# Investment Decisions under Ambiguity: Evidence from Mutual Fund Investor Behavior [§]

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We provide novel evidence on the role of ambiguity aversion in determining the response of mutual fund investors to fund performance. We present a model of ambiguity averse investors who receive multiple performance-based signals of uncertain precision about manager skill. A key implication of the model is that investors place a greater weight on the worst signal. We find strong empirical support for this prediction in the form of heightened sensitivity of investor fund flows to the worst performance measure across multiple horizons. This effect is particularly pronounced for retail funds in contrast to institutional funds.

*Keywords: Ambiguity aversion, Mutual fund performance, Investor behavior, Bayesian learning, Flow-performance sensitivity*

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# I. Introduction

Investors today operate in a world with increasing complexity requiring them to process large amounts of information while making decisions. However, information quality can often be difficult to judge for investors. As argued by Epstein and Schneider (2008), when faced by information signals of unknown quality, investors treat the signals as being ambiguous.[1] There is now considerable experimental evidence documenting that individuals are ambiguity averse (e.g., Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010), Ahn, Choi, Gale, and Karivl (2011)) in the sense that they prefer to bet on events with known, rather than unknown, objective probabilities. However, empirical evidence of ambiguity aversion in non-experimental settings is relatively scarce. In this study we provide novel evidence on this issue by examining the response of mutual fund investors to historical fund performance information.

Mutual funds offer an appealing setting in which to study the role of ambiguity on investor decisions for a number of reasons. First, mutual funds represent a very substantial component of U.S. household portfolios.[2] Second, the well-documented phenomenon of performance-chasing by fund investors suggests a natural link between performance-related information and the investment/divestment decisions of investors.[3] Third, funds typically make available performance statistics including relative rankings measured at various horizons (e.g., 1-year, 3-year, 5-year, and since-inception) which serve as multiple signals to investors about fund manager skill. These performance statistics are readily available in fund prospectuses and related marketing materials. Each of the multiple signals reflects the performance of a particular fund relative to the available pool of funds, albeit over different time horizons. In this sense, the signals are comparable and studying the response of investors to various signals allows for a natural test of decision making under ambiguity in a non-experimental setting. Furthermore, as we subsequently discuss, our framework allows us to distinguish the impact of ambiguity aversion from the implications of standard Bayesian learning under uncertainty and/or models that allow for behavioral biases such as loss aversion.

[1] The distinction between risk and ambiguity/uncertainty was highlighted by Knight (1921) who defined uncertain events as those for which the probability distribution of outcomes is unknown. According to Knight, "The practical difference between risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation a priori or from statistics of past experiences), while in the case of uncertainty this is not true, the reason being that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique." (p. 103).

[2] According to the 2012 Investment Company Institute Fact Book, 44% of all U.S. households owned mutual funds.

[3] See, for example, Ippolito (1992); Gruber (1996); Chevalier and Ellison (1997); Sirri and Tufano (1998).

It is reasonable to believe that individual investors are faced with considerable ambiguity when it comes to their fund investment decision. Investors face a dizzying array of choices. For example, according to the 2012 Investment Company Fact Book, there were more than 4,500 U.S. equity mutual funds in existence at the end of 2011. Investors are also subjected to a barrage of performance statistics on the funds. While these performance data provide signals of the fund manager skill, the investors clearly face a great deal of uncertainty about the quality of the signals. Past performance is at best a noisy signal of managerial skill. How do ambiguity averse investors interpret and respond to such signals? The goal of this study is to provide some answers to this question as a way to further our understanding of mutual fund investor behavior.

We present a simple model of ambiguity averse investors who receive multiple performance-based signals of uncertain precision about manager skill or fund alpha. The model relies on the framework of Epstein and Schneider (2008), Klibanoff, Marinacci, and Mukerji (2005), and Ju and Miao (2012). A key implication of the model is that when investors receive multiple signals of uncertain quality, they place a greater weight on the worst signal. In practical terms this implies that ambiguity averse investors are more sensitive to the worst-case scenario when evaluating funds. We find strong empirical support for this prediction in the data. Specifically, we examine the sensitivity of fund flows to past performance measured over multiple time horizons: 1 year, 3 years, and 5 years. We find that fund flows display significantly higher sensitivity to the worst performance measure even after controlling for performance volatility and a host of other relevant explanatory variables. This heightened sensitivity holds regardless of whether fund performance is measured by raw return or the Carhart 4-factor alpha. This effect is particularly pronounced in the case of retail funds whose investors are likely to face a higher degree of uncertainty regarding the quality of performance-related signals they observe, compared to the institutional fund investors.[4]

We use a number of fund characteristics as proxies for the degree of ambiguity about fund performance/manager skill. These include the fund's investment strategy shifts, return volatility, fund flow volatility, and family size. We consistently find that in cases with higher degree of ambiguity as captured by our proxy measures, fund flows display significantly higher sensitivity to

[4] Instead of using performance measured at different time horizons as signals, in unreported tests we also consider various performance measures (including the CAPM alpha, Fama-French 3-factor alpha, Carhart 4-factor alpha, raw return, and Morningstar fund rating) over the identical time horizon (1-, 3-, or 5-year) as performance signals and obtain qualitatively similar results. These results are available upon request.

the worst performance measure. Our results are robust to the use of additional controls including the convexity in the flow-performance relation, Morningstar fund ratings, and fund performance since inception.

We next examine the implications of the potential differences in the degree of ambiguity faced by retail and institutional investors. The latter are typically viewed as being relatively sophisticated investors with a better understanding of the fund industry and who are therefore better able to interpret performance-related signals. Consequently, such investors face less ambiguity and may update their beliefs after observing the signals in a manner consistent with Bayes' rules. In a recent paper, Huang, Wei, and Yan (2012) show that investor fund flow sensitivity to past performance is decreasing with fund return volatility. Our analysis confirms their findings. An increase in fund return volatility implies that past performance is a less precise signal of skill or ability and hence investors rationally moderate their response to the signal, consistent with Bayesian learning. The dampening effect of return volatility on flow-performance sensitivity is more pronounced for institutional investors, which is consistent with the notion of such investors being more sophisticated.

While high performance volatility is naturally associated with low signal precision, high volatility could also imply a high degree of ambiguity surrounding the signal for ambiguity averse agents. Hence, the impact of volatility on the flow-performance sensitivity is likely to reflect not only the impact of signal precision, but also the ambiguity aversion of the investors.

Interestingly, we find that an increase in performance volatility, while dampening the flow-performance sensitivity, also leads to an increase in flow sensitivity to the worst performance signal at the margin for both groups of investors, although the increase is more pronounced for retail investors. This suggests that investor behavior is best characterized as reflecting both Bayesian updating and ambiguity aversion with the two groups of investors displaying interesting differences in their response behavior. Institutional investors appear to behave in a manner more consistent with Bayesian updating whereas ambiguity aversion appears to play a relatively bigger role in the fund investment decisions of retail investors.

Our primary contribution is to provide empirical evidence of ambiguity averse behavior in a non-experimental setting using data on mutual funds. As we discuss in the next section, there is now an extensive literature which provides theoretical predictions about the impact of ambiguity on asset markets. However, there is at best, limited empirical evidence in support of these predictions. Most of the evidence comes from experimental studies in this area. It is clearly important to provide evidence on these issues using a setting that captures the richness and complexity of the decision environment facing the typical market participant. Our study helps address this gap in the literature. In addition, our paper makes several contributions to the mutual fund literature. One, to our knowledge, it is the first study that explores the impact of ambiguity on fund investor decisions. Two, the paper complements recent findings on how fund investors respond to information about past fund performance.

## II. Ambiguity Aversion

The work of Ellsberg (1961) famously provided evidence of individual aversion to ambiguity or uncertainty (in contrast to risk). Subsequently, a large theoretical literature has evolved to formally develop models that accommodate ambiguity averse behavior and its implications. The notion of ambiguity is captured in such models by assuming that the agent considers a range of probability distributions, rather than a single distribution, for the potential outcomes. Ambiguity averse agents seek to maximize the minimal expected utility across the range of outcome distributions considered. Intuitively, an ambiguity averse agent seeks a course of action that is robust to the uncertainty inherent in the multiple probability distributions of potential outcomes. Examples include the studies by Gilboa and Schmeidler (1989) and Epstein and Schneider (2003) who provide theoretical foundations for maxmin expected utility, and Epstein and Wang (1994) who extend the Lucas (1978) model of asset prices by incorporating ambiguity averse preferences.

A number of subsequent theoretical studies have relied on ambiguity aversion to explain well-known market phenomena. For example, Illeditsch (2011) considers a setting in which investors with maxmin preferences receive a public signal with unknown precision. He shows that small shocks to news about cash flows can cause significant changes in how investors interpret the news leading to large price movements and the appearance of excess volatility. Further, he shows that there is a range of prices over which investors are reluctant to deviate from (risky) portfolio positions that hedge against ambiguity, i.e., they exhibit a form of portfolio inertia. Epstein and Schneider (2010) show that a sufficiently large increase in uncertainty about asset returns can make it optimal for the investor to withdraw from the market altogether, i.e., induce nonparticipation.[5]

Leippold, Trojani, and Vanini (2008) consider a model of learning under ambiguity in which a representative investor observes the realized dividends along with a noisy signal. The investor does not know the dividend-generating process and faces ambiguity regarding the expected dividend growth rate. The model is able to generate a high equity premium, a low interest rate and excess volatility for reasonable parameter values. Similarly, Ju and Miao (2012), consider a model in which an ambiguity averse representative agent seeks to learn about the latent economic state based on the observed consumption and dividends which follow a hidden Markov regime-switching process. They show that the calibrated model can match the empirically observed equity premium, the mean risk-free rate and the volatility of the equity premium. Easley and O'Hara (2009) examine the impact of regulation when some investors are ambiguity averse. They show that regulation such as deposit insurance, by limiting the perceived maximum variance of asset payoffs or by improving the perceived minimum mean payoff, can mitigate the effects of ambiguity. In turn, this can lead to increased market participation by agents and generate welfare gains.

Another set of papers has explored the link between ambiguity and liquidity. Caballero and Krishnamurthy (2008) analyze a model in which an increase in Knightian uncertainty or decrease in the aggregate liquidity available can lead to a flight to quality, in the form of liquidity hoarding, that is socially costly. Routledge and Zin (2009) consider a model with an ambiguity averse intermediary who sets bid and ask prices. They show that ambiguity aversion can cause bid-ask spreads to widen and hence reduce liquidity. Easley and O'Hara (2010) consider a model in which traders are unable to rank order certain portfolios due to extreme uncertainty following ambiguous shocks to asset values, e.g., during periods of financial crises. The result is a market freeze during which price quotes may exist but no trading takes place.

In an experimental setting Bossaerts et al. (2010) find evidence of heterogeneity in investor attitudes towards ambiguity. In contrast to the above mentioned theoretical and/or experimental studies, there is limited empirical evidence regarding the ambiguity averse behavior of investors. Two recent exceptions are the studies by Anderson, Ghysels, and Juergens (2009) and Viale, Garcia-Feijoo, and Giannetti (2014) that provide empirical evidence of an uncertainty-return trade-off in equity markets.

[5] See also, Cao, Wang, and Zhang (2005), and Epstein and Schneider (2007).

Our paper contributes to the ambiguity literature by providing empirical evidence consistent with ambiguity averse behavior on the part of mutual fund investors. To our knowledge ours is the first attempt at using an ambiguity aversion framework to study the response of fund investors to fund performance based signals. An important feature of our study is that the evidence of ambiguity aversion that we provide is based on the buy/sell decisions of fund investors, rather than on observed asset prices. As argued by Condie (2008), it is difficult to rely solely on asset price data to assess the significance of ambiguity aversion since the impact of ambiguity averse investors on prices is empirically indistinguishable from that of expected utility maximizers with potentially biased beliefs.[6] Our solution to this challenge is to directly examine the decisions of the investors, in particular, the decision of whether or not to invest with a fund.

## III. The Model and Empirical Predictions

In order to formally motivate the empirical analysis that follows we present a model of a representative ambiguity averse investor who chooses how much to invest in a fund after observing multiple signals regarding the fund manager's skill. Here we briefly discuss the intuition underlying the model and present the details in Appendix A. In the model, the investor receives two noisy signals related to fund manager's skill. The investor is uncertain about the precision of the signals. Specifically, each signal can have either a high or a low precision. Given that there are two signals, this gives rise to four potential scenarios for the investor to consider. The first scenario is the one in which both signals have high precision, while the second scenario represents the case in which both signals have low precision. The third scenario refers to the case in which the first signal has high precision and the second signal has low precision, while the fourth scenario represents the case in which the first signal has low precision while the second signal has high precision.

Due to ambiguity aversion, the investor will always worry the most about the worst case scenario regarding the fund manager's skill. Given the signal realizations, the worst case across the four potential scenarios is the one in which the signal with the lower realization has a high precision

[6]To see this, note that ambiguity averse investors behave as if they have several probability distributions in mind for the future outcomes. When solving their decision problem, they may attach subjective weights to the potential distributions. The ambiguity averse investors' subjective weights can be viewed as being biased, from the perspective of a rational expected utility (REU) optimizing investor who relies on an 'objective' distribution of outcomes. In this sense, the impact of an ambiguity averse investor would be observationally equivalent to that of a REU investor with biased beliefs.

while the higher-valued signal has low precision. Such an outcome would imply a lower expected return from the candidate fund, compared for example, to a scenario in which the higher-valued signal has high precision while the lower-valued signal has low precision. As a consequence, the investor's fund allocation decision is more sensitive to the signal with the lower realization as it is deemed as being more precise under the worst case scenario. Conversely, the investor's fund allocation decision is less sensitive to the higher-valued signal as it is deemed as being less precise. Thus, a key implication of the model is that when facing multiple signals, ambiguity averse investors display greater sensitivity to the worst signal, i.e., signal with the lowest realization (Proposition 1, Appendix A). Further, the sensitivity to the worst signal is increasing in the degree of ambiguity faced by the investors (Corollary 1, Appendix A).

The discussion in Appendix A also contrasts the implications of our model to that of a standard Bayesian benchmark model with risk averse investors who receive signals of uncertain precision regarding fund manager skill. Consider the benchmark case with a Bayesian investor who faces uncertainty regarding the precision of the signals. In this case the investor's posterior beliefs will be formed in accordance with the rules of Bayesian updating and a signal with a relatively lower realized value would not necessarily carry a relatively higher weight in the investor's decision process. The model presented in Appendix A formalizes the above intuition. Furthermore, the analysis confirms that the implications of our model in terms of the investor fund flow-performance sensitivity are distinct from that of the Bayesian benchmark model as well as models incorporating behavioral biases like loss aversion or overconfidence.

In this section, we develop several testable hypotheses from the model concerning the impact of ambiguity aversion on mutual fund investor flow-performance sensitivity. Our model (see Proposition 1, Appendix A) implies that given signals of unknown quality, ambiguity averse investors' fund flow response is more sensitive to the worst signal, i.e., the signal with the worst realization. In our empirical analysis we use a fund's past performance measured over different horizons as multiple signals of the fund manager's skill.[7] Mutual funds typically publicize the performance figures for each share class over the previous 1 year, 3 years, 5 years, as well as the entire period since inception. However, the degree to which past performance is informative of fund manager skill is unknown

[7] Our analysis makes use of the salient performance-chasing pattern in fund flows to examine the investors' behavior. We are agnostic on the issue of whether the performance-chasing behavior is justified.

to investors. Also, these performance data have different realizations since a fund's performance may fluctuate over time. Thus, these performance statistics over different time horizons serve as multiple comparable signals with unknown quality and different realizations from which investors learn about fund manager skill. Hence, we have the following hypothesis.

HYPOTHESIS 1: *When a fund's past performance is measured over multiple time horizons, fund flows display additional sensitivity to the minimum performance measure in the presence of ambiguity averse investors.*

A direct implication of Corollary 1 (Appendix A) is that the marginal fund flow sensitivity to the worst signal is increasing in the level of signal ambiguity, *ceteris paribus*. To empirically test this implication of the model, we consider a number of proxy measures for a fund's ambiguity from the perspective of fund investors. Using the different proxy measures allows us to set up the following testable hypotheses.

HYPOTHESIS 2: *Individual investors experience greater ambiguity compared to institutional investors, as measured by a higher marginal sensitivity to the minimum performance measure.*

The above hypothesis reflects the notion that individual (retail) investors are believed to be less sophisticated compared to institutional investors. As a result, they may be less confident about how much the past performance is indicative about fund manager skill. They may therefore be subject to greater ambiguity in terms of interpreting such information. On the other hand, institutional investors, who are believed to be much more sophisticated, may be more confident about the precision of signals when they look at past performance measures.[8] As a result, consistent with Corollary 1, we expect to see a higher marginal sensitivity to the minimum performance measure in the sample of retail investors compared to institutional investors.

HYPOTHESIS 3: *Funds that change their investment strategy more aggressively and/or frequently are characterized by a higher degree of ambiguity. Investor fund flows in such funds will display a higher marginal sensitivity to the minimum performance measure.*

[8] We note that our comparison of individual investors and institutional investors is based solely on a consideration of the information environment faced by each group of investors. We make no reference to any potential difference in their preferences, i.e., whether one type of investor is more ambiguity averse than another type. Instead, if an investor has higher information processing ability, has more experience, and/or has better knowledge/understanding of the general information environment, then this investor will face less ambiguity.

Fund investment strategy changes can be viewed as a proxy for a fund's ambiguity level. If a fund constantly switches its investment strategy, e.g., from a passive diversification strategy to active factor timing or stock selection, investors may find it hard to evaluate its relative performance and to form a concrete expectation of future return. Thus, the more aggressively and/or frequently a fund changes its strategy, the more ambiguous it is from the investors' perspective.

HYPOTHESIS 4: *Funds with more volatile cash flows are characterized by a higher degree of ambiguity. Investor fund flows in such funds will display a higher marginal sensitivity to the minimum performance measure.*

Fund inflows reflect a general positive view among investors about the fund's future prospects and outflows are indicative of a negative view about the fund. Thus, flow volatility can be viewed as a proxy for the degree of variability or uncertainty of investor opinion with respect to the fund's prospects. In this sense, highly volatile investor fund flows imply greater uncertainty about the fund's future performance. Hence, funds with more volatile flows are likely to be the ones that are more ambiguous to investors.

HYPOTHESIS 5: *Funds that belong to a smaller family appear more ambiguous to investors. Investor fund flows to such funds will display a higher marginal sensitivity to the minimum performance measure.*

If a fund belongs to a family with a large asset base, it is more recognizable and enjoys the reputation built up by the entire fund family. On the other hand, if a fund belongs to a small and little known family, investors may be more conservative when making their investment decisions. It may also be harder for them to rely on past performance of such funds in drawing inference about manager skill. In other words, funds belonging to smaller families may appear more ambiguous to investors.

## IV. Data and Methodology

### *A. Data*

We use data from the Center for Research in Security Prices (CRSP) Survivorship Bias Free Mutual Fund Database, which includes information on the funds' total net assets, returns and characteristics. We focus on actively managed U.S. equity mutual funds, hence, we exclude index funds and funds that are closed to new investors. To be consistent with prior studies, we exclude sector funds, international funds, bond funds, and balanced funds from our analysis. We classify funds into five categories based on their objective codes:[9] aggressive growth, growth, growth and income, income and others.

We primarily study the period from January 1993 through December 2011, since the CRSP database does not report 12b-1 fees until 1992 and institutional funds begin to mushroom in the 1990s. However, as a robustness check we confirm that our results are qualitatively unchanged when we extend the sample to the period: January 1985 through December 2011. We examine fund flows and other characteristics at the quarterly frequency. Consistent with prior studies, we define quarterly net flow into a fund as

$$Flow_{i,t} = \frac{TNA_{i,t} - TNA_{i,t-1}(1 + R_{i,t})}{TNA_{i,t-1}}, \tag{1}$$

where $R_{i,t}$ denotes fund $i'$s return during quarter $t$, and $TNA_{i,t}$ is the fund's total net asset value at the end of quarter $t$. Thus, our definition of flows reflects the percentage growth of the fund's assets in quarter $t$. To prevent the potential impact of extreme values of flows resulting from the errors associated with mutual fund mergers and splits in CRSP mutual fund database, we filter out the top and bottom 1% tails of the net flow data. To further guard against this issue, we delete records of funds from our analysis before their total net asset value first hits the $3 million mark.

Table I reports the summary statistics of mutual funds characteristics. We note that since we are interested in studying the fund flow behavior of retail as well as institutional investors we treat each fund share class as an individual fund, consistent with the research design employed by Huang, Wei, and Yan (2007); Huang et al. (2012).[10] In 1993, there are 707 distinct fund share classes in our sample and 63 of them are open only to institutional investors. In 2011, the number of funds in

[9]We categorize funds according to the following criteria. First, funds with Lipper objective codes G, LCGE, MCGE, MLGE, SCGE, with Wiesenberger objective codes G, G-S, S-G, GRO, LTG, SCG, or with Strategic Insight objective codes GRO, SCG are classified as growth funds. Second, funds with Wiesenberger objective code AGG or with Strategic Insight objective code AGG are classified as aggressive growth. Third, funds with Lipper objective code GI, with Wiesenberger objective codes G-I-S, G-S-I, I-G, I-G-S, GCI, G-I, I-S-G, S-G-I, S-I-G, GRI, or with Strategic Insight objective code GRI are classified as growth and income funds. Fourth, funds with Lipper objective codes EI, EIEI, I, with Wiesenberger objective codes I, I-S, IEQ, ING, or with Strategic Insight objective codes ING are classified as income funds. Fifth, all the other actively managed equity funds in our sample are classified as others.

[10]As emphasized in Huang et al. (2007), since our focus is on analyzing fund flows, treating each fund share class separately will not lead to the double-counting problem. Also, since most of our tests require a separation between retail shares and institutional shares, we conduct all tests at the fund share class level.

our sample grows to 4,242 with 785 institutional funds. In total, our sample includes 7,020 distinct fund share classes and 216,366 fund share class-quarters. In an average quarter, the sample includes 2,847 fund share classes with average total net assets (TNA) of $678.06 million and an average net flow of 1.51%. Following Sirri and Tufano (1998) we measure total expense as the expense ratio plus one-seventh of the front-end load. The 12b-1 fees are the part of fund expenses that cover distribution expenses and sometimes shareholder service expenses. Distribution expenses include marketing, advertising, and compensation paid for brokers who sell the funds. As may be seen from Table I, the 12b-1 fees for retail funds are 0.57%, which is nearly three times of that for institutional funds.

### B. *Empirical Methodology*

We formally analyze the relationship between fund flows and performance measured over multiple time horizons when controlling for other factors. We estimate the following model using 76 quarters of fund-level data over the period 1993 to 2011 to test our baseline hypothesis:

$$\begin{aligned} Flow_{i,t} =& a + b_1 Perf\_1yr_{i,t} + b_2 Perf\_3yr_{i,t} + b_3 Perf\_5yr_{i,t} + cMin\_Rank_{i,t} \\ &+ Controls + \epsilon_{i,t}. \end{aligned} \tag{2}$$

Following Sirri and Tufano (1998), the variables $Perf\_1yr$, $Perf\_3yr$, and $Perf\_5yr$ represent fractional performance ranks ranging from 0 to 1 based on fund $i'$s performance during past 12 months, 36 months, and 60 months, respectively. The variable $Min\_Rank$ is defined as:

$$Min\_Rank_{i,t} = Min\left(Perf\_1yr_{i,t}, Perf\_3yr_{i,t}, Perf\_5yr_{i,t}\right). \tag{3}$$

Thus, the coefficient $c$ in Equation (2) captures the additional flow sensitivity to performance measured over the particular horizon during which the fund had the worst performance ranking. This coefficient is the focus of our tests. The specification here is different from the typical regression specification used in the fund performance chasing literature, where the purpose of the regression is often to establish a positive relation between fund flows and past fund performance. Our specification on the other hand allows us to examine in greater detail the functional form of the relation between the fund flows and past fund performance. It resembles in form the renowned Henriksson and Merton (1981) specification of a regression based test of market timing ability. As we discuss below, the additional flow-performance sensitivity captured by the coefficient $c$, is significant in both economic and statistical sense in the population of retail investors. We note that in the above specification we include all three performance rank variables in order to be conservative. If we change the specification to include any one of the three variables, or include only an average of the three performance ranks, our results will in fact be stronger. [11]

A relevant issue to consider at this stage is the nature of the fund performance related information available to the typical retail investor. Mutual funds disclose the net asset value of their shares on a daily basis. Additionally, firms like Morningstar and Lipper provide performance statistics that are widely distributed through the financial press and various media outlets. The focus is on relative performance ranking of funds within the respective categories — primarily based on returns. For example, the *Wall Street Journal* publishes a quarterly list of "Category Kings" which highlights the top 10 mutual funds within the different categories ranked by one-year performance as of quarter-end. The reported statistics include the 3-year, 5-year, and 10-year annualized returns. Similarly, Morningstar assigns a rating, commonly called the "star rating", to funds based on their within-category rank according to their risk-adjusted returns.[12] The statistics available from firms like Morningstar include the total return figures computed over a historical window including the past 1-year, 3-year, 5-year, and 10-year periods. Among other statistics, the one-factor fund alpha, calculated with respect to a standard benchmark like the S&P 500 or a "best-fit" benchmark, is also included in the detailed fund analysis. Given this information environment, it is reasonable to believe that the typical retail investor pays attention to returns-based performance rankings for a fund. On the other hand, institutional investors are likely to rely on additional measures of performance including multi-factor alphas.

In our analysis we consider two alternative fund performance measures. The first measure is the average monthly raw return measured over a specified time horizon, i.e., 12 months, 36 months or 60 months. The second measure is the fund's factor-adjusted performance using the Carhart (1997) 4-factor model. In order to estimate the 4-factor model, we first calculate fund $i$'s factor loadings in quarter $t$ by regressing the past 60 months' excess returns on the four factors:

$$R_{i,\tau} - R_{f,\tau} = \alpha_i + \beta_i^{MKT} MKT_\tau + \beta_i^{SMB} SMB_\tau + \beta_i^{HML} HML_\tau + \beta_i^{UMD} UMD_\tau + \epsilon_{i,\tau}, \qquad (4)$$

[11]We also note that the 1-year, 3-year, and 5-year performance ranks serve only as control variables in our study, and are not themselves variables of interest. Furthermore, their correlations are positive but moderate, thereby mitigating any potential multi-collinearity concerns.

[12]See $http://www.morningstar.com/help/data.html\#RatingCalc$ for additional details.

where $R_{i,\tau}$ is the return for fund $i$ and $R_{f,\tau}$ is the one-month T-bill rate in month $\tau$. The market factor, $MKT_\tau$, represents the monthly excess market return. The factors $SMB_\tau$, $HML_\tau$, and $UMD_\tau$ represent the monthly returns on the size, value, and momentum factor mimicking portfolios, respectively. We obtain the factor returns from Kenneth French's website. We then calculate the fund's factor adjusted alpha each month using the monthly fund excess returns, and the factor loadings estimated as above. We compute the average of these monthly alphas over distinct horizons of 12 months, 36 months, and 60 months, respectively.

To obtain fractional performance ranks ($Perf\_1yr$, $Perf\_3yr$, and $Perf\_5yr$) ranging from 0 to 1, we apply different approaches to rank funds based on the two performance measures. In the case of the raw return measure, we rank funds every quarter within fund objective categories based on their average raw returns over each of the three lagged time horizons. For rankings based on the 4-factor alphas, we rank all funds each quarter according to their Carhart (1997) alphas over each of the three time horizons considered.

The control variables employed in Equation (2) include a number of fund characteristics that have been shown to affect fund flows. In particular, we control for the logarithm of one plus fund age, previous quarter's flow, previous quarter's raw return, fund size as measured by the natural logarithm of fund total net assets in the previous quarter, volatility of monthly raw returns during the prior 12 months, the lagged total expense ratio, and the maximum drawdown of a fund's monthly return over the previous 60 months. Finally, following Sirri and Tufano (1998) we also include the category flow, defined as the percentage quarterly net asset growth of the fund's objective category.

We estimate the model in Equation (2) by conducting a cross-sectional linear regression each quarter and reporting the time-series means and the related Newey-West t-statistics of the coefficients following Fama and MacBeth (1973). We follow this approach throughout the paper.

A confounding factor in assessing fund performance is that fund returns are related to business cycles and general macroeconomic conditions. Note that we rely on relative performance measures which helps mitigate this concern. In the case of raw returns-based performance ranks, funds are ranked within their own category. This relative ranking method naturally controls for the influence of common factors that affect performance within a particular category of funds. For fund rankings based on the alphas, the 4-factor model serves to control for differences in investment styles across funds. Again, the use of relative performance ranks in this case, helps control for the influence of common factors.

# V. Empirical Results

## A. *Ambiguity Aversion*

In this subsection, we test Hypotheses 1 and 2 by studying investor ambiguity aversion behavior within subsamples of retail funds and institutional funds respectively. Our model implies that in the presence of ambiguity averse investors, fund flows will display additional sensitivity to a fund's minimum performance ranking. This marginal sensitivity is in addition to the general response of flows to past performance measures.

To test this implication, we estimate the baseline model in Equation (2) for retail funds and institutional funds separately. According to Hypothesis 2, we expect to see a significant coefficient, $c$, for the variable $Min\_Rank$ for the subsample of retail funds only. The results are reported in Table II. Columns 1 and 2 report results using raw return as the performance measure and Columns 3 and 4 report results using the Carhart (1997) alpha as the performance measure.

Consistent with Hypothesis 2, we find that the coefficient for $Min\_Rank$ is positive and significant at the 1% level for both measures of performance in the retail funds sample. However, for institutional funds, the coefficient is positive but insignificant. Focusing on the results for retail funds in Column 1 where the performance is measured in terms of raw returns, the coefficient for the $Min\_Rank$ variable is 0.047. The coefficient for performance measured over the 1-year horizon is 0.037. This implies that for retail funds, a 14% (the average absolute change in the 1-year performance ranking for retail funds in our sample) increase in the fund's 1-year performance rank will result in a $0.037 \times 14\% = 0.5\%$ increase in the fund's assets. However, if the 1-year performance rank happens to be the worst among the three horizons, a 14% increase in the fund's (1-year) ranking results in an inflow equal to $(0.047 + 0.037) \times 14\% = 1.18\%$ of the fund's assets. Thus, a 14% improvement in the worst performance rank results in a doubling of the fund flows enjoyed by the fund relative to the normal increase in flows from general performance improvement. The economic magnitude is quite significant given the average quarterly fund flow in the retail sample is 1.32%. In Column 3 when performance is measured using Carhart (1997) 4-factor alpha, we observe a similar coefficient for the *Min_Rank*. As seen from the results presented in Columns 2 and 4 of Table II, in the case of institutional funds, even though the coefficients for *Min_Rank* are positive, they are not statistically significant and are much smaller in magnitude than their retail counterparts.

We note that the coefficients on the other control variables included in Equation (2) are consistent with previous findings in literature. The generally positive and significant coefficients for all three performance measures in Columns 1 through 4 conform to the performance chasing behavior as documented in Chevalier and Ellison (1997) and Sirri and Tufano (1998), among others. Sirri and Tufano (1998) also document the negative impact of volatility on flows. The positive coefficient on *Previous Quarterly Flow* confirms the persistence in fund flows. Similarly, the negative coefficients for *Total Expense* in Columns 1 through 4 are consistent with previous studies by Barber, Odean, and Zheng (2005) and Sirri and Tufano (1998), among others.

After controlling for above factors, the significant coefficients for *Min_Rank* in Columns 1 and 3 indicate that minimum performance ranks have significant explanatory power for fund flows. In sum, Table II provides evidence for ambiguity aversion behavior among retail funds investors as shown by the significant positive coefficient for the minimum performance rank, which is consistent with Hypotheses 1 and 2. Accordingly, we focus on retail funds in conducting the tests reported for the remainder of the section.

*B. Strategy Changes as a Proxy for Fund Ambiguity*

Hypothesis 3 states that fund strategy changes could be viewed as a proxy for a fund's ambiguity level. Investors are likely to face a higher degree of ambiguity with regard to a fund that switches its investment strategy too aggressively/frequently. We adopt two ways to measure a fund's strategy shifts. The first measure is the fund's average absolute change in its factor loadings, while the second measure is the fund's R-squared computed from a time series regression using the entire history of fund returns.

The first proxy is motivated by Lynch and Musto (2003). Each quarter $t$, we compute a fund's factor loadings with respect to the four Carhart (1997) model factors over two non-overlapping 30-month periods, namely, the prior 1-30 and 31-60 month periods. We then compute the average absolute change in the factor loadings from the initial 30-month period to the most recent 30-month period as

$$LDEL_{i,t} = \frac{1}{4} \sum_{f} \left| \beta^{f}_{i,t,1-30} - \beta^{f}_{i,t,31-60} \right|, \tag{5}$$

for $f = MKT, HML, SML$ and $UMD$. A higher loading-change indicates a more aggressive shift in the fund's strategy.

The second proxy is the fund's R-squared from a time series regression of the fund's monthly returns on the four Carhart (1997) factors. A low R-squared value implies that the 4-factor model is a poor performance attribution model for the fund. A potential reason could be that the factor loadings of the fund are not constant over the sample period implying frequent shifts in the factor exposures or the investment style. Such shifts would make it harder for investors to interpret past performance related signals and contribute to an enhanced level of ambiguity in interpreting such signals. Each quarter, we divide the sample of funds into three groups, $Low, Mid$ and $High$ based on their $LDEL$ or R-squared values. We then we apply the baseline model described in Equation (2) to each group and report the time-series average of the coefficients. The results are reported in Table III.

Performance in Table III is measured by raw return (Column 1-3) or the Carhart (1997) 4-factor model alpha (Column 4-6). Panel A of Table III reports results when strategy changes are measured using the average absolute change in factor loadings ($LDEL$). As shown in Column 1, for the group of funds with low loading-change, an indication of less aggressive strategy shifts, the additional flow sensitivity to the minimum performance rank, as captured by the $Min\_Rank$ coefficient, is only 0.014 and it is statistically insignificant. However, for funds which shift their strategies more aggressively, as shown in Column 3, the marginal sensitivity to minimum performance is nearly six times higher at 0.058 which is significant at the 1% level. The $F$-test statistic rejects the null hypothesis that the $Min\_Rank$ coefficients across the three groups are equal. Columns 4-6 report qualitatively similar flow-performance sensitivity results when performance is measured using the Carhart (1997) 4-factor alpha.

Panel B of Table III presents results using a fund's 4-factor model R-squared values as a proxy for the fund's ambiguity. Columns 1-3 report results when a fund's performance is measured by the ranking of its raw return. Flows of funds with low R-squared values show additional sensitivity to the minimum performance rank as seen by the $Min\_Rank$ coefficient in Column 1 (0.051), which is significant at 1% level. In Column 3, however, flows of funds with high R-squared values, an indication of relatively stable investment strategy, have marginal sensitivity to minimum performance of only 0.023 which is statistically significant only at the 10% level. Columns 4-6 report qualitatively similar flow-performance sensitivity results when performance is measured using the Carhart (1997) 4-factor alpha.

In conclusion, the results in Table III show that funds that change investment strategy more aggressively/frequently are more ambiguous to investors, as evidenced by the greater marginal sensitivity of investor fund flows to the minimum performance measure.

*C. Flow Volatility as a Proxy for Fund Ambiguity*

Hypothesis 4 states that the flow volatility could be used as a proxy for a fund's ambiguity level. Note that fund flows are the consequence of investors' asset allocation decisions. A net inflow is an indication of an overall positive view of the fund while a net outflow represents an overall negative view. Thus, flow volatility captures the uncertainty about the funds' future performance from the perspective of an average investor. In this sense, flow volatility is a direct measure of the fund's ambiguity level. We expect to observe stronger ambiguity aversion behavior among investors of funds with more volatile past flows. We measure flow volatility ($Flow\_Vol$) as the standard deviation of a fund's previous 12 quarters' fund flows.

To test this hypothesis, we estimate the following regression model:

$$\begin{aligned}
Flow_{i,t} =& a + b_{11}Low_Flowvol_{i,t} \times Perf_1yr_{i,t} + b_{12}Mid_Flowvol_{i,t} \times Perf_1yr_{i,t} \\
&+ b_{13}High_Flowvol_{i,t} \times Perf_1yr_{i,t} + b_{21}Low_Flowvol_{i,t} \times Perf_3yr_{i,t} \\
&+ b_{22}Mid_Flowvol_{i,t} \times Perf_3yr_{i,t} + b_{23}High_Flowvol_{i,t} \times Perf_3yr_{i,t} \\
&+ b_{31}Low_Flowvol_{i,t} \times Perf_5yr_{i,t} + b_{32}Mid_Flowvol_{i,t} \times Perf_5yr_{i,t} \\
&+ b_{33}High_Flowvol_{i,t} \times Perf_5yr_{i,t} + c_1Low_Flowvol_{i,t} \times Min_Rank_{i,t} \\
&+ c_2Mid_Flowvol_{i,t} \times Min_Rank_{i,t} + c_3High_Flowvol_{i,t} \times Min_Rank_{i,t} \\
&+ Flow_Vol_{i,t} + Controls + \epsilon_{i,t},
\end{aligned} \tag{6}$$

where $Low\_Flowvol_{i,t}$ is a dummy variable that equals one if the fund $i'$s flow volatility falls into the bottom tercile in quarter $t$, $Mid\_Flowvol_{i,t}$ is a dummy variable that equals one if it belongs to the medium tercile, and $High\_Flowvol_{i,t}$ is a dummy variable that equals one if the flow volatility is ranked in the top tercile. Each quarter we conduct a cross-sectional regression of flows on the interaction of the three dummy variables with $Perf\_1yr$, $Perf\_3yr$, $Perf\_5yr$ and

$Min\_Rank$, respectively, together with the set of control variables and report the time-series mean and Newey-West t-statistics of the coefficients. The control variables are the same as those in Equation (2). The coefficients on the interaction terms capture the differential flow sensitivity to a certain performance horizon or to the minimum performance for funds with low, medium or high flow volatility. For example, the coefficient of $High\_Flowvol \times Min\_Rank$, i.e., $c_3$, captures the additional sensitivity to the $Min\_Rank$ for funds with high flow volatility. Given Hypothesis 4, we expect a large and positive coefficient for $High\_Flowvol \times Min\_Rank$, and a small coefficient for $Low\_Flowvol \times Min\_Rank$.

Table IV presents the flow-performance results based on flow volatility as a proxy for fund ambiguity level. Columns 1 and 3 report results using raw return and Carhart (1997) alpha as performance measures, respectively. As expected, in Column 1, the coefficients for the three interaction terms $Low\_Flowvol \times Min\_Rank$, $Mid\_Flowvol \times Min\_Rank$ and $High\_Flowvol \times Min\_Rank$ increase monotonically from -0.004 (statistically insignificant) to 0.108 (significant at the 1% level). The $F$-test statistic rejects the null that the coefficients of $Low\_Flowvol \times Min\_Rank$, $Mid\_Flowvol \times Min\_Rank$ and $High\_Flowvol \times Min\_Rank$ are equal. In Column 3 of Table IV, where fund performance is measured using the Carhart (1997) alpha, we observe a similar increase in the value of the coefficients of the interaction terms across the three flow volatility terciles.

In conclusion, the results in Table IV are consistent with Hypothesis 4 that funds with more volatile flow appear to be more ambiguous to investors.

### *D. Family Size as a Proxy for Fund Ambiguity*

Hypothesis 5 states that family size can also be used as a proxy for a fund's ambiguity level. In this subsection, we use the sum of total net assets for each fund within a fund family as a measure of family size. We then test whether a larger family size is associated with a reduction in the marginal flow sensitivity to a fund's minimum performance measure.

To test this hypothesis, we estimate the following model:

$$
\begin{aligned}
Flow_{i,t} =& a + b_{11} Low\_Famsize_{i,t} \times Perf\_1yr_{i,t} + b_{12} Mid\_Famsize_{i,t} \times Perf\_1yr_{i,t} \\
&+ b_{13} High\_Famsize_{i,t} \times Perf\_1yr_{i,t} + b_{21} Low\_Famsize_{i,t} \times Perf\_3yr_{i,t} \\
&+ b_{22} Mid\_Famsize_{i,t} \times Perf\_3yr_{i,t} + b_{23} High\_Famsize_{i,t} \times Perf\_3yr_{i,t} \\
&+ b_{31} Low\_Famsize_{i,t} \times Perf\_5yr_{i,t} + b_{32} Mid\_Famsize_{i,t} \times Perf\_5yr_{i,t} \\
&+ b_{33} High\_Famsize_{i,t} \times Perf\_5yr_{i,t} + c_1 Low\_Famsize_{i,t} \times Min\_Rank_{i,t} \\
&+ c_2 Mid\_Famsize_{i,t} \times Min\_Rank_{i,t} + c_3 High\_Famsize_{i,t} \times Min\_Rank_{i,t} \\
&+ D\_FamilySize_{i,t} + Controls + \epsilon_{i,t},
\end{aligned} \tag{7}
$$

where $Low\_Famsize_{i,t}$ is a dummy variable that equals one if fund $i$ belongs to a fund family whose size falls into the bottom tercile in quarter $t$, $Mid\_Famsize_{i,t}$ is a dummy variable that equals one if family size belongs to the medium tercile and $High\_Famsize_{i,t}$ is a dummy variable that equals one if the family size is in the top tercile. The variable $D\_Familysize_{i,t}$ is a dummy variable that equals 1 if the fund family size is above the median value for that quarter. We regress quarterly flows on the interaction of the three dummy variables with $Perf\_1yr$, $Perf\_3yr$, $Perf\_5yr$ and $Min\_Rank$, respectively. The coefficients on the interaction terms represent the differential flow sensitivity to a certain performance horizon or to the minimum performance rank for funds that belong to a small-sized family, medium-sized family and large-sized family, respectively. For example, the coefficient $c_1$ for the variable $Low\_Famsize \times Min\_Rank$ in Equation (7) captures the additional flow sensitivity to the minimum performance rank for funds in a small family. Given Hypothesis 5, we expect a large and positive coefficient for $Low\_Famsize \times Min\_Rank$, but a small coefficient for $High\_Famsize \times Min\_Rank$.

Table V presents results using fund family size as a proxy for the fund's ambiguity level. Columns 1 and 3 report results using raw returns and the Carhart (1997) alpha as performance measures, respectively. Consistent with our expectation, in Column 1, the coefficients for the three interaction terms $Low\_Famsize \times Min\_Rank$, $Mid\_Famsize \times Min\_Rank$ and $High\_Famsize \times Min\_Rank$ decrease monotonically from 0.069 (significant at the 1% level) to 0.018 (significant at the 10% level). This means that the flow sensitivity to minimum performance rank for funds in a small family is more than three times that for funds belonging to a large family. The $F$-test statistic rejects the null that the coefficients of $Low\_Famsize \times Min\_Rank$, $Mid\_Famsize \times Min\_Rank$ and $High\_Famsize \times Min\_Rank$ are equal. In Column 3, when performance is measure in terms of the Carhart (1997) alpha, the coefficient for $Low\_Famsize \times Min\_Rank$ (0.066) is about twice the coefficient for $High\_Famsize \times Min\_Rank$ (0.034), both significant at the 1% level.

As seen from the estimated coefficients for the variable, $D\_FamilySize$, family size has a positive and significant impact on fund flows, consistent with the findings of Sirri and Tufano (1998). This suggests that funds belong to bigger families experience a faster growth in assets. As noted by Gallaher, Kaniel, and Starks (2005), strategic decisions regarding advertising, and distribution channels are made at the fund family level. Funds that belong to a large family have more resources in terms of both management and reputation, allowing them to grow at a faster rate. In conclusion, the results in Table V are consistent with Hypothesis 5 that funds belonging to smaller families appear more ambiguous to investors. [13]

# VI. Contrast between the Response of Ambiguity Averse Investors and Bayesian Investors

Finally, we develop a test to distinguish ambiguity averse investor behavior from the Bayesian learning benchmark. Following earlier discussion, we argue that a fund's performance volatility is another proxy for the fund's ambiguity level in addition to the flow volatility and family size. The more volatile the fund's past performance, the harder it is for investors to learn about the fund's future performance. Thus, a fund with a higher degree of performance volatility is more ambiguous from an investor's perspective. Accordingly, we expect to observe greater marginal flow sensitivity to the minimum performance rank for such funds. By contrast, Huang et al. (2012) hypothesize that the volatility of funds' past performance should have a dampening effect on flow-performance sensitivity if investors update their beliefs in Bayesian fashion. As we note below, the two seemingly contradictory hypotheses can in fact be reconciled.

We want to first document the two effects by performing two separate tests. We apply the following model to test our baseline hypothesis of ambiguity aversion:

$$\begin{aligned} Flow_{i,t} =& a + b_1 Min\_Rank_{i,t} + e_1 Low\_3yr_{i,t} + e_2 Mid\_3yr_{i,t} + e_3 High\_3yr \\ &+ Controls + \epsilon_{i,t}. \end{aligned} \tag{8}$$

For the purpose of comparison, we adopt similar measures and time periods as in Huang et al.

[13] In unreported tests, we also employ the level of marketing effort, as reflected in the amount of 12b-1 fees borne by a fund, as a proxy for the fund's ambiguity. We hypothesize that funds that spend more on marketing appear to be less ambiguous to investors since investors are likely to be more familiar with funds that advertise more. Our results confirm that 12b-1 fees spent on marketing and advertising do help reduce the ambiguity of funds from the investors' perspective.

(2012). For example, in this section, we expand our sample to include both retail and institutional funds. Also, to be consistent with Huang et al. (2012) study, we focus on a fund's previous 3-year performance ($Perf\_3yr$), defined as a fund's performance measured by its raw return rankings within its objective category over the past 36 months. We define the following fractional performance rankings over the low, medium and high performance ranges[14]. The fractional rank for funds in the bottom performance quintile ($Low\_3yr$) is $Min(Perf\_3yr, 0.2)$, in the three medium quintiles ($Mid\_3yr$) is $Min(0.6, Perf\_3yr - Low\_3yr)$, and in the top quintiles ($High\_3yr$) is defined as ($Perf\_3yr - Mid\_3yr - Low\_3yr$). The variable $Min\_Rank$ is the same as defined in Equation (2). We also include the identical set of control variables as in our baseline model specified in Equation (2).

In the above specification, the coefficient $b_1$ captures the ambiguity aversion effect. We expect $b_1$ to be positive and significant. The results are presented in Column 1 of Table VI. The coefficient $b_1$ is estimated to be 0.085 which is highly significant in both statistical and economic terms. It is worth noting that we do observe the convexity in the flow-performance relationship reflected in the respective coefficients for the performance ranges ($Low\_3yr$, $Mid\_3yr$ and $High\_3yr$). In particular, the coefficient for the high performance range (0.255) is nearly 10 times the coefficient for the low performance range (0.026) suggesting a convex flow-performance relationship.

Next, we replicate the Huang et al. (2012) results using the following test:

$$\begin{aligned} Flow_{i,t} =& a + cVol_{i,t} \times Perf\_3yr_{i,t} + e_1 Low\_3yr_{i,t} + e_2 Mid\_3yr_{i,t} + e_3 High\_3yr_{i,t} \\ &+ Controls + \epsilon_{i,t}. \end{aligned} \tag{9}$$

The variable $Vol_{i,t}$ is the fund $i'$s previous 36 months' raw return volatility in quarter $t$. If the dampening effect of performance volatility on the flow-performance relationship exists, we expect to see a significant negative coefficient for the interaction term, $Vol \times Perf\_3yr$. As shown in Column 2 of Table VI, this coefficient is estimated to be -0.969, which is significant at the 1% level. All of the other coefficients are also qualitatively similar to the values reported by Huang et al. (2012).

Finally, it is of interest to show that ambiguity aversion and the dampening effect of performance volatility could co-exist. We distinguish our ambiguity aversion phenomenon using the following

[14]See, for example, Huang et al. (2007, 2012) and Sirri and Tufano (1998).

model:

$$Flow_{i,t} = a + b_1 Min\_Rank_{i,t} + b_2 Min\_Ind_{i,t} \times Vol_{i,t} \times Perf\_3yr_{i,t} + cVol_{i,t} \times Perf_3yr_{i,t} \quad (10)$$
$$+ e_1 Low\_3yr_{i,t} + e_2 Mid\_3yr_{i,t} + e_3 High\_3yr_{i,t} + fVol_{i,t} + Controls + \epsilon_{i,t}.$$

In the above specification, the variable $Min\_Ind_{i,t}$ is a dummy variable that equals one if fund $i'$s 3-year performance happens to be the worst among 1-year, 3-year and 5-year performance measures in quarter $t$. The coefficient $b_1$ captures the ambiguity aversion effect. Here $b_2$ captures the effect of performance volatility on 3-year performance if it happens to be the minimum performance rank and $c$ captures the impact of performance volatility on the 3-year performance, on average. Under ambiguity aversion, we expect $b_2$ to be positive and significant, since past performance volatility is expected to increase the fund's ambiguity level. However, if investors are Bayesian learners as modeled in Huang et al. (2012), we also expect to observe a negative value for the coefficient $c$, since high signal noise should dampen flow sensitivity, in general.

The results from estimating Equation (10) are reported in Column 3 of Table VI. We find that the coefficient $b_2$ has a value equal to 0.410 while the coefficient $c$ equals -0.868, and both coefficients are significant at the 1% level. The coefficients imply, in economic terms, a 1 unit increase in performance volatility will **decrease** sensitivity to performance by $0.868 \times 1 = 0.868$, on average. However, at the margin, it will **increase** sensitivity to the minimum performance rank by $0.410 \times 1 = 0.410$.

These findings show that the dampening effect of performance volatility and the impact of ambiguity aversion on the flow performance relation are not contradictory to each other and may actually co-exist. This is intuitive, since it is reasonable to conjecture that there are two distinct types of fund investors. The first type is the sophisticated investors who have a better understanding of the mutual fund industry. Upon observing past fund performance, these investors face less ambiguity and are able to update their beliefs in a manner that is largely consistent with Bayesian rules. The other type of investors views the situation as being more ambiguous, possibly due to lack of experience and knowledge of the investment. The behavior of such investors may be influenced more by their ambiguity aversion. As long as a fund's investor base includes both types of investors, the fund flow patterns will reflect both the dampening effect of performance volatility due to the Bayesian investors, and the sensitivity to the worst performance signal due to the ambiguity averse investors. The relative importance of the two effects may depend on the relative population weights of the two types of investors in a fund's investor base.

We further examine the above intuition by including a dummy variable $Inst_i$, which equals 1 if a fund is only open to institutional investors. We use the following specification to examine the flow-performance response for the two types of investors:

$$\begin{aligned} Flow_{i,t} =& a + b_1 Min\_Rank_{i,t} + b_2 Min\_Ind_{i,t} \times Vol_{i,t} \times Perf\_3yr_{i,t} + cVol_{i,t} \times Perf\_3yr_{i,t} \\ &+ d_1 Inst_i \times Min\_Ind_{i,t} \times Vol_{i,t} \times Perf\_3yr_{i,t} + d_2 Inst_i \times Vol_{i,t} \times Perf\_3yr_{i,t} + d_3 Inst_i \\ &+ e_1 Low\_3yr_{i,t} + e_2 Mid\_3yr_{i,t} + e_3 High\_3yr_{i,t} + Controls + \epsilon_{i,t}. \end{aligned} \tag{11}$$

We expect to observe a stronger volatility dampening effect in institutional fund flows and a weaker ambiguity aversion effect. The results of this test are presented in Column 4 of Table VI. Similar to Equation (10), the coefficient, $b_2$, captures the impact of ambiguity aversion for retail investors and the coefficient, $d_1$, captures the marginal impact of ambiguity aversion for the institutional investors. The coefficient $b_2$ is estimated to be 0.434 which is significant at the 1% level while $d_1$ is estimated to be -0.182 which is not significantly different from zero. This implies that, in economic terms, the impact of ambiguity aversion on the flow performance sensitivity is lowered by nearly 40% for every unit increase in performance volatility for institutional investors compared to retail investors. The coefficient $c$, captures the dampening effect of increased performance volatility for retail investors while $d_2$ captures the marginal volatility dampening effect for institutional investors. The coefficient $c$ is estimated to be -0.850 which is significant at the 1% level while $d_2$ equals -0.355 which is significant at the 5% level. This implies that the dampening effect of volatility on flow performance sensitivity is magnified by 42% for institutional investors. Thus, we find stronger evidence of Bayesian updating behavior on the part of institutional investors compared to retail investors. On the other hand, we find stronger evidence of ambiguity aversion on the part of retail investors compared to institutional investors.

In order to more closely match the results of Huang et al. (2012) we repeat our tests using an identical sample period: 1983-2006. We find qualitatively similar results and report them in

Columns 5-8 in Table VI.

# VII. Robustness

In previous tests we investigated the asymmetric sensitivity of fund flows to performance measured over 1-, 3-, and 5-year horizons caused by investor ambiguity aversion. In particular, we find that the more ambiguous a fund appears to its investors, the greater the flow-performance sensitivity to the minimum performance over the three time horizons. Of course, investors have access to additional information beyond the above three measures of performance. Other information available to investors includes the performance of the fund since inception and the Morningstar fund ratings. In fact, previous studies show that investor flows do respond to Morningstar ratings.[15] A natural question is whether the additional sensitivity to minimum performance documented in our tests is simply due to the omission of long term performance measures and/or Morningstar ratings. To address this concern we reexamine the baseline model in Equation (2) while controlling for the funds' performance since inception and the Morningstar fund ratings.

We measure a fund's performance since its inception in the same manner as its 1-, 3- and 5-year performance. We rank funds based on their average monthly raw returns since inception within their objective category. The resulting ranking is a number between 0 and 1. In terms of the Morningstar fund ratings, according to Sharpe (1997), Morningstar ranks funds within four asset classes before 1996 and within smaller categories thereafter. We replicate the latter rating scheme because it covers most of our sample period. Since we do not have access to the criteria that Morningstar uses to classify funds into style categories, we continue to employ the classifications used for our primary tests. Morningstar rates all funds based on their 3-, 5- and 10-year return and risk, respectively and then a weighted overall rating is determined. We follow the procedure described in Nanda, Wang, and Zheng (2004) to replicate the funds' Morningstar fund ratings.

Our robustness tests are based on the following specification:

$$\begin{aligned} Flow_{i,t} =& a + b_1 Perf\_1yr_{i,t} + b_2 Perf\_3yr_{i,t} + b_3 Perf\_5yr_{i,t} + b_4 Perf\_Incep_{i,t} + \\ & b_5 MorningstarRating_{i,t} + cMin\_Rank_{i,t} + Controls + \epsilon_{i,t}, \end{aligned} \tag{12}$$

The *Morningstar Rating* represents fund $i'$s rating based on the Morningstar fund rating scheme in quarter $t$. The variable $Perf\_Incep_{i,t}$ captures a fund $i'$s performance since inception in quarter

[15] See, e.g., Del Guercio and Tkac (2002).

$t$.

Table VII presents results of the robustness tests. We note that the coefficient of $Min\_Rank$ is consistently positive and significant for the sample of retail funds when we control for performance since inception and/or Morningstar rating, while it is insignificant for the institutional sample. Moreover, the coefficient of $Perf\_Incep$ is negative and insignificant in all columns. However, the coefficient for $Morningstar\ Rating$ is consistently positive and significant. For example, based on results in Column 5, if a retail fund's Morningstar rating increases by 1 star, there is a corresponding increase of 1.6% in terms of fund flows. By contrast, for an institutional fund, a 1 star increase in the Morningstar rating is associated with a fund flow increase of only 0.7%, based on the results in Column 6. These results suggest that the Morningstar rating is an important signal for retail investors when making investment decisions. On the other hand, institutional investors seem to rely to a much lesser extent on this rating.

One of the most well-known findings in mutual fund literature is the convex relationship between investor flows and past fund performance. Previous studies (e.g. Sirri and Tufano (1998), Chevalier and Ellison (1997)) find that investor flows respond strongly to good past performance while being less sensitive to bad performance. As we note in Appendix A (see footnote 22), in the context of our model, the flow-performance convexity is potentially complementary to the effect of investor ambiguity aversion. We now show that our baseline result is robust after controlling for the convex flow-performance relationship. Table VIII presents results of this test. As in the baseline model in Equation (2), we regress quarterly flow on past performance measured over 1-, 3-, and 5-year. We adopt fractional ranks for each of the 1-, 3-, and 5-year performance (same as $Low\_3yr$, $Mid\_3yr$ and $High\_3yr$ defined in Equation (8) to capture the convexity in flow-performance relationship. The results presented in Table VIII confirm that the additional sensitivity to the minimum performance measure remains positive and significant in the retail fund sample even after controlling for the convexity of the flow-performance relationship.

Another possible concern is that some time variant factors, such as a change in the fund manager, may cause differential flow sensitivity to performance measured over different time horizons. To deal with this issue, in unreported tests, we consider various performance measures calculated over a fixed time horizon (1-, 3- or 5-year). The various performance measures considered include the CAPM alpha, Fama-French 3-factor alpha, Carhart 4-factor alpha, raw return, and the Morningstar fund rating. The various performance measures calculated over the identical time horizon may be viewed as multiple performance signals. We find that our results are qualitatively unchanged. In particular, we find that for each of the three time horizons considered, retail fund flows display additional sensitivity to the minimum performance measure.

In conclusion, the results of the robustness tests confirm that mutual fund flows display an additional sensitivity to the minimum performance rank even after controlling for performance since inception, the funds' Morningstar star ratings, and convexity in the flow-performance relation.

## VIII. Concluding Remarks

This paper presents a model of an ambiguity averse mutual fund investor who faces multiple signals about the performance of the fund. The model implies that an ambiguity averse investor, in attempting to learn about manager skill, always puts additional weight on the worst signal. We empirically test the key implications of the model by using a fund's past performance measured over multiple time horizons, as multiple signals about manager skill observed by investors.

We find that consistent with our model, fund flows display heightened sensitivity to the minimum performance measure. Further, we observe this ambiguity averse behavior only among retail fund investors. By contrast, fund flows in institutional funds appear to be more consistent with standard Bayesian learning behavior. We also find that funds with more frequent/aggressive strategy changes, funds that have more volatile past flows, and funds belonging to smaller fund families appear more ambiguous to investors. Furthermore, we distinguish between the effect of increased performance volatility on ambiguity averse investors and on investors whose behavior is more consistent with Bayesian learning. We find that fund volatility increases ambiguity averse investors' response to the minimum performance measure while it dampens the Bayesian investors' sensitivity to past performance in general. Taken together, these results suggest that fund investor behavior is best characterized as reflecting both Bayesian learning and ambiguity aversion.

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**Table I Summary Statistics of the Equity Mutual Fund Sample**

This table reports the time-series averages of quarterly cross-sectional averages of fund characteristics for the period 1993-2011. TNA is the total net assets. Flow is the percentage change in TNA. Expense Ratio is the total annual management and administrative expenses divided by average TNA. Total Expense is estimated as expense ratio plus 1/7 of maximum front-end load. 12-b1 Fee are fees paid by the fund out of fund assets to cover marketing expenses, distribution expenses and sometimes shareholder service expenses. The 12-b1 fees are only available since 1992. Raw Return is the average monthly raw return during the prior 12 months and Vol. of Raw Return is the corresponding standard deviation. The statistics are reported for all funds (i.e., share classes), funds open only to retail investors, and funds available only to institutional investors.

| | Share Classes | | |
|---|---|---|---|
| | All Funds | Retail Shares | Institutional Shares |
| Fund Share Classes | 7,020 | 5,781 | 1,239 |
| Average # of Fund Share Classes | 2,847 | 2,359 | 466 |
| Flow (in % per quarter) | 1.51 | 1.32 | 2.33 |
| Age (in years) | 11.60 | 12.07 | 8.67 |
| TNA (in millions) | 678.06 | 739.36 | 330.49 |
| Expense Ratio (in %) | 1.41 | 1.49 | 0.96 |
| Total Expense (in %) | 1.61 | 1.73 | 0.99 |
| 12b-1 Fee (in %) | 0.56 | 0.57 | 0.20 |
| Raw Return (in % per month) | 0.73 | 0.72 | 0.79 |
| Vol. of Raw Return (in % per month) | 4.60 | 4.59 | 4.66 |

**Table II Flow-Performance Sensitivity of Retail and Institutional Investors**

This table examines the flow-performance sensitivity of both retail and institutional investors during period 1993-2011. The sample is divided into retail shares and institutional shares. Each quarter, funds are assigned ranks between zero and one according to their performance during the past 12 months (Perf_1yr), 36 months (Perf_3yr) and 60 months (Perf_5yr) respectively. Performance is measured by the average monthly raw returns or the Carhart (1997) 4-factor alphas. Funds are ranked based on raw returns within the funds' objective category while ranks based on alphas are computed across all funds in the sample. Min_Rank is defined as the minimum performance rank among three periods. A linear regression model is estimated by regressing quarterly flows on funds' three performance ranks (Perf_1yr, Perf_3yr and Perf_5yr) and the minimum rank (Min_Rank). The control variables include fund age (Age), defined as log (1+age), quarterly flow in previous quarter (Previous Quarter Flow), previous quarter raw return (Previous Quarter Return), logarithm of lagged fund TNA (Size), volatility of monthly raw return in prior 12 months (Volatility), lagged total expense (Total Expense), aggregate flow to the fund' objective category (Category Flow), and the maximum drawdown of a fund's monthly return over the previous 60 months (Maximum Drawdown). Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

| Performance Measured by | Raw Return | | 4-Factor Alpha | |
|---|---|---|---|---|
| | Retail | Institutional | Retail | Institutional |
| Intercept | 0.023*** | 0.084*** | -0.005 | 0.037*** |
| | (3.35) | (4.99) | (-0.65) | (2.98) |
| Perf_1yr | 0.037*** | 0.046*** | 0.018*** | 0.030*** |
| | (6.96) | (3.16) | (3.57 ) | (3.26) |
| Perf_3yr | 0.040*** | 0.039*** | 0.044*** | 0.063*** |
| | (7.50) | (2.96) | (10.36 ) | (5.95) |
| Perf_5yr | 0.013*** | 0.045*** | -0.005 | 0.026*** |
| | (2.93) | (3.34) | (-0.81 ) | (2.57) |
| Min_Rank | 0.047*** | 0.012 | 0.057*** | 0.009 |
| | (6.99) | (0.63) | (9.17) | (0.81) |
| Age | -0.016*** | -0.025*** | -0.012*** | -0.018*** |
| | (-9.36) | (-6.77) | (-8.82) | (-5.48) |
| Previous Quarter Flow | 0.177*** | 0.143*** | 0.182*** | 0.158*** |
| | (7.51) | (5.62) | (7.39) | (5.35) |
| Previous Quarter Return | 0.246*** | 0.120** | 0.308*** | 0.217*** |
| | (6.49) | (2.63) | (9.77) | (5.17) |
| Size | -0.004*** | -0.010*** | -0.004*** | -0.009*** |
| | (-6.40) | (-7.46) | (-7.97) | (-7.90) |
| Volatility | -0.066 | -0.209 | 0.307* | 0.273 |
| | (-0.52) | (-1.27) | (1.90) | (1.21) |
| Total Expense | -0.369*** | -1.823*** | -0.455** | -1.818*** |
| | (-2.35) | (-3.12) | (-2.87) | (-3.04) |
| Category Flow | 0.261*** | 0.089 | 0.128** | 0.051 |
| | (3.41) | (0.55) | (2.33) | (0.41) |
| Maximum Drawdown | 0.261*** | 0.089 | 0.128** | 0.051 |
| | (3.41) | (0.55) | (2.33) | (0.41) |

## Table III Flow-Performance Sensitivity and Changes in Fund Strategy

This table presents results of tests that use a measure of a fund's strategy changes as a proxy for the fund's ambiguity level. The sample includes only the retail funds. Each quarter, the sample is divided into three groups, Low, Mid, and High based on the value of the strategy change measure. Panel A reports results when strategy changes are measured by the average absolute change in the fund's (Carhart (1997) model) factor loadings between the prior 1-30 and 31-60 months. In Panel B, fund strategy changes are measured by a fund's R-squared value over its lifetime based on the Carhart (1997) 4-factor model. The table presents coefficient estimates obtained by regressing quarterly fund flows on the funds' 1-year, 3-year, and 5-year performance ranks (Perf_1yr, Perf_3yr and Perf_5yr) and the minimum rank (Min_Rank). Performance is measured by the average monthly raw returns or the Carhart (1997) 4-factor alphas. The control variables are identical to those in Table II. Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

Panel A: Strategy Changes Proxied by Change in Factor Loadings

| Performance Measured by | Raw Return | | | 4-Factor Alpha | | |
|---|---|---|---|---|---|---|
| | Low | Mid | High | Low | Mid | High |
| Intercept | 0.027** | 0.028*** | 0.016* | -0.022* | -0.001 | 0.007 |
| | (2.36) | (2.85) | (1.75) | (-1.96) | (-0.12) | (0.81) |
| Perf_1yr | 0.039*** | 0.032*** | 0.033*** | 0.018** | 0.014** | 0.020*** |
| | (4.68) | (5.02) | (5.19) | (2.17) | (2.19) | (3.63) |
| Perf_3yr | 0.036*** | 0.032*** | 0.037*** | 0.049*** | 0.043*** | 0.033*** |
| | (6.13) | (5.73) | (4.70) | (6.05) | (8.52) | (4.36) |
| Perf_5yr | 0.032*** | 0.012** | 0.011* | 0.011 | -0.008 | -0.004 |
| | (6.19) | (2.04) | (1.84) | (1.53) | (-1.08) | (-0.97) |
| Min_Rank[a] | 0.014 | 0.044*** | 0.058*** | 0.029*** | 0.053*** | 0.060*** |
| | (1.53) | (4.01) | (6.06) | (2.58) | (5.39) | (5.36) |
| Control Variables | | | | Yes | | |

Panel B: Panel B: Strategy Changes Proxied by Fund R-Squared Values

| Performance Measured by | Raw Return | | | 4-Factor Alpha | | |
|---|---|---|---|---|---|---|
| | Low | Mid | High | Low | Mid | High |
| Intercept | 0.016* | 0.036*** | 0.019 | 0.003 | -0.001 | -0.019 |
| | (1.98) | (3.42) | (1.62) | (0.31) | (-0.06) | (-1.33) |
| Perf_1yr | 0.036*** | 0.040*** | 0.034*** | 0.019*** | 0.021*** | 0.011 |
| | (7.61) | (5.05) | (3.37) | (3.66) | (3.40) | (1.19) |
| Perf_3yr | 0.034*** | 0.034*** | 0.049*** | 0.036*** | 0.041*** | 0.060*** |
| | (4.33) | (5.36) | (6.36) | (4.87) | (6.94) | (4.89) |
| Perf_5yr | 0.007 | 0.020*** | 0.015* | -0.012** | 0.003 | -0.005 |
| | (1.46) | (3.29) | (1.89) | (-2.35) | (0.58) | (-0.53) |
| Min_Rank[b] | 0.051*** | 0.033*** | 0.023* | 0.058*** | 0.038*** | 0.043*** |
| | (5.82) | (3.70) | (1.70) | (6.73) | (4.60) | (3.40) |
| Control Variables | | | | Yes | | |

[a] The value of the $F$-test statistic (*p-value*) corresponding to a test of equality of coefficients of Min_Rank across the three groups, Low, Mid, and High, equals 5.16 (*0.006*) when funds are ranked by raw return and it equals 2.64 (*0.073*) for the case when funds are ranked by 4-factor alphas.

[b] The value of the $F$-test statistic (*p-value*) corresponding to a test of equality of coefficients of Min_Rank across the three groups, Low, Mid, and High, equals 1.81 (*0.166*) when funds are ranked by raw return and it equals 1.06 (*0.349*) for the case when funds are ranked by 4-factor alphas.

## Table IV Flow-Performance Sensitivity and Fund Flow Volatility

This table presents results of tests that use flow volatility as a proxy for a fund's ambiguity level. The sample includes only the retail funds. Flow_Vol is the standard deviation of quarterly flows over prior 12 quarters. Each quarter, a dummy variable Low_Flowvol equals one if the flow volatility falls in the bottom tercile, a dummy variable Mid_Flowvol equals one if the flow volatility belongs to the medium tercile and a dummy variable High_Flowvol equals one if the flow volatility is in the top tercile. A linear regression is performed by regressing quarterly flows on funds' three performance ranks (Perf_1yr, Perf_3yr and Perf_5yr) and the minimum rank (Min_Rank) interacted with the three dummy variables. The control variables are identical to those in Table II. Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

| Performance Measured by | Raw Return | t-statistic | 4-Factor Alpha | t-statistic |
|---|---|---|---|---|
| Intercept | -0.001 | (-0.20) | -0.020*** | (-2.81) |
| Low_Flowvol×Perf_1yr | 0.034*** | (8.82) | 0.017*** | (3.83) |
| Mid_Flowvol×Perf_1yr | 0.039*** | (5.99) | 0.026*** | (3.98) |
| High_Flowvol×Perf_1yr | 0.059*** | (6.25) | 0.024*** | (2.78) |
| Low_Flowvol×Perf_3yr | 0.027*** | (6.02) | 0.026*** | (5.16) |
| Mid_Flowvol×Perf_3yr | 0.038*** | (6.90) | 0.046*** | (8.10) |
| High_Flowvol×Perf_3yr | 0.048*** | (4.89) | 0.060*** | (6.71) |
| Low_Flowvol×Perf_5yr | 0.037*** | (9.65) | 0.022*** | (4.32) |
| Mid_Flowvol×Perf_5yr | 0.010* | (1.88) | -0.011*** | (-1.45) |
| High_Flowvol×Perf_5yr | -0.031*** | (-3.72) | -0.050*** | (-5.87) |
| Low_Flowvol×Min_Rank[1] | -0.004 | (-0.85) | 0.005 | (1.02) |
| Mid_Flowvol×Min_Rank | 0.038*** | (4.76) | 0.040*** | (4.64) |
| High_Flowvol×Min_Rank | 0.108*** | (8.09) | 0.124*** | (8.83) |
| Flow_Vol | 0.057*** | (5.76) | 0.040*** | (4.18) |
| Control Variables | | Yes | | |

[a]The value of the $F$-test statistic (*p-value*) corresponding to a test of equality of coefficients of Low_Flowvol×Min_Rank, Mid_Flowvol×Min_Rank and High_Flowvol×Min_Rank, equals 35.04 (*0.000*) when funds are ranked by raw return and it equals 32.03 (*0.000*) for the case when funds are ranked by 4-factor alphas.

## Table V Flow-Performance Sensitivity and Fund Family Size

This table presents results of tests using family size as a proxy for a fund's ambiguity level. The sample includes only retail funds. Family size represents the total net assets of all fund share classes that belong to the same fund family. Each quarter, a dummy variable Low_FamSize equals one if fund $i$ belongs to a family whose size falls into the bottom tercile, a dummy variable Mid_FamSize equals one if family size belongs to the medium tercile, and a dummy variable High_FamSize equals one if the family size is in the top tercile. D_FamliySize is a dummy variable that equals one if family size is above the median value for that quarter. A linear regression is performed by regressing quarterly flows on funds' three performance ranks (Perf_1yr, Perf_3yr and Perf_5yr) and the minimum rank (Min_Rank) interacted with the three dummy variables. The control variables are identical to those in Table II. Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

| Performance Measured by | Raw Return | t-statistic | 4-Factor Alpha | t-statistic |
|---|---|---|---|---|
| Intercept | 0.039*** | (5.53) | 0.013 | (1.65) |
| Low_Famsize×Perf_1yr | 0.032*** | (5.70) | 0.016** | (2.37) |
| Mid_Famsize×Perf_1yr | 0.032*** | (6.17) | 0.017*** | (3.57) |
| High_Famsize×Perf_1yr | 0.049*** | (4.55) | 0.026*** | (3.04) |
| Low_Famsize×Perf_3yr | 0.027*** | (4.80) | 0.035*** | (5.31) |
| Mid_Famsize×Perf_3yr | 0.039*** | (4.58) | 0.045*** | (5.58) |
| High_Famsize×Perf_3yr | 0.047*** | (5.78) | 0.051*** | (7.99) |
| Low_Famsize×Perf_5yr | -0.002 | (-0.39) | -0.015 | (-1.63) |
| Mid_Famsize×Perf_5yr | 0.010 | (1.63) | -0.008 | (-1.24) |
| High_Famsize×Perf_5yr | 0.033*** | (6.24) | 0.016** | (2.48) |
| Low_Famsize×Min_Rank[1] | 0.069*** | (6.26) | 0.066*** | (5.65) |
| Mid_Famsize×Min_Rank | 0.058*** | (6.33) | 0.065*** | (5.70) |
| High_Famsize×Min_Rank | 0.018* | (1.78) | 0.034*** | (4.48) |
| D_FamilySize | 0.008*** | (4.00) | 0.008*** | (3.32) |
| Control Variables | | Yes | | |

[a] The value of the $F$-test statistic (*p-value*) corresponding to a test of equality of coefficients of Low_Famsize×Min_Rank, Mid_Famsize×Min_Rank and High_Famsize×Min_Rank, equals 7.02 (*0.001*) when funds are ranked by raw return and it equals 3.18 (*0.043*) for the case when funds are ranked by 4-factor alphas.

## Table VI Contrast between the Response of Ambiguity Averse Investors and Bayesian Investors

This table examines the differences in the response of ambiguity averse investors and Bayesian investors during the periods: 1983-2006 and 1993-2011. The sample includes all retail and institutional funds. Min_Rank is the same as defined in Table 2. Perf_3yr is a fractional rank ranging from 0 to 1 reflecting a fund's ranking within an objective category based on the average monthly raw returns over prior 36 months. The fractional rank for funds in the bottom performance quintile (Low_3yr) is Min(Perf_3yr,0.2), in the three medium quintiles (Mid_3yr) is Min(0.6, Perf_3yr-Low_3yr), and in the top quintiles (High_3yr) is Perf_3yr-Mid_3yr-Low_3yr. Vol is the standard deviation of raw returns over prior 36 months. The dummy variable Min_Ind equals one if the 3-year performance rank is the minimum among the performance ranks during the 1-year, 3-year and 5-year ranking periods. The dummy variable Inst equals one if the fund is open only to institutional investors. Each quarter, a linear regression is performed by regressing quarterly flows on funds' Perf_3yr and its interaction with Min, Vol, and Inst. The control variables are identical to those in Table II. Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

| | Panel A: 1993-2011 | | | | Panel B: 1983-2006 | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Intercept | 0.052*** | 0.033*** | 0.031*** | 0.032*** | 0.038*** | 0.030** | 0.029** | 0.032** |
| | (6.36) | (4.90) | (4.87) | (4.70) | (3.32) | (2.41) | (2.37) | (2.58) |
| Min_Rank | 0.085*** | . | 0.072*** | 0.073*** | 0.090*** | . | 0.078*** | 0.078*** |
| | (12.68) | . | (11.99) | (12.30) | (13.09) | . | (11.32) | (11.35) |
| Min_Ind×Vol×Perf_3yr | . | . | 0.410*** | 0.434*** | . | . | 0.411*** | 0.439*** |
| | . | . | (5.32) | (5.17) | . | . | (2.96) | (3.12) |
| Vol×Perf_3yr | . | -0.969*** | -0.868*** | -0.850*** | . | -0.631** | -0.534* | -0.524* |
| | . | (-3.30) | (-3.11) | (-2.99) | . | (-2.14) | (-1.89) | (-1.83) |
| Inst×Min_Ind×Vol×Perf_3yr | . | . | . | -0.182 | . | . | . | -0.378* |
| | . | . | . | (-1.16) | . | . | . | (-1.90) |
| Inst×Vol×Perf_3yr | . | . | . | -0.355** | . | . | . | -0.280 |
| | . | . | . | (-2.06) | . | . | . | (-1.66) |
| Inst | . | . | . | 0.004 | . | . | . | -0.005 |
| | . | . | . | (1.22) | . | . | . | (-1.31) |
| Low_3yr | 0.026 | 0.132*** | 0.078*** | 0.076*** | 0.021 | 0.107*** | 0.051* | 0.048* |
| | (1.61) | (6.23) | (3.62) | (3.52) | (1.00) | (3.85) | (1.82) | (1.71) |
| Mid_3yr | 0.026*** | 0.129*** | 0.077*** | 0.079*** | 0.009 | 0.096*** | 0.040*** | 0.042*** |
| | (4.90) | (8.42) | (4.62) | (4.82) | (1.52) | (6.99) | (3.07) | (3.22) |
| High_3yr | 0.255*** | 0.416*** | 0.342*** | 0.343*** | 0.214*** | 0.345*** | 0.266*** | 0.269*** |
| | (10.81) | (12.90) | (10.69) | (10.94) | (7.51) | (10.37) | (8.69) | (8.60) |
| Control Variables | | | | Yes | | | | |

## Table VII Robustness Test Controlling for Morningstar Rating and/or Performance since Inception

This table reexamines the baseline model that studies ambiguity aversion behavior of both retail and institutional investors when controlling for performance since inception and/or the Morningstar star rating during the period 1993-2011. The sample includes all retail and institutional shares. Each quarter, funds are assigned ranks between zero and one within their objective category according to their performance during past 12 months (Perf_1yr), 36 months (Perf_3yr), 60 months (Perf_5yr) and since inception (Perf_Incep), respectively. Performance is measured by the ranking within category of the average monthly raw returns. Morningstar overall rating (Morningstar Rating) is a weighted average of a fund's 3, 5 and 10 years category star rating ranging from 1 to 5 (See Nanda et al. (2004) for details). Min_Rank is defined as the minimum performance rank among 1-year, 3-year and 5-year performance ranks. A linear regression is performed by regressing quarterly flows on funds' three performance ranks and the minimum rank. The control variables are identical to those in Table II. Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

| | Retail | Institutional | Retail | Institutional | Retail | Institutional |
|---|---|---|---|---|---|---|
| Intercept | 0.009 | 0.076*** | -0.020*** | 0.062*** | -0.030*** | 0.062*** |
| | (1.15) | (4.81) | (-2.23) | (3.33) | (-3.67) | (3.52) |
| Perf_1yr | 0.035*** | 0.049*** | 0.036*** | 0.053*** | 0.036*** | 0.052*** |
| | (7.61) | (3.24) | (7.70) | (3.29) | (7.68) | (3.26) |
| Perf_3yr | 0.035*** | 0.032 | 0.019*** | 0.025 | 0.019*** | 0.023 |
| | (6.74) | (2.06) | (3.69) | (1.38) | (3.85) | (1.25) |
| Perf_5yr | 0.019*** | 0.051*** | -0.007 | 0.033** | -0.007 | 0.040*** |
| | (3.69) | (3.01) | (-1.08) | (2.41) | (-1.10) | (2.73) |
| Perf_Incep | -0.001 | -0.013 | . | . | -0.009* | -0.016* |
| | (-0.35) | (-1.35) | . | . | (-1.91) | (-1.68) |
| Morningstar Rating | . | . | 0.015*** | 0.005 | 0.016*** | 0.007* |
| | . | . | (8.74) | (1.47) | (9.57) | (1.72) |
| Min_Rank | 0.050*** | 0.019 | 0.044*** | 0.014 | 0.044*** | 0.014 |
| | (7.49) | (1.07) | (6.81) | (0.69) | (6.76) | (0.72) |
| Control Variables | | | | Yes | | |

**Table VIII Robustness Test Controlling for Flow-Performance Convexity**

This table reexamines the baseline model that studies ambiguity aversion behavior of both retail and institutional investors when controlling for the convexity in flow-performance relationship during the period 1993-2011. The sample includes all retail and institutional shares. Each quarter, we adopt fractional ranks for funds' performance measured over past 1-, 3-, and 5-year horizon. For example, the 1-year performance of a fund in the bottom quintile (Low_1yr) is defined as Min(Perf_1yr, 0.2), in the three medium quintiles (Mid_1yr) is defined as Min(0.6, Perf_1yr-Low_1yr), and in the top quintiles (High_1yr) is defined as Perf_1yr-Low_1yr-Mid_1yr. Performance is measured by the ranking within category of the average monthly raw returns. Min_Rank is defined as the minimum performance rank among 1-year, 3-year and 5-year performance ranks. A linear regression is performed by regressing quarterly flows on funds' fractional performance ranks and the minimum rank. The control variables are identical to those in Table II. Time-series averages of coefficients and the Newey-West t-statistics (in parentheses) are reported. The symbols *, **, and *** denote significance at the 10%, 5% and 1% level, respectively.

| Performance Measured by | Raw return | | 4-Factor Alpha | |
|---|---|---|---|---|
| | Retail | Institutional | Retail | Institutional |
| Intercept | 0.043*** | 0.125*** | 0.014 | 0.061*** |
| | (5.61) | (8.06) | (1.44) | (4.09) |
| Low_1yr | 0.048*** | 0.073** | 0.043** | -0.013 |
| | (3.19) | (2.42) | (2.61) | (-0.35) |
| Mid_1yr | 0.021*** | 0.046*** | 0.006 | 0.031** |
| | (4.28) | (2.82) | (1.37) | (2.06) |
| High_1yr | 0.209*** | 0.077* | 0.165*** | 0.078** |
| | (8.73) | (1.92) | (9.18) | (2.02) |
| Low_3yr | 0.042*** | 0.032 | 0.028*** | 0.158** |
| | (2.89) | (0.52) | (2.75) | (2.57) |
| Mid_3yr | 0.026*** | 0.032** | 0.032*** | 0.050*** |
| | (4.49) | (2.33) | (7.09) | (4.66) |
| High_3yr | 0.181*** | 0.142*** | 0.201*** | 0.206*** |
| | (6.45) | (2.67) | (8.73) | (3.36) |
| Low_5yr | -0.036** | -0.172*** | -0.017 | -0.081 |
| | (-2.13) | (-3.26) | (-1.13) | (-1.42) |
| Mid_5yr | 0.015*** | 0.053*** | -0.003 | 0.025** |
| | (3.40) | (3.48) | (-0.37) | (2.03) |
| High_5yr | 0.061** | 0.123*** | 0.022 | 0.074 |
| | (2.50) | (2.82) | (0.95) | (1.58) |
| Min_Rank | 0.036*** | 0.001 | 0.040*** | -0.005 |
| | (5.00) | (0.05) | (6.99) | (-0.31) |
| Control Variables | | Yes | | |

## Appendix A. A Model of an Ambiguity Averse Fund Investor

In this section, we build a model to analyze the important features of the flow-performance sensitivity for an ambiguity averse mutual fund investor and derive testable empirical predictions.

**The Model**

We model the potential capital pool for a fund by assuming that there is a representative investor who has increasing marginal cost of capital. The supply of capital is captured by a strictly increasing function $F(k)$ with support in $[0, \infty)$, where $F(k)$ represents the amount of capital with the opportunity cost below $k$. The investor decides how much to invest in a fund by assessing whether the prospect of future performance of the fund is high enough to justify the opportunity cost of capital.

The fund's return, $R$, is governed by

$$R = \mu + \alpha + \epsilon, \tag{A.1}$$

where $\alpha$ denotes the fund manager skill, $\mu$ is the market risk premium given the fund's risk profile, and $\epsilon \sim N\left(0, \sigma_\epsilon^2\right)$ represents the noisy component of the fund's return. The risk-free rate is normalized to zero. We note that the managerial skill, $\alpha$, is not directly observable by the investor. The investor is assumed to have knowledge about the distribution of skill in the population of the fund managers, i.e., investor knows a priori: $\alpha \sim N\left(\mu_\alpha, \sigma_\alpha^2\right)$. The investor, at the time of making the fund investment decision, observes two signals, $s_1$ and $s_2$, about managerial ability $\alpha$. More specifically, $s_i = (\alpha - \mu_\alpha) + \eta_i$, with

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}\right).^{16} \tag{A.2}$$

We define the signal-to-noise ratio by $H_i = \sigma_\alpha^2/\sigma_i^2$ which captures the precision of signal $i$ $(i = 1, 2)$. Through the standard Bayesian updating, we have the posterior distribution of $\alpha$, after observing the two signals, as

$$N\left(\mu_\alpha + \beta^T s, \sigma_\alpha^2/H\right) \equiv N\left(\mu_p, \sigma_p^2\right), \tag{A.3}$$

where $\beta = (H_1/H, H_2/H)^T$, and $H = 1 + H_1 + H_2$.

[16] There may be some potential interest in analyzing the ambiguity of the signal correlation. But this feature is turned off here by assuming the noise terms in the two signals are uncorrelated.

What differentiates our model from the standard Bayesian model is that we assume the investor is uncertain about the precision of the signals. Specifically, the precision of signal $i$ ($i = 1, 2$), denoted as $H_i$, can take on one of two values, denoted as $h$ for high, and $l$ for low, i.e., $H_i = h$, or $H_i = l$ (with $l < h$). Given that there are two signals, this leads to four possible scenarios for the investor to consider. The first scenario is the one in which both signals have high precision (i.e., $H_1 = H_2 = h$), while the second scenario represents the case in which both signals have low precision (i.e., $H_1 = H_2 = l$). The third scenario refers to the case in which the first signal has high precision and the second signal has low precision (i.e., $H_1 = h$, and $H_2 = l$), while the fourth scenario represents the case in which the first signal has low precision while the second signal has high precision (i.e., $H_1 = l$, $H_2 = h$). For each of the four scenarios, there is a corresponding joint probability distribution of the two signals, $s_1$ and $s_2$, and the fund return $R$, which is denoted in general by the term, $\pi_m$ ($m = 1, 2, 3, 4$). Hence, the investor faces not one but four candidate probability distributions for the outcomes. This is the essential source of ambiguity faced by the investor.

To capture the investor's attitude about this ambiguity, we adopt the ambiguity model framework in Klibanoff et al. (2005) and Ju and Miao (2012), as described below. In deciding to invest with a fund, the investor evaluates the perceived return from the fund adjusted for the level of ambiguity she is facing. We model the investor's certainty equivalent rate of return, $R_{CEU}$, as:

$$R_{CEU} = -\gamma \ln E_\theta \left[ \exp\left( -\frac{1}{\gamma} E_{\pi_m}(R) \right) \right]. \tag{A.4}$$

This specification deserves some explanation. As described above, due to the presence of ambiguity about the signals' precisions, the investor faces an environment with multiple scenarios characterized by differing perceived precisions of the two signals. The evaluation of the investor's utility in such a case can be broken down into two steps. First, the investor calculates the expected return under each of the four plausible scenarios. This corresponds to the term $E_{\pi_m}(R)$ in the expression for the certainty equivalent rate of return above. To the extent that the investor cares only about the conditional expected return, she behaves in a risk neutral fashion, in this step.

In the second step, the investor weights the outcomes across all of the four plausible scenarios. To fix the idea, we assume that the investor assigns equal weights, represented by the term $\theta$, to the four possible cases, as ex ante the investor considers all of them as being plausible. Then, instead of taking the average of the expected returns across the four cases, the investor averages a concave function, in this case, the negative exponential function, i.e., $-\exp(-\frac{1}{\gamma}(.))$, of the expected return. The notation $E_\theta(\cdot)$ denotes the averaging across scenarios, where the subscript $\theta$ indicates the assigned weights for the four scenarios. The final adjustment, by applying the transformation $-\gamma \ln(-(.))$ to the resulting value, which is the inverse function of the negative exponential function, serves to convert the resulting value back to the returns scale. This calculation yields a constant return that the investor would view as utility-equivalent to that of the ambiguous return from investing in the fund.

The investor's attitude towards the ambiguity in the environment is characterized by the concavity of the negative exponential function, $-\exp(-\frac{1}{\gamma}(.))$, in Equation (A.4). Due to Jensen's inequality, the effect of inserting a concave function (the negative exponential function in our case) between the expected return calculation in the first step, and the final averaging across the four scenarios, is that the certainty equivalent return, $R_{CEU}$, calculated in Equation (A.4) is lower than the simple average of the expected returns, $E_{\pi_m}(R)$, across the four scenarios. In other words, the investor is averse to the variation in expected returns across the four scenarios.

To gain further insight into the effect of the concave function in Equation (A.4) we may ask the question: how much weight does each scenario carry in the investor's decision process? To answer this question we can examine how the investor's certainty equivalent return changes in response to changes in the expected returns, $E_{\pi_m}(R)$, for the various scenarios. The overall change in the certainty equivalent return, $R_{CEU}$, is determined by total differentiation of Equation (A.4). As a result, the change in the certainty equivalent return can be expressed as a weighted average of the changes in the expected returns across the four scenarios, with the weight of each scenario, $\lambda_m$, being negatively correlated with the expected return in that scenario. That is,

$$dR_{CEU} = \sum_{m=1}^{4} \lambda_m d\left[E_{\pi_m}(R)\right], \tag{A.5}$$

where $\sum_{m=1}^{4} \lambda_m = 1$ and $\lambda_m > \lambda_n$, if $E_{\pi_m}(R) < E_{\pi_n}(R)$.[17] This representation of the change in the certainty equivalent return makes it clear that the investor behaves as if she puts more weight

[17]Note that, $\lambda_m = \frac{exp\left[-\frac{1}{\gamma}E_{\pi_m}(R)\right]}{\sum_{n=1}^{4} exp\left[-\frac{1}{\gamma}E_{\pi_n}(R)\right]}$, via total differentiation of Equation (A.4) and identifying terms from Equation (A.5). From this expression it may be seen that $\lambda_m$ is negatively correlated with the expected return, $E_{\pi_m}(R)$.

on the worst case scenario. In this sense the investor acts conservatively and exhibits ambiguity aversion. The strength of this effect is directly related to the value of the parameter $\gamma$.

As a point of comparison, the maxmin utility approach considers the four possible scenarios, and then effectively focuses on the case that gives the lowest utility outcome as the only relevant case. Such an approach captures extreme ambiguity aversion: the investor is so concerned about the worst case scenario that she completely ignores all other possible cases which she had deemed plausible ex ante. Here, we follow Klibanoff et al. (2005) by considering a less extreme weighting scheme to model the investor's ambiguity aversion. Under our approach the investor behaves as if she assigns positive, albeit smaller, weights to the cases with outcomes other than the worst case outcome.

Note that due to the nonlinear function (the negative exponential function) between the two expectation calculations, namely $E_\pi$ and $E_\theta$ in Equation (A.4), the investor's certainty equivalent return in (A.4) cannot be reduced to a single, composite expectation. In other words, the behavior of a fund investor with the preference specification in Equation (A.4) cannot be consistently represented by some form of expected utility function, and therefore, her behavior deviates from the expected utility paradigm. Such deviation highlights the ambiguity averse behavior captured by the modeling technique of Klibanoff et al. (2005) that is intrinsically different from the traditional risk aversion in the expected utility paradigm. Also note that in our context, the degree of ambiguity faced by the investor is directly related to the difference between the two possible levels of signal precision, namely $h$ (high) and $l$ (low).[18] In the limiting case, as this difference converges to zero, we have the standard Bayesian learning framework.

The investor's objective is to maximize the net gain from investing in a fund. The net gain is the total dollar return from investing in the fund net of the cost of capital invested. The investor's optimization problem boils down to choosing an optimal scale of investment in the fund conditional on the observed signals regarding the fund manager skill. The solution to the problem is provided in the Appendix B. We note the following proposition regarding the fund flow-performance sensitivity.

PROPOSITION 1: *The flow-performance sensitivity is higher for the signal with relatively lower*

[18]In this framework, the investor is unable to learn about the signal precision over time, possibly due to cognitive or informational constraints. For more detailed discussion and justification of the time-invariant nature of ambiguity, see Epstein and Schneider (2007).

*realized value:*

$$\frac{dF(k^\star)}{ds_1} > \frac{dF(k^\star)}{ds_2} \tag{A.6}$$

*if and only if $s_1 < s_2$, i.e., in relative terms, the signal, $s_1$, conveys bad news, while the signal, $s_2$, conveys good news.*[19]

*Proof:* See Appendix B.

Thus, the model implies that ambiguity averse investors' fund flow is more responsive to the worst signal. In other words, in the population of ambiguity averse investors that face multiple signals, we expect to observe heightened flow sensitivity towards the signal that conveys bad news.

Denote the (scaled) gap between the high and low values of the signal precisions, $(h - l)/2$, by $\delta$. Clearly, the higher the parameter value for $\delta$, the wider the gap between the two possible values for the signal precision, and therefore the higher the ambiguity the investor faces. As a result, we would expect the investor to display stronger ambiguity aversion as the gap between the signal precisions widens. We have the following corollary.

COROLLARY 1: *The extra sensitivity to the worst signal is higher when the level of ambiguity is higher, keeping the average precision of the signal constant (i.e., $(h + l)/2$ is fixed).*[20] *That is:*

$$\frac{d}{d\delta}\left(\frac{dF}{ds_1} - \frac{dF}{ds_2}\right) > 0 \tag{A.7}$$

*if $s_1 < s_2$.*

*Proof:* See Appendix B.

**Bayesian Benchmark**

Next, we contrast our model with the standard Bayesian benchmark case with risk averse utility.

*Bayesian Benchmark with No Signal Precision Uncertainty*

Consider first the simplest case of a risk averse investor with exponential utility and risk aversion

[19]Remark: In the model, the shape of the flow-performance relation (i.e., whether or not it is convex) depends on the specification of the cumulative distribution function $F(k)$. On the one hand, this implies that the model is unable to explain why the flow-performance relation has a specific functional form, because such a relation is driven by direct assumption. On the other hand, the model is flexible enough to allow for such a relation. Our point is that our key result, namely, that the flow-performance sensitivity is higher for the signal with relatively lower realized value, is logically consistent and potentially complementary to the convexity in the flow-performance relation.

[20]It is also straightforward to show that the extra sensitivity to the worst signal is increasing in the degree of investor ambiguity aversion, $\gamma$.

coefficient $a$, i.e., $U(R) = -e^{aR}$, who receives two performance-related signals with known precision. Specifically, consider the case when $h = l \equiv H$. The flow-performance sensitivity is captured by $dF(k^{\star})/ds_i$, where

$$\frac{dF(k^{\star})}{ds_1} = \frac{dF(k^{\star})}{ds_2} = F' \times \left(\frac{H}{1+2H}\right). \tag{A.8}$$

From the above expression, it is clear that in this case the flow-performance sensitivity depends only on signal precision $H$, and is independent of the level of the signal realization. This implies we would not expect to observe additional flow sensitivity to the worst signal in the population of Bayesian investors who do not face signal precision uncertainty.

*Bayesian Benchmark with Signal Precision Uncertainty*

We next consider the case where risk averse investors are uncertain about the precision of the realized signals, $s_1$ and $s_2$. Specifically, the precision of each signal may take on one of two values: $h$ (high) and $l$ (low).

Note that we assume $s_1 < s_2$, as in the ambiguity aversion model. In this case, the investor faces the following possible scenarios: (1) both signal realizations are negative, i.e., $s_1 < s_2 < 0$; (2) both signal realizations are positive; and (3) the signal realizations are opposite in sign, i.e., $s_1 < 0$, and $s_2 > 0$.

To explore these cases we numerically analyze the investor flow sensitivity to the respective performance related signals over a range of plausible values of the signals. We calibrate the key parameters for this analysis based on the sample of equity mutual funds described in detail in Section IV. In particular, we set the investor's prior regarding the average fund manager skill, $\mu_\alpha$, equal to zero. The value of the parameter $\sigma_\alpha^2$ is set equal to the cross-sectional variance of the Carhart 4-factor alphas across all funds in the sample. Appendix B provides additional details of the numerical analysis.

Intuitively, the Bayesian investor's sensitivity to a signal is a function of the inferred precision of the observed signal. As a result, the further away a signal is from the investor's prior regarding the signal, the lower the signal's implied precision. Recall, that the investor's prior beliefs about the distribution of the signals are centered on zero. Accordingly, if both signal realizations are negative then the investor attaches a lower precision to the signal further away from zero, i.e., the more negative (worst) signal $s_1$. On the other hand, if both signal realizations are positive then the investor attaches a lower precision to the more positive signal, i.e., $s_2$.

Figure 1 shows the difference in the flow-performance sensitivity of the signal, $s_1$, relative to the signal, $s_2$, as a function of the respective signal realizations. Panel A displays the case when both signals have negative realizations, i.e., $s_1 < s_2 < 0$. As is evident, in this case the flow-performance sensitivity to the worst signal, $s_1$, is consistently lower across the entire range of signal realizations. This is in stark contrast to our earlier result for the case with ambiguity aversion.

Panel B shows the difference in the flow-performance sensitivity of signal $s_1$ relative to the signal, $s_2$, when both signals have positive realizations, i.e., $0 < s_1 < s_2$. In this case the flow-performance sensitivity to the worst signal, $s_1$, is higher over a certain range of signal realizations and it is lower outside of this range. This result is also different from the case with ambiguity aversion where the flow-performance sensitivity is consistently higher for the worst signal.

Panel C shows the difference in flow-performance sensitivity when $s_1 < 0 < s_2$. Consistent with intuition, the flow-performance sensitivity is always lower for the signal further away from zero. As a result, there is a certain range of signal realizations in which the flow-performance to the worst signal is higher. However, outside this range it is lower.

In summary, the results of this subsection confirm that the ambiguity aversion model leads to predictions that differ from the Bayesian benchmark case with respect to the investor flow-performance sensitivity to the worst signal. We further confirm that the above contrasting results for the Bayesian benchmark case are qualitatively unchanged even under extreme values of the risk aversion parameter ($a \approx 50$).

**Distinction with Other Behavioral Biases**

In the behavioral finance literature, a number of alternatives have been suggested as departures from the traditional rational agent (Savage utility) paradigm. Examples include loss-averse preferences as well as behavioral biases such as overconfidence.[21] It is worth noting that the hypothesis developed in Proposition 1 is uniquely attributable to the existence of ambiguity aversion on the part of investors. In particular, neither loss aversion, nor overconfidence will lead to a differential sensitivity of investor fund flows to the lower realization signal in the absence of ambiguity aversion. To illustrate this, we next formally examine the implications of loss aversion and overconfidence

[21] Bailey, Kumar, and Ng (2011) provide evidence of the impact of behavioral biases including overconfidence, on the decisions of mutual fund investors.

respectively, on the behavior of investors who face multiple noisy signals. For this analysis we abstract from the effect of ambiguity aversion by imposing the restriction that the two signals have equal precision, i.e., $H_1 = H_2 = H$.

First, consider the case of loss-averse utility preferences. The case with habit formation utility follows in similar fashion. Assume, as in Kahneman and Tversky (1979), the investor displays loss aversion in her utility function but she is not ambiguous about the signal precision. Thus, given the posterior distribution of $\alpha$, we assume the utility function takes the following form:

$$U(R) = E_\pi\left(\alpha \mathbb{1}[0,\infty)\right) + \lambda E_\pi\left(\alpha \mathbb{1}(-\infty,0)\right), \tag{A.9}$$

where $\lambda > 1$. Writing the above equation in explicit form, we have:

$$\begin{aligned} U(R) =& \int_0^\infty \frac{\alpha}{\sqrt{2\pi\sigma_p^2}} e^{\frac{-(\alpha-\mu_p)^2}{2\sigma_p^2}} d\alpha + \lambda \int_{-\infty}^0 \frac{\alpha}{\sqrt{2\pi\sigma_p^2}} e^{\frac{-(\alpha-\mu_p)^2}{2\sigma_p^2}} d\alpha \\ =& \frac{\sigma_p(1-\lambda)}{\sqrt{2\pi}} e^{\frac{-\mu_p^2}{2\sigma_p^2}}, \end{aligned} \tag{A.10}$$

where $\alpha_p$ and $\mu_p$ are given in Equation (A.3) with $H_1 = H_2 \equiv H$. Thus, the flow-performance sensitivity under the assumption of loss aversion is:

$$\frac{dF(k^\star)}{ds_1} = \frac{dF(k^\star)}{ds_2} = \frac{(\lambda-1)H\mu_p^2}{\sigma_p(1+2H)\sqrt{2\pi}} F' e^{\frac{-\mu_p^2}{2\sigma_p^2}}, \tag{A.11}$$

which is independent of whether or not $s_1 < s_2$.

Second, we note that overconfidence cannot by itself result in the asymmetric sensitivity to signals with different realizations. Overconfidence is the belief on the part of the investor that a certain signal is more precise than it actually is. For example, suppose that an overconfident investor receives a signal, $s_1$, from a private channel, while another signal, $s_2$, is publicly available. She is confident that her private signal, $s_1$, is always more reliable than the public signal, $s_2$. As a result, when making investment decisions, the investor allocates additional capital to the fund whenever $s_1$ is sufficiently positive, and conversely she withdraws money from the fund whenever the signal, $s_1$, is negative. The public signal, $s_2$, on the other hand, regardless of its realization, will have a lesser influence on the investor's fund investment decisions. Simply put, under the assumption of overconfidence, regardless of whether $s_2 > s_1$, or $s_2 \leq s_1$, we always have:

$$\frac{dF(k^\star)}{ds_1} > \frac{dF(k^\star)}{ds_2}. \tag{A.12}$$

Obviously under this setting of overconfidence we cannot arrive at Proposition 1. In an environment devoid of ambiguity, a similar argument would apply to investors who are "under-confident" in the sense that they believe their private signal is less precise than it actually is.

## Appendix B. Proofs

**Proof of Proposition 1:** Given that $F(k)$ is strictly increasing, the investor's problem of deciding how much to optimally invest in a fund is equivalent to determining the cutoff level $k$, which we will denote by $k^\star$, and then invest an amount equal to $F(k^\star)$ in the fund. The total cost of capital of investing an amount $F(k^\star)$ in the fund is given by the integral, $\int_0^{k^\star} kdF(k)$. The corresponding total certainty equivalent gain for the investor is: $F(k^\star)R_{CEU}$. Thus, the investor's overall objective is to choose the cutoff $k^\star$ to maximize the gain net of the cost: i.e.,

$$\max_{k^\star} F(k^\star)R_{CEU} - \int_0^{k^\star} kdF(k). \tag{B.1}$$

From Equations (A.1) and (A.3), we have that $E_\pi(R) = \mu + \mu_\alpha + \beta^T s$. Via direct calculation, the investor's certainty equivalent rate of return of investing with the fund, defined in Equation (A.4), is:

$$R_{CEU} = \gamma \log 4 + \mu + \mu_\alpha - \gamma \log \left[ e^{\frac{-h(s_1+s_2)}{\gamma(1+2h)}} + e^{\frac{-l(s_1+s_2)}{\gamma(1+2l)}} + e^{\frac{-(hs_1+ls_2)}{\gamma(1+h+l)}} + e^{\frac{-(ls_1+hs_2)}{\gamma(1+h+l)}} \right].$$

The optimization problem in (B.1) is straightforward, and the necessary and sufficient condition for its unique solution is that the above certainty equivalent return equals the marginal cost of capital at the cutoff level, $k^\star$. That is,

$$k^\star = \gamma \log 4 + \mu + \mu_\alpha - \gamma \log \left[ e^{\frac{-h(s_1+s_2)}{\gamma(1+2h)}} + e^{\frac{-l(s_1+s_2)}{\gamma(1+2l)}} + e^{\frac{-(hs_1+ls_2)}{\gamma(1+h+l)}} + e^{\frac{-(ls_1+hs_2)}{\gamma(1+h+l)}} \right]. \tag{B.2}$$

Thus, the amount of capital under the management of the fund is $F(k^\star)$. Then taking the derivative of $F(k^\star)$ with respect to $s_1$ and $s_2$ respectively, we have the flow-performance sensitivity which is captured by $dF(k^\star)/ds_i$ as

$$\frac{dF(k^\star)}{ds_i} = F' e^{\frac{(k^\star-\mu-\mu_\alpha-\gamma\log 4)}{\gamma}} \Bigg[ \frac{h}{1+2h} e^{-\frac{h(s_1+s_2)}{\gamma(1+2h)}} + \frac{l}{1+2l} e^{-\frac{l(s_1+s_2)}{\gamma(1+2l)}} + \frac{l}{1+h+l} \left( e^{-\frac{(hs_1+ls_2)}{\gamma(1+h+l)}} + e^{-\frac{(ls_1+hs_2)}{\gamma(1+h+l)}} \right) + \frac{h-l}{1+h+l} e^{-\frac{(h-l)s_i+l(s_1+s_2)}{\gamma(1+h+l)}} \Bigg]. \tag{B.3}$$

As a consequence of Equation (B.3), we have

$$\frac{dF(k^\star)}{ds_1} - \frac{dF(k^\star)}{ds_2} = F' e^{\frac{(k^\star-\mu-\mu_\alpha-\gamma\log 4)}{\gamma}} \left( \frac{h-l}{1+h+l} \right) \left[ e^{\frac{-(hs_1+ls_2)}{\gamma(1+h+l)}} - e^{\frac{-(ls_1+hs_2)}{\gamma(1+h+l)}} \right]. \tag{B.4}$$

When $h > l$, we have $hs_1 + ls_2 < ls_1 + hs_2$, and thus the above expression is always positive.

**Proof of Corollary 1:** Taking the derivative of Equation (B.4) with respect to $\delta$, defined as $(h-l)/2$ yields:

$$\frac{d}{d\delta}\left(\frac{dF}{ds_1} - \frac{dF}{ds_2}\right) = \frac{1}{2(1+h+l)X} F' e^{\left[\frac{k^\star-\mu-\mu_\alpha-\gamma\log 4}{\gamma} - \frac{(h+l)(s_1+s_2)}{2(1+h+l)\gamma}\right]} \left[ \left(X^2-1\right) + \delta \frac{s_2-s_1}{1+h+l} \left(X^2+1\right) \right] > 0.$$

**Numerical Analysis of Bayesian Benchmark Case with Signal Precision Uncertainty:** We next provide details for the numerical analysis for the Bayesian benchmark case with signal precision uncertainty discussed in Appendix A. We assume the investors are risk averse with the exponential utility function $U(R) = -e^{aR}$ and are uncertain about the precision of the realized signals, $s_1$ and $s_2$. Specifically, the precision of each signal may take on one of two values: $h$ and $l$. This gives rise to the following four potential models: $M_1$: both $s_1$ and $s_2$ have high precision $h$, i.e., $H_1 = H_2 = h$; $M_2$: both $s_1$ and $s_2$ have low precision $l$, i.e., $H_1 = H_2 = l$; $M_3$: the worst signal $s_1$ has high precision and $s_2$ has low precision, i.e., $H_1 = h, H_2 = l$; $M_4$: the worst signal $s_1$ has low precision and $s_2$ has high precision, i.e., $H_1 = l, H_2 = h$. According to Bayes' rule, the posterior probabilities of the four models, denoted as $P_1 - P_4$, are as follows: $P_1 = \frac{A}{A+B+C+D}, P_2 = \frac{B}{A+B+C+D}, P_3 = \frac{C}{A+B+C+D}, P_4 = \frac{D}{A+B+C+D}$, where

$$A = \frac{h}{\sqrt{1+2h}} \exp\left\{ -\frac{1}{2\sigma_\alpha^2} \left[ h(s_1^2+s_2^2) - \frac{h^2}{1+2h}(s_1+s_2)^2 \right] \right\},$$

$$B = \frac{l}{\sqrt{1+2l}} \exp\left\{ -\frac{1}{2\sigma_\alpha^2} \left[ l(s_1^2+s_2^2) - \frac{l^2}{1+2l}(s_1+s_2)^2 \right] \right\},$$

$$C = \frac{\sqrt{hl}}{\sqrt{1+h+l}} \exp\left\{ -\frac{1}{2\sigma_\alpha^2} \left[ hs_1^2 + ls_2^2 - \frac{1}{1+h+l}(hs_1+ls_2)^2 \right] \right\},$$

$$D = \frac{\sqrt{hl}}{\sqrt{1+h+l}} \exp\left\{ -\frac{1}{2\sigma_\alpha^2} \left[ ls_1^2 + hs_2^2 - \frac{1}{1+h+l}(ls_1+hs_2)^2 \right] \right\}.$$

The investor's expected utility is thus:

$$\begin{aligned}E_\pi(U(R)) &= P_1 \times E(U(R)|M_1) + P_2 \times E(U(R)|M_2) + P_3 \times E(U(R)|M_3) + P_4 \times E(U(R)|M_4) \\ &= P_1 \times (-e^{Y_1}) + P_2 \times (-e^{Y_2}) + P_3 \times (-e^{Y_3}) + P_t \times (-e^{Y_4}),\end{aligned}$$

where

$$\begin{aligned}Y_1 &= -a\left[\mu + \mu_\alpha + \frac{h}{1+2h}(s_1 + s_2)\right] + \frac{1}{2}a^2\left(\sigma_\epsilon^2 + \frac{\sigma_\alpha^2}{1+2H}\right), \\ Y_2 &= -a\left[\mu + \mu_\alpha + \frac{l}{1+2l}(s_1 + s_2)\right] + \frac{1}{2}a^2\left(\sigma_\epsilon^2 + \frac{\sigma_\alpha^2}{1+2H}\right), \\ Y_3 &= -a\left[\mu + \mu_\alpha + \frac{1}{1+h+l}(hs_1 + ls_2)\right] + \frac{1}{2}a^2\left(\sigma_\epsilon^2 + \frac{\sigma_\alpha^2}{1+2H}\right), \\ Y_4 &= -a\left[\mu + \mu_\alpha + \frac{1}{1+h+l}(ls_1 + hs_2)\right] + \frac{1}{2}a^2\left(\sigma_\epsilon^2 + \frac{\sigma_\alpha^2}{1+2H}\right).\end{aligned}$$

We have

$$k^* = \frac{1}{a}\left[\log(A+B+C+D) - \log(Ae^{Y_1} + Be^{Y_2} + Ce^{Y_3} + De^{Y_4})\right]. \tag{B.5}$$

The difference in the flow-performance sensitivity to the signal $s_1$ relative to signal $s_2$ is:

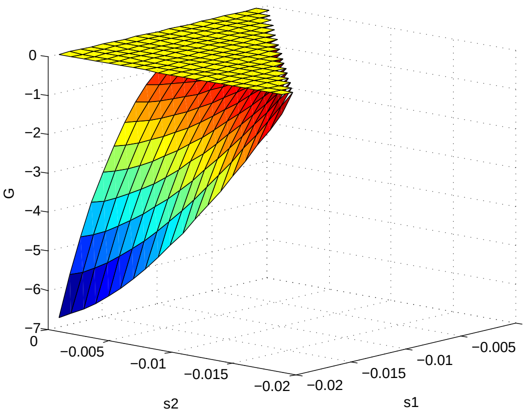
$$\begin{aligned}&\frac{dF(k^\star)}{ds_1} - \frac{dF(k^\star)}{ds_2} \\ &= F' \times \left(\frac{dk^\star}{ds_1} - \frac{dk^\star}{ds_2}\right) \\ &= \frac{F'}{a} \times \left\{\frac{P_1}{\sigma_\alpha^2}(hs_2 - hs_1) + \frac{P_2}{\sigma_\alpha^2}(ls_2 - ls_1) + \frac{P_3}{\sigma_\alpha^2}\left[\frac{(h-l)(hs_1 + ls_2)}{1+h+l} + ls_2 - hs_1\right]\right. \\ &+ \frac{P_4}{\sigma_\alpha^2}\left[\frac{(l-h)(ls_1 + hs_2)}{1+h+l} + hs_2 - ls_1\right] + \frac{Q_1}{\sigma_\alpha^2}(hs_1 - hs_2) + \frac{Q_2}{\sigma_\alpha^2}(ls_1 - ls_2) \\ &+ \frac{Q_3}{\sigma_\alpha^2}\left[\frac{(l-h)(hs_1 + ls_2)}{1+h+l} + \frac{a\sigma_\alpha^2(h-l)}{1+h+l} + hs_1 - ls_2\right] + \frac{Q_4}{\sigma_\alpha^2}\left[\frac{(h-l)(ls_1 + hs_2)}{1+h+l} + \frac{a\sigma_\alpha^2(l-h)}{1+h+l}\right. \\ &\left.\left. + ls_1 - hs_2\right]\right\}\end{aligned}$$

where $Q_1 = Ae^{Y_1}/(Ae^{Y_1} + Be^{Y_2} + Ce^{Y_3} + De^{Y_4}), Q_2 = Be^{Y_2}/(Ae^{Y_1} + Be^{Y_2} + Ce^{Y_3} + De^{Y_4}), Q_3 = Ce^{Y_3}/(Ae^{Y_1} + Be^{Y_2} + Ce^{Y_3} + De^{Y_4})$, and $Q_4 = De^{Y_4}/(Ae^{Y_1} + Be^{Y_2} + Ce^{Y_3} + De^{Y_4})$. We denote

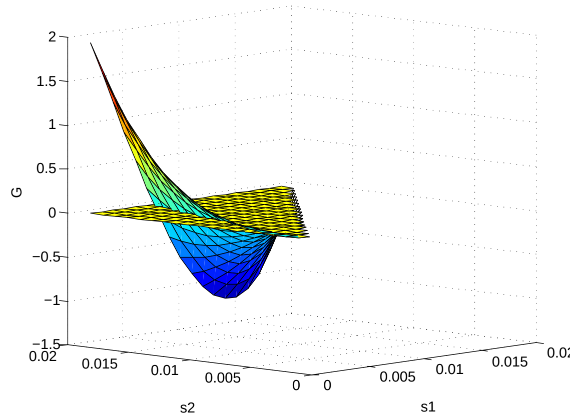
$$\frac{dF(k^\star)}{ds_1} - \frac{dF(k^\star)}{ds_2} \equiv \frac{F'}{a} \times G. \tag{B.6}$$

Since both $F'$ and the degree of risk aversion $a$ are positive, the sign of $dF(k^\star)/ds_1 - dF(k^\star)/ds_2$ is determined by the sign of the term $G$.

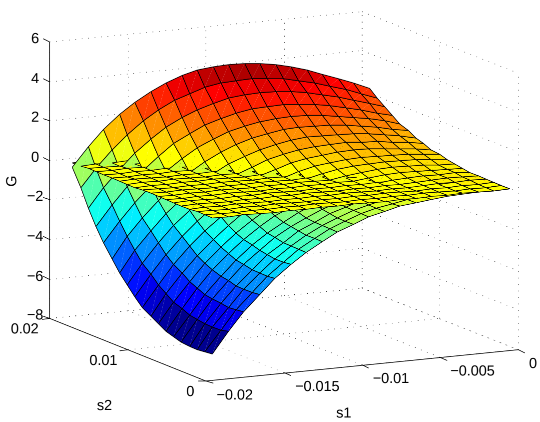
We next calibrate the key parameters in the term $G$ based on the sample of mutual funds described in detail in Section IV. In particular, the investor's prior regarding the average fund managers skill, $\mu_\alpha$, is set equal to zero. The prior variance, $\sigma_\alpha^2$, is set equal to the cross-sectional variance of the Carhart 4-factor alphas across all funds. We use the cross-sectional average of the Carhart 4-factor model error variance across all funds as a measure of the noise in the performance-related signal.We compute this measure for two non-overlapping, equal sub-periods: 1991–2000 and 2001–2010. We set the high signal noise ($\sigma_h^2$) equal to the value of the measure computed over the period over the period 1991–2000. Similarly $\sigma_l^2$ is set equal to the value of the above measure obtained using data for 2001–2010. Thus, the high signal-to-noise ratio, $h = \sigma_\alpha^2/\sigma_l^2$, and the low signal-to-noise ratio, $l = \sigma_\alpha^2/\sigma_h^2$. Moreover, $\sigma_\epsilon^2$ is computed as the cross-sectional average of the Carhart 4-factor model error variance across all funds from 1983 to 2011. The degree of risk aversion $a$ is set equal to 2. The market risk premium, $\mu$, is the average market excess return from 1983 to 2011. Finally, the range of the signal realizations, i.e., $s_1$ and $s_2$, is set equal to one standard deviation, $\sigma_\epsilon$, around zero. All parameters are chosen on a monthly basis. Based on the above discussion, we obtain the following values for the key parameters: $\sigma_\alpha^2 = 0.0007\%$, $\sigma_l^2 = 0.0282\%$, $\sigma_h^2 = 0.0845\%$, $\sigma_\epsilon^2 = 0.0467\%$, and $\mu = 0.5702\%$. Figure 1 shows the results of this numerical analysis.




Panel A: $s_1 < s_2 < 0$




Panel B: $0 < s1 < s2$




Panel C: $s_1 < 0 < s_2$

**Figure 1. Difference in Flow-Performance Sensitivity to Signal $\mathbf{s_1}$ Relative to $\mathbf{s_2}$ across Different Signal Realizations**

This figure plots the difference in the flow-performance sensitivity to the lower-valued signal, $s_1$, relative to the higher-valued signal, $s_2$. In particular, it graphs the value of the term G in Equation (B.6) as we vary the signals $s_1$ and $s_2$. Panels A, B, and C plot the difference in the flow-performance sensitivities for three cases of interest, namely, when (a) $s_1 < s_2 < 0$, (b) $0 < s_1 < s_2$, and (c) $s_1 < 0 < s_2$, respectively.