generating operator, i.e.,

$$\begin{aligned}
A^{A,g} V = {} & V_t + (V_W + V_N)(rW + A^T h) + V_N\left(-c(A, W, t) - \frac{1}{2} R_a g^T g\right) + V_P \mathrm{diag}(P_t)\mu \\
& + \frac{1}{2}(V_{WW} + V_{NN}) A^T \sigma\sigma^T A + \frac{1}{2} V_{NN} g^T g - V_{NN} A^T \sigma g + V_{WN}\left(A^T \sigma\sigma^T A - A^T \sigma g\right) \\
& + \frac{1}{2}\mathrm{trace}\left(V_{PP} \mathrm{diag}(P_t)\sigma\sigma^T \mathrm{diag}(P_t)\right) + (V_{WP}^T + V_{NP}^T)\mathrm{diag}(P_t)\sigma\sigma^T A \\
& - V_{NP}^T \mathrm{diag}(P_t)\sigma g.
\end{aligned} \tag{A.23}$$

The first-order conditions of the Bellman equation with respect to the control variables A and g are

$$\begin{aligned}
0 = {} & (V_W + V_N)h - V_N c_A + (V_{WW} + 2V_{WN} + V_{NN})\sigma\sigma^T A - (V_{WN} + V_{NN})\sigma g, \\
& + \sigma\sigma^T P^d (V_{WP}^T + V_{NP}^T)
\end{aligned} \tag{A.24}$$

and

$$0 = -V_N R_a g + V_{NN} g - (V_{NN} + V_{WN})\sigma^T A + \sigma^T P^d V_{NP}^T. \tag{A.25}$$

Conjecture that $V(t, W_t, N_t, P_t)$ takes the following form:

$$V(t, W_t, N_t, P_t) = -\frac{1}{R_p}\exp\left\{-R_p\left[f_1(t)W_t + f_2(t)N_t + f_3(t)\right]\right\}, \tag{A.26}$$

with the boundary conditions $f_1(T) = f_3(T) = 0$, and $f_2(T) = 1$. Then we have that $V_W = V \cdot (-R_p f_1)$, $V_N = V \cdot (-R_p f_2)$, $V_{WW} = V \cdot (R_p f_1)^2$, $V_{NN} = V \cdot (R_p f_2)^2$, $V_{WN} = V \cdot (R_p^2 f_1 f_2)$, and all the derivatives with respect to *P* are zeroes. Substituting all of these into the first-order conditions above, we get

$$g_t = \frac{R_p}{R_a + R_p f_2}(f_1 + f_2)\sigma^T A, \tag{A.27}$$

and

$$A_t = (f_1(t) + f_2(t))\left[f_2(t)k_t + \frac{R_a R_p}{R_a + f_2 R_p}\left(f_1(t) + f_2(t)\right)^2 \sigma\sigma^T \right]^{-1} h\,. \tag{A.28}$$

To determine the values of $f_1(t), f_2(t)$, and $f_3(t)$, we substitute *V* back into the Bellman equation:

$$\begin{aligned} 0 = {} & W_t\left[f_1' - (f_1 + f_2)r + f_2\gamma \right] + N_t f_2' + f_3' - (f_1 + f_2)A_t^T h + f_2\left[A_t^T k_t A_t + \frac{1}{2}R_a g_t^T g_t \right] \\ & + \frac{1}{2}R_p f_2^2 g_t^T g_t + \frac{1}{2}R_p (f_1 + f_2)^2 A_t^T \sigma\sigma^T A_t - R_p f_2 (f_1 + f_2) A_t^T \sigma g_t\,. \end{aligned} \tag{A.29}$$

Notice that both A_t and g_t do not depend on N_t or W_t. To eliminate the N_t term from the right-hand side of the above equation, we must have $f_2' = 0$ with the boundary condition, $f_2(T) = 1$. Therefore, $f_2(t) = 1$. To eliminate the W_t term, we must have

$$f_1' + f_1 r + r - \gamma = 0, \tag{A.30}$$

with the boundary condition $f_1(T) = 0$. Therefore,

$$f_1(t) = (1 - \frac{\gamma}{r})(e^{r(T-t)} - 1). \tag{A.31}$$

The function $f(t)$ in theorem 1 is given by $f(t) = f_1(t) + 1$. The Bellman equation is satisfied by setting

$$f_3(s) = -\int_s^T \left(\begin{aligned} & f_2\left[A_t^T k_t A_t + \frac{1}{2}R_a g_t^T g_t \right] - (f_1 + f_2)A_t^T h + \frac{1}{2}R_p f_2^2 g_t^T g_t \\ & + \frac{1}{2}R_p (f_1 + f_2)^2 A_t^T \sigma\sigma^T A_t - R_p f_2 (f_1 + f_2) A_t^T \sigma g_t \end{aligned} \right) dt. \tag{A.32}$$

The following computation is not necessary for our derivation of the first-best solution and for the derivation of the optimal contract. It is presented here solely for the purpose of fully replicating Equation (10) in Ou-Yang's Theorem 1. Given the portfolio strategy $\{A_t^*\}$, the wealth process is determined by the stochastic differential equation:

$$dW_t^* = (rW_t^* + A_t^{*T} h)dt + A_t^{*T} \sigma dB_t . \tag{A.33}$$

Using Ito's lemma, we can rewrite the equation as:

$$d\left(e^{r(T-t)} \cdot W_t^*\right) = \left(e^{r(T-t)} \cdot A_t^{*T} h\right)dt + \left(e^{r(T-t)} \cdot A_t^{*T} \sigma\right)dB_t . \tag{A.34}$$

Therefore,

$$W_t^* = e^{rT} W_0 + \int_0^t \left(e^{r(T-s)} \cdot A_s^{*T} h\right)ds + \int_0^t \left(e^{r(T-s)} \cdot A_s^{*T} \sigma\right)dB_s \text{ , for } t \in [0, T]. \tag{A.35}$$

Notice that

$$S_T^* = \int_0^T \left(c(A_t^*, W_t^*, t) + \frac{1}{2} R_a g_t^{*T} g_t^* \right)dt + \int_0^T g_t^{*T} dB_t . \tag{A.36}$$

We thus have

$$\begin{aligned}
& S_T^* - W_T^* - \left[Z_0 - e^{rT} W_0^* + \int_0^T \left(\frac{1}{2} A_t^{*T} k(t) A_t^* + \frac{1}{2} R_a g_t^{*T} g_t^* - e^{r(T-t)} A_t^{*T} h \right)dt \right] \\
&= \int_0^T \gamma W_t^* dt + \int_0^T \left(f(t) - e^{r(T-t)} \right) A_t^{*T} \sigma dB_t - \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \sigma dB_t \\
&= \frac{\gamma}{r} \left(\int_0^T r W_t^* dt + \int_0^T \left(1 - e^{r(T-t)}\right)(dW_t^* - rW_t^* dt) \right) - \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \sigma dB_t \qquad \text{(A.37)} \\
&= \frac{\gamma}{r} \int_0^T d\left((1 - e^{r(T-t)}) W_t^* \right) - \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \sigma dB_t \\
&= \frac{\gamma}{r} (1 - e^{rT}) W_0 + \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \mu dt - \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \operatorname{diag}(P_t)^{-1} dP_t .
\end{aligned}$$

We choose the constant, F, as in Ou-Yang (2003), to absorb all the constant terms in the above equation, and write the result in short as:

$$S_T^* - W_T^* = F - \frac{R_a}{R_a + R_p} \int_0^T f(t) A_t^{*T} \operatorname{diag}(P_t)^{-1} dP_t . \tag{A.38}$$

Hence, the proposition is proved.

Appendix II

Proof of Proposition 1. The proof is developed in two steps. First, if $\phi = 0$, the contract is the constant contract. Clearly, the agent will then choose $A = 0$, which is strictly less than the first-best case outcome. This contradicts the fact that the shadow price of the incentive compatibility constraint is zero (i.e., $\phi = 0$). Second, assume that $\phi < 0$. That is, the shadow price of the incentive compatibility constraint is negative. We thus have the agent's portfolio allocation $A^* > A_{FB}$. The contract form is specified in Expression (8). The first order derivative for the agent's objective function at the solution, A^*, is given by:

$$\phi E\left[\frac{(r-r_f)^2}{A^{*2}(1-\rho^2)\sigma^2 C(w,r_b)}\middle| C(w,r_b) > c\right] - V'(A^*) < 0 .$$

This contradicts the claim that $A^* > A_{FB}$, which is an interior solution. Hence, we conclude that $\phi > 0$.

Proof of Proposition 2. Under the assumptions of the proposition, the principal's payoff in the first-best scenario, W_P^*, depends on A^*, R, and R_f. In other words, W_P^* is based only on public information given the assumption about the first-best allocation, A^*. Therefore, the contract in (10) is feasible. It is clear that under the compensation scheme, S^*, described in Equation (10), the principal's payoff, W_P, is given by, $W_P = W - S^* = W_P^*$, which is in fact the first-best outcome. Therefore, the first-best outcome is achieved. The fact that the agent chooses the first-best portfolio allocation can be shown accordingly.

The result in Proposition 2 does not apply to the framework in Stoughton (1993).

We use Stoughton's framework to illustrate the point. The manager has access to a risk-free asset with gross return R_f and a risky asset with gross return, $\mu + R^e$, where μ is the mean and thus $E(R^e) = 0$. One of the key assumptions made by Stoughton is that the manager can observe a signal that is correlated with the true return:[14]

$$I = R^e + \varepsilon ,$$

where all variables are assumed to be jointly normal. Furthermore, $E(\varepsilon) = 0$, and $E(R\varepsilon) = 0$. The information precision is captured by, $\rho = \sigma_R^2 / \sigma_\varepsilon^2$, which is also identified as the manager's effort. The utility functions of the manager and the investor are assumed to be exponential:

$$U_a(W_A) = -\exp\{-aW_A + V(\rho)\},$$

[14] We adopt notation that differs from Stoughton (1993) in order to be consistent with the other part of this paper.

$$U_p(W_B) = -\exp\left(-bW_B\right),$$

where W_A and W_B are the end-of-period wealth of the agent and the principal, respectively, and $V(\rho)$ is the disutility of effort. At the first-best outcome, the payoff to the principal is the following function of R and I :

$$\begin{aligned} W_P^* &= \frac{1/b}{(1/a)+(1/b)} W^* - \Phi \\ &= \frac{1/b}{(1/a)+(1/b)} \left[W_0 R_f + \frac{a+b}{ab} \frac{\rho^*+1}{\sigma_R^2} \left(\mu + \frac{\rho^*}{\rho^*+1} I - R_f \right) (R - R_f) \right] - \Phi \ , \end{aligned}$$

where Φ is a constant such that the manager's participation constraint is satisfied. All the parameters and variables in the last row of the above expression are either known or observable at the end of the game to the investor and therefore, can in principle be contracted on, with the exception of the manager's signal, I. Due to this fact, the contract in Equation (10) is not feasible in Stoughton's (1993) framework. Based on a similar setting as Stoughton, Li and Tiwari (2009) show that all forms of symmetric contracts are suboptimal. They demonstrate that an appropriately designed option-type bonus fee contract can be used to improve efficiency and such a contract dominates all symmetric contracts in this setting.